VŠB - TECHNICAL UNIVERSITY OF OSTRAVA
Faculty of Economics, Department of Finance

Managing and Modelling of Financial Risks

9th International Scientific Conference

PROCEEDINGS

(Part I.)

5th – 6th September 2018
Ostrava, Czech Republic

ORGANIZED BY
VŠB - Technical University of Ostrava, Faculty of Economics, Department of Finance
EDITED BY
Miroslav Čulík
TITLE
Managing and Modelling of Financial Risks
ISSUED IN
Ostrava, Czech Republic, 2018, 1st Edition
PAGES
586
ISSUED BY
VŠB - Technical University of Ostrava
PRINTED IN
BELISA Advertising, s.r.o., Hlubinská 32, 702 00 Ostrava, Czech Republic

ISBN 978-80-248-4225-7 (BOOK OF PROCEEDINGS)
ISBN 978-80-248-4223-3 (CD)
ISSN 2464-6970 (BOOK OF PROCEEDINGS)
ISSN 2464-6989 (ON-LINE)

PROGRAM COMMITTEE

EDITED BY

doc. Ing. Miroslav Čulík, Ph.D. VŠB - Technical University of Ostrava, Czech Republic

REVIEWED BY

doc. Ing. Miroslav Čulík, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
prof. Dr. Ing. Dana Dluhošová	VŠB - Technical University of Ostrava, Czech Republic
Ing. Petr Gurný, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
doc. Ing. Aleš Kresta, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
prof. Ing. Lumír Kulhánek, CSc.	VŠB - Technical University of Ostrava, Czech Republic
prof. Dr. Sergio Ortobelli Lozza	University of Bergamo, Italy
prof. Ing. Martin Macháček, Ph.D. et Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
Ing. Martina Novotná, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
doc. Ing. Martin Svoboda, Ph.D.	Masaryk University in Brno, Czech Republic
prof. Ing. Tomáš Tichý, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
Ing. Jiří Valecký, Ph.D.	VŠB - Technical University of Ostrava, Czech Republic
prof. Dr. Ing. Zdeněk Zmeškal	VŠB - Technical University of Ostrava, Czech Republic

CONTENTS

Part I.

Part II.

Categorization of the Risks Related to Payment and Taxation of Dividend Income in Conditions of the Czech Republic

Kristýna Bělušová[1], Karel Brychta[2]

Abstract

The aim of the paper is to identify and systematize the risks related to payment and taxation of dividend incomes in the terms of the rules as stated by the Czech law. Taking account of the complexity of the issue, just the position of the subject paying the dividends is covered in this respect. The qualitative research realized was based on the study of expert literature and above all on the text of relevant legal norms. Based on the lessons learned the authors identified operations related to payment and taxation of dividend income. These operations were subsequently subordinated, taking their affinity into consideration, under particular general phases associated with the transaction in question. The authors also summarized, at a general level, sanctions that arise in the case of non-compliance with the conditions as set by law. In conclusion of the paper there is presented a matrix in which the identified risks are assessed from the point of view of tax risks categories as specified by PwC (2004).

Key words

Czech Republic, dividend payment, dividends, risk identification, tax risk

JEL Classification: H25, G32

1 Introduction

Risk management is currently an area of corporate management that is so complex and complicated that the need for specialization arose: quite deservedly, financial risk management became one of the separate established parts of risk management (Dionne, 2013). This subcategory of risk management includes tax risks. Those are globally described as „*the separate subgroup of financial risks because taxes are an integral part of finance, and therefore, tax risks are part of financial risks*" (Liapis et al., 2013; Nechaev and Antipina, 2016; Hapsoro and Suryanto, 2017 as all stated *in* Artemenko *et al.*, 2017). Lavermicocca (2011) defines tax risk in a broader context, as *„any event, action, or inaction in tax strategy, operations, financial reporting, or compliance that adversely affects either the company's tax or business operations or results in an unanticipated or unacceptable level of monetary, financial statement or reputational exposure".* The relevance of tax risks as a separate subject of study is evidenced by the existence of numerous studies (see e.g. (Lavermicocca and Buchan, 2015), (Weisbach, 2004), (Alltizer and Hamill, 1999)). Worthy of special attention at present are tax risks relating to the "passive income" category, which includes dividend, interest and royalties (Sojka *et al.*, 2017), (Dunbar, 2009), (Greggi, 2009). Compared to active income, this income category exhibits a number of specific features and options for tax optimization (see e.g. (Monsour, 2016)) and is, as is only logical, associated with different tax risks as well. Multinational enterprises, whose role is to support, control and budget their

[1] Ing. et Ing. Kristýna Bělušová, Brno University of Technology, Kolejní 2906/4, 612 00 Brno, Czech Republic, Kristyna.Belusova@vutbr.cz.
[2] doc. Mgr. Ing. Karel Brychta, Ph.D., Brno University of Technology, Kolejní 2906/4, 612 00 Brno, Czech Republic, brychta@fbm.vutbr.cz.

subsidiaries (Hanafizadeh et al., 2008) and to optimize their global effective tax rate while taking advantage of differences in tax rules around the world are also specific in terms of their behavior with regard to this income category. By suitable positioning of companies within the holding, and through a number of other operations, these supranational companies minimize their tax burden, and maximize their profits at the same time (De Wolf, 2017).

2 Aim and Methodology

The aim of this paper is to identify and classify tax risks associated with dividend income from the perspective of the rules on dividend distribution and taxation, as provided for in legal regulations of the Czech Republic. The risks are identified in relation to actions within basic phases relating to dividend flow (*the phase prior to the distribution of dividends, the phase of taxation and distribution of dividends, the phase after dividend distribution and taxation*), and then classified according to categories defined by PwC (2004); the categories and specifications of individual risks are provided in Table 1 below.

Table 1 Specification of individual risk categories

Category of the risk	Description
Transactional	*The risks and exposures associated with specific transactions undertaken by a company. In any transaction there may be uncertainty as to how the relevant tax law will apply and uncertainty arising from specific judgement calls – particularly in the more complex areas.*
Operational	*Risk concerns the underlying risks of applying the tax laws, regulations and decisions to the routine every day business operations of a company.*
Compliance	*Compliance risk would primarily relate to the preparation, completion and review of an organisation's tax returns (of whatever type and not only corporate tax returns) and the risks within those processes.*
Financial accounting	Risks resulting from, "*the figures included in the tax accounts, at the time the financial statements are issued, are 'estimates'.*"
Portfolio	*Portfolio risk concerns the overall aggregate level of risk when looking at transactional, operational and compliance risks as a whole and considers the interaction of these three different specific risk areas.*
Management	*Risk is one of not properly managing the various risks set out above.*
Reputation	*Reputational risk concerns the wider impact on the organisation that might arise from an organisation's actions if they become a matter of public knowledge. By their very nature such risks will impact wider business interests.*

Source: Own elaboration using PwC (2004).

The paper is based on qualitative research, the aim of which is to glean a better understanding of the phenomenon of risk in relation to dividends as an important passive income category in the legislative conditions of the Czech Republic. The research method used is a case study carried out in order to identify and classify risks relating to the distribution and taxation of dividend income. Existing and potential risks relating to dividend distribution and taxation are the subject of research. Czech, mainly tax, legal regulations, *Council Directive (EU) 2016/1164 of 12 July 2016 laying down rules against tax avoidance practices that directly affect the functioning of the internal market* (hereinafter referred to as ATAD Directive only) as well as selected OECD materials pertaining to BEPS Action Plans (for more details see e. g. (OECD, 2018)) regulating international dividend taxation, are the object of research. The study of wording of legal regulations is the data collection technique employed.

3 Rules for Distribution and Taxation of Dividends

Dividends can be viewed as the means through which owners/investors pay profits on their investments to themselves. However, a dividend distribution is not always possible, even where the condition of positive results of operations is met. Czech law, same as the laws of other countries, sets forth rules for the protection of creditors, and by extension, for the prevention of abuse of the sovereign position of the owners in terms of control over the company and disposal with its funds. Rules other than tax rules must therefore be also taken into account in the context of dividend distribution. In the conditions of the Czech Republic, this means in particular Act No. 90/2012 Coll., on Business Corporations, as amended (hereinafter referred to as BCA only). The BCA sets forth rules, on the satisfaction of which the actual dividend distribution is conditioned.

Table 2: Rules providing for the option of dividend distribution

Provision of the BCA	Rule
Section 34 Section 35 Section 40	***Insolvency test*** Profit share/dividend or funds from other own resources (including advance payments) must not be paid, if such distribution might result in the company's insolvency under another legal regulation.
Section 350	***Own equity test*** The company may not distribute profit/dividends if, as at the last date of the accounting period ended, its equity as shown in the annual or extraordinary closing financial statements or its equity after such profit distribution would fall below the amount of the subscribed registered capital increased by funds not eligible for distribution to the shareholders pursuant to the BCA or the company's Articles of Association.
Section 40 Section 161	***Distributable profit amount test*** The maximum amount of profit share/dividend to be distributed must not exceed the latest operating result, increased by retained profits from previous years, less accumulated losses from previous years and allocations made to the reserve or other funds.
Section 167 *et seq.* Section 398 *et seq.*	***Conditions for approval of dividend distribution*** The dividend distribution approval process (term for the convocation of the supreme body of the company, form of minutes of the meeting, quorum, etc.)
Section 34	***Dividend due date*** Profit shares/dividends are payable within three months from the supreme body's decision on profit distribution.

Source: Own elaboration using the BCA.

A breach of the conditions for dividend distribution may result in the liability of the members of the statutory body for the compensation for damages (for more detail, see Section 53 of the BCA and related provisions). Depending on the circumstances, the company may ultimately become insolvent, which - even with "mere" gross negligence – is qualified as the criminal act of causing insolvency (see Section 224 of Act No. 40/2009 Coll., the Criminal Code, ad amended (hereinafter referred to as CC only)).

If the above conditions are met, *the actual distribution and taxation of dividends* follows. Taxation rules are set forth at both EU and international levels (in the form of double taxation treaties (hereinafter referred to as DTT only), and naturally also in national legislation. In the EU law, this is not addressed merely by a secondary legal regulation, specifically *Council Directive 2011/96/EU of 30 November 2011 on the common system of taxation applicable in the case of parent companies and subsidiaries of different Member States* (hereinafter referred to as Council Directive 2011/96/EU), but also by primary EU law standards: according to the case law of the Court of Justice of the EU (formerly the European Court of Justice), certain domestic rules are capable of infringing on fundamental freedoms in this income category (see e.g. Bělušová, Brychta, 2017). The rules in secondary EU law (see Council Directive 2011/96/EU) are naturally reflected in national laws of the member states; in the context of the Czech Republic, in Act No. 586/1992 Coll., on Income Taxes, as amended (hereinafter referred to as ITA only). It needs to be pointed out that the above mentioned Council Directive 2011/96/EU gives the member states some degree of discretion in case of international taxation of this category of incomes, and does not cover certain aspects at all – the Council Directive 2011/96/EU targets tax relations with an international element, and moreover, only within the EU. For this reason, the rules for the taxation of such income set forth in national laws may vary, and the results of the study carried out support this conclusion (see e.g. Bělušová, 2017). The provisions of law contained in international law then merit special consideration - specifically, those found in particular DTTs which, thanks to the rule as stated under Article 10 of the constitutional act, the Constitution of the Czech Republic, as amended, enjoy an application priority. The DTTs to which the Czech Republic has been a party respect the concept and structure of the OECD Model Tax Convention (for the current wording of the OECD Model Convention, see (OECD, 2017). Nevertheless, as noted by Brychta and Bělušová (2018), there is some variability as regards the taxation rules in the source state even in these treaties. A summary of basic rules for dividend taxation from the perspective of the entity distributing the dividends can be seen in Table 3 below.

Table 3: Rules in conjunction with dividend taxation on the part of the distributing entity

	Rule
Exemption from taxation pursuant to ITA	Conditions pertaining to: a) legal form (see Directive 2011/96/EU), b) residence of the receiving and distributing companies, and c) size of the share and time of holding. must be satisfied cumulatively. The exemption arises from the rules as stated under Section 19 (1)(ze) of the ITA; the conditions are set forth in Section 19 (3) and related provisions of the ITA.
Exemption from taxation pursuant to DTT	The relevant DTT does not entitle the source state to tax the income. There is a need to provide proof of residence in the other contracting state, and further, the fact that the recipient of the dividend is its beneficial owner at the same time.

	Rule
Taxation pursuant to rules set forth in the DTT (*in case the state source has the right to tax*)	If the dividend income is not exempt and if dividends are paid to a contracting state, then pursuant to the applicable DTT, such income is subject to tax in the source state at the rate limited by the DTT (if provided for). The need to provide proof of residence in the other contracting state, and further, the fact that the recipient of the dividend is its beneficial owner at the same time.
Taxation pursuant to ITA	a) The rule for conversion if distribution of dividends is made in a currency other than CZK (*accounting exchange rate is used - see Section 38* of the ITA) b) Applied withholding tax rate (*35% for a non-contracting state, provided there is not at least a Tax Information Exchange Agreement between the relevant state and the Czech Republic; 15%, unless the rate is limited by the DTT, or lower set by the DTT (if limited) - see Section 36* of the ITA) c) Withholding date (*upon distribution, but in any case by the end of the third month following after the distribution month at the latest – see Section 38d* of the ITA) d) Date of the obligation to pay tax to the tax administrator (*by the end of the month following after the month in which the payer was obliged to make the tax withholding – see Section 38d* of the ITA) e) Date of payer's notification of tax collected by way of withholding (*by the end of the month following after the month in which the payer was obliged to make the tax withholding – see Section 38d* of the ITA) f) Date of filing of the statement of tax account for tax collected by way of withholding at a special income tax rate (*within three months from the end of the calendar year – see Section 38d* of the ITA)

Source: Own elaboration using OECD (2017), Council Directive 2011/96/EU and the ITA.

If the duration of holding of the share is not met subsequently (see part *Exemption from taxation pursuant to* ITA in Table 3 above), Section 38s of the ITA must be applied – when determining the amount of the withholding tax, income gross up is used (the amount paid to the shareholder represents the amount distributed after the withholding is made). If the conditions for the withholding, payment and notification are not met, *ex lege* sanctions provided for in Act No. 280/2009 Coll., the Tax Procedure Code, as amended (hereinafter referred to as TPC only) apply; specifically, the provisions from Section 250 to Section 254.

Deliberate tax evasion or non-payment triggers criminal liability (for more detail, see Section 240 and Section 242 of the CC).

3.1 Novelties in dividend taxation - rules under the ATAD Directive

OECD also addresses the issue of passive income taxation in its BEPS project (*Base Erosion and Profit Shifting*). As regards the relevant income category, this involves Action 3 – CFC Rules (OECD, 2018). Said rules have been incorporated into the ATAD Directive. However, given the defined scope of this articles, it needs to be noted that the new CFC Rules focus on companies collecting dividends. Therefore, changes and risks at the level of indirect impact can be expected on the part of distributing companies.

4 Classification of Risks

Based on the study of the relevant legal regulations, results of previous research conducted by the authors (Bělušová, Brychta, 2017) and findings provided in PwC (2004), the authors of the article have identified the basic phases relating to the distribution and taxation of dividends, and the related risks. These are summarized in Table 4 below.

Table 4: Classification of identified risks

Phase	Identified risk	Risk according to PwC (2004)						
		Specific risk area				General risks		
		1	2	3	4	5	6	7
Pre-distribution	Omission to carry out the insolvency test.		X	X	X		X	X
	Unauthorized distribution of dividends and the related law violations.		X	X	X		X	X
	Failure to bear the burden of proof resulting from inadequate evidentiary documentation proving the possibility to distribute dividends.		X	X	X		X	
	Lack of quorum resulting from the absence of shareholders.						X	
	Risk of non-approval of dividend distribution (e.g., due to fragmented ownership structure and difference of opinion).						X	
	Decision on distribution of dividends in an unauthorized amount.			X	X		X	
Taxation and distribution of dividends	Risk of omission and/or failure to prove facts relevant to the possibility to invoke advantages under EU/international/national law.		X	X	X	X	X	
	Sanctions resulting from failure to withhold tax from dividend income							X
Post-dividend distribution and taxation	Failure to meet the deadline for payment of tax to the tax administrator			X	X		X	X
	Failure to meet the deadline for notification and statement tax account			X	X			
	Sanctions relating to belated filing of tax			X	X			

Phase	Identified risk	Risk according to PwC (2004)						
		Specific risk area				General risks		
		1	2	3	4	5	6	7
	statement and payment of tax to the tax administrator							
	Failure to bear the burden of proof with regard to tax advantages invoked under EU/international/national law during tax controls, and related sanctions		X	X	X		X	

Source: Own elaboration using PwC (2004), BCA, ITA, OECD (2017), TPC.

Explanatory notes: 1 – transactional, 2 – operational, 3 – compliance, 4 – financial accouting, 5 – portfolio, 6 – management, 7 – reputation.

5 Discussion and Conclusion

This paper aimed to identify, systemize and classify risks relating to the distribution of dividends in the Czech Republic. Given the breadth of this topic, the paper focuses on entities distributing dividends. The paper summarizes basic findings obtained mainly by the study of legal regulations and the obligations resulting therefrom, together with sanctions for non-compliance with statutory conditions. Based on these findings, it can be noted in summary that the distribution and taxation of dividends (if they are not exempt from taxation) is a rather demanding and complex process. The new private law provisions put an emphasis on creditor protection: the current provisions of law contained in the BCA set forth three tests on which dividend distribution is conditioned. These are the *insolvency test, the equity test* and *the distributable profit test*. In this regard, the legislation can be deemed adequate and affording protection to creditors, provided there is transparent and well-maintained accounting. Together with the approval process, which may also be administration and cost-intensive, depending on specific conditions, this phase can be referred to as the *pre- distribution phase.* The violation of conditions set forth in the BCA may result in both private law and public law sanctions.

The phase associated with the distribution of dividends is connected with the taxation of this income category. Every potential situation (application of EU law, international law and/or national law standards) is associated with certain requirements in terms of proving the requisite conditions, and therefore also with the risks of failure to bear the burden of proof of satisfaction of such conditions. Nonetheless, there is a positive aspect on the part of the entity distributing the dividend in that the introduction of new conditions (and thus also risks) cannot be directly expected on account of the rules introduced by the ATAD Directive – the change occurs on the part of the entity receiving the dividends. However, there are certain pitfalls to such a general assessment as the amended rules may have an indirect impact on subsidiaries (subordinated companies).

In the phase associated with the taxation and distribution of dividends, sanctions connected with the violation may have the form of compensation for damages, sanctions provided for in tax laws (*fine, penalty, default interest* under the TPC), or a gross up pursuant to Section 38s of the ITA and may even have criminal nature consequences.

To put the identified risks in a broader context and understanding, the authors of the article carried out an assessment of selected risks using the PwC (2004) classification at the end. Although the assessment relies on subjective judgment of the authors, the conclusion that from the perspective of dividend distribution and taxation, the general risk of poor management is a key area, can be expected to be a valid conclusion. It may result in a breach of rules for the entitlement to distribute dividends, and the related risk of unauthorized distribution which can be deemed to be of fundamental importance with a view to causing the company's insolvency, which, even if not caused deliberately, could trigger criminal liability (in additional to compensation for damages) for the members of the statutory body. Thus, the distribution of dividends and related acts need to be viewed in a holistic manner: the company should not lack a person who is cognizant of the area and who is responsible for this area and its management.

References

[1] Alltizer, R. L., Hamill, J. R. (1999). The effects of tax rates and dividend payouts on common stock returns. *American Business Review*, [e-journal] 17(2), p. 24-33. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[2] Artemenko, D. et al. (2017). Causes of tax risks and ways to reduce them. *European Research Studies*, [e-journal] 20(3), p. 453-459. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[3] Bělušová, K. (2017). Zdaňování dividend v rámci Evropské unie - Srovnávací studie. In: In: *DANĚ -TEORIE A PRAXE 2017, sborník referátů a abstraktů XX. ročníku mezinárodní odborné conference.* Brno, Czech Republic, 14-15 September 2017. Brno: Akademie Sting, o. p. s.

[4] Bělušová, K., Brychta K. (2017). Selected aspects of international dividend taxation within the case-law of the Court of Justice of the European Union. In: *Účtovníctvo a audítorstvo v procese svetovej harmonizáci*. Zuberec Slovakia, 11-13 September 2017. Bratislava: Ekonóm.

[5] Brychta, K., Bělušová, K. (2018). International Taxation of Dividends as Regulated in Double Tax Treaties – a Case of the Czech Republic. In: *Financial Management of Firms and Financial Institution, 11th International Scientific Conference: Proceedings*. Ostrava: Czech Republic, 6-7 September 2017. Ostrava: VŠB Technická univerzita Ostrava.

[6] *Council Directive 2011/96/EU of 30 November 2011 on the common system of taxation applicable in the case of parent companies and subsidiaries of different Member State*

[7] *Council Directive (EU) 2016/1164 of 12 July 2016 laying down rules against tax avoidance practices that directly affect the functioning of the internal market.*

[8] De Wolf, D. (2017). BEPS case study: Treaty abuse. *International Tax Journal*, [e-journal] 43(6), p. 45-51. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[9] Dionne, G. (2013). Risk management: History, definition, and critique. *Risk Management and Insurance Review*, [e-journal] 16(2), p. 147-166. http://dx.doi.org/10.1111/rmir.12016

[10] Dunbar, D. G. (2009). New zealand CFC REFORM OPTIONS. *Journal of International Taxation*, [e-journal] 20(12), p.40-51. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[11] Greggi, M. (2009): European taxation of dividends, interests and royalties: the impact on third countries investors. *Current Issues of Business*, vol. 3, p. 118-140.

[12] Hanafizadeh, P. et al. (2008). Defining CSFs for information systems strategic planning in holding companies: A case study of an iranian managerial holding company (system Group1). *Journal of Information Technology Case and Application Research*, [e-journal] 10(1), p. 7-25. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[13] Lavermicocca, C. (2011). Tax risk management practices and their impact on tax compliance behaviour - the views of tax executives from large australian companies. *EJournal of Tax Research*, [e-journal], 9(1), 89-115. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[14] Lavermicocca, C., Buchan, J. (2015). Role of reputational risk in tax decision making by large companies. *EJournal of Tax Research*, [e-journal]13(1), p. 5-50. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[15] Monsour, E. L. (2016). Tax-motivated income shifting for multinational firms in U.S. *International Research Journal of Applied Finance*, [e-journal], 7(9), p. 220-225. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[16] OECD. (2017). *Model Tax Convention on Income and on Capital: Condensed Version 2017* [online]. Available at: <https://doi.org/10.1787/mtc_cond-2017-en> [Accessed 26 July 2018].

[17] OECD. (2018). *BEPS Actions* [online]. Available at: < http://www.oecd.org/ctp/beps-actions.htm> [Accessed 20 July 2018].

[18] PricewaterhouseCoopers. (2004). *Tax risk management.* [online] London: PricewatersCoopers. Available at: < https://www.pwc.com/gx/en/tax-management-strategy/pdf/tax-risk-management-guide.pdf> [Accessed 20 July 2018].

[19] Sojka, V. et al. (2017). *Mezinárodní zdanění příjmů*. 4th ed. Praha: Wolters Kluwer ČR.

[20] *Ústavní zákon č. 1/1993 Sb., ve znění pozdějších předpisů.* [Constitution of the Czech Republic *in Czech langugage*].

[21] Weisbach, D. A. (2004). Taxation and risk-taking with multiple tax rates. *National Tax Journal*, [e-journal] 57(2), p. 229-243. Available through: Proquest Central < https://search-proquest-com> [Accessed 15 July 2018].

[22] *Zákon č. 40/2009 Sb., trestní zákoník, as amended.* [Penal Code *in Czech langugage*].

[23] *Zákon č. 90/2012 Sb., o obchodních korporacích, as amended.* [Business Corporations Act *in Czech langugage*].

[24] *Zákon č. 280/2009 Sb., daňový řád, as amended.* [Tax Procedure Code *in Czech langugage*].

[25] *Zákon č. 586/1992 Sb. o daních z příjmů, as amended.* [Act on Income Taxes *in Czech langugage*].

Modelling life insurance risk for teaching purposes

Mária Bilíková, Ingrid Ondrejková Krčová [1]

Abstract

Before it issues a new type of contract a life insurance company needs to assess the risks associated with that contract in particular the sensitivity of the profit expected to changes in future demographic and financial experience. The paper describes the main risks involved and the main profit criteria that a life insurance company can use to assess the expected profitability. With the help of a simple interactive computer model set up to illustrate the principles involved for the benefit of students on a university actuarial science course the authors analyse the effect on the profitability of some basic life insurance contracts due to variations in future financial experience and how the adverse effects of such experience can be mitigated using profit sharing between the company and the policyholders.

Key words

Endowment insurance without and with profit contract, profit criteria, risks in life insurance.

JEL Classification: C 65, G 22

1. Introduction

When considering the problem of risk in the context of life insurance companies it is usual to approach this from the point of view of the policyholders and the emphasis is therefore on minimising or at least controlling the risk that a company will become insolvent and the policyholders will be left with no cover. Little attention is however paid to risk from the point of view of the shareholders. Without shareholders prepared to put up capital to finance the life insurance business members of the public would not have the possibility of providing security for their families in the event of their untimely death. In this paper we look at risk in terms of the products that are being sold. It is these products that will provide the profit required by the shareholders, but associated with this profit is the risk that it may not be attained. In Section 2 we discuss briefly the main risks associated with the products issued by life insurance companies and in Section 3 outline the profit-test method for assessing the profitability of a given product. Historically the method has been used only to quantify in some way the amount of the expected profit as we describe in Section 4, but little thought has been given to the riskiness of this profit. Using the model set out in Section 5 we set out in Section 6 the results of carrying out simulations of this expected profit, where we quantify not just the amount of the profit, but also the variance of this profit in terms of the standard deviation.

2. Main risks involved with life insurance contracts

In life insurance as in any other commercial enterprise there is no such thing as a free lunch. If the company wishes to make a profit it must take on risk. It therefore needs to know

[1] doc. RNDr. Mária Bilíková, PhD., maria.bilikova@euba.sk, Ing. Mgr. Ingrid Ondrejková Krčová, PhD., ingrid.krcova@euba.sk, Department of Mathematics and Actuarial Science, University of Economics, Dolnozemská cesta 1, 852 35 Bratislava, Slovakia.

what these risks are and so manage them that the shareholders receive an adequate return on their investment whilst reducing to an acceptable level the danger that the risks taken on lead to the insolvency of the company. In this section we will briefly describe the main risks associated with the life insurance contracts themselves in particular those associated with the assumptions used to price such contracts. We will not consider the risks associated with the enterprise as a whole.

Given that the future is unknown models are used to represent what might happen in the future with regard to the assumptions required in product pricing. As in all contexts in which models are used we can identify three types of risk.

- a "model" risk, that is the model itself may not be appropriate for the purpose it is being used;
- a "parameter" risk, that is the parameters used with the model may not adequately reflect what might happen in the future, even though the underlying model may be appropriate;
- a "random fluctuations" risk, that is the actual future experience may not correspond with the model and parameters adopted, even though the model is appropriate and the parameters have been suitably chosen.

The mortality assumptions used in pricing will be subject to all three of the above risks with their extent dependent amongst other things on the nature of the contracts being priced. In this paper we will concern ourselves with endowment contracts, i.e. savings contracts which provide a capital sum on survival to the end of a fixed period or on earlier death, where in general the risk from mortality is not significant[2]. A discussion of the effect of mortality risk on profitability can be found in [1].

For endowment assurances the most significant assumption with regard to both risk and profitability is that regarding the future investment performance expected from the assets in which the premiums paid under the contracts will be invested. The three basic risks described above apply also here. In the past a very simple deterministic model was used which expressed the investment return as a constant rate of interest with the rate chosen so as to be conservative with regard to then investment returns and those in the recent past. Future fluctuations in the return were therefore ignored in pricing. The widespread use of computers in actuarial work led to the introduction of more sophisticated models, first by having the deterministic interest rate vary over time albeit in a simplistic way and then by introducing stochastic models for the rate of investment return. In Section 6 of this paper we will show the effect on profitability of some of the risks involved with the use of stochastic models by means of a simple model for the investment return which has been designed primarily for teaching purposes.

The insurance company will recoup its operational expenses from the premiums it receives and therefore it requires a model for these which it can incorporate into its pricing structure. As with the investment return the model used here in the past was a deterministic one based on an analysis of the company's recent expenses allowing for past trends and future fluctuations in the expenses, primarily but not only due to inflation, were ignored. Computers have permitted the use of stochastic models to represent these fluctuations and we have again the three risks outlined earlier. We illustrate this also in Section 6 of the paper.

The holders of most life insurance contracts, annuities being the exception, have the right to cancel their contracts and this needs to be modelled for pricing purposes. In the past as with the investment and expense assumptions the model was very simple. In this case the possibility of cancellation was simply ignored on the base that any amount that may be paid to

[2] A detailed discussion about it can be found in [4] Špirková, Urbaníková, (2012).

the cancelling policyholder would be less than that which the company was holding as a reserve for the contract. This assumption was in general conservative, but did not allow for the fact that in general a cancellation in the early life of a contract can lead to a loss for the company due to the high initial costs incurred and of course it did not allow for quantification of the profit that would thus arise from cancellations in later years. The use of spread sheets for pricing purposes has permitted the use of an explicit model for cancellations to be incorporated into the pricing process and in this case only the parameter and random fluctuations risks apply.

Although not directly entering into the pricing process an important consideration is whether the resulting product is competitive, i.e. giving the benefits to be provided will the price of the product be such as to attract an adequate number of customers to purchase it.

3. The profit-test approach to pricing life insurance contracts[3]

In current life insurance practice the profit-test approach is the favoured method for pricing life insurance products. The main idea of this method is to determine such amount of the premium for the contract so as to meet the company's profit requirement. Proposed premium rates are tested by projecting the various cash-flows associated with the contract making certain assumptions with regard to those factors that will affect the level of those cash-flows. The process ends when the expected profit from the contract meets the requirements of the insurance company's management. These requirements are often expressed in terms of the amount of the profit expressed in various ways as we set out in Section 4. In this paper we will extend this to consider the riskiness of that profit.

In Section 6 we will illustrate this using an endowment assurance contract with sum assured S, payable immediately on death, if it occurs within n years or at the end of n years if the life assured, aged x at the start of the contract, is still alive at that time. The contract will be paid by regular constant annual premiums P[4]. The net cash flow ignoring reserves at time t (in years) after payment of the premium expected to be received at time t, P_t , can be calculated using the formula:

$$NCF_t = P_t - E_t - S_{t-1} \cdot q_{x+t-1} \cdot (1+i_{t-1})^{1/2} - SV_t \tag{1}$$

where E_t are the expected expenses associated with the payment of the premium, S_{t-1} is the sum assured payable in the year up to time t including the associated expected expenses, q_{x+t-1} is the expected mortality rate in the year up to time t, i_{t-1} is the expected investment return in the year up to time t and SV_t is the expected amount paid on cancellation in the year up to time t assumed paid at the end of that year. Appropriate modifications of this formula apply where t is equal to 0 and n.

To calculate the expected profit at time t allowance needs to be made for the reserves that the company is required to hold in respect of the contract so that we get the expected profit at time t, Pr_t:

$$Pr_t = NCF_t + {}_{t-1}V_x \cdot (1+i_{t-1}) - {}_tV_x \tag{2}$$

where the ${}_tV_x$ are the technical reserves required at time t after the premium then due has been paid and allow for survivorship up to time t.

[3] A detailed discussion of this topic from an alternative viewpoint can be found in [3] Sakálová, (2001).

[4] In practice premiums are paid monthly, but for ease of computation this has been ignored.

4. Profit criteria used to assess expected profitability

For measuring the expected profitability of the contract it is not sufficient to consider just the profit that will emerge at each time *t*. Instead insurers use various profit criteria[5]. A profit criterion is a single figure, which tries to summarise the relative efficiency of the contracts with different profit signature $\sigma_t = Pr_t;\ 0 \le t \le n$.

Net present value: Discounting the profit signature at the risk discount rate *r* we obtain the accumulated present value of expected future profits or the first criterion − net present value (*NPV*). So

$$NPV = \sum_{t=0}^{n} Pr_t \cdot (1+r)^{-t} \tag{3}$$

NPV is considered the best profit criterion to use, but there exist also some practical problems connected with it. It is subject to the law of diminishing returns and it says nothing about competition.

Net present value as a percentage of the initial commission: Because initial commission can partly reflect the work required to sell the contract, it is usual to express the *NPV* as a percentage of the initial commission, which the life insurance company intends paying. The company's management will have a requirement as to the level of this percentage.

nternal rate of return: This is the rate of investment return that when used to discount the profit signature produces a value of zero. So if this rate is i_{IRR} we have

$$\sum_{t=0}^{n} Pr_t \cdot (1+i_{IRR})^{-t} = 0 \tag{4}$$

This criterion has the disadvantage that it may not always exist, for example if the profit figures in the profit signature are all positive. In practice it is unlikely, that the profit figures will all be positive due to the heavy expenses incurred at the inception of a contract plus the need at that time to set up technical reserves and thus the internal rate of return represents the rate of return on the amount that the company needs to invest into the contract when it is written. If this is used as a criterion the management will have a requirement for its level.

Profit margin – *NPV* as a percentage of the present value of the future premiums: The *NPV* can, alternatively, be expressed as a percentage of the present value of the future premiums, which will be paid under the contract. So the appropriate formula in the case of a regular premium is

$$\frac{\sum_{t=0}^{n} Pr_t \cdot (1+r)^{-t}}{\sum_{t=0}^{n-1} P_t \cdot (1+r)^{-t}} \tag{5}$$

Again the management will have a requirement for the level of the profit margin.

Discounted payback period: The discounted payback period is the policy duration at which the emerging profits first have a discounted value of at least zero. That means, it is the time it takes for the insurer to recover its initial investment allowing for interest at the risk discount rate. So it is the first time *k* for which $PVFP_k$ is non-negative, where

[5] More about this can be found in [2] Sakálová, Ondrejková Krčová, (2016).

$$PVFP_k = \sum_{t=0}^{k} Pr_t \cdot (1+r)^{-t} \tag{6}$$

Note that depending on the values Pr_t the discounted payback period may be zero, if the values are all positive, or may not exist at all, for example if the net present value is negative.

5. The model

The authors have at their disposal an interactive computer model in *Excel* which makes use of *Visual Basic*. The model permits a choice of contracts. Given the orientation of this paper towards financial risk the contracts we will consider are endowment assurances on both a with and a without profit basis. In the case of a without profit contract the benefit payable is constant over the whole duration of the contract.

For a with profit contract, i.e. one that participates in the profit made by the insurance company, there needs to be a method for determining how much profit can be distributed to policyholders and how it can be distributed. The profit from the profit-test calculations cannot be used for this purpose. Instead use is made of the technical profit TP_t arising in terms of the technical reserves. This can be calculated as follows

$$TP_t = (P_{t-1} - E_{t-1} + TR_{t-1}) \cdot (1 + i_{t-1}) - D_{t-1} \cdot (1 + i_{t-1})^{1/2} - TR_t \tag{7}$$

where the TR_t are the technical reserves at time t before payment of the premium at that time and D_{t-1} is the amount paid with respect to deaths in the year to time t and includes the appropriate expenses. In European practice it is usual to divide this into parts – the so-called savings profit SP_t and risk profit RP_t, representing the profit from sources other than the investment return, and we have

$$SP_t = TR_t \cdot (i_{t-1} - i^T) \cdot (1 + i^T)^{-1} \tag{8}$$

where i^T is the interest rate used to calculate the technical reserves.

The risk profit is then

$$RP_t = TP_t - SP_t \tag{9}$$

How much of each of these two profits is given to the policyholders depends on the philosophy of the company and in particular what it told policyholders when they purchased their contracts. The amount given to each policyholder is finally converted to an addition to the benefit payable on death or survival.

For our model contract we chose an endowment assurance for a term of 10 years issued to a life aged 35 at the start of the contract and with an initial sum assured of €10 000. The assumptions used for our profit-test calculations and for the calculation of the technical reserves are hypothetical ones, but the authors do not consider them to be unreasonable in current conditions. To generate future investment returns and expense inflation rates a simple stochastic model was used.

6. Results of the model

We first modelled the profit-test results for a without profit contract and used for this purpose a premium of €925.00 and 1 000 simulations. The results are summarised in Table 1.

Table 1: Profit test results for without profit contract

	Mean	**Standard deviation**
Profit as percentage of initial commission	25.19%	47.32%
Rate of return on capital*	17.63%	10.86%
Profit margin	1.17%	2.20%
Discounted payback period (years)*	7	5
Maturity value	10 000.00	0.00

* The means and standard deviations for these two criteria have been calculated only in respect of those simulations for which the corresponding values exist. In the case of the rate of return on capital there were 144 non-existent values and in the case of the discounted payback period there were 312 non-existent values.

We can see from these results that even though the mean values of the various profit criteria might be considered acceptable the risk as illustrated by the standard deviations would almost certainly be considered too high. In particular it should be noted that in almost one third of the simulations there did not exist a discounted payback period, i.e. the initial capital injected into the product was never repaid. It is not therefore surprising that in many European countries without profit contracts are not issued. In the United Kingdom where such contracts have existed at least in the past they were priced allowing for significant risk margins making the resulting premiums not particularly attractive for potential customers.

For our second set of simulations we used a with-profit contract using the same parameters as for the without profit contract, but with a premium of €980.00. In terms of profit sharing we assumed that the company would allocate 100% of the savings profit to policyholders and 100% of the risk profit to shareholders. The results of the simulations are shown in Table 2. Comparing the mean values here with those in Table 1 we see that they are better from the point of view of the shareholders, in particular the simulations produced quite low values for the discounted payback period, meaning that the initial capital invested in the contract was recouped very quickly.

Table 2: Profit test results for with profit contract – 100% savings profit to policyholders

	Mean	**Standard deviation**
Profit as percentage of initial commission	25.65%	1.72%
Rate of return on capital*	33.60%	1.58%
Profit margin	1.19%	0.08%
Discounted payback period (years)*	3	0
Maturity value	10 854.78	384.23

* The means and standard deviations for these two criteria have been calculated only in respect of those simulations for which such the values exist. For both criteria there were only 2 non-existent values.

Of more significance however are the very low values for the standard deviations. By distributing all the savings profit to the policyholders the company has managed almost to eliminate the risk to shareholders. The small remaining risk is that associated with expenses, which for a contract of only 10 year duration is not important as the majority of the expenses associated with the contract are suffered up front and are not therefore subject to inflation.

As can be seen the risk has been largely passed on to the policyholder where the expected total addition to benefits at maturity is €854.78 with a standard deviation of €384.23.

In practice life insurance companies do not usually allocate the whole of the savings profit to the policyholders. More common is to allocate something of the order of 90%. The effect of this is shown in Table 3, where we have retained the same premium as used in Table 2.

Table 3: Profit test results for with profit contract – 90% savings profit to policyholders

	Mean	**Standard deviation**
Profit as percentage of initial commission	37.09%	5.98%
Rate of return on capital*	38.79%	2.34%
Profit margin	1.72%	0.28%
Discounted payback period (years)	3	0
Maturity value	10751.62	347.47

* There was one non-existent value.

The results here are as to be expected: the mean values of the profit criteria have increased, but at the cost of an increase in the standard deviations, i.e. the risk. The opposite effect applies to the policyholders' benefit at maturity, although it is less marked than in the case of the profit criteria.

7. Conclusion

By using a simple model that has been developed for teaching purposes we have shown how we can extend the standard profit-test method for pricing to provide not just values for various profit criteria, but also to calculate the standard deviations of these values, which can be taken as a proxy for the risk associated with the profit. The results for the without profit endowment contract show that, although the expected profit to the shareholders may be acceptable, the level of risk associated with it is not. In order to reduce the risk to acceptable levels the company can use profit sharing whereby the savings profit, i.e. the profit from investment experience, is given in part or in whole to the policyholders, with the higher the policyholder share the lower the risk to shareholder profits, but at the cost of a lower shareholder profit.

References

[1] Bilíková, M. and Luffrum, G., (2018). *Mortality modelling using a modified Makeham formula for term assurance in Excel* (in Slovak). In Software support in economic-mathematical and actuarial models. Bratislava: Ekonom publishers.

[2] Sakálová, K. and Ondrejková Krčová, I., (2016). *Measures of profitability in life insurance product management.* In Strategic Management: international Journal of Strategic Management and Decision Support Systems in Strategic Management. Subotica: Faculty of Economics in Subotica University of Novi Sad, vol. 21, no. 1.

[3] Sakálová, K., (2001). *Product pricing in life insurance.* (in Slovak) Bratislava: Ekonom publishers, 156 pp.

[4] Špirková, J. and Urbaníková, M., (2012). *Actuarial mathematics – life insurance.* (in Slovak) Bratislava: Iura Edition. 241 pp.

Acknowledgements:

This paper is supported by the Grant Agency of Slovak Republic − VEGA, grant no. 1/0618/17 entitled *Modern tools for modelling and managing risks in life insurance*.

Period and threshold rebalancing of blended big-cap growth and big-cap value portfolios

Martin Boďa[1], Mária Kanderová [2]

Abstract

On the understanding that many investors invest into diverse classes of stocks and investing into big-cap stocks has sound reasons, the paper studies whether there is some merit in investing into big-cap growth stocks and big-cap small stocks simultaneously. In the set-up of periodic and threshold rebalancing strategies, the paper conducts a case study oriented on the US stock market and examines how changing proportions between big-cap growth stocks and big-cap small stocks affect the performance of the portfolio. Oriented on a small investor, the case study accounts for different historical samples and accommodates several designs aiming at tracking the S&P 500 Index on the basis of its big-cap constituents. The results suggest useful practical advice for investing into big-cap stocks.

Key words

periodic rebalancing, threshold rebalancing, big-cap stocks, growth and value stocks, mixed investment style, performance, S&P 500 Index

JEL Classification: G11

1. Introduction

When an investment portfolio of stocks is to be created with a view to tracking the performance of a general market index, there is a good rationale to invest into big-cap stocks that are represented in the index as partial replication. Since big caps have a substantial say in a market index, the resulting portfolio with the weights derived from the constituent shares may be expected to track the underlying index to a great degree of precision. Nonetheless, big caps do not create a uniform universe, but may be further classified on the basis of industrial sectors or pricing (fundamental) characteristics. The latter differentiation gives rise to singling out big-cap growth and big-cap value stocks. When market price is high/low relative to a fundamental (such as book value, earnings, revenue), then the stock is classified as a growth/value stock. Growth stocks are typically credited with above-average growth prospects and offer frequently superior returns, whereas value stocks represent in fact "cheap" investing opportunities capable of maintaining value in a near future (Christopherson and Williams, 1995, pp. 4-5). Their inexpensiveness is owing to their undervaluation in respect to the chosen fundamental. The assessment and classification is based usually on a multiple such as the P/B (price-to-book) or P/E (price-to-earnings) ratio. There is also a link between the pricing classification into growth/value stocks and economic sectors. It seems that value big caps are mostly associated with financials, utilities, consumer cyclicals and energy, whereas growth big

[1] doc. Mgr. Ing. Martin Boďa, PhD., Matej Bel University in Banská Bystrica, Ekonomická fakulta, Department of Quantitative Methods and Information Systems, Tajovského 10, 975 90 Banská Bystrica, Slovakia, e-mail: martin.boda@umb.sk.
[2] Ing. Mária Kanderová, PhD., Matej Bel University in Banská Bystrica, Ekonomická fakulta, Department of Quantitative Methods and Information Systems, Tajovského 10, 975 90 Banská Bystrica, Slovakia, e-mail: maria.kanderova@umb.sk.

caps fall more often into consumer staples, consumer services and health care (Brown and Mott, 1995, p. 34). In addition, there are also noticeable differences in performance between these two groups (Brown and Mott, 1995, p. 34-35). Institutional investors (fund managers) tend to diversify among a variety of styles defined alongside to axes: market capitalization and pricing status. Although the complete picture that arises therewith is growth small caps, value small caps, growth big caps and value big caps, the focus here is only on growth and value big caps, and the paper studies whether diversifying among these two opposite classes of big caps adds value to a small investor in terms of performance. The paper is a systematic continuation of the research owing to the authors who investigated the usefulness of diversifying between small and big caps as well as between growth and value stocks (Boďa and Kanderová, 2018a) and between growth small caps and value small caps (Boďa and Kanderová, 2018b). Now the attention is moved to growth big caps and value big caps. To the best knowledge of the authors, the research in this field is scarce although it is fully recognized in investment community that there are diverse investment styles and investing into different classes of stocks has numerous merits. This at least is a legacy of the practitioner's compendium by Coggin and Fabozzi (1995) which nowadays seems in oblivion.

A preference for an investment style is relevant both to asset pre-selection (which is the initial stage of investing when some assets are identified in a universe of assets for later portfolio optimization) and to rebalancing (when the composition of the portfolio is adjusted so that its risk-return characteristics remain consistent with the investor's tastes). Considering both the pre-selection and rebalancing stages of investing, the paper explores a possibility of simultaneous investing into growth and value big-caps in an attempt to improve the risk-return characteristics of the resulting portfolio and to take advantage of different market patterns embodied in growth and value segments of the stock market (see Fabozzi, 1998, pp. 101-109). In this, the paper develops a rebalancing strategy that blends big-cap growth and value investment styles into current rebalancing strategies. In distinguishing growth and value stocks the P/B ratio was utilized so as to identify distant classes of big-cap stocks. Blending disparate categories of stocks that are attributed different patterns and performance characteristics should be obviously beneficial to diversification and possibly economical in terms of transaction costs. A similar procedure is frequently applied by practitioners for different categories of assets (such as equities and bonds) and then some pre-determined proportions of shares between these categories are maintained over the investment horizon (e.g. Tokat and Wicas, 2007). The idea explored here is to invest into big-cap stocks in some proportions allocated to the subcategories of growth and value stocks and to maintain the pre-defined proportions over time using traditional methods of rebalancing.

The features of the proposed rebalancing approach are assessed experimentally through a case study oriented on the US stock market. Using the quadratic formulation, the case study tracks the S&P 500 Index using its big-cap constituents and its growth and value subcategories identified using the P/B ratio. Several portfolios are fictively created, rebalanced under different configurations and compared in terms of their performance. The different configurations are adopted in order to suppress subjectivity as well as to improve generalizability; therefore, they vary by investment style, rebalancing strategy, portfolio size and sample period. In defining new rebalancing strategies, two rebalancing approaches are put to use: periodic rebalancing (realized at regularly spaced time intervals regardless of the actual market situation) and threshold rebalancing (initiated only if there is a deviation from the desired portfolio composition due to market development). Portfolio size of nominal 10, 20, 30 and 40 stocks is chosen to fit the situation of a small investor. Finally, portfolios are constructed and evaluated for four different consecutive data samples represented by a two-year long in-sample period and a two-year investment horizon.

Using monthly frequency of data and considering the existence of transaction costs, the paper studies the performance of 720 tracking portfolios and finds that threshold rebalancing is recommendable for portfolios that have at least a 50 % share of "BG" stocks as it is more reliable regardless of the chosen intervention threshold.

The remainder of the paper is organized into three more sections. Section 2 gives explanations and enlightens the adopted methodological set-up. Section 3 describes the experimental design and presents the obtained results. Finally, Section 4 concludes.

2. Methodology

Big caps are usually safer (i.e. less volatile) than small caps. Hence, the size criterion of market capitalization may be deemed as associated with riskiness. As far as the growth/value classification is concerned, growth stocks display promising growth prospects whereas low stocks are usually deemed as undervalued.

Judgements as to whether a stock belongs to the growth or value category are most frequently based upon confronting the market value of the stock and the book value of a fundamental that is believed to contribute to performance (usually the book value of equity, net earnings or revenue). In fact, comparing market and book values is linked with pricing, and the pricing status is in this study assessed through P/B (price-to-book) ratio. A stock with a high/low value of the P/B ratio is tagged as a growth/low stock. Investing into value stocks means purchasing stocks that are cheap relative to their fundaments and is based on the stylized fact that the market can be beaten by stocks that have prices low relative to their earnings, dividends, historical prices , book values (hence the P/B ratio) or other measures of their value. The role of investment styles recognized by the pricing criterion is not negligible (see e.g. Chan *et al*., 1991, or Chan and Lakonishok, 2004).

Big caps ("B") are identified by ordering stocks by their market capitalization and selecting those above the 50 % quantile. Growth big caps ("BG") and value big caps ("BV") are singled out by ordering big-cap stocks first by their P/B ratios and then dividing them by their accumulated market capitalization. This is the procedure advanced by Fabozzi (1998, p. 60).

Once the portfolio is optimized using some suitable allocation rule and created, the investor faces a decision whether he should keep the portfolio intact over the investment horizon or he should rebalance it whenever its return-risk profile is eroded. Rebalancing yields numerous benefits. First, in comparison to the buy-and-hold strategy it reduces volatility and has little or no unfavourable effect upon mean returns (see Dichtl *et al*., 2013). Second, it decreases risk concentration and downside risk (see Bouchey *et al*., 2012). The other side of rebalancing is that it induces transaction costs that off-set the input of rebalancing to preserving or improving performance over the investment horizon. Dichtl *et al*. (2013) divided rebalancing strategies into two chief groups: periodic and interval rebalancing. Periodic rebalancing means regular reallocation of assets with respect to pre-determined weights set at the very outset of the investment horizon. One approach to interval rebalancing is to set a non-trade region around target weights defined by means of a suitable threshold and to undertake a revision only if the portfolio deviates from these weights. For example, traditionally a certain proportion is invested into stocks and the rest into bonds. Say that these weights at the moment of portfolio creation are v and $1 - v$, respectively, and say that a threshold θ is introduced (such that $v \in [0,1]$ and $\theta \in (0,1)$ being relatively small). Over time as the market value of the portfolio develops, the proportions v and $1 - v$ change and the threshold θ controls the timing of an intervention. Usually at regularly spaced time intervals the portfolio is monitored and checked whether the actual proportions do not deviate from the intended proportions by more than $\pm\theta$. Whenever this is the case, the portfolio weights are optimized and reset to v and $1 - v$ for stock and bonds, respectively.

The proportions to be invested into "BG" and "BV" stocks are denoted as v and $1 - v$, respectively. The same number m of equities is chosen in each category and the weights in both classes of assets are determined separately. Denote the respective $(m \times 1)$ vectors of weights for "BG" and "BV" stocks by $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$. Both vectors are required to satisfy $\boldsymbol{\omega}_1'\mathbf{1} = 1$ and $\boldsymbol{\omega}_2'\mathbf{1} = 1$, wherein $\mathbf{1}$ is an $(m \times 1)$ vector of ones. The budget allocated into "BG" stocks is then $v \cdot \boldsymbol{\omega}_1$ and into "BV" stocks $(1 - v) \cdot \boldsymbol{\omega}_2$. The nominal portfolio size is $k = 2m$ since the portfolio is to be composed of m "BG" and m "BV" stocks whenever a non-trivial case $v \in (0,1)$ occurs. To ensure comparability and hold the portfolio nominal size intact, in the trivial case $v \in \{0,1\}$ k is reset to $2m$. Throughout this process, the weights $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are optimized independently using the formulation of quadratic tracking for the m "BG" stocks and the m "BV" stocks, respectively.

The task of quadratic tracking is detailed e.g. in Boďa and Kanderová (2016, p. 48) and presumes that a benchmark index is available. This function is fulfilled by the S&P 500 Index as is clarified in the next section. The model of computing portfolio holdings and calculating transaction costs is detailed in Boďa and Kanderová (2018a, 2018b). A grave simplification associated with this model in comparison to practical conditions is that it overlooks the transaction costs that must be paid at revision times and that decrease the value of the portfolio. Somewhat simplistically it is assumed here that there exists a separate account that covers these transaction costs. Only the final value of the tracking portfolio is confronted with the volume of transaction costs (in an inflation-free world), and the net value of the investment is computed by subtracting the transaction costs total from the portfolio value.

3. Empirical configuration and results

The study takes the S&P 500 Index as benchmark for a non-institutional investor who desires to form a small portfolio of S&P 500 constituents. In order to control for the amount of transaction costs affecting ultimate performance, the investor is willing only to form a tracking portfolio of no more than 40 stocks. To this end, he uses 24 historical monthly logarithmic returns for a period of two years (the in-sample period) to identify the weights of portfolios that mimic the underlying S&P 500 in the sense of quadratic tracking. Using the procedure described in the preceding section, the investor first identifies in the universe of S&P 500 constituents big caps and afterwards classifies them into growth ("BG") and ("BV") value stocks. He then sets up portfolios of different nominal sizes counting $5 + 5$, $10 + 10$, $15 + 15$ and $20 + 20$ stocks of polar categories and of varying proportions $v \in \{0, 0.25, 0.50, 0.75, 1\}$ of allocation into "BG" and "BV" stocks. When a zero sum is invested into "BG" stocks (i.e. when $v = 0$), then "BV" stocks count 10, 20, 30 and 40 stocks, and vice versa. All portfolios are formed on the last day of the in-sample period that coincides with year-end. Afterwards, the portfolios are held and monitored in the course of the next two years (the out-of-sample period or investment horizon) and possibly held without change or rebalanced in conformity with the strategy applied.

A total of four samples are employed and are referred to as “periods”. Each of these samples spans a period of 4 years, with the first two years representing the in-sample period of 24 monthly returns for portfolio selection and the last two years standing for the out-of-sample horizon of active investing and rebalancing. The samples start in 2011 (the start of the first in-sample period) and end in 2017 (the end of the last out-of-sample period), which is the reason they are denoted as "20112014" to "20142017". At the end of each in-sample period, the S&P 500 was screened for its constituents and the effective number of constituents was somewhat smaller than 500 (owing to a lack of data or changes in the index over time). In consequence, the effective universe of S&P 500 constituents ranged from 450 (with period “20122015”) to 458 (with periods “20112014” and “20142017”).

For each sample at the end of the in-sample period portfolios blending "BG" and "BV" stocks are identified and created using as many as five mixing proportions and four nominal sizes. These portfolios are optimized under a ban on short sales and made by investing the initial budget of US $ 10,000 with the rate of transaction costs set to $\varphi = 0.4$ %. These portfolios are then held intact over the out-of-sample period, which amounts to applying the buy-and-hold strategy, or rebalanced. Four periodic rebalancing strategies are considered differentiated by whether revisions are undertaken periodically every month (1M), every quarter (3M), every half-year (6M) and every year (12M/1Y). Moreover, these periodic revisions are complemented by threshold rebalancing strategies with four different choices for threshold θ given as $\theta = \{0.005, 0.01, 0.015, 0.02\}$. Note that when a portfolio is rebalanced under a certain value for threshold θ, it is also rebalanced for any smaller threshold. In total, the experiment resulted in a total of $4 \times 5 \times 4 \times (1 + 4 + 4) = 720$ blending portfolios. The multiples here are sample – mixing proportion – nominal size – rebalancing strategy, respectively. It is worthwhile noting that at the end of the out-of-sample period no portfolio was actually liquidated and so transaction costs were spared. Only the terminal value of the portfolio was ascertained and adjusted by the total amount of transaction costs. Three measures of portfolio performance were allowed for in the assessment: mean return, volatility and net cumulative return, all being expressed in annualized form (as p.a.). Net cumulative return differs from mean return by being calculated as a compound annual growth rate inclusive of transaction costs.

In computations and preparing graphical presentations, the software R version 3.4.1 (R Core Team, 2017) was employed with several of its libraries, `quantmod`, `quadprog`, `timeSeries` and `PerformanceAnalytics`.

The experiment produced an exhaustive computational output, which is in fact not reported for its substantial size. The computational output is available on request from the authors, the following text is confined only to interpretations.

For period "20112014" there are not significant differences between mean returns and volatilities of blending portfolios and all the rebalancing strategies as well as the buy-and-hold strategy appear to yield comparable returns regardless of the number of assets and the blending proportion. Nonetheless, the volatilities for portfolios rebalanced on a threshold basis are uniformly lower. For period "20122015" the general pattern is that for portfolios up to 30 stocks investing only into "BV" stocks periodic rebalancing performs better than threshold rebalancing or the buy-and-hold strategy in terms of higher mean returns, but it is offset by somewhat higher volatilities. For other blending portfolios (i.e. with a non-zero weight on "BG" caps and also those formed out of 40 stocks), no firm regularity may be ascertained. Periodic rebalancing sometimes yields extraordinary mean returns, but other times mean returns that are comparatively minute. Threshold rebalancing seems in this respect more reliable, because in period "20122015" it did not lead to considerable fluctuations in mean returns regardless of the number of assets or blending proportion. There are also no noticeable differences in volatilities. Period "20132016" is characteristic by the dominance of blending portfolios rebalanced on a threshold basis in terms of mean returns, but this holds only for portfolios investing some portion into "BG" stocks. For those investing all funds into "BV" stocks merely, periodic rebalancing seems more favourable. The link between mean return and volatility is generally contrasting. Higher mean returns are accompanied usually with higher volatilities; yet, there again are no apparent differences in volatilities of diverse blending portfolios. Eventually, for period "20142017" portfolios investing 0 %, 25 % or

100 % into "BG" stocks seem more preferable in terms of mean return when rebalanced periodically than when rebalanced using a threshold. For other cases, in which the proportion of "BG" stocks was either 50 % or 75 % periodic rebalancing appear comparatively better than threshold rebalancing so the pattern is reverse. Periodic rebalancing has a tendency to smaller volatilities.

In addition to the previous analysis, the factors of performance of portfolios blending "BG" and "BV" stocks were examined with the aid of traditional regression techniques, in a somewhat mechanical manner. The "best" regression models were selected using LASSO, optimization of the Bayesian information criterion (BIC) and the small-sample corrected Akaike information criterion (AICC). The search was performed for all 720 blending portfolios separately for mean return, volatility and net cumulative return (whilst all of them were expressed as percentages per annum). The search accounted for all explanatory factors, i.e. weight on "BG" stocks, nominal portfolio size (i.e. # assets), rebalancing approach and period. With the exception of nominal portfolio size, all the explanatory factors were represented through dummy variables, which is the reason why the scrutinized models included no intercept. The fitted regression models identified as optimal by the individual three criteria are reported in Table 1 alongside with adjusted coefficients of determination (R-squared) affording quick assessments of the quality of the fits.

Table 1: Explanatory factors of portfolio performance identified in a regression context (source: the authors)

Variable	**Mean return (% p.a.)**			**Volatility (% p.a.)**			**Net cumulative return (% p.a.)**		
	LASSO	BIC	AICC	LASSO	BIC	AICC	LASSO	BIC	AICC
Weight 0 on "BG" stocks	0.34		-0.78	1.38	1.40	1.40			
Weight 0.25 on "BG" stocks	4.51	4.58	3.39				4.25	4.39	4.44
Weight 0.50 on "BG" stocks			-1.28	1.02	1.03	1.03	-0.03		
Weight 0.75 on "BG" stocks	1.12	1.19		0.19	0.20	0.20	1.08	1.22	1.27
Weight 1 on "BG" stocks	2.23	2.30	1.11				2.20	2.34	2.39
# assets	0.21	0.21	0.18	-0.01	-0.03	-0.03	0.16	0.16	0.17
Buy-and-hold strategy	0.44			0.32	0.28	0.26	1.43	1.39	1.53
1M-rebalanced	0.08			0.01			0.26		
3M-rebalanced	0.07			0.01			0.09		
6M-rebalanced						-0.14			
1Y-rebalanced						-0.17			
Rebalanced at 0.005				0.09					
Rebalanced at 0.01	0.78		0.73	0.34	0.30	0.28	1.60	1.56	1.70
Rebalanced at 0.015	-0.05		-0.58	0.19		0.13			
Rebalanced at 0.02	0.14			0.15			0.74		0.85
Period "20112014"	12.04	12.36	13.79	8.99	9.39	9.42	12.35	12.55	12.30
Period "20122015"	0.59	0.90	2.33	10.80	11.21	11.23	0.07		
Period "20132016"			1.69	12.13	12.53	12.56	-0.32		-0.59
Period "20142017"	5.77	6.08	7.51	7.83	8.24	8.26	5.57	5.77	5.52
Adjusted R-squared	**0.93**	**0.93**	**0.93**	**0.99**	**0.99**	**0.99**	**0.91**	**0.91**	**0.91**

Note: Since the variables weight on "BG" stocks, strategy (buy-and hold strategy, periodic rebalancing, threshold rebalancing) and period are all categorical variables, during their transformation to dummy variables the intercept was neglected to prevent the issue of perfect collinearity.

The results in Table 1 can only be interpreted in comparative terms for there are visible disparities between the regression fits and it makes little sense to emphasize a particular value of a regression coefficient. It seems that the most favourable in terms of (mean and net cumulative) return are on average portfolios that arise by investing 25 % and 75 % into "BG" and "BV" stocks, respectively, and that the least favourable volatilities are for portfolios composed of "BV" stocks only or those investing an equal share into "BG" and "BV" stocks. At large, portfolio size is a factor that increases (mean and net cumulative) return and simultaneously, if very slightly, decreases volatility. On the other hand, rebalancing with threshold set to 0.01 is associated generally with the highest (mean and net cumulative) returns, but also volatilities. The buy-and-hold strategy is a runner-up that shares this pattern (of high returns and volatilities). Of recognizable effect is also the period chosen for investing and rebalancing. In general, whereas period "20112014" was most fortunate for attaining comparatively higher (mean and net cumulative) returns, periods "20122015" and "20132016" were typical of higher volatilities. It also seems that in period "20132016", transaction costs were an issue as they on average pushed down cumulative return. The patterns related to data sample are obviously linked with the trends exhibited by the US stock market as attested by Boďa and Kanderová (2017, pp. 1856-1857).

4. Conclusion

The obvious hindrance to generalizing the attained results is the confinement of the case study to the US stock market and to a certain historical period. Any such case study may operate only with a limited configuration of settings and can only exhaust only some of the choices. Here the case study focused upon a small investor who contemplated investing into growth and value big-cap constituents of the S&P 500 in an attempt to track it. The portfolio is created in certain proportions investing into "BG" and "BV" big caps and held unaltered over a two-year period (the buy-and-hold strategy) or rebalanced (periodic and threshold rebalancing strategies). Notwithstanding the limitations of the case study, some advice can be safely drawn for a small investor who is considering an investment into big-cap equities.

1. There is an obvious trade-off between return and volatility. The rebalancing strategies considered in this study in the context of investing into small-cap assets prove themselves as factors that improve just one aspect of the return-risk profile. In consequence, the investor must choose whether he will favor return or risk. It seems that both factors cannot be controlled for at a time.
2. Threshold rebalancing seems suitable for big-cap portfolios that have a share of "BG" stocks of 50 % at least since they are more reliable in terms of mean return. It does not mean that they would always attain the highest mean return, but there is much little variation of mean returns regardless of the threshold chosen in comparison to period rebalancing strategies.
3. Portfolio size acts as another relevant factor of controlling simultaneously for return and risk in big-cap investing. More populous blending portfolios tend to have higher returns and smaller volatility.
4. It also seems that there are not substantial differences in portfolio volatilities generated by different rebalancing strategies. There is no universal pattern and neither period nor threshold rebalancing can be typified as preferable.

Outstandingly, points 1 and 3 of the previous enumeration are identical also for similar small-cap portfolios and were established for that different set-up by the authors (Boďa and Kanderová, 2017b).

It must be fully understood that it would be useful to repeat the study utilizing more exhaustive configurations regarding the target market, historical period, and methods of portfolio optimization or rebalancing.

The paper arose in fulfilment of the tasks associated with grant scheme VEGA # 1/0554/16.

References

[1] Boďa, M. and Kanderová, M. (2016). Comparison of approaches for asset pre-selection in portfolio tracking. In Boďa, M. and Mendelová, V. (eds.) *Applications of Mathematics and Statistics in Economics 2016. Conference Proceedings*. Banská Bystrica: Občianske združenie Financ, pp. 44-57.

[2] Boďa, M. and Kanderová, M. (2017). Investment style preference and its effect upon performance of tracking portfolios. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 65(6), pp. 1851-1863.

[3] Boďa, M. and Kanderová, M. (2018a). *Periodic and threshold rebalancing based on polar investment styles.* Paper presented at the 21st Applications of Mathematics and Statistics in Economics [AMSE] 2018 in Kutná Hora, the Czech Republic, on 29 August 2018 – 2 September 2018.

[4] Boďa, M. and Kanderová, M. (2018b). *Blending small-cap growth and value stocks: effect upon a periodic and threshold rebalancing strategy* Paper presented at the 10th Economics & Finance Conference in Rome, Italy, on 10 September 2018 – 13 September 2018.

[5] Bouchey, P. et al. (2012). Volatility harvesting: why does diversifying and rebalancing create portfolio growth? *Journal of Wealth Management*, 15(2), pp. 26-35.

[6] Brown, M. R. and Mott, C. E. (1995). Understanding the differences and similarities of equity style indexes. In Coggin, T. D. and Fabozzi, F. J. (eds.) *The handbook of equity style management*. New Hope (PA): Frank J. Fabozzi Associates, pp. 23-50.

[7] Chan, L. K. C., Hamao, Y. and Lakonishok, J. (1991). Fundamentals and stock returns in Japan. *Journal of Finance*, 46(5), pp. 1739-1789.

[8] Chan, L. K. C. and Lakonishok, J. (2004). Value and growth investing: review and update. *Financial Analysts Journal*, 60(1), pp. 71-86.

[9] Chrisopherson, J. A. and Williams, C. N. (1995). Equity style: What it is and why it matters. In Coggin, T. D. and Fabozzi, F. J. (eds.) *The handbook of equity style management*. New Hope (PA): Frank J. Fabozzi Associates, pp. 1-22.

[10] Coggin, T. D. and Fabozzi, F. J. (1995) *The handbook of equity style management*. New Hope (PA): Frank J. Fabozzi Associates.

[11] Dichtl, H., Drobets, W. and Wambach, M. (2013). Testing rebalancing strategies for stock-bond portfolios: what is the optimal rebalancing strategy? *Applied Economics*, 48(9), pp. 772-788.

[12] Fabozzi, F. J. (1998). Overview of equity style management. In Fabozzi, F. J. (ed.). *Active equity portfolio management*. New Hope (PA): Frank J. Fabozzi Associates, pp. 57-70.

[13] Penman, S. H. (2013). *Financial statement analysis and security valuation*. 5th ed. New York: McGraw-Hill.

[14] R Core Team (2017). *R: a language and environment for statistical computing*. Vienna: R Foundation for Statistical Computing, http://www.r-project.org/.

[15] Tokat,Y. and Wicas, N. W. (2007). Portfolio rebalancing in theory and practice. *Journal of Investing*, 16(2), pp.52-59.

Insurance Market: Analysis, Development, Risks

Martina Borovcová, Daniela Hasoňová [1]

Abstract

The insurance market is very much intertwined in the financial markets, therefore assessment of its level is important. The assessment and analysis of the insurance market is done by using selected indicators.
This paper is focused on the description, verification and application of the multi-attribute decomposition methods AHP and ANP based on the Saaty pair comparison approach. We consider importance of setting the ratio for evaluation indicators of the development of the insurance market by applying Saaty methods in the framework of decomposition methods AHP and ANP (insurance penetration ratio, claims frequency ratio, concentration ratio, premium indicator, benefit indicator and more). The development of selected indicators is illustrated. The risks of the development of the insurance market are mentioned.

Key words

Multi-attribute methods, AHP, ANP, Key Assessment Indicators, Saaty Pair Comparison Approach, Regression Analysis

JEL Classification: C02, C4, G2, G11

1. Introduction

Multicriteria decision-making is one of options, how to choose optimal variant of certain sets of variants. Only very rarely it is possible to find the very optimal variant which meets all specified criteria. The solution of decision-making problem is more often a compromise variant, which meets just the most important criteria, while it does not meet all the specified criteria the best.

It is preferable to take into account more than one decision-making criterion when making the decision. Although may arise a situation, where the choice of options has been used only a single evaluation criteria. Conditions for the quantitative nature of the criteria would then be enough to organize a variant according to the values of the criteria and the variant with the highest or the lowest value would be the best (optimal) option. However, there are relatively a few decision-making problems with monocriterial character. More and more frequently, it is possible to meet with problems, when the solution variants should be assessed using a larger number of evaluation criteria. Such decision-making problems then have the character of multicriteria decision-making. It is necessary to determine the goals of decision-making for the application of methods of function evaluation of variants and criteria of decision-making.

The aim of the article is to describe the multi-attribute methods AHP (analytic hierarchy process) and ANP (analytic network process), and their applications to verify the simplified example of determining weights partial indicators of the evaluation of the development of the insurance market. Qualitative, quantitative and other indicators of the level of the insurance market are considered in the study.

[1] Martina Borovcová, Ing. Ph.D., Daniela Hasoňová, Ing., Department of Finance, Faculty of Economics, VŠB-Technical University of Ostrava, e-mail: martina.borovcova@vsb.cz

2. Methodology Description of the multicriteria decision making evaluation of alternatives

The aim of the application of the multi-criteria decision making evaluation of variants is primary finding the best (optimal) variant and layout of these variants from the best to the worst. The best option is usually a variant of the compromise. The compromise solution is the least distant one from the ideal variant, or the furthest away from the variants of basal, while the ideal option is the one that has all the criteria with the best possible value. On the contrary, variant with the worst values of the criteria is the basal variant. Ideal and basal variants are usually hypothetical. If the ideal variant really existed, it would be at the same time, a variant of the optimal solution. However, this situation usually does not occur and therefore any selected solution is the solution to the compromise. Compromise variant must be undominated in all tasks, which means that there is no dominating variant among decision-making variants (Ramík, 1999).

2.1 Criteria and methods of determining the values of the criteria

Alternatives are specified by using variants and the measurement of satisfaction depends on each variant. Determination of the criteria is difficult process, which requires certain knowledge of the area. The criteria used to selection of the most appropriate variants can be classified according to several aspect. Firstly it's possible to divide criteria as maximizing (income, profit) or minimizing (cost, loss) according to the level of desirable values. According to the type Secondly it is possible to divide criteria into qualitative and quantitative. These are expressed in the units of measurement.

For calculations and comparison it is usually desirable for specified criteria values y_{ij} to be normalized the unit interval, i.e.

$$x_{ij} \in [0;1] \tag{1}$$

Generally it is possible to obtain these values of the criteria from the sub-functions of the utility (value) as

$$x_{ij} = u(y_{ij}) \tag{2}$$

Utility of the criteria, which acquire the worst values is equal to 0 or close to 0, and the utility of the criteria with the best value is equal to 1.

Saaty method AHP and ANP will be used in the application part of the study, therefore the following description will be focused on these methods.

2.2 Saaty's method of pairwise comparison

The Saaty's method of weights determination of the criteria can be divided into two steps. The first step consists of a pairwise comparison when finding the preferential relations of criteria pairs. It is presented as so-called Saaty's matrix S. This matrix is symmetric with elements s_{ij}. It is possible to determine also the size of this preference expressed by a certain number of points from the selected point scale in addition to the direction of the preference of pair of criteria. Scale of relative importance (descriptors) was recommended by Saaty and it is shown in *Table 1*. Other values can be used to express sub-preferences. The strength of preferences is expressed in the interval $s_{i,j} \in [0;9]$. The result of this step is to obtain the right upper triangular part of the matrix size preferences (Saaty's matrix). The diagonal element have to be $s_{i,j} = 1$ and for the inverse elements (in the lower left triangular part of matrix) is true the following:

$$s_{i,j} = \frac{1}{s_{j,i}}. \quad (3)$$

The elements $s_{i,j}$ Saaty matrix are estimated shares of weights of criteria v_i and v_j, so:

$$s_{i,j} \cong \frac{v_i}{v_j}. \quad (4)$$

The scales can be obtained in the following manner:

$$\min F = \sum_i^n \sum_j^n \left(s_{i,j} - \frac{v_i}{v_j} \right)^2, \quad (5)$$

with the condition $\sum_i^n v_i = 1.$

Because of difficulty it is possible to obtain the weights using an algorithm based on the geometric average.

$$\min F = \sum_{i=1}^n \sum_{j>i}^n \left[\ln s_{i,j} - (\ln v_i - \ln v_j) \right]^2, \quad (6)$$

with the condition $\sum_i^n v_i = 1.$

The final solution is based on the geometric mean of rows (Saaty, 2010):

$$w_i = \frac{v_i}{\sum_i^N v_i} = \frac{\left[\prod_j^N s_{i,j} \right]^{\frac{1}{N}}}{\sum_i^N \left[\prod_j^N s_{i,j} \right]^{\frac{1}{N}}}, \quad (7)$$

Table 1: Recommended point of scale with the descriptors by Saaty

The number of points	Descriptor
1	Element A and B are equally important
3	Element A is moderately more important than element B
5	Element A is strongly more important than element B
7	Element A is very strongly more important than element B
9	Element A is extremely more important than element B

Source: Saaty (2006), own processing

The sign of relevant evaluation is the consistency of Saaty's matrix, in other words when the elements satisfy the condition of transitivity the most. It should be emphasized that in many methods this aspect is not accounted. Consistency can be measured using the coefficient of consistency CR (Consistency Ratio). The coefficient for consistent evaluation should be $CR \leq 0{,}1$. Consistency ratio is calculated as following $CR = \frac{CI}{RI}$, where $CI = \frac{\lambda_{max} - N}{N-1}$, (Saaty, 2010). The characteristic number of the matrix λ_{max} can be determined by various procedures. One option is $\lambda_{max} = \frac{1}{N} \sum_i^N (S \cdot w)_i / w_i$, while w is a vector and $(S \cdot w)_i$ is the *i-th* element of the vector. Furtherly RI (Random Index) is derived from an empirical examination and reaches the following values depending on the number of criteria (Zmeškal, 2014), see in *Table 2*.

Table 2: The value RI according to the number of criteria

N	1	2	3	4	5	6	7	8	9	10
RI	0,00	0,00	0,52	0,89	1,11	1,25	1,35	1,40	1,45	1,49

Source: Saaty (2009), own processing

2.3 Multi-attribute methods AHP and ANP

Weights or values of criteria are in the case of decomposition tasks set by gradual decomposition from the goal, global groups of criteria, sub-groups, to the the initial sub-criteria and variants. For AHP method these linkages may be linear and for ANP method in the shape of a pyramid or nonlinear with feedbacks. Evaluation of preferences (weights) of the criteria is carried out using the Saaty's method of pairwise comparison.

Local weights (preferences) of the subgroups or indicators with regard to the specified purpose are determined by using Saaty's method of pairwise comparison. The next step is calculation of the global weights including the initial sub-weights. The sum of all sub-weights is equal to one.

In AHP method can be used analytical procedure and also method of supermatrix. In the ANP, it is possible to calculate global weights by using only the method of supermatrix (Saaty, 2010).

For analytical method AHP the indicator subgroup weights are obtained as follows, $w_{i,j} = v_i \cdot v_{i,j}$ where $w_{i,j}$ is global weight of *j-th* indicator and *i-th* group, v_i is local weight of *i-th* group and $v_{i,j}$ is local weight of *j-th* indicator and *i-th* group. By this way we can gradually get all the global weights of primary indicators.

The procedure for the calculation of sought weights in case of AHP and ANP supermatrix method can be divided into three steps:

- First step is determination of default supermatrix W. The local weights $v_{i,j}$ are typed to the columns inside this supermatrix W. The weights of criteria are highlighted from $e_{2,1}$ to $e_{2,n2}$ according to the purpose (criteria) $e_{1,2}$.
- Subsequently the default supermatrix is transformed into the weighted supermatrix $\overline{W}$ so, that sums of columns are equal 1.
- The last step is the calculation of limit (final) supermatrix $\overline{W}^{\propto}$. This supermatrix can be calculated like acyclical weighted matrix as following $\overline{W}^{\propto} = \lim_{k \to \propto} \overline{W}^{k}$,where $\overline{W}^{\propto}$ is limit (final) supermatrix, $\overline{W}^{k}$ is weighted supermatrix without existence cycle, and this supermatrix is *k* times amplified. Global weights are found in the first column considering the goal.

3. Determining the preferences of the indicators assessment of the level insurance market according to the AHP and ANP

We consider a task to set the weights (preferences) of evaluation indicators of the development of the insurance market to apply Saaty's method in the framework of decomposition methods AHP and ANP.

3.1 Decomposition for AHP and ANP

Decomposition is designed in two ways from the goal, categories of indicators and each indicator of the assessment of the level of the insurance market is. The first way is linear system of AHP, and subsequently the second is nonlinear system ANP with typical feedbacks. The intention is to determine the weights of individual indicators of the assessment of the level of the insurance market by using AHP method and ANP method. Analytical procedure and supermatrix method can be used in the case of AHP method.

Qualitative and quantitative indicators of the assessment of the level of the insurance market are taken into account for the purposes of the article. Following indicators are included among qualitative indicators according to the subjective opinion of an expert: insurance penetration and claims frequency. Following indicators are included among quantitative indicators: gross premium, insurance benefit, number of concluded insurance contracts, average insurance premium on one insured contract, number of settled insurance claims, the average insurance benefit on one insured contract, number of employees, number of commercial insurance companies, concentration of the insurance market.

3.2 Solution and the result for the method of supermatrix AHP and ANP

Local and global weights based on Saaty's method of paired comparison are calculated in this subchapter. At first the local weights are established and then the global weights. Global weights are calculated by both AHP and ANP method. Analytical procedure and supermatrix method were used in case of AHP. In the second case ANP, was used the only possible way and it was supermatrix method.

Table 3: The result of AHP and ANP

Goal	Local	Groups	Global - analytical method	Global-supermatrix method	
			AHP	AHP	ANP
Qualitative	75,00 %				
Quantitative	25,00 %	0			
Insurance penetration	33,33 %		25,00 %	**25,00 %**	**19,44 %**
Claim frequency	66,67 %	75,00 %	50,00 %	**50,00 %**	**38,89 %**
Gross premium	28,36 %		7,09 %	**7,09 %**	**11,81 %**
Insurance benefit	23,24 %		5,81 %	**5,81 %**	**9,68 %**
The number of concluded insurance contracts	14,81 %		3,70 %	**3,70 %**	**6,17 %**
The average insurance premium	10,58 %		2,64 %	**2,64 %**	**4,41 %**
The number of settled insurance claims	9,31 %		2,33 %	**2,33 %**	**3,88 %**
The average insurance benefit	5,17 %		1,29 %	**1,29 %**	**2,15 %**
The number of employees	1,83 %		0,46 %	**0,46 %**	**0,76 %**
The number of commercial companies	4,02 %		1,00 %	**1,00 %**	**1,67 %**
Concentration of the insurance market	2,69 %	25,00 %	0,67 %	**0,67 %**	**1,12 %**
∑			100,00 %	100,00 %	100,0 %

Source: own processing

Table 4: Final order of the key assessment indicators

Indicators	Order
Claims frequency	1.
Insurance penetration	2.
Gross premium	3.
Insurance benefit	4.
The number of concluded insurance contracts	5.

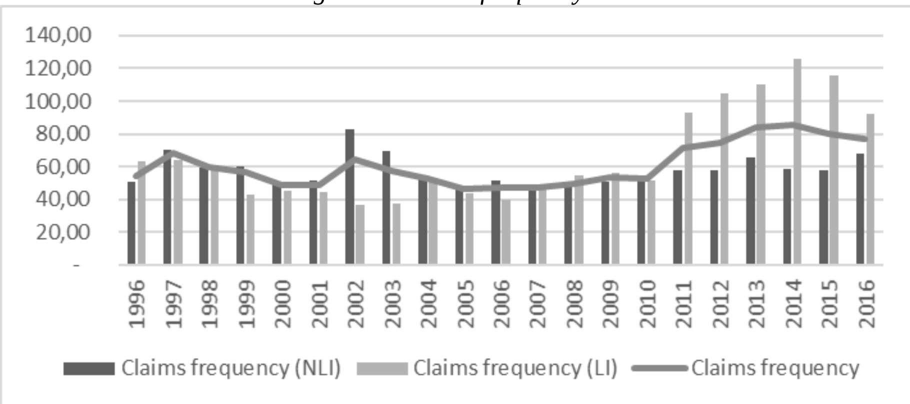
The average insurance premium	6.
The number of settled insurance claims	7.
The average insurance benefit	8.
The number of commercial insurance companies	9.
Concentration of the insurance market	10.
The number of employees	11.

Source: own processing

3.3 Development of the insurance market in the most important indicators

The following is the development of indicators with the highest importance according to multicriteria analysis. Specifically, claims frequency, insurance penetration, gross premium, insurance benefit, the number of concluded insurance contracts and the average insurance premium.

Figure 1: Claims frequency

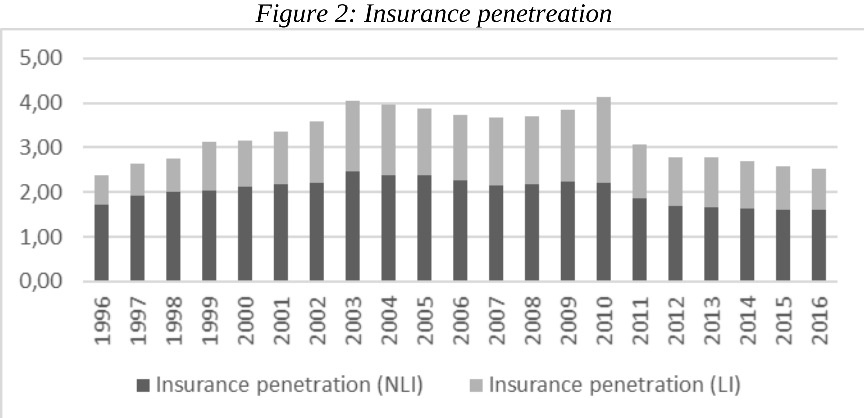


Source: own processing

The indicator of claims frequency expressed as the ratio between the value of insurance benefit and gross premium. Desirable is the declining or constant development of the indicator. The problem is greater than 100%. The evolution is not optimal, see *Figure 1*. A major problem is in life insurance.

The calculation of the insurance penetration constructed as a proportion of the gross premium to the gross domestic product. Even in this case, the indicator expressed in percent. Indicator growth is desirable. However, such a development does not follow from *Figure 2*.

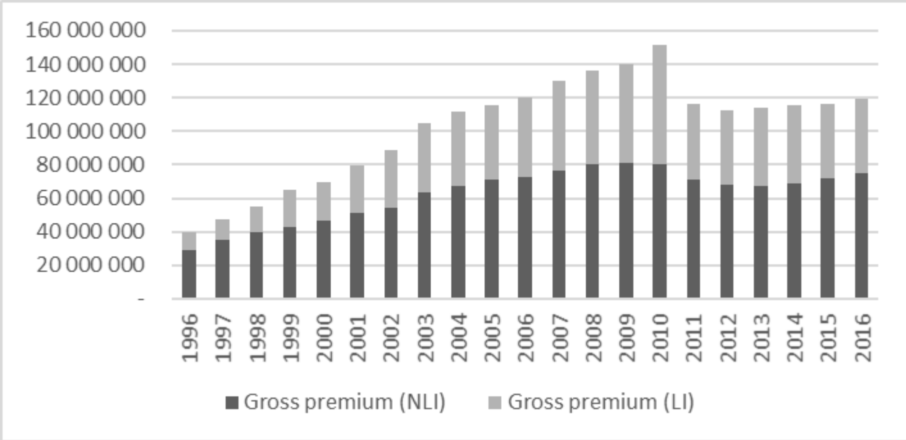
Figure 2: Insurance penetreation



Source: own processing

The gross premium indicator is another important indicator. The indicator expressed in thousands of Czech crowns. Its value should grow over time. Such a development, however, does not capture *Figure 3*.

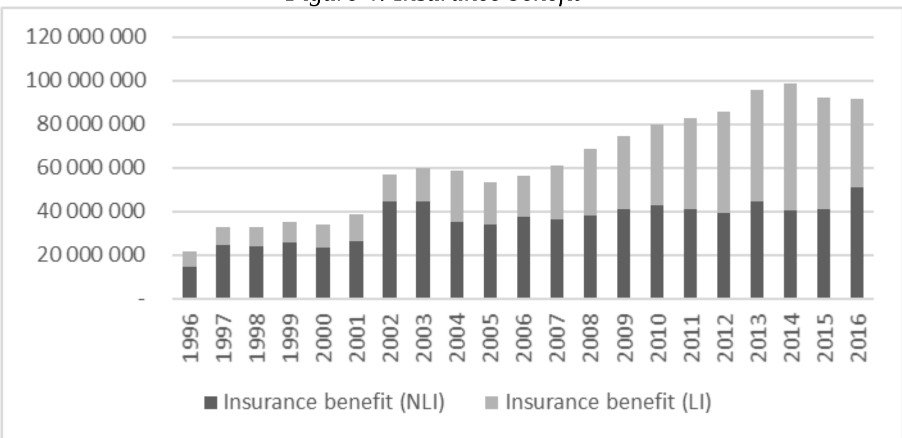
Figure 3: Gross premium



Source: own processing

The pointer expresses all the amounts paid out of the incurred claims. The effort is to reduce the amounts of indemnities paid. As *Figure 4* shows, development is not desirable.

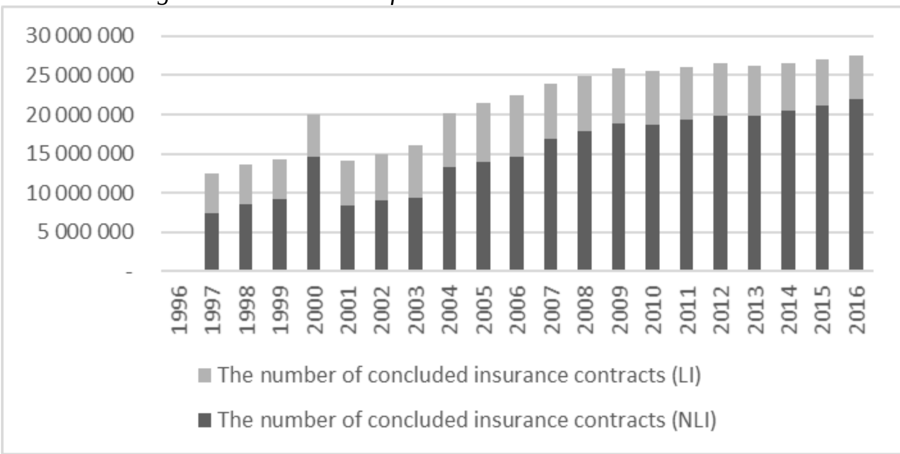
Figure 4: Insurance benefit



Source: own processing

Based on the number of insurance contracts concluded, the efficiency of work in the insurance and insurance markets assessed. Desirability is to increase the pointer value. According to *Figure 5* it is possible to evaluate the development positively.

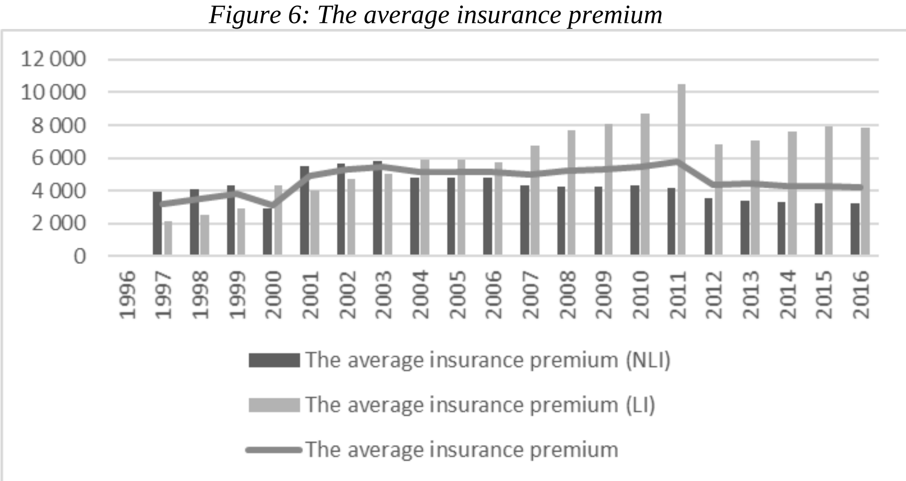
Figure 5: The number of concluded insurance contracts



Source: own processing

The quantification of the average insurance premium indicator measures the value of the total gross premiums and the number of insurance contracts concluded. The aim is to increase the average premiums year on year. The indicator expressed in Czech crowns. However, such a development does not show *Figure 6*.

Figure 6: The average insurance premium



Source: own processing

3.4 The risks of the development of the insurance market

The development of the insurance market situation entails a number of risks. It reacts both the insurance market and the individual insurance companies. Based on the analysis of the development of important indicators of the insurance market, it is possible to point out possible future risks. These may result from the development of gross premium, indemnity, loss, insurance, and average premium for the insured event. The analysis of the development shows that the growth in the number of insurance contracts concluded does not cause a corresponding increase in gross premiums. This is reflected in the indicator of the average premium on the concluded contract. The impact is also on the indicator of loss. Negative developments, evident from the development of these indicators, confirm the values of the insurance indicator.

Insurance risk monitoring and management is an important aspect of insurance management. In order for these risks to have no fatal consequences for the clients of the insurance companies, the control of the insurance market enters into control and subsequent measures throughout the process.

4. Conclusions

The aim of this paper was to determine the preferences of indicators of evaluation of the level of insurance market using multi-attribute methods AHP and ANP on the basis of Saaty's method of paired comparison. Methods including multi-attribute decomposition AHP and ANP on the basis of Saaty's method of paired comparison were described. Subsequently analytical method and supermatrix method AHP were applied on the example of determination of preferences of indicators assessment of the level of insurance market. It was found that both approaches AHP and ANP lead to the same results and at the same time, that the results obtained on the basis of linear methods of AHP and non-linear method ANP differ significantly.

The most important indicators, indicators with the highest preferences analyzed. The development results were present in the charts and commented on.

Acknowledgments

This paper was supported by the SGS Project VŠB – TU Ostrava SP2018/154 "Finanční rozhodování podniků a finančních institucí za rizika”.

References

[1.] ČAP (1997-2017). Výroční zprávy ČAP. http://www.cap.cz/o-nas/vyrocni-zpravy

[2.] Fotr, J., Dědina, J. a Hrůzová, H. (2010). Manažerské rozhodování. 2. aktualiz. vyd. Praha: Ekopress.

[3.] Hasoňová, D. (2018) Hodnocení úrovně pojistného trhu aplikací vícekriteriálních dekompozičních metod. Ostrava 2018. Diplomová práce. VŠB – Technická univerzita Ostrava.

[4.] Ramík, J. (1999). Vícekriteriální rozhodování – analytický hierarchický process (AHP). 1. vyd. Karviná: Slezská univerzita v Opavě.

[5.] Saaty, T. L. (2012). Decision making form leaders: the analytic hierarchy process for decisions in a complex world. 3rd rev. ed. Pittsburgh: RWS Publications.

[6.] Saaty, T. L. (2010). Mathematical principles of decision making: generalization of the analytic network process to neural firing and synthesis. 1st ed. Pittsburgh: RWS Publications.

[7.] Saaty, T. L. (2009). Theory and applications of the analytic network process: decision making with benefits, opportunities, costs, and risks. 3rd ed. Pittsburgh: RWS Publications.

[8.] Zmeškal, Z., Dluhošová, D. a Tichý T. (2013). Finanční modely: koncepty, metody, aplikace. 3., přeprac. a rozš. vyd. Praha: Ekopress.

[9.] Zmeškal, Z. (2014). Application of the group multi-attribute decision-making methods in financial management. *Managing and Modelling of Financial Risks 7th international scientific conference*. Ostrava: VŠB-TU Ostrava, Ekf.

Sparsity and regularization in portfolio selection problems

Martin Branda [1]

Abstract

Sparse portfolios, i.e. portfolios with limited number of assets, are preferred by investors for their good out-of-sample performance and reduction of transaction costs. Exact sparsity can be ensured by adding so called cardinality constraints into the portfolio selection problems. However, the resulting problems lead to mixed-integer programs which are difficult to solve in general. Therefore we will discuss several alternative ways how to ensure sparsity including very popular l1 "lasso" regularization, Scholtes-type regularization or general NCP-functions. Moreover, we will add a penalty term which ensures zero-one values of additional variables. The discussed approaches will be compared in a numerical study.

Key words

Portfolio selection, sparsity, cardinality constraints, regularization, penalty

JEL Classification: C44, C61, D81, G11

1. Introduction

Sparse solutions, i.e. solutions with limited number of nonzero elements, are required in many areas of optimization, operations research, mathematical statistics and others. For example, Adam and Branda (2016a) were constructing sparse solutions of inverse modelling problems in atmosphere. Many data-mining and machine learning methods rely on regularizations, cf. Hastie et al. (2009). In general, the sparsity can be ensured by mixed-integer programming techniques leading to demanding problems.

We give a short overview of the approaches to portfolio selection problems with sparsity. Fastrich et al. (2015) proposed several types of penalties that explicitly consider financial information and at the same time improve the standard l_1-regularization approach. Gao and Li (2013) focused on the mean-variance problems with cardinality constraints and developed a new lower bounding procedures based on Lagrangian relaxation and semidefinite programming which were shown to be effective within the branch-and-bound algorithm. Teng et al. (2017) introduced a penalty proximal alternating linearized minimization (PALM) method in which a sequence of penalty problems is solved. They were able to prove the convergence of solutions to a Karush-Kuhn-Tucker point of the sparse optimization problem. Mutunge and Haugland (2018) studied the problem of selecting a restricted number of assets included in a stock market index using the tracking error. They proposed a construction heuristic to improve the lower bounds and computational performance. Branda and Kopa (2014, 2016) investigated relations of the risk measures and stochastic dominance efficiency tests which can also include sparsity of the portfolios.

There is a series of papers that deal with general optimization problems with cardinality constraints. Červinka et al. (2016) proposed constraint qualification and first-order optimality

[1] The Czech Academy of Sciences, Institute of Information Theory and Automation, Pod Vodárenskou věží 4, 182 08 Prague 8, Czech Republic
Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
branda@karlin.mff.cuni.cz

conditions for the cardinality constrained problems, whereas Bucher and Schwartz (2018) provided the second-order optimality conditions. Burdakov et al. (2016) investigated various regularizations together with their convergence properties and performed a numerical experiment on the mean-variance portfolio selection problems with the cardinality constraints. Finally, Branda et al. (2018) focused on the Scholtes regularization and confirmed its strong convergence properties as well as good numerical performance. Similar approach based on the regularization was successfully applied to nonlinear chance constrained problems by Adam and Branda (2016b).

The basic portfolio optimization problem with cardinality (sparsity) constraint can be formulates as

$$\begin{aligned} min \quad & R(x) \\ s.t. \quad & x \in X_r \\ & \|x\|_0 \leq \kappa. \end{aligned}$$

where we denote a risk measure depending directly on portfolio weights x by $R(x)$, i.e.

$$R(x) = \rho\left(\sum_{i=1}^{n} x_i R_i\right),$$

where ρ is a risk measure on a set of random portfolio returns and R_i is the random return of asset i. The risk measure can belong, e.g., to the class of coherent risk measures, cf. Artzner et al. (1999), or general deviation measures, cf. Rockafellar et al. (2006). We assume that the portfolio weights are restricted by the set which contains the minimal required expected return r as one of the constraints:

$$X_r = \left\{x \in \mathrm{R}_+^n : \sum_{i=1}^{n} x_i = 1, E\left(\sum_{i=1}^{n} x_i R_i\right) \geq r\right\}.$$

We will denote the cardinality of the decision vector by

$$\|x\|_0 = number\ of\ nonzero\ elements\ of\ x.$$

Let κ be the maximal allowed number of nonzero portfolio weights. Then the cardinality constraint $\|x\|_0 \leq \kappa$ can be exactly reformulated using n binary variables leading to mixed-integer problems by adding the following set of constraints to the problem:

$$\begin{aligned} & \sum_{i=1}^{n} y_i \geq n - \kappa, \\ & x_i y_i = 0, i = 1, \dots, n, \\ & y_i \in \{0,1\}, i = 1, \dots, n. \end{aligned}$$

The binary variable y_i can be equal to one only if the corresponding portfolio weight x_i is equal to zero. Then, the first constraint ensures that at least $n - \kappa$ portfolio weights are equal to zero. However, $x_i y_i$ is a nonlinear (bilinear) term which would cause serious problems to standard optimization solvers. Therefore, if there are some natural lower l_i and upper u_i bounds for the portfolio weights, the bilinear terms can be substituted by linear constraints

$$l_i y_i \leq x_i \leq u_i y_i, i = 1, \dots, n.$$

Note that the bounds need not to be positive, e.g., if the short sales are allowed, then l_i can be equal to -1.

The remainder of the paper is organized as follows. In Section 2, we discuss various approaches to regularization in portfolio optimization problems. First, we review the traditional *l1* and *l2* regularizations. Then, we focus on several approaches which are motivated by handling the general complementarity conditions. In Section 3, we compare the approaches on several test instances of the mean-variance portfolio selection problem with the cardinality constraint. Section 4 concludes the paper.

2. Regularized approximations

In this section, we will discuss several approaches based on regularizations which are popular in many areas of mathematical programming and its applications. We will start with the most common approaches based on the *l1* and *l2* norms of the decision vector.

2.1 Traditional *l1* and *l2* regularized approximations

Boyd and Vandenberghe (2004) reviewed the basic approaches to the regularization as bi-criterion formulations of the least square problems. In general, this can be seen as a multiobjective (bi-objective) problem, where we minimize the risk objective and at the same time $p \in \{1,2\}$ norm of the decision vector:

$$\begin{aligned} \text{"}min\text{"} \quad & \left(R(x), \|x\|_p\right) \\ s.t. \quad & x \in X_r. \end{aligned}$$

Moreover, in our case, it can be further generalized to three objectives by considering the expected return as the third criterion, but we will stay with the two objectives for simplicity. A traditional method for obtaining the efficient solutions is based on the aggregate function approach where the objective functions are aggregated into a single one. In this case, the norm objective can be even seen as a penalty term. Let $\lambda > 0$ be the aggregation or penalty parameter, then

$$\begin{aligned} min \quad & R(x) + \lambda\|x\|_2^2 \\ s.t. \quad & x \in X_r. \end{aligned}$$

So called lasso (least absolute shrinkage and selection operator) regularization, cf. Tibshirani (1996), is the most popular approach used in mathematical statistics and machine learning. It can be applied to our portfolio selection problem as

$$\begin{aligned} min \quad & R(x) + \lambda\|x\|_1 \\ s.t. \quad & x \in X_r. \end{aligned}$$

The main problem in using this traditional regularization in portfolio selection problems lies in the restrictions of the portfolio weights. Since the sum of the portfolio weights is already equal to one, the *l1* norm can only restrict the (large) short selling.

Another possibility, which is also based on general principles of the multiobjective optimization, is to prescribe an upper bound on the norm, i.e. $\|x\|_p \leq \varepsilon$, leading to so called ε-constrained approach. Finally, note that we performed a short numerical experiment to verify that none of these approaches can ensure the sparsity of the portfolio weights.

2.2 Regularizations based on complementarity conditions

Using the idea by Burdakovet al. (2016), who relaxed the binary variables y_i suggested above, we can arrive at a nonlinear programming problem with continuous variables

$$\begin{aligned} min \quad & R(x) \\ s.t. \quad & x \in X_r \\ & \sum_{i=1}^{n} z_i \geq n - \kappa, \\ & x_i z_i = 0, i = 1, \dots, n, \\ & 0 \leq z_i \leq 1, i = 1, \dots, n. \end{aligned}$$

Obviously, this problem has larger set of feasible solutions than the original mixed-integer one. However, Burdakov et al. (2016) showed that there are strong relations between the global and local minima and stationary points of these problems.

2.2.1 Scholtes-type regularization

Scholtes (2001) introduced particular type of regularization for solving difficult problems with equilibria and complementarity constraints. This regularization inspired the approach for the cardinality constrained problems which was deeply elaborated by Branda et al. (2018). The regularized problem depending on parameter $t \geq 0$ can be formulated as

$$\begin{aligned} min \quad & R(x) \\ s.t. \quad & x \in X_r \\ & \sum_{i=1}^{n} z_i \geq n - \kappa, \\ & -t \leq x_i z_i \leq t, i = 1, \dots, n, \\ & 0 \leq z_i \leq 1, i = 1, \dots, n. \end{aligned}$$

The algorithm starts with a point (x^0, z^0) and decreases the parameter t using the previous optimal point as a starting point for the new iteration. Burdakovet al. (2016) investigated a different regularization called Kanzow-Schwartz based on particular NCP-function, see below. Note that they did not obtain such strong results as Branda et al. (2018) for the Scholtes regularization.

Branda et al. (2018) observed some numerical problems for several test instances and particular starting points when using the Scholtes regularization. Therefore, we will employ a smooth penalty function suitable for solving zero-one programming problems, see Rinaldi (2009), Lucidi and Rinaldi (2010), who investigated various penalty terms and their properties. In particular, we will use the following penalty term:

$$\varphi(z, \epsilon) = \sum_{i=1}^{n} [\log(z_i + \epsilon) + \log(1 - z_i + \epsilon)],$$

where ϵ is a positive parameter. This term is added to the objective function to ensure that by solving the resulting nonlinear programming problem with bounded variables $0 \leq z_i \leq 1$, we obtain binary values of z_i for any choice of the parameter $\epsilon \in (0, \bar{\epsilon}]$ for particular value $\bar{\epsilon} > 0$, i.e. this corresponds to so called exact penalty property.

2.2.2 NCP functions

General nonlinear complementarity problems (NCP) are difficult to solve. Therefore, NCP-functions, cf. Sun and Qi (1999), Chen et al. (2011), were introduced and investigated. These NCP-functions θ fulfil

$$a, b \geq 0, ab = 0 \;\Leftrightarrow\; \theta(a, b) = 0,$$

i.e. the complementarity condition is equivalent to $\theta(a, b) = 0$. An example of such function is $\theta_1(a, b) = \min(a, b)$. However, this function is nondifferentiable, thus the standard solvers can have problems with it. In the numerical study, we will use the differentiable NCP-function:

$$\theta_2(a, b) = \sqrt{a^2 + b^2} - (a + b).$$

This means that if we consider nonnegative portfolio weights, we can solve the problem

$$\begin{aligned} min \quad & R(x) \\ s.t. \quad & x \in X_r \\ & \sum_{i=1}^{n} z_i \geq n - \kappa, \\ & \theta(x_i, z_i) = 0, i = 1, \dots, n, \\ & 0 \leq z_i \leq 1, i = 1, \dots, n. \end{aligned}$$

Note that general portfolio weights can be taken into account by applying the NCP-function θ to both positive and negative parts. Thus the generalization is straightforward.

To improve the performance of the numerical method based on the NCP-functions, we bound the regularization function by *t* (as in the case of the Scholtes regularization) instead of setting the regularization term directly to zero. In particular, we consider the constraints

$$\theta(x_i, z_i) \leq t, i = 1, \dots, n,$$

and decrease the parameter *t* to zero in the iterations.

3. Numerical study

In this section, we use the test instances which were proposed by Frangioni and Gentile (2007) who implemented and tested valid inequalities for general mixed-integer quadratic programs. In particular, we consider five instances of the mean-variance problems with 200 assets: orl200-005-a, -b, -c, -d, -e. We set $\kappa = 10$.

There are two critical issues which much be taken into account when using the proposed regularizations. Since the algorithms construct exterior points with respect to the original set of feasible solutions, we must prescribe a tolerance when the solution is accepted as feasible for the original problem. We consider an element of x as nonzero, if it is greater than this tolerance level. The tolerance is set to 10^{-5}. The other issue concerns the starting points which can influence the performance of the method significantly, see, e.g., Branda et al. (2018). We will use several starting points for the algorithms bases on the regularization, in particular we start the iterations from:

1. the optimal solution of the mean-variance problem without sparsity (MV),
2. $x_i = 0$, $z_i = 1, i = 1, \dots, n$, (0-1),
3. $x_i = 0$, $z_i = 0, i = 1, \dots, n$, (0-0).

Table 1: Numerical results for the test instances – optimal value, computational time (s), number of positive weights (with a tolerance) – comparison of the solutions based on MIP and regularizations. The best results obtained by the regularizations are in bold.

orl200-005	**MIQP**	**Scholtes regularization**			**Scholtes r. with penalty**			**NCP-function**		
Start. point	-	MV	0-1	0-0	MV	0-1	0-0	MV	0-1	0-0
a	76.076	87.82	90.03	80.73	**76.57**	83.82	89.31	83.99	108.23	81.75
	60.0	12.0	12.4	8.5	**25.5**	112.7	39.7	18.4	16.6	17.9
	10	10	10	10	**10**	9*	10	9*	9*	10*
b	75.85	109.83	88.45	89.78	**77.17**	126.98	104.76	84.65	92.50	87.60
	60.0	11.9	9.4	9.6	**30.9**	114.1	169.7	15.5	18.1	17.1
	10	10	10	10	**10**	6	8	9*	9*	9*
c	76.22	82.64	81.59	80.42	84.39	101.13	**77.43**	76.89	91.04	84.99
	60.0	9.5	10.2	8.2	126.9	113.5	**31.0**	22.6	30.1	23.3
	10	10	10	10	9*	8*	**10**	10*	9*	9*
d	76.73	90.08	81.53	86.44	84.19	86.17	**76.96**	80.73	76.76	77.14
	60.0	10.6	10.2	13.3	99.8	125.9	**82.0**	22.8	17.5	15.5
	10	10	10	10	9	9*	**10**	10*	10*	10*
e	75.91	**84.54**	87.63	88.26	84.26	87.98	87.98	99.23	93.66	87.73
	60.0	**9.7**	9.7	9.6	130.1	112.6	126.5	19.5	19.5	17.5
	10	**10**	10	10	9*	9*	9*	9	9*	9*

We will compare the following three regularized approximations using the proposed settings of the parameters:

1. Scholtes regularization: $t = 0.1^k$, $k = 1, \dots, 6$.
2. Scholtes regularization with the penalty term: $\epsilon = t$, $t = 0.1^k$, $k = 1, \dots, 4$.
3. NCP-function regularization: $t = 0.1 \cdot 0.01^k$, $k = 1, \dots, 4$.

We employed the modelling system GAMS and the interior point solver ICOPT, cf. Wächter and Biegler (2006), for solving the nonlinear programming problems which appeared in the iterations. For completeness, we solved also the mixed-integer quadratic program (MIQP) using the solver Cplex with the time limit set to 60s.

Table 1 contains the results. The best results were obtained by the Scholtes regularization with the binomial penalty term in four out of the five test instances. The standard Scholtes regularization was better in one case only. However, in some cases the regularizations had problems to reach the prescribed tolerance 10^{-5}, therefore we used 10^{-4}, which is market by *. This problem was observed for almost all applications of the NCP-function. If we compare the computational time, the basic Scholtes regularization is really quick (around 10s), whereas the additional penalty term increased the computational time significantly (to more than 100s in most cases).

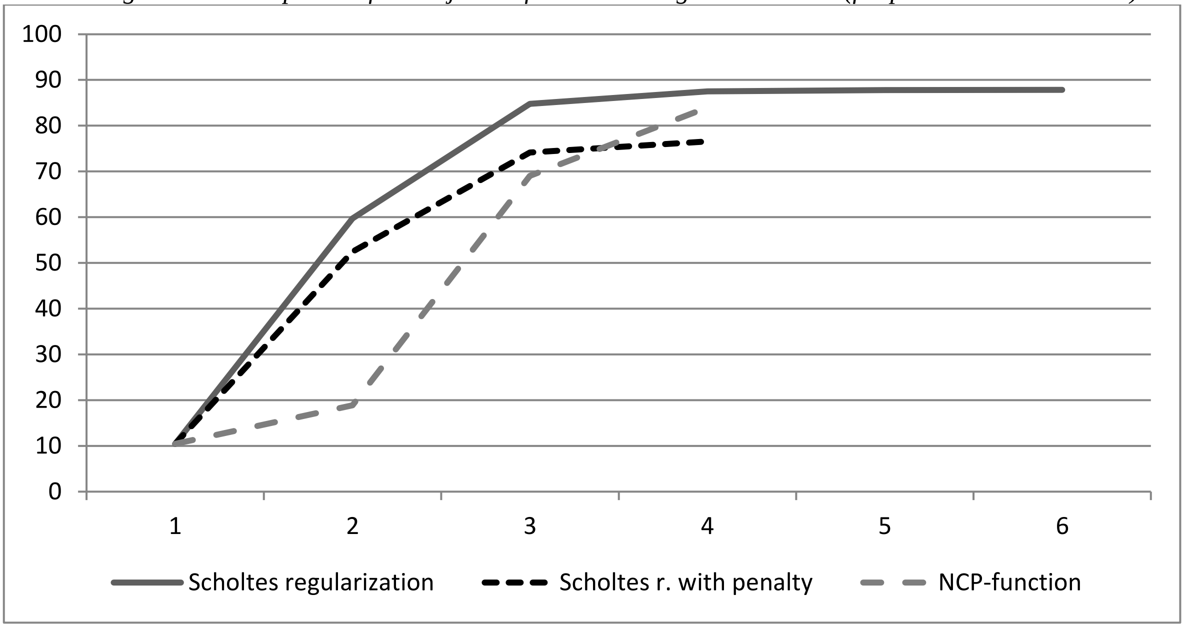
Figure 1 shows the development of the objective value (portfolio variance) during the iterations. We can observe that the objective value increases as the optimal solutions are closer to fulfill the cardinality constraint.

4. Conclusions and future research

In this paper, we have discussed several approaches how to avoid solving difficult portfolio optimization problems with cardinality constraints by the mixed-integer programming algorithms. We have mainly focused on the approximations of the solution sparsity using various regularizations. In particular, we have compared the Scholtes, penalized Scholtes and

NCP-function regularizations. Especially the Scholtes approach with the penalty term provided interesting results and deserves further elaboration. The convergence of the methods based on the NCP- functions and penalty terms is left as a topic for future research.

Figure 1: Development of the objective function during the iterations (for problem orl200-005-a)



Acknowledgement The research was supported by the Czech Science Foundation (GAČR) under the projects 18-04145S, 18-05631S.

References

[1] Adam, L., Branda, M. (2016a). Nonlinear chance constrained problems: optimality conditions, regularization and solvers. *Journal of Optimization Theory and Applications*, 170(2), p. 419–436.

[2] Adam, L., Branda, M. (2016b). Sparse optimization for inverse problems in atmospheric modelling. *Environmental Modelling & Software*, 79, p. 256–266.

[3] Artzner, P., Delbaen, F., Eber, J.-M. and Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9, p. 203–228.

[4] Bienstock, D. (1995). Computational study of a family of mixed-integer quadratic programming problems. *Mathematical Programming*, 74, p. 121–140.

[5] Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge University Press.

[6] Branda, M. and Kopa, M. (2014). On relations between DEA-risk models and stochastic dominance efficiency tests. *Central European Journal of Operations Research*, 22(1), p. 13–35.

[7] Branda, M. and Kopa, M. (2016). DEA models equivalent to general N-th order stochastic dominance efficiency tests. *Operations Research Letters*, 44(2), p. 285–289.

[8] Branda, M., Bucher, M., Červinka, M. and Schwartz, A. (2018). Convergence of a Scholtes-type regularization method for cardinality-constrained optimization problems with an application in sparse robust portfolio optimization. *Computational Optimization and Applications*, 70(2), p. 503–530.

[9] Bucher, M. and Schwartz, A. (2018). Second-order optimality conditions and improved convergence results for regularization methods for cardinality-constrained optimization problems. *Journal of Optimization Theory and Applications*, 178(2), p. 383–410.

[10] Burdakov, O.P., Kanzow, Ch. and Schwartz, A. (2016). Mathematical programs with cardinality constraints: Reformulation by complementarity-type conditions and a regularization method. *SIAM Journal on Optimization*, 26(1), p. 397–425.

[11] Červinka, M., Kanzow, Ch. and Schwartz, A. (2016). Constraint qualifications and optimality conditions for optimization problems with cardinality constraints. *Mathematical Programming*, 160(1), p. 353–377.

[12] Chen, J.-S., Huang, Z.-H. and She, C.-Y. (2011). A new class of penalized NCP-functions and its properties. *Computational Optimization and Applications*, 50(1), p. 49–73

[13] Fastrich, B., Paterlini, S. and Winkler, P. (2015). Constructing optimal sparse portfolios using regularization methods. *Computational Management Science*, 12(3), p. 417–434.

[14] Frangioni, A. and Gentile, C. (2007). SDP diagonalizations and perspective cuts for a class of nonseparable MIQP. *Operations Research Letters*, 35(2), p. 181–185,

[15] Gao, J.J. and Li, D. (2013). Optimal cardinality constrained portfolio selection. *Operations Research*, 61(3), p. 745–761.

[16] Hastie, T., Tibshirani, R. and Friedman, J. (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer Series in Statistics, 2nd edition.

[17] Lucidi, S. and Rinaldi, F. (2010). Exact penalty functions for nonlinear integer programming problems. *Journal of Optimization Theory Applications*, 145(3), p. 479–488.

[18] Mutunge, P. and Haugland, D. (2018). Minimizing the tracking error of cardinality constrained portfolios. *Computers and Operations Research*, 90, p. 33–41.

[19] Rinaldi, F. (2009). New results on the equivalence between zero-one programming and continuous concave programming. *Optimization Letters*, 3(3), p. 377–386.

[20] Rockafellar, R.T., Uryasev, S. and Zabarankin, M. (2006). Generalized deviations in risk analysis. *Finance and Stochastics*, 10, p. 51–74.

[21] Scholtes, S. (2001). Convergence properties of a regularization scheme for mathematical programs with complementarity constraints. *SIAM Journal on Optimization*, 11, p. 918–936.

[22] Sun, D. and Qi, L. (1999). On NCP-functions. *Computational Optimization and Applications*, 13, p. 201–220.

[23] Teng, Y., Yang, L., Yu, B. and Song, X. (2017). A penalty PALM method for sparse portfolio selection problems. *Optimization Methods and Software*, 32(1), p. 126–147.

[24] Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society*, *Series B*, 58(1), p. 267–288.

[25] Wächter, A. and Biegler, L. T. (2006). On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming. *Mathematical Programming*, 106(1), p. 25–57.

Influence of the macroeconomic indicators on the stock market

Jiří Branžovský[1]

Abstract

This paper is dedicated to the macroeconomic indicators affecting the US stock index S&P 500 in the after-crisis period since September 2008 to September 2017. Among the observed indicators there are included the currency-basket dollar index, a production manufacturing index and University of Michigan Consumer Sentiment index. The main goal was to analyse their influence on the US stock market on the monthly basis while using vector autoregressive model that would help to predict the stock index. However three time-lagged VAR model does not predict stock market evolution, but exactly the opposite. Endogenous variable of stock index is actually leading all other macroeconomic variables for 3 months. This conclusion actually supports the economic theory that stock markets make the step first and all the others follow in advance.

Key words

Multivariate vector autoregressive model, eViews, macroeconomic indicators, stock index, post-crisis period.

JEL Classification: C01, C32, E4, E5, G10

1 Introduction

This paper examines look up for the well-known macroeconomical indicators reported by universities, central banks and corporations. These are usually released on the monthly basis. Monthly observed period is gathered by financial data post-Financial Recession in 2008. There are specific discussions about structural changes in the economy itself occurring during past decades hence the main focus will be devoted to the recent decade. This period is definitely specific for its historically-unconventional monetary and fiscal policy decisions.

The main goal of this paper is to identify relevant influentional indicators affecting the stock index.

Vector autoregressive (VAR) stochastic model is being used in this paper for analysing, simulating and predicting linear interdependencies among several time series. Compared to the regression model, multivariate VAR model provides more information, for example impulse-response analysis, variance decomposition or causality between regressors. Observation is realised on the monthly basis in the after-crisis period since September 2017.

2 VAR model

Standard vector autoregressive (VAR) stochastic model is appropriate option for analysing, simulating and predicting linear interdependencies among several time series. Husek (1997) supports VAR model as it fits economical conditions these days when many variables use to be inter-related. Compared to the regression model, multivariate VAR model provides more

[1] Ing. Jiří Branžovský, Ekonomická fakulta VŠB-TU Ostrava, jiri.branzovsky@vsb.cz

information, for example impulse-response analysis, variance decomposition, inner-causality and possibly even cointegration while still being able to describe their behaviour in the past. Every variable plays a symmetric part not only as an exogenous, but also endogenous variable as well. VAR as a multi-equational model is due to possible cointegration among regressors considered mainly as the short-run model.

VAR models are well known and widely used for their relative easy use but still better analytical capability of prediction.

A *p*-order VAR model, described as a VAR(*p*) is a set of *k* variables and their p time-lagged values. The specific model for endogenous variable explained by its own time lags and one another variable could be as follows:

$$\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{A}\boldsymbol{y}_{t-p} + \boldsymbol{B}\boldsymbol{x}_{t-p} + \ldots + \boldsymbol{e}$$
$$\boldsymbol{x} = \boldsymbol{c} + \boldsymbol{A}\boldsymbol{y}_{t-p} + \boldsymbol{B}\boldsymbol{x}_{t-p} + \ldots + \boldsymbol{e} \qquad \forall\, t, p = 1, 2, \ldots, T$$

where $\boldsymbol{y}$ is a *k . 1* column vector of observed endogenous variables, $\boldsymbol{c}$ is a *k . 1* column vector of intercepts (constants), **A** is a *k . k* column vector of endogenous variables parameters and their own time lagged values, **B** is a *k . k* column vector of exogenous variables parameters and possibly their own time lagged values and $\boldsymbol{e}$ is a *k . 1* column vector of normal-distributed standard errors and *p* is the maximum time lag. The optimum maximum time of lags to be used may be found out by t-tests, F-test and mainly by information criteria.

Granger causality (1969) is the statistical predictive concept for stochastic linear measuring whether lagged value of a stationary variable X does/doesn´t improve an explanation of another stationary variable Y. It is not implication of that causality cause-result. It is in fact to see what proportion of Y is explained by its time lags and then to see whether adding lagged values of X might improve their relationship. If Y is Granger-caused by X, there are some information in variable X that help predict Y.

$$x_t = c_1 + \sum_{i=1}^{p} A_i \cdot x_{t-i} + \sum_{i=1}^{p} B_i \cdot y_{t-i} + e_{1t}$$
$$y_t = c_2 + \sum_{i=1}^{p} A_i \cdot x_{t-i} + \sum_{i=1}^{p} B_i \cdot y_{t-i} + e_{2t}$$

Impulse-Response analysis is being performed to verify the inner-behaviour among the variables in the VAR model during the period of time. Orthogonalised innovations of Impulse-response using Choleski´s decomposition of the residual covariance matrix are being used for dynamic analysis of particular unexpected unit shocks of the regressors with their effect on the endogenous variable error, their magnitude, responsiveness and continuous normalising then. That actually describes the reaction of the variable on the time horizon.

Variance decomposition works sort of as a dynamic coefficient of determination. That helps to explain proportion of the endogenous variable´ variance based on own shocks and cross-variables shocks. It is clear that own series shocks are the most important of the error variances.

Residual tests of normal distribution, homoscedasticity and no autocorrelation will be performed too. Potential interdependent long-run equilibrium relationships will not be processed in this paper as it is going to be analysed in another paper.

Time series stationarity is considered such a characteristic that during the observed period there is no volatility in mean value and no trend that represents autocorrelation. Stationarity assessment of the time series is to be identified by graphics method of a correlogram or by unit root tests. In this paper it is processed by Augmented Dickey-Fuller (ADF) test and by Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. Cipra (2008) mentions constant level, variance and covariance structure in time. Under the condition that correlation absolute value

$|\rho| \sim 1$ (meaning strong positive or negative correlation), time serie is being nonstationary (having a unit root). Stationarisation of time series in original level is usually processed by natural logarithmisation, by first or higher-order differentiation, or by both. Non-stationary time series can be subjects to the spurious regression. There is an advanced model called Vector Error Correction Model (VECM) that furthermore solves an issue of the model with cointegrated regressors.

3 Data

There have been several academic papers searching for specific indicators and analyzing their effect on the developed, but recently even emerging, stock markets. As stocks are considered as real economy assistants, these can be deeply overwatched by fiscal and monetary politicians that have common ultimate objectives. Stocks are generally sensitive to the published information (mainly financial ones) that determines the future evolution of the market. Fundamental information can be evaluated as micro or macroeconomical.

King (1966), Lintner (1973), Musílek (1997), Ross & Chen (1986) or Flannery & Protopapadakis (2001) concluded macroeconomical (global) factors as those explaining greater part of the dependent variable variance than segment or specific corporate information. Sirucek (2015) found out risk-free rate, CPI & PPI, monetary supply, GDP, manufacturing indices, unemployment rate, current account, FX rate or net capital flows as the influentional factors. And that is why top-down analytical approach is in the foreground of this paper. Mainly the monetary policy´ tools are being usually mentioned by economists as the most important ones, eventhough they are not of the same opinion in their research for indicators whether interest rates or monetary aggregates matter more. On the other hand, Hussainey and Le Khanh Ngoc (2009) identified real economical indicators only affecting stock markets and not the interest rates. This surprising conclusion was also found out in this paper that effective fed fund rate did not influence the stock index. Oppositely, monetary supply M2 was positively tested in common with Michigan consumer sentiment, GDP and unemployment rate by Maskay (2007).

3.1 Time series

Our empirical research was processed on the montly basis from Sept 2008 to Sept 2017. That means 109 observations over the US market, gathered at St. Louis Fed portal and Yahoo agency. Time series are collected at the end of each month and were decomposed by seasonal adjustment by multiplicative Henderson Census X12 method. All financial series reported their trend meaning those were non-stationary with a unit root hence first differentiation of naturally logarithms (DL_X) was used to obtain the stationarity in the first order I(1). ADF and KPSS tests were performed for identifying this issue at 10% significance level.

The main and critical endogenous variable is world-widely known and used US stock market index S&P 500 (DL_SPX) of 500 market capitalised largest corporations publically traded in the USA.

Among the observed explaining seasonally adjusted regressors in their first-order differencies (monthly returns) of logarithms there have been spot effective fed fund rate DL_EFFR, spot dollar index DL_DI_SA, one month ex-post reported manufacturing index DL_MI_SA and one month ex-post reported University of Michigan Consumer Sentiment

DL_UMCS_SA. Out of them EFFR, DI and UR ought to have the inverse relationship to the stocks. VAR model is then suitable as all regressors are stationary at I(1).

3.2 Results

The lowest information criteria is showed by Akaike criteria that points out three time lags to be involved into the most sufficient model VAR(3). The model was made originally from four upper-mentioned variables, but in the end effective fed fund rate DL_EFFR was dismissed.

$$\mathbf{DL\_SPX}_{t-3} = a_1 + \sum_{i=1}^{3} B_i \,.\, \mathrm{DL\_SPX}_{t-3-i} + \sum_{i=1}^{3} C_i \,.\, \mathrm{DL\_UMCS\_SA}_{t-1-i} + \sum_{i=1}^{3} D_i \,.\, \mathrm{DL\_DI\_SA}_{t-i} + \sum_{i=1}^{3} E_i \,.\, \mathrm{DL\_MI\_SA}_{t-i} + u_{t-3}$$

$$\mathrm{DL\_UMCS\_SA}_{t-1} = a_2 + \sum_{i=1}^{3} B_i \,.\, \mathrm{DL\_UMCS\_SA}_{t-1-i} + \sum_{i=1}^{3} C_i \,.\, \mathrm{DL\_SPX}_{t-3-i} + \sum_{i=1}^{3} D_i \,.\, \mathrm{DL\_DI\_SA}_{t-i} + \sum_{i=1}^{3} E_i \,.\, \mathrm{DL\_MI\_SA}_{t-i} + u_{t-1}$$

$$\mathrm{DL\_DI\_SA}_{t} = a_3 + \sum_{i=1}^{3} B_i \,.\, \mathrm{DL\_DI\_SA}_{t-i} + \sum_{i=1}^{3} C_i \,.\, \mathrm{DL\_SPX}_{t-3-i} + \sum_{i=1}^{3} D_i \,.\, \mathrm{DL\_UMCS\_SA}_{t-1-i} + \sum_{i=1}^{3} E_i \,.\, \mathrm{DL\_MI\_SA}_{t-i} + u_{t}$$

$$\mathrm{DL\_MI\_SA}_{t} = a_4 + \sum_{i=1}^{3} B_i \,.\, \mathrm{DL\_MI\_SA}_{t-i} + \sum_{i=1}^{3} C_i \,.\, \mathrm{DL\_DI\_SA}_{t-i} + \sum_{i=1}^{3} D_i \,.\, \mathrm{DL\_SPX}_{t-3-i} + \sum_{i=1}^{3} E_i \,.\, \mathrm{DL\_UMCS\_SA}_{t-1-i} + u_{t}$$

There was a request in looking for the predictive model that would help to prognose the stock index, however three time-lagged VAR model does not predict stock market evolution so much. Endogenous variable of stock index is slightly leading all other macroeconomic variables for 3 months. This conclusion actually supports the economic theory that stock markets make the step first and all the others come in advance.

Parameters estimation of the regressors were processed by OLS method. Particular parameters were rounded to two decimals. Intercept was remained in the VAR model with its ** statistical significance however its existence depends on the economical interpretation.

$$\begin{aligned}\mathrm{DL\_SPX}_{t-3} = {} & 0{,}01^{**} - 0{,}25 \,.\, \mathrm{DL\_SPX}_{t-4}{}^{***} - 0{,}17 \,.\, \mathrm{DL\_SPX}_{t-5}{}^{**} + 0{,}06 \,.\, \mathrm{DL\_SPX}_{t-6} \\ & + 0{,}18 \,.\, \mathrm{DL\_UMCS\_SA}_{t-2}{}^{**} + 0{,}28 \,.\, \mathrm{DL\_UMCS\_SA}_{t-3}{}^{***} + 0{,}17 \,.\, \mathrm{DL\_UMCS\_SA}_{t-4}{}^{**} \\ & + 0{,}50 \,.\, \mathrm{DL\_DI\_SA}_{t-1}{}^{*} - 0{,}60 \,.\, \mathrm{DL\_DI\_SA}_{t-2}{}^{*} - 1{,}35 \,.\, \mathrm{DL\_DI\_SA}_{t-3}{}^{***} \\ & + 0{,}58 \,.\, \mathrm{DL\_MI\_SA}_{t-1} + 1{,}28 \,.\, \mathrm{DL\_MI\_SA}_{t-2}{}^{**} + 0{,}86 \,.\, \mathrm{DL\_MI\_SA}_{t-3}\end{aligned}$$

This VAR(3) model helps to explain 46,8 % of dependent variable variance by adjusted coefficient of determination R^2. Results of the VAR(3) model do not actually predict stock market DL_SPX evolution by other macroeconomic regressors. There is statistical significant dependence of stock index on its own 1-2 months-lagged values. Consumer sentiment

DL_UMCS_SA and manufacturing index DL_MI_SA are originally reported with one month delay. Consumer sentiment plays statistical significant role on the stock market already two months earlier, one month and even in the spot time from the stationarity state and that makes this variable itself actually predicting approximately 15 % of the stock index variability. Dollar index DL_DI_SA is the most important negative indicator at the spot time with its release that explains proportion of stocks variability but does not really help to predict it. Commonly, manufacturing index DL_MI_SA plays the statistically significant role only immediately in the present time.

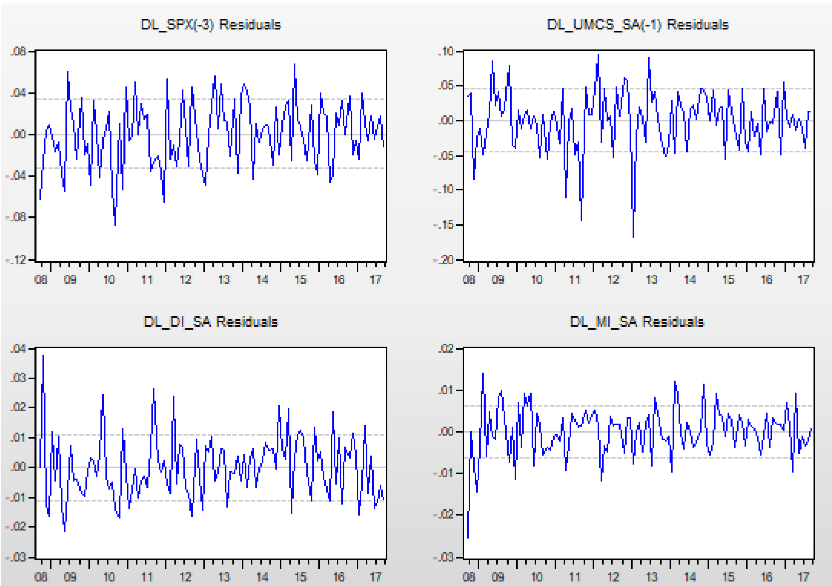
In terms of **Granger causality** it may be seen all three exogenous regressors do Granger cause DL_SPX(-3) because the null hypothesis of no causality was rejected and rather alternative hypothesis was accepted. On the other hand, stock index does G-cause the dollar index. Manufacturing index is the variable that G-causes and is G-caused by all other variables as well.

Picture 1: Granger pair-causality among regressors [eViews]

Null Hypothesis:	Obs	F-Statistic	Prob.
DL_UMCS_SA(-1) does not Granger Cause DL_SPX(-3)	111	5.85036	0.0010
DL_SPX(-3) does not Granger Cause DL_UMCS_SA(-1)		1.80000	0.1518
DL_DI_SA does not Granger Cause DL_SPX(-3)	110	14.5390	6.E-08
DL_SPX(-3) does not Granger Cause DL_DI_SA		2.84017	0.0416
DL_MI_SA does not Granger Cause DL_SPX(-3)	109	6.00639	0.0008
DL_SPX(-3) does not Granger Cause DL_MI_SA		0.39840	0.7544
DL_DI_SA does not Granger Cause DL_UMCS_SA(-1)	110	1.07700	0.3623
DL_UMCS_SA(-1) does not Granger Cause DL_DI_SA		1.31368	0.2740
DL_MI_SA does not Granger Cause DL_UMCS_SA(-1)	109	2.31926	0.0798
DL_UMCS_SA(-1) does not Granger Cause DL_MI_SA		3.89072	0.0112
DL_MI_SA does not Granger Cause DL_DI_SA	109	3.00406	0.0339
DL_DI_SA does not Granger Cause DL_MI_SA		6.13850	0.0007

Regarding to **residual econometric verification**, there is no residual autocorrelation identified in the model by Autocorrelation LM test. P-value for 3 time lags is 47,8 % which forbids to deny the null hypothesis of no autocorrelation at 10% significance level.

Picture 2: Regressors´ residuals [eViews]

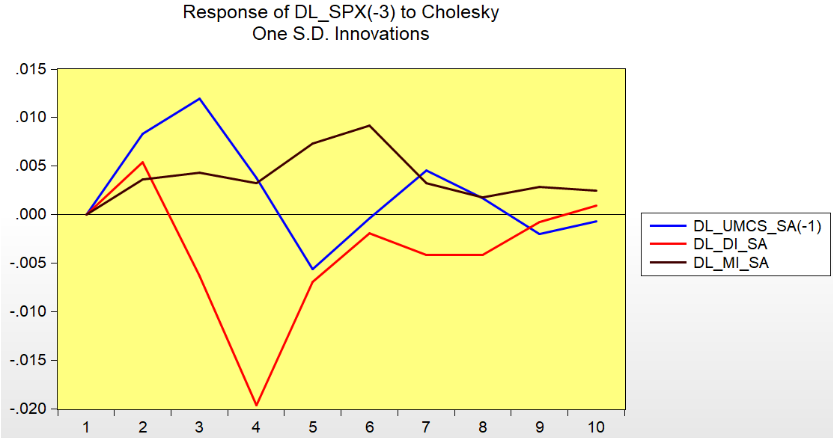


P-value of White test of residual heteroscedasticity with cross terms is being 9,96 % that is slightly lower than 10% significance level (denying of null hypothesis representing residual heteroscedasticity) but higher than 5% significance level representing homoscedastic model.

VAR model residuals do not originally come from the normal distribution, they are skewed and kurtosed as well.

In order to oversee the sensitivity of the main endogenous variable at ten-months period there was also **Impulse response** function with usage of Cholesky´s decomposition method (as see below) being applied to the VAR model. Regarding to the estimated model, the interpretation itself is a rather difficult task. That is because a dependent variable three months ago is being explained by other variables today which little bit points out the causality problem among the regressors.

Picture 3: Response of the regressors to the stock index [eViews]



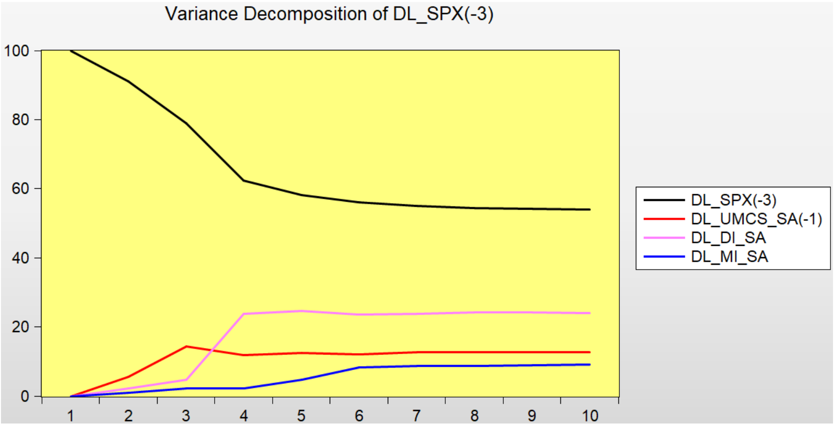
As visible, an one standard deviation shock (innovation) of *dollar index* today immediately increases past DL_SPX(-3) in second period, and then rapidly falling into the negative values until reaching the down in the period four and then arising the dependent variable again while still in the negative sector until tenth period. The more simple interpretation would be that stock markets react negatively already one and two months earlier obviously based on expectations while correct positively today. That means any currency intervention of monetary authority today has been already previously included in the stock market and that strong negative effect recently opposited and within a month whole negative shock will be mostly mitigated.

One standard deviation shock today of the one-month ex-post reported DL_MI_SA continuously positively affects the stock index. More correct economic interpretation is that stocks already predicted and implemented greater proportion of *manufacturing index* three months sooner and react then positively immediately in the present time and following one month as well.

Consumer sentiment index is also reported a month ex-post that makes the period difference between this variable and stock index only two months. As seen above, an one standard deviation of DL_UMCS_SA(-1) has been already predicted by the stock index a month earlier and is also positively impacted immediately in the present. During two following months stocks are correcting previous trend.

In terms of Cholesky´s **Variance decomposition** there may be seen that dollar index affects the greatest proportion of stock index´s volatility. In its fourth period the dollar index explains almost 24 % of dependent´s error variance. Variation of consumer sentiment is accounted for another 14% part in the third time period while manufacturing index takes another approximately 8 %.

Picture 4: Variance decomposition of the stock index [eViews]



4 Conclusion

This research paper was done through application of vector autoregressive model. Research study was processed on the monthly financial time series since Sept 2008 to Sept 2017. VAR model included three regressors with their three time lags: dollar index, manufacturing index and University of Michigan consumer sentiment index that explained 47 % of total S&P 500 stock price index volatility in the after-crisis nearly decade. VAR model was not enriched by effective fed fund rate after all. The methodological part of study was devoted to the econometric techniques used at financial time series analysis. Next to the ordinary residual verification tests there were impulse-response analysis, variance decomposition and Granger causality applied to the VAR model. Long-term potential cointegration of the variables was not tested in this paper as that may be separately tested in another study.

The relationship among several macroeconomic indicators and stock market in the USA was addressed but was not so predictive as originally thought. VAR model itself was predictive based on endogenous time-lagged values and sentiment index only. But impulse-response analysis pointed out that our endogenous stock market index is actually leading all other variables by two to three months in advance.

References

[1] CIPRA, T.. *Financni ekonometrie*. Praha: Ekopress (2008). ISBN 978-80-86929-43-9.

[2] Critical Values for the Durbin-Watson Test: 10% Significance Level. *Stanford* [online]. Stanford, USA: Stanford University [cit. 2018-08-02]. Available: https://web.stanford.edu/~clint/bench/dw05a.htm

[3] EViews*: VAR model* [online]. [cit. 2018-08-02]. Available: www.eviews.com

[4] Finance Yahoo*: Historical Data* [online]. New York, USA [cit. 2017-12-31]. Available: www.finance.yahoo.com

[5] FLANNERY, M., PROTOPAPADAKIS, A. (2001). *Macroeconomic factors do influence aggregate stock returns.* Oxford university press: The review of financial study.

[6] FRED Economic Data*: Economic Research* [online]. St. Louis, USA: Federal Reserve Bank St. Louis [cit. 2017-12-31]. Available: https://fred.stlouisfed.org/categories

[7] HANCLOVA, J. (2012) *Ekonometricke modelovani: klasicke pristupy s aplikacemi*. 1. vyd. Praha: Professional Publishing. ISBN 978-80-7431-088-1.

[8] HUSEK, R. (1997). *Zaklady ekonometricke analyzy*. 1.vyd., dotisk. Praha: VSE. ISBN 80-707-9102-0.

[9] CHEN, N., ROLL, R. a ROSS, S. (1986). *Economic Forces and the Stock Market.* The Journal of Business, 59(3).

[10] KING, B. (1966). *Market and industry factors in stock price behaviour.* The journal of Business, University of Chicago Press. January 1966.

[11] KULHÁNEK, L. (2012). The relationship between stock markets and gross domestic product in the Central and Eastern Europe.

[12] LINTNER, J. (1973). *Inflation and common stock prices in a cyclical context.* National Bureau of Economic Research Annual Report.

[13] MASKAY, B. (2007) Analyzing the Relationship between change in Money Supply and Stock Market Prices. Honors Projects. Paper 35.

[14] MUSÍLEK, P. (1997). Změny makroekonomických veličin a akciové kurzy. Finance a úvěr, 47.

[15] MONTGOMERY, D. C., PECK E. A. a VINING G. G. (2012). *Introduction to linear regression analysis*. 5th Edition, New York, John Wiley & Sons. ISBN 978-0-470-54281-1.

[16] ŠIRŮČEK, Martin. *Kauzální vztah peněžní nabídky a amerického akciového trhu*. Brno: Mendelova univerzita v Brně, 2015. ISBN 978-80-7509-304-2.

The Beginning of Sustainable Financial System in Europe

Ondřej Buben, Jiřina Bokšová[1]

Abstract

The signature of the Paris Agreement on 12th December, 2015 is one of the most discussed events regarding global economy and environment in recent years, mostly due to United States withdrawal from the Paris Agreement announced by the president Donald J. Trump. The public sees focus of the agreement only as moving towards a low-carbon society, where renewable energy increase our quality of life. The truth is that the impact can be significantly greater affecting all segments, including financial reporting and regulatory framework of financial institutions.
The aim of this paper is to describe reasons that led to signing the Paris agreement and steps that were taken by European Commission since then towards the Sustainable Finance, focusing on High Level Expert Group and its work.

Key words

Sustainable Finance, HLEG, environment, Paris Agreement, Kyoto Protocol, Greenhouse gasses emissions

JEL Classification: Q54, Q56, O1, O16,

1. Introduction

The signature of the Paris Agreement on 12th December 2015 is one of the most discussed events regarding global economy and environment in last couple of years. The main reason why it got into the perception of the public is United States withdrawal from the Paris Agreement announced by the president Donald J. Trump on 1st June, 2017. The Paris Agreement was initially signed by Trump's predecessor Barack Obama. (Trump, 2017)

Due to US withdrawal, the Paris Agreement became a generally known topic, but the public sees focus of the agreement only as moving towards a low-carbon society, where renewable energy increase our quality of life. The truth is that the impact can be significantly greater when taking into account other aspects such as United Nations the 2030 Agenda for Sustainable Development, which overlaps with the Paris agreement and the decision-making about future strategy should be always based on both those agreements rather than each one separately.

The aim of this paper is to describe reasons that led to signing the Paris agreement and steps that were taken by European Commission since then, focusing on High Level Expert Group and its work.

The introduction is followed by describing the path that led to the Paris agreement. Second half of the paper is dedicated to High Level Expert Group that was formed by European Commission to come up with the strategy towards the Sustainable Financial System based on Paris Agreement and the 2030 Agenda for Sustainable Development. It contains the

[1] Ondřej Buben, University of Economics in Prague, Buben.Ondrej@gmail.com.
doc. Jiřina Bokšová, Ph.D., University of Economics in Prague, boksova@vse.cz.

recommendations as to how to accomplish the Sustainable financial system. The paper is closed by conclusion and discussion over its possible future impacts including financial reporting and regulatory framework of financial institutions.

2. The path to the Paris Agreement

2.1 Kyoto Protocol – First Commitment period

The Paris Agreement is not the first attempt towards a low-carbon society by United Nations, but it is definitely the most discussed. On 11th December 1997 The Kyoto Protocol was adopted in Kyoto, Japan and entered into force on 16th February 2005. It is an international agreement under the United Nations Framework Convention on Climate Change, which commits its Parties by setting internationally binding emission reduction targets. The Kyoto Protocol was seen as an important first step towards a truly global emission reduction regime that will stabilize greenhouse gasses emissions. (United Nations, 2018b)

The Kyoto Protocol was signed and ratified by 127 countries with exception of the United States. The countries had agreed to reduce their greenhouse gas emissions by an average of 5.2 per cent by 2012 based on 1990 levels as first commitment period. According to Norwegian researchers, U.S. president Bill Clinton could not sign the agreement due to U.S. Congress adoption of a resolution stating that the U.S. should not be signatory to any agreement that would mandate new commitments to limit or reduce greenhouse gas emissions, because it would result in serious harm to the U.S. economy. Some could say that it was not coincidence that the resolution was made just couple of months before the UN climate change conference in Kyoto. (Falkner, 2016).

The European Union Emissions Trading Scheme (EU ETS) was launched 13 years ago in the Czech Republic and other European countries as result to comply with the Kyoto Protocol. EU ETS is based on "cap and trade" concept that is designed to encourage participants in reduction of emission level. For each period, only a certain amount of allowances are issued by the government, therefore a certain, maximum allowable greenhouse gas emissions. It is typical to issue the emission allowances free of charge to the participants. Based on yearly emissions participants can buy or sell the emission allowances on the active market, but the total number of allowances in circulation is non-volitile. Every year, lower number of emission allowances is distributed, therefore participants are forced to decrease their emissions or to pay higher price in order to get the allowances. The mechanism help to stimulate green investments and help Parties meet their emission targets in a cost-effective way.

The important question is whether the first commitment period was success. The opinions differ depending whether it focuses only to the countries that participated in the first period, or worldwide impact. The European Union and its member countries (15 countries at that time) set the target to an 8% cut for the block as a whole (which is 3 p.p. more than initially agreed) for the first commitment period. In the end of this period, the European Union achieved an overall cut of 11.7% domestically. [9] Based on this statement, everyone would say, it is success. On the other hand, according to World Bank report, global emissions of carbon dioxide have increased by 19 per cent from 1990 to 2007, which is completely different message. What is behind those two contradictory claims? While the Kyoto Protocol for first period was ratified by many countries, United States, Russia and China were not part of it. Those countries also emit the largest share of emissions in the World. Therefore it is not possible to reduce the world pollution if those big players are not contributing to those targets. (Buanawaty and Hastiadi, 2017)

There are also some claims challenging the success of parties that were part of this period, claiming that substantial part of decrease in emissions was due to drop in economic activity in response to the economic crisis, or there was an emission shifting to developing countries such as China that were not part of the Kyoto Protocol. (Shishlov, Morel and Bellassen, 2016). Of course these claims are very hard to measure and therefore in research conducted by Grunewald and Martinez-Zarzoso on emissions of CO2, countries that ratified under the Kyoto Protocol emit in average about 7 per cent less CO2 than similar countries that did not ratify the Protocol. (Grunewald and Martinez-Zarzoso, 2015)

2.2 Kyoto Protocol – Doha Amendment

The second commitment period from 2013 to 2020 was discussed on 8th December 2012 in Doha, Qatar and the parties had committed to reduce the greenhouse gases by at least 18 per cent below 1990 levels in this eight-year period, where the European Union set its target to 20 per cent. However the composition of countries is not the same as for the first commitment period (e.g. Canada did not ratify the second period).

Apart from the set target and different composition of countries, the second commitment period was extended by one more greenhouse gas (nitrogen trifluoride). Also, there are new rules as to how developed countries are to account for emissions from land use & forestry.

As the first commitment period was fulfilled, there are some companies in these countries that transferred some of allocated emission allowances to the second period as a hedge against higher targets. This behavior could endanger the target for the second committed period, therefore there is a limit on how much could be carried over from first period and they cannot be sold to some potential buyers as Australia, Japan, Monaco, Switzerland, Norway and Liechstein. (European Commission, 2).

2.3 Paris Agreement

Nearly twenty years from signing the Kyoto Protocol that targets on reduction of greenhouse gas emissions, on 12th December 2015, 195 countries come to an agreement on new climate treaty and the Paris Agreement was signed. As the Kyoto Protocol target ends in 2020, the Paris agreement is the effective successor treaty. As to February 2018, 195 countries have already ratified with the Paris Agreement, but the United States headed by the President Donald Trump, are not one of them, although the pressure on the United States is strong.

The central aim of the Paris agreement is to keep the global temperature rise this century well bellow two degrees Celsius. Also the agreement aims to strengthen the ability of countries to deal with climate change and its impacts. It is necessary to plan appropriate financial flows, new technology framework and comprehensive strategy to reach desired goals.

The Paris Agreement addresses the following crucial areas necessary to combat climate changes: Long-term temperature goal, Global peaking and climate neutrality, Mitigation, Sinks and reservoirs, Voluntary cooperation, Adaptation, Loss and damage, Finance, technology and capacity-building support, Climate change education, training, public awareness, public participation and public access to information, Transparency, implementation and compliance, Global Stocktake, Decision 1/CP.21. All of those topics are described on United Nations Climate Change website. (United Nations, 2018)

The goal is definitely not negligible and targets larger economic area than according to general public perception. Paris Agreement is mostly known for aiming on reduction of greenhouse gasses emission, but this is completely different level. Is it even possible to meet those targets, or is the Paris Agreement destined to fail without participation of United States and other big polluters?

3. High Level Expert Group

Based on all the topics in the Paris Agreement, it is not possible to address each one of them separately and some comprehensive framework is necessary. European Commission took this role upon itself and in December 2016 established a High-Level Expert Group (HLEG) on sustainable finance. The HLEG's objective was to come up with a comprehensive EU strategy on sustainable finance to integrate sustainability into EU financial policy by the end of 2017. The HLEG was formed by experts with diverse profiles representing different approaches to this complex topic. According to HLEG, to reach the Paris agreement goals, it requires no less than a complete transformation of the entire financial system, its culture and its incentives.

This part of the paper focuses mainly on HLEG's key recommendation disclose in Final Report from January 2018 that contains their recommendations on how to establish and maintain a sustainable financial system in Europe. HLEG also encourages other countries outside Europe to draw on their findings. Apart from key recommendations, HLEG also provided other cross-cutting recommendations that targets financial institutions and other sectors (banking, insurance companies, asset management, pension funds, credit ratings and sustainability ratings, stock exchanges and financial centers, investment consultants and investment banks).

The High-level Expert Group (HLEG) mandate was ended with publication of the final report which is briefly introduced in this section. The HLEG may gather only informally to take stock of progress on sustainable finance.

The expression "sustainable finance" is used multiple times and is almost in every article in the Final report. What is the meaning of this phrase? According to the HLEG, it means creating economic prosperity that will last for a long time while taking into account the social aspects of the society, environmental issues and less dependence on non-renewable resources.

3.1 Key recommendations

Establishing and maintain a common sustainability taxanomy at the EU level. Requires creating a classification system which identifies activities, assets and revenue segments that contributes to the EU's sustainability goals based on conditions or criteria set out by the taxonomy. Therefore if the classification system for sustainable activities is accurate, understandable and shared in EU, it enables market participants to invest in sustainability with certainty. In the future, the best case scenario would be to have a common sustainability taxanomy at the World level. The HLEG developed a framework for a full sustainability taxanomy and proposed screening criteria for assets and projects to be listed in the taxanomy for climate change mitigation goals.

Clarify investor duties to better embrace long-term horizon and sustainability preferences. It is important to make investors responsible when making investment decisions or engaging with investees in their portfolios. Investor duties are more or less codified into key EU financial services directives, such as MiFID II and Solvency II, but none of these directives include requirements for Sustainability or Environmental, Social and Governance factors, therefore no standard requiring disclosing information about the way these issues were considered during investment process. The HLEG recommends clarifying the duties of institutional investors as well as their asset managers.

Upgrade disclosure rules to make sustainability risks fully transparent, starting with climate change. Risks reported in financial disclosures are too short-term, making it nearly impossible to define the full scale of sustainability risks when most analyses do not exceed three-year horizon. Another problem with disclosures according to HLEG, is relying on qualitative data rather than quantitative elements, which prevents measurement over time and across sectors. Therefore comparison between two years is not possible if measurement should be made retrospectively.

Key elements of a retail strategy on sustainable finance: investment advice, ecolabel and SRI minimum standards. According to survey in twenty two countries made by Natixis Global Asset Management in 2017, there is around 70% of retail investors that consider social and environmental issues as an important factor when making decision about their investments. However most retail investors do not have the opportunity to invest according to these preferences and investment advisors have just few possibilities to even respond to these preferences.

Develop and implement official European sustainability standards and labels, starting with green bonds. To insure increase in investments in green projects and activities, it is recommended by HLEG to establish official EU sustainability standards and official EU Green Bond Standard. According to standards in combination with sustainable taxanomy, there is an expectation of increased demand from green investors if the assets are ensured as high priority assets in a low-carbon and climate-resilient economy.

Establish "Sustainable Infrastructure Europe". Much progress has been made in Europe regarding infrastructure investments, but several member states remains a concern. Therefore the infrastructure varies throughout the Europe. To ensure the sustainable infrastructure Europe it is necessary to reduce the differences, which can be done only by organization designed to support development of sustainable infrastructure projects across all member states.

Governance and Leadership. The HLEG proposes that a clear commitment into the sustainability is one of the required duties of company directors and part of the governance rules related to company management and expects that clients' sustainability preferences will be taken into account. The HLEG questions whether financial sector members are able to anticipate longer-term risk and sustainability challenges.

Include sustainability in the supervisory mandate of the ESAs and extend the horizon of risk monitoring. The HLEG suggest that the European Supervisory Agencies (ESAs) monitors financial stability and orderly functioning of financial markets, should include sustainability in their mandate and extend the time horizon of risk monitoring to climate-related risks and other long-term, non-linear, non-cyclical risks. The Group pointed out that the mismatch of investment horizons between issuers of securities that are usually long-term, intermediaries that focus on short-term revenues, and beneficiaries that are mostly long-term horizon, leads to a short-term focus on risk metrics, which may cause the mispricing of actual risk, opportunities and lower degree of diversity in investments as well. (European Commission, 2018c)

4. Conclusion and Discussion

The Europe Union and its members stand on the threshold of new Sustainable Financial System and we might expect minor or major adjustments in regulatory frameworks of individual segments that can seem insignificant or even unnecessary at the time, but can have a significant impact in a big picture as a whole.

The path to this moment started about twenty years ago in Japan with Kyoto Protocol which was the first important step towards a truly global emission reduction regime that should have stabilized greenhouse gasses emissions in the World. Due to absence of significant polluters such as United States, China and Russia in the agreement, the amount of greenhouse gasses emissions is still growing and the required limit was not met. On the other hand the participants of the first amendment of Kyoto Protocol fulfilled the established limit and in the EU even exceeded expectations. That should count as a victory and signal to the big polluters, that it is possible to go towards a low-carbon Earth. But there are suggestions that the success was caused by economic recession and migration of production to countries that was not part of the first amendment. Therefore it is nearly impossible to calculate whether the first amendment was a success or not, but it definitely started put pressure on companies to take the environmental impact in certain countries into the account.

As the Kyoto Protocol target ends in 2020, the Paris agreement is the effective successor treaty. The central aim of the Paris agreement is to keep the global temperature rise this century well bellow two degrees Celsius. Also the agreement aims to strengthen the ability of countries to deal with climate change and its impacts. Paris Agreement with the 2030 Agenda for Sustainable Development together creates space for discussion regarding sustainable finance and low-carbon and environmental friendly society.

In December 2016, European Commission founded the High-Level Expert Group (HLEG) that should come up with comprehensive strategy to achieve goals of those agreements by the end of 2017. The HLEG issued their final report in January 2018 where all their work and recommendation for the EU how to proceed towards the more sustainable financial system. The recommendation if followed will influence all segments and regulation changes are inevitable. Not just all regulations, but the recent once as well, e.g. Solvency II works with investor duties and risks, but does not include a long-term view (more than five years) on sustainability and most of all, does not take into account Environmental, Social and Governance factor.

There are numerous questions arising from this activity. If all the recommendation will apply in the EU, what will be the impact on the World economy and environment, if the big polluters will not take part in these actions? Is it possible that the regulation would cause the transfer of production to other countries where are no restrictions and instead of creating new jobs, cancel them?

There is a huge pressure on financial institutions and their investment management that should be more Environmental, Social and most of all long-term driven. In combination with requests to change financial disclosures, portfolio management and more, will be the added value higher than all the costs related to regulation changes, system changes and other negative externality caused by this upcoming changes?

If the EU wants to set an example to other countries of how to create a sustainable financial system, but by doing so, creating a restriction through new regulations and requirements in the Europe, is there also a possibility that it will lead companies in other countries to re-evaluate their business intentions with European partners and maybe find themselves a new ones somewhere else, where such restrictions do not apply.

References

[1] Amundsen, B. and Lie, E. (2010). *Why the Kyoto agreement failed*. The research Council of Norway. Retrieved from: https://www.forskningsradet.no/en/Newsarticle/Why_the_Kyoto_agreement_failed/1253963392536

[2] Buanawaty, P. and Hastiadi, F . (2017). *The Impact of Kyoto Protocol on Environment Quality in the Free Trade Era: The Case of G20 Countries*. International Journal of Economics and Financial Issues. 7 (3). p. 36-42. Retrieved from: http://dergipark.gov.tr/ijefi/issue/32021/354189

[3] European Commission. 2018a. First Commitment period (2008-12). Retrieved from: https://ec.europa.eu/clima/policies/strategies/progress/kyoto_1_en

[4] European Commission. 2018b. Kyoto Second commitment period (2013-20). Retrieved from: https://ec.europa.eu/clima/policies/strategies/progress/kyoto_2_en

[5] European Commission 2018c. Financing a *Sustainable European Economy*. Final report 2018. Retrieved from https://ec.europa.eu/info/business-economy-euro/banking-and-finance/sustainable-finance_en

[6] Falkner, R. (2016). *The Paris Agreement and the new logic of international climate politics, International Affairs,* Volume 92, Issue 5, p. 1107–1125.

[7] Grunewald, N. and Martinez-Zarzoso, I. (2015). *Did the Kyoto Protocol Fail? An evaluation of the effect of the Kyoto Protocol on* CO_2 *emissions*. Environmental and Development Economics. p. 1-22

[8] Shishlov, I., Morel, R. and Bellassen,V. (2016). *Compliance of the Parties to the Kyoto Protocol in the first commitment period*. Climate Policy, 16:6, p. 768-782.

[9] Trump, D. J. (2017). *Statement by President Trump on the Paris Climate Accord*. Retrieved from: https://www.whitehouse.gov/briefings-statements/statement-president-trump-paris-climate-accord/

[10] United Nations (2018a). What is the Paris Agreement. Retrieved from: https://unfccc.int/process-and-meetings/the-paris-agreement/what-is-the-paris-agreement

[11] United Nations (2018b). Kyoto Protocol Introduction. Retrieved from: https://unfccc.int/process/the-kyoto-protocol

Stability problems of limit cycles in the keynesian business cycle models

Dominika Byrska[1], Aleksandra Borowska[2]

Abstract

For theoretical study of irregularities appearing in an economy we use the theory of dynamical systems. For this type of systems the causes of cyclic behaviour have endogenous character. In the paper we present the Kaldor business cycle model, in which cyclic solution is caused by nonlinearity of the investment function. There is a limit cycle solution as a result of a Hopf bifurcation. The main result is calculation of a stability index.

Key words

business cycle - Hopf bifurcation - Kaldor model - limit cycle – stability analysis

JEL Classification: E32, C61, C63

1 Introduction

Initially, linear deterministic systems were used in economic analyzes, which translated into the existence of a single critical point, which is a stable point. From the analysis of deterministic linear systems, occurred also the existence of a boundary cycle, however, the probability of its occurrence was small. Linear systems did not reflect the character of economic phenomena, because they prevented the formation of fluctuations with irregular amplitudes, which is typical for data describing existing economies.

In the case of a dynamic economic model, the occurrence of cyclical solutions informs that in the long term macroeconomic variables are subject to fluctuations. This translates into fluctuations around the long-term trend - growth cycles. The occurrence of cyclical behavior due to The aim of the work is to present the impact of model parameters on the properties of economic processes described by means of a non-linear dynamic system. The goal defined in this way determined the structure and content of the work. the existence of bifurcation, a mechanism in which a small change in the model parameter causes a significant change in the model's property.

The aim of the work is to present the impact of model parameters on the properties of economic processes described by means of a non-linear dynamic system. The goal defined in this way determined the structure and content of the work. The scope of the research method consisted of literature studies and mathematical analysis of the proposed model indicating a wide range of potential borderline behaviors of the economic system. In the theoretical work, the issue of empirical verification of the obtained results was omitted.

At the beginning, the basic conventions and designations used to analyze the stability of limit cycles in the case of non-linear dynamic systems will be presented. The basic definitions used in this work were reviewed. The role and significance of the occurrence of border cycles from the empirical point of view are discussed. This part presents the application of the theory

[1] Mgr Dominika Byrska, Institute of Economics, Finance and Management, Jagiellonian University, Łojasiewicza 4, 30-348 Kraków, e-mail: byrska.dominika@gmail.com
[2] Mgr Aleksandra Borowska, Institute of Economics, Finance and Management, Jagiellonian University, Łojasiewicza 4, 30-348 Kraków, e-mail: ola.borowska91@gmail.com

of bifurcation of economic models and their impact on the emergence of areas characterized by different volatility in the dynamic system.

Then an analysis of the non-linear model of the business cycle proposed by Kaldor was carried out. A modification of the model has been introduced consisting in the use of a generating function. Then it was shown that the source of cyclicality are endogenous factors, and the variability in the system is caused by the appearance of bifurcation. Later, the stability of the limit cycle appearing in the Kaldor model was examined. The stability index was calculated and the condition for the stability of the boundary cycle was indicated.

2 Kaldor Model

In 1940, Nicolas Kaldor [9] proposed a business cycle, in which there is an endogenous limit cycle. It is Keynesian model of the business cycle, similar to Kalecki's model, however focuses on the non-linearity of the investment function, and there is no time implementation of the investment [10,11]. Kaldor's business cycle model is a simple closed model economy [17]. The basic assumptions of this model are the non-linearity of the investment function I and savings function S [5].

We assume that investment function has the form:

$$I(Y(t), K(t)) = s\mu + \gamma\left(\frac{s\mu}{\delta} - K(t)\right) + \operatorname{arctg}(Y(t) - \mu),$$

where μ - expected income $\frac{s\mu}{\delta}$ and -expected level of physical capital. It means that $\gamma\left(\frac{s\mu}{\delta} - K(t)\right)$ is a surplus of the expected stock of physical capital over the current level of physical capital adjusted by the coefficient $\delta > 0$ (the cost of adjustment). The second part of the investment function fulfills the necessary conditions of the S-shaped function:

$$f(t) = \operatorname{arctg}(Y(t) - \mu), \qquad f(0) = 0, \qquad f'(0) = 0.$$

This part of the investment function represents the difference between current and expected value of income. Inclination to investment decreases with dismissal the level of national income from its Y*-balance. Also, at a fixed level of income Y, the level of investment decreases with increasing physical capital. It means, that $I_Y > 0$ and $I_K < 0$. There is also a level of income such that:

$$\begin{aligned} I_{YY} &> 0 \quad \text{dla} \quad Y < Y_0, \\ I_{YY} &< 0 \quad \text{dla} \quad Y > Y_0. \end{aligned}$$

The S savings function is a growing function relative to income and physical capital, i.e. $S_Y > 0$ and $S_K < 0$. Without loss of generality, we assume the saving function is only a linear income function.

$$S(Y(t), K(t)) = sY(t), \;\; 0 \leqslant s \leq 1.$$

The increase in income with the unchanged production method results in a proportional increase in savings in the economy.

Increment of income at any time t ϵ [0, + 1) is the product of the coefficient $\alpha > 0$ and the difference between planned investments and savings *(I(Y(t),K(t))-S(Y(t),K(t)))*. Coefficient α we call speed adjustment of supply to demand in the analyzed economy. In turn, the increase in the amount of physical capital K in the economy is equal ongoing investment outlays less depreciation (consumption) of physical capital.

It means, that:

$$\begin{cases} \dot{Y}(t) = \alpha[I(Y(t), K(t)) - sY(t)] \\ \dot{K}(t) = I(Y(t), K(t)) - \delta K(t) \end{cases}.$$

$$\begin{cases} \dot{Y}(t) = \alpha[s\mu + \gamma\left(\frac{s\mu}{\delta} - K\right) + \operatorname{arctg}(Y - \mu) - sY] \\ \dot{K}(t) = s\mu + \gamma\left(\frac{s\mu}{\delta} - K\right) + \operatorname{arctg}(Y - \mu) - \delta K \end{cases}.$$

Taking into account the detailed assumptions about the savings function and investment functions we get the following arrangement:

$$K^* = \frac{sY^*}{\delta}.$$

Therefore, we can describe the amount of physical capital in equilibrium help with the following equation:

$$s\mu\left(1+\frac{\gamma}{\delta}\right) + \mathrm{arctg}(Y-\mu) = \frac{\delta+\gamma}{\delta}sY.$$

We have the equilibrium point $E^*(Y^*, K^*)$ in the following form:

$$\begin{cases} K^* = \frac{sY^*}{\delta} \\ (Y^*-\mu)s\left(1+\frac{\gamma}{\delta}\right) = \mathrm{arctg}\,(Y^*-\mu) \end{cases}.$$

This means that the balance of physical capital in an equilibrium situation to income is equal to the ratio of the saving rate to the rate of depreciation of physical capital. We can determine the equilibrium points only graphically. Alternatively, critical points can be determined for the Kaldor system with the approximated investment function. To this end, we will develop the cyclometric functions in the Maclaurin series:

$$\mathrm{arctg}(Y^*-\mu) = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}(Y^*-\mu)^{2n+1}.$$

$$\mathrm{arctg}(Y^*-\mu) = (Y^*-\mu) - \frac{1}{3}(Y^*-\mu)^3 + \dots$$

We have:

$$\begin{cases} Y(t) = \alpha[s\mu + \gamma\left(\frac{s\mu}{\delta} - K\right) + (Y-\mu) - \frac{1}{3}(Y-\mu)^3 - sY] \\ K(t) = s\mu + \gamma\left(\frac{s\mu}{\delta} - K\right) + (Y-\mu) - \frac{1}{3}(Y-\mu)^3 - \delta K \end{cases}.$$

Suppose, that $x = (Y^* - \mu)$. In this case we have:

$$\frac{1}{3}x^3 + x\left|s\left(1+\frac{\gamma}{\delta}\right) - 1\right| = 0.$$

When:

$$3 - 3s\left(1+\frac{\gamma}{\delta}\right) < 0$$

we have one critical point:

$$\begin{aligned} Y^* &= \mu \\ K^* &= \frac{s\mu}{\delta}. \end{aligned}$$

When:

$$3 - 3s\left(1+\frac{\gamma}{\delta}\right) > 0$$

we have three critical points:

$$\begin{cases} Y_1^* = \mu \\ K_1^* = \frac{s\mu}{\delta} \end{cases},$$

$$\begin{cases} Y_2^* = \sqrt{3-3s(1+\frac{\gamma}{\delta})} + \mu \\ K_2^* = \frac{s}{\delta}\sqrt{3-3s(1+\frac{\gamma}{\delta})} + \frac{s}{\delta}\mu \end{cases}.$$

$$\Big\{\; Y_3^* = -\sqrt{3-3s(1+\frac{\gamma}{c})} + \mu$$

Jacobian in critical point has the form:

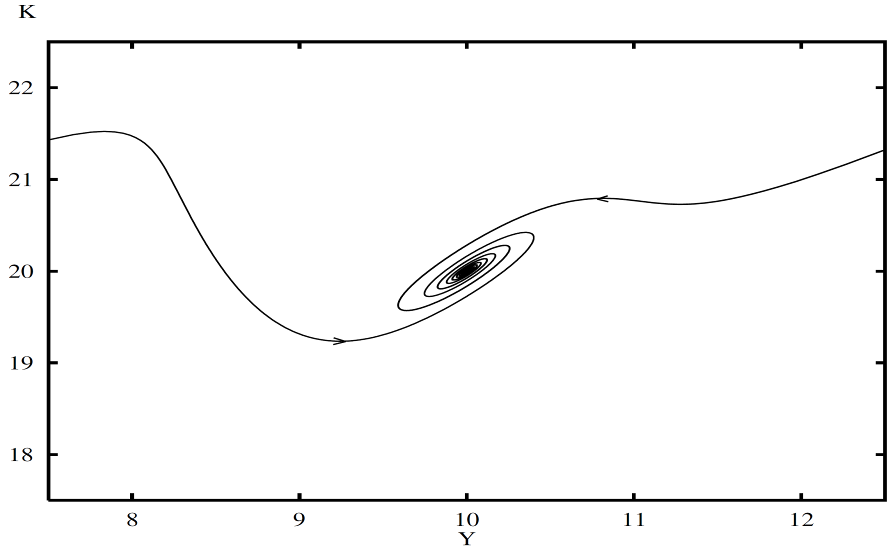
$$J = \begin{bmatrix} \alpha Y^* - \alpha(Y^*-\mu)^2 - s & -\alpha\gamma \\ Y^* - (Y^*-\mu)^2 & -\gamma-\delta \end{bmatrix}.$$

For the first critical point Jacobian has the following form:

$$J = \begin{bmatrix} \alpha\mu - s & -\alpha\gamma \\ \mu & -\gamma - \delta \end{bmatrix}.$$

If s > αμ, then the determinant of Jacobian is positive, its trace is negative, so we are dealing with a sink, that is, a stable node or a stable focus. The numerical analysis shows that this point is stable focus. Figure 1 presents a phase portrait with this critical point.

Figure 1: Phase portrait in Kaldor model for α = 1.2, δ = 0.2, γ = 0.6, s = 0.4



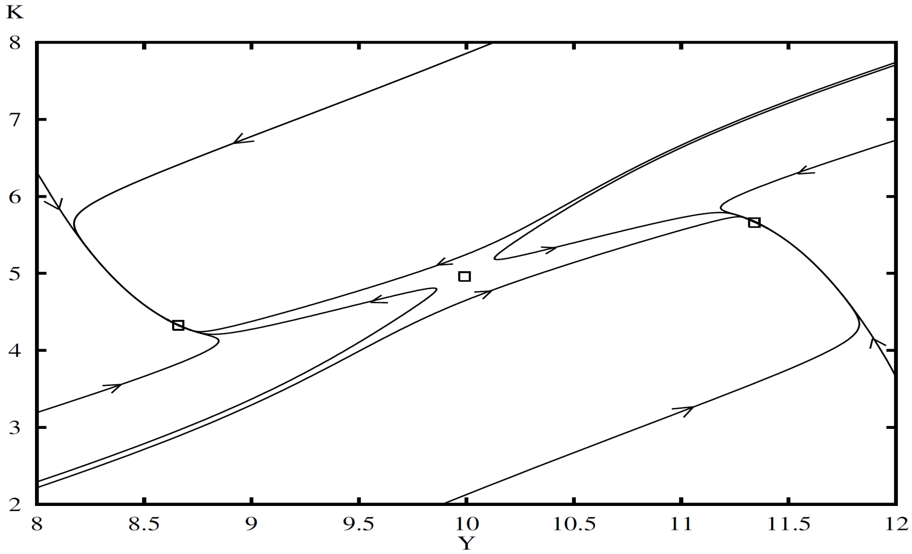
For the next points Jacobian has the form:

$$J = \begin{bmatrix} \alpha\left(\sqrt{3 - 3s\left(1 + \frac{\gamma}{\delta}\right)} + \mu\right) - \alpha\left(3 - 3s\left(1 + \frac{\gamma}{\delta}\right)\right) - s & -\alpha\gamma \\ \sqrt{3 - 3s\left(1 + \frac{\gamma}{\delta}\right)} - \left(3 - 3s\left(1 + \frac{\gamma}{\delta}\right)\right) + \mu & -\gamma - \delta \end{bmatrix}$$

$$J = \begin{bmatrix} \alpha\left(-\sqrt{3 - 3s\left(1 + \frac{\gamma}{\delta}\right)} + \mu\right) - \alpha\left(3 - 3s\left(1 + \frac{\gamma}{\delta}\right)\right) - s & -\alpha\gamma \\ -\sqrt{3 - 3s\left(1 + \frac{\gamma}{\delta}\right)} - \left(3 - 3s\left(1 + \frac{\gamma}{\delta}\right)\right) + \mu & -\gamma - \delta \end{bmatrix}.$$

From numerical analysis it follows that critical points in this case there is a saddle and two stable nodes. Phase portrait of these points has been presented in figure 2.

Figure 2: Phase portrait in Kaldor model for α = 1.2, δ = 0.2, γ = 0.6, s =0.1



2.1 The existence of limit cycles

Let us now consider the possibility of a limit cycle in the considered model. In the analysis of the stability of the Kaldor model, we are interested in the critical point (y^*, k^*). We have:

$$\begin{cases} \frac{d}{dt}(y-y^*) = [I_y(y^*) - S_y(y^*)](y-y^*) + (I_k(k^*) - S_k(k^*))(k-k^*) \\ \frac{d}{dt}(k-k^*) = I_y(y^*)(y-y^*) + [I_k(k^*) - \delta)](k-k^*) \end{cases}.$$

We assume, without loss of generality, that:

$$S_y = s = \text{const},\ Y - y^* = y,\ k - k^* = k.$$

We have:

$$\begin{cases} \dot{y} = [\alpha(I_y(0) - s)]y(t) + I_k(0)k(t) \\ \dot{k} = I_y(0)y(t) + [I_k(0) - \delta]k(t) \end{cases}$$

In the next step we introduce characteristic polynomial of the system:

$$f(\lambda) = \lambda^2 - [I_y(0) - s + I_k(0) - \delta]\lambda +$$
$$+ [I_y(0) - s][I_k(0) - \delta] - I_y(0)I_k(0).$$

Let:

$$A = -[I_y(0) - s + I_k(0) - \delta],$$
$$B = [I_y(0) - s][I_k(0) - \delta] - I_y(0)I_k(0).$$

Then:

$$f(\lambda) = \lambda^2 + A\lambda + B.$$

Using the Routh-Hurwitz criterion, we assume that:

$$\begin{cases} -[I_y(0) - s + I_k(0) - \delta] > 0 \\ -[I_y(0) - s][I_k(0) - \delta] - I_y(0)I_k(0) > 0 \end{cases}.$$

Then all the elements of the equation have the following form:

$$\lambda^2 - [I_y(0) - s + I_k(0) - \delta]\lambda +$$
$$+ [I_y(0) - s][I_k(0) - \delta] - I_y(0)I_k(0) = 0,$$

with negative real part. Thus, the equilibrium point $E^*(Y^*, K^*)$ is stable point. In this case, the critical point loses its stability. It becomes unstable, and a periodic orbit is formed around it. We will now show the existence of a limit cycle.

2.2 Hopf theorem

Using the Hopf theorem we will show that there is a limit cycle in considered Kaldor model. Recall that the characteristic equation of the system has a form

$$\lambda^2 + A\lambda + B = 0.$$

Let's assume that $\lambda_{1,2} = \pm ia$ is a pair of compressed complex elements with a real part equal to zero.

Let:

$$f(ia) = -a^2 + Aia + B = 0.$$

The solution of the system are pairs: $a = \pm\sqrt{B}.$

Therefore, the bifurcation equation has the form:

$$A\sqrt{B} = 0,$$

Then:

$$AB = 0,$$

Therefore:

$$-[I_y(0) - s + I_k(0) - \delta] = 0,$$

So we have:

$$[I_y(0) - s][I_k(0) - \delta] - I_y(0)I_k(0) = 0.$$

After differentiation A and B by α we get the system:

$$\begin{cases} \frac{dA}{d\alpha} = -I_y(0) \\ \frac{dB}{d\alpha} = -I_y(0)(g+\delta) \end{cases}$$

Therefore:

$$\frac{d\lambda}{a\alpha} = \frac{-\frac{dB}{d\alpha} - i\sqrt{B}\frac{dA}{a\alpha}}{2i\sqrt{B} + A} = \frac{\left(-\frac{dB}{d\alpha} - i\sqrt{B}\frac{dA}{d\alpha}\right)\left(A - 2i\sqrt{B}\right)}{A^2 - 4B}$$
$$= \frac{\left(I_y(0)(g+\delta) + i\sqrt{B}I_y(0)\right)\left(A - 2i\sqrt{B}\right)}{A^2 - 4B}.$$

Thus, the real part of the equation is:

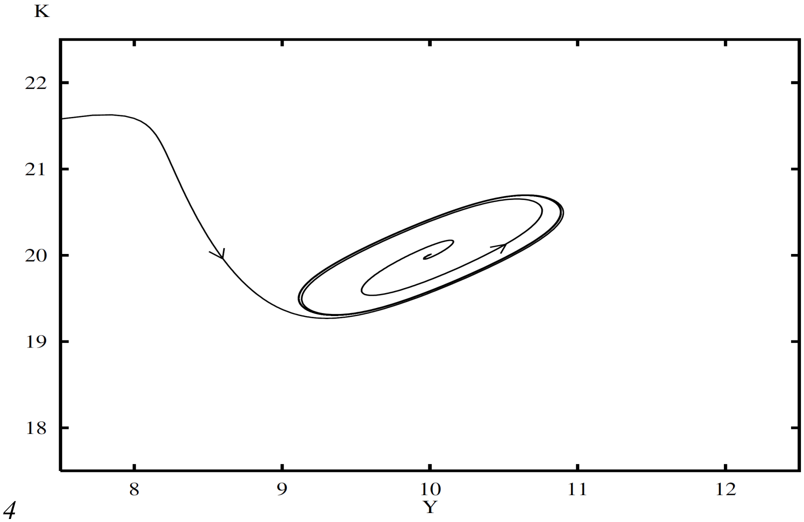
$$Re\left(\frac{d\lambda}{d\alpha}\right) = \frac{A\left(-\frac{dB}{d\alpha}\right) + 2B\frac{dA}{d\alpha}}{A^2 - 4B} = \frac{A\left(I_y(0)(g+\delta)\right) - 2BI_y(0)}{A^2 - 4B}.$$

$Re\left(\frac{d\lambda}{d\alpha}\right)|_{\alpha=const} \neq 0,$ $A\left(-\frac{dB}{d\alpha}\right) + 2B\frac{dA}{d\alpha} =$

If what's happening thanks $A(I_y(0)(g+\delta)) - 2BI_y(0) \neq 0$ and $A^2 - 4B > 0,$ this and is the Hopf bifurcation and a periodic solution may occur near the equilibrium point $E^*(Y^*, K^*)$.

Figure 3 presents the phase portrait of the limit cycle in the Kaldor model. Comparing this portrait with Figure 2, we see thatvstable focus lost its stability and around the unstable outbreak was formed limit cycle when the parameter value αchanged from 1.2 to 2.0.

Figure 3: Phase portrait in Kaldor model for α = 2.0, δ = 0.2, γ = 0.6, s =



0.4

We have proved the existence of a limit cycle in the Kaldor model in the result of Hopf's bifurcation.

3 Summary

In the work was considered the model of the Kaldor business cycle with a specific one investment function:

$$I(Y,K) = s\mu + \gamma\left(\frac{s\mu}{\delta} - K(t)\right) + (Y(t) - \mu) - \frac{1}{3}(Y-\mu)^3.$$

The main result of the work is to show that there is a stable boundary cycle in this model, which has been proven using the modified Guckenheimer method.

The examined model is an example of Keynesian models of the business cycle. This model is a two-dimensional, non-linear dynamic system in which the cyclic solution is endogenous. The advantage of this model is its simplicity. It allows modeling the cyclical behavior of two macroeconomic variables: income and capital in kind.

The Kaldor model is a dynamical system in the form of a two-dimensional system of differential equations. The work presents the most important definitions related to the existence of solutions and their stability for this type of systems. In the context of the occurrence of limit cycles, criteria and statements about their stability have been presented.

The source of cyclical behavior in the Kaldor model is the non-linearity of the investment function. In the work a special form of investment function was adopted that met Kaldor's assumptions. Critical points have been determined for this model and numerical analysis has been carried out. For selected values of the model parameters, the phase portraits of the vector field showing the behavior of the system were made. Based on these portraits, the nature of critical points was determined. The analysis shows that the change in parameter values affects the structure of the phase space. In this model, as a result of Hopf bifurcation, the stable focus loses stability and a limit cycle is created around it when the bifurcation value of the α parameter is exceeded.

Taking advantage of the fact that the boundary cycle exists in the model under consideration, its stability has been examined. The method presented by Liu [tide], which is a development of the Guckenheimer method, was used. The corresponding index was calculated and the condition for the stability of the boundary cycle was indicated. As a result of this analysis, it was obtained that stability depends only on the γ parameter.

References

[1] Betounes D., *Differential Equations: Theory and Applications*, Springer, New York, 2010.

[2] Bischi G.I., Dieci R., Rodano G., Saltari E., *Multiple attractors and global bifurcations in a Kaldor-type business cycle model*, Journal Evolutionary Economic 11 (2001), s. 527-554.

[3] Chang W.W., Smyth D.J., *The existence and persistence of cycles in a non-linear model: Kaldor's 1940 model re-examined*, Review of Economic Studies, 38, (1971) s. 37-44.

[4] Dohtani A., *Existence of Stable Quasi-Periodic Solutions and Presistence of Business Fluctuations*, SIAM Journal on Applied Mathematics 53, (1993) s. 1776-1784.

[5] Gabisch G., Lorenz H.-W., *Business Cycle Theory 2nd ed.*, Springer, Berlin, 1989.

[6] Guckenheimer J., Holmes P., *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields*, Springer, Berlin, 1983.

[7] Guillemin V., Pollack A., *Differential Topology*, Prentice Hall, Englewood Cliffs, NJ, 1974.

[8] Hirsch M.W., Smale S., Davaney R.L., *Differential Equations, Dynamical System, and an Introduction to Chaos*, Elsevier Press, 2004.

[9] Kaldor N., *A model of economic growth*, Economic Journal, 67, (1957) s. 591-624.

[10] Kalecki M., *A Macro-Economic Theory of Business Cycles*, Econometrica 3 (1935), s. 327–344.

[11] Kalecki M., *A theory of the business cycle. Essays in the Theory of Economic Fluctuation*, Allen-Unwin, London 1972.

[12] Milnor J., *Topology from the Differentiable Viewpoint*, University of Virginia Press, Charlottesville 1965.

[13] Palis J., Smale S., *Structural Stability Theorems. In Global Analysis*, Proc. Symp. Pure Math, 14, (1970) 223-231.

[14] Peixoto M. M., *Structural Stability on Two-Dimensional Manifolds*, Topology, 2 (1962), s. 101-121.

[15] Perko L., *Differential Equations and Dynamical Systems*, Springer, Berlin 2001.

[16] Rodano G., *Lezioni Sulle Teorie Della Crescita e Sulle Teorie Del Ciclo, Dipartimento di teoria economica e metodi quantitativi*, Universitia di Roma Ła Sapienza", 1997.

[17] Torre V., *Existence of Limit Cycles and Control in Complete Keynesian System by Theory of Bifurcations*, Econometrica, 45 (1977), s. 1457-1466.

Wavelet method for sensitivity analysis of options under the Black-Scholes model

Dana Černá [1]

Abstract

The paper is concerned with the numerical computation of the Greeks for options on one asset under the Black-Scholes model. Since analytic forms of the Greeks are known only in simple cases, numerical methods have to be employed. In this paper we use a method based on wavelets, because due to well-known compression property of wavelets the solution in a wavelet basis is represented by a small number of coefficients and thus the computation of the solution can be carried out with the small number of parameters. We use the Crank-Nicolson scheme for time discretization and an adaptive wavelet method for spatial discretization. Numerical example is presented for a European vanilla option.

Key words

Greeks, option, Black-Scholes model, adaptive method, spline wavelet.

JEL Classification: C63, G13.

1. Introduction

The paper is concerned with the numerical computation of the Greeks, i.e. derivatives of the option price with respect to underlying parameters such as the underlying asset price, time to expiration, volatility, and interest rates. The Greeks measure the sensitivity of the option price to these parameters and their computation is important for hedging. In this paper, we focus on the computation of the Greeks of one-asset options under the Black-Scholes model.

Various numerical methods are available for the solution of the equation representing the Black-Scholes model and the computation of the Greeks, see e.g. (Achdou and Pironneau, 2005; Hager et al., 2010; Hilber et al., 2013) and references therein. We use an approach based on wavelets that is a modification of the method from (Cohen et al., 2002; Černá, 2016; Černá and Finěk, 2017). In these papers the number of basis function was increasing successively, while in the method presented in this paper the number of basis functions is fixed. The advantage of wavelet methods lies in the high order accuracy and a sparse representation of the solution.

First we introduce the Black-Scholes model and we discretize the Black-Scholes equation in time using the Crank-Nicolson scheme. Then we construct a cubic spline wavelet basis on the interval and we propose an adaptive method with this basis for spatial discretization. We recall the definitions of the Greeks and we present methods for their numerical computation. Finally we provide a numerical example.

2. Black-Scholes model

Let $S > 0$ represent the price of the underlying asset, t represent time to maturity, r be a risk-free rate, and σ be volatility of the asset price. In (Black and Scholes, 1973) it was

[1] RNDr. Dana Černá, Ph.D., Department of Mathematics and Didactics of Mathematics, Technical University of Liberec, Studentská 2, 461 17 Liberec, Czech Republic, e-mail: dana.cerna@tul.cz.
This work was supported by grant No. GA16-09541S of the Czech Science Foundation.

derived that for several types of options the market price $V(S,t)$ can be computed as the solution of the equation

$$\frac{\partial V}{\partial t} - \mathcal{L}_{BS}(V) = 0, S \in (0,\infty), t \in (0,T),$$

where the operator $\mathcal{L}_{BS}$ is given by

$$\mathcal{L}_{BS}(V) = \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV.$$

We approximate the unbounded domain $(0,\infty)$ by $\Omega = (0, S^{max})$, where S^{max} is a large enough constant. We assume that initial and boundary conditions are of the form $V(S,0) = g(S), S \in \Omega$, and $V(0,t) = h(t)$, $V(S^{max},t) = k(t)$, $t \in (0,T)$. The functions g, h, and k depend on the type of an option.

Let W be a smooth enough function such that $W(0,t) = h(t)$ and $W(S^{max},t) = k(t)$ for $t \in (0,T)$ and let us set $\tilde{V} = V - W$. Then we can restrict ourselves to a problem

$$\frac{\partial \tilde{V}}{\partial t} - \mathcal{L}_{BS}(\tilde{V}) = f, \qquad f = -\frac{\partial W}{\partial t} + \mathcal{L}_{BS}(W),$$

with homogeneous Dirichlet boundary conditions and an initial condition $\tilde{V}(S,0) = g(S) - W(0,S)$, $S \in \Omega$.

For time discretization we use the Crank-Nicolson scheme. Let $M \in \mathbb{N}$, $\tau = T/M$, $t_l = l\tau$, $l = 0,\cdots,M$, and let us denote $\tilde{V}_l(S) = \tilde{V}(S,t_l)$, and $f_l(S) = f(S,t_l)$. The scheme has the form

$$\frac{\tilde{V}_{l+1} - \tilde{V}_l}{\tau} - \frac{\mathcal{L}_{BS}(\tilde{V}_{l+1})}{2} - \frac{\mathcal{L}_{BS}(\tilde{V}_l)}{2} = \frac{f_{l+1} + f_l}{2}.$$

The Crank-Nicolson scheme can be combined with Richardson extrapolation for increasing efficiency of the scheme, see e.g. (Finěk, 2017).

3. Adaptive wavelet method

The standard variational formulation has the form:

$$\frac{(\tilde{V}_{l+1}, v)}{\tau} - \frac{a(\tilde{V}_{l+1}, v)}{2} - \frac{a(\tilde{V}_l, v)}{2} = \frac{(\tilde{V}_l, v)}{\tau} + \frac{(f_{l+1} + f_l, v)}{2},$$

where $v \in H_0^1(\Omega)$, $a(u,v) = (\mathcal{L}_{BS}(u), v)$, and $(.,.)$ denotes the L^2-inner product.

For spatial discretization we use an adaptive method based on wavelets. This method uses some ideas of the adaptive wavelet method from (Cohen et al., 2002). The main difference is that the number of basis functions in our method is fixed which simplifies the algorithm, reduces the number of iterations and decreases the computational time. We briefly review a concept of wavelets, for more details one can see e.g. (Urban, 2009). Let J be an index set such that each index $\lambda \in J$ takes the form $\lambda = (j,k)$, $|\lambda| := j$ denotes a level and k represents a spatial location.

Definition: A family $\Psi = \{\psi_\lambda, \lambda \in J\}$ is called a wavelet basis of the Hilbert space H if it satisfies the following conditions:

- Ψ is a Riesz basis for the space H.
- For all $\lambda \in J$ a diameter of the support of ψ_λ is bounded by $C2^{-|\lambda|}$ with the constant C independent on $|\lambda|$ and at a given level the supports of only finitely many wavelets overlap at any point.

A wavelet basis Ψ has typically a hierarchical structure

$$\Psi = \Phi_{j_0} \cup \bigcup_{j=j_0}^{\infty} \Psi_{j.}$$

The functions $\varphi_{j,k} \in \Phi_j$ are called scaling functions and the functions $\psi_{j,k} \in \Psi_j$ are called wavelets. They are typically translations and dyadic dilations of one or several functions.

In this paper, we use a cubic spline-wavelet basis from (Chen, 1995; Han and Shen, 2006) adapted to the bounded interval. The scaling basis is the same as in (Černá and Finěk, 2015; Chui and Quak, 1992). Let φ be a cubic B-spline defined on knots [0,1,2,3,4], φ_{b1} and φ_{b2} be cubic B-splines defined on knots [0,0,0,1,2] and [0,0,1,2,3], respectively. Then the scaling basis $\Phi_3 = \{\varphi_{3,k}, k = 1, \cdots, 9\}$ contains functions defined by

$$\varphi_{3,k}(S) = \sqrt{8}\varphi\left(\frac{8S}{S^{max}} - k + 3\right), k = 3, \cdots, 7,$$
$$\varphi_{3,1}(S) = \sqrt{8}\varphi_{b1}\left(\frac{8S}{S^{max}}\right), \varphi_{3,9}(x) = \sqrt{8}\varphi_{b1}\left(8 - \frac{8S}{S^{max}}\right),$$
$$\varphi_{3,2}(S) = \sqrt{8}\varphi_{b2}\left(\frac{8S}{S^{max}}\right), \varphi_{3,8}(x) = \sqrt{8}\varphi_{b2}\left(8 - \frac{8S}{S^{max}}\right),$$

for $S \in (0, S^{max})$. We define a wavelet ψ as

$$\psi(x) = \varphi(2x-1) - 4\varphi(2x-2) + 6\varphi(2x-3) - 4\varphi(2x-4) + \varphi(2x-5)$$

and boundary wavelets ψ_{b1} and ψ_{b2} as

$$\psi_{b1}(x) = 6\varphi_{b1}(2x) - \frac{57\varphi_{b2}(2x)}{5} + \frac{919\varphi(2x)}{100} - \frac{116\varphi(2x-1)}{25} + \varphi(2x-2),$$
$$\psi_{b2}(x) = \frac{7\varphi_{b2}(2x)}{3} - \frac{319\varphi(2x)}{60} + \frac{101\varphi(2x-1)}{15} - \frac{25\varphi(2x-2)}{6} + \varphi(2x-3).$$

Then all wavelets have four vanishing moments, i.e.

$$\int_{0.5}^{4.5} x^k \, \psi(x)dx = 0, \quad \int_{0}^{3.5} x^k \, \psi_{bi}(x)dx = 0, \ i = 1,2, \ k = 0,1,2,3.$$

This property is crucial in adaptive methods, because it enables a sparse representation of the right-hand side. Wavelet basis on the interval $(0, S^{max})$ on the level $j \geq 3$ is defined by $\Psi_j = \{\psi_{j,k}, k = 1, \cdots, 2^j\}$ with

$$\psi_{j,k}(S) = 2^{\frac{j}{2}}\psi\left(\frac{2^j S}{S^{max}} - k + 2\right), \qquad k = 3, \cdots, 2^j - 2,$$
$$\psi_{j,1}(S) = 2^{\frac{j}{2}}\psi_{b1}\left(\frac{2^j S}{S^{max}}\right), \qquad \psi_{j,2^j}(S) = 2^{\frac{j}{2}}\psi_{b1}\left(2^j - \frac{2^j S}{S^{max}}\right),$$
$$\psi_{j,2}(S) = 2^{\frac{j}{2}}\psi_{b2}\left(\frac{2^j S}{S^{max}}\right), \qquad \psi_{j,2^j-1}(S) = 2^{\frac{j}{2}}\psi_{b2}\left(2^j - \frac{2^j S}{S^{max}}\right).$$

Then

$$\Psi = \Phi_3 \cup \bigcup_{j=3}^{\infty} \Psi_j$$

is a wavelet basis for the space $L^2(0, S^{max})$. We set $\psi_{2,k} = \varphi_{3,k}$, and $J = \{(2,k), k = 1, \dots, 9\} \cup \{(j,k), j \geq 3, k = 1, \dots, 2^j\}$. We expand the solution $\tilde{V}_{l+1}$ in the basis Ψ, i.e.

$$\tilde{V}_{l+1} = \sum_{\lambda \in J} u_\lambda \, \psi_\lambda.$$

We substitute this expansion to the variational formulation and we obtain the biinfinite matrix equation

$$\mathbf{A}\mathbf{u}^{l+1} = \mathbf{f}^l$$

with $\mathbf{u}^{l+1} = \{u_\lambda^{l+1}\}_{\lambda\in J}$, $\mathbf{A} = \{A_{\mu,\lambda}\}_{\mu,\lambda\in J}$, $\mathbf{f}^l = \{f_\mu^l\}_{\mu\in J}$, and

$$A_{\mu,\lambda} = \frac{(\psi_\lambda, \psi_\mu)}{\tau} - \frac{a(\psi_\lambda, \psi_\mu)}{2}, \qquad \lambda, \mu \in J,$$
$$f_\mu^l = \frac{a(\tilde{V}_l, \psi_\mu)}{2} + \frac{(\tilde{V}_l, \psi_\mu)}{\tau} + \frac{(f_{l+1} + f_l, \psi_\mu)}{2}, \qquad \mu \in J.$$

The biinfinite matrix **A** has so called finger-band pattern, see e.g.(Černá and Finěk, 2017). We solve the resulting system by the method of generalized residuals (GMRES) with diagonal preconditioning. The algorithm comprises the following steps:

1. Choose the number of basis functions N and the number of time steps M.
2. Compute the vector of coefficients $\mathbf{u}^0$ for the function $\tilde{V}_0$ and do $\mathbf{u}^0 = \mathrm{COARSE}(\mathbf{u}^0, N)$.
3. For $l = 0, 1, 2, \ldots, M-1$ compute the right-hand side $\mathbf{f}^l$, do $\mathbf{u}^{l+1} = \mathrm{GMRES}(\mathbf{A}, \mathbf{f}^l, \mathbf{u}^l)$, and $\mathbf{u}^{l+1} = \mathrm{COARSE}(\mathbf{u}^{l+1}, N)$.
4. Compute the approximate solution at time T as $V_T(S) = V(S,T) = \tilde{V}_M(S) + W(S,T)$, $S \in (0, S^{max})$.

In this algorithm, $\mathbf{u}^{l+1} = \mathrm{GMRES}(\mathbf{A}, \mathbf{f}^l, \mathbf{u}^l)$ means that $\mathbf{u}^{l+1}$ is the solution of the system of linear algebraic equations with the matrix **A** and the right-hand side $\mathbf{f}^l$ using GMRES method with initial vector $\mathbf{u}^l$. The routine $\mathbf{u}^{l+1} = \mathrm{COARSE}(\mathbf{u}^{l+1}, N)$ consists in thresholding, i.e. N entries from the vector $\mathbf{u}^{l+1}$ that are largest in the absolute value are kept and the others are set to zero. Thus the output parameter $\mathbf{u}^{l+1}$ contains N nonzero entries. The routine GMRES requires multiplication of the infinite-dimensional matrix **A** with a finitely supported vector. It is computed approximately by the method from (Černá and Finěk, 2013). Since we work with the sparse representation of the right-hand side and a sparse representation of the vector representing the solution, the method is adaptive.

4. Computation of Greeks

We focus on the key Greeks: Delta, Gamma, Theta, Vega, and Rho, but other Greeks can be computed using the similar approach. For the computation of Delta and Gamma we exploit the fact that the approximate solution on the given time level is a cubic spline and thus it is two-times differentiable. Therefore we compute Delta and Gamma directly as derivatives of the approximate solution. For the computation of Theta, Vega, and Rho we use finite differences.

Delta is defined as the derivative of the option price with respect to the underlying asset price, i.e.

$$\Delta = \frac{\partial V}{\partial S}.$$

Therefore it characterizes how much the option price is expected to change if the price of the underlying asset changes. Since we have

$$V(S, t_l) = \sum_{\lambda\in J} u_\lambda^l \, \psi_\lambda(S) + W(S, t_l),$$

we can compute Delta as

$$\Delta(S,t_l) = \frac{\partial V(S,t_l)}{\partial S} = \sum_{\lambda\in J} u_\lambda^l\, \psi'_\lambda(S) + \frac{\partial W(S,t_l)}{\partial S}.$$

Gamma measures how much Delta is expected to change if the price of the underlying asset changes and thus it is defined as the second derivative of the option price with respect to the underlying asset price, i.e.

$$\Gamma = \frac{\partial^2 V}{\partial S^2}.$$

Similarly as Delta we can compute it directly as

$$\Gamma(S,t_l) = \frac{\partial^2 V(S,t_l)}{\partial S^2} = \sum_{\lambda\in J} u_\lambda^l\, \psi''_\lambda(S) + \frac{\partial^2 W(S,t_l)}{\partial S^2}.$$

Theta measures the rate of decline in the option price due to the passage of time. It is defined as

$$\Theta = \frac{\partial V}{\partial t}.$$

We compute it numerically from the solution V using the central finite difference

$$\Theta(S,t) = \frac{\partial V(S,t)}{\partial t} \approx \frac{V(S,t+\tau) - V(S,t-\tau)}{2\tau}.$$

Vega measures the sensitivity of the option price to changes in the volatility. Up to now we have assumed that volatility σ and the interest rate r are given constants. Now we consider the price of the option V as a function of four variables $V = V(S,t,\sigma,r)$. Then Vega is defined by

$$\mathcal{V} = \frac{\partial V}{\partial \sigma}.$$

We approximate the derivative with the central finite difference and we obtain

$$\mathcal{V}(S,t) = \frac{\partial V(S,t,\sigma,r)}{\partial \sigma} \approx \frac{V(S,t,\sigma+h_\sigma,r) - V(S,t,\sigma-h_\sigma,r)}{2h_\sigma}.$$

Rho measures the sensitivity of the option price to changes in the interest rate. It is defined as

$$\rho = \frac{\partial V}{\partial r}.$$

We compute it numerically as

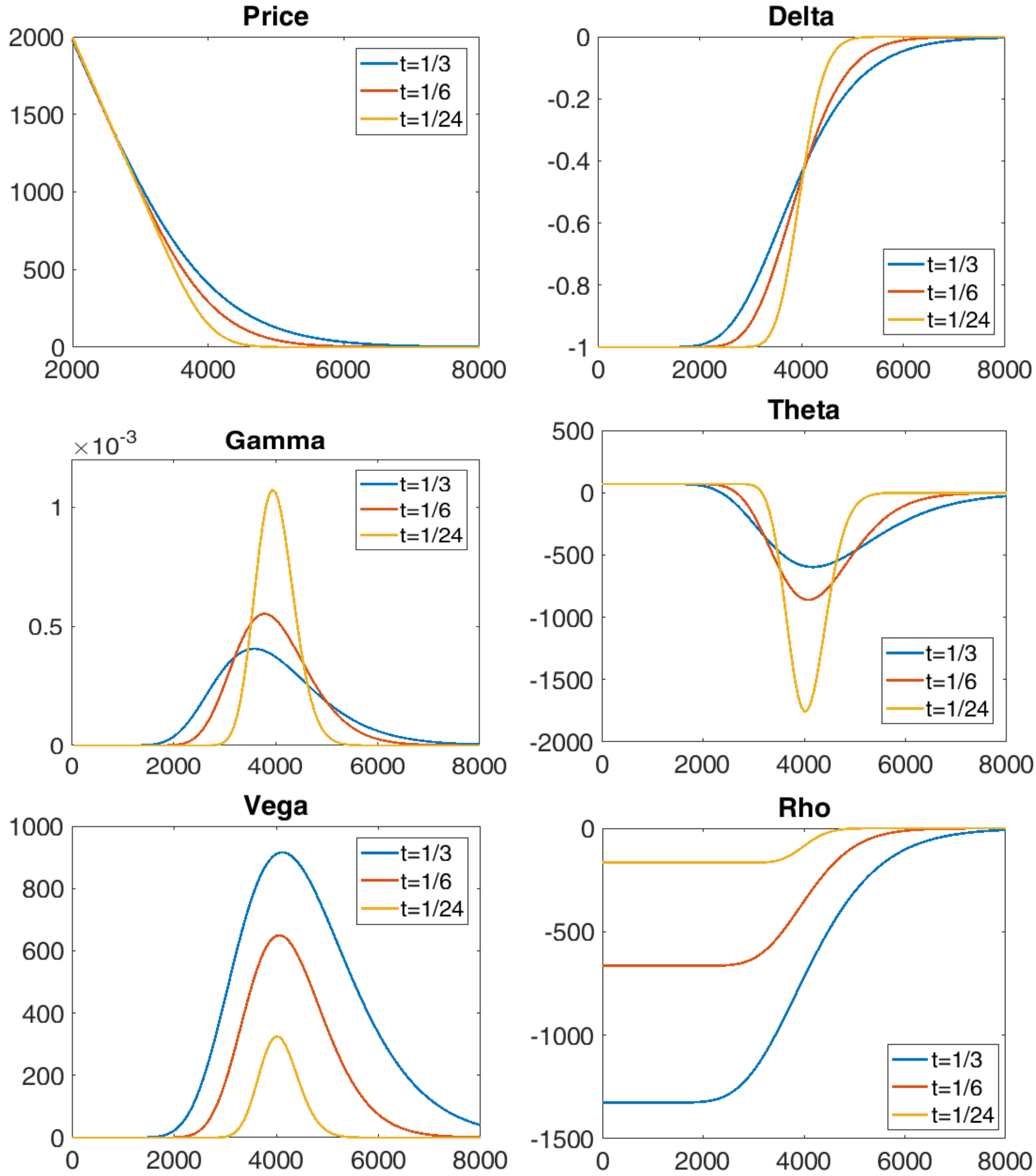
$$\rho(S,t) = \frac{\partial V(S,t,\sigma,r)}{\partial r} \approx \frac{V(S,t,\sigma,r+h_r) - V(S,t,\sigma,r-h_r)}{2h_r}.$$

5. Numerical example

We consider a European vanilla put option with the real data of the German DAX on 15 September 2011 as in (Černá, 2016; Hozman and Tichý, 2014). The strike price is $K = 4000$ Euro and the option is maturing in 4 months, i.e. $T = 1/3$. The corresponding volatility is $\sigma = 0.4594$ and the risk-free interest rate is $r = 0.0176$.

We set $S^{max} = 4K = 16000$, because for this choice the error is not significantly influenced by approximate boundary conditions that are for a European put option of the form $V(0,t) = Ke^{-rt}$, $V(S^{max},t) = 0$. We set W= $Ke^{-rt}(1 - S/S^{max})$ and we choose $N = 2^k$, $M = 2^{2k}/16$, $k = 4,\cdots,7$. For the computation of Vega and Rho we use $h_\sigma = 0.002$ and $h_r = 0.001$. The resulting approximate solution and the corresponding Greeks are for several choices of time to maturity t displayed in Figure 1.

Figure 1: Price of the option and the Greeks for several choices of time to maturity t.



We denote the exact solution at time to maturity T by V_T^{ex}. Recall that an approximate solution at time to maturity T is denoted by V_T. Let $\|.\|_2$ and $\|.\|_\infty$ denote the L^2 and the L^∞-norm, respectively. We compute the relative errors

$$\rho_\infty = \frac{\|V_T^{ex} - V_T\|_\infty}{\|V_T^{ex}\|_\infty}, \quad \rho_2 = \frac{\|V_T^{ex} - V_T\|_2}{\|V_T^{ex}\|_2},$$

and by similar approach we compute relative errors for the Greeks at time T.

The errors typically depend on the number of basis functions N and the number of time steps M. In the optimal case the Crank-Nicolson scheme is second order accurate in time and a cubic spline approximation is fourth order accurate. Therefore the optimal error estimate for our scheme is $\rho_i \approx C_1 M^{-2} + C_2 N^{-4}$, $i = \infty, 2$, with the constants C_1 and C_2 independent on M and N, i.e. $\rho_i \approx C_1 M^{-r/2} + C_2 N^{-r}$ for the rate of convergence $r = 4$. The errors for the first derivatives of the solution are typically one order smaller and the errors for the second derivatives are typically two orders smaller. We compute the estimates of the rate of convergence numerically. If $\rho_i(M, N)$ is a computed error for N functions and M time steps, then the approximate rate of convergence is

$$r_i(M, N) \approx \frac{\log \rho_i(M, N) - \log \rho_i\left(\frac{M}{4}, \frac{N}{2}\right)}{\log 2}, \qquad i = \infty, 2.$$

Relative errors and rates of convergence computed by the proposed method are presented in Table 1. We can see that the relative errors are quite small for option prices and all the Greeks. As expected computed convergence rates are smaller for Greeks than for the option price and the convergence rate is smallest for Gamma, because it is defined as the second derivative while other computed Greeks are the first derivatives of a function representing the option price.

Table 1: Relative errors for the Greeks computed by the adaptive wavelet method with the number of time steps M and the number of basis functions N.

		Price				Delta			
N	M	ρ_∞	r_∞	ρ_2	r_2	ρ_∞	r_∞	ρ_2	r_2
16	16	2.69e-4		3.05e-4		3.17e-3		2.07e-3	
32	64	3.37e-6	6.32	3.16e-6	6.59	1.65e-4	4.26	7.55e-5	4.78
64	256	1.32e-7	4.67	1.04e-7	4.93	1.08e-5	3.93	4.85e-6	3.96
128	1024	7.22e-9	4.19	6.48e-9	4.00	1.01e-6	3.41	4.81e-7	3.33
		Gamma				Theta			
N	M	ρ_∞	r_∞	ρ_2	r_2	ρ_∞	r_∞	ρ_2	r_2
16	16	4.91e-2		3.04e-2		1.69e-2		1.12e-2	
32	64	8.40e-3	2.55	3.69e-3	3.04	2.12e-3	2.99	8.70e-4	3.69
64	256	1.61e-3	2.38	6.82e-4	2.44	7.82e-5	4.76	2.31e-5	5.24
128	1024	4.02e-4	2.00	1.62e-4	2.07	3.32e-5	1.24	6.94e-6	1.73
		Vega				Rho			
N	M	ρ_∞	r_∞	ρ_2	r_2	ρ_∞	r_∞	ρ_2	r_2
16	16	1.57e-2		1.07e-2		3.52e-3		2.25e-3	
32	64	3.63e-3	2.11	1.53e-3	2.81	6.53e-5	5.75	2.75e-5	6.35
64	256	7.57e-5	5.58	3.44e-5	5.48	1.81e-5	1.85	5.62e-6	2.29
128	1024	6.11e-6	3.63	5.73e-6	2.59	6.78e-6	1.42	1.35e-6	2.06

Conclusion

In this paper, we presented an adaptive wavelet method for the numerical computation of the price of an option and the Greeks under the Black-Scholes model. Since we used cubic spline wavelets, the method is high-order accurate with respect to asset price and thus the small number of parameters was needed to obtain the sufficiently accurate solution. We provided a numerical example of a European vanilla put option, because in this case explicit formulas for an option price and for the Greeks are known, which enabled us to study the errors of approximation. However, described methods can be used for pricing options that are represented by the Black-Scholes model and they can be simply generalized to multi-asset options.

References

[1] Achdou, Y., and Pironneau, O. (2005). *Computational Methods for Option Pricing*. Society for Industrial and Applied Mathematics, Philadelphia.

[2] Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. *J. Polit. Econ.*, 81, p. 637-659.

[3] Cohen, A., Dahmen, W., and DeVore, R. (2002). Adaptive wavelet methods II - beyond the elliptic case. *Found. Math.*, 2, p. 203-245.

[4] Černá, D., (2016). Wavelet methods for pricing European options. In: Proceedings of the 8th International Scientific Conference Managing and Modelling of Financial Risks, ed. M. Čulík, Ostrava, p. 129–136.

[5] Černá, D., and Finěk, V. (2013). Approximate multiplication in adaptive wavelet methods. *Cent. Eur. J. Math.*, 11, p. 972-983.

[6] Černá, D., and Finěk, V. (2015). Wavelet basis of cubic splines on the hypercube satisfying homogeneous boundary conditions. *Int. J. Wavelets Multiresolut. Inf. Process.*, 3, 1550014 (21 pages).

[7] Černá, D., and Finěk, V. (2017). Sparse wavelet representation of differential operators with piecewise polynomial coefficients. *Axioms*, 6, (21 pages).

[8] Chen, D., (1995), Spline wavelets of small support, *SIAM J. Math. Anal.*, 26, p. 500–517.

[9] Chui, C., and Quak, E., (1992) Wavelets on a bounded interval, In: Numerical Methods of Approximation Theory, eds. D. Braess and L.L. Schumaker, (Birkhäuser), p. 53-75.

[10] Finěk, V., (2017). Fourth order scheme for wavelet based solution of Black-Scholes equation. In: 43rd International Conference Applications of Mathematics in Engineering and Economics, eds. Pasheva, V., Popivanov, N., and Venkov, G., *AIP Conference Proceedings* 1910, No. 030004, (4 pages).

[11] Hager, C., Hüeber, S., and Wohlmuth, B.I., (2010). Numerical techniques for the valuation of basket options and their Greeks. *J. Comput. Finance*, 13, p. 3-33.

[12] Hilber, N., Reichmann, O., Schwab, C., and Winter, C. (2013). *Computational Methods for Quantitative Finance.* Berlin: Springer.

[13] Han, B., and Shen, Z. (2006) Wavelets with short support. *SIAM J. Math. Anal.*, 38, p. 530-556.

[14] Hozman, J., and Tichý, T. (2014). Black-Scholes option pricing model: comparison of h-convergence of the DG method with respect to boundary condition treatment. *ECON - Journal of Economics Management and Business*, 24, p. 141-152.

[15] Urban, K. (2009). *Wavelet methods for elliptic partial differential equations*, Oxford University Press, Oxford.

EU Funds Implementation Risks in the Czech Republic in 2007-2013 Programming Period

Petra Daníšek Matušková, Štěpánka Vyskočilová [1]

Abstract

The cohesion policy is one of the key elements of the European integration process and the European structural funds represent an important financial instrument, supporting EU regions development, growth and competitiveness. The 2007-2013 programming period meant for the Czech Republic an opportunity to implement over 26 billion EUR from the European funds. However many of the applied projects were, for different reasons, not realized, and it was feared that the Czech Republic would not be able to implement the allocated EU help fully. The objective of the paper is to point up the problems of the projects management and EU funds absorption, and to evaluate how individual cohesion regions of the Czech Republic dealt with the EU funds implementation in the 2007-2013 programming period.

Key words

Structural Funds, European Union, Cohesion Policy, Risks of Project Management

JEL Classification: O22, G32, R10

1. Introduction

The cohesion policy is one of the key elements of the European integration process and its main purpose is to promote economic growth and to speed up the economic convergence among EU member states and regions (European Communities, 2006). The importance of the cohesion policy increased in connection with the deepening of disparities among states of the Central and Eastern Europe at the end of 1990s. The interest in regional policy was further raised in 2004, in the context with the biggest European Union („EU“) enlargement, leading to further regional disparities deepening (Pénzes, 2013).

The main financial instrument of the cohesion policy are European structural and investment funds (ESIF). There is five of them. European Regional Development Fund supports investment (infrastructure) projects; European Social Fund (ESF) focuses on activities related to employment and human resources, and other non-investment projects; Cohesion Fund aims to support the development of poorer countries, not regions, and provides finances for large investment projects; European Agricultural Fund for Rural Development (EAFRD) supports European policy on rural development and finances are used to increase agriculture and forestry competitiveness, and to improve the environment and landscape, etc.; European Maritime and Fisheries Fund (EMFF) represents a financial instrument to support fisheries. Apart from Structural and Investment Funds there is a whole range of other theme-oriented multinational funds established by the European Union.

The fundamental priorities and objectives of the cohesion policy are specified in the Community Strategic Guidelines for Cohesion policy. All member states must respect these objectives and priorities, when setting up their own national operational programs. Each

[1] Ing. Petra Daníšek Matušková, VŠB-TU Ostrava, petra.matuskova@vsb.cz.
Ing. Štěpánka Vyskočilová, VŠB-TU Ostrava, stepanka.vyskocilova@vsb.cz.

member state must prepare, based on a national analysis, a national development plan (for the 2007-2013 period called the National Strategic Reference Framework), that sets the state strategy towards the EU cohesion policy fulfilment and defines the national operational programs, forming the basis for the EU funds absorption. At the same time, a national institutional framework has to be established, and the co-financing from national public budget secured (in the 2007-2013 programming period the co-financing scheme was set to EU85:CZ15).

The EU regional policy had three main objectives defined in the 2007-2013 period: Convergence objective; Regional Competitiveness and Employment objective; European Territorial Cooperation objective. An amount of 347 billion EUR was allocated via the structural and investment funds to pursue these objectives. The Czech Republic laid down four strategic objectives for the 2007-2013 programming period: Competitive Czech economy; Open, flexible and cohesive society; Attractive environment; Balanced territorial development; and defined 26 operational programs, out of which 8 operational programs were thematic, 7 were regional, and the rest of the operational programs focused on a territorial cooperation. The financial allocation for the cohesion policy in the 2007-2013 period is summarized in the Table 1.

Table 1: Financial allocations for cohesion policy

Objective	Funds for EU27 in billion EUR	%	Funds for Czech Republic in billion EUR	%
Convergence	283	81.54	25.88	96.68
Regional Competitiveness and Employment	54.96	15.95	419.09	1.56
European Territorial Cooperation	8.72	2.52	389.05	1.46
Total	347	**100**	**26.69**	**100**

Source: www.strukturalni-fondy.cz

In the given programming period the Czech Republic had the opportunity to spend 26.7 billion EUR. The financial support from these funds became for many potential applicants and their projects very attractive. However many of the applicants, and later on also of the beneficiaries, did not realize the risks related to the funds absorption.

2. EU funds implementation risks

Risks influencing on EU funds absorption were, in the past programming period, many times derived already from the implementation framework itself. A high number of operational programmes caused a confusion about offered forms of support, lead to a significant administrative burden and a number of errors on both sides; applicants’ as well as providers‘ side.

Controlling systems were focused primarily on formal requirements fulfilment; hence the evaluation of the results usefulness and investment effectiveness was neglected. The inappropriately defined monitoring indicators, that were not addressing projects quality, but only project quantitative measures, together with the absence of a systematic approach to significant risks identification, evaluation, mitigation, monitoring and reporting, resulted into an illusion that the funding can be drawn on basically anything. However problems were not arising only because of the system shortcomings, but also for other reasons, related to a lax attitude to projects administration or to national rules setting (e.g. unnecessarily rigid rules, complex structure of programmes, etc.).

In the following text we aim to point out the most frequent problems (irregularities) of the 2007-2013 programming period, that lead to a project application rejection, or to a status where funded money had to be, partially or fully, returned. Irregularity means "any infringement of a provision of Community law resulting from an act or omission by an economic operator, which has, or would have, the effect of prejudicing the general budget of the Communities or budgets managed by them, either by reducing or losing revenue accruing from own resources collected directly on behalf of the Communities, or by an unjustified item of expenditure" (Transparency International, 2016). It has to be noted, that it is meant to be an overview of these irregularities, not a comprehensive list of them, since the topic is quite complex. The included irregularities, together with the related risks, have been grouped based on the process phases.

2.1 Projects selection and evaluation

One of the first challenges, that the potential applicants encountered, was a correctly prepared funding application. Besides the necessity to comply with all formal requirements, it is important that the designed project must remain viable also without subsidy, and its content must be aligned with the given programme. The funding application processing system is not simple, and without sufficient knowledge of it, it is appropriate to turn to a specialized company, that helps with the project application handling. The most frequent causes for the application rejection were content-wise: a missing or insufficient description of the project, realization team, chosen technologies, project realization, project sustainability; a lack of neediness or usefulness of the project; underestimated or overestimated economic and other analyses; insufficient marketing study; groundless information and data included; unclear timeline, with missing or conflicting deadlines; a project that does not fulfil even basic target of the monitoring indicators, insufficient experience with project realization; and others. From the economic point of view, the risks factors to be pointed out are: insufficiently secured financial envelope for the project duration and project sustainability, financial data inconsistency; unrealistic outcomes of financial and economic analyses; overestimated investment and operational expenditures or unjustified costs, etc.

In this phase irregularities concern mainly those applicants that either don't have sufficient knowledge, or those that intentionally paint it in a better light, in order to get their application approved. On the side of the contracting authority, the biggest risk factor is the prioritization of an applicant, that would be, under normal circumstances, not eligible for the funding. In this way the EU principles (of transparency, equal treatment, zero profit, co-financing rules, wide competition, proportionality and proper financial management for public procurement) are violated.

2.2 Projects realization

In case of a successful application, the applicant becomes a beneficiary. However also in this position there are further risks to be dealt with, that can lead to the funding to be cut off or withdrawn. In the project realization phase the highest number of irregularities and fraudulent behaviour occurs.

The most frequent cause for the funding being cut off or withdrawn are errors in the administration of selection procedures that are realized as public tenders. The basic problem here is the fact that several regulations (law No. 137/2006 Public procurement law, internal regulations of the public purchaser, regulations for the given programme and call) have to be followed at the same time in these cases, and the a risk emerges that any of them gets violated, either due to a lack of knowledge or due to discrepancies between those directives. In relation to the public procurement procedure the shortcomings appear in the procurement specification, i.e. disclosing information that breaches the transparency of the procurement procedure; in applying potentially discriminating criteria and in non-objective evaluation; in non-compliance with the deadlines for contract signing; in selection of a supplier not corresponding with the evaluation outcomes; in material or personal connections among individual participants; in contract signing with other than winning participant.

The beneficiaries face the risk that the public-law controlling authority may discover any of the irregularities listed in the Table 2, and as a result of that the funding is being put on hold.

There are also problems concerning project pay-as-you-go financing securing, or, by analogy, delayed payments from the contracting authority, and this applies not only for financially demanding projects. Beneficiaries should take that risk into consideration and be ready to mitigate it by own financial resources. This risk is especially valid for situations that can appear after consuming the initial payment or when the interim payments from the contracting authority are, for different reasons, delayed.

Furthermore, subsidy beneficiaries face risks derived from the mandatory publicity of the project that is monitored by controlling authorities. In this area it is necessary to secure not only the correct promotion form but also the promotion duration.

During the project lifecycle it is common to deal with smaller or bigger changes in the project. It is a duty of the applicant to inform the contracting authority about every planned change before it is realized. Certain changes can be only declared, others have to be evaluated by the contracting authority based on a formal request, and are then approved or rejected. Most frequently the changes affect the project timeline, either the deadline of the complete project or some of the project phases. Often the applicant executes such a change without a declaration/formal request. In such cases the funding is being cut off/not reimbursed.

2.3 Sustainability phase

The 2007-2013 programming period is characterized by the fact that the most of the funded projects had to prove sustainable operations with own financial resources, for 5 years after the EU funds financing was finished. During this period every beneficiary must secure the sustainability of the selected activities that he committed to. The financing is secured by the beneficiary, hence the related risk shall be handled already in the project preparation phase, where these activities were supposed to be financially covered in the plan. The following table shows the areas of most frequently committed errors by beneficiaries.

Table 2: Financial impact and numbers of findings in programming period 2007-2013

Infringement area	Financial impact (mil. CZK)	Number of findings
Public procurement rules	4 881.5	3 405
Unauthorised/ineligible expenditure	1 267.08	769
Others	602.31	151
Missing, incomplete documents	442.79	612
Accounting	192.3	347
Suspicion of fraud	56.88	8
Archiving (fulfilling the obligation to keep documents linked to project implementation)	39.67	235
Work contracts, work statements, wages, pay	38.77	484
Rules 3E (economy, effectiveness, efficiency)	26.59	107
Control work (e.g. insufficient control work)	20.7	203
Failure to achieve monitoring indicator	19.78	68
Rules for revenue-generating projects	11.77	40
Missing deadlines (e.g. calls)	4.17	110
Other legislation	3.2	146

State aid rules	2.49	51
Publicity	0.14	121
Total	**7 609.51**	**6 857**

Source: own elaboration, NKÚ ČR 2017

In the Table 2 there are summarized the numbers of irregularities, that were revealed in audits during the 2007-2013 programming period, executed by the appointed auditing body. As shown, the highest number of errors occurred in connection to the public procurement rules; also the related financial impact is biggest there. The second most important category of errors, frequency-wise as well as money-wise, are ineligible expenditures. In eight cases the auditing body suspected a fraud; although it is a low number of cases, the financial impact for EU budget is very significant. From the Table 2 it can be also concluded that some of the problems appeared quite frequently, but their financial impact was negligible.

3. Data and methodology

The analysis that follows is based on the data published at web pages www.strukturalni-fondy.cz, www.europa.eu, www.nku.cz (monitoring reports, studies on funding implementation progress, etc.) and the Czech Statistical Office. The objective of this paper is to analyse, how successful the NUTS II regions in the Czech Republic („cohesion regions“) were in the process of EU funds implementation, via an evaluation of the regional operational programmes („ROP“). We will focus on evaluating project phases based on the error level and compare the individual regions of the Czech Republic in this respect. In the first part of the analysis the total numbers per region are compared, in the later part the analysed numbers are re-calculated per region inhabitant, because the population is always connected with various activities in the given spatial framework. The number of inhabitants can be thus perceived as an approximate indicator of the volume of activities in the analysed area and various territorial policies should take it into account (Sucháček, 2008).

In the given programming period, altogether over 4.5 billion EUR had been allocated for the cohesion regions in the Czech Republic, in means of the regional operational programmes. These programmes were prepared and implemented by the cohesion regions, with focus on the priorities and specific needs of each region. The total sum of funding was split among individual cohesion regions relatively evenly; however minor differences can be observed, mainly due to a bigger support for the lagging regions. Whereas to Central Bohemia cohesion region was allocated little under 15 billion CZK, for the Moravia Silesia region it was close to 20 billion CZK (see Table 3). Here it shall be noted that almost all the cohesion regions were able to implement the allocated sum fully.

Table 3: ROP funding overview per cohesion regions

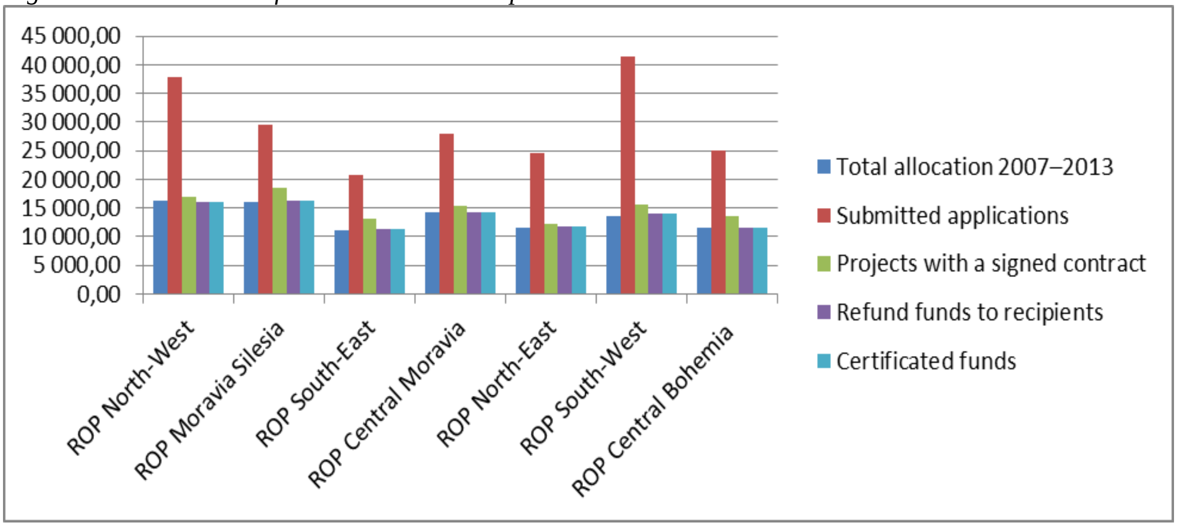
Operational Programme	Total allocation 2007–2013	Submitted applications			Projects with a signed contract			Refund funds to recipients		Certificated funds	
		Quantity	mil. CZK	%*	Quantity	mil. CZK	%*	mil. CZK	%*	mil. CZK	%*
North-West	18 298.10	1 390	42 654.50	233.1	541	19 170.30	104.8	18 174.00	99.3	18 169.90	99.3
Moravia Silesia	19 626.70	1 894	36 321.20	185.1	1 177	22 667.30	115.5	19 923.70	101.5	19 923.70	101.5
South-East	18 761.40	1 701	34 845.70	185.7	940	22 113.10	117.9	19 118.10	101.9	19 118.10	101.9
Central Moravia	17 575.60	2 487	34 215.50	194.7	1 260	18 724.30	106.5	17 602.40	100.2	17 602.40	100.2
North-East	17 404.00	1 422	37 251.40	214	751	18 533.10	106.5	17 624.20	101.3	17 624.20	101.3
South-West	16 563.40	3 430	50 158.30	302.8	1 150	18 882.60	114	16 886.80	102	16 886.80	102
Central Bohemia	14 922.70	2 080	32 239.90	216	1 077	17 695.20	118.6	14 963.50	100.3	14 904.20	99.9

*Source: own calculation, monitoring report MMR CR, * share of the allocation*

The highest number of applications (3 430), with the total sum of 50 billion CZK, was submitted in South-West region. At the same time, this cohesion region also had the highest number of unsuccesfull applications; only 1150 applications were approved, which is less than half of the initial number. The financial amount of these approved applications, i.e. signed contracts, was 18,8 billion CZK. At this point it is necessary to point out one problem that is typical for regional operational programmes. In comparisson to the thematic programmes, the allocations for the regional operational programmes are quite modest, and therefore the applications were not rejected only for the irregularities, but also due to a lack of financial resources. The lowest number of applications was submitted in North-West region, however the financial sum for these applications was the second highest. In the perspective of the total financial sum submitted per region the lowest number belongs to the Central Bohemia, but the number of applications was the third highest here. The biggest number of approved applications was in Central Moravia region, but the highest sum of signed contracts can be observed in the Moravia Sileasia region. When it comes to an average size of an approved project, it was 30,5 mil. CZK in North-West region, compared with 13,8 mil. CZK in Central Moravia. In four regions a higher number of less expensive projects was approved.

When looking at the comparisson between the signed contract and certified funding phases, there is no point in evaluating the number of cases any more, since it remains constant for the rest of the process. From the financial point of view the biggest cuts (due to initial allocation) were made in Central Bohemia, where the sum decreased by almost 19 percentage points. Similarly it evolved in the South-East (-16 p.p.) and Moravia Silesia (-14 p.p.) regions. Concerning the refund sum the South-East and Moravia Silesia regions are placed as the most sucessful. In contrast to that, as the least succesful, are North-West and Central Bohemia. In the project realization phase, the most errors were made in the South East and Central Moravia regions, where close to 3 billion CZK were not refunded. The smallest financial cuts were made in North-East, where only 900 mil. CZK were not re-imbursed. Based on the above stated, it can be concluded that in the project realization phase there was not a high number of irregularities, or if so their financial impact was not large. The risk of failure was much higher in the project approval phase, where applications could be rejected for both reasons – applicants errors as well as insufficient allocated amount.

Figure 1: ROP subsidies financial sum in CZK per inhabitant



Source: own calculation, monitoring report MMR CR

The outcomes of the subsidy analysis per region inhabitant are diplayed on the graph in Figure 1. When it comes to allocation amount, the number per capita is highest in North-West, with the allocated sum of 16 215 CZK per inhabitant, whereas the lowest number is revealed for South-East 11 168 CZK per inhabitant. In the stage of submitted applications, the total financial sum of all projects per capita makes 41 477 CZK in South-West, while in the South East region reaches only 20 743 CZK. In overall the order of cohesion regions changed based on the numbers per capita, compared to the previous part of the analysis based on amounts per region. In detail those order changes will be elaborated for the stage of signed contracts. In that phase the highest sum of subsidy per inhabitant is shown for Moravia Silesia region (18 479 CZK), the lowest number belongs to the North-East region (12 290 CZK). In this region it was also submitted the lowest number of applications per inhabitant, whereas in the South-West region the number of applications per inhabitant was the highest. Whilst the total funding sum of approved projects was lowest for the Central Bohemia region, after the recalculation per capita this region was placed before the South-East and also the already mentioned North-East region. For the certified funds, the amount per inhabitant is almost identical in Moravia Silesia and North-West regions. The lowest amount of certified subsidy goes to a citizen of Central Bohemi, North-East and South East regions. From the error rate point of view, the order of regions remains almost unchanged. The only switch observed is between North-East and Central Moravia mutually. During the project realization is then the highest error rate shown in Moravia Silesia region, where it is represented by amount of 2 236 CZK per inhabitant, and the lowest reduction (602 CZK per inhabitant) goes to the North-East region.

4. Conclusion

The most important objective of the Cohesion Policy is to secure EU regions development and competitiveness, and to reduce disparities among them. These objectives are supported by means of investment and structural funds, that enable to allocate financial subsidy to weak regions.

Applying for the subsidy is not an easy process and does not always end by a success. The application process includes a significant administrative burden and together with the following project realization process, they create a space for different types of errors, that can violate the legislation and that are subject of auditing authorities. In result of that, the contracted funding is cutt off, which might lead to financing delays and complications in the project realization, in case the own financial resources are not sufficient. From another point of view, these errors may also lead to a situation where the allocated funding is not implemented, which has also an impact on the next programming periods.

The objective of this paper was to analyze, how individual cohesion regions in the Czech Republic succeeded in the process of EU funds implementation, when it comes to the regional operational programmes.

The most applications for these programmes were submitted in South-West region and also their financial sum was highest here. The least submitted applications were registered in the Norh-West region, despite the fact that this region had the second highest allocation. The most rejected applications are observed in the South-West, and also Central Moravia and Central Bohemia regions. However at that phase the rejection can not be linked to application errors only, but also to the fact that financial sum of applied projects exceeded the allocated sum, in some cases even multiple times. In the context with the overall allocation the most succesful regions were South-East and Moravia Silesia. In contrast to that stands the Nort-West, as the least successful cohesion region, known also for several corruption affairs. The biggest cuts during the project realizations are reported in South-East; the modest reductions belongs to the North East.

From the subsidy analysis per capita it was concluded that the South-West region had the highest financial sum of rejected projects, whereas the South-East region was at the opposite end of this scale. The irregularities in the project realization phase impacted, per capita, most significantly in the Moravia Silesia and Central Bohemia regions. The best results from this perspective had the North-East region.

Acknowledgements

This work was supported by the SGS project of VSB-TU Ostrava under No. SP2018/92. The support is greatly acknowledged.

References

[1] Centrum investic, rozvoje a inovací (2016). *Čerpání dotací ze strukturálních fondů EU v Královéhradeckém kraji v programovém období 2007 – 2013*. [pdf] Královehradecký kraj: Centrum investic, rozvoje a inovací. Available at: < http://www.cirihk.cz/dotace-2007-2013.html> [Accessed 20 July, 2018].

[2] *Council Regulation (EC) No. 1083/2006 of 11 July 2006*. Official Journal of the European Union, Article 3(a).

[3] Český statistický úřad. (2018). *Český statistický úřad.* [online]. Available at: <https://www.czso.cz/> [Accessed 21 July, 2018].

[4] Evropská unie. (2018). *Europa.eu.* [online]. Available at: <https://europa.eu/european-union/index_cs> [Accessed 5 July, 2018].

[5] Ministerstvo pro místní rozvoj ČR. (2016). *Čtvrtletní monitorovací zpráva o průběhu čerpání strukturálních fondů, fondu soudržnosti v programovém období 2007-2013.* [pdf] MMR ČR. Available at: <https://www.strukturalni-fondy.cz/Dotace/media/SF/NOK/Dokumenty/Analytick%C3%A9%20v%C3%BDstupy/Pravideln%C3%A9%20monitorovac%C3%AD%20zpr%C3%A1vy/%C4%8Ctvrtletn%C3%AD%20monitorovac%C3%AD%20zpr%C3%A1va/CMZ_2016_IIQ_elektronicka.pdf> [Accessed 15 July, 2018].

[6] Ministerstvo pro místní rozvoj ČR. (2018). *DotaceEU.cz.* [online]. Available at: < https://www.strukturalni-fondy.cz/cs/uvod> [Accessed 10 July, 2018].

[7] Národní kontrolní úřad. (2017). *Zpráva o finančním řízení prosředků Evropské Unie v ČR, II. díl*. [pdf] Národní kontrolní úřad. Available at: <https://www.nku.cz/assets/publikace-a-dokumenty/eu-report/eu-report-2017-cz-oddil2.pdf > [Accessed 20 July, 2018].

[8] Pénzes, J. (2013). The Dimensions of Peripheral Areas and Their Restructuring in Central Europe. *Hungarian Geographical Bulletin*, 62(4), p. 373-386.

[9] Sucháček, J. (2008). *Territorial development reconsidered.* Ostrava [online]. Available at: <http://www.researchgate.net/publication/47043899 > [Accessed 2 August, 2018].

[10] Transparency International - Česká republika. (2016). *Hodnocení rizik podvodů a jejich možná prevence*. [pdf] Transparency International - Česká republika. Available at: <https://www.transparency.cz/wp-content/uploads/Hodnocen%C3%AD-rizik-a-podvod%C5%AF-a-jejich-mo%C5%BEn%C3%A1-prevence-2016.pdf> [Accessed 1 August, 2018].

[11] Turečková, K. and Nevima J. (2017). Evropské fondy-management rizik v oblasti veřejného školství. *Scientific papers of the University of Pardubice. Series D, Faculty of Economics and Administration*, 2017(41). <http://hdl.handle.net/10195/69606> [Accessed 15 June, 2018].

Increasing the stability of the national currency of Kazakhstan

Gulnur Dautaliyeva[1]

Abstract

The modern currency market is characterized by high volatility of exchange rates, which is the forerunner of the beginning of the crisis phenomena. The uncertainty of such important parameters as the frequency, amplitude and rate of change in the exchange rate, which has increased in recent years, significantly reduces the predictability of the conditions in which business decisions are made, and also provokes huge currency risks for economic entities of any level. Instability of the exchange rate of the national currency, characteristic for the Republic of Kazakhstan today, negatively affects the economy of the country, creating uncertainty for Kazakh banks and business entities in terms of estimating future cash flows, threatening them with large losses and even bankruptcy.

Key words

national currency, data analysis, currency risk, correlation, exchange rate, tenge, ruble, USD dollar

JEL Classification: C19, C53, D83, E31, H 50, F 31

1. History of the national currency

National money represents one of the key elements of the market economy and serves as an important lever of the mechanism of state regulation of its effective development. As its national money, the Republic of Kazakhstan introduced its own money symbol - tenge, which has been successfully used for 15 years. The Tenge occupies a special place in the system of key elements and mechanisms of the mechanism by which the state regulates the market economy in order to ensure its normal functioning and effective development.

In its essence, tenge is a rather complex socio-economic phenomenon; it acts as an indulgence and quintessence of the success of functioning of a market economy in Kazakhstan, its direct creation as a kind of visiting card.

As you know, economically developed countries have a stable national currency. In addition, for weak countries, the national currency is often unstable. Hence, the natural relationship is obvious - a stable currency contributes to the formation of a strong economy. National money is mediated by virtually all commodity, investment, financial flows that take place in a market economy. Even where money is not directly used, for example, in transactions of natural exchange between enterprises, they continue to perform the function of a counting unit.

The national currency that symbolizes Kazakhstan's independence is tenge. It was put into circulation on November 15, 1993, according to the Decree of the President of the Republic "On the Introduction of the National Currency of the Republic of Kazakhstan" of November 12, 1993. The name "Tenge" originated from the Turkic language meaning money with

[1] researcher, graduate Peoples' Friendship University of Russia, Tax financial manager LLC “UNITED TRADING”, gulnur_dautalina@mail.ru

variants of "tanga", "donga", "tenga". Its development took place in a short time and under the status of secrecy. Harrison and Sons printed the first batch of tenge in the UK. It was this order that brought the companies' world fame and special award to the Golden Sign of the Queen of Great Britain. Tenge was recognized as the best export product in England; therefore, in the office of the company, a shield with a picture of the national currency of Kazakhstan hangs in a place of honor. Today, tenge is considered the most beautiful and protected currency in the world. It possesses 18 degrees of protection, two of which are used in printing dollars and euros.

In this phenomenon, the following stages of the devaluation of the Kazakhstani tenge are highlighted (see Table 1).

Table 1: History of the tenge devaluation period

1994	the dollar rate increased from 4.70 tenge, set at the time of introduction of the national currency, to 54.26 tenge, or more than 11.5 times
1999	exchange rate of the national currency decreased by 64.6% to 138.25 tenge per dollar
2009	The dollar exchange rate increased from 120 tenge to 150 tenge. The National Bank pointed to two reasons for the devaluation: it is a drop in world oil prices and the devaluation of national currencies against the dollar in the trading partners of Kazakhstan.
2014	The rate of tenge was fixed in the range from 155.43 to 185.99. Among the main reasons are the bad economic situation in the BRICS countries, the flow of capital from developing countries and the free exchange rate formation of the Russian ruble.
2015	in August, it was announced that the tenge was released into the free market rate of education, and the rate exceeded the level of 350 tenge per dollar.

National Bank of Kazakhstan identifies three reasons for devaluation of tenge:

1. The devaluation is a fall in oil prices. Since oil is about 60% of the total exports of Kazakhstan and is the main source of income on the market.
2. Allocate trade partners of Kazakhstan and states whose economy depends on the export of raw materials.
3. Decrease in the tenge is the need to preserve the gold and foreign exchange reserves of the National Bank of the Republic of Kazakhstan.

Inflation is also considered one of the most unfavorable and dangerous process. In Kazakhstan, inflation is due to the "excess" of money supply in the country, due to monopolistic and oligopolistic price inflation, because of a lack of competition and state regulation of the economy (see Table 2).

Table 2. Inflation rates in Kazakhstan in 1991-2018 in %[2].

1991 – 147,1	1992 – 2960,8	1993 – 2165,0	1994 – 1160,2
1995 – 59,9	1996 – 28,7	1997 – 11,3	1998 – 1,9
1999 – 18,1	2000 – 10,0	2001 – 6,5	2002 – 6,7
2003 – 7,0	2004 – 6,8	2005 – 7,9	2006 – 8,4
2007 – 18,8	2008 – 9,5	2009 –6,3	2010 – 7,9
2011 – 7,4	2012 – 6,0	2013 – 4,9	2014 – 7,5
2015 – 13,6	2016 – 8,3	2017 – 7,2	2018 for 1st half-year – 5,9

Inflation Review. As we see, the highest level of inflation falls on 1992, it amounted to 2 960.8%. To reduce and stabilize it, the National Bank of the Republic of Kazakhstan (the NBK - *further*) has pursued a tight monetary policy in the state. It manifested itself in the control of the money supply, the gold and foreign exchange reserves and government securities. The result was a decrease in inflation by 1998 to 1.9%. Nevertheless, by 1999 there has been an increase to 18.1%. From 2000 to 2006, there is an average fluctuation of inflation.

By 2007, there has been a sharp jump in inflation, in the first place, it was due to external factors. These factors include the global financial crisis, food and energy crises. This process was influenced by the availability of credit resources, both for entrepreneurs and the general population, the depreciation of the dollar against the euro. Also influenced by the increase in prices for grain, oil, hydrocarbons and other products of common use. During this period, there is an increase in prices for food products by 26.6%, non-food products by 10.5%, paid services by 15.4%.

From 2008 to 2013, there was a decrease in inflation from 9.5% to 4.9%. However, in 2014, there is an increase, and it was 7.4%. This is associated, primarily, with the fall in the exchange rate of the national currency.

In 2013, Kazakhstan moves to using the mechanism of binding the tenge to a multi-currency basket, which includes the dollar, euro and ruble. The purpose of this mechanism is to change the exchange rate policy of the state, i.e. The NBK will be guided by the dynamics of changes in the three currencies. The structure of this basket includes 70% of the dollar, 20% of the euro and 10% of the Russian ruble. The advantage of the multicurrency basket is a gradual decrease in the dollar's impact on the country's economy as a whole, a decrease in speculative sentiment in the foreign exchange market and mitigation of the negative consequences from the inflow and outflow of speculative capital.

Kazakhstan remains a commodity country and therefore the main external factor of influence on the exchange rate of the national currency is the price of oil on world markets. The price conjuncture for June-July 2014 was favorable, and then the price for Brent oil was higher than $ 100 per barrel, which contributes to foreign capital inflow to the country's domestic currency market. Nevertheless, to date, the price of oil has fallen to 56.12 dollars per barrel, which does not have a very good impact on the economy of Kazakhstan. In addition, the situation on the foreign exchange market was influenced by the situation of the country's main trading partner, Russia. Reduction of the ruble to tenge led to the fact that it caused an increase in imports and a reduction in production within Kazakhstan.

The main reasons why central banks have to devalue are the worsening of the country's trade balance and high inflation, when imports prevail over exports. As a result of devaluation, imports are becoming more expensive, and exports are cheaper. This helps the

[2] Tables of Annual Inflation in Kazakhstan - https://www.statbureau.org/ru/kazakhstan/inflation-tables

state fulfill several functions and solve several macroeconomic and external economic problems:

- improve the country's balance of payments, devaluing its national currency,
- to increase the competitiveness of domestic goods in international trade,
- to stimulate domestic production, which starts to work for substitution of imports.

To date, the national currency has a huge impact on the development of the economy of Kazakhstan. If the tenge rate goes up, this is a favorable factor, as its purchasing power increases, hence the economic security of the country increases. If the rate of tenge decreases, this leads to a decrease in the economic security of Kazakhstan. To prevent this, the state should conduct measures to increase the stability of the national currency. In general, it is necessary to conduct a strict monetary policy, to keep inflation within the established corridor, which now is 68%. We also need to ensure price stability, reduce the outflow of speculative capital, and reduce the dollarization of the economy and the volatility of interest rates.

As a tool to improve the accuracy of forecasting the exchange rate, taking into account the multiplicity of exchange rate factors and the instability (variability) of the values of their quantitative parameters, one can use the methods of economic-mathematical modeling and forecasting: correlation regression analysis and construction of econometric models, whereas the resultant indicator will be to act as the official exchange rate of Kazakhstan tenge in relation to major currencies.

In our opinion, the most important quantifiable factors directly or indirectly affecting the exchange rate are: the volume of GDP, the amount of money in circulation, the level of inflation, the level of interest rates, the balance of the country's balance of payments, the volume of the country's gold and foreign exchange reserves.

In addition, as a factor that takes into account the specifics of exchange rate formation in the Republic of Kazakhstan under the conditions of a peg to a basket of currencies, the main share of which is currently formed by the Russian ruble is the level of oil prices. The choice of this factor is due to its direct impact on the stability of the Russian ruble in view of the current structural imbalances in the Russian economy, which is highly dependent on energy exports. The Russian ruble, in turn, as the integration processes intensify, increasingly affects the positions of the national currency of our country.

For the purposes of this study, we used stochastic factor analysis, since it examines the influence of factors whose relationship to the resulting indicator is incomplete or probabilistic (correlation), and the method of mathematical predictive extrapolation.

To build the correlation-regression model, we analyzed the monthly data for the last six years (2010-2016) (the number of observations is 10 times greater than the number of variables).

As an example, consider the correlation-regression parameters for the US dollar. The significance of the coefficients of the multiple regression equation was estimated using the t-test of Student. The table value of Student's t-test at 95% significance level and 65 degrees of freedom is 1.27. The condition for calculating the table corresponds to all the coefficients, respectively, the coefficients $\alpha1$, $\alpha2$ and $\alpha3$ are significant (significant).

The coefficient of multiple correlation is 0.97, shows the high closeness of the relationship of the dependent variable to the three explanatory factors included in the model. The coefficient of determination shows that about 94% of the variation of the dependent variable is taken into account in the model and is due to the influence of the included factors.

We estimated the significance of the regression equation based on the calculation of Fisher's F-criterion. According to the calculations, F = 335.2. The table value of the F-test with a confidence probability of 0.95 is 19.48. Since F> Ftable, the regression equation should be considered statistically reliable and suitable for the purposes of our research. The equation of regression of the dependence of the exchange rate of the tenge against the dollar on the money supply, the refinancing rate and the state of the trade balance is presented in the following form:

$$Y = 471 + 37\,X1 + 85X\,2 + 36\,X3 \text{, tenge, (1)}$$

where Y is the exchange rate of tenge to dollar; X1 - money supply; X2 - refinancing rate; X3 - balance of trade balance.

The extrapolation of the trend line for each indicator is shown in Fig. 1-3.

Figure 1. Extrapolation of the polynomial trend of the money supply in the Republic of Kazakhstan for the first quarter of 2017.

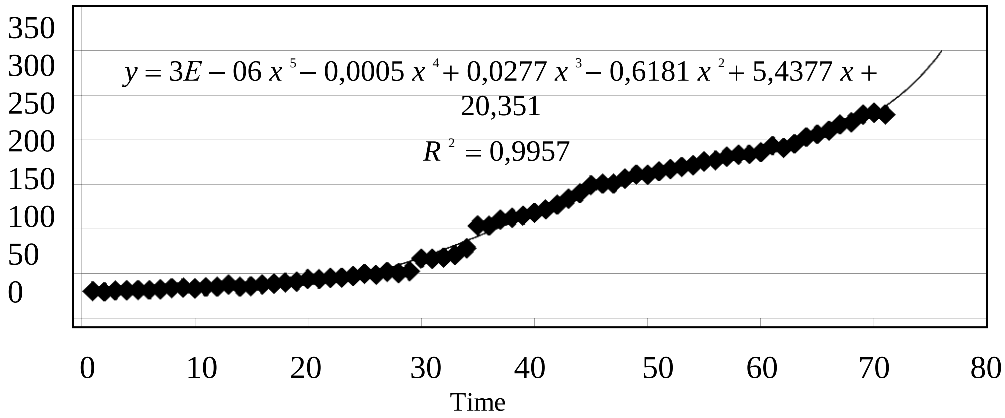


Figure 2. Extrapolation of the polynomial trend line of the change in the refinancing rate in the Republic of Kazakhstan for the 1st quarter of 2017.

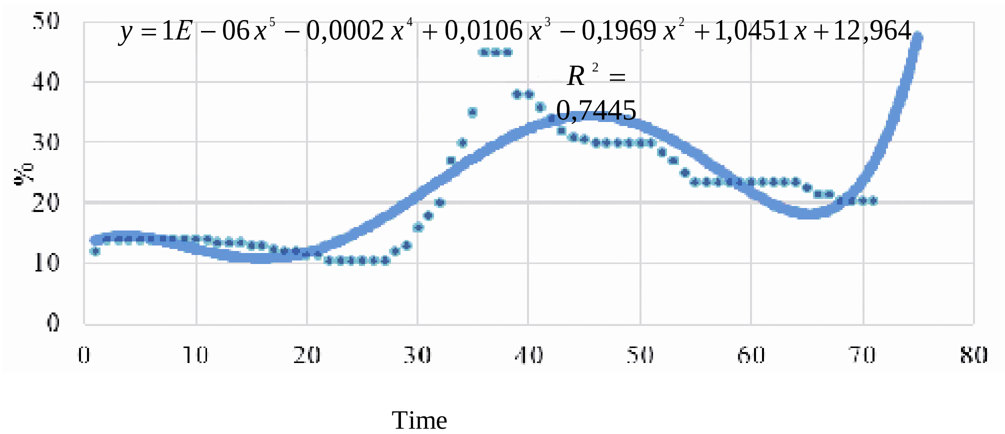
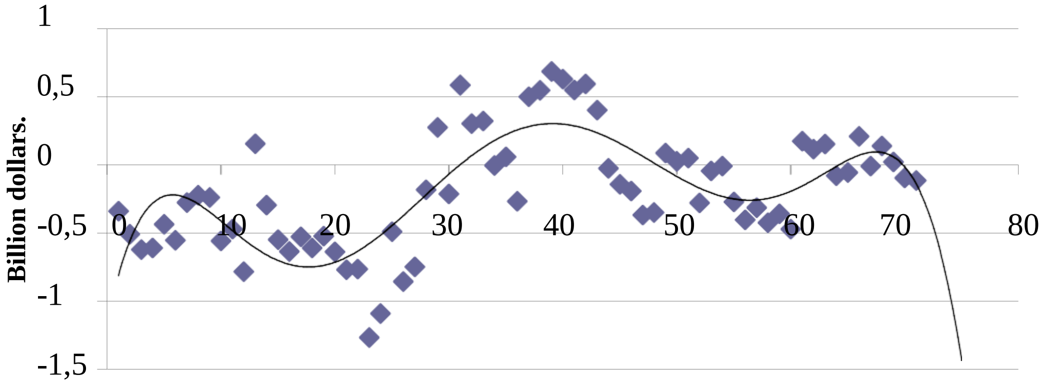


Figure 3. Extrapolation of the polynomial trend line of the change in the trade balance of the Republic of Kazakhstan for the 1st quarter of 2017.



On the basis of the data obtained, we can predict the values of the money supply, refinancing rates and trade balance for the first quarter of 2017 (Table 3).

Table 3: Forecast values of factors influencing the exchange rate of tenge to major currencies for the 1st quarter of 2017.

Index	On 01.01.2017		On 01.02.2017		On 01.03.2017	
	plan	fact	plan	fact	plan	fact
Money supply, trillion tenge	255	240	287	270	300	270
Refinancing rate, %	13	10	16	12	15	11
Trade balance, billion dollars	–0,62	0,62	–0,99	0,06	–1,49	0,94
Exchange price for BRENT oil, $ per barrel	72	85	68	65	60	49

** A similar situation develops with other currencies in the currency basket (euro, ruble).*

The forecasting technique described above is reduced to the implementation of the following main stages:

- determination of the most important factors affecting the exchange rate of the national currency;

- the establishment of the actual presence of a correlation between the exchange rate of the national currency and the factors under consideration;

– construction of the regression equation, verification of its significance and reliability, elimination of multicollinear factors;

- forecasting the future value of the factors included in the regression equation using mathematical predictive extrapolation; if necessary, the application of the method of exponential smoothing of time series;

– determination of the forecasted value of the exchange rate of the national currency by substitution in the regression equation of the predicted indicators.

Under these conditions, it becomes obvious that the government needs to focus its attention on regulating the most important macroeconomic parameters that affect the positions of tenge:

– to establish target restrictions on the growth of the money supply in the country for a specific period, relying on the possibility of regulating the velocity of money circulation in the economy (which follows from the Fisher equation);

- Flexibly use such an instrument of monetary and monetary policy as the refinancing rate, which is an indicator of inflationary processes occurring today in the Republic of Kazakhstan;

- to ensure and do everything possible to maintain the positive balance of foreign trade as the main source of foreign currency for the development of the country's economy.

Thus, the proposed methodology allows not only to predict the exchange rate, but also points to the specific causes (factors) of the crisis phenomena in the foreign exchange market, the analysis of which is necessary to develop a set of measures to improve the currency sphere. At the same time, the accuracy of the forecast depends on the international position of the dollar, euro and ruble. Therefore, possible sharp fluctuations of these world currencies because of political or economic shocks cannot be taken into account in forecasting and can reduce the accuracy of the calculations.

An example is the sharp weakening of the Russian ruble in world markets since the fall of 2014 against the backdrop of economic and political sanctions of the European Union. As a result, the exchange rate of the Tenge is symmetrically decreasing relative to the Russian ruble, which, however, does not mean a real appreciation of the national currency.

References

[1] Chinn, M. (2006) Real Exchange rates / M. D. Chinn // University of Wisconsin. – MadisonAnd National Bureau of Economic Research, – 7 p.

[2] Engel, C. Exchange-Rate Models / C. Engel // The National Bureau of Economic Research Reporter: Research Summary, 2006. – 4 p.

[3] Currency of the Republic of Kazakhstan http://www.nationalbank.kz/?docid=3321&switch=english

[4] Base interest rate of the NBK http://www.nationalbank.kz/?docid=107&switch=english

[5] Bakanov MI, Melnik MV, Sheremet AD (2005) Theory of economic analysis. Textbook 5-eizd., Pererab. и доп.-M .: Finances and Statistics

[6] Artemenko VG, Blendir M.V. (2009) Financial Analysis: Textbook. - 2 nd ed., Pererab. and additional. - M .: "Business and Service"; Novosibirsk: "The Siberian Agreement".

[7] The official internet-resource of the Committee of statistics http://stat.gov.kz/faces/homePage;jsessionid=B_tEaEu05nzpXsacS86jm5p9lWapuN_IfYwoRmS3HRpCl1VC27J0!1738043929!-1234667160?_afrLoop=1650195653783849#%40%3F_afrLoop%3D1650195653783849%26_adf.ctrl-state%3Dw47bayfpy_4

On the issue of commercial insurance market as a part of financial market in the Slovak Republic

Barbora Drugdová[1]

Abstract

The insurance industry in the Slovak Republic has become an important dynamically developing area of economy. Insurance affects all the activities into the national economy, touching every company, business, citizen, society, and foreign countries. The Slovak insurance market is developed. As at 31. 12. 2017, there operated 21 commercial insurance companies on the Slovak commercial insurance market. The evolution of the market in life insurance in recent years is more dynamic than in the non-life insurance.

Key words

key insurance, commercial insurance market, life insurance, non-life insurance, indicators of the insurance market

JEL Classification: JEL22

1. Introduction

Insurance as a non-productive sector is of a vital importance to the national economy of every advanced and modern economy. Insurance, as one of the ways of covering the risks, is part of the finances, namely the finances of all the entities of the national economy. Insurance plays an important role not only in the insurance of business risks, but also in international risks, which we include according to Insurance Act no. 39/2016 Coll. on Insurance in the Slovak Republic among non-life risks. Insurance and insurance intervene directly or indirectly in all areas of human activity, so they affect every business, enterprise, citizen, society, and foreign. All changes in the company are also in some way reflected in the insurance business. The development of science, technology and culture brings new modern products, new activities and new risks to the market. These risks must be taken into consideration and calculated by businesses and entrepreneurs, citizens, the state, and insurance companies. Therefore, commercial insurers have to prepare new insurance products for the insurance market, while monitoring developments throughout the economy (e.g., unemployment, GDP growth, inflation, price growth, money income movements, demographic data, etc.). While explaining the essence of insurance, we assume that the operation of the insurance market mechanism involves the transfer of risk from one entity to the Community and the joint bearing of losses under predetermined conditions by all members of that community.[2]
Since January 1, 2004, the Slovak insurance market is an inseparable part of the European Union's single insurance market. In practice, this means that commercial insurance companies in all EU Member States have the option to perform insurance and reinsurance activities in the Slovak Republic. This is also true of the reverse relation: domestic insurers in the Slovak

[1]Ing. Barbora Drugdová, PhD., University of Economics in Bratislava, Dolnozemská Rd. No. 1, 852 35 Bratislava, Slovak Republic; tel: 02/672 91 539,email: barbora.drugdova@euba.sk
[2] VAUGHAN,E.J. – VAUGHAN, T.M. 200. *Fundamental of Risk and Insurance.* New Jersey: John Willey and Sons, 2007, p. 34

Republic can offer their services to clients in other countries of the European Union. In connection with the integration into the insurance market of the European Union, it was necessary to adapt the legislation of the Slovak Republic in the field of insurance and the insurance industry to the legislation of the European Union, which affected both life and non-life insurance. The last law, which came into force on 1 January 2016 was Act No. 39/2015 Coll. on insurance. Under this law, there are changes that affect individual sectors in both life and non-life insurance.

2. Characteristic of the Insurance and its Delimitation/Definition

The insurance market is the place where supply and demand for insurance come together. Insurance is a specific commodity, the essence of which is the transfer of risk. The insurance market operates on the basis of market principles. The insurance market has a significant position in each market economy with specific roles, principles and meanings. pretože ponúka nástroje na riadenie rizík, ktoré súčasná „riziková spoločnosť“[3] produkuje. The insurance industry, covering the insurance market, has an irreplaceable position in every economy because it offers the risk management tools that the current "venture company" produces. As a result, it is also referred to as a secondary sector of economic activity because it deals with the negative consequences that would have occurred even if the insurance had not existed.

In the literature, we can find various definitions and characteristics of the insurance market. Let us just mention some of them in the Czech, Polish and Slovak sources. For example, in the Czech literature, J. Daňhel states: "The insurance market has roughly the same features as any other market and for the smooth meeting the demand, the so-called intermediaries" enter between the clients and the insurers. V. Čejková expanded the definition with another dimension by using in her monograph: "The Insurance Market" in defining the insurance market by means of comparing it to "a living and developing organism that is constantly changing and developing". In the Polish literature, A. Szromnik defines the insurance market as follows: "The insurance market is a place where supply and demand for insurance come together. The demand side consists of residents and businesses that, by buying insurances, transfer risks to insurance institutions, which represent the supply side". In the Slovak literature, A. Majtánová states, "the insurance market is a market that can be divided into a real insurance market and an investment insurance market"[4].

The subject of the insurance market is insurance and reinsurance. Insurance, as one of the important sectors of the market economy, offers its services in the insurance market, or insurance and reinsurance. The insurance industry includes insurance houses, insurance companies, insurance companies from other Member States, branches of an insurance company from another Member State, foreign insurance and reinsurance undertakings from another Member State, insurance intermediaries, insurance supervision, insurance association, insurers' office, insurance institutions offering insurance apart from their main business (banks, leasing companies, car services, etc.) and other entities. In the Slovak commercial insurance market, similarly as in other European Union countries and in the advanced world in a strong competitive environment, commercial insurance companies and their intermediaries are applying for clients – the insured. Similarly, as in other markets, the condition of fair competition includes the creation of quality insurance products and their price evaluation (reasonable price for optimum insurance protection).

[3] BECK, U. 1992. *Risk Society: Towards a New Modernity*. London: Sage, 1992. 260 s. ISBN 97-808-039-8345-8
[4] MAJTÁNOVÁ, A.: *Poistný trh*. Bratislava: Elita, 1993, p. 11. ISBN 80-85323-32-X

The commercial insurance market, its scope, efficiency, and the importance in terms of the national economy can be characterized by several indicators. Among the most frequent indicators we encounter in the literature are the following: the number of commercial insurers, technical premiums, the share of life and non-life insurance in GDP, the ratio of life and non-life insurance, the number of employees of commercial insurers (number of internal and external employees), insurance products, and other indicators.
In the next part, we analyze selected indicators of the commercial insurance market for 2017.

3. Analysis of Selected Indicators of the Commercial Insurance Market in the Slovak Republic for the Year 2017

In 2117, twenty-one commercial insurance companies operated in the Slovak insurance market, creating a relatively strong competitive environment among them. Of the total number of insurers operating on the Slovak insurance market, 21 commercial insurance companies were associated in the Slovak Insurance Association as of 31 December 2017. Of the members of the Slovak Insurance Association, fourteen commercial insurance companies had universal/general licence; life insurance was provided by four commercial insurers and non-life insurance by three commercial insurance companies. (Spring: Slovak Insurance Association (SAP) Internal Materials, Bratislava, 2018).
The most important indicators of the commercial insurance market include technical premiums. Table 1 shows the technical premiums as at 31 December 2016 in thousands of euros and that as at 31 December 2017 thousands of euros. We present both years to compare the development of this indicator of the insurance market.

Tab. 1 Technical premium insurance premium as at 31 December 2016 and as at 31 December 2017 in EUR thousands.

Year	Life insurance	share	Non-life insurance	share	total
2016	1,145,414	54.3 %	964,579	45.7 %	2,109,993
2017	1,279,400	52.3 %	1,167,881	47.7 %	2,447,281

Source: Internal materials, SAP, Bratislava 2016

In the next part of the paper, we compare the years 2016 and 2017. We will analyze the total technical premium as at 31 December 2016 and as at 31 December 2017 and the technical premium of life insurance and that of non-life insurance as at 31 December 2016 and 31 December 2017.
The total technical premium as at 31 December achieved the amount of 2,109,993 thousand euros.
Of this, the technical life-insurance non-insurance premium amounted to 1,145,414 thousand euros and the non-life technical insurance premium was 964,579 tis. euros.
As at 31 December 2017, the total technical premium achieved the amount of 2,447,281 thousand euros.
Of this, the technical premium of life insurance amounted to 1,279,400 thousand euros, and the technical premium of non-life insurance was 1,167,881 thousand euros.
The total technical premium in 2017, was 2,447,281 thousand euros in comparison with the total technical premium in the year 2016, which was 2,109,993 thousand euros – it was higher in comparison with preceding year.

The share of life insurance on the total technical premium in the year 2017 was 52.3% and in the year 2016 it was 54.3% – it was a decline in comparison with the preceding year.
The share of non-life insurance on the total technical premium in the year 2017 was 47.7% and in the year 2016 the amount was 45.7 % – an increase in comparison with the previous year.
The ratio of life insurance and non-life insurance in the year 2017 was 52.3 : 47.7, and in the year 2016 it was 54.3 : 45.7 – in favour of life insurance.

Table 2 depicts the technical premium in life and non-life insurance as at 31 December 2017 in thousand euros and the participation of individual commercial insurance companies in the total technical insurance premium on the Slovak insurance market.

Tab. 2 . Technical insurance premium in life insurance and non-life insurance as at 31 December 2017 in EUR thousands.

	Technical insurance premium (in EUR thousands) as at 31 December 2017		
No.	**Commercial insurance company**	**Technical insurance premium in life insurance**	**Technical insurance premium in non-life insurance**
1	AEGON Life Insurance company, a.s.	48,690	0
2	Allianz – Slovenská poisťovňa/Slovak Insurance company , a. s.	239,329	364,039
3	ČSOB Poisťovňa/Insurance company, a. s.	48,724	37,619
4	ERGO Life Insurance company, a.s.	6,509	870
5	Generali Slovensko insurance company, a. s.	94,735	130,280
6	NN Life Insurance company, a. s.	74,570	0
7	KOMUNÁLNA insurance company, a.s.VIG	108 202	87,229
8	KOOPERATIVA insurance company, a. s., VIG	202,263	226,431
9	Insurance company Cardif Slovakia, a.s.	6,447	17,750
10	Poštová poisťovňa/Post Office Insurance company, a.s.	12 494	1,784
11	Insurance company of the Slovak Savings Bank, a. s. VIG	154,122	214
12	UNION Insurance company, a. s.	9 894	43 221
13	UNIQA Insurance company, a. s.	37,594	92,062
14	Wüstenrot Insurance company, a. s..	31,992	25 198
15	AXA, Life-insurance company, a. s. branch of insurance from another Member State	55,214	13,219
16	AXA, insurance company, a. s. branch/agency from another Member State	0	0
17	BASLER, Sachversicherungs, insurance company, a. s. from another Member State	0	4 669
18	Collonade Insurance, S.A., insurance company, a. s. from another Member State	0	23,388
19	D.A.S.	0	2,862
20	Groupama insurance company, a.s., branch/agency	370	7,628
21	MetLife, Europe	120,326	6,317

22	Slovak Office of Insurers	0	11
	Total	**1,229,254**	**1,167,881**

Source: Internal materials, SAP, Bratislava 2018

The total technical premium in 2017 was 2,447,281 thous. euros. As can be seen from Table 2, technical insurance in life and non-life insurance as at 31 December 2017 (ths. Euro) of the insurance company Allianz – Slovenská poisťovňa, a. s., indicates it was the leader in the Slovak insurance market in the life insurance business with 239,329 thousand euros and in non-life insurance with CZK 364,039 thousand euros.
The second position was occupied by KOOPERATÍVA, insurance company, a. s. in the area of life insurance amounting to 202,263 thous. euros and also in non-life insurance amounting to 226,431 thous. euros; the third place in life insurance business was taken by the Insurance company of the Slovak Savings Bank, a. s. VIG with 154,122 thous. euros and in non-life insurance by the insurance company Generali Slovensko, insurance company, a. s. with 130,280 thous. euros.

4. Conclusion

We can state in conclusion that the Slovak insurance market is a dynamically developing market, which forms part of the financial market. The total technical premium in the year was 2,447,281 thousand euros. The share of life insurance of the total technical premium in the year 2017 was 52.3% and in the year 2016, it was 54.3% – a decline in comparison with the preceding year. The share of non-life insurance on the total technical insurance premium in the year 2017 was 47.7% and in the year 2016, it was 45.7% – an increase in comparison with the preceding year. The ratio of life insurance to non-life insurance in the year 2017 was 52.3: 47.7 and in the year 2016, it was 54.3: 45.7 – in favour of life insurance. The population is gradually becoming aware of life risks and uses insurance products of life insurance. The offer of insurance products in the area of life insurance as well as non-life insurance via commercial insurance companies on the Slovak insurance market is flexible.

References

[1] Comité Européen des Asurances European. Paris, 2012.

[2] DAŇHEL, J a kol.: Pojistná teórie. Praha: PROFESIONAL PUBLISHING, 2005. 205 s. ISBN 80-86419-92-0.

[3] ČEJKOVÁ,V.: Pojistný trh. Praha: GRADA, Publishing, s.r.o., 2002, s. 20. ISBN 80-247-0137-5

[4] DRUGDOVÁ, B.: POISTENIE A POISŤOVNÍCTVO, nemecko-slovenský a slovensko – nemecký slovník, ELITA, 1995, Bratislava, 183 s. ISBN 80-85323-96-6

[5] DRUGDOVÁ, B.: Neživotné poistenie v teórii a praxi. Bratislava: ŠEVT, 2012. 189 s. ISBN 987-80-8106-051-9

[6] PASTORÁKOVÁ, E., VESELOVSKÁ, S., DRUGDOVÁ, B.: Vybrané problémy poistného a zaistného trhu. Bratislava: Vydavateľstvo EKONÓM, 2006. 160 s. ISBN 80-225-2273-2.

[7] MAJTÁNOVÁ, A.: Poistný trh. Bratislava: Elita, 1993, s. 11. ISBN 80-85323-32-X

[8] SIVÁK, R. a kol.: Riziko a neistota vo financiách. / vybrané problémy/. 185 s. Bratislava: EKONÓM. 2009 ISBN 80-225.1956-1

[9] SZROMNIK, A.: Rynek ubezpieczeniowy. Krakov: Wydawnictwo Akademii Ekonomicznej v Krakowie 2001, 62 s. ISBN 83 – 7252 – 084 -4

[10] 10.VAUGHAN,E.J. – VAUGHAN, T.M. 200, Fundamental of Risk and Insurance. New Jersey: John Wlley and Sons, 2007, p.252. ISBN 978-0-470-08753-4

The paper is an outcome of the research finnaced by the Ministry of Education, SR, research project VEGA No. 1/0776/16; project title: “Taxation of the Finance Sector and Harmonisation Trends in the European Union”.

Disposition Effect in the Experimental Currency Trading

Hana Dvořáčková[1], Marek Jochec[2]

Abstract

The disposition effect has been described in the stock-investing context as a behavioral tendency of investors to hold on to losing stocks for too long and sell winning stocks too soon. In this paper it is examined, whether the disposition effect can be confirmed also in the experimental currency trading data set. The presence of the disposition effect leads to the conclusion that students, despite using demo money, were trading with real behavioral bias.

Key words

Behavioral finance, currency trading, experimental finance

JEL: G41, J93

1. Introduction

The disposition effect has been described in the stock-investing context as a behavioral tendency for investors to hold on to losing stocks for too long and sell winning stocks too soon. It is one implication of the Kahneman and Tversky´s (1979) prospect theory to investment. According to them, a person who has not made peace with his losses tend to accept gambles, which would not be acceptable otherwise. According to Odean (1998) the most obvious explanations—explanations based on informed trading, rebalancing, or transaction costs—fail to capture important features of the data. For his paper there were analyzed records for 10 000 accounts at a large discount brokerage house to prove the tendency of investors to hold losing investments too long and sell winning investments too soon. First paper, labeling this behavioral effect as the disposition effect, was published by Shefrin and Statman (1985).

Over time there were published many studies focused on the tarder´s behavior and the disposition effect. For example Barberis and Xiong (2009) investigated whether prospect theory preferences can predict a disposition effect, Kaustia (2010) included the chapter focused on the disposition effect into the book Behavioral Finance: Investors, Corporations, and Markets. In the Choe and Eom (2009) was examined, whether the disposition effect exists in the Korean stock index futures market. There was found the strong evidence for the disposition effect and explained in terms of of investor characteristics. Chen, Kim, Nofsinger and Rui (2007) studies the investment decision making in an emerging market. According to them Chinese investors besides other tend to sell stocks that have appreciated in price, but not those that have depreciated in price, consistent with a disposition effect, acknowledging gains but not losses. Results of Marciukaityte and Szewczyk (2012) are consistent with the proposition that the disposition effect increases the supply of winning stocks and depresses their prices. Contrary Locke and Mann (2005) found no evidence of any contemporaneous measurable costs associated with the Disposition effect.

The aim of the paper is to determine, whether the disposition effect is present in the experimental data sample, while it is supposed, that its presence supports the presumption that

[1]Hana Dvořáčková, VŠB-TUO, hana.dvorackova@vsb.cz
[2] Marek Jochec, Nazarbayev University, Graduate School of Business, marek.jochec@nu.edu.kz

students had the same, or at least similar biases, as real traders. The upcoming section 2 is focused on the data set information and the methodology of the data collection and then in section 3 are presented results of the research in respect with the aim of the paper.

2. Data and Methodology

The data was collected by Jochec from 2009 to 2015 during his lectures. Students were trading on the OANDA FX Trade Practice platform with currency pairs and CDFs. Initially, the students were given 100 000 US, meanwhile the trading period was standardized and it took three months. As the students did not trade with real money, they were motivated to achieve as good result as possible by a financial reward and extra points for the exam for the winner (student with the highest account balance at the end of the trading period). One of the learning objective was to experience first-hand currency trading. At the end of the trading period students had to submit detailed recorded trading history together with answers to a short questionnaire and demographic information. There was collected also the student's login and password of his/her official game account, therefore it was not possible to change the account later, reset the losses, use more accounts etc. Based on the collected information there was created a unique dataset of experimental trading data linked to individual student traders. There were 292 students involved in the research over the time, who made in total 12 416 trades with the total volume of over three billion units. From the above-mentioned students, 43% (125 students) became profit makers. That is to say that their account balance at the end of trading period was higher than 100 000 USD. Regarding the gender diversity of traders, there were involved 120 females and 172 males. Overall, there were 29% of trades made by females, 71% by males.

The setting of the game in the currency markets is convenient; currency markets are liquid and close to efficient. It is difficult to make meaningful price predictions and trading is more a matter of luck then skill. Thus, the skill component does not distort the picture and the trading patterns and strategies tend to be more behavioral in nature.

The data generation and collection process proceeded as follows: The course was started with a series of lectures and assignments designed to explain the currency trading basics and the use of the trading platform. The game was launched sometime in the second to fourth week, and was running for the rest of the semester (60 to 90 days). Soon the focus shifted on other topics and the game continued in the background. The rules and the interference with the students were minimal, students were not asked for any specific strategy, neither encouraged nor discouraged the use of fundamental or technical analysis; there was no "desired" amount, frequency, size, or currency of trades. The winner was the student with the highest trading account balance in the end. The ending profit or loss did not affect the course grade except that the winner (and only the winner) earned few extra points towards the final course grade and in some cases a voucher to a bookshop.

The experimental setting has obvious disadvantage that the money is not real and thus the joy of winning (pain of losing) is moderated. This should be slightly counteracted by awarding the winner. The counterargument might be that the "winner takes it all" reward scheme is problematic, there is no incentive for scoring second (third...); similarly, scoring low does not bear any penalty. This and the fact that it is hard to predict currency rates even for professional traders, means that the students were in effect encouraged taking higher-than-normal risk and engage in "all or nothing" gamble. It was not possible to rule those problems out, however, there is no indication of more frequent occurrence of large bets on the last few days of the game, which would pointing out a tendency towards gambling. It can be assumed that the students derived some benefits also from simply doing well, even if not the best. This could result from the long-term continuance of the experiment and the psychological benefit (cost) of favorable (unfavorable) comparison with the peers, and, perhaps most importantly, by keeping the current winner and his/her balance at strict confidentiality. This conjecture is supported by student comments and informal feedback. The assumption is, that in spite of the singular incentive, the students chose investment strategies without trying to "game the system"[3] or engaging in an ultimate all-deciding gamble. The indirect evidence will be shown in subsequent sections.

There are also advantages of the experimental setting; firstly, participants do not self-select into roles and thus the sample is less biased: take, for example, an effect of gender on trading decisions. To compare behavior of actual female and male traders is problematic because the female traders might have some male characteristics which made them select the trading profession and helped them succeed in it (self-selection and survivorship bias). The sample of students/traders does not represent the population ideally, because all of the decided to study business, but not all of them would like to do the trading career. Second advantage of the experimental setting is the homogeneity of objectives. In real situation, someone might set up a currency position as a hedge for some other asset. For example, if somebody's savings are predominantly in Euros but expected spending in dollars, that person might want to open a short position in EUR/USD in order to hedge the Euro exposure in the savings account. The loss on currency position offsets the gain on the savings account (and vice versa), thus decreasing the volatility of wealth. In case that the real traders behavior is analyzed, their goals are not known, are they speculating or hedging? The different objectives would lead to different trading strategies. Moreover, in the real life, the trader's wealth is given by the sum of different assets, therefore a loss in Oanda may be compensated by gain in another asset, and a high net asset value person may trade differently from a low net asset value person.

In spite of not being offered monetary compensation, the data shows that students took the trading game seriously. It is supposed that it was partly caused by the fact, that students found the game interesting, as shown by the questionnaire result, 75% of students answered that they traded because it was a course requirement and also interesting for them, 5% traded because it was purely interesting and 20% of students traded only because it was a course requirement[4]. According to another question[5], 44% of students actually had at least some feelings of addiction during the game. Partly because they played the game while being introduced to the world of international finance as the course progressed (thus seeing its relevance and connection with the real world). The other part of their motivation could have been the peer pride. Students were often discussing their trading success and failures or boastfully showing others their impressive results on their Oanda-enabled smart phones. The fact that they competed with and benchmarked their results against their classmates and friends over an extended period possibly made the game more interesting (compared to some short laboratory experiment with strangers).

3. Results

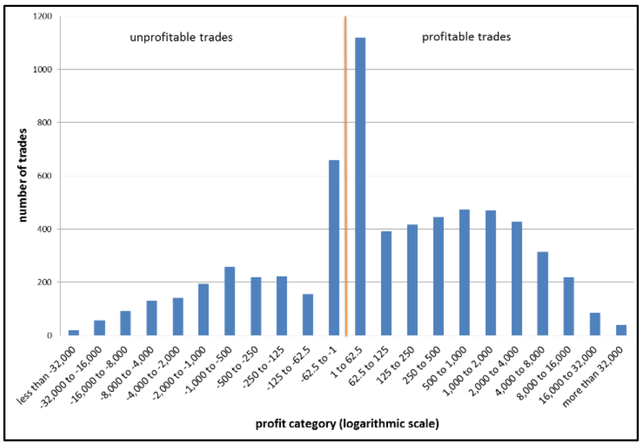
For the analysis purpose there are used all actively closed trades with the absolute value of profit/loss at least $1. For the individual traders analysis is set up also requirement for at least ten trades throughout the entire game. Actively closed trade is a trade that was closed by the student action and not by the system (a trade may be closed automatically if a stop loss or take profit order is attached to it). The requirement of at least ten trade´s filters out less active students who might have not enjoyed the game and played it only formally, as it was a course requirement.

[3] There was collected each student's login and password of his/her official game account; it was not possible to change the account later, reset the losses, to use more accounts etc.
[4] Did you play the game because it was a course requirement, or because it was fun to play? Or both?
[5] Have you ever had a feeling that you became "hooked-up" (addicted) to the game?

Out of about 13 000 trades in the dataset, 5 300 (40%) used attached stop loss or take profit orders, the remaining 7 700 (60%) were actively closed. Out of all trades, 1 500 had profit/loss smaller than $1. Applying both filters simultaneously, there is obtained 6 500 trades. Out of 300 traders, 160 traders had both profitable and unprofitable trades and ten or more trades with profit/loss at least $1.

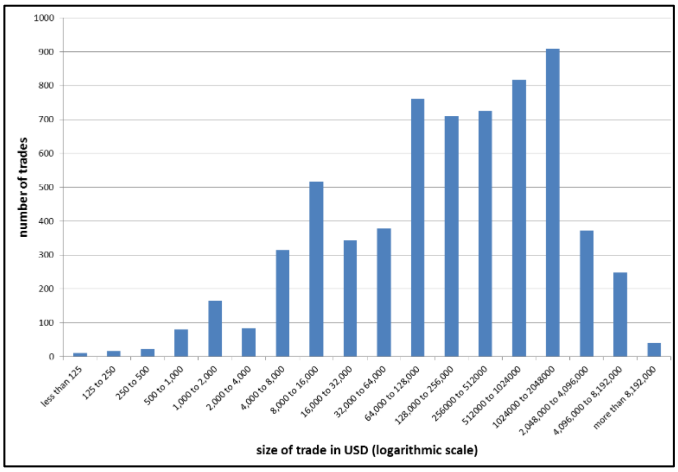
Average profit on a trade is $850 (on average trade size $800K, or 0.1%), where average loss on unprofitable trades is $ 2 400 and average profit on profitable trades is $2 450. In the following Figure No.1 is shown the histogram of profit/loss for 6 543 actively closed trades with profit/loss higher than $1.

Figure 1: Distribution of the P/L on a Trade.



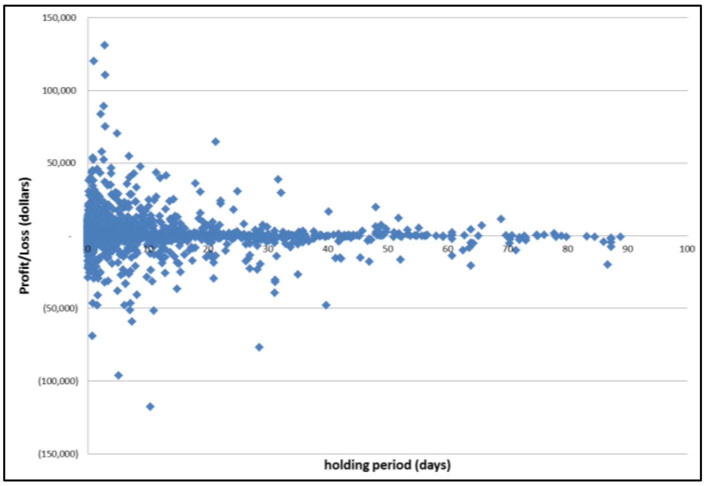
The figure no. 2 represents the distribution the trade sizes in USD, obviously the most of trades were between $64 000 and $ 2 048 000.

Figure 2: Histogram of Trade Sizes in USD



The relatively high positive average profit is a result of applying two data filters, firstly only actively closed trades (average Profit/Loss on all trades with P/L larger than $1, closed actively or through take profit and stop loss orders, is $650), secondly system issued margin closeouts were excluded, which, if included, make average P/L per transaction negative $120 (there were 520 trades closed in margin closeouts, with average P/L $ -18 000). Margin closeout is the broker's way to ensure that a client has sufficient funds to cover the losses (positive Net Asset Value) in case of adverse price trend. In the case of Oanda, the margin closeout is issued if the margin used by all open positions becomes higher than trader's NAV multiplied by two. This happens when the unrealized P/L on open positions sufficiently deteriorates as a result of unfavorable move in exchange rate. When margin closeout happens, all positions are closed. This always results in a substantial aggregate realized loss, and such losses were excluded from the data set. The negative $120 average P/L is close to zero (.015% of the trade size), as expected in an uninformed zero-sum game. Slightly negative result is consistent with the presence of transaction costs. Excluding the margin closeouts from the analysis biases the results against the disposition effect: margin closeout more likely happens for trades with long holding periods (caused by trades that were allowed to deteriorate too much for too long), and results in a loss. If they were included in the data set, the average holding period of an unprofitable trade would be higher, while the average holding period of profitable trades would be unaffected. The figure No. 3 presents the relationship between profit/loss on a trade and holding period.

Figure 3: Profit/Loss Function of the Holding Period



Is there observed the disposition effect overall? Disposition effect was confirmed by many studies in the past, e.g. Barberis and Xiong (2009), Choe and Eom (2009), Marciukaityte and Szewczyk (2012). Our results goes together with them. In the Table 1 is presented number of trades, average holding period in days, and timeLtoP (ratio of the average holding period of unprofitable trades to average holding period of profitable trades) for profitable/ unprofitable actively closed trades with profit/loss $1 and higher. The reported p-value is for independent two-sample t-test for the difference in means, assuming unequal variance.

Table 1: Holding Period of Trades

	N	**mean (days)**	**mean, weighted by |P/L|**	**mean, weighted by trade size**
profitable trades	4 400	3.4	4.4	1.5
unprofitable trades	2 100	5.5	10.5	1.9
all trades	6 500	4.1	6.2	1.6
p-value (mean difference)		<.0001	<.0001	0.004
timeLtoP		1.6	2.4	1.3

Obviously, there is a disposition effect in all three specifications, results from the fact, that the average holding period of unprofitable trades is significantly longer than that of profitable trades. Looking at the basic mean, the average holding period for profitable trades is 3.4 days, whereas for unprofitable trades it is 5.5 days, which is 62% higher value. Concerning the mean, weighted by P/L, the difference is even more significant. Traders in average held profitable trades 4.4 days, but unprofitable trades 10.5 days, which is 2.4 times longer holding period.

According to this result, it is assumed that students took the trading seriously and they were afraid of big losses. The last category, average holding period weighted by the trade size also shows the presence of the disposition effect. The unprofitable trades holding period (1.9 days) is by 26% higher than for profitable trades (1.5 days).

4. Conclusion

The aim of the paper is to determine, whether the disposition effect is present in the experimental data sample, while it is supposed, that its presence supports the presumption, that students had the same, or at least similar biases, as real traders. Results, presented in section 3, indicate that the holding period of unprofitable trades is significantly longer than the holding period of profitable trades, which is considered as a sign of the disposition effect. Specifically the average holding period for profitable trades was 3.4 days, whereas for unprofitable trades it was 5.5 days, which is 62% higher value. Concerning the mean weighted by P/L, the difference is even more significant, traders in average held profitable trades 4.4 days, while unprofitable trades 10.5 days, which is 2.4 times longer holding period. For the upcoming research it would be enhanced to compare the strength of the disposition effect for example based on gender or education.

Acknowledgements

This work has been supported by the Czech Science Foundation (GACR) under project 17-19981S and SP2018/34, an SGS research project of VSB-TU Ostrava, and within RRC/10/2017 support scheme of the Moravian-Silesian Region.

References

[1] Chen, G., Kim, K. A., Nofsinger, J. R. and Rui, O. M. (2007) Trading performance, disposition effect, overconfidence, representativeness bias, and experience of emerging market investors. *J. Behav. Decis. Making*, 20: 425–451. doi:10.1002/bdm.561

[2] Choe, H. and Eom, Y. (2009), The disposition effect and investment performance in the futures market. *J. Fut. Mark.*, 29: 496–522. doi:10.1002/fut.20398

[3] Kahneman, Daniel, and Amos Tversky (1979) Prospect theory: An analysis of decision under risk, *Econometrica* 46, 171–185.

[4] Kaustia, M. (2010) *Disposition Effect, in Behavioral Finance: Investors, Corporations, and Markets* (eds H. K. Baker and J. R. Nofsinger), John Wiley & Sons, Inc., Hoboken, NJ, USA. doi: 10.1002/9781118258415.ch10

[5] Marciukaityte, D. and Szewczyk, S. H. (2012), Disposition Effect and Public Secondary Equity Offerings. *Journal of Financial Research*, 35: 55–77. doi:10.1111/j.1475-6803.2011.01309.x

[6] Odean, Terrance, 1998, Are investors reluctant to realize their losses, *Journal of Finance*, 53, 1775–1798.

[7] van Dooren B. and R. Galema (2018), Socially responsible investors and the disposition effect, *Journal of Behavioral and Experimental Finance*, Vol 17, Pages 42-52

[8] Pelster M. and A. Hofmann (2018), About the fear of reputational loss: Social trading and the disposition effect, *Journal of Banking and Finance*, Vol 94, pages 75-88

[9] Shefrin, Hersh, and Meir Statman (1985) The disposition to sell winners too early and ride losers too long: Theory and evidence, *Journal of Finance* 40, 777–790.

Identification of institutional development of the local government unit – the risk of dissatisfaction of stakeholders as barrier to development

Aldona Frączkiewicz-Wronka [1] Anna Kozak[2]

Abstract

The purpose of this article is to find answer to the following research question: how institutional improvements can support local development. The authors adopted the assumption that lack of satisfaction of the stakeholders (any group or individual who can affect or is affected by the achievement of the organization's objectives) causes a risk of inhibiting local development. In this paper, by investors we mean only investors. Their dissatisfaction with administrative services in the municipality may result in the lack of investment in it, what is equal to the lower income of the commune and, consequently, it can constitute a barrier to local development. For that reason institutional development (understood as the creation of principles and mechanisms and their application in order to improve the efficiency of local government administration) is extremely important.

Key words

Public management, institutional development, the risk of dissatisfaction of stakeholders

JEL Classification: D02, D04, D78, H41

1. Introduction

A contemporary state as an organization has a special role to play in the process of socio-economic development of subordinate societies. The growing size, the number of functions performed by and the scope of state power stimulate the need to effectively manage the increasing stream of public funds allocated to meet social needs deemed important for the development of the state [1]. F. Fukuyama [1] divided the state functions into three groups – minimal, intermediate and activist. In addition, he subdivided them into the functions counteracting the effects of market failure and those that result from concern for social justice. According to Fukuyama, the state's minimal functions are: providing “pure public goods", defense, law and order, property rights, macroeconomic management, public health, improving equity, protecting the poor. The intermediate level functions include: external costs, education and the environmental protection, antitrust rights, overcoming imperfect information, insurance, financial regulations, social insurance. The activist functions of state include the strategy of industrial development and the redistribution of goods. The latter area include activities which are implemented as part of the institutional development program supported by European funds in Poland.

The scope and method of state functions implementation are influenced, among others, by such factors as: progressive globalization, level of economic and civilization development of a given country, adopted political solutions, implemented social doctrine, ruling political

[1] Aldona Frączkiewicz-Wronka, *Prof.*, Faculty of Economics, University of Economics in Katowice, 1 Maja 50 Street, 40-287 Katowice, POLAND, e-mail: afw@ue.katowice.pl

[2] Anna Kozak, *M.Sc.*, Faculty of Economics, University of Economics in Katowice, 1 Maja 50 Street, 40-287 Katowice, POLAND, e-mail: anna.kozak@edu.uekat.pl, phone: +48 606 677 759

parties, influence of the European Union [2]. What is increasingly noticed, is the negative consequence of failure to satisfy stakeholders' in their understanding of value.

According to the classic approach proposed by Freeman [3], the stakeholder is an individual or a group that can influence the organization's (institution's) ability to achieve its goals or which can be influenced by the organization (institution) achieving its goals. Referring to the key assumptions of the interest of the stakeholder theory, it can be pointed out that: (1) the organization has relationships with many groups, that is, stakeholders; (2) stakeholder theory is focused on the nature of relationships, both in the area of the process as well as the outcome for the organization; (3) the interests of all authorized stakeholders have a significant value and none of them should dominate others [4].

The lack of satisfaction of stakeholders may be manifested, inter alia, in the lack or withdrawal of support for the ruling political group or lowering the scope of interest in investing in a given territory. The last outcome contributes to the reduction of income of local government units and, as a result, the quality of life of residents decreases.

The purpose of the functioning of the public sector and the reason for its existence is the implementation of public tasks entrusted to it. By public tasks we mean the obligations of the state and local government unit determined by law [5].

Effective planning and implementation of development strategies and complying with the principles of social justice, the provision of public goods requires the existence of efficient institutions at every level of management. The efficient functioning of the institutions (i.e. government administration and local government units) appointed to meet the needs indicated by citizens as important for shaping the principles of socio-economic development reduces the risk of the failure to satisfy the stakeholders. It is also an element that promotes the stimulation of economic activity, which contributes to the growth of affluence of both citizens and the state. The report *Introduction to the Institutional Development Program [9] states that Poland's experiences after 1989 indicate that if the development activities undertaken at the central level are to be effective, they must be supplemented with a range of activities aimed at stimulating economic development by local governments. The lack of necessary synergy in this area leads inevitably to the situation where foreign investors, that are encouraged to locate their production in Poland, do not find places that would meet the basic boundary conditions. It is similar with the development of domestic business activity, which has the right to expect – especially at the local level – the support and creating conditions for its development.*

The control of the results of the institutional development program is carried out systematically and in a standardized manner, and the resulting conclusions contribute to formulating the directions of national public policies and local development policies.

2. Theoretical background – the concept of institutional development

When in the 1970s the growing economic recession has undermined public confidence in the institutions of the welfare state and public administration, the activities aimed at reforming the public sector became very popular. The model of administrative management of the economy was questioned and the extensive public sector was criticized. The scope of the allegations made against it was very wide and the most important of them concerned, among others, the way public administration is characterized by ineffectiveness, inefficiency, lack of flexibility and low level of meeting the stakeholders' needs. This has led to the necessity to look for alternative ways of organizing the public sector (including public administration) and to develop new mechanisms of its operation [10].

The concepts of institutional development and institutionalism have become synonymous with actions for reforming the public sector in many countries. Supporters of the institutional approach emphasize the role of socio-cultural determinants (standards, models, rules) in the economic activities of man "[10]. Institutions influence specific rules of behaviour, which is why state institutions guarding the legal and economic order are of such great importance [11]. Mutual interactions of institutions, game rules and market players shape and give the direction to the evolution of the economy [12]. Therefore, one of the problems that the institutional trend is trying to solve is to assess the role of institutions and their regional differentiation in economic development [13].

The institutional development does not have one recognized definition. In the field of sociology, the influence of patterns, norms and values (formal and informal) on the way the organization operates is emphasized [10].

On the basis of Polish literature, the concept institutional development corresponds to the term institutional development, which means perfecting the operation of the organization, so that it can better use the opportunities created by the existing institutional order [14]. The institutional development is also understood as creating rules and mechanisms and their application in order to improve the efficiency of local government administration in the basic areas of their activity, including provision of public services [10]. In international literature institutional development was defined as the reinforcement or creation of "the capacity of an organisation to generate, allocate and use human and financial resources effectively to attain development objectives, public or private". It includes the building and strengthening of institutions and also their retrenchment or liquidation in the pursuit of sectoral, institutional or government-wide rationalisation of expenditure [15]. Capacity is understood as a series of activities related to the development of an institution, ranging from planning to ending with an impact assessment [16].

Buyck suggests that operationally institutional development is typically aimed at improving and strengthening: internal organisational structures; management systems, including monitoring and evaluation; financial management and planning systems; personnel management, staff development and training; inter-institutional relationships; institutional structures of subsectors or sectors; legal framework; government regulations and procedures [15]. Among the specific objectives, it is possible to indicate the improvement of public administration procedures from the perspective of achieving strategic objectives and increasing the level of public service[10]. The principles and mechanisms of institutional development are intended to create conditions for the good functioning of public administration and to improve its operation – their application is to improve the efficiency of public administration in the dimensions of effectiveness, efficiency, cost-effectiveness and benefits of offices [9] [10] [17].

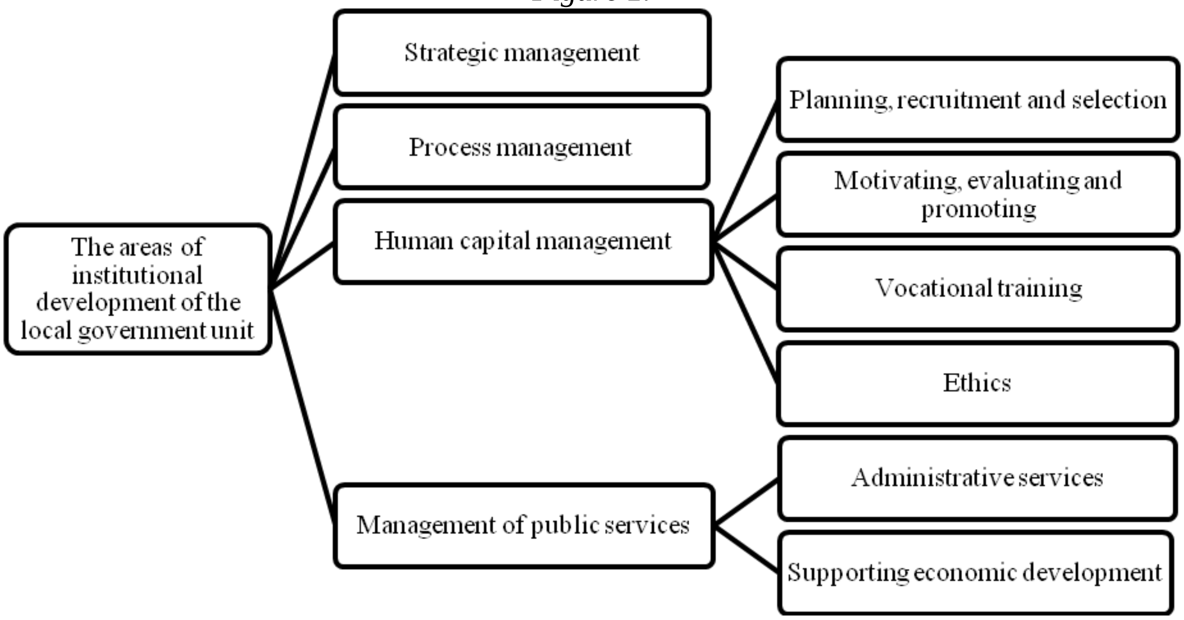
Institutions may be defined as „the rules of the game in a society or, more formally, are the humanly devised constraints that shape human interaction". Three important features of institutions are apparent in this definition: (1) that they are "humanly devised", which contrasts with other potential fundamental causes, like geographic factors, which are outside human control; (2) that they are "the rules of the game" setting "constraints" on human behaviour; (3) that their major effect will be through incentives The notion that incentives matter is second nature to economists, and institutions, if they are a key determinant of incentives, should have a major effect on economic outcomes, including economic development, growth, inequality, and poverty [18].

3. Methodology end research findings

The constant measurement of institutional development is of great importance for shaping public policies at the micro and macro levels. At the macro level, it allows for the preparation of a comprehensive assessment of the results of the activities carried out so far, and selecting the directions of public intervention adequate to the diagnosed needs. A properly prepared assessment may contribute to the selection of such priorities and goals in negotiations regarding the directions of public funds allocation under the ESF, which will support/stimulate the achievement of higher stages of local government development. At the micro level, the measurement of institutional development is used to plan activities related to the development of a specific office (municipality or county).

For the purposes of assessing the measurement of institutional development, the so-called Barometer of institutional development of local government units is organized (in 2013 [19] and 2017 [20]). Generally, the methodology of institutional development research inspired by the Institutional Development Planning Method, which was developed by experts of the Cracow University of Economics and the Małopolska School of Public Administration in Krakow, was used in the Barometer. In each of the conducted research – in response to the revealed cognitive needs – the methodology indicated underwent minor modifications. However, the areas of the local government units functions that are always evaluated are following: strategic management, process management, human capital management in the organization and management of public services.

Figure 1:



Source: Piróg, K. (Eds.) (2013). *Barometer of institutional development of local government units*, Report on the study for the Ministry of Administration and Digitization, Warsaw: Department of Cooperation with Local Government Units, p. 7.

Within each of the management areas, a set of institutional development indicators (IDI) was set. Each of the IDI has been assigned to one of four levels of institutional development. For each area it is possible to achieve one of four stages of development (1: the lowest stage and 4: the highest level). Based on the verification of individual IDI outcome, synthetic measures (indexes) of institutional development relating to the distinguished areas of the office's operation have been developed.

The Local Government Units Barometer 2017 [20] analysed in this study is a partial continuation of the study carried out in 2013, however, more emphasis was put on issues related to supporting economic development in local governments units as a sub-area for the assessment of public service management. This is justified in the context of the scope of interventions planned under the 11 Priority for Investment [21] and the Operational Program Knowledge Education Development [22]. The planned activities are related to the implementation of the solutions in the public administration that improve the efficiency of management of domain services in areas relevant to running a business.

The sub-area "Economic development support" includes 9 indicators that relate to three phases: defining measures to support economic development, implementing measures for economic development and monitoring and evaluation of measures for economic development. On the basis of the institutional development barometer of local government units from 2017 [20], the authors analysed the index of institutional development of local government units in view of the risk of dissatisfaction of investors.

The first indicator within the index of the institutional development of local government units in the supporting economic development sub-area is the percentage of local government units that implement economic development policy and use tools belonging to at least two of the following categories: provision of space for conducting business, utilities of the investment areas, advisory or training assistance, tax preferences, facilitating access to financing, human resources development programs in the municipality, marketing and promotional activities, programs and initiatives supporting the development of entrepreneurship. In comparison to 2013, the value of the index decreased by as much as 48 percentage points and now amounts to 29%. On a regional basis, the value of the indicator is lower than the rest of the country in the following voivodeships: Świętokrzyskie (16.1%), Podlaskie (16.3%), Masovia (20.3%), Lublin (22.2%) and Łódź (23 , 9%), and therefore in the eastern and central parts of Poland. The highest values of the indicator were recorded in the Lower Silesia (44%) Opole (42%) and Warmia-Masuria (39.2%) voivodships.

The percentage of local government units that implement the economic development program being the operational plan for the development strategy was 14%, which was lower by 2 percentage points in comparison to the 2013 result. In regional terms, the highest value of the indicator was achieved by LGUs from Warmia-Masuria (24.3%) and Lubusz (23.7%) voivodships. The lowest percentage of LGUs implementing the economic development program being the operational plan for the development strategy occurred in the Podlaskie (7.6%), Łódź (8.3%) and Silesia (9.3%) voivodships.

The next indicator refers to the percentage of local government units implementing the economic development policy and using tools belonging to at least three of the following categories: provision of space for conducting business, utilities of the investment areas, advisory or training assistance, tax preferences, facilitating access to financing, human resources development programs in the municipality, marketing and promotional activities, programs and initiatives supporting the development of entrepreneurship. In comparison to 2013, the value of the index decreased by as much as 42 percentage points and currently amounts to 20%. On a regional basis, the value of the indicator is lower than the rest of the country in the following provinces: Podlaskie (7.6%), Łódź (8.3%), Silesia (9.3%) and Masovia (10.1%). The highest valuesof the indicator were recorded in Lubusz (23.7%), Warmia-Masuria (24.3%) and Podkarpackie (18.7%) voivodships.

Compared to 2013, the percentage of LGUs that took into account in the multi-annual financial plan or multi-annual investment plan funds for the implementation of the economic development program decreased by 36 percentage points. The indicator value decreased in all types of local government units. Among voivodships, the highest percentage of LGUs meeting the analysed criterion occurred in Warmia-Masuria (16.2%) and Lubusz (13.6%) voivodships. The lowest value of the indicator was recorded in Podlaskie (2.2%), Opole (6%) and Łódź (6.4%) voivodships.

The next indicator refers to the percentage of local government units implementing the economic development policy and using tools belonging to at least four of the following categories: provision of space for conducting business, utilities of the investment areas, advisory or training assistance, tax preferences, facilitating access to financing, human resources development programs in the municipality, marketing and promotional activities, programs and initiatives supporting the development of entrepreneurship. In comparison to 2013, the value of the index decreased by 2 percentage points and now amounts to 11%. On a regional basis, the value of the indicator is lower than the rest of the country in the following provinces: Świętokrzyskie (4.8%), Masovia (5.1%), Podlaskie (5.4%) and Podkarpackie (5.5%). The highest values of the indicator were recorded in the following provinces: Opole (20%) and Lower Silesia (20.7%).

The percentage of local government units that monitor and periodically evaluate the implementation of the economic development program increased in comparison to 2013 by 6 percentage points and amounted to 7%. On a regional basis, the lowest percentage of LGUs monitoring and using periodic evaluation of the implementation of the economic development program occurred in the Podlaskie voivodship (1.1%). The highest values of the indicator were recorded in the Lower Silesia voivodship (12.1%). Percentage of units that coordinate the activities aimed at economic development along with neighbouring local government units in order to increase the joint development potential or a better supplement thereof, increased compared to 2013 by 44 percentage points and amounted to 45%. On a regional basis, the lowest value of the indicator was recorded for the following voivodships: Podlaskie (33.7%) and Greater Poland (30.4%). The highest values are characteristic of the Podkarpackie (53.8%), Pomerania (53.7%), Lublin (51.9%) and Lesser Poland (51.5%) voivodships.

Another indicator within the index of the institutional development of local government units in the supporting economic development area is the percentage of local government units that implement economic development policy and use tools belonging to at least five of the following categories: provision of space for conducting business, utilities of the investment areas, advisory or training assistance, tax preferences, facilitating access to financing, human resources development programs in the municipality, marketing and promotional activities, programs and initiatives supporting the development of entrepreneurship. In comparison to 2013, the value of the index decreased by as much as 42 percentage points and currently amounts to 6%. On a regional basis, the value of the indicator is lower than the rest of the country in the following provinces: Podlaskie (0%), Podkarpackie (2.2%), Masovia(2.8), Łódź (28%) and Świętokrzyskie (3.2%), therefore, in the Eastern and Central parts of Poland. The highest values of the indicator were recorded in the Lower Silesia (12.1%) and Silesia (12.7%) voivodships.

The percentage of local government units that conduct cyclical assessment of the effects of activities in the field of supporting economic development and all tools and procedures used in this area, with the results of the evaluation being used to improve the mechanisms stimulating economic development, amounted to 3%. In comparison to 2013, it was higher by 3 percentage points. The highest value of the indicator was recorded in the case of cities with poviat rights (21%). On a regional basis, the highest valuesof the indicator were obtained by Pomerania (6.1%), Greater Poland (6.1%), and Lower Silesia voivodships (6%).

4. Conclusion

LGUs have a large number of important stakeholders. Stakeholders are also referred to as strategic partners of organizations that have shares, rights or other interests related to it. Among the stakeholders the key stakeholders can be distinguished, i.e. those who have a direct impact on the success and an extraordinary importance for its achievement. The identified stakeholders may have a positive, negative or neutral attitude towards the organization. Their attitude is largely dependent on the effectiveness of the organization's activities and their evaluation. In other words, we can identify supportive stakeholders who support project implementation, anti-counteracting (blocking) stakeholders who disagree with the activities of the organization and neutral stakeholders who do not have a clear opinion or opinion that can not be determined [23]. The key stakeholders of the local government unit are investors, who may invest invest funds in its area, ipso facto contributing to stimulating the economic development. Similarly, if investors are not satisfied, at least to a minimal degree", by public institution's standards, it will only be natural that they do not invest in the territorial self-government unit represented by this institution and, consequently, it can constitute a barrier to the local development.

Summarize, the results of the institutional development measurement carried out in 2017 concern public services in terms of supporting economic development decreased, which may result in a smaller tendency to invest in specific local government units. The effects of processes occurring in public organizations should balance the interests of different stakeholder groups [24]. Moreover, stakeholder satisfaction involves meeting these needs which are perceived as valuable in the hierarchy of a particular stakeholder [25].

Therefore, as we read in the research report, it is necessary to intensify activities aimed at supporting local government units in the creation of infrastructure as well as substantive and human resources for the investor's support, as well as supporting local entrepreneurs.

References

[1] Fukuyama, F. (2005). *Budowanie państwa. Władza i ład międzynarodowy w XXI wieku.* Poznań: Rebis.

[2] Lubińska, T. (2010). *Budżet a finanse publiczne*. Warszawa: Difin.

[3] Freeman, R.E. (1984). *Strategic Management: A stakeholder approach*, Boston: Pitman.

[4] Jones, T. and Wicks, A. (1999), Convergent Stakeholder Theory, *Academy of Management Review*, 24(2).

[5] Ziębicki, B. (2014). *Efektywność organizacyjna podmiotów sektora publicznego.* Kraków: Wydawnictwo Uniwersytetu Ekonomicznego.

[6] Bossak, J.W. (2006). *Systemy gospodarcze a globalna konkurencja*. Warszawa: Wydawnictwo SGH.

[7] Kieżun, W. (2004). *Struktury i kierunki zarządzania państwem*. In Kieżun, W. and Kubin, J. (Eds.), *Dobre państwo*. Warszawa: Wyższa Szkoła Przedsiębiorczości i Zarządzania im. Leona Koźmińskiego.

[8] Osiński, J. (2011). *Niezbędność państwa w dobie globalizacji oraz jego współczesne modele*. In Oniszczuk, J. (Eds), *Współczesne państwo w teorii i praktyce*. Warszawa: SGH.

[9] Kijowski, D., Misiągm W., Prutis, S., Stec, M., Szlachta, J. and Zaleski J. (2004). *Wprowadzenie do Programu Rozwoju Instytucjonalnego*, Warszawa: Instytut Badań nad Gospodarką Rynkową. (http://www.dobrepraktyki.pl/zalaczniki/wprowadzenie_PRI.pdf)

[10] Mazur, S. (eds.) (2004). *Rozwój instytucjonalny*. Kraków: Małopolska Szkoła Administracji Publicznej Akademii Ekonomicznej.

[11] Nowak, P. (2011). *Nowy paradygmat rozwoju regionalnego na przykładzie wybranych regionów Europy Zachodniej*. Rozprawa doktorska. Poznań: Uniwersytet Ekonomiczny. (http://www.wbc.poznan.pl/Content/178703/NowakPiotrS4240.pdf)

[12] Chmielewski, P. (1995). *Ludzie i instytucje. Z historii i teorii nowego instytucjonalizmu*, Warszawa: Instytut Pracy i Spraw Socjalnych.

[13] Martin, R. (2000). *Institutional Approaches in Economic Geography*. In Sheperd and E., Barnes, T.J., *A Companion in Economic Geography*, Oxford: Blackwell.

[14] Zawicki, M. (2004). Metoda analizy instytucjonalnej urzędu gminy, *Zeszyty Naukowe Akademii Ekonomicznej w Krakowie*, No. 667.

[15] Buyck, B. (1991). The Bank's Use of Technical Assistance for Institutional Development, *World Bank Working Paper*, No. 578, Washington, DC: World Bank.

[16] Honadle, B.W. (1981). A Capacity-Building Framework: A Search for Concept and Purpose, *Public Administration Review*, 41(5).

[17] Bober, J. (2008). Metoda rozwoju instytucjonalnego jako narzędzie doskonalenia administracji publicznej na przykładzie strategii rozwoju samorządów terytorialnych, *Zarządzanie Publiczne*, 1(3).

[18] Acemoglu, D. and Robinson, J. (2008). *The Role of Institutions in Growth and Development*. Working Paper No. 10. The International Bank for Reconstruction and Development. Washington: The World Bank.

[19] Piróg, K. (Eds.) (2013). *Barometr rozwoju instytucjonalnego jednostek samorządu terytorialnego*, Raport z badania dla Ministerstwa Administracji i Cyfryzacji, Warszawa. (http://ip.mswia.gov.pl/ip2/program-operacyjny-kapi/dokumenty/badania-i-raporty/badania-ewaluacyjne/7622,Wyniki-badania-quotBarometr-rozwoju-instytucjonalnego-jednostek-samorzadu-teryto.html)

[20] Frączkiewicz-Wronka, A., Szemiako, Ł. And Gutta, K. (2017). *Barometr rozwoju instytucjonalnego jednostek samorządu terytorialnego*, Raport z badania dla Ministerstwa Administracji i Cyfryzacji, Gdańsk. (http://ip.mswia.gov.pl/ip2/program-operacyjny-wied/ewaluacja/10563,Barometr-rozwoju-instytucjonalnego-jednostek-samorzadu-terytorialnego-2017.html)

[21] http://ec.europa.eu/regional_policy/en/policy/how/priorities

[22] https://www.power.gov.pl/strony/o-programie/zasady/

[23] Frączkiewicz-Wronka, A., Kotowski, P. and Wronka, M. (2009). *Koncentracja na zewnętrznym otoczeniu jednostki ochrony zdrowia – wyznaczanie kierunków działania przez przywódcę*, In Frączkiewicz-Wronka, A. and Austen-Tynda, A. (Eds.), *Przywództwo w ochronie zdrowia. Idee i instrumenty*, Warszawa: Wolters Kluwer.

[24] Hillman, A. J. and Keim, G. D. (2001). Shareholder Value, Stakeholder Management, and Social Issues: What's the Bottom Line?, *Strategic Management Journal*, No. 22.

[25] Austen, A. and Frączkiewicz-Wronka, A. (2018), Stakeholders and resources in public hospitals. Towards an integrated view, *Argumenta Oeconomica*, 1(40), p.

Application of Classification Trees for Credit Rating Prediction

Taťána Funioková[1], Martina Novotná[2]

Abstract

This paper is focused on credit rating modelling using the technique of recursive partitioning (CART). In this study, we apply the methods of decision tree learning to create models for credit rating prediction. The models can be used to assess and monitor credit quality of companies and determine the contribution of financial predictors, especially when a certified rating is not available. In this study we present one nominal and four ordinal models based upon different splitting and pruning rules. The focus is paid to the performance comparison of various algorithms for this ordinal classification problem in terms of Somers' delta and ROC curves. Main findings of our study suggest that for our data both approaches perform well and provide comparable results in terms of classification ability.

Key words

Credit rating, prediction, decision trees, CART algorithm, ordinal response

JEL Classification: C52, C53, C38, G24, G32

1. Introduction

The general purpose of credit rating prediction is to build models that can extract knowledge of credit risk assessment based on past observations and apply it to evaluate the credit quality of a borrower. The estimated rating models can be used for credit rating assessment, especially in cases where there is no certified rating available. Thus, they may serve to a number of participants in the financial market, from investors and intermediaries to the bond issuers. Contribution of own credit models is for example evaluated by Rerolle and Rimaud (2009). As they confirm, research in credit risk area and credit models has in comparison with certified rating important value added, because it enables to react on changes and new information sooner than in the case of complete dependency. The analysis of variables with a significant impact on rating should be taken into account when assessing the corporate credit risk because they can provide signals about the potential change of credit quality. Although rating agencies emphasize the role of a subjective judgment in determining credit ratings, the area of credit rating modelling and prediction has attracted considerable attention in the field of research. Moreover, based on the recent investigation, such credit rating models can provide a satisfactory predictive ability and they are able to largely capture the characteristics of the credit rating process.

The main objective of this study is to develop predictive models for credit rating using the methods of decision tree learning and determine the contribution of individual financial variables for every model. In this study, we apply recursive partitioning methods with both, nominal and ordinal approach, and develop five classification tree models based upon different splitting and pruning rules. Their predictive accuracy is compared on the basis of the area under the receiver operating characteristic (ROC) curve (AUC) and Somers' delta (Somers, 1962).

[1] Mgr. Taťána Funioková, Ph.D., VŠB TU Ostrava, Faculty of Economics, tatana.funiokova@vsb.cz.
[2] Ing. Martina Novotná, Ph.D., VŠB TU Ostrava, Faculty of Economics, martina.novotna@vsb.cz.

The main contribution of this paper is the empirical study of modelling corporate credit rating of randomly selected non–financial companies from Central and Eastern Europe. The findings of this study can be used to assess the appropriateness of methods applied and emphasize the role of certain financial variables for corporate credit rating prediction.

The structure of this paper is as follows. Firstly, we provide a brief overview of recent research, followed by the description of our dataset and used methodology. We start with the nominal approach to find an optimal model based upon our training data sample. Then we use the ordinal approach and build four other models as complex as the nominal one. All the models are compared according to several criteria such as classification ability and importance of financial variables for a classifier construction. Finally, main findings of our study are summarized in the conclusion.

2. Literature Review

Decision tree models have been traditionally used for credit scoring, which is one of the most commonly applied assessment systems. For example, Carter and Catlett (1987) used this methodology for assessing credit card applications. More recently, Bensic, Sarlija and Zakic–Susac (2005) compared three different methods including logistic regression, neural networks and the CART algorithm for building a decision tree for modelling small–business credit scoring. The authors confirmed some limitations of logistic regression methodology and highlighted the ability of CART model to extract more features, however with a lower predictive ability. Bastos (2007) employed the method of boosted decision trees, a procedure that aggregates many weak classifiers in order to achieve a high classification performance, using the dataset of credit card applications. According to the author, this method outperformed the multilayer perceptron and the support vector machines methods, suggesting that boosted decision trees may be a competitive alternative to other techniques in credit scoring application. Gepp, Kumar and Bhattacharya (2009) compared different decision tree building algorithms (RPA, CART, See5) and multiple discriminant analysis (MDA) used for business failure prediction, extending the work of Frydman, Altman and Kao (1985). As the authors suggest, decision trees produced very similar results and could be superior predictors when compared to MDA. Cho, Hong and Ha (2009) applied decision trees also for bankruptcy prediction, whereas they proposed a hybrid method based on the combination of variable selection using decision trees and case–based reasoning utilizing the Mahalanobis distance with variable weight. The authors introduced an alternative model with a good performance, however as they accentuate there are some limitations related to the computational difficulty. Among other studies, Zhang, Zhou, Leung and Zheng (2010) focused on credit scoring models using the data of credit card applicants through the method of vertical bagging decision trees model and so improving the predictive accuracy of credit scoring.

Similarly to credit scoring, the decision tree learning methods seem to be a suitable technique for credit rating prediction. For example, Shin and Han (2001) examined the effectiveness of inductive learning approach to case indexing process for business classification tasks and proposed approach by application to corporate bond rating. As they suggest, the determination of decision trees using different stopping conditions has a critical impact on the performance of the system. Hüllermeier and Vanderlooy (2009) consider decision trees as the most extensively studied methods in machine learning and emphasize their ability to handle different types of attributes, compare well with the number of attributes and instances, and obtain outperforming classification performance when compared to more traditional techniques. However, there is a lack of research in terms of application related to Central and Eastern European countries, which is the purpose of this paper.

3. Description of Data and Methodology

The overall description of data in use is provided in this section, followed by a brief overview of methods applied in the practical part. The attention is paid to the description of classification and regression trees algorithm and the evaluation of developed models.

This study is focused on the analysis of corporate credit rating of eight countries from Central and Eastern Europe (CEE): Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia. The models are developed on the basis of MORE Rating which classifies companies similarly as rating agencies (Bureau van Dijk Electronic Publishing, 2008). The original sample that contains 3,083 observations for the period 2002–2008 is randomly split into training data set (2,055 obs.) and testing data set (1,028 obs.).

Given companies are classified according to five rating levels. That is, the cases fall into five classes. Table 1 shows details including (relative) frequencies of the classes across the data sets.

Table 1: (Relative) frequencies for training and testing data set

Rating: Class (j):	B 1	BB 2	BBB 3	A 4	AA 5
Training data set	96 (0.047)	363 (0.177)	1037 (0. 505)	450 (0.219)	109 (0.053)
Testing data set	34 (0.033)	176 (0.171)	494 (0.481)	271 (0.264)	53 (0.052)
Original data set	130 (0.042)	539 (0.175)	1531 (0.497)	721 (0.234)	162 (0.053)

The used financial variables reflect size, profitability, capitalization, liquidity and interest coverage of a company. In this study, we analyse the effect of the following company–based financial variables (nine explanatory variables): Total assets (TA), interest coverage (INTC), long–term debt to total assets (LTDTA), equity to total assets (EQTA), return on equity (ROE), return on capital employed (ROCE), return on assets (ROA), liquidity ratio (LIQD) and current ratio (CURR) on credit rating (response variable RATING).

Various decision tree algorithms can be used to build a classification tree (CT) representing rules to predict the target variable. In this work we construct five binary tree structured classifiers using the CART (Classification and Regression Trees) algorithm introduced by Breiman et al. (1993). In what follows, we use the open source software, R, and its implementation of the CART algorithm called `rpart` (Recursive Partitioning and Regression Trees) for nominal response variable (Therneau and Atkinson, 1997) and `rpartScore` for ordinal response variable (Galimberti, Soffritti, and Maso, 2012).

Every classifier is built in two phases. In the growth phase we recursively partition the training set (root node) into the descendant subsets (nodes) until each partition consists of cases from exactly one class. The binary splits are determined such that the data in each of the descendant subsets are "purer" than the data in the parent subset. A node can be terminal (it does not split) or nonterminal (it splits into two daughter nodes). Each of them is designated by a class label and proportions of observations from training set for all classes in question. The height of a tree is the number of splits on the longest path between its root node and a terminal node. This phase generates a CT referred to as the maximal tree.

In the second phase, the so–called prune phase, we look for a sub–tree, T, with the least true misclassification rate, the probability that the model will misclassify a new case drawn from the same distribution as the training sample. We present two estimators of the misclassification rate in this study. Given a tree T, the resubstitution estimate, $R^{RS}(T)$, is the proportion of poorly classified cases from the training data set, and the test sample estimate, $R^{TS}(T)$, is the proportion of poorly classified cases from the testing data set. As more splits in T result in lower values of $R^{RS}(T)$, the test sample estimates $R^{TS}(T)$ tend to grow for overfitted models. The aim of this phase is to reduce the risk of overfitting and thus improve the ability of the tree to generalize to unseen data.

In the growth phase we use a Gini impurity function to determine each split in the tree. Given a node t with proportions of observations in this node that belong to class j, $p(j|t), j = 1, \ldots, 5$, the general Gini measure of node impurity in our case of five classes has the form:

$$i_{GG}(t) = \sum_{i=1}^{5} \sum_{j=1}^{5} C(i|j) \cdot p(i|t) \cdot p(j|t), \quad (1)$$

where $C(i|j)$ is a misclassification cost, that is the cost of misclassifying a class j object as a class i object and satisfies: (i) $C(i|j) \geq 0$, for $i \neq j$, and (ii) $C(i|j) = 0$ otherwise. In `rpart` package the nominal misclassification cost $C_{nom}(i|j) = 1$, for $i \neq j$, is used. Whereas in `rpartScore` package we can choose between two values of parameter `split`: `abs` and `quad`, representing an absolute misclassification function $C_{abs}(i|j) = |i - j|$ and a quadratic misclassification function $C_{quad}(i|j) = (i - j)^2$, respectively. For a given misclassification cost $C(i|j)$, the total misclassification cost of a tree T, $R^{MC}(C, T)$, is a weighted proportion of poorly classified cases with weights $C(i|j)$.

In the prune phase we apply the so–called minimal cost–complexity pruning. Given a tree T with $card(T)$ terminal nodes, the cost–complexity measure is given by

$$R_\alpha(T) = R(T) + \alpha \cdot card(T), \quad (2)$$

where $\alpha \geq 0$, the complexity parameter (parameter `cp` in `rpart`/`rpartScore` package), is a tuning parameter that controls the trade–off between predictive performance and model complexity, and $R(T)$ is a predictive performance measure expressed in terms of $R^{RS}(T)$ or $R^{MC}(C, T)$. Package `rpart` employs the first one and we can choose between these two by setting a parameter `prune` to `mr` or `mc` in `rpartScore`, respectively. Given a pre–selected α we are looking for a (minimal) subtree $T(\alpha)$ that minimizes $R_\alpha(T)$. The whole process generates various levels of α together with possible candidates for optimal CT. The final (optimal) α is determined by a 10–fold cross–validation and applying Breiman's 1–SE rule to choose the minimal tree that has the predictive cross–validated error within 1 SE of the minimum cross–validated error (see Breiman (1993) for details). As we want to compare five single CTs, we determine an "optimal" α for the case of nominal CT and we use it to choose optimal ordinal CTs later, which makes the process of comparison reproducible.

To evaluate and compare the performance of CTs, the $R^{TS}(T)$ measure is applied together with Somers' delta, a non–parametric asymmetric measure of agreement between an ordinal variable and ordinal independent variable; predicted and observed ratings in this case. It takes values between –1 and 1, where 1 (–1) means that all pairs of the variables agree (disagree).

4. Construction of CT Models

In this section, we apply the methods introduced in previous section to develop classification tree models for credit rating prediction based on our data sample. In total, we find five classification trees: one nominal tree and four ordinal trees depending on the method used to build them. All the models are compared according to Somers' delta, AUC, and variable importance and the main results are summarized.

4.1 Nominal Classifier with `rpart`–package

In this section we treat the target variable, rating, as a nominal one. That is, for example, we evaluate the misclassification of rating BBB as AA equally as misclassifying rating BBB as A. Therefore we choose package `rpart`, with $C(i|j) = C_{nom}$ in (1) and $R(T) = R^{RS}(T)$ in (2) to construct an optimal CT.

We start with the root tree T_0, represented in Table 1 by (relative) frequencies in training data set, with predicted label BBB with probability of 0.5046, and the resubstitution misclassification rate $R^{RS}(T_0) = (96 + 363 + 450 + 109)/2055 = 0.4954$. The maximal tree includes 226 terminal nodes (see Table 2 for details). Employing Breiman's 1–SE rule to 2,000 randomly generated cross–validations, we get 10 different optimal CTs. Table 2 shows some of their characteristics including 95% confidence intervals for Somers' delta (SomerL and SomerU), and distribution (Appear.) for these 10 models along with the root and maximal tree. We can see that more than 55% leads to CT with 48 terminal nodes and `cp` of 0.002456. For this reason we consider this CT to be the optimal nominal model in this study. The same CT results for any value of α in (2) such that $0.002456 \leq \alpha < 0.002947$. As a geometric mean of these two values is used in cross–validation process, we set the "optimal" `cp` in `rpart` to 0.00269 to be able to compare this model with ordinal models later. The test sample estimate of 17.41% means that 179 cases out of 1,028 are poorly predicted by the optimal nominal model.

Table 2: CTs for 2,000 different cross–validations using Breinman's 1–SE rule

Card(T)	Appear.	cp	Height	R^{RS}	R^{TS}	SomerL	SomerU
1	–	0.178781	0	49.54%	51.95%	0	0
32	0.35%	0.004912	8	12.17%	18.77%	0.8033	0.8488
33	1.75%	0.004420	8	11.92%	18.39%	0.8091	0.8542
35	6.20%	0.003929	8	11.48%	18.00%	0.8110	0.8561
40	9.75%	0.002947	9	10.51%	17.41%	0.8146	0.8588
48	55.15%	0.002456	9	9.34%	17.41%	0.8174	0.8618
54	5.30%	0.002292	9	8.61%	17.32%	0.8153	0.8592
57	0.45%	0.002161	9	8.27%	17.22%	0.8157	0.8596
62	19.60%	0.001965	10	7.74%	16.93%	0.8216	0.8653
78	0.90%	0.001473	10	6.13%	16.63%	0.8236	0.8672
93	0.55%	0.000982	12	5.01%	15.66%	0.8294	0.8730
226	–	0	16	0%	16.15%	0.8322	0.8768

The area under the ROC curve for each level of rating for the tree with 48 terminal nodes can be seen in Table 3.

Table 3: Areas under ROC curves for nominal model

Class Rating:	B	BB	BBB	A	AA
AUC	0.9756	0.9363	0.9108	0.9404	0.9621

Although it is not possible to list all the classification rules, we are able to study variable importance in the tree. Table 4 shows how our explanatory variables participate in the tree structure of the optimal tree. The variable importance scores are scaled relatively to the best performing variable. These scores do not indicate absolute information value of a variable; however, they measure a variable's ability to perform in our optimal tree either as a primary or surrogate variable (see Breiman (1993) for details).

Table 4: Variable importance scores for optimal nominal model

EQTA	ROA	INTC	ROE	ROCE	CURR	LIQD	TA	LTDTA
100	94.65	92.59	90.11	77.44	39.98	28.81	7.03	6.53

4.2 Ordinal Classifier with `rpartScore`–package

In this section we treat the target variable rating as an ordinal one. That is, for example, it is twice as expensive to misclassify rating BBB as AA as it is to misclassify rating BBB as A. To express this nature, we will use package `rpartScore` with $C(i|j) = C_{abs}$ or $C(i|j) = C_{quad}$ in (1) and $R(T) = R^{RS}(T)$ or $R(T) = R^{MC}(C, T)$ in (2).

We start with the root tree T_0, again with the misclassification cost of $R^{MC}(C_{abs}, T_0) = (2 \cdot 96 + 1 \cdot 363 + 1 \cdot 450 + 2 \cdot 109)/2055 = 0.5951$ or $R^{MC}(C_{quad}, T_0) = (2^2 \cdot 96 + 1^2 \cdot$

$363 + 1^2 \cdot 450 + 2^2 \cdot 109)/2055 = 0.7946$, and so the growth phase results in two different maximal CTs. We prune them both with `mr` and `mc` parameter. The prune phase results in four different sequencies of `cp`s and corresponding trees. We choose all four optimal ordinal CTs according to the complexity parameter of 0.00269 derived in previous section. You can find details in Table 5, similar to Table 2, enriched by numbers of misclassified cases in the testing set.

Table 5: Comparison of ordinal models

Algorithm		Card(T)	cp	Height	R^{RS}	R^{TS}	Misclass. cases	Somers' delta	
growth	prune							lower	upper
abs	mr	49	0.002456	9	9.49%	16.63%	171	0.8209	0.8642
abs	mc	41	0.002453	9	10.66%	16.73%	172	0.8185	0.8619
quad	mr	55	0.002456	9	8.32%	16.25%	167	0.8272	0.8690
quad	mc	47	0.002453	9	9.68%	16.54%	170	0.8201	0.8622

Because there are no predictions misclassifying class i as class j such that $|i - j| > 1$ in the testing set for all the models, the total misclassification rate will be the same as R^{TS} for both C_{abs} and C_{quad}. AUCs for every rating–level across the models are shown in Table 6.

Table 6: Areas under ROC curves for ordinal models

model	B	BB	BBB	A	AA
abs mr	0.9758	0.9408	0.9149	0.9446	0.9615
abs mc	0.9758	0.9318	0.9144	0.9412	0.9582
quad mr	0.9748	0.9168	0.9227	0.9459	0.9312
quad mc	0.9744	0.9154	0.9158	0.9391	0.9305

Finally, Table 7 summarizes how the variable importance vary for our four ordinal classifiers and Figure 1 provides a comparison of the effect of individual variables in the construction of all five developed models.

Table 7: Variable importance for ordinal models

Model	Variable importance scores								
abs mr	EQTA 100.00	ROE 80.19	ROA 75.59	ROCE 74.94	INTC 70.44	LIQD 51.84	CURR 45.90	TA 24.70	LTDTA 14.88
abs mc	EQTA 100.00	INTC 80.58	ROE 80.08	ROA 78.38	ROCE 74.17	CURR 56.16	LIQUID 48.42	TA 21.05	LTDTA 8.53
quad mr	INTC 100.00	ROA 75.29	EQTA 59.79	ROE 57.36	ROCE 52.35	CURR 21.54	LIQD 20.49	LTDTA 3.66	TA 2.63
quad mc	INTC 100.0	ROA 73.70	EQTA 58.61	ROE 55.69	ROCE 51.04	CURR 20.72	LIQD 19.71	LTDTA 3.47	TA 2.23

5. Conclusion

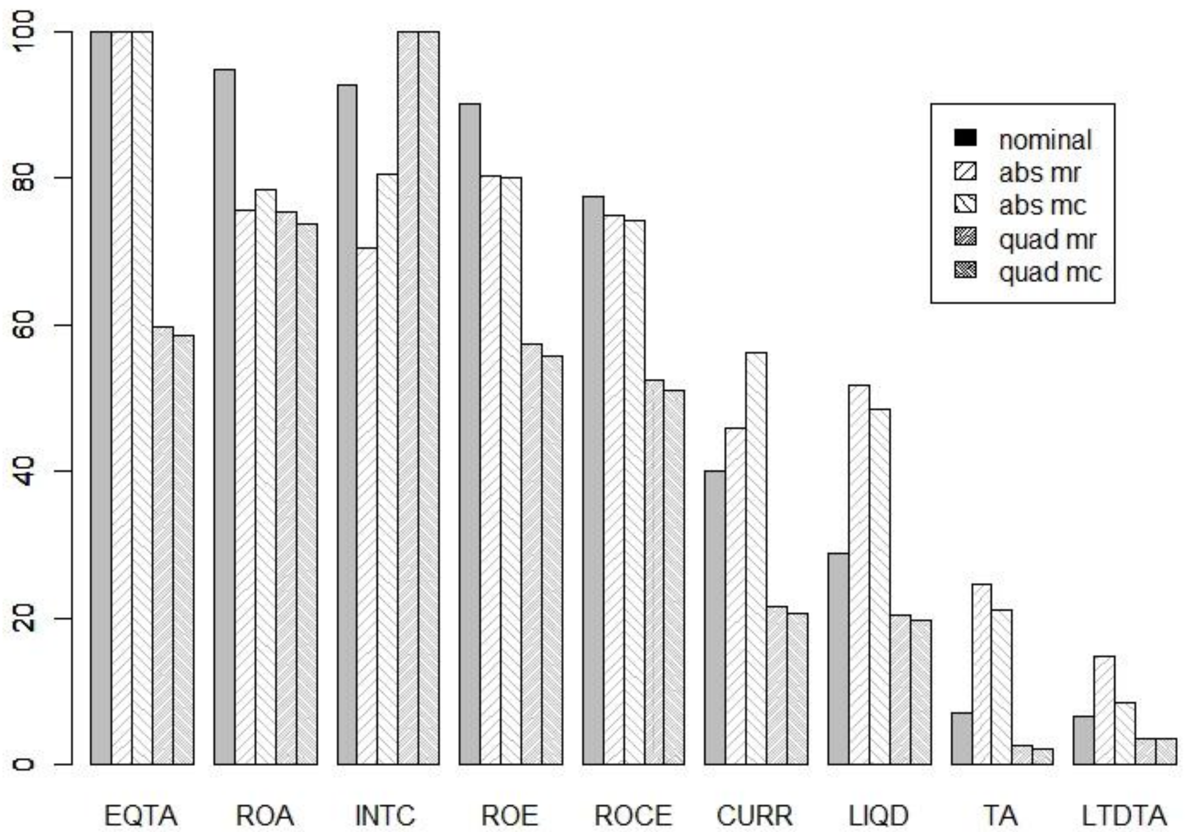
The purpose of this paper was to construct models for credit rating prediction using the decision tree learning methods and determine the contribution of used financial variables for corporate rating. In this study, two approaches to recursive partitioning methods (CART), nominal and ordinal, were applied to the data of selected non–financial companies from Central and Eastern European countries.

Overall, five different optimal models were developed using various methods of splitting and pruning on the basis of the same complexity parameter. From the point of view of overall classification ability, all five models are comparable according to R^{TS}. Although the nominal model exhibits a higher value of misclassification rate R^{TS} compared to ordinal models, differences among all of them are not statistically significant (we obtain p–value of 0.9690 for test of multiple proportions and minimal p–value for paired tests is 0.5196). This result is supported also through overlapping 95% confidence intervals for Somers' delta. To evaluate the classification ability of individual rating categories, ROC curves were constructed and the corresponding AUCs were computed. Since the AUCs range from 0.9108 to 0.9758, we conclude that all the models provide a high classification ability, however we observe minor differences among individual rating categories. Overall, the models perform the best ability to predict rating group B, and the worst ability to predict BBB (nominal and ordinal C_{abs}) or BB (ordinal C_{quad}).

Finally, based upon Figure1, the importance of individual variables for the construction of each model is comparable for nominal and ordinal models (C_{abs}), namely EQTA, ROA, INTC, ROE and ROCE mostly contribute to the construction of the tree. However, when we put more emphasis on misclassification in terms of C_{quad}, the impact of INTC significantly increases.

To summarize, we conclude that all models perform a high and comparable overall classification ability. From the practical point of view, for our data, taking the scores for misclassification into consideration does not significantly improve the predictive ability of binary tree classifiers. However, the reason can be the high accuracy of optimal nominal CT which offer little room for improvement.

Figure 1: Variable importance for a nominal CT and four ordinal CTs



Acknowledgement

This paper was supported by the Operational Programme for Competitiveness under Project CZ.1.07/2.3.00/20.0296 and Project SGS 2018/154.

References

[1] Bastos, J. (2007). *Credit Scoring with Boosted Decision Trees.* MPRA Munich Personal RePEc Archive MPRA Paper No. 8034, posted 2 April 2008.

[2] Bensic, M., Sarlija, N. and Zekic–Susac, M. (2005). Modelling Small–Business Credit Scoring by using Logistic Regression, Neural Networks and Decision Trees. *Intelligent Systems in Accounting, Finance and Management*, 13(3), p. 133.

[3] Breiman, L., Friedman, J.H., Olshen, R. A. and Stone, C.J. (1993). *Classification and regression trees.* New York: Chapman & Hall.

[4] Carter, Ch. and Catlett, J. (1987). Assessing Credit Card Applications Using Machine Learning. *IEEE EXPERT*, Fall 1987, p. 71.

[5] Cho, S., Hong, H. and Ha, B.–Ch. (2010). A Hybrid Approach Based on the Combination of Variable Selection using Decision Trees and Case–Based Reasoning using the Mahalanobis Distance: For Bankruptcy Prediction. *Expert Systems with Applications*, 37(4), p. 3482.

[6] Frydman, H., Altman, E. and Kao, D. (1985). Introducing Recursive Partitioning for Financial Classification: The Case of Financial Distress. *The Journal of Finance*, 40(1), p. 269.

[7] Galimberti, G., Soffritti, G. and Maso, M. (2012). Classification Trees for Ordinal Responses in R: The rpartScore Package. *Journal of Statistical Software*, 47(10), p. 1.

[8] Gepp, A., Kumar, K. and Bhattacharya, S. (2009). Business Failure Prediction using Decision Trees. *Journal of Forecasting*, 29(6), p. 536.

[9] Hüllermeier, E., Vanderlooy, S. (2009). Why Fuzzy Decision Trees are Good Rankers. *IEE Transactions on Fuzzy systems*, 17(6), p. 1233.

[10] Rerolle, J., Rimaud, C. (2009). Does Independent Credit Research Add Value? *Alternative Intelligence Quotient*, 32, p. 31.

[11] Shin, K–s., Han, I. (2001). A Case–Based Approach using Inductive Indexing for Corporate Bond Rating. *Decision Support Systems*, 32(1), p. 41.

[12] Somers, R.H. (1962). A New Asymmetric Measure of Association for Ordinal Variables. *American Sociological Review*, 27, p. 799.

[13] Therneau, T.M. and Atkinson, E.J. (1997). *An Introduction to Recursive Partitioning Using the rpart Routines.* Technical Report 61, Section of Biostatistics, Mayo Clinic, Rochester. URL http://www.mayo.edu/hsr/techrpt/61.pdf.

[14] Zhang, D., Zhou, X., Leung, S. C.H. and Zheng, J. (2010). Vertical Bagging Decision Trees Model for Credit Scoring. *Expert Systems with Applications*, 37(12), p. 7838.

Modelling Extreme values of the PX Index returns

Ján Gogola [1]

Abstract

In this contribution we focused on the daily log returns of investment in the Prague stock exchange index, PX-Index. We analysed data from January 1st, 1995 to June 30th, 2018. We can see that the data has fatter left and right-hand tails than the normal distribution. Conclusions of our basic analysis are that the daily log returns are leptokurtic and heavy tailed. They are not i.i.d. and volatility varies over time. Further we investigated extreme values of daily log returns. The focus is on how Extreme Value Theory fares in contrast to the assumption of normally distributed losses. Also we can say that extreme daily log returns appear in clusters.

Key words

PX index, daily log returns, extreme value theory, generalized extreme value distribution, risk measures

JEL Classification: C13, C40, C60

1. Introduction

Risk managements are motivated to search methodologies able to cope with rare events that may have serious consequences for them. The question arises: "If things go wrong, how wrong can they be? Extreme value theory (EVT) provides a firm theory on which we can build statistical models describing extreme events. In many fields of modern science, engineering, hydrology and insurance (see e.g. Embrechts (1999), Jindrová (2015), Jindrová (2015), Pacáková (2009)), extreme value theory is well suited. Our contribution deals with the behaviour of the daily returns of the PX Index. The PX Index is the official index of major stocks that trade on the Prague Stock Exchange.

Before we turn to the topic of modelling extreme values of PX Index returns, it is worthwhile to consider and review typical characteristics of financial market data. Typical characteristics of financial market data are summarized in the literature (see e.g. McNeil et. al. (2005)) as "stylized facts". These stylized facts are:

- time series data of returns, in particular daily return series, are in general not independent and identically distributed (i.i.d.);
- the volatility of return processes is not constant with respect to time;
- the absolute or squared returns are highly autocorrelated;
- the distribution of financial market returns is leptokurtic. The occurrence of extreme events is more likely compared to the normal distribution;
- extreme returns are observed closely in time (volatility clustering);
- the empirical distribution of returns is skewed to the left, negative returns are more likely to occur than positive.

First we will check whether these stylizes facts are valid to the returns of the PX Index. Then we continue with the EVT. Two approaches to modelling extreme values are presented:

[1] Ján Gogola, RNDr. Ph.D., Fakulta ekonomicko-správní, Univerzita Pardubice, jan.gogola@upce.cz

the block maxima method and the peaks-over-threshold method. More treatment of the EVT we can find in Pfaff (2013). Our results are accomplished by means of the R language (a free statistical software) and its currently available packages.

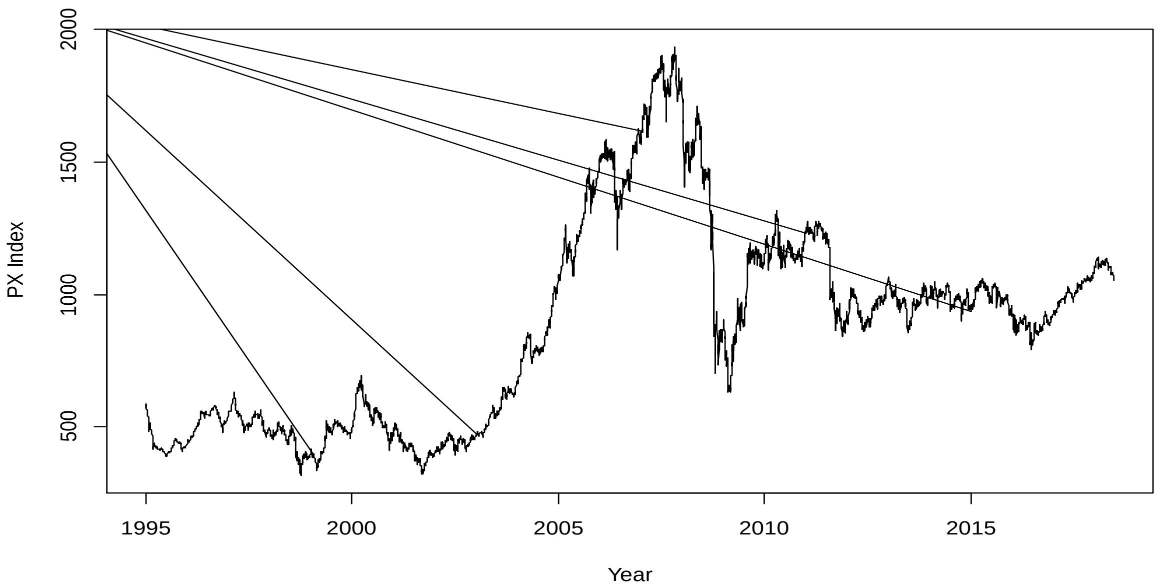
2. Data and methodology

The PX Index is the official index of major stocks that trade on the Prague Stock Exchange. The index was calculated for the first time on March 20, 2006 when it replaced the PX 50 and PX-D indices. The index took over the historical values of the PX50. The starting day of PX 50 was April 5, 1994 and its opening value was fixed at 1 000 points. At this time the index included 50 companies on the Prague Stock Exchange.

Since data in the year 1994 are irregular, we decided to analyse data from from January 1st 1995 to June 30th, 2018. Figure 1. shows the development of the PX Index. From the beginning of 1995 to about 2004 we can see something that looks like business cycles. Business cycles of this type might exist but the cycles are all of different lengths, the timing of the peaks and the lows are difficult to predict. The PX Index reaches its top on October 29, 2007 with 1936 points. As result of financial crisis reached 700 points on October 27, 2008 losing almost 50% of its value in two months.

In our contribution we are going to focus on the daily log returns (Figure 2.) and analyze these returns.

Figure 1: PX Index (in CZK) from 1. 1. 1995 to 30. 6. 2018. Source: www.pse.cz [6]



Particularly, the focus is on the use of extreme value theory to analyse losses (left tail) of the PX Index.

Our first approach considers the maximum the variable takes in successive periods. These selected observations constitute the extreme events, also called ***block maxima***. Assume that a sequence of random variables $X_1, X_2, \ldots, X_m$ over a time span of *n* periods is given. The time span could be a calendar period such as a month, quarter, half year, or year. With respect to EVT, the question arises as to which distribution the maximum of these random variables, $M_n = \max\{X_1, X_2, \ldots, X_k\}$, follows or, to put it more precisely, is asymptotically best approximated by. When modelling the maxima of a random variable, EVT plays the same fundamental role as the central limit theorem plays when modelling sums of random variables.

In both cases, the theory tells us what the limiting distributions are. The limit law for the block maxima M_n, is given by the following theorem Pfaff (2013):

Theorem A: Let $\{X_m\}$ be a sequence of i.i.d. random variables. If there exist constants $c_n > 0$, $d_n \in R$ and some nondegenerate distribution function G such that

$$P\left(\frac{M_n - d_n}{c_n} \le z\right) \to G(z), \text{ for } n \to \infty,$$

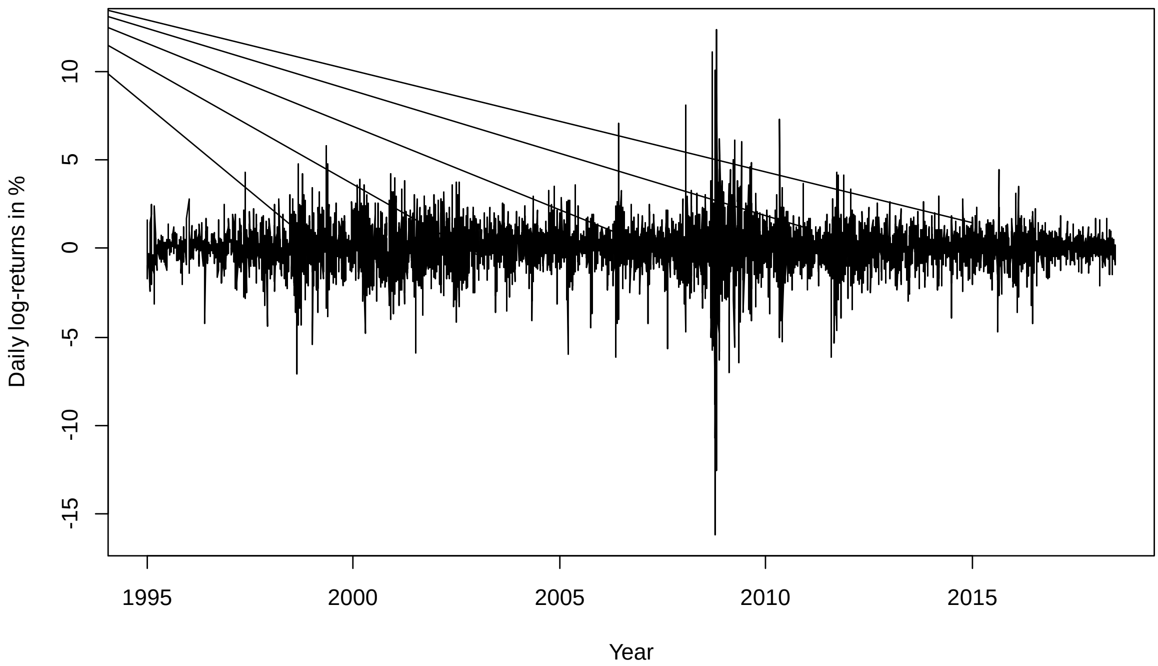
then G belongs to one of the following distributions: Gumbel, Fréchet or Weibull.

The three distributions can be subsumed into the ***generalized extreme value*** (GEV) distribution,

$$G(z) = \exp\left\{-\left[1+\xi\cdot\left(\frac{z-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}. \tag{1}$$

The GEV is a three-parameter distribution where μ is the location, σ the scale and ξ the shape parameter. For the limit $\xi \to 0$ the Gumbel distribution is obtained, for $\xi > 0$ the Fréchet, and for $\xi < 0$ the Weibull. The Weibull has a finite right point. The density is exponential in the case of Gumbel and polynomial for the Fréchet distribution. Hence, the characteristics and properties of the GEV can be deduced from the value of the shape parameter. Fréchet distribution has a polynomially decaying tail and therefore suits well heavy tailed distributions.

Figure 2: Percentage daily return on the PX Index (5876 observations). Source: Own calculation



Our second approach focuses on the realizations exceeding a given (high) threshold u - the peaks-over-threshold method. This can be summarized for a given threshold u by the following probability expression:

$$P(X > u + y \mid X > u) = \frac{1 - F(u+y)}{1 - F(u)}, \; y > 0. \tag{2}$$

In practice the distribution $F(x)$ is generally unknown and hence, similarly to the derivation of the GEV, one needs an approximative distribution for sufficiently large threshold values.

The law is given by the following theorem Pfaff (2013):

Theorem B: For a large class of underlying distribution function F the conditional excess distribution function $F_u(y) = P(X - u \le y \mid X > u)$, for u large is well approximated by $F_u(y) \approx G_{\xi,\sigma}(y),\ u \to \infty$,
where

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi}{\sigma} y\right)^{-\frac{1}{\xi}}, \text{if } \xi \ne 0 \\ 1 - e^{-\frac{y}{\sigma}}, \ \text{if } \xi = 0 \end{cases}. \tag{3}$$

$G_{\xi,\sigma}$ is the so called ***generalized Pareto distribution*** (GPD).

The shape parameter ξ gives an indication of the heaviness of the tail, the larger ξ, the heavier the tail.
In practice, a difficulty arises when an adequate threshold has to be selected. An adequate threshold value can be determined graphically by means of a mean residual life (MRL) plot. This kind of plot is based on the expected value of the GPD: $E(Y) = \dfrac{\sigma}{1-\xi}$. For a given range of thresholds u the conditional expected values

$$E(X - u \mid X > u) = \frac{\sigma_u + \xi \cdot u}{1 - \xi} \tag{4}$$

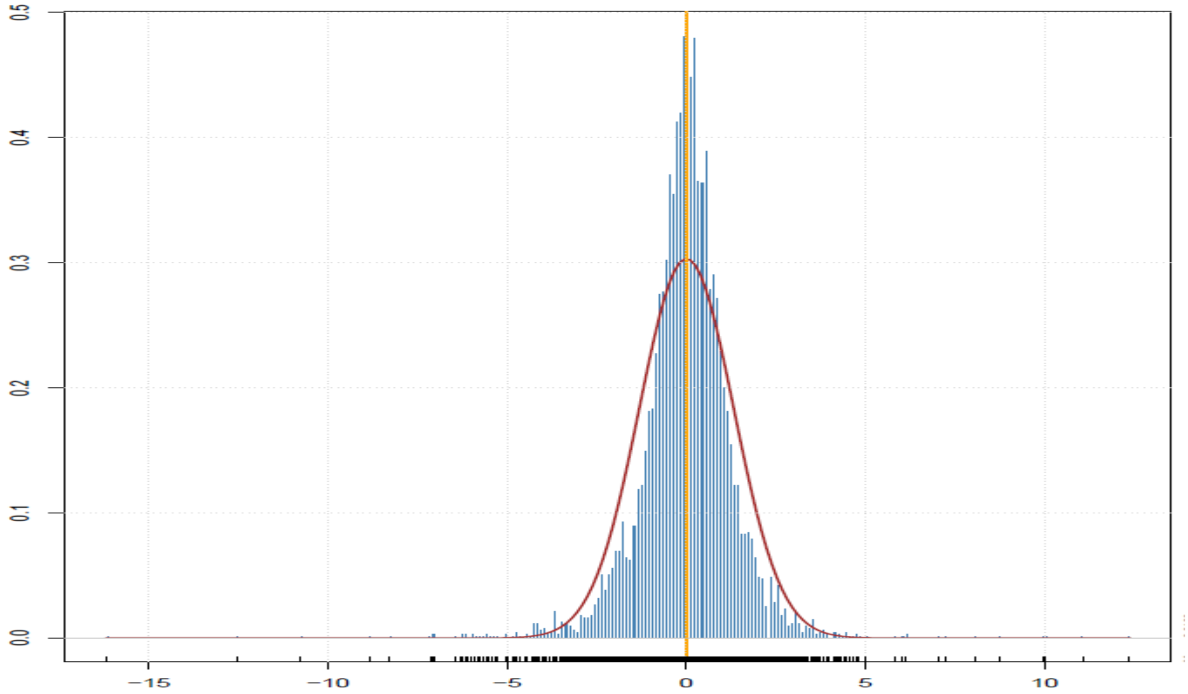
are plotted against u.
This equation is linear with respect to the threshold u. A suitable value for u is given when this line starts to become linear.

3. Results

First we will establish certain stylised facts about daily returns. The PX Index daily returns data has a skewness of $\sqrt{b} = -\ 0.455$ and a kurtosis of $k = 12.55$. These empirical coefficients look quite different from 0 and 3 respectively. The data is clearly non-normal from these analyses. We add to this by looking at the data using graphical techniques, such as histogram and Q-Q plot.

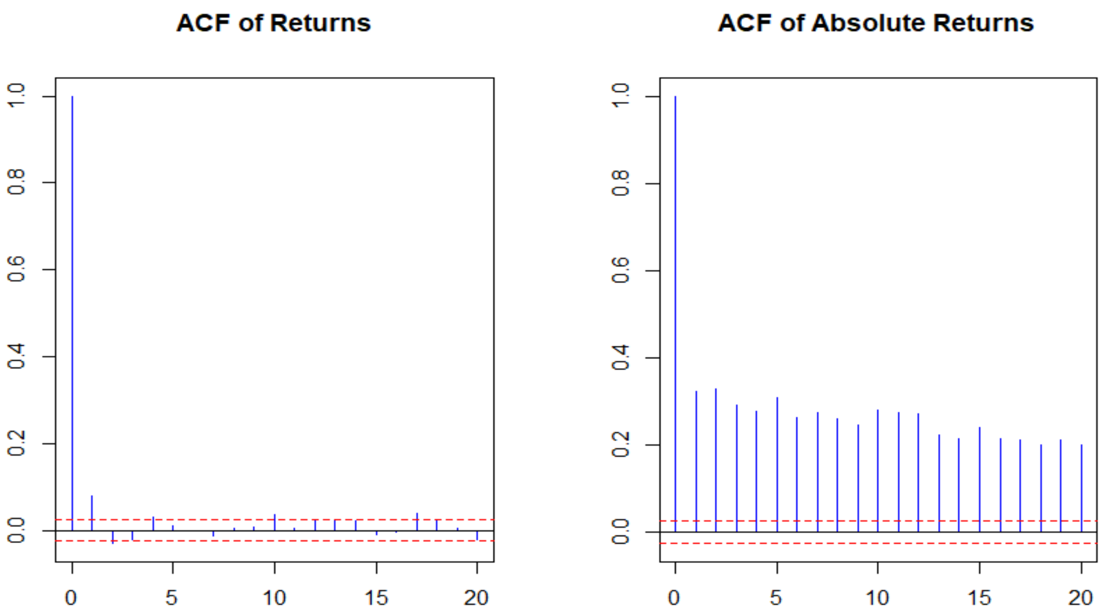
We have plotted in Figure 3. the histogram of the daily log returns on the PX Index. We have also drawn in the density function for the $N(\hat{\mu}, \hat{\sigma}^2)$ distribution. We can easily see from this that the data exhibit a narrower peak than the *best-fitting* normal distribution. Less obviously, but certainly a feature of the data is, that it has a fatter left and right-hand tails than the *best-fitting* normal distribution. A Q-Q plot is a dot plot that plots the ordered sample against the corresponding quantiles of the distribution that we are considering to model the data. If the data were genuinely normally distributed then we would expect to see the 5876 points much more in a straight line. The fact that Figure 5. (left) actually exhibits an inverted "S" shape means that the data has fat left and right-hand tails. The downturn in the plot at the left-hand end means that the left-hand tail is fatter than the normal distribution: in other words we should expect rather more large losses over time than we would predict using the Normal distribution. This inverted "S" shape therefore points to the data being leptokurtic.

Figure 3: Histogram of the percentage daily returns on the PX Index. Source: Own calculation



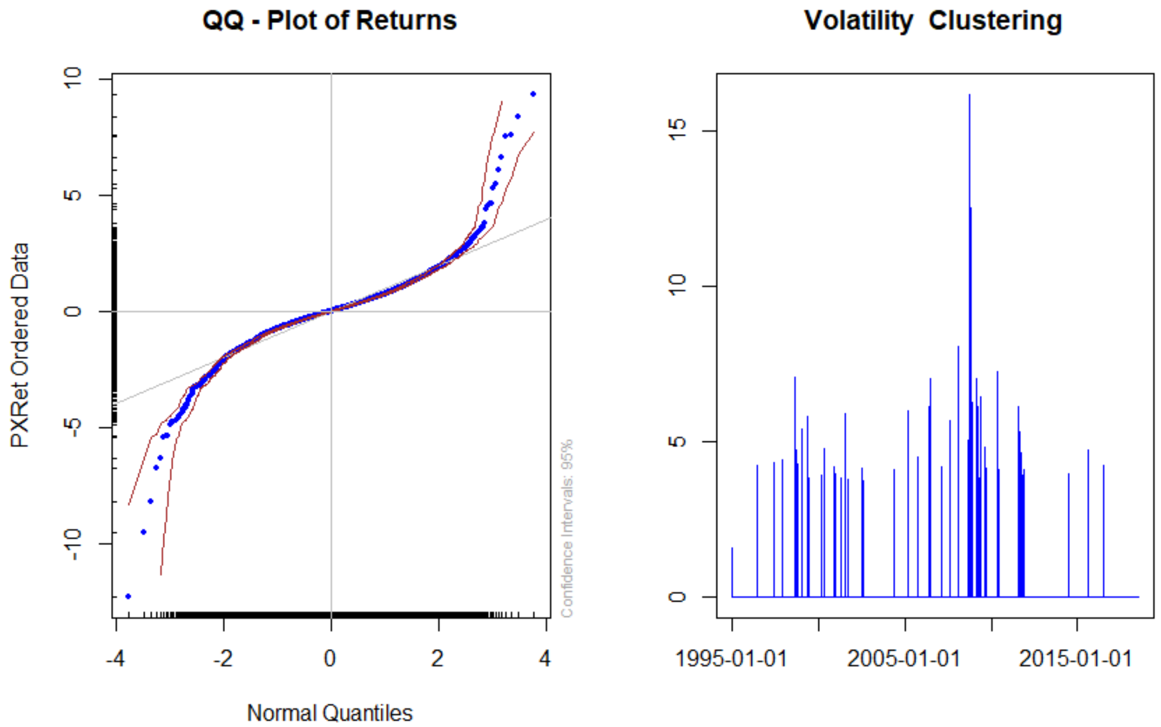
The less-formal graphical/diagnostic tests clearly indicate that the assumption that returns are normally distributed is not valid. Additionally Figure 5. (right) also suggests that the daily log returns are not i.i.d.. Instead, it looks like there are clear clusters of high and low volatility. The PX Index log returns have clusters of high volatility (e.g. in 2008) and low volatility (e.g. 2013).

Figure 4: Left: Sample autocorrelation function for the volatility-standardised residuals. Right: Sample autocorrelation function for absolute value of volatility-standardised residuals. Source: Own calculation



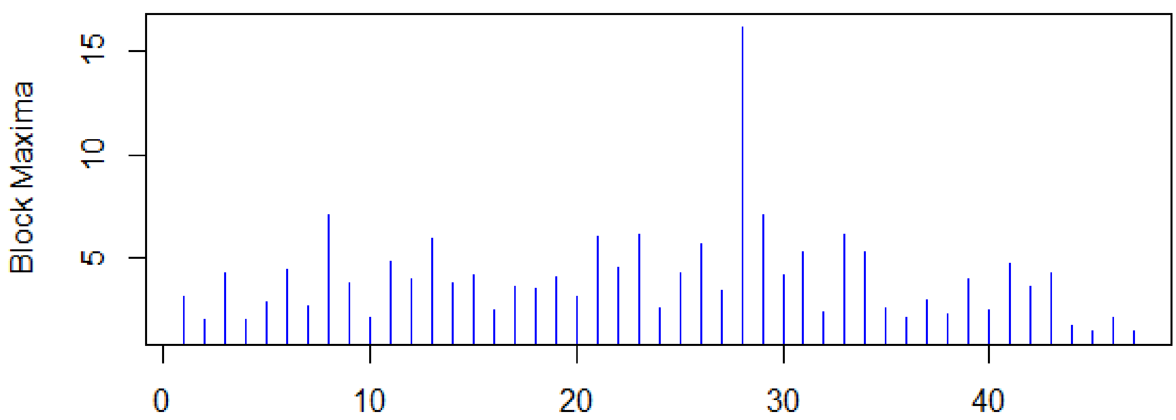
The autocorrelation function (ACF) for the daily returns (Figure 4. - left) hint at a slight autocorrelation of first order. The ACF of the absolute daily returns (Figure 4. - right) looks differently. There is a moderate, but nevertheless highly significant correlation between next days. Clearly, these are significantly different from zero and taper off only slowly. The significant autocorrelations in the absolute values of daily returns imply that the market goes through phases of high and low volatility.

Figure 5: Stylized facts for the PX Index. Source: Own calculation



Now we can apply the extreme value theory to the PX Index data. First the daily returns are converted to positive figures expressed as percentages. The application of the method of block maxima requires the following steps: divide the sample in n blocks of equal length, collect the maximum value in each block, fit the GEV distribution to the set of maxima. The key point of this method is the appropriate choice of the periods defining the blocks. We choose half-year periods to get 47 blocks (which are likely to be sufficiently large for *Theorem A* to hold).

Figure 6: Block maxima (left) for the PX Index. Source: Own calculation



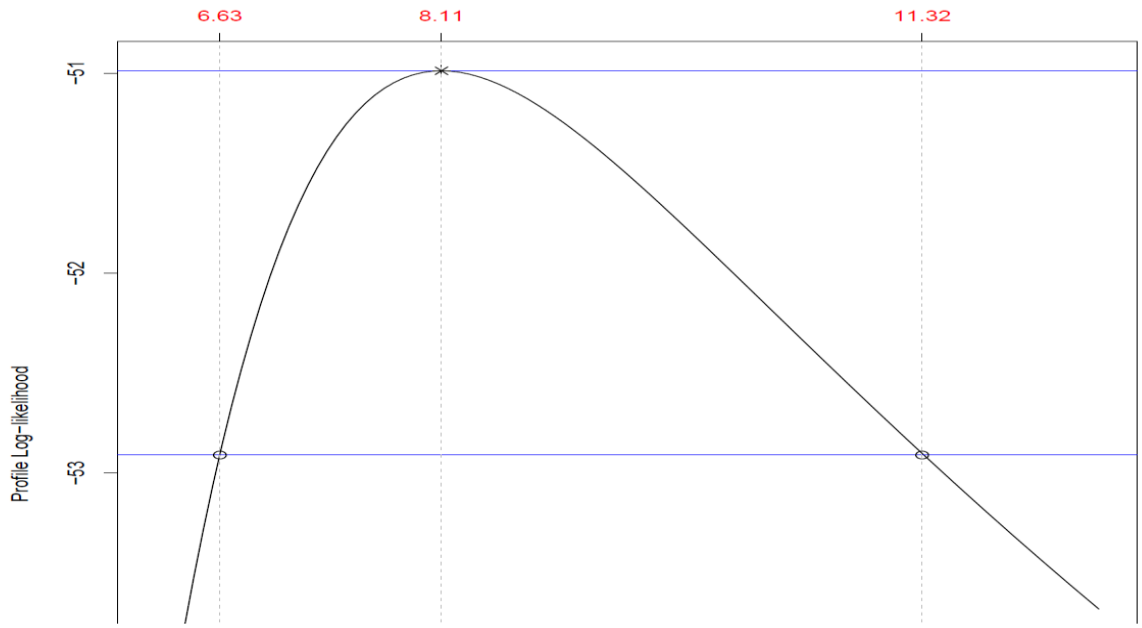
The maximum return in each of the blocks (Figure 6.) constitute the data points for the sample of maxima M which is used to estimate the GEV distribution. Point estimates for the parameters are given in Table 1. Positive value for ξ implies that the limiting distribution of maxima belongs to the Fréchet family.

Table 1: Fitted GEV to block maxima of PX Index

GEV	ξ	σ	μ
Estimate	0.178	1.263	3.010
Standard Error	0.119	0.165	0.210

Further inference from the model can be made using the profile log-likelihoods. Figure 7. shows those for a 10-year return level. A daily loss as high as 8.11 % would be observed once every 10 years. This point estimate falls within in a 95% confidence level ranging from 6.63 % to 11.32 %. As can be clearly seen, the confidence level (the horizontal line) is asymmetric.

Figure 7: Profile log-Likelihood for fitted GEV model



One way to better exploit information about extremes in the data sample is to use the peak-over-threshold (POT) method. The POT method requires the following steps:
select the threshold *u*, fit the GPD function to the exceedances over *u*.
The key point of POT method is the selection of the threshold *u*. Theory tells us that *u* should be high. But the higher the threshold the less observations are left for the estimation of the parameters of the tail distribution function. There is no clear algorithm for the selection of the threshold *u*. A graphical tool that is very helpful for the selection of the threshold *u* is the MRL plot. The sample mean excess function should be linear. This property can be used as a criterion for the selection of *u*. Figure 8. shows the MRL plot corresponding to the PX Index data. From a closer inspection of the plot we suggest the value *u* = 3 %. For the given threshold a total of 104 exceedances result. This data set corresponds roughly to the upper 98% quantiles of the empirical distribution function.

Table 2: Fitted GPD of PX Index, threshold = 3 %

GPD	ξ	σ
Estimate	0.153	1.300
Standard Error	0.099	0.180

We know that the distribution of the observation above the threshold in the tail should be a generalized Pareto distribution (GPD). We compute the distribution parameters (Table 2.) that maximize the log-likelihood function for the sample defined by the observations exceeding the threshold *u*.

Figure 8:MRL plot for PX Index losses

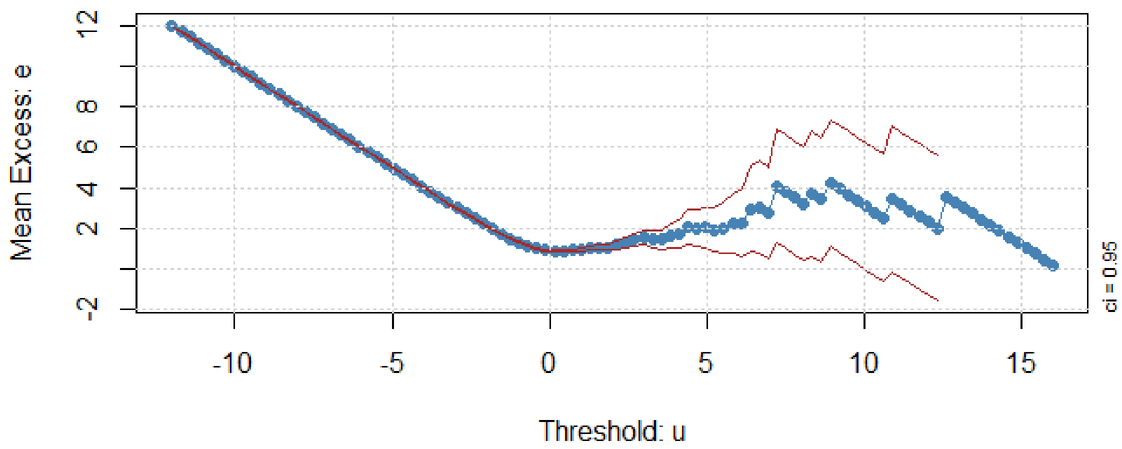


Table 4: Risk measures for PX Index

Confidence level	VaR	ES
95.0 %	1.752	3.061
99.0 %	3.775	5.451
99.5 %	4.813	6.676

4. Conclusion

We have illustrated how to analyse extreme values of daily returns of the PX Index. Our conclusion is that EVT can be useful for assessing the size of extreme events. The Value at Risk (VaR) and expected shortfall (ES) risk measures can be inferred directly from EVT. Point estimates for the VaR and ES risk measures (Table 4.) are computed for the 95%, 99% and 99.5% levels. These measures would qualify as unconditional risk assessment for the next business day.

References

[1] Embrechts, P., Klüppelberg, R. J., Mikosh, T. (1999). *Modelling Extremal Events for Insurance and Finance*, Application of Mathematics, Springer, New York, 2nd ed.

[2] Jindrová, P., Jakubínský, R. (2015). *Significance and possibilities of major accident insurance*. E + M Ekonomie a Management, 18(4), 121-131.

[3] Jindrová, P., Sipková, Ľ. (2014). Statistical Tools for Modeling Claim Severity, *11th International Scientific Conference on European Financial Systems 2014*, p. 288-294.

[4] McNeil A. J., Frey R., Embrechts P. (2005) *Quantitative Risk Management*. Princeton University Press, Princeton and Oxford, ISBN 13: 978-0-691-12255-7.

[5] Pacáková V., Linda B. (2009). *Simulation of Extreme Losses in Non-life insurance*. *E+M Ekonomie a management*, XII, no. 4., pp. 97-103, ISSN 1212-3609

[6] Pfaff, B. (2013). *Financial Risk Modelling and Portfolio Optimization with R*, John Wiley & Sons Ltd, West Sussex, UK, ISBN: 978-0-470-97870-2

[7] R Core Team. (2015). *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, http://www.R-project.org/ (accessed).

[8] www.pse.cz/indexy/hodnoty-indexu/historicka-data

Criteria Interaction in Regional Effectiveness Measurement

Roman Guliak [1]

Abstract

Effectiveness as the missing in the literature aspect of regional performance has been explored and measured by the ratio additive weighting method with regard to the interaction of criteria. Effectiveness is presented as the complementing aspect of regional performance which can enrich regional policy. To analyse interaction between criteria the fuzzy measures were identified and new computationally simple optimization model was proposed. Its specific feature is the processing of correlation between criteria. The new weights in the form of Shapley values counting the criteria interaction were obtained. New weights are able to change the development pattern being put as the benchmarking base and consequently to improve or worsen the effectiveness scores of regions being evaluated. The research received evidences that interaction between criteria is able to influence considerably the effectiveness scores of regions.

Key words

effectiveness, regional performance, MCDM, interacting criteria, fuzzy measures

JEL Classification: C44, R11, R58

1. Introduction

The MCDM methods are commonly able to provide decision-makers with the classification, ranking or utility values of alternatives – results of multidimensional aggregation within the reduction way of thinking. In this aspect, some authors claim that MCDM techniques are highly suitable in multidimensional frameworks when aggregating single indicators into a composite one [Gibari et al., 2018, Nardo et al, 2005, 2008; Jacobs et al. 2004; Freudenberg 2003; Saisana and Tarantola, 2002, Guliak[c], 2017]. Having studied recent articles [Booysen, 2002; Gibari, Gómez, 2018; Stamenković, Savić, 2017; Meyer, Jongh, 2016; Saltelli, 2006; OECD-JRC, 2008; Mazziotta, Pareto, 2013; Goletsis, Chletsos, 2011; Melecký, 2017] reviewing the use of aggregated indicators in the field of regional studies, we have come to conclusion that the effectiveness as a key aspect of regional performance is totally missing. This gap is been filled in by the variety of indicators, such as efficiency, level of development, competitiveness etc [Guliak[b], 2017; Guliak[d], Volkova, 2017].

More over another prominent characteristic of the research mentioned in reviewing papers is that application of MCDM methods does not relate to the analysis of the criteria dependency or interaction, this aspect is merely avoided. However there is one methodology called Pena Distance method (DP2) [Pena, 1977; Somarriba, Pena, Zarzosa, 1996] able to count the substitutability of criteria. The DP2 eliminates all the superfluous common variance and avoids redundant information by means of counting the determination coefficient from

[1] Mgr. Roman Guliak, VŠB - TU Ostrava, Faculty of Economics, Sokolská třída 33, 70121 Ostrava 1, e-mail: roman.guliak@vsb.cz

linear regressions. Thus, it permits higher headroom in the variables selection proves more robust than traditional approaches [Montero, Chasco, 2010]. All other methods applied assume that there is an independency of criteria and they can be treated as just additive sum.

Such a prominent lack of analysis of an important aspect of regional performance counting the interaction between criteria induced the current research. Thus the aim of the research is to find the proper way of treating dependent criteria by means of fuzzy measures and embed it into the suitable methodology for the effectiveness measurement of regional performance.

2. Ratio Additive Weighting method for the effectiveness measurement

Effectiveness of regional performance is going to be measured by the Ratio Decomposition Approach, which steps presented as follows [Guliak[a], 2017]:

1. to determine the initial set of alternatives A.
2. to select original criteria (elements) E and decompose it to the extended set of sub-elements $\bigcup_{i=2}^{n} E^*_{\tau(i)/\tau(v)}$ for $i, v = \overline{1,n} : i > v$, where $\tau(i), \tau(v)$ is the permutation on W, that is $w_{\tau(1)} < w_{\tau(i)} < w_{\tau(n)}$.
3. to form the performance matrix $X = [x_{ki}]_{m \times n}$ showing all values assigned to the alternatives relating to an each criterion.
4. to obtain weights of elements denoted by $W = \{ w_i | i = \overline{1,n} \}$ and decompose it using difference function to obtain new extended n-1 subsets $\underset{v=1}{\overset{i-1}{W}}{}_{\tau(i)/\tau(v)}$ for $i = \overline{2,n} : i > v$ with sub-weights $w_{\tau(i)/\tau(v)}$ calculated by:

$$w_{\tau(i)/\tau(v)} = w_{\tau(i)} - w_{\tau(v)} . \quad (1)$$

5. to normalise the new extended sub-weights $w_{\tau(i)/\tau(v)}$ to obtain 1 as the sum of $w^*_{\tau(i)/\tau(v)}$:

$$w^*_{\tau(i)-\tau(v)} = \frac{w_{\tau(i)} - w_{\tau(v)}}{\sum_{i=2}^{n} \sum_{v=1}^{i-1} (w_{\tau(i)} - w_{\tau(v)})} . \quad (2)$$

6. to normalise initial performance values x_{ki} to obtain l_{ki} for the k-th alternative and i-th criterion using modified linear max normalization producing values from 1 to 10:

$$l_{ki} = \begin{Bmatrix} 1 \, if \ (x_{ki} \cdot 10 / x_i^{\max}) < 1 \\ (x_{ki} \cdot 10 / x_i^{\max}) \\ 10 \, if \ (x_{ki} \cdot 10 / x_i^{\max}) > 100 \end{Bmatrix} . \quad (3)$$

7. to decompose levels of target achievement l_{ki} using ratio function to obtain new extended target achievements $l_{k,\tau(i)/\tau(v)}$, encompassing relations between two criteria:

$$l_{k,\tau(i)/\tau(v)} = l_{k,\tau(i)} / l_{k,\tau(v)} \text{ for } i = \overline{2,n} : i > v . \quad (4)$$

8. to standardise decomposed levels of target achievement to obtain their z-values $l^*_{k,\tau(i)/\tau(v)}$ for $\sigma(i)/\sigma(v)$-th criterion among m alternatives:

$$l^*_{k,\tau(i)/\tau(v)} = \frac{l_{k,\tau(i)/\tau(v)} - \bar{l}_{\tau(i)/\tau(v)}}{S_{\tau(i)/\tau(v)}} , \quad (5)$$

where $\bar{l}_{\tau(i)/\tau(v)}$ – average value of target achievement,

$S_{\tau(i)/\tau(v)}$ – standard deviation of target achievement.

9. to calculate effectiveness E_k representing the decision maker's preferences over k-th alternative considering the extended weighted target achievements:

$$E_k = \sum_{i=2}^{n}\sum_{v=1}^{i-1} l^*_{k,\tau(i)/\tau(v)} \times w^*_{\tau(i)/\tau(v)} \quad \text{for } i = \overline{2,n} : i > v. \tag{6}$$

3. Determination of interacting weights

By this section we are introducing the sequence of steps aimed at the determination of new set of criteria weights which is different from the original one by counting the interaction effect. The steps are the following:

1. to form the original set of weights $W = \{w_i | i = \overline{1,n}\}$;
2. to calculate the Pearson correlation between all pairs of weights $\rho_{X\,ij}\big| i \neq i$ and $i, j = \overline{1,n}$;
3. to determine the fuzzy measures;
4. to calculate Shapley values;

By these steps we are able to obtain by difference operator new sub-weights for effectiveness sub-utilities. Below we will reveal some details of each aspect.

Original set of weights can be obtained by different data driven (objective) or subjective methods. In this paper the weights are defined based on the ratio method, which requires subjectively defined reverse ranking order of all criteria:

$$w_i = \frac{rank_i}{\sum_{i=1}^{n} rank_i}. \tag{7}$$

This method perfectly suits the situation when a decision maker is aware of the targets to be reached and able to clearly form the preferences. As a ranking is reversed, the criteria with the highest value of a rank will get the highest importance. Obtaining the set of weights we receive the pattern of development to be corrected afterwards, when interaction effect is taken into account.

By relying on the following formula we are making the bridge between statistics and geometry needed later for the fuzzy measures identification:

$$\rho_{X\,ij} = \frac{\sum_{i=1}^{n} x_{ki}x_{kj}}{\sqrt{\sum_{i=1}^{n} x_{ki}^2}\sqrt{\sum_{i=1}^{n} x_{kj}^2}} = \frac{x_i x_j}{\|x_i\| \cdot \|x_j\|} = \cos(x_i, x_j). \tag{8}$$

The first expression in the formula is the ratio of two elements, where the numerator is the scalar product of two vectors and denominator is the product of its lengths. Thus, by this we have briefly shown the identity between correlation coefficient and the cosine of the angle between two random vectors.

Next step is to find the fuzzy measures. Let us briefly review basic definitions and properties of fuzzy measures. Considering a set of criteria (elements) $E = \{e_i | i = \overline{1,n}\}$ capacity [Choquet, 1953] or a fuzzy measure [Sugeno, 1974] on E is a mapping $\mu : 2^E \to [0,1]$ satisfying the following conditions:

1. $\mu(\varnothing) = 0, \mu(N) = 1$ (boundary condition);
2. if $\forall S \subseteq A \subseteq B$ implies $\mu(S) \leq \mu(B)$ (monotonicity condition).

In MCDM context each subset of criteria $A \subseteq B$, $\mu(A)$ can be considered as the weight or importance of A. Having $S \cap T = \varnothing$ fuzzy measure can be of three types depending on the interaction effect (λ_{st}) expressing the interdependence between subsets of criteria S and T:

1. capacity is an additive one, when $\mu(S \cup T) = \mu(S) + \mu(T)$ and $\lambda_{ST} = 0$;

2. capacity is non-additive, but super-additive when $\mu(S \cup T) > \mu(S) + \mu(T)$ or $\mu(S \cup T) = \mu(S) + \mu(T) + \lambda_{ST}$;

3. capacity is non-additive, but sub-additive when $\mu(S \cup T) < \mu(S) + \mu(T)$ or $\mu(S \cup T) = \mu(S) + \mu(T) + (-\lambda_{ST})$. The applied procedure for the determination of the interaction effect (λ_{ij}) between *i*-th and *j*-th criteria is based on the trigonometric expression called the "law of cosine" [Saville, Graham, 1991]:

$$|\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 - 2|\bar{a}||\bar{b}|\cos\gamma . \quad (9)$$

Based on this expression we can find the relative λ_{ij} as follows:

$$\lambda_{ij} = \sqrt{\mu(i)^2 + \mu(j)^2 - 2\cdot\mu(i)\cdot\mu(j)\cdot\rho_{Eij}} - \sqrt{\mu(i)^2 + \mu(j)^2 - 2\cdot\mu(i)\cdot\mu(j)\cdot\rho_{Eij}^{etalon}} . \quad (10)$$

The new variable (ρ_{Xij}^{etalon}) being the cosine varies from -1 to1. Thus, depending on its value fuzzy measures are determined. The last element in the presented procedure to add is the optimization model determining the λ_{ij}. Holding the monotonicity and boundary conditions optimization problem has the following view:

$$\text{objective function: } \mu(E) \to \max; \quad (11)$$

$$\mu(E) = \sum_{i=1}^{n}\mu_i + \sum_{i=1}^{n}\lambda_{ij} \le 1$$

$$\text{s.t.: } 0 \le \mu\{e_i, e_j\} = \mu_i + \mu_j + \lambda_{ij} \le 1, \text{ for all } \mu\{e_i, e_j\}\, i \ne j ,$$

$$\rho_{Eij}^{etalon} \in [-1,1].$$

It should be mentioned that the optimization problem presented by inequalities above has a unique solution with optimized ρ_{Eij}^{etalon} when $\mu(E)$ reaches 1.

On the concluding step we define new weights, namely Shapley importance indices (adopted from the cooperative game theory) [Shapley, 1953] relying on the obtained fuzzy measures:

$$\phi_i(\mu) = \sum_{A \subseteq E \setminus i} \frac{|A|!(n-|A|-1)!}{n!}(\mu(A \cup i) - \mu(A)) . \quad (12)$$

Newly obtained Shapley values can considerably be different from the initial set of weights and by this even the order of criteria can suffer from changes. It implies that for the comparison purpose the RAW method should be applied twice: first time with the initial weights and the second time with the Shapley values considering the interaction between criteria.

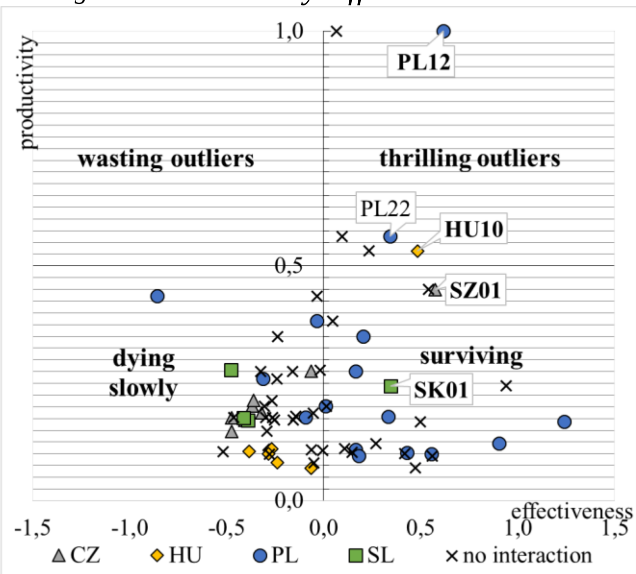
4. Measurement of Visegrad NUTS 2 regions effectiveness

To measure effectiveness with regard to the interaction between criteria the RAW method was applied for the data describing performance of four countries, in particular their 35 Visegrad NUTS 2 regions in 2015 year. The choice of data covering the most essential socio-economic aspects is influenced by the Eurostat database [5] availability. The set of criteria consists of the following original 7 indicators and their weights (defined by the ratio method) reflecting the preferences in the following development pattern: gross domestic product (PPS) (w_7 = 0.25); unemployment (persons) (w_6 = 0.21); employment (persons) (w_5 = 0.18); human resources in science and technology (persons with tertiary education and/or employed in science and technology) (w_4 = 0.14); total intramural R&D expenditure (euro) (w_3 = 0.11);

gross fixed capital formation (euro) (w_2 = 0.07); economically active population (persons) (w_1 = 0.04). Weights of criteria were established using subjective Ratio Method.

Due to the decomposition 7 original indicators have been extended to the 21 sub-criteria. The set of weights as well as set of criteria was extended by the decomposition principle. For the analysis of regional performance we have chosen two-dimensional space, which includes Productivity on axis Y (measured as ratio of Gross Value Added / working hours) and Effectiveness on axis X (fig. 1). This space presented as Productivity-Effectiveness matrix describes regional performance in four main quadrants. Regions belonging to "thrilling outliers" have got high productivity and high effectiveness, what makes them leaders within Visegrad group with. Wasting outliers, in other words regions considered as highly productive but not effective did not appear in the matrix.

Figure 1: Productivity-Effectiveness matrix



"Surviving" regions reach the "right targets" but they are not quite successful in productivity aspect, what points at the lack of resources to realise their relatively high potential of effectiveness in a full extent. Special attention should be given to regions falling to the "dying slowly" sector as they do not possess enough resources and moreover spend them in a not effective way. In comparison to "surviving" their resources are not used in a full extent and targets need to be re-established according to the development pattern reflected in the weights of initial criteria.

The main aspect of analysis, in particular interaction effect, has proven to be influential and made a quite considerable impact on many regions. However there are a few regions changing their quadrant under the interaction influence. It is also necessary to notice that for different countries changes in effectiveness level are different. For instance, on average relative change for countries are the following: all Slovakian regions changed their better effectiveness positions to worse; 5 from 7 Hungarian regions also worsened the scores except the region with capital city (HU10) and region HU32; the majority of Polish regions obtained much better positions, excluding two regions from 16, which have got much worse scores (PL41 and PL52); as concerns Czech Republic, all regions, excluding two (CZ06, CZ01), received less favourable positions.

5. Conclusion

To measure effectiveness as unprecedented in regional practice aspect of regional performance we applied RAW method. It is based on the decomposition of an original set to obtain an extended set of sub-criteria for the subsequent measurement of ration subutilities. However, the main methodological value of the current research is the way how to treat the interacting criteria based on their correlation. Such treatment after all can decrease or increase effectiveness scores. For the purpose of consideration interacting criteria fuzzy measures and theory of capacities are addressed allowing the evaluation of new weights of criteria, namely Shapley values originating from cooperative game theory. New weights being considerably different from the original ones can change the development pattern for the effectiveness measurement and therewith change the effectiveness scores of the regions. Relating the application part, not all regions from Visegrad benefited under the interaction consideration. Moreover such countries as Hungary, Slovakia and Czech Republic in general lost their better positions obtained without counting the interaction effect.

In addition it should be mentioned, that effectiveness measurement should just complement and enrich the classical analysis of regional performance with traditional indices, such efficiency, competitiveness etc. Concept of effectiveness plays the role of a source giving additional information and broadening modelled picture of the reality as it was pictured on the example of development of Visegrad regions. Author hopes that the research conducted in this article will motivate other authors to reveal new sound aspects of the regional performance. Similar close attention to so far not studied aspects of regional performance could make regional policy be more comprehensive and effective.

Acknowledgements

This article has been funded by Student Grant Competition project SGS SP2018/154 of Faculty of Economics, VŠB-Technical University of Ostrava.

References

[1] Booysen F. (2002). *An overview and evaluation of composite indices of development*, Social Indicators Research, 59, p. 115-151.

[2] Choquet, G. (1953). *Theory of capacities*. Annales de l'Institut Fourier, 5, p. 131–295.

[3] Saville, D. J., Graham, R. (1991). *Statistical methods: the geometric approach*. P. 560.

[4] Eurostat database: available from: http://ec.europa.eu/eurostat.

[5] Gibari, S., Gómez, T. & Ruiz, F. J Bus Econ (2018). *Building composite indicators using multicriteria methods: a review*. Journal of Business Economics. pp 1–24.

[6] Goletsis, Y., Chletsos, M., 2011. *Measurement of development and regional disparities in Greek periphery: A multivariate approach*. Socio Econ. Plan. Sci. 45 (4), 174–183. doi:10.1016/j.seps.2011.06.002.

[7] Guliak[a], R. (2017). *Effectiveness as a New Focus of the MAVT MCDM Methods*. Proceedings of the conference Mathematical Methods in Economics (MME 2017), Hradec Králové, Czech Republic, p. 207-2012.

[8] Guliak[b], R. (2017) New *Resonance Approach to Competitiveness Interventions in Lagging Regions: The case of Ukraine before the armed conflict*. Review of Economic Perspectives, Issue 1, 2017, p. 25-56.

[9] Guliak[c], R. (2017) Two*-factor outranking approach to MCDM methods for the lagging regions determination.* SMSIS 2017 - Proceedings of the 12th International Conference on Strategic Management and its Support by Information Systems 2017, Ostrava; Czech Republic, p. 219-230.

[10] Guliak[d], R, Volkova. M. (2017) Two-Based Approach to Regional Potential Measurement in Developing Countries. Actual problems of economics, Issue 2 (188), 2017, p. 333-343.

[11] Mazziotta, M., & Pareto, A. (2013). *Methods for constructing composite indices: one for all or all for one.* Rivista Italiana di Economia Demografia e Statistica, 67(2), 67-80.

[12] Melecký, L. (2017). *Meta-analyses of composite indices in the evaluation of European Union territory*. 20th International Colloquium on Regional Sciences: conference proceedings. p. 43-51.

[13] Meyer, D.F., DE Jongh, J., Meyer, N. (2016). T*he Formulation of a Composite Regional Development Index*. International Journal of Business and Management Studies, vol. 8, no.1, pp. 100-116. ISSN 1309-8047.

[14] Montero, J.-M., Chasco, C., & Larraz, B. (2010). *Building an environmental quality index for a big city: A spatial interpolation approach combined with a distance indicator*. Journal of Geographical Systems, 12, p. 435–459.

[15] OECD-JRC(2008), "*Handbook on Constructing Composite Indicators: Methodology and User Guide"*, by Nardo, M. M. Saisana, A. Saltelli and S. Tarantola (EC/JRC), A. Hoffman and E. Giovannini (OECD

[16] Pena, J. B. (1977). *Problemas de la medicio´n del bienestar y conceptos afines*. Una aplicacio´n al Caso Espanol. (I.N.E.: Madrid).

[17] Saisana, M., Tarantola, S., (2002). *State-of-the-Art Report on Current Methodologies and Practices for Composite Indicator Development. European Commission*: Joint Research Centre – Working paper EUR 20408 EN.

[18] Saltelli, A., Munda, G., Nardo, M., (2006). *From Complexity to Multi-dimensionality: the Role of Composite Indicators for Advocacy of EU Reform*. Tijdschrift voor Economie en Management, vol. LI, no 3, pp. 221-235. ISSN 0772-7674.

[19] Shapley, S. (1953). *A value for n-person games*. In H. W. Kuhn and A. W. Tucker, editors, Contributions to the Theory of Games, Vol. II, number 28 in Annals of Mathematics Studies, pages 307–317.

[20] Somarriba, N. & Pena, B. (2009). "*Synthetic Indicators of Quality of Life in Europe*", Soc. Indic. Res. 94(1): 115–133.

[21] Stamenković, M., & Savić, M. (2017). *Measuring regional economic disparities in Serbia: Multivariate statistical approach*. Industrija, 45(3), 101-130.

[22] Sugeno, M. (1974). *Theory of fuzzy integrals and its applications*. PhD thesis, Tokyo Institute of Technology.

[23] Zarzosa, P. (1996). *Aproximacio´n a la medicio´n del bienestar social*. Valladolid: Secretario de Publicaciones.

Volatility Model based EWMA Minimum Variance Hedging

Haochen Guo [1]

Abstract

In the risk management, volatility as the important parameter for estimation in the issue of hedging. Volatility model is the regression based forecasting model. EWMA (Exponentially Weighted Moving Average) model is one of volatility model, it improves on simple variance by assigning weights to the periodic returns, which means more recent returns have greater weight on the variance. EWMA model introduces lambda which smoothing parameter. Hence, there are many literatures supposed to use the EWMA minimum variance hedging the financial derivatives. In the paper, it estimates the minimum variance hedge based on an advanced econometric model (EWMA model).

Key words

Volatility Model, EWMA Model, Minimum Variance, Hedge Ratio

JEL Classification: C01, G17, G32

1. Introduction

The forecasting of volatility can be regarded as a significant problem of financial modelling. Because the volatility is an important parameter for financial risk management and it is applied in many issues such as option pricing, portfolio optimization, Value at Risk, hedging and so on. There are many literatures supposed to use the EWMA minimum variance hedging the financial derivatives. The are two volatility measurement approaches: historical and implied volatility. The historical approach assumes that past is prologue, the easiest method is simple variance. But the weakness with simple variance is all returns get the same weight. Hence the EWMA model improves on simple variance by assigning weights to the periodic returns. In this paper analyzes the volatility model applies to hedging - EWMA minimum variance hedging.

2. Methodology Volatility Model based EWMA Minimum Variance Hedge

This section describe the minimum variance hedge (section 2.1), the volatility forecasting - EWMA model (section 2.2) and EWMA minimum variance hedge ratio (section 2.3).

2.1 Minimum Variance Hedge

Suppose there is a maturity mismatch, so that the hedge position is closed at some time $t < T$, where T is the expiry date of the futures. The value of the hedged portfolio at time t is

$$P(t) = n \times N_F \times F(t,T) - N_S \times S(t). \quad (1)$$

The variance of this portfolio value is

[1] Ing.Haochen Guo Ph.D, VŠB-Technical University of Ostrava, Faculty of Economics, Department of Finance, Sokolská třída 33, 702 00 Ostrava, Czech Republic, haochen.guo@vsb.cz.

$Variance\big(P(t)\big) = n^2N_F^2Variance\big(F(t,T)\big) + N_S^2Variance\big(S(t)\big) - 2nN_FN_SCov(F(t,T)S(t)).$ (2)

Note that if $nN_F = N_S$ as in the one-for-one hedge then

$Standard\ Deviation\big(P(t)\big) = N_S \times \sqrt{Variance\big(F(t,T)\big) + Variance\big(S(t)\big) - 2Cov(F(t,T),S(t))}.$ (3)

The hedging criterion is to choose n at time 0 to minimize equation (2). Differentiating with respect to n and checking the second order condition gives the optimal number of contracts in the hedge as

$n^* = (\frac{N_S}{N_F}) \times \beta^*,$ (4)

where $\beta^* = \frac{Cov(F(t,T),S(t))}{V(F(t,T))}.$ (5)

The ratio equation is called the minimum variance hedge ratio.

Verification of the financial risk estimation approaches following the minimum variance hedging under partial risk, which expressed variable parameter $r \in (0;1]$. The parameter r means, what part of the risk should be hedged. If the r equal 1, it means the whole risk is hedged, if it is less than 1, it is only a partial hedged. Hence, the minimum variance partial hedge ratio expressed by following equation

$\beta_r^* = \frac{r*Cov(F(t,T),S(t))}{Variance(F(t,T))}.$ (6)

2.2 Estimating Volatility – The EWMA Model

Define σ_t as the volatility of a market variable on day *t*, as estimated at the end of day *t-1*. The square of the volatility on day t σ_t^2 is the variance rate.

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time.

$R_{i,t} = \ln(\frac{P_{i,t}}{P_{i,t}-1})$ for i=1,2,…,t. (7)

Where, t+1: Number of observations,

$P_{i,t}$: Stock price at end of ith interval (i=0,1,…,t).

Because $P_i = P_{i-1}e^{R_i}$, R_i is the continuously compounded return in the ith interval. The usual estimate, P, of the standard deviation of the σ_i ’s is given by

$$\sigma_i = \sqrt{\frac{1}{N-1}\sum_{t=1}^{n}[R_{i,t} - E(R_i)]^2} = \sqrt{\sigma_i^2} \ ,$$ (8)

Where $E(R_i)$ is the mean of the $R_{i,t}$ ’s.

$$\ln\frac{P_T}{P_0} \sim \phi[(R_{i,t} - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\,],$$ (9)

Following that σ itself can be estimated as σ^*.

$$\sigma^* = \frac{P}{\sqrt{\tau}},$$ (10)

The standard error of this estimate can be shown to be approximately $\sigma^* / \sqrt{2t}$.

Suppose that the value of the market variable at the end of day i is P_i. The variable R_i is defined as the continuously compounded return during day i .

$$R_i = \ln \frac{P_i}{P_i - 1}, \tag{11}$$

An unbiased estimate of the variance rate per day, σ_t^2 using the most recent *n* observations on the R_i is

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{N}[R_{t-1} - E(R_i)^2], \tag{12}$$

Where $E(R_i)$ is the mean of the R_i 's:

$$E(R_i) = \frac{1}{N}\sum_{i=1}^{N} R_{t,i} \ , \tag{13}$$

For the purposes of calculating Value at Risk, the formula in equation (13) is usually changed in a number of ways R_i, $E(R_i)$, *N-1*.

Where R_i is defined as the proportional change in the market variable between the end of day i-1 and the end of day i so that

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}}, \tag{14}$$

$E(R_i)$ is assumed to be zero and N-1 is replaced by t.

The formula for variance rate becomes

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N} R_{t-i}^2 \ , \tag{15}$$

Variances are modeled using the Exponentially Weighted Moving Average (EWMA) forecast. The EWMA model is a particular case of the model in follows equation (16).

$$\sigma_t^2 = \sum_{i=1}^{m} \alpha_i R_{t-i}^2 \ , \tag{16}$$

Where the weights, α_i, decrease exponentially as move back through time. EWMA model introduces lambda which smoothing parameter. Specifically, $\alpha_{i+1} = \lambda \alpha_i$ where λ is a constant between zero and one.

It turns out that this weighting scheme leads to a particularly simple formula for updating volatility estimates. The formula is

$$\sigma_t^2 = \lambda \sigma_{r-1}^2 + (1-\lambda) R_{r-1}^2, \tag{17}$$

The estimate, σ_n, of the volatility for day n is calculated from σ_{t-1} and R_{t-1}.

To understand why equation (17) corresponds to weights that decrease exponentially, it substitute for σ_{n-1}^2 to get

$$\sigma_t^2 = \lambda[\lambda \sigma_{t-2}^2 + (1-\lambda) R_{t-2}^2] + (1-\lambda) R_{t-1}^2, \tag{18}$$

or $\sigma_t^2 = (1-\lambda)(R_{t-1}^2 + \lambda R_{t-2}^2) + \lambda^2 \sigma_{t-2}^2$, (19)

Substituting in a similar way for σ_{t-2}^2 gives

$$\sigma_t^2 = (1-\lambda)(R_{t-1}^2 + \lambda R_{t-2}^2 + \lambda^2 R_{t-3}^2) + \lambda^3 \sigma_{t-3}^2, \tag{20}$$

Continuing in this way will see that

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{N} \lambda^{i-1} R_{t-i}^2 + \lambda^N \sigma_0^2 \ , \tag{21}$$

For a large m, the term $\lambda^N \sigma_0^2$ is sufficiently small to be ignored so that equation (17) is the same as equation (16) with $\alpha_i = (1-\lambda)\lambda^{i-1}$. The weights for the R_i's decline at rate λ as move back through time. Each weight is λ times the previous weight.

2.3 The EWMA Minimum Variance Hedge

This is the number of futures contracts in the hedge when minimize the variance of the hedge portfolio. Note that when beliefs are expressed in terms of returns normally express the minimum variance hedge ratio as

$$\tilde{\beta} = \rho \frac{\sigma_S}{\sigma_F}. \tag{22}$$

It estimated by performing a simply linear regression by OLS. The regression model is

$$r_{St} = \alpha + \tilde{\beta} r_{Ft} + \varepsilon_t, \tag{23}$$

where the dependent variance is the return on the portfolio to be hedged and the independent variable is the return on the hedging instrument.

To define the EWMA minimum variance ratio, if λ denotes the smoothing constant the EWMA estimate of the minimum variance hedge ratio at time t is

$$\beta_t^\lambda = \frac{Cov_\lambda(r_{St}, r_{Ft})}{V_\lambda(r_{Ft})}. \tag{24}$$

Verification of the financial risk estimation approaches following the EWMA minimum variance partial hedging under partial risk, which expressed variable parameter $r \in (0; 1]$. The parameter r means, what part of the risk should be hedged. If the r equal 1, it means the whole risk is hedged, if it is less than 1, it is only a partial hedged. Hence, the EWMA minimum variance partial hedge ratio expressed by following equation

$$\beta_t^\lambda = \frac{rCov_\lambda(r_{St}, r_{Ft})}{V_\lambda(r_{Ft})}. \tag{25}$$

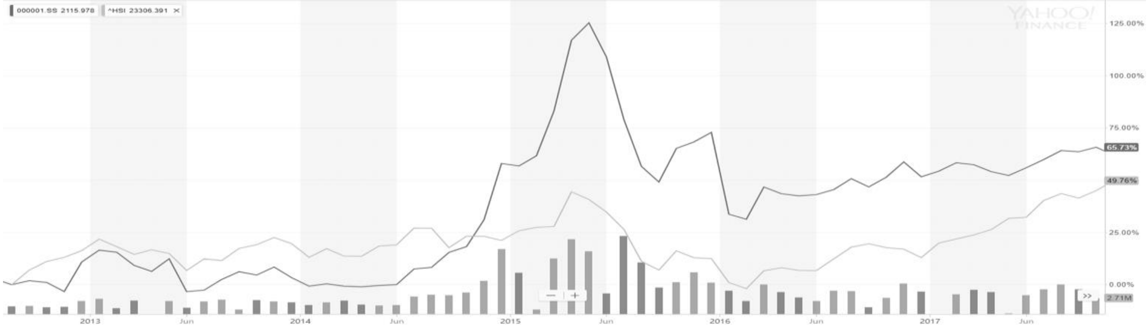
3. Empirical Study of Volatility Model based EWMA Minimum Variance Hedge

This section illustrates the minimum variance hedge ratio (section 3.1) and EWMA minimum variance hedge as the empirical research with two Chinese indices (section 3.2).

3.1 Minimum Variance Hedge Ratios

Illustrate that based on the equation (5), (6) and (21) to calculate the OLS minimum variance futures hedge ratios for the two Chinese indices (Figure 1) of SSE composite index (000001.SS), HANG SENG index (^HIS) that according to the hedging period of 1 day (daily data). The following Table 1 presents the results of minimum variance hedge ratios from 23th July 2012 – 13th October 2017.

Figure 1: Index data from 2012-2017



Source: Yahoo Finance

Table 1: Minimum Variance Hedge Ratio

Returns	SSE	HANG SENG
Spot volatility	23.51%	16.48%
Futures Volatility	26.49%	17.77%
Correlation	0.082	0.020
Minimum Variance Hedge Ratio	0.073	0.018

Source: own calculation

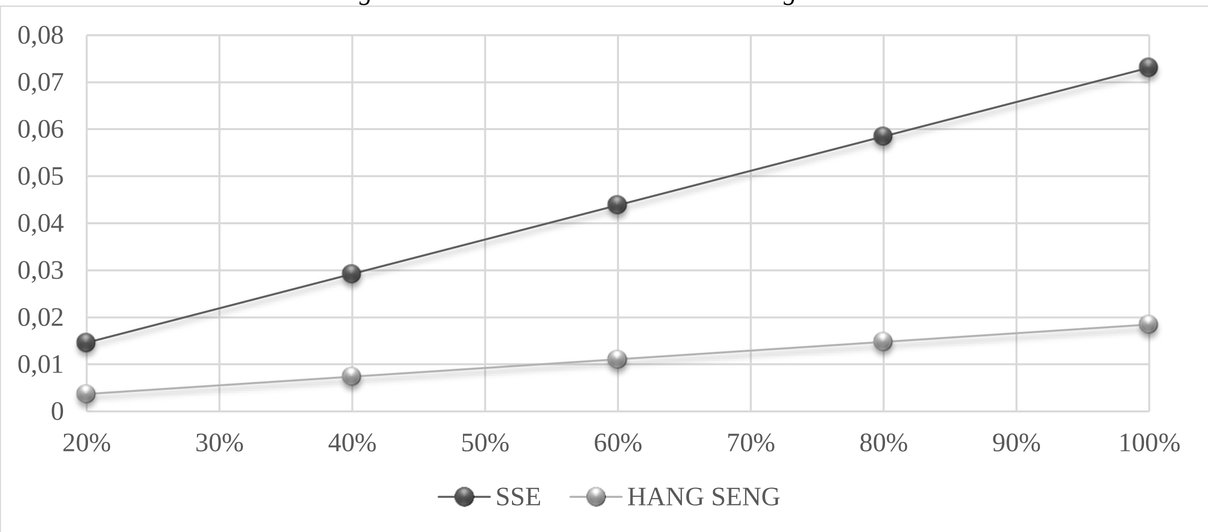
The minimum variance hedge ratio is the product of the correlation coefficient between the changes in the spot and futures prices and the ratio of the standard deviation of the changes in the spot price to the standard deviation of the futures prices. The size of the minimum variance hedge ratio increases with the spot-futures correlation. When the spot and futures are very highly correlated the minimum variance hedge ratios are very close to 1. Otherwise might opposite.

Table 2: Minimum Variance Partial Hedge Ratio

	100% (full hedge)	80%	60%	40%	20%
SSE	0.073	0.058	0.044	0.029	0.015
HANG SENG	0.018	0.015	0.011	0.007	0.004

Source: own calculation

Figure 2: Minimum Variance Partial Hedge Ratio



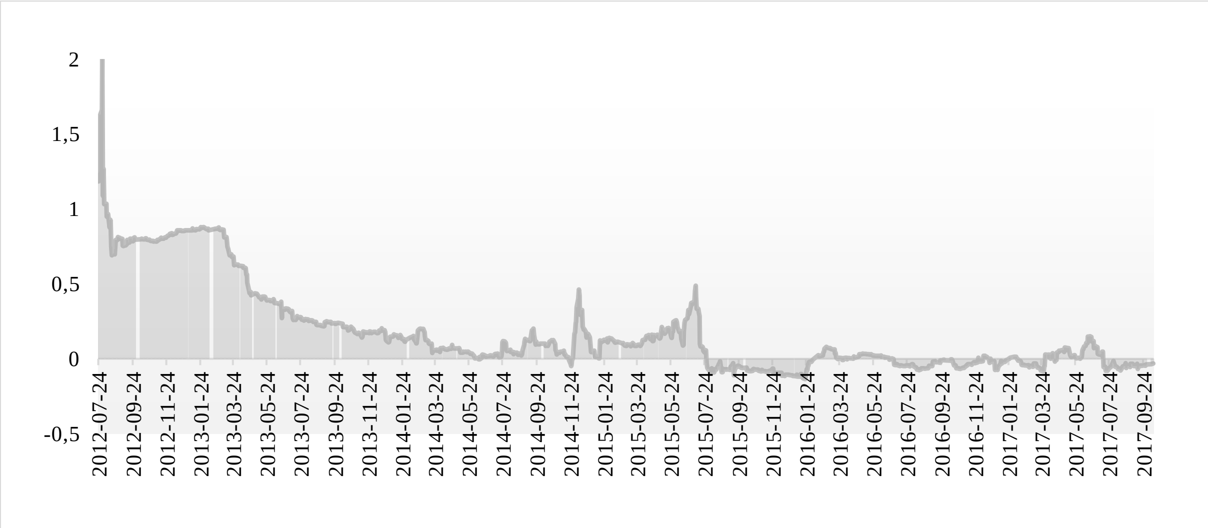
Source: own calculation

The result under the risk of 20%, the partial hedging with SSE is 0.015 and HSI is 0.004; under the risk of 40%, the partial hedging with SSE is 0.029 and HSI is 0.07; under the risk of 60%, the partial hedging with SSE is 0.044 and HSI is 0.011; under the risk of 80%, the partial hedging with SSE is 0.058 and HSI is 0.015; under the risk of 100% (full hedge), the hedging with SSE is 0.073 and HSI is 0.018. Though the increase of risk the partial hedging optimization is going up that is proof in the Figure 2.

3.2 The EWMA Minimum Variance Hedge

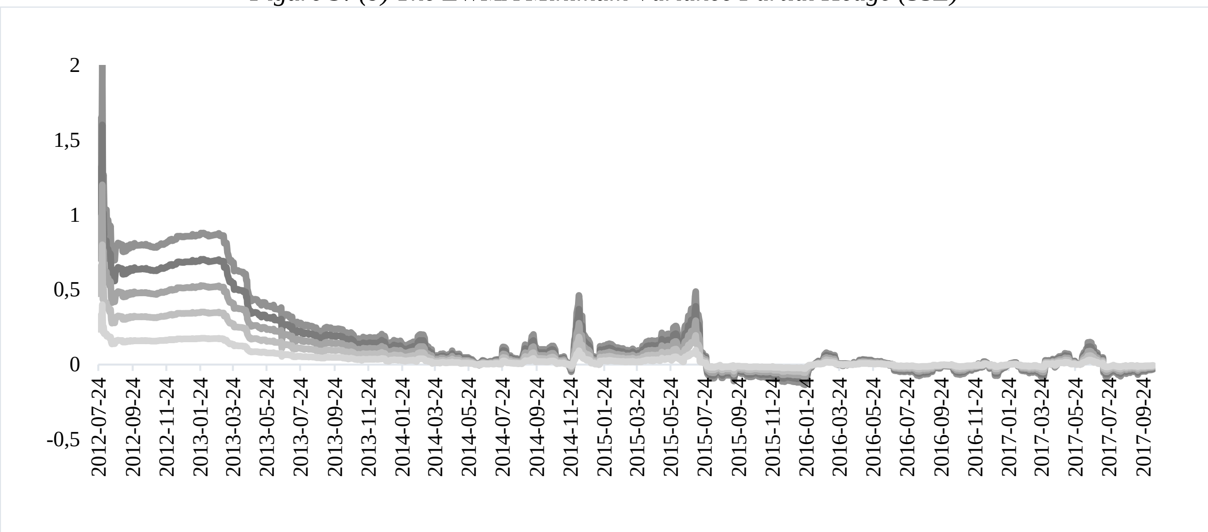
Illustrate that the case of SSE composite index and Hang Seng index apply EWMA minimum variance hedge which based on the equation (24) and (25). Following Figure 3 and Figure 4 present the EWMA minimum variance hedge ratio with full hedge risk and partial risk in SSE composite index (000001.SS) and HANG SENG index (^HSI).

Figure 3: (a) The EWMA Minimum Variance Hedge (SSE)



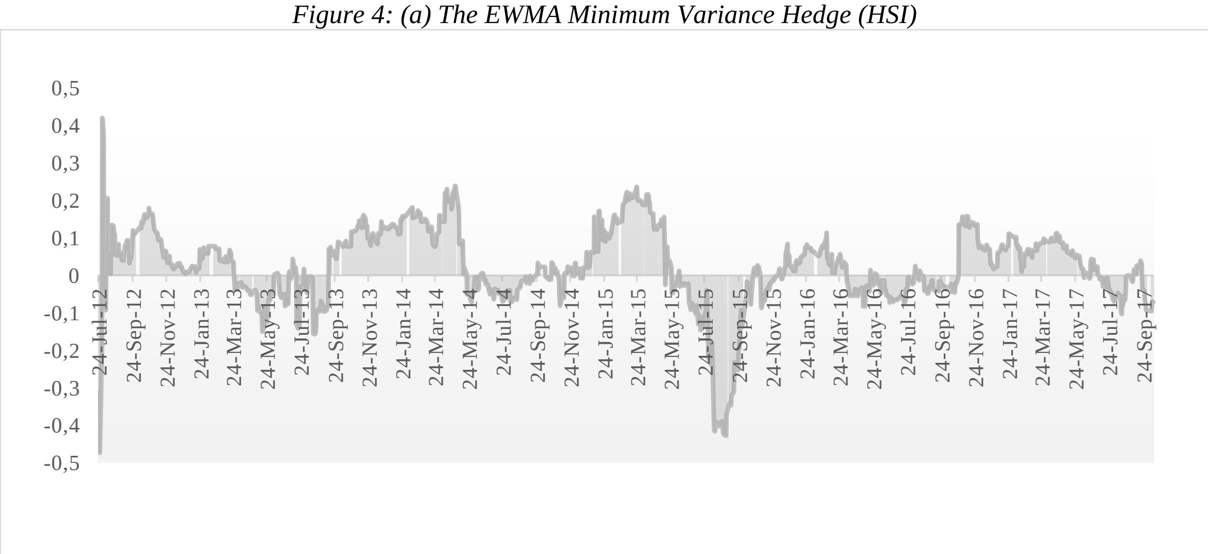
Source: own calculation

Figure 3: (b) The EWMA Minimum Variance Partial Hedge (SSE)



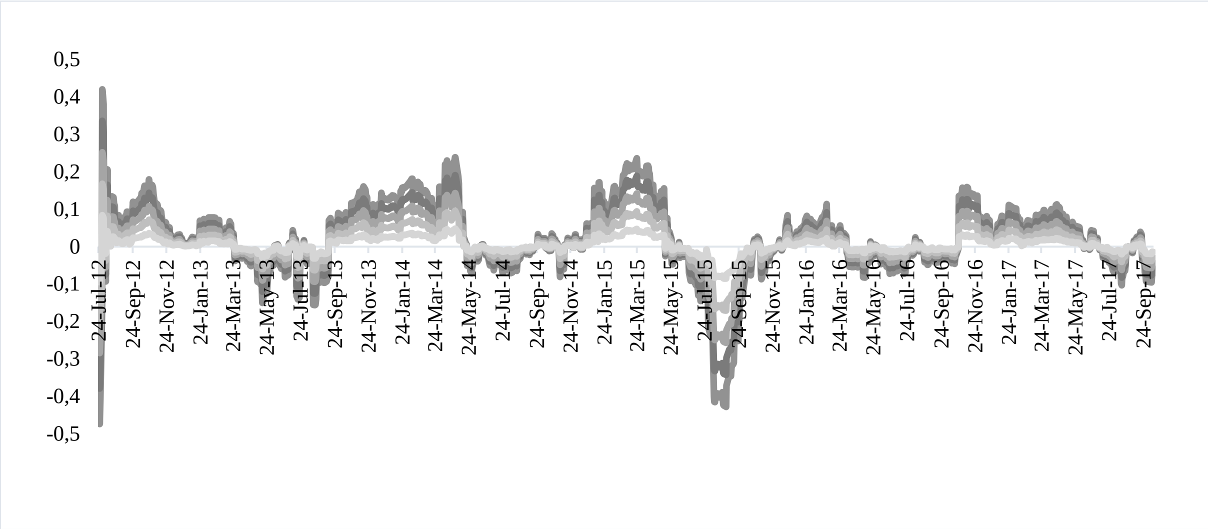
Source: own calculation

Figure 4: (a) The EWMA Minimum Variance Hedge (HSI)



Source: own calculation

Figure 4: (b) The EWMA Minimum Variance Partial Hedge (HSI)



Source: own calculation

Figure 3 and Figure 4 present the EWMA minimum variance hedge ratios. The OLS minimum variance hedge ratio is equal to the average of the EWMA hedge ratios over the sample period.

4. Conclusions

Volatility is the instantaneous standard deviation of a stock and the most common risk metric. The are two volatility measurement approaches: historical and implied volatility. The historical approach assumes that past is prologue, the easiest method is simple variance. But the weakness with simple variance is all returns get the same weight. Hence the EWMA model improves on simple variance by assigning weights to the periodic returns.

This paper analyzes the volatility model (EWMA model) to estimate minimum variance hedge ratio when the hedge is placed over a very short horizon (daily data) with SSE composite index (000001.SS) and HANG SENG index (^HSI). In principle, the GARCH model is better compared with the EWMA model for short term forecasting. Hence, the following step of empirical study will the volatility model based EWMA minimum variance hedging. To compare EWMA model from GARCH model, which states with a proper specification of the dynamics of σ_t^2 and then proceeds to estimate the parameters of this model. Apply to the EWMA covariance, EWMA as a simplistic version of bivariate normal GARCH. Similarly, the EWMA volatility forecast for all risk horizons is simply set at the current EWMA estimate of volatility. The base horizon for the forecast is given by the frequency of the data-daily returns will give the one-day covariance matrix forecast, weekly returns will give the one-week covariance matrix forecast etc. Then, since the returns are independent and identically distributed, the square root of time rule applies. Hence, to compare with the EWMA hedge ratio and GARCH hedge ratio, where the model hedge ratio is constant and only its estimate varies over time.

References

[1] Alexander, C. (2008). Market Risk Analysis Volume I: Quantitative Methods in Finance. John Wiley & Sons.

[2] Alexander, C. (2008). Market Risk Analysis Volume III: pricing, Hedging and Trading Financial Instruments. John Wiley & Sons.

[3] Billio,M., Casarin,R., Osuntuyi,A. (2018). *Markov switching GARCH models for Bayesian hedging on energy futures markets*. Energy Economics. 70(2018).545-562.

[4] Chang,C., Gonzalez-Serrano,L., Jimenez-Martin, J. (2013). Currency hedging strategies using dynamic multivariate GARCH. Mathematics and Computers in Simulation. 94(2013). 164-182.

[5] Chung,S. (2009). Bivariate mixed normal GARCH models and out-of-sample hedge performances. Finance Research Letters. 6(2009). 130-137.

[6] Focardi,M,S., Fabozzi,J,F. (2004). The Mathematics of Financial Modeling & Investment Management.

[7] Frommel,M. (2010). Volatility regimes in central and eastern european countries‘ exchange rates. Czech Journal of Economics and Finance. 1(2010). 2-21.

[8] Guo,H. (2012). Estimating Volatilities by the GARCH and the EWMA model of PetroChina and TCL in the stock exchange market of China. 6th Internatioal Scientific Conference Managing and Modeling of Financial Risks. Ostrava.

[9] Hull,C,J. (2006). *Option, Futures, and other Derivatives.* Prentice-Hall International.

[10] Lien,D. (2009). *A note on the hedging effectiveness of GARCH models*. International Review of Economics & Finance. 18(2009). 110-112.

[11] Neftci,N,S. (2000). An Introduction to the Mathematics of Financial Derivatives. Academic press. Neftci,N,S.(2004). Principles of financial engineering. Elsevier academic press.

[12] Sengupta,N,A. (2005). Pricing Derivatives: the Financial Concepts Underlying the Mathematics of Pricing Derivatives. McGraw-Hall.

[13] Yan,Z., Li,S. (2018). Hedge ratio on Markov regime-switching diagonal Bekk-Garch model. Financial Research Letters. 24(2018). 49-55.

[14] Zmeskal, Z., Dluhosova, D., Tichy, T. (2004). *Financial Models.* Ostrava.

Evaluation of the Influence of Chosen Value Indicators on the Value Creation of Selected Czech Companies

Petr Gurný, Miroslav Čulík, Martin Popp[1]

Abstract

This study focuses on evaluation of influence of selected value indicators on the change of company value. The study analyses 10 companies traded on the Prague Stock Exchange. The goal of the paper is to evaluate the impact of EVA and SVA indicators on the change in company value. First, the relationship is evaluated by graphical analysis, followed by the Pearson, Spearman and Kendall correlation coefficients, as well as linear regression. For most companies, the relationship between selected value indicators and a change in value was not found.

Key words

Company valuation, value indicators, economic value added, market value added, cash value added, value creation, correlation coefficient.

JEL Classification: G12, G32

1. Introduction

This paper examines the impact of selected indicators affecting the value of a company. This study is inspired by the paper of Pablo Fernandéz entitled *“EVA and Cash value added do NOT measure shareholder value creation.”* In this paper, the relationship between *EVA* and market value of the firms using graphs is studied. The author found some correlations, for example for Coca Cola Company, but for most of firms, the correlation was not so evident. Next, he provides the evidence regarding the lack of correlation between *EVA* and *MVA* by analysing 582 firms in the US between years 1988 and 1997. Fernandéz has found that only 18 firms have correlations significantly higher than 80 % and 210 companies have negative correlation. The intention of this study was to find out if the statements of Stern Stewart & Co are valid. Stern Stewart & Co, which invented EVA, provided following statements:

- *“EVA is the measure that correctly takes into account value creation or destruction in a company.”*
- *“There is evidence that increasing EVA is the key for increasing the company’s value creation.”*, etc.

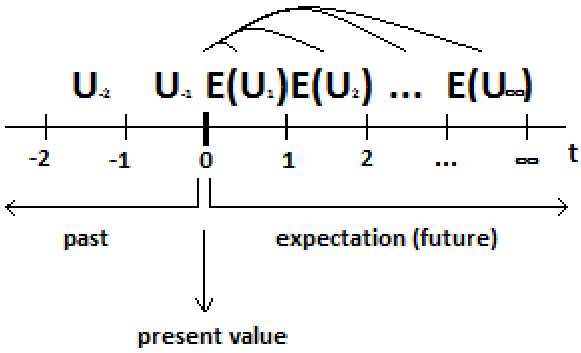
Next, the relationship between increase of *CVA* and Shareholder return between years 1994 and 1998 was analysed. In Fernandez’s analysis 100 the most profitable companies from 5316 largest listed companies in the world. In the whole sample, the mean of correlation is 1.7 %.

These results were expected, because in general the value is given as a present value of the expected (future) benefits. These benefits are returns, cash flows, etc. In this case, the change in the value (value increment) is given by present value of the changes in these indicators.

[1] Ing. Petr Gurný, Ph.D., doc. Ing. Miroslav Čulík, Ph.D. VSB-Technical University of Ostrava, fakulty of Economics, Finance deparment, 70121 Ostrava, Czech Republic. Emails: petr.gurny@vsb.cz, miroslav.culik@vsb.cz, martin.popp.st@vsb.cz

Therefore, the value or change in the value of these companies is not affected by the past changes in these indicators. This is shown in the figure 1, where *U* is benefit, *E* is expected and *t* is time.

Figure 1: General Concept of Value Creation



This study differs from Fernandez's original paper in some aspects. One of the differences is geographic difference, the Czech companies are analysed in this paper particularly. The next difference is in applied methods because it is not evident whether the time series are normally distributed. Therefore, not only parametric correlation coefficients are used, but both parametric Pearson's correlation coefficient and nonparametric Spearman's and Kendall's correlation coefficient are used as well. The normal distribution of time series is tested by Shapiro-Wilk test but we cannot be sure that the results of this test are correct because very short time series is analysed. Next difference is in the calculation of indicators per year but moving after the quarters and it is calculated at the end of the reporting period. However, changes in value of company are calculated on the day when this information is publicly available. There is also an analysis of extreme values and after careful consideration, these values are omitted or replaced with linear interpolation method. Analysed value indicators and relevant dependent variables are listed in table 1.

Table 1 Analysed Relationships

Indicator	Measure of change in value
EVA equity (EVA)	Difference of market value (dMV)
Value range of EVA equity (rEVA)	Relative change in market value (rMV)
EVA equity (EVA)	Change in Market Value Added (dMVA)
SVA equity (SVA)	Difference of market value (dMV)
Value range of SVA equity (rSVA)	Relative change in market value (rMV)

2. Measure of Indicators, Change in Value and Used Methods

2.1 Indicators

Economic Value Added is indicator developed by Stern Stewart & Co. *EVA* should eliminate certain shortcomings of traditional indicators, e.g. *ROE* or *ROA*. In this study, two different concepts of *EVA* are used. The first one is *EVA* based equity given by the formula:

$$EAT = EAT - E_{bv} \cdot R_E \,, \tag{1}$$

where *EVA* represents Economic Value Added based equity, *EAT* stands for profit after tax, E_{bv} is book value of equity and R_E is the cost of equity. When *EVA* is positive, the value of equity is created and *vice versa*.

The next concept used in this paper is the value range of *EVA*. The relative version of this indicator is given by the following formula:

$$rEVA = \frac{EVA}{E_{bv}} = \frac{EAT}{E_{bv}} - R_E . \tag{2}$$

Shareholder Value Added relies on a concept similar to *EVA* but with different measure of capital. This measure works with the market value of equity, i.e.

$$SVA = EAT - E \cdot R_E , \tag{3}$$

where *E* is the market value of equity.

The value range of *SVA* is calculated in this way,

$$rSVA = \frac{EVA}{E} = \frac{EAT}{E} - R_E . \tag{4}$$

2.2 Measure of the Change in Value

The first measure of change in value in this study is *dMV*, which is calculates as:

$$dMV = E_t - E_{t-1}, \tag{5}$$

where *t* is time when the financial reports are publicly available and *t-1* is the previous period.

The second measure is the relative change in market value of equity and is given by the following formula,

$$rMV = \frac{dMV}{E_{t-1}} . \tag{6}$$

The last indicator used is *dMVA*. In general, *MVA* is calculated as,

$$MVA = E - E_{bv} . \tag{7}$$

However, in this study it is necessary to find *MVA* only for one period, therefore, we calculate it as,

$$dMVA = (E_t - (E_{bv})_t) - (E_{t-1} - (E_{bv})_{t-1}) . \tag{8}$$

For *dMVA* we find the relationship only with *EVA* because (according to Stern Stewart & Co), these two indicators should provide the same results. Moreover, both indicators operates with the book value of equity.

2.3 Correlation Coefficients Applied

The Pearson's correlation coefficient is parametric coefficient, which requires normality in distribution of time series. The formula is,

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y} . \tag{9}$$

where σ_{xy} is covariace, σ_x and σ_y are standard deviations of *x* and *y*.

The Kendall's correlation coefficient is a nonparametric coefficient, which works with number of concordant and discordant pairs. The formula is as follows,

$$\tau = \frac{P - Q}{n \cdot (n-1)/2} . \tag{10}$$

where *P* stands for the number of concordant pairs, *Q* is the number of discordant pairs and *n* is the number of pairs.

The Spearman's correlation coefficient is a nonparametric coefficient working with the ranking of two variables. It is defined as,

$$\rho = 1 - \frac{6}{n \cdot (n^2 - 1)} \cdot \sum_{i=1}^{n} (x_{ri} - y_{ri})^2 . \tag{11}$$

where *n* is the number of observation and x_{ri} and y_{ri} are each ranked observations.

3. Data

In this paper, 10 companies traded on the Prague stock exchange are analysed. The low number of selected companies is due to size of the Czech capital market, which is relatively small. Only companies with the time series of stock price longer than 5 years are chosen. The variables as *EAT* and E_{bv} needed for the calculation are obtained from the reported and publicly available financial reports of each company. The market variables value of equity are obtained from the Prague stock exchange database. Each company that publish financial reports on a quarterly basis time series between the 1st quarter of 2013 and 3rd quarter of 2017 are used. The firms that publish financial results on a semi-annually basis, time series between 1st half of 2013 and 1st half of 2017 are used. *CETV* is the only exception because this company had the negative book value of equity. Therefore, this company is analysed only from the 1st quarter of 2013 to 1st quarter of 2016. The cost of equity is calculated by employing CAPM model. All time series are adjusted and converted to per one share.

All variables are obtained as a sum of 4 quarters moving forward. It means that the first data is from the 1st quarter of 2013 to 4th quarter of 2013, the next is from the 2nd quarter of 2013 to 1st quarter 2014 and so on. This adjustment helps to eliminate the seasonality in the short time data series.

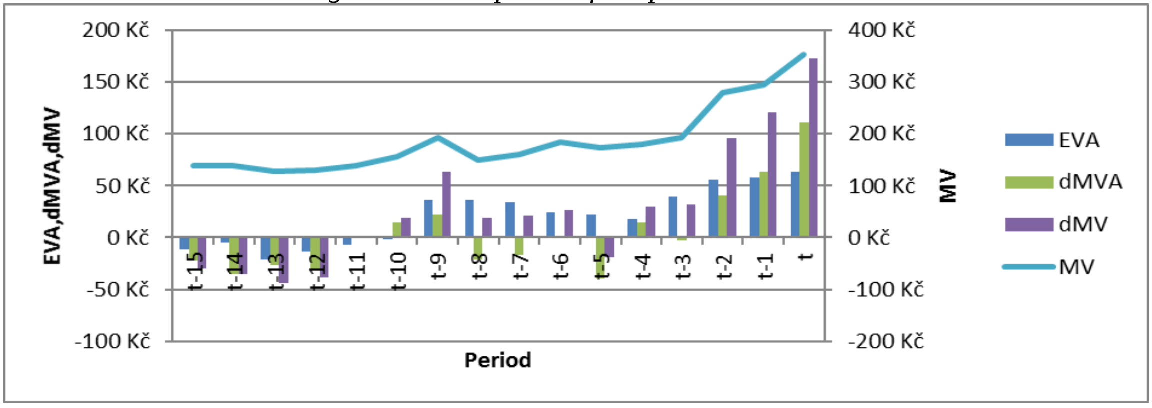
A crucial problem in this analysis is in the reporting date. Particularly, indicators *EVA* and *SVA* are reported to the end of accounting period but financial reports were published later. Therefore, investors cannot react immediately to arriving information and price of the stock cannot be affected by *EVA* or *SVA* indicators. Consequently, *EAT* is for given accounting period whereas E_{bv} or E (used for calculating *SVA*) are the values at the end of given period. On the top, E value (used for calculating change of market value of equity) is the value valid on the day the financials reports were published. We abstracted such problems with the time discrepancy from the analysis.

Data used to calculate the Pearson’s correlations coefficient and linear regression should be normally distributed. Data are tested by the Shapiro-Wilk test. Insomuch as we cannot be sure that the results of this test are correct, interpretation of the results requires caution. Further (after careful consideration) the extreme values of time series are removed from the data series and replaced by the linearly interpolated values. The determination of the Spearman’s and Kendall’s correlation coefficients do not require normality in the value distributions and that is why any adjustment in the time series values are not necessary. Some of time series are delayed based on the results of the cross correlation.

4. Results analysis

First, the relationship between the above mentioned concept of *EVA* and change in the market value of company is examined. Figure 2 shows the correlation between *EVA* per share and *dMVA* per share or *dMV* per share and evolution of the market value (*MV*) of the company, as well. We can observe that there is a positive correlation between these relationships, but only in a few examined companies. In figure 3, correlation is not very explicit.

Figure 2 Development of Unipetrol Indicators



Next, these relations and relations of relative indicators are analysed by the correlation coefficients. The results of these calculations are shown in table 2. In this table, Evans scale of the correlation power can be found. We can observe that there is not any universal conclusion of correlation. Only a few companies have very strong correlations and some companies have negative correlations. However, there is a problem with the significance of the results because more than 70% of the results is statistically insignificant on alfa level of 5%. The results (after reduction) alfa to 10% are shown in table 3. In this table, values, which are not significant, are presented as a zero correlation.

Figure 3 Development of Erste Group Bank Indicators

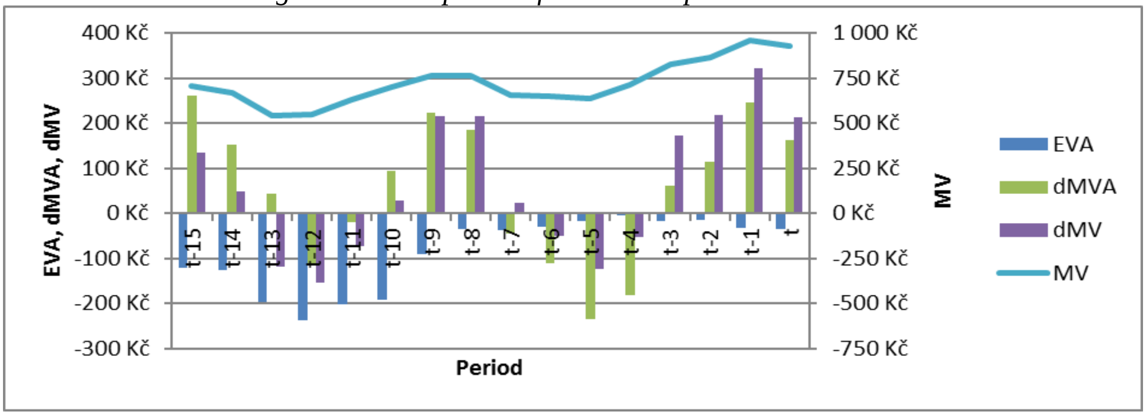


Table 2 Strength of Correlations between EVA Indicators and Change in Value Indicators

Strength of correlation		*Relation of EVA and dMV*			*Relation of rEVA and rMV*			*Relation of EVA and dMVA*		
Range		ρ_p	τ	ρ_s	ρ_p	τ	ρ_s	ρ_p	τ	ρ_s
0.8 to 1.0	**Very strong**	2	0	2	1	0	1	0	0	1
0.6 to 0.8	**Strong**	1	2	1	1	1	1	3	1	1
0.4 to 0.6	**Moderate**	3	1	3	4	2	4	3	2	2
0.2 to 0.4	**Weak**	0	4	1	0	4	1	0	4	3
0.0 to 0.2	**Very weak**	2	1	1	2	1	1	1	1	1
0.0	**Zero**	0	0	0	0	0	0	0	0	0
0.0 to - 0.2	**Very weak**	0	1	1	0	0	0	2	1	1
-0.2 to -0.4	**Weak**	1	1	0	1	1	1	1	1	1
-0.4 to -0.6	**Moderate**	1	0	1	1	1	1	0	0	0
-0.6 to -0.8	**Strong**	0	0	0	0	0	0	0	0	0
-0.8 to -1.0	**Very strong**	0	0	0	0	0	0	0	0	0

Table 3 shows the best relations is relation between *EVA* and *dMV*. Here, the amount of insignificant values is the lowest and the amount of very strong correlations is the highest. However, the conclusion is the same as in table 2, the results cannot be generalized and strong correlation applies only for a few companies.

Table 4 contains the results of the linear regression and statistical significance. There are only models with *R* square higher than 0.5 displayed. All these models have insignificant constant; the regression coefficients show that changes in the market value have the same course as *EVA*, but the volatility of the change in value is higher.

Table 3 Strength of Correlations between EVA Indicators and Change in Value Indicators (alfa = 10 %)

Strength of correlation		**Relation of EVA and dMV**			**Relation of rEVA and rMV**			**Relation of EVA and dMVA**		
Range		ρ_p	τ	ρ_s	ρ_p	τ	ρ_s	ρ_p	τ	ρ_s
0.8 to 1.0	**Very strong**	2	0	2	1	0	1	0	0	1
0.6 to 0.8	**Strong**	1	2	1	1	1	1	3	1	1
0.4 to 0.6	**Moderate**	3	1	2	3	1	2	1	2	2
0.2 to 0.4	**Weak**	0	2	0	0	2	0	0	0	0
0.0 to 0.2	**Very weak**	0	0	0	0	0	0	0	0	0
0.0	**Zero**	4	4	4	4	5	6	6	7	6
0.0 to - 0.2	**Very weak**	0	0	0	0	0	0	2	1	1
-0.2 to -0.4	**Weak**	0	1	0	0	0	0	1	1	1
-0.4 to -0.6	**Moderate**	0	0	1	1	1	0	0	0	0
-0.6 to -0.8	**Strong**	0	0	0	0	0	0	0	0	0
-0.8 to -1.0	**Very strong**	0	0	0	0	0	0	0	0	0

Table 4 Regression Models of EVA Indicators and Change in Value Indicators

Company	Model	*T* test β_0	*T* test β_1	*F* test	R^2
Philip Morris	$dMV_t = -5843.344 + 7.850 \cdot EVA_t + \hat{u}_t$	H_0	H_A	H_A	0.509
Unipetrol	$dMV_t = -12.562 + 1.928 \cdot EVA_t + \hat{u}_t$	H_0	H_A	H_A	0.771
VIG	$dMV_t = 103.444 + 8.522 \cdot EVA_t + \hat{u}_t$	H_0	H_A	H_A	0.678
Unipetrol	$rMV_t = -0.042 + 1.913 \cdot rEVA_t + \hat{u}_t$	H_0	H_A	H_A	0.722
VIG	$rMV_t = 0.124 + 9.898 \cdot rEVA_t + \hat{u}_t$	H_0	H_A	H_A	0.639

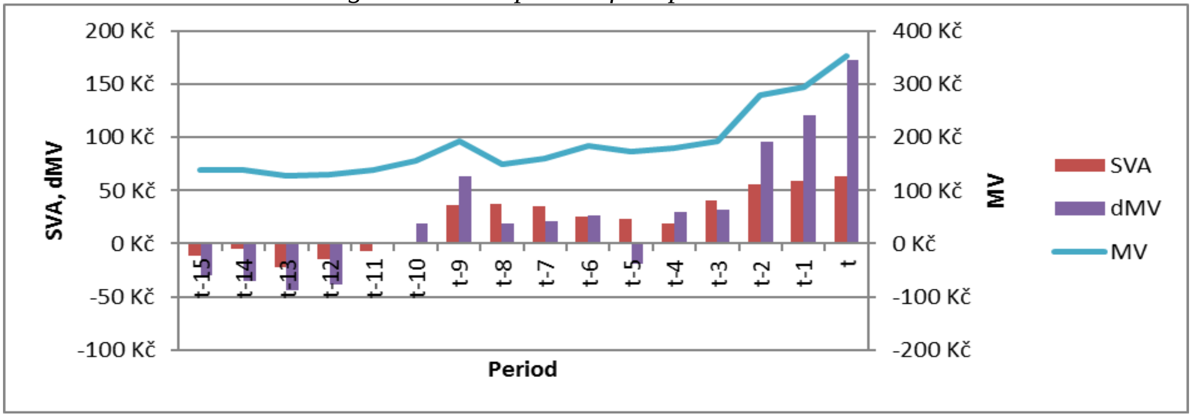
A data file that contains every company is created, as well. This dataset should serve as the summary results, but the problem is that all companies have different value level of *EAT*, E_{bv} and *E*. Therefore, when we calculate the necessary indicators, we have datasets with a lot of extremes values and these datasets are not normally distributed. After conversion of indicators as per share, this problem cannot be solved because companies have different value of share. When we analyse relative version of indicators (*rEVA* and *rMV*), there is similar problem because each of the firms have other proportions. Therefore, the results of parametric correlation coefficient should be taken cautiously. The scores of analysed coefficients are presented in table 5.

Table 5 Strength of Correlations between EVA Indicators and Change in Value Indicators of All Firms

	ρ_p	τ	ρ_s
Correlation (EVA* and *dMV)	0.604	0.239	0.329
***p* value**	0.000	0.000	0.000
Correlation (EVA* and *dMVA)	0.588	0.133	0.186
***p* value**	0.421	0.502	0.603
Correlation (rEVA* and *rMV)	0.146	0.244	0.350
***p* value**	0.087	0.000	0.000

The next relationships analysed are the relationships between *SVA* per share indicators and change in the market value of share indicators. Correlation between *SVA* and *dMVA* is not analysed because *SVA* does not come out from E_{bv} like *EVA*, hence analysis is irrational. For some companies, there is evident correlation between *SVA* and *dMV* and for some is not. For example, in figures 4 and 5 is shown the development of Unipetrol and Erste Group Bank indicators.

Figure 4 Development of Unipetrol Indicators



After the graphic analysis, the correlation analysis is performed. The conclusion of correlation analysis is presented in table 6. We can say that *SVA* indicators are strongly correlated with the change in the value than *EVA* indicators, still strong and very strong levels of correlations have less than 50% of the companies. No company have negative correlation. These results are statistically insignificant. In table 7, significant results on alfa level of 10% are demonstrated.

Figure 5 Development of Erste Group Bank Indicators

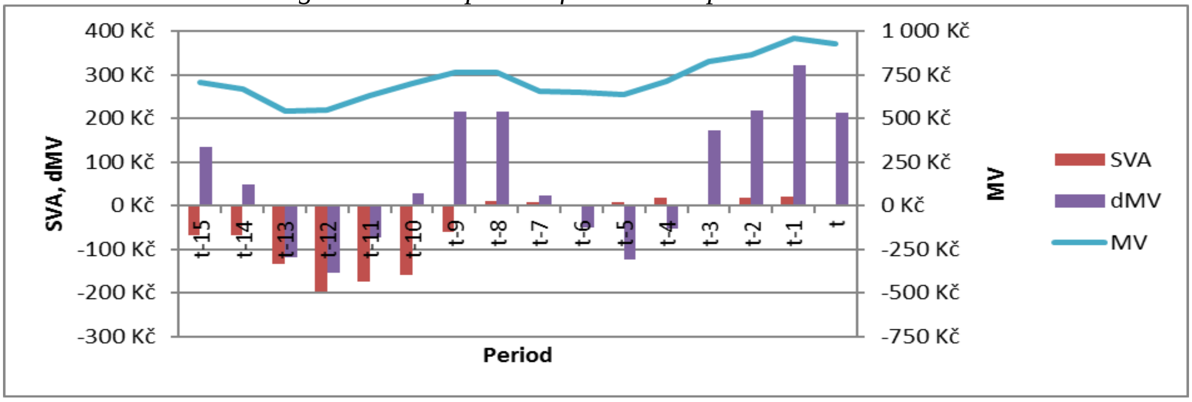


Table 6 Strength of Correlations between SVA Indicators and Change in Value Indicators

Strength of correlation		***Relation of EVA and dMV***			***Relation of rEVA and rMV***		
Range		ρ_p	τ	ρ_s	ρ_p	τ	ρ_s
0.8 to 1.0	**Very strong**	3	0	4	2	0	3
0.6 to 0.8	**Strong**	1	4	0	4	4	1
0.4 to 0.6	**Moderate**	5	1	3	3	1	4
0.2 to 0.4	**Weak**	0	4	2	1	5	2
0.0 to 0.2	**Very weak**	1	0	1	0	0	0
0.0	**Zero**	0	1	0	0	0	0

Table 7 Strength of Correlations between SVA Indicators and Change in Value Indicators (alfa = 10 %)

Strength of correlation		***Relation of EVA and dMV***			***Relation of rEVA and rMV***		
Range		ρ_p	τ	ρ_s	ρ_p	τ	ρ_s
0.8 to 1.0	**Very strong**	3	0	4	2	0	3
0.6 to 0.8	**Strong**	1	4	0	4	4	1
0.4 to 0.6	**Moderate**	4	1	2	2	1	4
0.2 to 0.4	**Weak**	0	2	0	0	3	0
0.0 to 0.2	**Very weak**	0	0	0	0	0	0
0.0	**Zero**	2	3	4	2	2	2

Further the regression analysis is examined. Final models and statistical significance are displayed in table 8. There are only models with *R* square higher than 0.5. More models were found than for *EVA* indicator. Without the model of *CETV* relative indicators, models have insignificant constant and higher regression coefficient than 1, it means there is higher volatility of market value than *SVA*. Every regression coefficients indicates the same trend *SVA* as the market value.

Table 8 Regression Models of SVA Indicators and Change in Value Indicators

Company	**Model**	***t* test β_0**	***t* test β_1**	***F* test**	R^2
ČEZ	$dMV_t = -29.935 + 9.524 \cdot SVA_{t-2} + \hat{u}_t$	H_0	H_A	H_A	0.657
Philip Morris	$dMV_t = -3243.446 + 6.584 \cdot SVA_t + \hat{u}_t$	H_0	H_A	H_A	0.566
Unipetrol	$dMV_t = -12.638 + 1.902 \cdot SVA_t + \hat{u}_t$	H_0	H_A	H_A	0.766
VIG	$dMV_t = -59.270 + 7.029 \cdot SVA_t + \hat{u}_t$	H_0	H_A	H_A	0.756
CETV	$rMV_t = 0.403 + 0.729 \cdot rSVA_{t-2} + \hat{u}_t$	H_A	H_A	H_A	0.532
ČEZ	$rMV_t = -0.029 + 9.538 \cdot rSVA_{t-2} + \hat{u}_t$	H_0	H_A	H_A	0.633
Philip Morris	$rMV_t = -0.397 + 7.844 \cdot rSVA_t + \hat{u}_t$	H_0	H_A	H_A	0.541
Unipetrol	$rMV_t = -0.064 + 1.692 \cdot rSVA_t + \hat{u}_t$	H_0	H_A	H_A	0.687
VIG	$rMV_t = -0.140 + 6.215 \cdot rSVA_t + \hat{u}_t$	H_0	H_A	H_A	0.751

In the end, the correlation for dataset with all analysed companies is found. For *SVA* we face similar problem as for *EVA*, so there are again the results of the Pearson's correlation coefficient affected by the data distribution. The values of correlation coefficients are summarized in table 9. We can observe that correlation of *SVA* indicators and the change in the market value indicators are higher than for *EVA*.

Table 9 Strength of correlations between SVA indicators and change in value indicators of all firms

	ρ_p	τ	ρ_s
Correlation (*SVA* and *dMV*)	0.659	0.345	0.471
***p* value**	0.000	0.000	0.000
Correlation (*rSVA* and *rMV*)	0.366	0.367	0.504
***p* value**	0.000	0.000	0.000

5. Conclusion

This study focuses on the examination of the influence of various versions of indicators *EVA* and *SVA* on different conceptions of change in selected Czech companies' market value. It follows from the findings that similar results as in Fernandez's article (2015) are reached, Fernandez in his article examines the U.S. market. The relationship between *EVA* and the change in the market value is not detected for most companies. For companies with strong and very strong form of the correlation is not possible to state any explicit reasonable conclusion. Any rational relationship among the companies with the strong correlation is not found. The companies could not be compared with companies in similar sector since the benchmark on Prague Stock Exchange is missing. In total, the results are for the relation between *SVA* and change in the company market value insignificantly higher; nevertheless, the conclusion is the same as it is for *EVA*. In summary, it is not possible to confirm any general relationship between the mentioned indicators.

Acknowledgment

This paper is supported by the SGS Project VŠB – TU Ostrava SP2018/154 "Finanční rozhodování podniků a finančních institucí za rizika" and project ESF CZ.1.07/2.3.00/20.0296 "Výzkumný tým pro modelování ekonomických a finančních procesů na VŠB-TU Ostrava".

References

Books

[1] CAMPBELL, John Y. The econometrics of financial markets. Princeton: Princeton University Press, 1997. ISBN 0-691-04301-9.

[2] CIPRA, Tomáš. Finanční ekonometrie. 2. vol. Praha: Ekopress, 2013. ISBN 978-80-86929-93-4.

[3] DLUHOŠOVÁ, Dana a kol. Finanční řízení a rozhodování podniku. 3. vol. Praha: Ekopress, 2010. ISBN 978-80-86929-68-2.

[4] EVANS, J. D. Straightfoward Statistics for the Behavioral Sciences. California: Brooks/Cole Publishing Company, 1996. ISBN 9780534231002.

[5] FERNANDEZ, Pablo. Valuation Methods and Shareholder Value Creation. 1. vol. Academic Press, 2002. ISBN 978-0-12-253841-4.

[6] HANČLOVÁ, Jana. Ekonometrické modelování. 1. vol. Praha: Professional Publishing, 2012. ISBN 978-80-7431-088-1.

[7] HENDL, Jan. Přehled statistických metod. 5. vol. Praha: Portál, 2015. ISBN 978-80-262-0981-2.

[8] KOLLER, T. GOEDHART, M. a WESSELS, D. Valuation: measuring and managing the value of companies. 6. vol. New York: McKinsey & Company Inc., 2015. ISBN 978-1118873700.

[9] MAŘÍK, M. a Maříková, P. Moderní metody hodnocení výkonnosti a oceňování podniku. 2. vol. Praha: Ekopress, 2005. ISBN 80-86119-61-0.

[10] MAŘÍK, Miloš. Metody oceňování podniku pro pokročilé. 1. vol. Praha: Ekopress, 2011. ISBN 978-80-86929-80-4.

[11] MAŘÍK, Miloš. Metody oceňování podniku. 3. vol. Praha: Ekopress, 2011. ISBN 978-80-86929-67-5.

[12] RAPPAPORT, Alfred. Creating shareholder value: the new standard for business performance. Michigan: Free Press, 1986. ISBN: 0029257204.

[13] SHAPIRO, S. S., WILK, M. B. An Analysis of Variance Test for Normality. In: Biometrika. Vol. 52, No. 3/4. Oxford University Press, 1965. DOI: 10.2307/2333709.

Web pages

[14] BURZA CENNÝCH PAPÍRŮ PRAHA. PSE: Kurzy akciového trhu [online]. [cit. 2018-03- 30]. Link: https://www.pse.cz/udaje-o-trhu/akcie/prime-market/.

[15] CENTRAL EUROPIAN MEDIA ENTEPRISES. CETV: Investors [online]. [cit. 2018-03- 30]. Link: https://www.cetv-net.com/investors/default.aspx.

[16] ČESKÁ NÁRODNÍ BANKA. ČNB: Kurzy devizového trhu [online]. [2018-03-30]. Link: http://www.cnb.cz/cs/financni_trhy/devizovy_trh/kurzy_devizoveho_trhu/denni_kurz.jsp.

[17] ČESKÁ NÁRODNÍ BANKA. ČNB: Trh státních dluhopisů [online]. [2018-03-30]. Link: http://www.cnb.cz/cs/financni_trhy/trh_statnich_dluhopisu/sd/.

[18] DAMODARAM ONLINE: Risk Premiums for Other Markets [online]. [2018-03-30]. Link: http://pages.stern.nyu.edu/~adamodar/.

[19] ERSTE GROUP. ERSTE: Investor Relations [online]. [2018-03-30]. Link: https://www.erstegroup.com/en/investors.

[20] FERNANDEZ, Pablo. EVA and Cash Value Added Do Not Measure Shareholder Value Creation, 2015. [online]. [2018-03-30]. Link: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=270799.

[21] FORTUNA ENTERTAINMENT GROUP. FORTUNA: Investoři [online]. [cit. 2018-03- 30]. Link: http://www.fortunagroup.eu/cz/investori/novinky/index.html.

[22] KOMERČNÍ BANKA. KB: Vztahy s investory [online]. [2018-03- 30]. Link: https://www.kb.cz/cs/o-bance/pro-investory-rozcestnik.

[23] O2 CZECH REPUBLIC. O2: Vztahy s investory [online]. [2018-03-30]. Link: https://www.o2.cz/spolecnost/akcie/.

[24] PEGAS NONWOVENS. PEGAS: Investoři a média [online]. [2018-03-30]. Link: http://www.pegas.cz/investori-a-media.

[25] PHILIP MORRIS ČR. PM: Pro akcionáře [online]. [2018-03-30]. Link: https://www.pmi.com/markets/czech-republic/cs/investor-relations/shareholder-information.

[26] SKUPINA ČEZ. ČEZ: Pro investory [online]. [2018-03-30]. Link: https://www.cez.cz/cs/pro-investory.html.

[27] UNIPETROL: Vztahy s investory [online]. [2018-03-30]. Link: http://www.unipetrol.cz/cs/VztahySInvestory/Stranky/default.aspx.

[28] VIENNA INSURANCE GROUP. VIG: Investor relations [online]. [2018-03-30]. Link: https://www.vig.com/en/investor-relations.html.

Risks connected with undisclosed financial statements

Josef Horák[1], Jiřina Bokšová [2]

Abstract

The submitted paper deals with the analysis of risks connected with undisclosed financial statements. Stakeholders such as investors, employees, financial institutions face many risks and need true and fair information about the company to protect their own interests. For this reason, companies have the obligation to publish their financial statements as it is based by the Directive 2013/34/EU of the European Parliament and of the Council on the annual financial statements, consolidated financial statements and related reports of certain types of undertakings. The goal of this directive is to protect the stakeholders and provide them with the information about the accounting entity. The research results will be based on the sample of all licensed companies operating in the arms industry in the Czech Republic.

Key words

arms industry, risks connected with undisclosed financial statements, statistical information

JEL Classification: M41, M42

1. Introduction

The main aim of the paper is to highlight the risks connected with undisclosed financial statements and present the results of research that was realised in SKODA AUTO University in 2017.

All accounting entities operating in the Czech Republic that are registered in the Business register must disclose their financial statements in the Collection of Documents. [2] The Collection of Documents is part of a public register and it is freely available and everyone has the right to access it regardless of whether or not he/she has a legal interest.

Financial statements consist of Statement of Financial Position, Statement of Financial Performance, Cash Flow Statement and Statement of Changes in Equity and Notes to the Financial Statements. [1] The financial statements must be prepared as a complete set of financial statements (in case of medium or large companies) or a condensed set of financial statements in the Czech Republic if the company fulfils the criterions for a micro or a small accounting entity without obligation of the inspection of the financial statements by an external audit. Those entities publish only the Statement of Financial Position known as a balance sheet.

The financial statements are prepared under Czech legislation or under International Financial Reporting Standards (IFRS) in the Czech Republic. The obligation to report financial statements under IFRS is when the company is listed in the European Union or European Economic Area (Iceland, Norway, and Liechtenstein) securities market. [8], [9]

The annual report must be published too in the Collection of Documents if the company had the obligation to inspect their financial statements by an external audit. The financial

[1] Ing. Josef Horák, Ph.D., ŠKODA AUTO UNIVERSITY, josef.horak@savs.cz
[2] doc. Ing. Jiřina Bokšová, Ph.D., ŠKODA AUTO UNIVERSITY, jirina.boksova@savs.cz

statements of the large or middle-sized companies are always subject to obligatory external audit. The financial statements of small companies must be inspected by the external audit if the company meets at least 2 of 3 criterions: assets higher than 40,000,000 CZK; net turnover higher than 80,000,000 CZK and the annual average number of employees higher than 50 employees. [1]

The purpose of publishing of financial statements is to inform external users of accounting information and enable them to analyse whether the company is financially sound or not. The accounting experts are able to analyse individual financial statements and possibly find some mistakes (unintentional or intentional) or identify some illegal activities such as frauds due to the special interconnections between the financial statements. [6]

The undertaking is very risky. For this reason, investors and other external users face a lot of risks. This is one of the reasons why the companies have obligation to publish their financial statements.

Then, they are available to everyone who is interested in them. The company publishes very important information about its financial position and external users are able to assess if it is appropriate to invest in the company or on the other hand to reduce activities in that company. [7]

Accounting entities that do not publish their financial statements do not allow the analysis of their financial statements to the external users. In this way, they harm the external users, because external users are not able to check the financial position of the company and are not able to discover some possible discrepancies reported in the financial statements. [3], [5]

Moreover, the companies that published their financial statements are harmed too, because other competitors can assess their financial position in comparison with the companies that did not publish their financial statements and use this information in the competitive fight of the businesses.

Due to the importance of this issue, the government of the Czech Republic is currently considering several radical measures, such as the cancellation of the company by the court or significant increase of fines, which should force more companies to disclose their financial statements.

As is evident from the text, the situation is very serious. For the purposes of this paper, the research team analysed the publishing of financial statements of the licensed companies in the arms industry in the Czech Republic with the aim to compare the situation connected with the publishing of financial statements in this industry with the national average consisting of all segments of the economy in the Czech Republic.

2. Methodology of research

We analysed the research sample that consisted of 237 companies operating in the military industry in the Czech Republic. All those companies were licensed to produce or trade with arms, ammunition or other military products that could not be traded without a special license issued by the Ministry of the Interior of the Czech Republic and Ministry of Foreign Affairs of the Czech Republic.

The research was realised in January and February 2018 and was focused on the analysis of disclosed financial statements in the Collection of Documents in the Business register for the year 2016.

At first, the research team analysed the structure of the research sample (mainly the legal form of the companies).

Secondly, we checked if those companies fulfilled their obligation and published their financial statements. After that, we compared obtained results with the national average consisting of all segments of economy that was published by the company CRIF – Czech Credit Bureau in 2013 and by the company Bisnode for years 2015 and 2016.

The next step of the research was focused on the answer to the question if the companies were audited or not and then we analysed the rate between audited and non-audited companies operating in the military industry. We were able to use only published data and that is why we had to reduce the research sample to 184 companies.

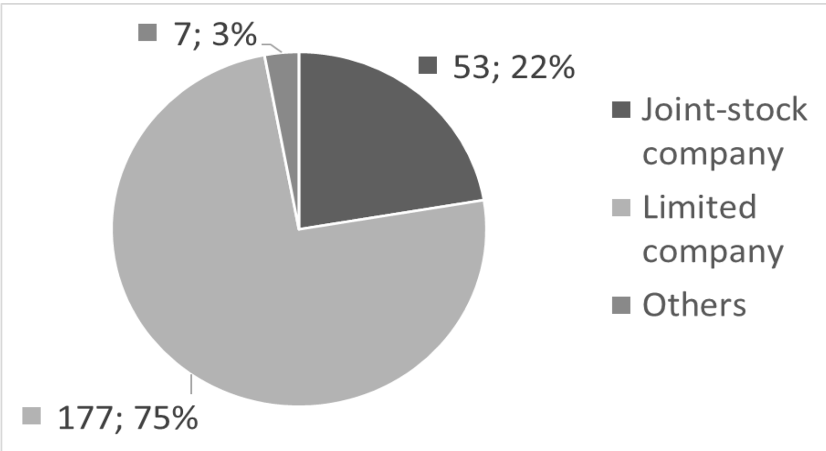
The last part of the research answered the question if the published financial statements were based on the Czech accounting legislation or International Financial Reporting Standards (IFRS).

3. Research results

We analysed the legal forms of business of all licensed companies (totally 237 accounting entities) in the military industry operating in the Czech Republic regardless of whether they published they financial statements in the Collection of Documents in the business register or not.

We found out that 75 % of those companies (177 accounting entities) were limited companies, 22 % of the analysed sample were joint-stock companies and the rest were other legal forms of business mainly like individual entrepreneurs as it is presented in the Figure 1.

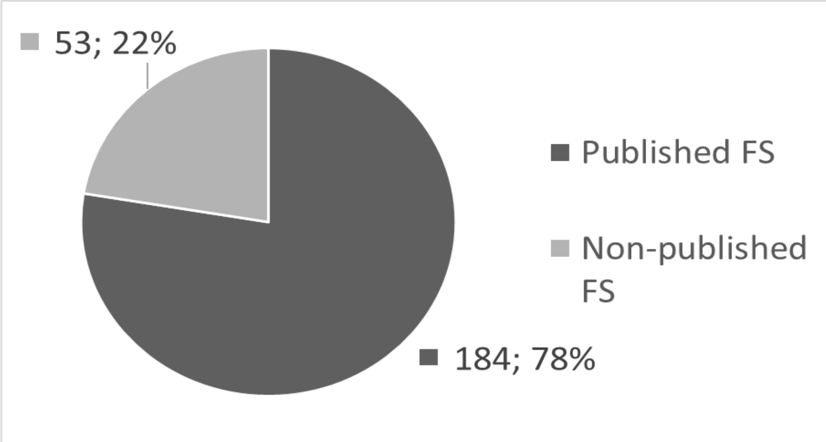
Figure 1: Legal forms of business of analysed companies



Source: own elaboration

The main aim of the research was focused on the analysis of the published financial statements for the year 2016 in the Collection of Documents. Each individual company was searched in the Business register and the findings of the research are shown in the Figure 2.

Figure 2: Publishing of financial statements in the arms industry



Source: own elaboration

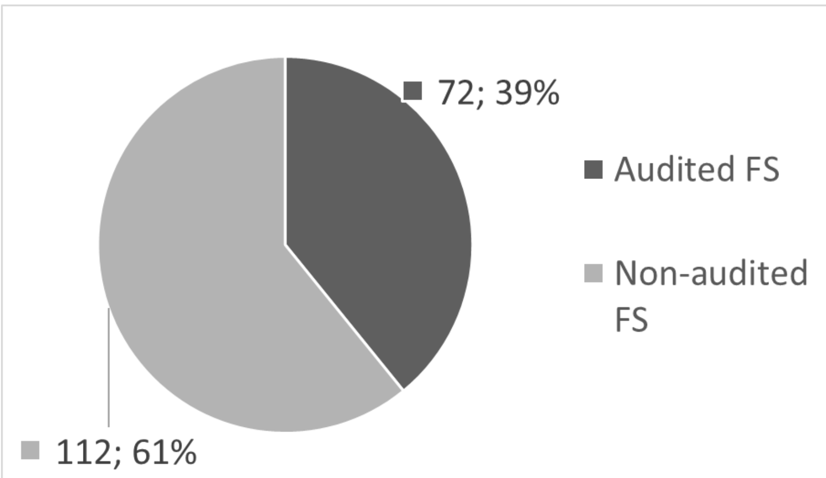
78 % of the research sample i.e. 184 companies operating in the military industry in the Czech Republic published their financial statements for 2016 in the Collection of Documents and fulfilled their statutory and legally binding obligation. The rest 53 accounting entities did not disclose their financial statements in the term.

The question is why they did not meet this obligation. Those firms can be very risky as business partners because they do not publish information about their assets, equity, liabilities, expenses, revenues, profit/loss, cash flow etc. For this reason, the users of accounting information are not able to analyse those companies and eventually detect various discrepancies. But those companies may be riskier if it is taken into account the business sector in which they operate.

The next step of the research consisted of the analysis of the published financial statements (184 companies) if they were inspected by the external audit or not. The external audit reduces the risk of incorrectly reported information in financial statements but on the other hand, this risk is not 100 % eliminated.

The obligation of inspection of financial statements by the external audit in the Czech Republic was specified in the introduction of this paper. The research results are shown in the Figure 3.

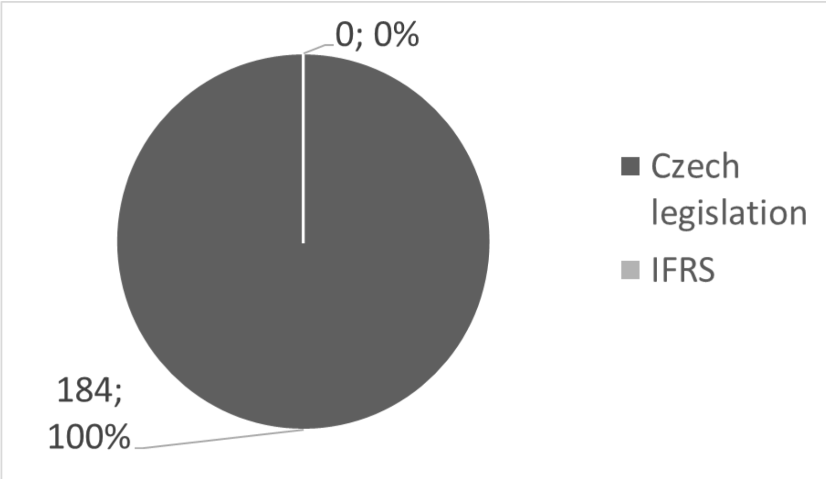
Figure 3: Audited and non-audited financial statements in the arms industry in 2016



Source: own elaboration

39 % of companies (72 accounting entities) involved in the research sample that disclosed their financial statements had the duty to inspect their financial statements by the external audit. The rest of financial statements (112 companies) were not audited, because these companies did not have this obligation in 2016.

Figure 4: Financial statements under Czech legislation or IFRS



Source: own elaboration

Accounting entities operating in the Czech Republic can report their financial statements under Czech legislation or International Financial Reporting Standards (IFRS) as it was described in the introduction. The research team found out that all analysed companies reported their financial statements under Czech legislation from the published financial statements in the Collection of Documents.

4. Conclusion

All EU member countries have own accounting legislation that specifically improves rules of the national accounting system. The harmonization process of financial accounting will be a very long-term process within all European Union member countries and it is possible to state it will be a very difficult process. Moreover, the protection of external users of accounting information against possible frauds caused by accounting entities is very important and that is why the European Union wanted to prepare some same basic rules of accounting that had to be valid in all EU member countries.

For this reason, the Directive 2013/34/EU of the European Parliament and of the Council on the annual financial statements, consolidated financial statements and related reports of certain types of undertakings came into force and is obligatory for all companies operating in the EU member countries.

One of these obligations is the obligation to publish the financial statements. Unfortunately, the situation of publishing of the financial statements is not good in the Czech Republic. The company CRIF – Czech Credit Bureau realised a research and it found that 52 % of all companies making business in the Czech Republic did not publish their financial statements in 2013. [10] It means that only 48 % of companies complied with the law. But the situation is getting worse because the company Bisnode found that 64 % of companies undertaking in the Czech Republic did not fulfil their obligation to disclose their financial statements in 2015 and incredibly 84 % of accounting entities did not publish their financial statements for the year 2016. [4]

We found out that 78 % of accounting entities operating in arms industry published their financial statements. The rest companies did not fulfil their statutory obligation. The situation in the arms industry is better in comparison with the national average but it is not still satisfactory. The better results of publishing of financial statements in arms industry could be probably achieved by the stricter rules of undertaking in this licenced industrial sector but there are still 22 % of companies that did not comply with the law.

Acknowledgment

This paper was written as one of the outputs of the research project entitled "Analysis of violations of going concern assumption in the financial statements of business entities in the Czech Republic" which was conducted at the ŠKODA AUTO University.

References

[1] 563/1991 Coll., Act on Accounting (1991).

[2] Bokšová, J. (2010). Účetní výkazy pod lupou I – Základy účetního výkaznictví. Praha: Linde Praha.

[3] Haller, A., & van Staden, C. (2014). The value added statement–an appropriate instrument for Integrated Reporting. *Accounting, Auditing & Accountability Journal, 27*(7), 1190-1216.

[4] Heller, J. (2018, August 15). Finananční kondici tají pět firem z šesti, stát tratí až stovky miliard. Retrieved August 15, 2018, from https://zpravy.idnes.cz/informacni-povinnost-bisnode-narodni-fond-proti-korupci-firmy-financni-udaje-obchodni-rejstrik-ie0-/domaci.aspx?c=A180815_114425_domaci_hell.

[5] Kanapickienė, R., & Grundienė, Ž. (2015). The model of fraud detection in financial statements by means of financial ratios. *Procedia-Social and Behavioral Sciences*, pp. 321-327.

[6] Kwok, B. K. (2017). Accounting irregularities in financial statements: A definitive guide for litigators, auditors and fraud investigators. Routledge.

[7] Robinson, T. R., Henry, E., Pirie, W. L., Broihahn, M. A., & Cope, A. T. (2015). *International Financial Statement Analysis, (CFA Institute Investment Series)*. John Wiley & Sons.

[8] Strouhal, J. (2012). Applicability of IFRS in the Practice of Czech SMEs: Insight of Czech Accounting Profession Representatives. *European Financial Systems 2012*, 214-219.

[9] Strouhal, J., & Bonaci C. G. (2014). Facing current trends in accounting harmonization: on case of Czech professional accountants. 8th International conference on accounting, auditing, and taxation (ICAAT 2014), 315-323.

[10] Účetní závěrky chronicky nezveřejňuje 52 % společností. (2015, August 03). Retrieved August 14, 2018, from http://www.informaceofirmach.cz/ucetni-zaverky-chronicky-nezverejnuje-52-spolecnosti/

Optimal reinsurance of very large losses using the semi-variance

Galina Horáková, František Slaninka [1]

Abstract

Insurance companies determine reinsurance programmes with the aim of increasing their stability also in the context of very large losses. The cost of reinsurance is important, judged from the point of view of balancing the gains and expenses therefrom. The aim of the paper is to incorporate into the method for determining an optimal reinsurance chain an approach for determining optimal reinsurance also allowing for very large losses. Thus not just to look at very large losses on their own, but to reduce both usual and very large risks at the same time. In this approach the solution for the relationship between the risk undergone by the insurer and the economic return of the expenses of reinsurance is based on Value-at-Risk measures and the Expected Shortfall substituted for by the semi-variance, a special case of LPM (Lower Partial Moments).

Key words

Reinsurance, optimisation, very large losses, Value-at-Risk, expected shortfall, lower partial moments, excess of loss, stop loss.

JEL Classification: G22

1 Introduction

The aim of the paper is to set out and compare some methods for determining the part of a risk that an insurer will retain, in the framework of determining an optimal reinsurance chain. Emphasis will be placed on the methodology of determining the reinsurance of an optimal layer of very large losses by maximising the risk-adjusted profit from the written risk in the framework of the classical standard deviation. In all cases the approach used is based on use of the properties of the collective risk model, the method of assembling reinsurance protection and assessing the function of the total expenses and profits. By buying reinsurance, including that for very large risks, the insurer increases the stability of its cash-flows. Reinsurance though has a cost attached to it and therefore it is important to maintain a balance between the perceived advantage of buying reinsurance and the related expenses of doing so. In most cases very large risks are dealt with separately. In this paper we will consider all risks together – the usual and the very large ones. Various optimisation criteria will be used, but the approach of measuring risk using lower partial moments will be dealt with in detail. According to Froot (2001) this is a more sensible and more flexible measure of risk than the traditional deviation and value at risk.

Reinsurance of very large risks serves to protect the insurer's profit from the effect of serious fluctuations due to large catastrophe losses. By buying reinsurance in respect of very large claims the insurer must give up part of its profit in order to ensure stability. Catastrophe

[1] Doc. RNDr. Galina Horáková, CSc., Mgr. František Slaninka, PhD., University of Economics in Bratislava, Faculty of Economic Informatics, Dolnozemská cesta 1, 852 35 Bratislava, Slovakia, e-mail: horáková@euba.sk, slaninka@euba.sk,

losses are independent of losses from other sources. In a perfect market the price of reinsuring very large losses should equal the expected expense of reinsuring. Gajek a Zagrodny (2004) reached the conclusion that if the reinsured has adequate funds to buy full protection, aggregate stop loss is the optimal reinsurance for very large losses with the highest possible priority.

In practice however financial markets are not perfect and the price of reinsurance is significantly higher than that according to the above mentioned theory and it makes no sense for an insurer to buy total protection against insolvency. Insurers are reluctant to hand over part of their profit even with regard to the limited reinsurance of very large losses. Whatever criterion we might use to determine optimal reinsurance cover the basic idea is one and the same: maximise profit while minimising risk.

2 Optimal non-proportional reinsurance

In order to determine how much of a risk the insurer wishes to retain one must use one of the risk measures.

2.1 Lower Partial Moments

One of the most popular methods of judging risk is to setting the value at risk *VaR*, the worst possible loss given a certain level of certainty and time horizon. Due to the limitations and weak points of this value new methods have been developed. One such method is to set the conditional value at risk *CVaR*. Instead of estimating what we expect to happen, the expected shortfall takes into account how the bad situation will develop, if it actually happens. Risk management often concentrates on the extreme risks and these risks are in fact caught in the *CVaR*. Optimisation criteria based on these risk measures have been presented and developed in for example [3].

One of the risk measures is also the standard deviation used there where a simple risk measure is needed. This measures deviations about the average. But deviations about the average need not be all bad if they are under a certain level. According to Markowitz the deviation under a given reference point should be a better measure of risk. To solve this limitation of the deviation the semi-variance was introduced as a special case of the risk measure in the form of the lower partial moment. We will use this form of measuring risk for its optimisation and in the conclusion we will compare the results with those obtained from the already mentioned criteria. We will show the differences between optimising a portfolio on the basis of the standard deviation and on the basis of the lower partial moment. The average and standard deviation are the first and second order moments and meet the following definition.

Definition 1.1 Let X be a random variable with distribution function $F_X(x)$ and reference level τ, the generalised k th order moment is defined by the equation

$$\mu_{\tau,k}(X) = \int_{-\infty}^{\infty} (\tau - x)^k \, \mathrm{d}F_X(x) \tag{2.1}$$

The special case, where the reference level τ equals the mean value of the distribution and $k = 2$, is called the central moment of the second order, the first moment at the beginning is that with reference level $\tau = 0$ with $k = 1$ is called the mean value. The lower partial moments *LPM*, first defined by Fishburn, examine the kth moment under a certain threshold. So the *LPM* values combine for example the advantages of the deviation with those of the *CVaR*.

Definition 1.2 Let τ be a chosen reference level, k the moment order and X a random variable with distribution function $F_X(x)$, the lower partial moments are defined by the equation

$$LPM_{\tau,k}(X) = \int_{-\infty}^{\tau} (\tau - x)^k \, \mathrm{d}F_X(x) \tag{2.2}$$

In practice we can estimate the *LPM* given m observations by means of the following

$$LPM^{*}_{m,\tau,k}(X) = \frac{1}{m}\sum_{t=1}^{m} \max\left(0, \left(\tau - X_{i,t}\right)^k\right) \tag{2.3}$$

Definition 1.3 Let X be a random variable with distribution function $F_X(x)$ and let the reference level be τ, then the semi-variance is defined as follows

$$SV_{\tau}(X) = \int_{-\infty}^{\tau} (\tau - x)^2 \, \mathrm{d}F_X(x) \tag{2.4}$$

The *LPM* are a group of risk measures specified by the parameters τ a *k*. The parameter τ is often set at a risk free rate or simply 0. By choosing the order of the moment one can determine which measure suits aversion to risk. Intuitively, large values of *k* penalise large deviations more than low values do. The semi-variance is a special case of *LPM*, for which the order of the moment *k* = 2. Thus for example in a symmetrical distribution with reference level $\tau = E(X)$ and $k = 2$ we have

$$SV_{\tau} = 0{,}5 \cdot D(X) \tag{2.5}$$

From this we deduce that in general it is expected that a decision based on the semi-variance may be of more concern.

2.2 Optimisation criteria

Every investor, insurance companies included, aims to maximise the expected profit from their business or to minimise the riskiness of that profit (standard deviation, *VaR, CVaR,* probability of ruin). In relation to reinsurance both paths lead to the same result. Transfer of risk from the direct writer to one or more reinsurers takes place on the assumption that the premium received should be greater than the expected total claim amounts together with the expenses of reinsuring. We will express the risk retained by the original writer after applying reinsurance or the application of a chain of protections. We will limit the ceded risk and express the expenses of reinsurance. Reinsurance protections can be divided into various layers, whereby each layer has its own conditions set out in the reinsurance agreement. This also sets out the level to which the reinsurer is prepared to accept risks. A risk exceeding this level goes back to the writer who passed it to the reinsurer.

For the implementation of optimisation criteria based on setting bound extremes we will construct a Lagrange function $L(\overline{x}, \gamma)$, in general a function of *n* variables on the basis of knowledge of the expected profit of the insurer and its associated dispersion. With the help of Lagrange multipliers we will calculate the extremes of this function and state conclusions regarding the examined reinsurance chain. In the case of the maximisation of expected profit with constant dispersion criterion the Lagrange function takes the form

$$L(\overline{x}, \gamma) = E(Z) + \gamma\left(D(Z) - k^2\right) \tag{2.6}$$

We can express the expected profit of the insurer using the risk premium *RP* and the insurer's and reinsurer's expected claims according to the type of reinsurance

$$E(Z) = RP - E({}^{P}S^{C}) - \Pi({}^{Z}S) \tag{2.7}$$

For example in the case of non-proportional excess-of-loss reinsurance with priority β and with a reinsurer's limit

$$E(Z) = RP - E({}^{P}S^{C}_{\beta,L}) - \sum_{i=1}^{2}(1+\zeta_i)\cdot E({}^{Z_i}S_{\beta,L}) \tag{2.8}$$

where ζ_i are the risk additions for the particular type of reinsurance. From the characteristics of the dispersion we have

$$D(Z) = D\left({}^{P}S^{C}_{\beta,L}\right) = k^2 \tag{2.9}$$

By solving this system of equations we get according to the set up Lagrange function stationary points whose coordinates form potential values for example of an insurer's own retention, priorities, limits, etc. The approach shown, where the insurer sets a fixed level for the dispersion and determines a level for its own retention which maximises the expected profit, can be replaced by the equivalent of the previous criterion. This is the minimisation of the dispersion of the total profit given a constant expected profit. We will use the approach shown with the same notation, but in this case the Lagrange function has the following form

$$L(\bar{x},\gamma) = D(Z) + \gamma\left(E(Z) - k\right) \tag{2.10}$$

which we will minimise.

Remark: One can also set the level of own retention on the basis of the use of ruin theory. We will consider the vector of own retentions to be optimal if it minimises the insurer's probability of ruin after taking into account reinsurance subject to the constraint that the insurer's expected profit in a unit of time is equal to a given constant. Thus the difference compared with equation (2.6) is that we exchange the dispersion as a measure of risk for the probability of ruin.

Following the classical mean-variance method let us set out an optimisation criterion which will maximise the risk-adjusted profit de

$$RAZ = E(Z) + \theta \cdot D(Z) \tag{2.11}$$

where Z is the net profit from the written risk, $E(Z)$ is the expected profit, $D(Z)$ is its deviation and θ is the penalty on risk coefficient. The higher is the risk measure θ the greater is the aversion to the reinsurance risk. If $\theta = 0$ the reinsurance risk is neutral. Using this criterion the insurer maximises profit. It does not give away profit but this optimisation variant reflects the variability of the results. It gives information about the divergences of the profit about the mean. It therefore considers all divergences both the desirable and undesirable. The insurer has no problem with the desirable divergences, but even these affect the total result. As compared with the total standard deviation, as we have already shown, the *LPM* measures only the undesirable deviations and does not take into account the favourable ones. For this reason it can be considered as a better measure than for example the standard deviation. Lindenberg proposed the use of the *LPM* as a replacement for the total standard deviation for optimisation purposes. We get the downside risk adjusted profit by maximising the function *DRAZ*

$$DRAZ = E(Z) - \theta \cdot LPM_{\tau,k}(Z) \tag{2.12}$$

The three parameters τ, k, θ in equation (2.12) reflect interactively the perception of risk and aversion to risk. The flexibility provided by these three parameters in meeting the insurer's needs provides the possibility of capturing its toleration of risk. It permits the choosing of a reference point for the profit, below which it would be considered as unacceptable. τ can be the aimed level of profit, the riskless profit, zero or even a negative value according to the perception of risk.

2.3 Non-proportional reinsurance and the risk premium

Non-proportional reinsurance is characterised by the fact that it divides the liability between the ceding and reinsuring companies based on the occurrence and size of a claim. Stop-loss is a non-proportional type of reinsurance, which operates similarly to excess of loss except that with the latter the ceded amount depends on the size of each individual claim whereas stop-loss covers the total claims in a year usually in combination with quota reinsurance. It is therefore usual for the reinsurer's liability to be limited to a certain amount. We can therefore express the random variables, representing the size of individual claims in connection with the aggregate total claims of the insurer and reinsurer with excess of loss reinsurance allowing for the reinsurer's limit, as follows

$$ {}^{P}X_{\beta,L} = \begin{cases} X & x \le \beta \\ \beta & \beta < x \le \beta + L \\ X - L & x > \beta + L \end{cases} \qquad {}^{Z}X_{\beta,L} = \begin{cases} 0 & x \le \beta \\ X - \beta & \beta < x \le \beta + L \\ L & x > \beta + L \end{cases} \tag{2.13} $$

In the case of the reinsurance of the *i*th very large claim, we can divide the random variable *X*, which describes it using a compound distribution, between the insurer and reinsurer, whereby for the reinsurer's share of the risk we have

$$ {}^{Z}X_{\beta,L} = \begin{cases} 0 & x \le \beta \\ (X - \beta)\cdot\mu & \beta < x \le \beta + L \\ L\cdot\mu & x > \beta + L \end{cases} \tag{2.14} $$

where $X = \sum_{i=1}^{N} Y_i$ is a compound random variable, representing the total claims in a year, *N* is the number of claims and Y_i the distribution of each individual claim amount, $i = 1, 2, ..., N$.

We can specify the risk premium in respect of reinsurance of very large claims. If we describe the total annual claim amount in respect of claims in a year for a given portfolio by the random variable *X* with distribution function $F_X(x)$, the risk premium for a non-proportional reinsurance with priority β and reinsurer's limit *L* will be

$$ \prod({}^{Z}X_{\beta,L}) = \int_{\beta}^{\beta+L} (x-\beta)\cdot \mathrm{d}F_X(x) + \int_{\beta+L}^{\infty} L\cdot \mathrm{d}F_X(x) \tag{2.15} $$

On integrating we get

$$ \prod({}^{Z}X_{\beta,L}) = L\cdot F_X(\beta+L) - \int_{\beta}^{\beta+L} F_X(x)\,\mathrm{d}x + L - L\cdot F_X(\beta+L) \tag{2.16} $$

So we can express the net risk premium as follows

$$ \prod({}^{Z}X_{\beta,L}) = \int_{\beta}^{\beta+L} (1-F_X(x))\,\mathrm{d}x = \prod({}^{Z}X_{\beta}) - \prod({}^{Z}X_{L}) \tag{2.17} $$

where $\prod({}^{Z}X_{\beta})$ is the reinsurance premium for an unlimited stop-loss cover above the priority β. Estimates of the premium, respectively reinsurance premium, for a stop-loss reinsurance can be based on information about the distribution function of the sum of all the individual claims on the assumption that the reinsurance covers a period of one year.

In the case that the random variable X has the lognormal distribution with mean value μ and various σ^2 we have

$$\prod({}^{Z}X_{\beta}) = E(X)\cdot\left(1-\Phi\left(\frac{\ln\beta-\mu-\sigma^2}{\sigma}\right)\right)-\beta\left(1-\Phi\left(\frac{\ln\beta-\mu}{\sigma}\right)\right) \qquad (2.18)$$

where Φ is the distribution function of a standardised normal distribution. The stop-loss risk premium with priority β is

$$\prod({}^{Z}X_{\beta}) \le 0,5\sigma\left(\sqrt{1+\frac{\left(\beta-E(X)\right)^2}{\sigma^2}}-\frac{\beta-E(X)}{\sigma}\right) \qquad (2.19)$$

for $\beta \ge E(X)$, where the simple approximation by the normal distribution was used

$$F_X(x) = \Phi\left(\frac{x-E(X)}{\sigma(X)}\right) \qquad (2.20)$$

In the case where $\beta \ge E(X)+\sigma(X)$, the stop loss risk premium $\prod({}^{Z}X_{\beta})$ without reinsurer's limit can be approximated by the following

$$\prod({}^{Z}X_{\beta}) = \sigma(X)\cdot\left(1+\frac{\gamma}{6}y_{\beta}\right)\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{(y_{\beta})^2}{2}}-\left(\beta-E(X)\right)\cdot\left(1-\Phi\left(y_{\beta}\right)\right) \qquad (2.21)$$

where $E(X)$ is the net premium, i.e. the mean value of the total claim amounts, $\sigma(X)$ is the standard deviation and γ_1 is a coefficient representing the raggedness of the total claim amounts. y_{β} depends on the priority β by means of a more suitable approximation than the Bower's one, namely the Normal Power approximation, where

$$y_{\beta} = v_{\gamma_1}^{-1}\left(\frac{\beta-E(X)}{\sigma(X)}\right) \qquad (2.22)$$

$$v_{\gamma_1}^{-1}(x) = \sqrt{1+\frac{9}{\gamma_1^2}+\frac{6x}{\gamma_1}}-\frac{3}{\gamma_1} \qquad (2.23)$$

γ_1 is the coefficient of kurtosis and

$$F_X(x) \approx \Phi\left(v_{\gamma_1}^{-1}(x)\right) \qquad (2.24)$$

2.4 Very large losses

Let *RP* be the gross premium, *M* the total number of very large claims in the time period, for example a year, $\prod({}^{Z}X_{\beta,L})$ the reinsurance premium, which is a decreasing function of the priority β and a non-decreasing function of the reinsurance limit, and *RI* is the new reinsurance premium in respect of the risk being reinsured. We can express the profit function from the insurance risk accepted as follows

$$Z = RP - S^C - \sum_{i=1}^{N} Y_i \qquad (2.25)$$

where S^C is the compound random variable representing the regular claims and the random variable X represents the very large losses, thus $X \sim Co(N;Y_i)$, whereby one can suppose that such claims arise at most twice in the given time period. We can denote the newly-arising reinsurance premium after the happening of a very large loss as $\Pi(X,\beta,L)$, whereby $\Pi(X,\beta,L)=\prod({}^{Z}X_{\beta,L})\cdot {}^{Z}Y_{\beta,L}\,|\,L$. The net profit after applying reinsurance can be expressed as follows

$$Z = RP - S^C - \sum_{i=1}^{N} Y_i - \prod({}^{Z}X_{\beta,L}) + \sum_{i=1}^{N} {}^{Z}Y_{i(\beta,L)} - \sum_{i=1}^{N} \Pi(Y_i,\beta,L) \qquad (2.26)$$

In this case we get the optimal layer with a combination of the priority β and the reinsurance limit L by maximising the following

$$\max_{\beta,L} E(Z) - \theta \cdot LPM(\tau;k) \qquad (2.27)$$

Whereby the insurer's expected reinsurance expenses are then

$$\prod({}^{Z}X_{\beta,L}) + \sum_{i=1}^{N} \Pi(Y_i,\beta,L) \qquad (2.28)$$

In this case using reinsurance the standard deviation is significantly reduced, but so is also the expected profit.

2.5 Results of applying the various approaches

In the theoretical part we presented optimisation criteria based on the lower partial moments and the difference between these criteria and those based on the mean-variance with bound extremes. The difference between the results from using these criteria will be shown by an example of independent risks, where the normal losses follow a compound geometric distribution and the size of the very large losses in the given period has a lognormal distribution, the values shown are in 10,000s. First we will show the results of the approach based on setting optimal non-proportional reinsurance for normal losses. $S^C \sim CoGe(0{,}8; F_V(x) = 1 - e^{-0{,}1x})$, the risk premium will be calculated with the help of the mean value principle, risk addition 0,2. We will look for an optimal non-proportional reinsurance with priority β, $\varsigma_\beta = 0{,}22.$ On the basis of the characteristics of the collective risk model we get the pseudo probability density of the profit for one time period

$$f_Z(x) = \frac{2 \cdot e^{-2(-x+3)/25}}{125},\ x < 3$$

The expected profit, if we ignore the reserves from the standard portfolio without reinsurance, is 0,5 with standard deviation 56,25, the $VaR_{0{,}01}(\mathrm{Z}) = -34{,}4467,$ the conditional value at risk $CVaR_{0{,}01}(\mathrm{Z}) = -46{,}9471.$ The specific values of the *LPM* are calculated without reinsurance according to (2.2), also after applying excess of loss reinsurance in accordance with (2.13) without a reinsurer's limit

Table 1: *Table of values of the lower partial moments*

LPM	$\tau = E(\mathrm{Z})$	$\tau = VaR_{0{,}01}(\mathrm{Z})$	$\tau = 0$
$k=1$	2,0468	0,125	1,9666
$k=2$	51,1707	3,125	49,1641

The values shown in Table 1 can be used as the basis for the optimisation criterion (2.12). Resulting from that we have for example that a suitable priority is $\beta = 1{,}0829$ for a reference level $\tau = 0,5$ and $\theta = 4, k = 2$, whereas using the optimisation criterion (2.6) we have $\beta = 1,8541$. Resulting from this we have that the expected profit using criterion (2.12) is less than that using the criterion (2.6), but the level of security is higher as the insurer's standard distribution is lower.

If we generalise this approach for very large losses, described by the lognormal distribution $Y \sim LN(10; 0,9^2)$, and the number of catastrophe claims has a binomial distribution with parameter $\mu = 0,02, n = 1$, then the gross premium for regular and very losses is $RP = 1516$. So the very large losses have, $E(Z)$ = 856,13272 and $D(Z)$ = $4,859542832 \cdot 10^7$. The portfolio of regular insurance contracts was described using such a variable which allowed us to bring out the individual steps. In principle that is repeated also when considering very large losses, which leads to the setting of a profit function in accordance with relationship (2.28).

Table 2: *Characteristics of the profit without reinsurance and with non-proportional reinsurance* $\theta = 5$

Retention, Upper Limit	$E({}^{P}Z_{\beta,L})$	$D({}^{P}Z_{\beta,L})$	*Retention, Upper Limit*	$E({}^{P}Z_{\beta,L})$	$SV_{\tau=E({}^{P}Z_{\beta,L})}({}^{P}Z_{\beta,L})$
$\beta = 77, L = 900$	42,8066	$1,31694 \cdot 10^5$	$\beta = 69, L = 835$	37,121629	$3,6446 \cdot 10^4$

3 Conclusion

The approach shown is suitable for pricing the whole of a very large loss, to a sensible extent it covers it. In the case of very large losses the insurer's choice of reinsurance layers is a matter of deciding between profit and risk. Insurers do not want to give up too large a part of their profit by buying too high a level of reinsurance, but they also have to balance the risk of a large loss if the reinsurance cover is inadequate. In the paper we have tried to show an approach using LPM and allowing for only undesirable deviations, which takes into account the insurer's risk aversion and determines such a reinsurance layer which maximises the relevant balance between risk and profit.

References

[1] Panjer H. H.: (2006) Operational Risk. WILEY INTERSCIENCE. New Jersey. IBN-13 978-0-471-76089-4.

[2] Horáková, G., Páleš, M., Slaninka, F. (2015) Insurance risk theory. (In Slovak) Wolters Kluwer Bratislava. ISBN 978-80-8168-273-5

[3] Horáková, G., Poljovka, J. (2010) Optimal reinsurance based on the values. (In Slovak) 5th International conference Management and modelling of financial risk, KF Ekonomická fakulta VŠB-TU Ostrava 2010

[4] Bawa, V. S., Lindenberg E. B.(1977) Capital Market Equilibrium in a Mean- Lower partial Moment Framework. Journal of Financial Economics 5, pp.189-200

[5] Barker, R., Kirk, J. and Munday, R.J., (1988). Narrative analysis. 3rd ed. Bloomington: Indiana University Press.

[6] Wolf,V., Rudiger, H., Fundamentals of Risk Theory, vyd. Verlag Versicherungswirtschaft. . V. Karlsruhe, 288 s., ISBN 3-88487-151-X

DG method for sensitivity measurement of Black-Scholes option prices

Jiří Hozman[1], Tomáš Tichý[2]

Abstract

Option pricing is a very important discipline which has a lot of applications in financial engineering. Its purpose is not only to determine the price of an option, but also to measure the sensitivity of the option price to the underlying parameters, such as asset prices, volatilities, interest rates, times to maturity — simply called the Greeks of an option. Since changes in the values of these parameters lead to changing values of options, the analysis of Greeks for the options is particularly important. In this paper we focus on a one-factor Black-Scholes model and its sensitivity measurement. The main objective is to present a numerical scheme to solve a pricing equation with emphasis on the model parameters. The numerical approach is based on the discontinuous Galerkin approximation in spatial domain together with the Crank-Nicolson scheme for the time discretization, and leads to a sparse linear algebraic problems at each time level. The presented scheme is tested on several numerical experiments and the obtained results are compared with the analytical ones.

Keywords

Option pricing, discontinuous Galerkin method, Black-Scholes model, vanilla option, sensitivity measures.

JEL Classification: G22

[1]RNDr. Jiří Hozman, Ph.D., Department of Mathematics and Didactics of Mathematics, Faculty of Science, Humanities and Education, Technical University of Liberec, Studentská 2, 461 17 Liberec, Czech Republic, e-mail: jiri.hozman@tul.cz.

[2]prof. Ing. Tomáš Tichý, Ph.D., Department of Finance, Faculty of Economics, VŠB-TU Ostrava, Sokolská 33, 701 21 Ostrava, Czech Republic. e-mail: tomas.tichy@vsb.cz.
Both authors were supported through the Czech Science Foundation (GACR) under project 16-09541S. Furthermore, the second author also acknowledges support provided within SP2018/34, an SGS research project of VSB-TU Ostrava. The support is greatly acknowledged.

1. Introduction

The Black-Scholes (BS) model was introduced in [2, 12] in order to simplify pricing of plain vanilla options under selected circumstances. Since than various options become commonly traded not only at OTC markets, but even at organized exchange markets. Obviously, the BS model is quite simplifying and thus during the decades, many classes of advanced models for efficient pricing of options have been introduced. This fact is reflected also in the literature devoted to the numerical studies of the option pricing problems, from binomial models, over finite difference methods [4], to finite element approaches [1, 13], and in references cited therein.

In this contribution we continue in our previous research on numerical option pricing via discontinuous Galerkin (DG) approximation technique, see, e.g., [8]. However, instead of the price itself, here we study the sensitivities of option prices on selected factors, such as the underlying asset price, time to maturity, volatility, etc. These factors are crucial especially when sensitivities of large option portfolios are analyzed and risk limits are considered. Given the nature of options, these factors are the most sensitive for ATM or close to ATM options.

In this paper we specifically focus on a relatively simple problem of an option sensitivities under BS setting since in such case a closed-form solution is available. In particular, we analyze the computational error in the standard L^2-norm and discrete l^∞-norm and study the behavior of the sensitivity functions.

We proceed as follows. Section 2 is devoted to the formulation of relevant pricing equations within the BS model. In Section 3 we develop discontinuous Galerkin approach for complex treatment of sensitivity parameters. Finally, in Section 4 numerical examples based on real market data are provided.

2. Black-Scholes equation and the Greeks

Let us consider European-style options on an underlying asset S, i.e., options exercising of which is permitted only at maturity time T. Such options exist either as put or call options and their value at maturity can be represented as a payoff function

$$\max(\mathcal{K} - S, 0) \quad \text{(put)}, \qquad \max(S - \mathcal{K}, 0) \quad \text{(call)}, \tag{1}$$

where $\mathcal{K}$ denotes the specified price at which an option contract can be exercised, usually called the strike price.

Using the Black-Scholes framework [2], the values of an option at the remaining time instants $t \in (0, T)$ can be found as solutions of the following backward PDE, known as Black-Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \quad \text{for } S > 0,\ t \in (0, T), \tag{2}$$

where the non-negative function $V(S, t, \sigma, r)$ denotes the option price, which value depends also on the volatility σ of the underlying asset returns and on the risk-free interest rate r. The pricing equation (2) equipped with one of the terminal conditions (1) constitutes the Cauchy problem in (S,t)-domain whose analytical solutions are given by the Black–Scholes formula as (see [5])

$$V(S,t,\sigma,r) = \begin{cases} S\Phi(d_1) - \mathcal{K}e^{-r(T-t)}\Phi(d_2), & \text{for a call,} \\ \mathcal{K}e^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1), & \text{for a put,} \end{cases} \tag{3}$$

where

$$d_1 = \frac{\ln(S/\mathcal{K}) + (r+\sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}}, \quad d_2 = \frac{\ln(S/\mathcal{K}) + (r-\sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}},$$

and Φ stands for the standardized cumulative normal distribution function.

Let us note that option price (3) depends on several underlying parameters. It is obvious that changes in the values of these parameters change consequently values of options. The sensitivity analysis and measurement give us the answer, how significant these resulting changes are. The availability of analytical solutions in the closed form (3) implies the possibility to derive the corresponding closed-form representations for the sensitivity measures of the European call and put options — named after Greek letters, and simply called the Greeks of an option, see [14].

The simplest sensitivity measures are the first and second derivatives with respect to the underlying asset

$$\Delta_V = \frac{\partial V}{\partial S} = \eta\Phi(\eta d_1) \quad \text{and} \quad \Gamma_V = \frac{\partial^2 V}{\partial S^2} = \frac{\phi(d_1)}{S\sigma\sqrt{T-t}}, \tag{4}$$

where ϕ is a density function of the standard normal distribution and η indicates the option type as $\eta = 1$ for a call option and $\eta = -1$ for a put one. Since Delta (Δ_V) measures the sensitivity of the theoretical option value with respect to changes in the underlying assets price, its values are particularly important in hedging portfolios consisting of options. On the other hand, Gamma (Γ_V) belongs to the second-order Greeks, it measures the rate of change in Delta as underlying asset changes and plays an important role in the corrections for the convexity of an option value.

Further, the rate of change of an option value due to the passage of time is measured by Theta

$$\Theta_V = \frac{\partial V}{\partial t} = -\eta r\mathcal{K}e^{-r(T-t)}\Phi(\eta d_2) - \frac{\sigma S\phi(d_1)}{2\sqrt{T-t}}, \tag{5}$$

which is also referred to as an option's time decay, because the option loses its value as time goes to the maturity, supposing the remaining parameters as constants.

In volatile markets the value of some option strategies can be particularly sensitive to changes in volatility. Therefore the derivative of the option value with respect to the volatility of the underlying asset — Vega is introduced

$$\mathcal{V}_V = \frac{\partial V}{\partial \sigma} = S\sqrt{T-t}\,\phi(d_1). \tag{6}$$

The last of the first-order Greeks is Rho

$$\rho_V(S,t,r) = \frac{\partial V}{\partial r} = \eta(T-t)\mathcal{K}e^{-r(T-t)}\Phi(\eta d_2), \tag{7}$$

which measures the sensitivity to the risk-free interest rate. Since the value of an option is less sensitive to changes in the risk-free interest rate than to changes in other parameters, Rho is the least used of the first-order Greeks.

Finally, note that it is also possible to evaluate the sensitivity to other parameters in the Black-Scholes formula, e.g., the strike price or the dividend yield, but this is not the subject of our study. For further research we refer to the book [14].

3. Numerical approach

Since analytical option pricing formulae are not available for a wide class of pricing problems, numerical methods provide an alternative approach. In our study, we employ the DG method, recently and successfully used also in the field of financial engineering, see, e.g., [9, 10, 11]. The utilization of this technique significantly improves the valuation process for options and the corresponding sensitivity measures with respect to the spatial discretization parameter.

We proceed as follows in this section. At first, we change the variables and restrict the option pricing problem to a bounded spatial domain. Next, we mention the standard discretization steps and recall the numerical scheme. Finally, we present a numerical evaluation of the investigated Greeks.

3.1 Reformulation to bounded domain

We begin with the change of variables and the unknown. From the numerical point of view it is suitable to use the scaled asset values and forward time running, i.e.,

$$x = S/\mathcal{K}, \quad \hat{t} = T - t, \quad u(x, \hat{t}) = V(S, t)/\mathcal{K}, \tag{8}$$

where a new price function u also depends on σ and r, but this dependence is not explicitly denoted. Further, to localize the problem (2) to a bounded domain, we set $x_{max} = S_{max}/\mathcal{K}$ as the maximal asset price. Then we obtain the new governing equation, localized on bounded domain and written in the divergence form as

$$\frac{\partial u}{\partial \hat{t}} - \frac{\partial}{\partial x}\left(\frac{1}{2}\sigma^2 x^2 \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x}\left((\sigma^2 - r)xu\right) + (2r - \sigma^2)u = 0 \quad \text{in } (0, x_{max}) \times (0, T), \tag{9}$$

which is equipped with the initial conditions given by the transformed payoff functions (1). Moreover, we have to impose values of u on both endpoints of interval $(0, x_{max})$. These values are set in the accordance with asymptotic values of BS formula (3) and the so-called put-call parity relation (see [5]), i.e.,

$$u(0, \hat{t}) = \begin{cases} 0, & \text{for a call,} \\ e^{-r\hat{t}}, & \text{for a put,} \end{cases} \qquad u(x_{max}, \hat{t}) = \begin{cases} x_{max} - e^{-r\hat{t}}, & \text{for a call,} \\ 0, & \text{for a put.} \end{cases} \tag{10}$$

Note that artificial boundary conditions (10) are of Dirichlet type and it is also possible to prescribe another types, for more details see [9].

Finally, let us mention that the initial-boundary value probelm (9)–(10) represents a degenerate parabolic problem that exhibits a convection-diffusion character. Therefore, the numerical schemes for solving such equations have to take this property into account.

3.2 Discretization

We recall the utilization of the discontinuous Galerkin framework together with the Crank-Nicolson scheme to numerical solving of the studied option pricing problem, see our previous results in [8]. Within the first phase, we construct the semi-discrete solution

$$u_{DG}(\hat{t}) \in S_h^p = \left\{ v_h \in L^2((0, x_{max}));\ v_h\big|_{\mathcal{I}} \in P_p(\mathcal{I})\ \forall \mathcal{I} \in \mathcal{T}_h \right\} \tag{11}$$

from the finite dimensional space S_h^p of discontinuous piecewise polynomial functions of the p-th order defined on the partition $\mathcal{T}_h = \{\mathcal{I}\}_{\mathcal{I}\in\mathcal{T}_h}$ of domain $(0, x_{max})$.

Based on similar techniques, cf. [9], this semi-discrete solution is defined using a method of lines leading to the system of the ordinary differential equations as follows

$$\frac{d}{d\hat{t}} \big(u_{DG}(\hat{t}), v_h\big) + \mathcal{A}_h(u_{DG}(\hat{t}), v_h) = l_h(v_h)(\hat{t}) \quad \forall\, v_h \in S_h^p,\ \forall\, \hat{t} \in (0, T), \tag{12}$$

where $u_h(0)$ is given by a payoff function, $(\cdot,\cdot)$ denotes the L^2-inner product, the form $\mathcal{A}_h$ stands for the DG semi-discrete variant of the degenerate parabolic partial differential operator from (9), and the form l_h balances boundary conditions, for more details see [7].

The second phase aims to discretize the problem (12) on the time interval with a constant time step τ using the Crank-Nicolson method to obtain the numerical scheme of a high accuracy with respect to both discretization parameters. Denote u_{DG}^m as the approximation of the solution u_{DG} at time level $\hat{t}_m \in [0, T]$, then the approximate solution at the subsequent time level is found according to the following numerical scheme

$$\begin{aligned} \frac{1}{\tau}\left(u_{DG}^{m+1}, v_h\right) + \frac{1}{2}\mathcal{A}_h\big(u_{DG}^{m+1}, v_h\big) &= \frac{1}{\tau}\left(u_{DG}^{m}, v_h\right) - \frac{1}{2}\mathcal{A}_h(u_{DG}^{m}, v_h) \\ &+ \frac{1}{2}\Big(\ell_h(v_h)(\hat{t}_{m+1}) + \ell_h(v_h)(\hat{t}_m)\Big) \quad \forall\, v_h \in S_h^p,\ m = 0, 1, \ldots \end{aligned} \tag{13}$$

with the starting data $u_{DG}^0 \approx u_{DG}(0)$.

Moreover, one can easily identify that the discrete scheme (13) corresponds to the system of linear equations. More precisely, we rewrite the discrete DG solution as a linear combination of basis functions $\{\varphi_k\}_{k=1}^{DOF}$ of space S_h^p, $DOF = \dim(S_h^p)$, i.e., $u_{DG}^m(x) = \sum_{k=1}^{DOF} \xi_k^m \varphi_k(x)$, $x \in (0, x_{max})$, and set the vector of real coefficients $U_m = \left\{\xi_k^m\right\}_{k=1}^{DOF} \in \mathbb{R}^{DOF}$, then we receive a sequence of the sparse linear algebraic problems

$$\Big(\mathbf{M} + \frac{\tau}{2}\,\mathbf{A}\Big)\, U_{m+1} = \Big(\mathbf{M} - \frac{\tau}{2}\,\mathbf{A}\Big)\, U_m + \frac{\tau}{2}\,\left(F_{m+1} + F_m\right), \quad m = 0, 1, \ldots \tag{14}$$

where the matrix $\mathbf{M}$ is related to L^2-inner product, the matrix $\mathbf{A}$ to the form $\mathcal{A}_h$ and the vectors F_m, F_{m+1} represent the right-hand side form l_h, respectively. The solvability of the sparse linear algebraic system (14) is proven in [10].

3.3 Computation of the Greeks

In order to illustrate the robustness of the presented approach the numerical scheme (14) is used not only for the evaluation of option prices but also of their sensitivity measures.

Consider polynomial approximation at least of the second order (quadratic), then Delta and Gamma can be directly computed from the derivatives of the basis functions

$$\Delta_V(T-\hat{t}_m) \approx \frac{\partial u_{DG}^m}{\partial x} = \sum_{k=1}^{DOF} \xi_k^m \varphi_k'(x), \tag{15}$$

$$\Gamma_V(T-\hat{t}_m) \approx \frac{1}{\mathcal{K}}\frac{\partial^2 u_{DG}^m}{\partial x^2} = \frac{1}{\mathcal{K}} \sum_{k=1}^{DOF} \xi_k^m \varphi_k''(x). \tag{16}$$

On the other hand, the remaining Greeks are numerically computed using the central finite difference

$$\Theta(T-\hat{t}_m) \approx -\mathcal{K}\frac{\partial u_{DG}(\hat{t}_m)}{\partial \hat{t}} \approx -\mathcal{K}\frac{u_{DG}(\hat{t}_m+\delta_{\hat{t}}) - u_{DG}(\hat{t}_m-\delta_{\hat{t}})}{2\delta_{\hat{t}}},\ 0<\delta_{\hat{t}} \ll 1, \tag{17}$$

$$\mathcal{V}_V(T-\hat{t}_m,\sigma) \approx \mathcal{K}\frac{u_{DG}(\hat{t}_m,\sigma+\delta_\sigma) - u_{DG}(\hat{t}_m,\sigma-\delta_\sigma)}{2\delta_\sigma}, \quad 0<\delta_\sigma \ll 1, \tag{18}$$

$$\rho_V(T-\hat{t}_m,r) \approx \mathcal{K}\frac{u_{DG}(\hat{t}_m,r+\delta_r) - u_{DG}(\hat{t}_m,r-\delta_r)}{2\delta_r}, \quad 0<\delta_r \ll 1, \tag{19}$$

which involves to solve the option pricing problem twice. Note that this approach provides only the point-wise approximation with respect to the underlying parameter t, σ and r, respectively.

4. Numerical example

In this section we shall numerically study the dependency of the approximate option prices and selected Greeks on the degrees of freedom DOF for fixed quadratic polynomial approximations on uniformly refined grids.

Therefore, the relative errors are evaluated at final state in the standard L^2-norm and discrete l^∞-norm

$$e_2 = \frac{\|u_{DG}^* - u(T)\|}{\|u(T)\|} \qquad \text{and} \qquad e_\infty = \frac{\|u_{DG}^* - u(T)\|_\infty}{\|u(T)\|_\infty}, \tag{20}$$

where $u(T)$ is the exact solution at time T and u_{DG}^* the numerical one at the same time level obtained by the scheme (13). Next, the experimental orders of convergence are defined by

$$r_i \approx \frac{\log\big(e_i(DOF)/e_i(2DOF)\big)}{\log 2}, \quad i = 2,\infty, \tag{21}$$

where $e_i(DOF)$ and $e_i(2DOF)$ denote the corresponding spatial errors for discretization with DOF and $2DOF$ degrees of freedom, respectively. In order to restrain the discretization errors with respect to the time discretization, a sufficiently small time step $\tau = 1/3600$ (i.e., 1/10 of a calendar day) with combination of GMRES solver is used in scheme (14). All computations are carried out with an algorithm implemented in the solver Freefem++, for more details see [6].

According to the theoretical results [3], it is known that the DG technique produces (in general) suboptimal convergence of approximate solutions and optimal one of their spatial derivatives, all measured in L^2-norm for sufficiently regular problems. In the case of the remaining first order derivatives (not with respect to the underlying asset) the similar convergence results in L^2-norm could be expected. On the other hand, the general results of point-wise error estimates and a convergence in l^∞-norm for the discontinuous Galerkin methods are not available for these problems. Therefore, the forthcoming experiments provide the deeper insight to the properties of the presented the DG technique in connection with the option pricing problem and sensitivity measurement.

The convergence of the method within this sensitivity analysis is investigated on the idealized problem of pricing a vanilla put option with the real data of the German DAX on 15 September 2011 with implied volatilities, firstly introduced in [9]. To be consistent with the reference experiment, we consider the following data: $S_{max} = 16\,000$, $\mathcal{K} = 4\,000$, $T = 1/3$, $\sigma = 0.4594$ and $r = 0.0176$.

The calculations are performed on a sequence of the consecutive uniformly refined meshes with $DOF \in \{12, 24, 48, 96, 192, 384\}$. The relative errors and experimental orders of convergence for option values and their sensitivity measures with respect to the underlying asset are recorded in Table 1 and Table 2. One can easily observe that relative errors in both norms increase with the order of derivatives and decrease with the mesh refinement. In line with the theory, the convergences of Delta and Gamma values in L^2-norm are optimal, i.e., $r_2 \approx 2$ (Delta) and $r_2 \approx 1$ (Gamma) for piecewise quadratic approximation. Moreover, the convergence of option prices in L^2-norm is suboptimal ($r_2 \approx 2$). Further, the point-wise errors and convergence in l^∞-norm exhibit the same behavior as results measured in L^2-norm.

Table 1: Computational errors in L^2-norm and experimental orders of convergence of option, Delta and Gamma values for piecewise quadratic approximation with different degrees of freedom

DOF	Option value		Delta value		Gamma value	
	e_2	r_2	e_2	r_2	e_2	r_2
12	1.4886e-2	—	1.1574e-1	—	6.9733e-1	—
24	3.1160e-3	2.26	4.0387e-2	1.52	4.2118e-1	0.73
48	4.3537e-4	2.84	9.0636e-3	2.16	2.0542e-1	1.04
96	9.2099e-5	2.24	2.2549e-3	2.01	1.0480e-1	0.97
192	2.1688e-5	2.09	5.6787e-4	1.99	5.2994e-2	0.98
384	5.3266e-6	2.03	1.4252e-4	1.99	2.6632e-2	0.99

Next, we analyze the first-order Greeks — Theta, Vega and Rho. These sensitivity measures are computed using the central finite differences (17)–(19) with steps $\delta_{\hat{t}} = \tau$ and $\delta_\sigma = \delta_r = 0.001$. The corresponding errors and experimental orders are shown in Table 3 and Table 4. As expected the results obtained are of the same quality and experimental orders are consistent with the Delta ones, i.e., $r_2 \approx 2$ and $r_\infty \approx 2$.

Finally, for a complete overview, the approximate option values and selected sensitivity measures with respect to the different maturities are depicted at the final state in Figure 1.

Table 2: Computational errors in l^∞-norm and experimental orders of convergence of option, Delta and Gamma values for piecewise quadratic approximation with different degrees of freedom

DOF	Option value		Delta value		Gamma value	
	e_∞	r_∞	e_∞	r_∞	e_∞	r_∞
12	4.7493e-2	—	4.3037e-1	—	8.5154e-1	—
24	1.2700e-2	1.90	2.5323e-1	0.77	8.9755e-1	—
48	2.0421e-3	2.64	6.4593e-2	1.97	4.4686e-1	1.01
96	4.1469e-4	2.30	1.7659e-2	1.87	2.5125e-1	0.83
192	1.0072e-4	2.04	4.2759e-3	2.05	1.2562e-1	1.00
384	2.3913e-5	2.07	1.0620e-3	2.01	6.3104e-2	0.99

Table 3: Computational errors in L^2-norm and experimental orders of convergence of Theta, Vega and Rho values for piecewise quadratic approximation with different degrees of freedom

DOF	Theta value		Vega value		Rho value	
	e_2	r_2	e_2	r_2	e_2	r_2
12	1.4588e-1	—	1.5243e-1	—	1.3900e-1	—
24	6.2534e-2	1.22	6.5211e-2	1.22	4.6091e-2	1.59
48	8.8693e-3	2.82	9.1883e-3	2.83	1.1367e-2	2.02
96	1.6768e-3	2.40	1.7449e-3	2.40	2.2854e-3	2.32
192	3.8322e-4	2.12	3.9860e-4	2.13	8.6232e-4	1.40
384	9.3426e-5	2.04	9.7063e-5	2.04	7.4220e-4	0.23

It is apparent that these observable values change most for ATM or near ATM options as is consistent with analytical formulae (3)–(7).

5. Conclusions

The accurate calculation of option sensitivity factors, commonly known as the Greeks, constitutes a crucial step of risk management of large option portfolios. Since the computations of sensitivity factors often require the usage of advanced methods, in this contribution, we have develop the general discontinuous Galerkin scheme considering simplified BS setting so that the results can easily be compared with analytical formulae. The numerical experiments with DAX market data illustrate a huge potency of the DG method.

References

[1] Achdou, Y. and Pironneau, O. (2005). *Computational Methods for Option Pricing.* Society for Industrial and Applied Mathematics, Philadelphia.

[2] Black, F. and Scholes, M. (1973). The Valuation of Options and Corporate Liabilities. *Journal of Political Economy* **81**, pp. 637-654.

Figure 1: The approximate prices and selected sensitivity measures at different maturities on fixed mesh with $\#\mathcal{T}_h = 128$. The piecewise constant approximation of Gamma values is easily observable due the default piecewise quadratic approximation of option prices. The rest of graphs uses piecewise linear plots.

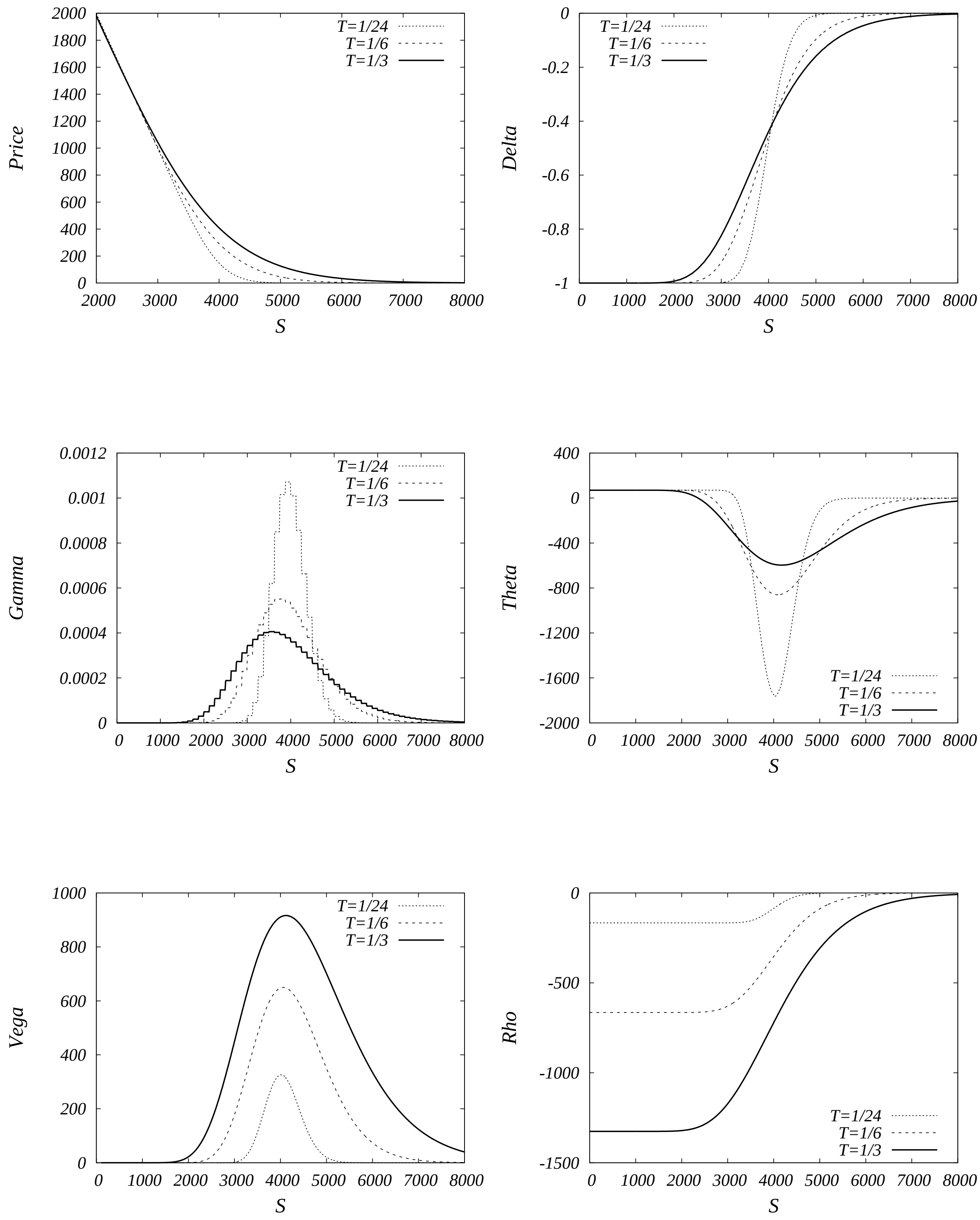


Table 4: Computational errors in l^∞-norm and experimental orders of convergence of Theta, Vega and Rho values for piecewise quadratic approximation with different degrees of freedom

DOF	Theta value		Vega value		Rho value	
	e_∞	r_∞	e_∞	r_∞	e_∞	r_∞
12	2.5196e-1	—	2.5779e-1	—	1.5412e-1	—
24	1.7291e-1	0.54	1.7280e-1	0.58	1.0461e-1	0.56
48	1.7667e-2	3.30	1.7244e-2	3.32	3.5662e-2	1.55
96	5.7123e-3	1.63	5.7881e-3	1.58	1.2341e-2	1.53
192	1.1865e-3	2.27	1.1187e-3	2.29	2.4995e-3	2.30
384	2.7285e-4	2.12	2.7507e-4	2.11	9.1783e-4	1.45

[3] Dolejší, V. and Feistauer, M. (2015). *Discontinuous Galerkin Method. Analysis and Applications to Compressible Flow.* Springer Series in Computational Mathematics **48**, Springer, Cham.

[4] Duffy, D.J. (2006). *Finite Difference Methods in Financial Engineering.* Wiley.

[5] Haug, E.G. (2006). *The Complete Guide to Option Pricing Formulas.* McGraw-Hill.

[6] Hecht, F. (2012). New development in FreeFem++. *Journal of Numerical Mathematics* **20**, no. 3-4, pp. 251-265.

[7] Hozman, J. (2012). Discontinous Galerkin method for the numerical solution of option pricig. *Aplimat – Journal of Apllied Mathematics* **5**(2), pp. 271-280.

[8] Hozman, J. and Tichý, T. (2016). DG solver for one-factor and two-factor Black-Scholes models. *Managing and Modelling of Financial Risks*, pp. 323-332, Ostrava: VSB-TUO.

[9] Hozman, J. and Tichý, T. (2016). On the Impact of Various Formulations of the Boundary Condition within Numerical Option Valuation by DG Method. *Filomat* **30**(15), pp. 4253-4263.

[10] Hozman, J. and Tichý, T. (2017). DG method for the numerical pricing of two-asset European-style Asian options with fixed strike. *Applications of Mathematics* **62**(6), pp. 607-632.

[11] Hozman, J. and Tichý, T. (2018). DG framework for pricing European options under one-factor stochastic volatility models. *Journal of Computational and Applied Mathematics* **344**, pp. 585-600.

[12] Merton, R. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science* **4**, pp. 141-183.

[13] Topper, J. (2005). *Financial Engineering with Finite Elements.* Wiley.

[14] Ugur, Ö. (2008). *An Introduction to Computational Finance.* Series in Quantitative Finance **1**. Imperial College Press, London.

Data Envelopment Analysis in Small and Medium Enterprises in hospitality industry

Lucie Chytilová [1]

Abstract

An important segment of any developed economy are small and medium-sized enterprises, which are considered to be the backbone of national economies. There are some studies which evaluate efficiency of hospitality industry. Generally, these studies evaluate the efficiency under certainty, for small sample in big region. The problem is missing data or the lack of data which may cause imprecise results. The aim of this paper is to determine the efficiency of the hospitality industry - restaurants in Moravian-Silesian region and include into the analysis the risk component for this problematic region. The technical efficiency of restaurants in Moravian-Silesian region is estimated for year 2016. The special type of DEA models with the risk component are used. The development of calculated technical efficiency is analysed in the cross section among the units. Also the influence of the risk variable is analysed as well.

Key words

DEA, risk variable, negative value

JEL Classification: C6, G21

1. Introduction

One of the keys for hospitality managers, especially restaurant managers, in conducting a proper assessment of their efficiency is through understanding the level of risk. That is why it is really important to understand and analyse the causal relationships between external environmental forces and internal capabilities of the firm, and then to see the efficiency of the restaurants.

The purpose of this article is to calculate the technical efficiency of some restaurants in Moravian-Silesian region for year 2016 with the influence of the risk variable. And also,to see and investigate the influence of risk variable in the restaurant industry's. This article is the first on this topic. The database of restaurant is small, but the influence of the risk variable may be already seen and analyse. This article proposes a contemporary framework that enables restaurant industry executives to develop a better understanding of the efficiency of restaurants and risk factors (macroeconomic and industry) that influence their firms.

The concept of risk is at the foundation of every firm as it seeks to compete in its business environment. According to financial theory, total risk is composed of two components, systematic and unsystematic risk. The examples of systematic risk could be changes in monetary and fiscal policies, the cost of energy, tax laws, and the demographics of the marketplace. This refers to the variability of a firm's stock returns that moves in unison with these macroeconomic influences as systematic, or stockholder, risk (Lubatkin and Chatterjee, 1994). On the other hand, a loss of a major customer as a result of its bankruptcy represents one source of unsystematic, or firm-specific risk (idiosyncratic or stakeholder risk). Other sources include the death of a high-ranking executive, a fire at a production facility, and the

[1] Lucie Chytilova, VSB-TU Ostrava, Faculty of Economic, lucie.chytilova@vsb.cz .

sudden obsolescence of a critical product technology (Lubatkin and Chatterjee, 1994). Other types of risk which are related to tax credits, bankruptcy, country, call options, inflation, interest rates etc. All these types of risks have to be take into acount in the analysis.

2. Theoretical principles

Data envelopment analysis (DEA) is non-parametric linear programming based technique for measuring the relative efficiency of a set of similar decision making units (DMUs). Since the work of Charnes et al (1978), DEA has demonstrated an effective technique for measuring the relative efficiency of set homogenous DMUs. In application, DMUs may include banks, hospitals, schools, different types of industries and other. Each DMU allocates its resources into a number of inputs to produce various outputs. The relative technical efficiency of the unit is define as the ratio of its total weighted output to its total weighted input or vice versa. DEA allows each production unit to choose its own weights of inputs and outputs in order to maximize its efficiency score. A technically efficient production unit is able to find such weights that it lies on the production frontier. The production frontier represents the maximum amounts of output that can be produced by given amounts of input in the output maximization model or, alternatively, the minimum amounts of inputs required to produce the given amount of output in the input minimization model. DEA calculates the efficiency score for each production unit and identifies peers for each production unit that is not technically efficient.

2.1 Classic DEA models

The first two known DEA models are called CCR and BCC. The CCR model is formulates for the assumption of constant return to scale (CRS). The origin model was extended by Banker et al (1984) for the assumption of variable return to scale (VRS). There are also other types of models – additive, super efficiency, two stage DEA or ZSG-DEA models. All of them are looking for an efficiency frontier that envelops data. DEA models are able to classify DMUs as efficient and inefficient.

The mathematical formulation of the origin DEA model was done by Charmes et al (1978). Suppose that there are T DMUs (DMU$_k$ for $k = 1, ..., T$), let input and output data for be $\boldsymbol{X} = \{ x_{ik}, i = 1, ...R; k = 1, ...T\}$ and $\boldsymbol{Y} = \{y_{jk}, , j = 1, ...S; k = 1, ...T\}$, u_i, for $i = 1, ...R$ and v_j, for $j = 1, ..., S$ be the weights of i-th input and j-th output, respectively. The mathematical model to measure the efficiency score of the under evaluation unit, DMU$_Q$ where $Q \in \{1, ..., T\}$ is flowed:

$$\begin{aligned} \max\ e_Q &= \sum_{j=1}^{S} v_j y_{jQ} \\ s.t. \quad & \sum_{i=1}^{R} u_i x_{iQ} = 1 \\ \sum_{j=1}^{S} v_j y_{jk} &\le \sum_{i=1}^{R} u_i x_{ik} \quad k = 1, \dots, T, \\ u_i \ge 0, v_j &\ge 0 \quad i = 1, \dots, R, j = 1, \dots, S, \end{aligned} \tag{1}$$

This model must be solved for each DMU. Notice that DMUQ is CCR-efficient if and only if e* = 1 and there exists at least one optimal solution (u_i*, v_j*) with u_i*>0 and v_j*>0. Inefficient units have a degree of relative efficiency less than one. The model (1) is called a multiplier input-oriented model.

However, for computing and data interpretation is preferable to work with a model that is dual associated to model (1). The model is referred as envelopment input oriented model is following:

$$\begin{aligned} & \min \theta_Q \\ s.t. \quad & \sum_{k=1}^{T} \lambda_k x_{ik} \le \theta_Q x_{iQ} \qquad i = 1,..R, \\ & \sum_{k=1}^{T} \lambda_k y_{jk} \ge y_{jQ} \qquad j = 1,..S, \qquad (2) \\ & \lambda_k \ge 0 \qquad \forall k, \\ & \theta_Q \in (-\infty, \infty), \end{aligned}$$

where λ_k is the weight for DMUk for $k = 1, ..., T$. It is dual-variable unit. The θ_Q represents the efficiency score of DMUQ. It can also be interpreted as a reduction rate of inputs to reach the efficient frontier. There also exist the multiplier output-oriented model, but this model is not in this used paper and presented.

BCC model by Banker et al (1984) has convex envelope of data and generally more units are efficient. The results of input-oriented and output-oriented models are same. Because the dual models are more useful, just the dual model is presented. The mathematical model of dual BCC model is following:

$$\begin{aligned} & \min \theta_Q \\ s.t. \quad & \sum_{k=1}^{T} \lambda_k x_{ik} \le \theta_Q x_{iQ} \qquad i = 1,..R, \\ & \sum_{k=1}^{T} \lambda_k y_{jk} \ge y_{jQ} \qquad j = 1,..S, \qquad (3) \\ & \lambda_k \ge 0 \qquad \forall k, \\ & \sum_{\kappa} \lambda_{\kappa} = 1, \end{aligned}$$

λ_k is the vector of the dual variables units. DMU_Q is BCC-efficient the radial variable θ_Q is equal to one, ie. the optimal value of the objective function of the model (6) $e^* = 1$, otherwise it is BCC-inefficient. Units that are not effective have a value of $e^* < 1$.

2.2 DEA models with negative numbers and zero values

Many times the data set have negative numbers. Basic DEA models are not capable of completing an analysis with negative numbers and all numbers must be non-negative and preferably strictly positive (no zero values). This has been defined as the “positivity” requirement of DEA. There exist many methods how to deal with this problem. In this article, the following and one of the more common is used.

The method is through the addition of a sufficiently large positive constant to the values of the input or output that has the non-positive number. Bowlin (1998) has advised to make the negative numbers or zero values a smaller number in magnitude than the other numbers in the data set. But there have been some problems - the results may still change depending on the scale (adjusting constant value) used by the models. This problem has been defined as “translation variance”. Ali and Seiford (1990) and Pastor (1996) have shown that a displacement does not alter the efficient frontier for certain DEA formulations. Specifically, the additive model for both inputs and outputs and the BCC model. These approaches have invariant translation.

3. Study case

This article presents just a study case of 21 restaurants from Ostrava (Moravian-Silesian region). In future the database should be extended. The data are taken for the year 2016 from the company statements. The chosen data for the basic analysis are: labor costs, suppliers, revenues and profit. First two variables should be as small as possible. So these two variables are define as input variables in this analysis. On the other hand, the second two variables should be as biggest as possible. So these two variables are define as output variables.

In Table 1 the descriptive statistics of input and output variables is seen. Also, the problem of these data are seen. The problem for the DEA models are negative values and values equal to zero. This problem is for there variables (at least) – labor cost has minim value zero, revenues and profits have even negative values. To be able use the DEA models. A large enough constant has been added to problematic values.

Table 1 Descriptive statistics of the classic input and output variables

	labor costs	supplies	revenues	profit
max	5 480 323	1 036 000	17 649 448	3 600 000
min	0	13 000	-316 000	-1 023 000
std.dev	1 591 299	269 607	5 337 812	855 622
average	1 678 277	299 762	3 180 056	520 238

Authors' calculation

As the risk component for this first study case was chosen the Return on Equity (ROE). ROE is the amount of net income returned as a percentage of shareholders' equity. Return on equity measures a corporation's profitability by revealing how much profit a company generates with the money shareholders have invested. ROE is expressed as a percentage and calculated as:

Return on Equity = Net Income/Shareholder's Equity.

Net income is for the full fiscal year (before dividends paid to common stockholders but after dividends to preferred stock.) Shareholders' equity does not include preferred shares. In future another variable should be used. ROE should be as bigger as possible. In other words, it should maximize. So this variable is define as another output variable.

Table 2 shows three types of the results. First are results for the DEA model with the constant return scale (CRS) assumption - model_CRS. To see which assumption is better, also the model with variable return to scale (VRS) assumption is calculated. This also tells if the model is correct (the model with VRS assumption should have more efficient DMUs). The model with VRS assumption is called model_VRS. Last used model is model VRS assumption and with the risk variable and this model is called model_VRS_risk.

Table 2 Efficiency score of all DMUs with the descriptive statistics

	model_CRS	model_VRS	model_VRS_risk
DMU01	0.8743	1.0000	1.0000
DMU02	0.9728	1.0000	1.0000
DMU03	0.0002	0.1143	0.1143
DMU04	0.8168	0.8954	0.9104
DMU05	0.6369	0.9189	1.0000
DMU06	0.8866	1.0000	1.0000
DMU07	0.5622	0.7597	0.7673
DMU08	0.7418	1.0000	1.0000
DMU09	1.0000	1.0000	1.0000
DMU10	1.0000	1.0000	1.0000
DMU11	0.3921	1.0000	1.0000
DMU12	0.4049	0.6212	0.6212
DMU13	1.0000	1.0000	1.0000

DMU14	0.8678	0.9114	0.9114
DMU15	1.0000	1.0000	1.0000
DMU16	0.1452	0.1685	0.1679
DMU17	0.3523	0.3533	0.3523
DMU18	0.1467	0.4532	0.4532
DMU19	0.5514	0.5557	0.5557
DMU20	0.3000	1.0000	1.0000
DMU21	0.0007	0.4003	0.4003
max	1.0000	1.0000	1.0000
min	0.0002	0.1143	0.1143
std.dev	0.3496	0.3048	0.3077
average	0.6025	0.7691	0.7740

Authors' calculation

The results show the basic idea of the model is correct. Model_CRS has four efficient DMUs and model_VRS has ten efficient DMUS. Generally, the efficiency score of the model_VRS is higher. So the basic idea of the model is correct. Also, the ides of VRS assumption is probably better for this industry. The efficiency scores of model_VRS and model_VRS_risk is very close. Model_VRS_risk has eleven DMUs efficient and higher average efficiency score. This means that the use of the risk component is useful. On the other hand this values is not too much higher, so probably another type of variable should be used.

4. Conclusion

This article deals with the DEA models and efficiency of hospitality industry. In the beginning, the idea was simple, but during calculation many problems has been found. First, a small database of the restaurants in Ostrava (hospitality industry in Moravian-Silesian region). So this article deals just with a study case before more data will be available. Second problem of this industry are negative or zero values in the database. In this article, just the basic solution is used. And lately, what is the proper variable to represent the risk variable – ROE or different? Overall, it is found that the DEA models may be used for this type of industry. Also, that the VRS assumption is probably better with the risk variable.

In future, another variable as the risk variable should be define and used (for different types of the risk). Even, some other technics as fuzzy variable should be explore. Also, the database of restaurants should be extended not just for restaurants in Ostrava or different model of DEA should be used.

Acknowledgements

This article was supported by Student Grant Competition project (SP2018/146) of Faculty of Economics, VŠB-Technical University of Ostrava.

References

[1] Ali, A.I. and Seiford, L.M. (1990). Translation Invariance in Data Envelopment Analysis. *Operations Research Letters*, 9, p. 403–405.

[2] Banker, R.D., Charnes, A., Cooper, W.W. (1984). Some models for estimating technical and scale inefficiency in data envelopment analysis. *Management Science*, 30(9), p. 1078–1092.

[3] Bowlin, W.F. (1998) Measuring Performance: An Introduction to Data Envelopment Analysis (DEA). *Journal of Cost Analysis*, 7, p. 3-27.

[4] Charmes, A., Cooper, W.W. and Rhodes, E. (1978). Measuring the inefficiency of decision making units. *European Journal of Operational Research*, 2, p. 429–444.

[5] Lubatkin, M. H., Chatterjee, S. (1994). Extending modern portfolio theory into the domain of corporate diversification: Does it apply? *Academy of Management Journal*, 37(1), p. 109- 137.

[6] Pastor, J.T. (1996) Translation Invariance in DEA: A Generalization. *Annals of Operations Research*, 66, p. 93-102.

The impact of moving the owners' registered office to tax havens on the reported values of land and structures

Michal Ištok, Mária Kanderová, Peter Krištofík[1]

Abstract

This work aims to analyze the impact of the transfer of the registered office of Slovak companies into tax havens on the reported values of land and structures. Analyzed and compared is an indicator of the share of structures and land on total non-current assets before and after the transfer of the registered office to the tax haven. There are 3 data sources used, respectively three databases. The first is an overview of Slovak companies that moved their registered office to selected jurisdictions that are referred to as tax havens (Bisnode database) from 2005 to 2015. The data for the financial statements of the Slovak companies are drawn from financial datasets provided by Finstat company. The change of the chosen indicator is evaluated regarding the type of jurisdiction, the size of equity and the NACE sector. The third database is the data published by Transparency International Slovakia – an overview of lands and structures in the Slovak Republic, which are owned through tax havens.

Keywords

Onshore and offshore companies. Structures and land. The anonymity of the UBO.

JEL Classification: F23, H25, H26

1. Introduction

The number of Slovak companies with owners in selected jurisdictions classified as tax havens in the first quarter of 2018 has fallen by 44 compared to the data from the end of 2017. Since 2005 there has been a steady increase in the number of these businesses. While in 2005 it was 1 510 Slovak companies, at the end of 2017 it was already 4 796 Slovak companies. Despite the constant year-on-year increase, it can be said that the growth rate is gradually slowing down, respectively when considering owners from offshore jurisdictions, we may even be talking about decline since the year 2016 (Bisnode, 2018).

Most academic analyzes on tax havens are currently primarily concerned with tax aspects, mainly international tax planning and tax aggression, from different perspectives. Zheng (2017) states that diversified companies use fewer tax evasion practices than so-called “stand-alone firms” (companies that focus on and emphasize only one business segment). Anken and Beasley (2012) deal with the issue of corporate decision-making at the level of the corporate-directed tree structure in order to maximize profits in the target parent company.

[1] Ing. Michal Ištok, PhD., Assistant professor of the Faculty of Economics, Matej Bel University in Banská Bystrica, michal.istok@umb.sk;
Ing. Mária Kanderová, PhD., Assistant professor of the Faculty of Economics, Matej Bel University in Banská Bystrica, maria.kanderova@umb.sk;
Doc. Ing. Peter Krištofík, Ph.D, Dean of the Faculty of Economics, Matej Bel University in Banská Bystrica, peter.kristofik@umb.sk.

Their analysis mainly implies the direct capital connection of parent company, subsidiary and other related companies. Mara (2015) stresses that to label jurisdiction a tax haven is except low taxation in a given jurisdiction, significant also the key level of the revenue share of the services sector from the attracted financial flows to their GDP.

Foad and Lundberg (2017) argue that the financial sector in offshore financial centers (OFC) is disproportionately higher than their domestic economy. Weizig and Dijk (2007) as offshore jurisdictions (real tax havens) refer mainly to small tropical states with a publicly inaccessible business register and a high degree of owner anonymity. As onshore jurisdictions, they put jurisdictions with a diversified economy and advanced legal system.

Holding companies are mainly used to hold and manage equity interests and to control subsidiaries. They are popular in the form of mainly international holding companies, respectively structures (use of foreign companies) that offer legal, business and tax use, in the field of e.g. protection of investments under international treaties, ensuring a high degree of anonymity and the use of special tax regimes (Liška, Sabolová, 2017). Dischinger and Riedel (2011) by using panel data in small and medium-sized European companies have confirmed that lower corporate tax rates within international holding companies result in the transfer of intangible assets, respectively investing them into these jurisdictions.

According to data published by Transparency International Slovakia (2016), companies in selected jurisdictions (Transparency International Slovakia identifies them as tax havens) or physical entities (the address of the participation relation in the tax haven) own 1 023 land and structures parcels in the Slovak Republic, which represents a total value of more than 1.3 mil. square meters (source: Geodesy, Cartography and Cadastre Authority of the Slovak Republic).

1.1. Analysis of changes in reported land and structures on total non-current assets

In the analysis, we focused on the indicator (land + structures)/total non-current assets. We have data available for 681[2] Slovak companies, where the data for the given indicator was known before and after the change of the owner. We have been focusing on a change in the indicator before and after the owner's transfer from the Slovak Republic to the tax haven. The possible change of the indicator was monitored regarding the type of jurisdiction, size of equity and NACE sector. We used a non-parametric Wilcoxon test to monitor a statistically significant difference in indicator values. We identified possible relationships by sorting available data into contingency tables and suitably chosen graphical tools. Statistical data processing was performed using statistical software SPSS.

The statistically significant difference in the value of the observed indicator before and after the change of the owner was tested using the nonparametric Wilcoxon test, which we performed separately for each type of jurisdiction[3]. The results of the test in all cases

[2] We matched the data provided by Bisnode (Slovak companies with owner from tax haven) with the available financial statements from financial datasets provided by Finstat. The large difference between the analysed number of companies and actual number of companies located in tax havens (4 282 by the end of 2015) was mainly due to the unavailability of data for both periods (before and after transfer to tax haven), respectively we excluded companies that reported zero values of total assets.

[3] Jurisdictions have been divided into three categories:

a) OFFSHORE JURISDICTIONS (OFF): Bahamas, Belize, Bermuda, British Virgin Islands, Gibraltar, Guernsey (United Kingdom), Jersey (United Kingdom), Cayman Islands, Marshall Islands, the Netherlands Antilles, Panama, Man Island, and Seychelles;
b) MIDSHORE JURISDICTIONS (MID): Hong Kong, Cyprus, Malta, United Arab Emirates, United States of America;

confirmed a statistically significant difference - Slovak companies after relocating their registered office to the tax haven reported a decrease in the share monitored (Table 1).

Table 1: Wilcoxon Signed Ranks Test

Test Statistics		
JURISDICTION		BEFORE - AFTER
ONSHORE	Z	-6,070[a]
	Asymp. Sig. (2-tailed)	,000
MIDSHORE	Z	-5,918[a]
	Asymp. Sig. (2-tailed)	,000
OFFSHORE	Z	-2,818[a]
	Asymp. Sig. (2-tailed)	,005
a. Based on negative ranks. *b. Wilcoxon Signed Ranks Test*		

Source: Own processing, 2018.

The test results confirmed a statistically significant difference in the value of the indicator before and after the change of the owner. A more detailed view of changing the indicator is shown in Table 2, where the division of companies is based on jurisdiction and also how the indicator has changed (the direction).

Table 2: Companies division by jurisdiction and change of the indicator

Jurisdiction	**rise**	**no change**	**fall**	**Total**	**rise**	**no change**	**fall**
ON	122	52	196	**370**	33%	14%	53%
MID	76	33	142	**251**	30%	13%	57%
OFF	14	14	32	**60**	24%	23%	53%
Sum	**212**	**99**	**370**	**681**	**31%**	**15%**	**54%**

Source: Own processing, 2018.

Regardless of the type of jurisdiction, there are dominant companies with a decline in the value of the indicator, in onshore and offshore categories it is 53% of companies and in midshore almost 57% of companies. From the analyzed set of Slovak companies, the lowest number was the number of companies that moved their registered address to some of the selected offshore jurisdictions (OFF). If the offshore company has the first-line ownership, the most observed is the achievement of the anonymity of the ultimate beneficial owner (UBO). Slovak companies owned by offshore companies are expected to show the largest share of companies without changing the indicator (23%), but on the other hand, the difference is only about 10 percentage points higher than if considering onshore and midshore companies. Onshore and midshore companies at first line of ownership offer mainly asset management benefits, flexible ownership arrangements, and potential tax optimization (savings). Furthermore, these companies may be owned by an offshore company, and thus the high anonymity of the real owner can be achieved. Especially on the results achieved in the onshore and midshore categories, it is relatively difficult to interpret the results more closely, given the many potential observed benefits and many potential transactions that may have occurred about land and structures (e.g., transfers, sales, purchases, etc.). The smallest share of non-changing companies was in the midshore category, where we ranked jurisdictions like Cyprus, Malta, USA, Hong Kong and the United Arab Emirates. These jurisdictions are the favorite destinations of Slovak entrepreneurs and their share in the total number of Slovak companies based in the tax haven was 50% at the 1st quarter of 2018. The most significant share was

c) ONSHORE JURISDICTIONS (ON): Liechtenstein, Latvia, Luxembourg, Monaco and the Netherlands.

reached by the jurisdiction of Cyprus (23%), which is used by Slovak companies mainly due to a good ratio of performance vs. price, that is, universal use at relatively low costs compared to onshore jurisdictions, as is for example Luxembourg.

The increase of indicator was anticipated, respectively keeping it at the same level. Analyzed data show that more often the transfer of Slovak companies into tax havens is only occurring at a time when the companies own land and structures compared to a move when it is expected that in the future the land and structures will be purchased by the company or transferred in another way.

Out of the total number of companies, there were 15% of them where the indicator has not changed. Checking closer these companies, almost all of them (94%) represented the value of this indicator 1 - their non-current assets consisted only of land and structures. It implies that this number represents the so-called "asset holding," that is, this degree of ownership/link in the holding was created mainly for anonymity of the real ultimate owner. In the group of companies, where the indicator had not changed regarding the size of the non-current assets before the change of the registered office, companies with a lower value of total assets dominated by 61% (Table 3).

Table 3: Division of enterprises without change of indicator according to the total assets

Total assets	**count**	**share**
to 1 million	60	61%
to 10 million	28	28%
greater 10 million	11	11%
Total	**99**	**100 %**

Source: Own processing, 2018.

The results of the statistical test, which confirmed a statistically significant difference in the value of the indicator observed before and after the change of the owner, could be affected by the fact that in the file were companies in which the share of land and structures on the total non-current assets was zero before the change, after the change of the registered office higher than 50% respectively where the indicator was above 50% prior to the change of residence and was zero after the change of residence. Table 4 shows the division of enterprises with a zero value of the given indicator before the change of owner and with a value higher than 0.5 after the change of the owner. Table 5 shows the breakdown of enterprises with a value higher than 0.5 before the change of owner and zero shares of land and structures on non-current assets after ownership has changed.

Table 4: Value of zero before the change

Jurisdiction	**count**	**% of the total sum**	**% of jurisd. sum**
ON	15	2%	4%
MID	13	2%	5%
OFF	4	1%	7%
Total	**32**	**5%**	

Source: Own processing, 2018.

Table5: Indicator value > 0,5 before the change

Jurisdiction	**count**	**% of the total sum**	**% of jurisd sum**
ON	57	8%	15%
MID	57	8%	23%
OFF	11	2%	18%
spolu	**125**	**18%**	

Source: Own processing, 2018.

Significantly higher was the share of enterprises that had a particular share of land and structures before entry and showed no land after entry (Table 5), with the most significant share with companies with a parent company in the midshore jurisdiction. Also, very interesting is the fact that out of these 125 companies there were 96 companies (77%), where the share of land and structures was 100% of the total non-current assets before (i.e., the non-current assets consisted of land and structures only) and after the entry the share on the total non-current assets was zero. At this level data testing, we are not able to justify these transactions deeper than by sales, respectively moving land and structures outside of the companies. After excluding the above "extreme" cases, we re-performed Wilcoxon's nonparametric test to verify a statistically significant difference between the values of the observed indicator before and after moving to selected jurisdictions. The test confirmed a statistically significant difference to 5% of significance level. The test results are shown in Table 6.

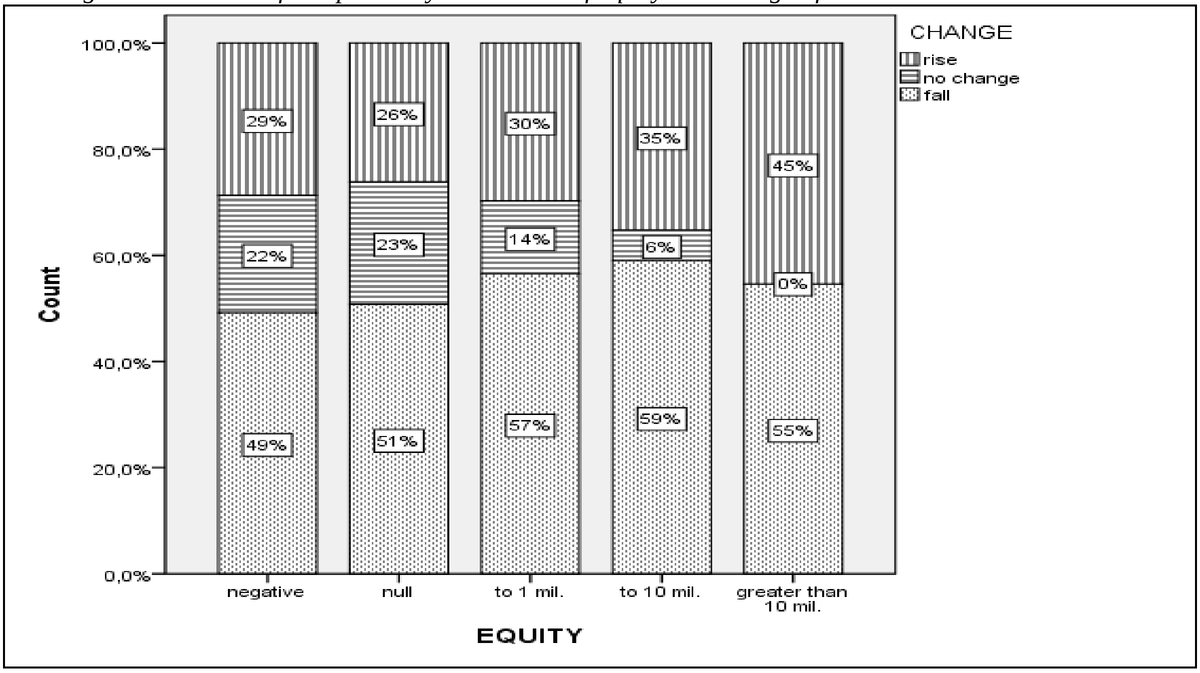
Table 6: Wilcoxon Signed Ranks Test

Test Statistics[b]		
JURISD_1		BEFORE_1 - AFTER_1
ON	Z	-3,392[a]
	Asymp. Sig. (2-tailed)	,001
MID	Z	-2,219[a]
	Asymp. Sig. (2-tailed)	,027
OFF	Z	-2,097[a]
	Asymp. Sig. (2-tailed)	,036
a. Based on negative ranks.		
b. Wilcoxon Signed Ranks Test		

Source: Own processing, 2018.

In the next analysis, we categorized the companies by the size of equity, and we investigated the behavioral activity of that indicator depending on the size of equity. The results of the sorting and graphical display of the studied dependence is shown in Figure 1.

Figure 1: Division of companies by the amount of equity and change of the indicator

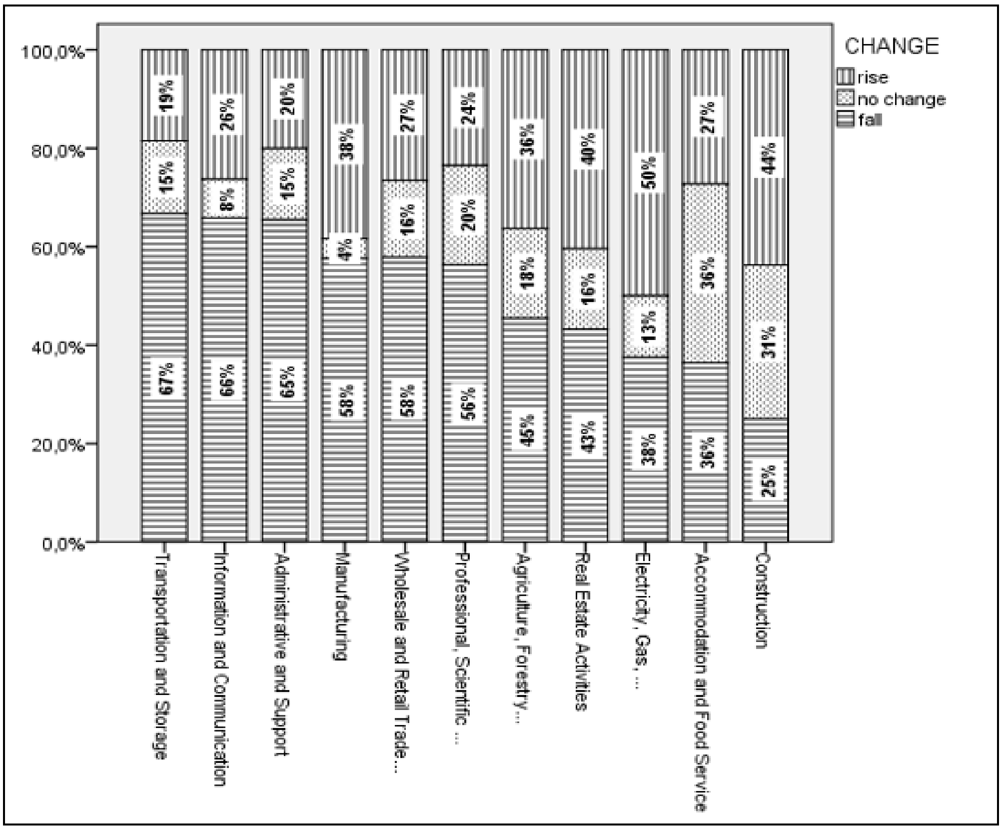


Source: Own processing, 2018

It is possible to identify from the figure that with the increasing size of a company measured by the size of the equity, the number of companies also increased experiencing the situation when the share of land and structures on the total non-current assets decreased. On the other hand, in companies with negative and zero reported value of equity, compared to other companies, there are a relatively high share and some companies when no change in the indicator was observed. In the category of companies with an equity capital above EUR 10 million, there was, in any case, either an increase or a decrease in the indicator and there was no single company where the indicator would not change.

In the next step, we analyzed the companies according to the observed indicator about the NACE sector. For better and complete clarity, we only showed those sectors where the proportion of companies in the total number of companies analyzed was more than 1%, thus reducing the number of companies by 24. We ranked businesses by NACE sector and, depending on whether the indicator fell, increased or remained unchanged before and after the change of residence. The results of the analysis are to be found in Figure 2.

Figure 2: Companies division by NACE sector and indicator change



Source: Own processing, 2018.

From the figure 2, it is possible to identify a higher share of companies in which we observed a decline of the indicator following a change of registered office in the sectors of Transportation and Storage, Information and Communication, Administrative and Support Activities, Manufacturing, Wholesale and Retail Trade. Sectors of Accommodation and Food Service Activities prove the highest share of companies when no change of indicator has been observed after the change of the registered office (it can be assumed that mainly the properties in the form of businesses were already active before the change of the registered office and there was no investment or sale of land and structures). In the Construction and Electricity, Gas, Steam and Air Condition Supply sectors, there is a higher ratio of companies that have increased their indicator after changing their registered office than those with a decline in the indicator after the change. In the Manufacturing and Real Estate Activities sectors, the frequent changes in the indicator were assumed, which was also confirmed in the analysis (frequent purchases and sales of land and structures).

1.2. Analysis of real estate properties in the Slovak Republic owned through tax havens

Based on the data published by Transparency International Slovensko (2016), 1,023 parcels[4] with a total area of 1.3 million square meters are currently owned through tax havens. There are two main opinion groups on real estate property ownership through tax havens. The first, a minority opinion group (in academic sphere) claims that real estate property ownership through tax havens has its specific benefits and is entirely rational when individuals or companies use these legal options. The main advantages are considered to be mainly the high anonymity of the real owner, the asset management, the flexible arrangement of ownership relations and, last but not least, the potential tax savings on selected business transactions. The second opinion group (e.g., Tax Justice Network, Transparency International) points out that, in particular, the anonymity of a real owner and very often suspicious transactions are conducive to crime, bribery of political leaders, and so on.

We again categorize the owners in the onshore, midshore and offshore categories. Compared to data from the Bisnode database, to onshore jurisdiction we also include Belgium, into midshore category Japan, and the offshore category is growing in countries like New Zealand, Lebanon, Botswana and South Africa. Regarding the size (area) of parcels owned in the tax haven, the most significant share of the total acreage is registered by the owners registered in Monaco (29.95%), Cyprus (20.8%), Luxembourg (8.11%), Lebanon (7.42%), Liechtenstein (4.81%), Belize (3.64%) and Singapore (3.30%).

We also analyzed these real-estate properties regarding the amount of ownership share of the parcel owned in the tax haven. According to the amount of the ownership, we divided the parcels into a category with a share of less than 1%, with a share of 1% to 25%, with a share of 25% to 75% and the last category being parcels with a 100% share. The shares sized between 50% to 75% we put into category with 25% - 75% range since this property scale only appeared three times. Ownership scaled 75% to 99.99% did not occur in either case. Using the Chi-Quadrate test, we verified the statistical dependence of the stake on selected parcels and the owner's registered office in the tax haven. The result of the statistical test is shown in Table 7.

Table 7: Result of Chi-Square test of independence

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	133,773[a]	6	,000
Likelihood Ratio	118,572	6	,000
N of Valid Cases	1023		

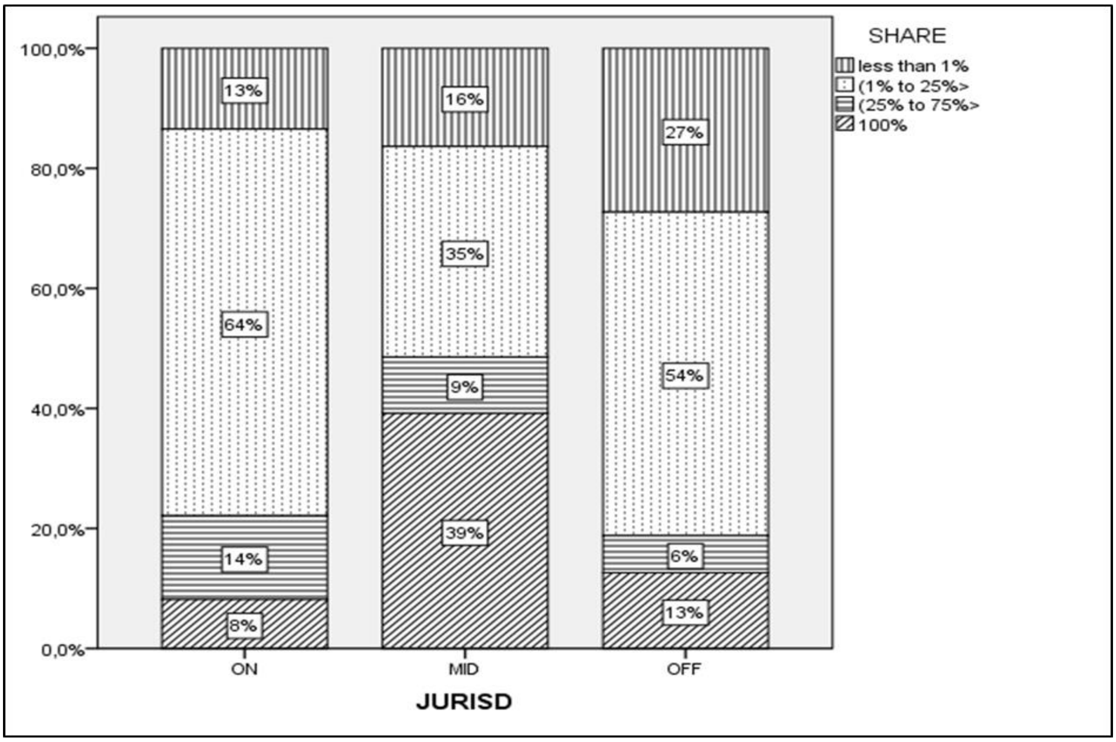
a. 0 cells (,0%) have expected count less than 5. The minimum expected count is 16,88.

Source: Own processing, 2018.

Results of the Chi-Quadrate test confirm the hypothesis of a statistically significant relationship between the size of the ownership share and jurisdiction of the owner. A more detailed view of the division of parcels by category of ownership and owner's jurisdiction is provided in Figure 3.

[4] Transparency International Slovensko includes in this number both land and structures, from this reason we take over the terminology "real estate properties". Out of the total area of 1.3 million square meters, 300 thousand square meters represents the area of structures, the rest is the area for land.

Figure 3: Percentage shares on parcels as per owner's jurisdiction



Source: Own processing, 2018.

Most parcels are owned through offshore jurisdictions (436), followed by onshore (416) and midshore jurisdictions (171). The 100% ownership of parcels is the most common with the category of midshore (39%), the ownership share of 1-25% is the most common in the case of onshore jurisdictions (64%). Ownership of 100% in offshore jurisdictions is only at the level of 13%. Ownership through offshore jurisdictions is the most common one, which leads us to the conclusion that in more than 42% of ownership structures, the primary incentive of the first line ownership was the anonymity motive of the ultimate beneficial owner. This fact can not eliminate either with categories of onshore or midshore (mainly at other levels of ownership), but in these jurisdictions it is also possible to use asset protection features, flexible ownership arrangements, tax benefits, and notable benefits can be a better outer image than offshore owners (even though considering the formal point of view of the ownership it does not matter which jurisdiction the owner belongs to).

2. Summary

This work aimed to analyze the indicator of the share of land and structures on the total non-current assets of Slovak companies before and after companies moved into jurisdictions considered as tax havens. Companies by relocating to tax havens expect a variety of benefits, notably in the field of real owner anonymity, asset management, flexible ownership arrangements, or tax optimization. Expected benefits, resp. the order of their importance is difficult to be identified by external view only. Only the owners of companies or their managers, partially lawyers or representatives of consultancy companies do know it. However, based on the available data, we can identify certain trends in corporate behavior, respectively to get selected financial flows and indicators. Our analysis shows that there is a statistically significant difference in the value of the reported indicator ((land + structures) / total non-current assets) before and after the transfer of the registered office to the tax haven. While only 31% of Slovak companies showed an increase in this share, 54% of Slovak companies showed a decrease in this indicator. Only 14.5% of Slovak enterprises show no change in the monitored indicator, where we can assume that the transfer was only due to the anonymity of the real owner and only passive ownership of the property is in progress. The statistical testing further confirmed the significant dependence between the type of jurisdiction and the amount of ownership share of the investigated parcels in the Slovak Republic. We are aware that the results of our analysis can be affected by a relatively low number of observations where we have encountered the problem of unavailability of tracked data for some enterprises in the databases used when creating the data base. For this reason, the results cannot be generalized, although certain tendencies in the behavior of the given indicator can be noticed by the analysis performed. The work brings only one of many insights into companies that have moved their headquarters to a tax haven. In further research, we would like to extend this analysis to look at the changes in the non-current financial assets of Slovak companies that have moved their registered office to the tax havens, mainly focused on equity securities and interests in a company with significant influence and other long-term securities and shares. The mid-term objective of this research is to analyze continually the impact of relocation to tax havens from different perspectives of tax optimization (e.g., tax payable and deferred tax, identification of methods used to transfer taxable profits) and financial analysis, whether ex-post or ex-ante.

Acknowledgment

The paper arose in fulfillment of the tasks associated with grant scheme VEGA 1/0767/18 SMART model - a decision support tool in the management of enterprises.

References

[1] Anken, F., Beasley, J.E. (2012). Corporate structure optimisation for multinational companies. In *Omega – The International Journal of management Science*, (40), p. 230-243. ISSN 0305-0483.

[2] Bisnode. (2018). *Daňové raje prišli o desiatky slovenských firiem*. Retrieved June 16, 2018, from https://www.bisnode.sk/o-bisnode/o-nas/novinky/danove-raje-prisli-o-desiatky-slovenskych-fririem/.

[3] Dischinger, M., Riedel, N. (2011). Corporate taxes and the location of intangible assets within multinational firms. In *Journal of Public Economics*, (95), p. 691-707. ISSN 0047-2727.

[4] Foad, H., Lundberg, C. (2017). The determinants of portfolio investment in offshore financial centers. In International Review of Financial Analysis, (54), p. 76-86. ISSN 1057-5219.

[5] Liška, L., Sabolová, L. (2017). *Holdingové struktury*. Contribution to the international conference Mezinárodní daňové plánování a správa majetku v digitálním světě, Prague, 10. 10. 2017.

[6] Mara, E., R. (2015). Determinants of tax havens. In *Procedia Economics and Finance* (32), p. 1638-1646. ISSN 2212-5671.

[7] Transparency International Slovensko. (2016). *Ktoré nehnuteľnosti vlastnia firmy z daňových rajov na Slovensku? Pozrite si zoznam.* Retrieved July, 19, 2018, from http://transparency.sk/sk/ktore-nehnutelnosti-vlastnia-firmy-z-danovych-rajov-na-slovensku-pozrite-si-zoznam/.

[8] Weizig, F., Dijk, M. (2007). *The global problem of tax havens: the case of the Netherlands*. SOMO – Centre for Research on Multinational Corporation. Retrieved June 16, 2018, from https://www.somo.nl/wp-content/uploads/2007/01/The-Case-of-the-Netherlands.pdf.

[9] Zheng, S. (2017). Can corporate diversification induce more tax avoidance? In the *Journal of Multinational Financial Management* (41), p. 47-60. ISSN 1042-444X.

Information asymmetry risk on Polish Stock Exchange on example of GetBack S.A.

Wojciech Kaczmarczyk [1]

Abstract

The paper focuses on information asymmetry risk occurring on Polish Stock Exchange basing on example of company GetBack S.A., which was described as a rising star of polish market until unexpectedly the company lost liquidity and after a few days quotes of all assets on Polish Stock Exchange were suspended (both shares and bonds). Methodologically the paper analyses the information published in official systems (EBI, EPSI), Internet portals as well as ratings and recommendations. Additionally basic statistical tools were used. Expected result of paper is valuation of market effectiveness and identification of information asymmetry risk on Polish Stock Exchange.

Key words

information efficiency, information asymmetry, market effectiveness

JEL Classification: G11, G12, G14

1. Introduction

According to Fama`s Theory of Efficient Markets (Fama, 1970) only on strong efficient market all information is fully reflected in assets prices, so there is no possibility of gaining extraordinary rate of return by using any kind of confidential or unpublic information. On effective market price includes investors preferences and risk associated with company (Shefrin, 2005 cited in Zielonka, 2008, p. 32).

In case of weak and semi-strong efficient market such information is not taken into account by most of investors and – despite of legal regulation – can be used by insiders for gaining extra profits. Of course, information asymmetry like this generates additional risk for typical investors (both individual and institutional), who have no access to confidential information.

Problem of Efficient Markets is well described in Polish literature, especially by Zielonka (2004, 2008), but there is visible lack of practical works focusing on Polish Stocks Market and the additional risk involved by unpublic information illegal use.

The case of GetBack S.A. seems to be a perfect subject for conducting research in mentioned area. Both shares and bonds of GetBack were listed on the Warsaw Stock Exchange. After initial public offering of GetBack shares in July 2017 the company had been described as a rising star until in 17th April stock exchange board decided to suspend quoting all GetBack securities (shares and 27 bond series).

Interestingly price of GetBack shares had begun to fall since October 2017, despite there were no official signals of financial liquidity problems until March 2018 when company management board briefly informed about possible plans for raising significant capital. In fact even in January and February 2018 rating agencies (Fitch, Standard&Poor`s) were raising ratings for GetBack bonds.

[1] Wojciech Kaczmarczyk, MA, University of Economics in Katowice, wojciech.kaczmarczyk@ue.katowice.pl.

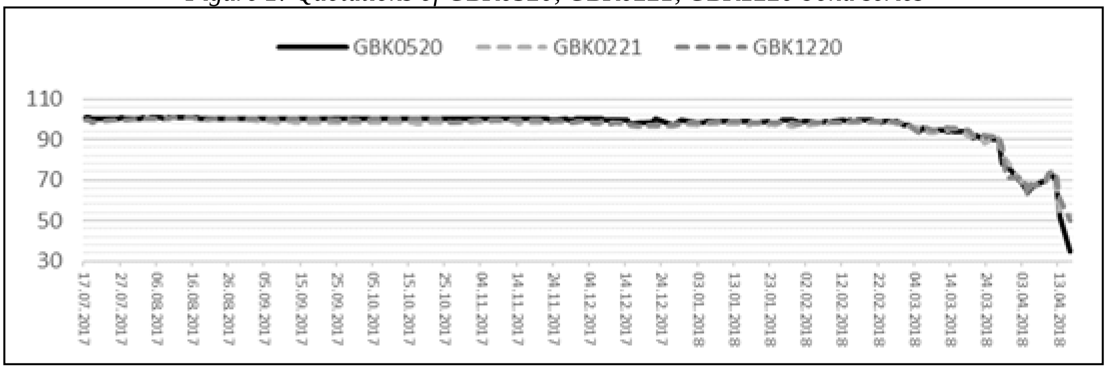
Such situation encourages for evaluation market effectiveness of Polish Stock Exchange and searching for an information asymmetry and related with it risk, which is the main aim of this paper.

2. Methodology

For conducting research it was necessary to acquire data containing quotes of all GetBack S.A. securities (data from bossa.pl were used). Despite the fact that Polish Stock Exchange was listing shares (GBK) and 27 series of GetBack bonds it was necessary to limit bonds number for further analyse. At first securities not listed on regulated market were rejected (22 of them).

On regulated market such bond series were listed: GBK0520, GBK0221, GBK1220, GBK0421, GBK0921. On the GBK0921 series there was only one transaction on 28th March, so it was unusable for analysing. The GBK0520 series had highest liquidity (transaction were made on most of trading days). Prices of series GBK0221, GBK1220, GBK0421 were behaving in similar way than in case of GBK520 (figure 1), so it was adequate to limit researches just for this one bond series.

Figure 1: Quotations of GBK0520, GBK0221, GBK1220 bond series



Source: own work.

As a second step several kind of events were gathered:

- investment recommendations for GetBack shares published by brokerage houses and banks;
- ratings for GetBack bonds published by rating agencies;
- official information published by GetBack S.A. in ESPI and EBI (official information systems for company being listed on Polish Stock);
- news and information published by media (press and investment portals).

Gathered events were rated as a positive, negative and neutral (except recommendations and ratings – there was no need for valuing them). Those which were rated as positive or negative were compared with prices (and their changes) of GetBack shares and GBK520 bond series.

Additionally impact of selected events (especially negative – there was a lot of positive news) on GetBack assets price was estimated by comparing rate of return form the event day with average rate of return and standard deviation for previous and next (including event day) 5 days (trading week). Logarithmic rate of returns were used.

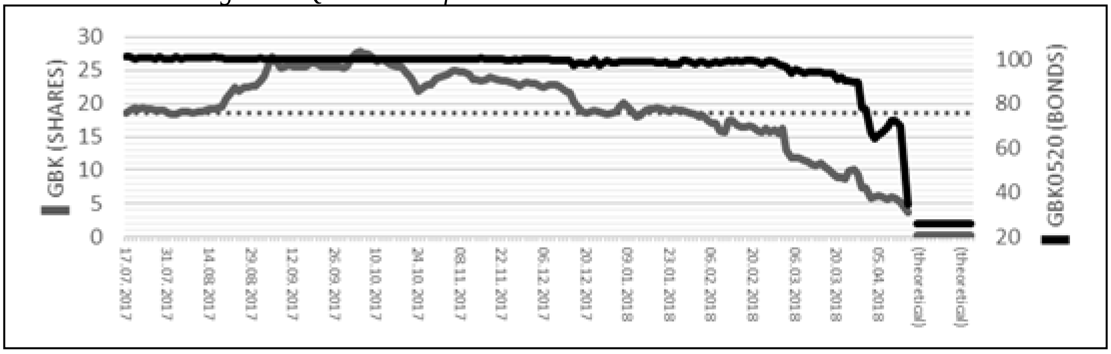
3. Results

In this section result of studies described above are presented: analysis of GetBack shares and bonds quotations, ratings, recommendations, official information, press news and their impact on GetBack assets price. Most useful result are presented in graphical and tabular form.

3.1 GetBack securities quotations

As can be seen (figure 2) value of GetBack shares had been increasing in first month of its listing on Polish Stock Exchange (17th July was the first day of share listing). Until end of January 2018 the value had been higher than the first offer price (18,50 PLN). The price started systematically to fall since October 2017 and from January 2018 was lower than offer price. However the value of GetBack bonds had been more stable and started small fluctuations in the end of December 2018. Significant value decrease was noticed scarcely at the end of February. Comparison of shares and bonds prices shows that bonds prices changes were slightly delayed.

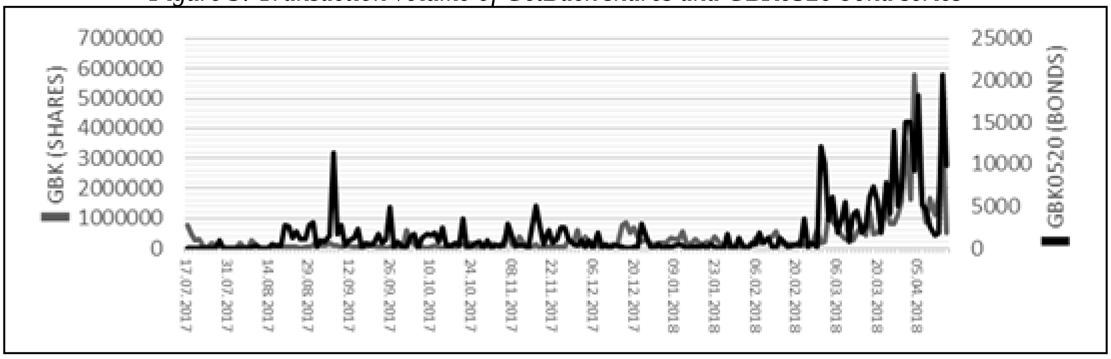
Figure 2: Quotations of GetBack shares and GBK0520 bond series



Source: own work.

Described prices movements were connected with change of transaction volume which noticed significant increase (both in case of shares and bonds) at the end of February (figure 3). Interestingly volume behaves similar in both assets, despite delayed reaction of bonds price changes.

Figure 3: Transaction volume of GetBack shares and GBK0520 bond series



Source: own work.

3.2 Investment recommendation and ratings

Investment recommendations are still significant source of information about company situation for investors, especially individuals. As table 1 shows all financial institutions were recommending to buy GetBack shares and prognosing price level close to 30 PLN per share. The first negative recommendation (sell) was published just on 23 March 2018 - 3 weeks before suspension listing of GetBack assets.

Similar situation was observed in case of rating agencies (table 2). Information provided by rating agencies were potentially useful for investors acting both on bonds and shares market. Three agencies were rating financial situations of GetBack S.A. Until the time of listing suspension agencies gave no signal of any additional risk connected with investing in Getback assets.

Table 1: Investment recommendations for GetBack shares

date	institution	type	market price	expected price
25.08.2017	Vestor Dom Maklerski	accumulate	22,5	26,6
04.09.2017	Raiffeisen	buy	24,2	34
11.09.2017	Haitong Bank	buy	25,6	30
27.09.2017	Wood & Company	buy	25,5	30
30.11.2017	Dom Maklerski mBank S.A.	buy	23	33,38
23.03.2018	Erste Bank	sell	8,9	7,6

Source: bankier.pl (n.d.).

In fact table 2 clearly shows that rating agencies did not react on time. Ratings started changing at the moment when listing were suspended and there was no way to sell both shares and bond on stock market.

Table 2: Ratings for GetBack bonds

date	rating agency	rating
20.01.2017	Eurorating	BB
17.03.2017	Standard&Poor's	B
26.01.2018	Fitch Ratings	B+
19.02.2018	Standard&Poor's	B
18.04.2018	Eurorating	B+
19.04.2018	Fitch Ratings	B
20.04.2018	Eurorating	CCC
25.04.2018	Standard&Poor's	suspended B
26.04.2018	Fitch Ratings	RD

Source: own work based on information from EBI & ESPI.

Comparison of recommendations with current GetBack situation confirms that financial institution were using similar data like typical investors. However it is easy to notice that investment recommendations were having small impact on GetBack shares values – visible price movement were observed only in case of Vestor DM recommendation.

Due to limited size of paper and similar (but delayed) behave of GetBack bonds price (GBK0520) detailed studies of events (ratings, recommendations, official information, news) impact on GetBack assets had been limited only to shares price analysis.

More details are presented in table 3. In case of first 3 recommendation there was noticeable increase of price on day when recommendations were published. However in case of Wood & Co. and DM mBank price was softly decreasing. Additionally despite the increase on first day in case of Vestor DM and Haitong Bank average rate of return was higher before recommendations were published. Ratings had rather no visible influence on shares price.

Analysis of observed standard deviations values shows that there was no typical influence of both recommendations and ratings on shares price stability. While looking at impact of positive information (despite sell recommendation of Erste Bank) – in 4 out of 7 times risk measured by standard deviation was decreasing and in 3 out of 7 times was increasing.

It is worth to notice that many funds were acting in the same way. For example Quercus TFI had invested in GetBack bonds approximately 80 million PLN. Finally all GetBack bonds owned by members of Quercus fund were rebought by Quercus fund and its CEO and owner – Sebastian Buczek who gave 40 million by himself (Stankunowicz, 2018).

Table 3: Influence of recommendations and ratings on GetBack share price

date	event	average daily rate of return			standard deviation	
		prev. 5 d	first d	next 5 d	prev 5 d	next 5 d
25.08.2017	Vestor DM recommendation	2,13%	4,26%	1,25%	3,27%	1,57%
04.09.2017	Raiffeisen recommendation	1,01%	3,11%	1,27%	1,12%	3,61%
11.09.2017	Haitong Bank recommendation	1,27%	3,54%	0,32%	3,61%	1,76%
27.09.2017	Wood & Co recommendation	0,22%	-0,20%	1,00%	1,29%	3,32%
30.11.2017	DM mBank S.A. recommendation	-0,17%	-0,70%	-1,55%	1,13%	2,57%
26.01.2018	Fitch Ratings rating	-0,04%	-1,06%	-1,08%	1,15%	0,45%
19.02.2018	Standard&Poor's rating	1,60%	-1,85%	-1,10%	3,68%	2,11%
23.03.2018	Erste Bank recommendation	-3,49%	-2,21%	-7,04%	2,41%	12,89%

Source: own work.

3.3 Information published by company and media

Seven information published by company in EPSI system seemed to be interesting for further analysis (table 4). Character of quarterly financial reports was rated as positive/neutral because GetBack had shown profits, but huge part of investors were expecting higher values. Information published in field of searching for capital has shown bad company financial condition and for this reason it was rated as negative. Information about engagement of PKO Bank and PFR was theoretically positive, but it was denied on the same day by both institution (Tychmanowicz, 2018). For transparency in further tables shorts (like I-1, N-1 etc.) are used.

Table 4: Official information published by GetBack

date	character	Information
20.09.2017	positive/ neutral	Quarterly financial report (I-1)
11.10.2017	positive	Connection of GetBack S.A. with EGB Investment S.A. (I-2)
23.10.2017	positive/ neutral	Quarterly financial report (I-3)
02.03.2018	negative	Information about the "review of strategic options", such as searching for a strategic investor, plan for further issue of shares worth 70 million PLN (I-4)
05.03.2018	negative	Information about negotiations on financing - capital up to 250 million PLN (I-5)
04.04.2018	negative	Information about loan agreement worth 50 million PLN (I-6)
16.04.2018	positive/ negative	Information about positive engagement in talks with Bank PKO BP S.A. and the Polski Fundusz Rozwoju S.A. on the subject financing GetBack by 250 million PLN (I-7)

Source: own work based on information from EBI & ESPI.

In case of media information there were hundreds of positives news about Getback S.A., therefore this paper focus only on negative information connected directly with company financial situation (first negative information was published in 19th February 2018) and news connected with bailiff affair which finally have no influence on GetBack situations but had caused fluctuations and price decrease both on shares and bonds (December 2017). Selected news are presented in table 5.

Table 5: Press news about GetBack

date	source	character	Information (title)
12.12.2017	wpolityce.pl	neutral	*Four people heard charges regarding the leak of ID numbers data from the bailiff's office* (N-1)
13.12.2017	money.pl	neutral	*They stole 350,000 data. Detained bailiffs* (N-2)
19.02.2018	parkiet.com	negative	*The market has forced to raise the margin in the new issue of GetBack* (N-3)
05.03.2018	bankier.pl	negative	*Getback getting cheaper. Money still missing?* (N-4)
26.03.2018	forsal.pl	negative	*GetBack needs capital. Debt collector performs the last chance issue (N-5)*
03.04.2018	forbes.pl	negative	*They were the second debt collector in the country. In 9 months they lost 2/3 of value (N-6)*
13.04.2018	bankier.pl	negative	*Analytical pat in GetBack (N-7)*

Source: own work based on press news (details in bibliography).

Details are presented in table 6. In case of positive and positive/neutral events small increase of average return rate was observed. News about affair connected with bailiffs resulted in visible decrease of average return rate and surprisingly decrease of risk measured by standard deviation. First definitely negative news (increase of margin, N-3) had similar influence (decrease both of return rate and standard deviation).

Such reactions were not observed in case of bonds prices which started to dramatically fall at the end of March 2018, which could be a result of offering GetBack bonds as an alternative to deposits to people not interested in Stock Exchange.

Table 6: Influence of official information and press news on GetBack share price

date	event	average daily rate of return			standard deviation	
		prev. 5 d	first d	next 5 d	prev 5 d	next 5 d
20.09.2017	I-1	-0,30%	-0,88%	0,22%	1,81%	1,29%
11.10.2017	I-2	-0,61%	-1,43%	-0,40%	2,70%	1,26%
23.10.2017	I-3	-1,87%	-2,80%	-0,65%	3,06%	5,37%
12.12.2017	N-1	-0,40%	-0,45%	-2,96%	4,39%	2,62%
13.12.2017	N-2	-0,36%	-1,27%	-3,40%	4,39%	2,33%
19.02.2018	N-3	1,60%	-1,85%	-1,10%	3,68%	2,11%
02.03.2018	I-4	0,00%	-18,10%	-6,38%	0,45%	8,50%
05.03.2018	I-5 / N-4	-3,67%	-14,85%	-3,74%	7,23%	6,19%
26.03.2018	N-5	-3,23%	16,13%	-7,56%	2,46%	12,73%
03.04.2018	N-6	-7,04%	-4,85%	-1,70%	12,89%	6,86%
04.04.2018	I-6	-7,56%	-11,05%	-0,56%	12,73%	6,72%
13.04.2018	N-7	-2,20%	-15,90%	-16,06%	4,47%	0,16%
16.04.2018	I-7	-5,03%	-16,21%	-	7,03%	-

Source: own work.

Information about the "review of strategic options" was the first official information which showed that GetBack had real problems with financial liquidity (it is worth to notice that it had been published in March – less than 2 months before listing of company assets was suspended). It had dramatical impact on GetBack shares value – on the first day their got cheaper by more than 18% and on next days – averagely by more than 6%. There was also visible increase on risk (from 0,45% to 8,50%).

On the same day (5th March) GetBack published information about negotiating on financing (eq. 250 million PLN) and press became interested in company situation, so it is difficult to distinguish impact of described events (they were evaluated together). Additionally news were published just after information about "review of strategic options", which surely still had big impact on shares value. On first day there was visible price decrease (equal to 15%), but it is difficult to measure impact on average return and risk (effects of previous information).

Information about next issue (N-5) was rated as negative but on first day caused increase of value by more than 16%, but averagely there was higher decrease on next days (-7,56 versus -3,23) and also visible increase on risk (from 2,46% to 12,73%). News about bad situation (N-6) had no visible effect on price (probably it was too late or maybe caused interest of buying GetBack assets in cheap price). At 4th April company had informed about loan agreement (50 million PLN), which caused visible shares sale on first day, but had no visible effects on next 5 days.

At the end price of GetBack shares was hugely falling probably as a result of analytics news (especially N-7) and finally false information about getting capital from PKO Bank and PFR. Measuring risk on last days of listing is rather pointless – every day there was a constant drop of value.

4. Summary

Conducted studies has revealed following observations. Price of GetBack shares had started loosing value in October 2017 and in January 2018 price was lower than offer price. Bonds value started fluctuating in December 2017, but its price was falling with significant delay.

First definitely negative information about GetBack situation was published by press in February 2018 and by company in March 2018. Until that time company and press were announcing only positive news (omitting bailiffs affair, which finally had no impact on GetBack).

Financial institution and rating agencies were recommending to buy GetBack assets until problems were clearly visible (first negative recommendation at the end of March 2018) or even listing of GetBack assets was suspended (all rating agencies).

In case of shares there were small or even none reactions while recommendations or ratings were published, but investors had visibly reacted on most events rated as negative, especially official communicates published by company (falls higher than 15% on first day). Reactions in case of bonds were visibly delayed.

Described observations directly leads to conclusion, that on Polish Stocks Exchange (in case of GetBack) there was two type on investors (owners): 1) those who had access only to official information and 2) those who had access both to official and unpublic or confidential information. Additionally it had been shown that financial institutions belonged to first type.

Finally, due to conclusion presented above, Polish Stock Market is not strongly efficient according to Fama definition (it is possible to use unpublic information) and investors acting on it has to consider additional risk of information asymmetry (which is hard to measure).

References

[1] Bankier.pl (n.d.), GETBACK S.A. (GETBACK) – notowania GPW. [online] Available at: <https://www.bankier.pl/gielda/notowania/akcje/GETBACK/komunikaty> [Accessed 20 July 2018].

[2] Ciszak P. (Money.pl, 2017), Wykradli dane 350 tys. osób. Zatrzymani komornicy sądowi. [online] Available at: <https://www.money.pl/gospodarka/wiadomosci/artykul/pesel-dane-komornik-wyciek,237,0,2393581.html> [Accessed 20 July 2018].

[3] Fama E. (1970). Efficient Capital Markets: A Review of Theory and Empirical Evidence. Journal of Finance, 25, p. 383-416.

[4] Hajdamowicz A. (Bankier.pl, 2018), Getback mocno tanieje. Pieniędzy wciąż brakuje? [online] Available at: <https://www.bankier.pl/wiadomosc/Getback-mocno-tanieje-Pieniedzy-wciaz-brakuje-7575796.html> [Accessed 20 July 2018].

[5] Jóźwik T. (Forsal.pl, 2018), GetBack potrzebuje kapitału. Windykator przeprowadza emisję ostatniej szansy. [online] Available at: <http://forsal.pl/artykuly/1113470> [Accessed 20 July 2018].

[6] Karnaszewski P. (Forbes, 2018), Byli drugim windykatorem w kraju. W 9 miesięcy stracili 2/3 wartości. [online] Available at: <https://www.forbes.pl/biznes/konrad-kakolewski-wzlot-i-upadek-getback/zr2tbjd> [Accessed 10 August 2018].

[7] Stankunowicz E. (Forbes, 2018), Wydałem 40 milionów, by móc patrzeć w lustro. [online] Available at: <https://www.forbes.pl/biznes/wywiad-z-sebastianem-buczkiem-prezesem-quercus-tfi-o-getbacku-i-prezesach-bankow/3njtvcc> [Accessed 10 August 2018].

[8] Szweda E. (Parkiet, 2018), Rynek wymusił podniesienie marży w nowej emisji GetBacku. [online] Available at: <http://www.parkiet.com/artykul/1528141.html&template=restricted> [Accessed 20 July 2018].

[9] Torchała A. (Bankier.pl, 2018), Analityczny pat w GetBacku. [online] Available at: <https://www.bankier.pl/wiadomosc/Analityczny-pat-w-GetBacku-7583174.html> [Accessed 10 August 2018].

[10] Tychmanowicz P. (Parkiet, 2018), GetBack: prowadzimy negocjacje w PFR i PKO BP w sprawie finansowania. PFR i PKO BP zaprzeczają. [online] Available at: <http://www.parkiet.com/Wierzytelnosci/304169975-GetBack-prowadzimy-negocjacje-w-PFR-i-PKO-BP-w-sprawie-finansowania-PFR-i-PKO-BP-zaprzeczaja.html> [Accessed 16 August 2018].

[11] wSumie.pl (wPolityce.pl, 2017), Cztery osoby usłyszały zarzuty w związku z wyciekiem danych PESEL z kancelarii komorniczej. Wśród nich jest komornik - został aresztowany. [online] Available at: <https://wpolityce.pl/kryminal/371598-cztery-osoby-uslyszaly-zarzuty-w-zwiazku-z-wyciekiem-danych-pesel-z-kancelarii-komorniczej-wsrod-nich-jest-komornik-zostal-aresztowany> [Accessed 20 July 2018].

[12] Zielonka P. (2004). Behawioralne finanse. In: T. Tyszka, ed. 2004. Psychologia ekonomiczna, Gdańsk: Gdańskie Wydawnictwo Psychologiczne.

[13] Zielonka P. (2008). Behawioralne aspekty inwestowania na rynku papierów wartościowych, 2nd ed. Warsaw: CeDeWu.

Validation of Technical Provisions in Solvency II

Natália Knošková, Jozef Fecenko [1]

Abstract

The article refers about the validation of technical provisions according to the requirements of Solvency II. It answers the questions how to get comfort with adequacy of technical provisions, who is responsible for caring out the validation, when it should be performed, which areas should be covered. Requirements regarding technical provisions are defined by the Directive of European Parliament and of the Council 2009/138/ES in Articles 76 - 85 and in Commission Delegated Regulation 2015/35 supplementing Directive in Articles 22 - 61. According to the requirements of Delegated Regulation should the system of governance of insurance and reinsurance undertakings include a validation process of the calculation of technical provisions with the aim to ensure that the valuation of technical provisions is carried out in compliance with Articles 76 - 85 of the Directive 2009/138/ES.

Key words

Solvency II, Validation of technical provision, Data Quality, Assumptions, Best estimate, Risk margin, Comparison against experience.

JEL Classification: G23, G28

1 Legislative background and meaning of the validation

Requirements regarding technical provisions are defined by the Directive of European Parliament and of the Council 2009/138/ES in Articles 76 - 85 and in Commission Delegated Regulation 2015/35 supplementing Directive in Articles 22 - 61. According to the requirements of Delegated Regulation should the system of governance of insurance and reinsurance undertakings include a validation process of the calculation of technical provisions with the aim to ensure that the valuation of technical provisions is carried out in compliance with Articles 76 - 85 of the Directive 2009/138/ES. The minimum requirements for validation of technical provisions are regulated by the Article 264 of Commission Delegated Regulation 2015/35.

According to the English Oxford dictionary the definition of the verb „validate“ is to „check or prove the validity or accuracy of“, „demonstrate or support the truth or value of“, „make or declare legally valid.“ The origin of the word „validate“ comes from medieval latin – validat – „ made legally valid“. The adjective „valid“ is explained by Oxford dictionary as „(of an argument or point) having a sound basis in logic or fact; reasonable or cogent“, „legally binding due to having been executed in compliance with the law“, „legally or officially acceptable.“ The origin of the word „valid“ comes from french „valide“ or latin „validus“ – „strong“, respectively „valere“– „to be strong“.

So the meaning and reason for validation is (reasonably, based on facts) to control and prove that the calculation of technical provisions was performed in compliance with legislation; make it „strong“ for the national competent authority.

[1] Natália Knošková, Ing. PhD., Slovak Society of Actuaries, natalia.knoskova@gmail.com
Jozef Fecenko, doc. RNDr., CSc., University of Economics in Bratislava, jozef.fecenko@euba.sk

2 Specific requirements of Commission Delegated Regulation

The Article 264 of Commission Delegated Regulation 2015/35 explains the requirement of technical provisions validation requested in its prolog in more details. According to it the insurance and reinsurance undertakings shall validate the calculation of technical provisions, in particular by comparison against experience, at least once a year and where there are indications that the data, assumptions or methods used in the calculation or the level of the technical provisions are no longer appropriate.

The validation shall cover the following:

a) the appropriateness, completeness and accuracy of data used in the calculation of technical provisions according to the requirements of SII regulation (defined in Article 19 of Commission Delegated Regulation 2015/35);

b) the appropriateness of any grouping of policies in accordance with SII regulation (defined in Article 34 of Commission Delegated Regulation 2015/35);

c) the remedies to limitations of the data referred to in SII regulation (defined in Article 20 of Commission Delegated Regulation 2015/35);

d) the appropriateness of approximations referred to in SII regulation (defined in Article 21 of Commission Delegated Regulation 2015/35) for the purposes of calculating the best estimate;

e) the adequacy and realism of assumptions used in the calculation of technical provisions for the purposes of meeting the requirements in SII regulation (defined in Articles 22 to 26 of Commission Delegated Regulation 2015/35);

f) the adequacy, applicability and relevance of the actuarial and statistical methods applied in the calculation of technical provisions;

g) the appropriateness of the level of the technical provisions as referred to in Article 84 of Directive 2009/138/EC necessary to comply with Article 76 of that Directive.

For the purposes of the validation the appropriateness of approximations, insurance and reinsurance undertakings shall assess the impact of changes in the assumptions on future management actions on the valuation of the technical provisions. Where changes in an assumption on future management action have a significant impact on the technical provisions, insurance and reinsurance undertakings shall be able to explain the reasons for this impact and how the impact is taken into account in their decision making process.

The validation shall be carried out separately for homogeneous risk groups. It shall be carried out separately for the best estimate, the risk margin and technical provisions calculated according to the market value of financial instruments which reliably replicate future cash flows. It shall be carried out separately for technical provisions where the matching adjustment is applied. In relation to the best estimate, it shall be carried out separately for the gross best estimate and amounts recoverable from reinsurance contracts and special purpose vehicles. In relation to non-life insurance obligations, it shall be carried out separately for premium provisions and provisions for claims outstanding.

3 Process and policy for technical provisions validation

The insurance or reinsurance undertakings have to issue a policy for the validation of technical provisions. The policy has to include the definitions of process, rules, responsibilities and frequency of the validation.

The validation of technical provisions is the role and responsibility of actuarial function. The process of validation helps the actuarial function to get the comfort that the calculation of technical provisions is performed in compliance with regulation and actuarial standards. The comfort can be obtained by considering appropriateness and adequacy of various inputs, outputs and applied actuarial and statistical methods and models.

4 Data quality

The first requirement of validation process is the verification of the quality of used data. Data are considered to be of quality if they are complete, accurate and appropriate.

Data used in the calculation of the technical provisions are considered by SII Delegated regulation to be complete in case they meet following criteria:

a) the data include sufficient historical information to assess the characteristics of the underlying risks and to identify trends in the risks;

b) the data are available for each of the relevant homogeneous risk groups used in the calculation of the technical provisions and no relevant data is excluded from being used in the calculation of the technical provisions without justification.

Data used in the calculation of the technical provisions are considered by SII Delegated regulation to be accurate in case they meet following criteria:

a) the data are free from material errors;

b) data from different time periods used for the same estimation are consistent;

c) the data are recorded in a timely manner and consistently over time.

Data used in the calculation of the technical provisions are considered by SII Delegated regulation to be appropriate in case they meet following criteria:

a) the data are consistent with the purposes for which they will be used;

b) the amount and nature of the data ensure that the estimations made in the calculation of the technical provisions on the basis of the data do not include a material estimation error;

c) the data are consistent with the assumptions underlying the actuarial and statistical techniques that are applied to them in the calculation of the technical provisions;

d) the data appropriately reflect the risks to which the insurance or reinsurance undertaking is exposed with regard to its insurance and reinsurance obligations;

e) the data were collected, processed and applied in a transparent and structured manner, based on a documented process that comprises all of the following:

- the definition of criteria for the quality of data and an assessment of the quality of data, including specific qualitative and quantitative standards for different data sets;
- the use of and setting of assumptions made in the collection, processing and application of data;
- the process for carrying out data updates, including the frequency of updates and the circumstances that trigger additional updates;

f) insurance or reinsurance undertakings shall ensure that their data are used consistently over time in the calculation of the technical provisions.

Insurance or reinsurance undertakings should adopt a Data Quality Policy and proceed accordingly. Part of the Data Quality Policy should be a documentation, that covers complete (end to end) data cycle, clearly defined quality criteria, regular data quality assessment and definition of clear data ownership.

This can be reached by following steps:

- creation of Data Directory; data directory includes all data that are used for calculation of technical provisions, specifies its sources (meta data file), characteristics and application in calculation;

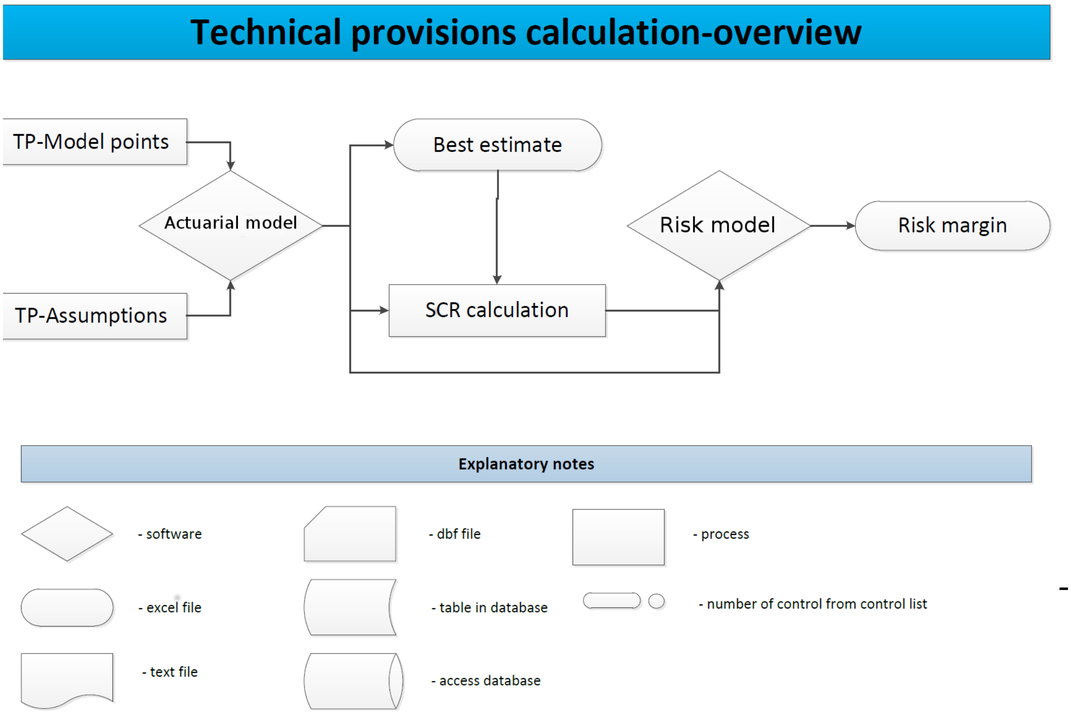
Example of Data Directory is in Table 1.

Table 1: Example of Data Directory

No.	Data Item	Explanation	Format	Possible values	Source	Use in calculation
1	premium	price for the insurance coverage paid by policyholder	number	Min. 20 eur	policy report	for modeling of cash in
2	sum assured	maximum pay out in case of claim	number	Min. 10 000 eur	policy report	for modeling of expected claims
3	age	entry age of insured	number	Min 18 years, Max 60 years	clients report	for modeling of expected claims and decrements
4	duration	duration of the contract	number	Min 5 years, Max 42 yeas	policy report	defines the length of projection

- creation of Data Flow Diagrams; data flow diagrams help to understand the main data points, processes and their interaction. Example of Data Flow Diagram is in Figure 1.

Figure 1: Example of Data Flow Diagram



performing regular Data Quality Assessments; data quality assessments ensure that internal and external data are complete, accurate and appropriate.

Example of Data Quality Assessments is in Table 2.

Table 2: Example of Data Quality Assessment

Data item	age	age	age
Mapping	product catalogue	client report	client report
Explanation	entry age of insured	entry age of insured	entry age of insured
Definition of the work	Age is correct based on definition in product catalogue	Transfer from core system to Model point file for actuarial software	- verification with Birth Number and core system Policy issue date
# COUNT	100 000	100 000	100 000
# NULL	0	0	0
# ZERO	0	0	0
# ERR	2	0	6
% of ERR	0,00%	0,00%	0,01%
Description	1000000001 Life Protect 61y 1000000002 Life Protect 65 y		1000000003 Life Protect 1000000004 Life Protect 1000000005 Life Protect 1000000006 Life Protect 1000000007 Life Protect 1000000008 Life Protect
Result	biznis error – check with product and IT department	no errors	data error check with IT
Query	BUSINESS_RULES_PRODUCT_KATALOGUEATACLOG_ERRORS	BUSINESS_RULES_PRODUCT_NOT_PAIRED_CONTRACTS_CATALOGUE_ERRORS	BUSINESS_RULES_AGE_BIRTH_NMB_ POLICY_ISSUE_ERRORS

- exercise the Data Deficiency Management; deficiencies in data have to be documented, evaluated, prioritized and corrected.

There are several tools how to get comfort with the appropriateness of data. The first choice is the evaluation of assumptions by backtesting. In this approach the originally modelled values are compared with real experience. The another tool is sensitivity testing, it helps the actuarial function to understand the sensitivity of results to assumptions. The comparison of modelled assumptions with available benchmark also improves the consideration of the appropriateness of assumptions. The assumptions have to be used in compliance with the purpose of the model. This compliance with the purpose of the model has to be controlled as well. Control of recent update of the data has to be performed. In case the data is comprised, then the results from policy by policy run have to be compared with the results of grouped data. Any expert judgments that are used in data processing have to be documented and kept up to date.

To consider the completeness of data following tools can be used:

- reconciliation of total number of records with the source system;
- reconciliation of total value of records with the source system;
- reconciliation of the total change with the source system (e.g. the difference between opening and closing values)
- control of data transfer logs;
- evaluation of missing fields.

To ensure the accuracy of data following tolls are applied:

- spot checks on randomly chosen data;
- evaluation of the reports about the exceptions from data validation (e.g. any data outside of predefined value ranges);
- analysis of data trends/ratios

Actuarial function gets comfort with data quality by a system of controls that are performed on regular basis. Data quality does not relate only to model points (database snap shot of existing portfolio), but also to data, that are input for assumptions derivation. The qualitative criteria for the controls of key data have to be exactly defined (e.g. limits for entry ages, sums assured), controls against accounting and controlling figures, etc. Controls have to be performed on source data, as well on the final data point file entering the actuarial software for technical provisions calculation. The complete data flow from source to input files has to be documented.

As a result, and documentation of performed tests and controls it is recommended to create a data quality dashboard – by control of success percentage. It should be predefined what level of control success percentage is acceptable. It can be e.g. derived from the risk appetite of the insurance company.

5 Appropriateness of grouping

In next step of validation process the appropriateness of grouping should be assessed.

Where a calculation method is based on grouped policy data, insurance company has to ensure that the grouping of policies creates homogeneous risk groups that appropriately reflect the risks of the individual policies included in those groups.

The cash flow projections used in the calculation of best estimates for life insurance obligations shall be made separately for each policy. Where the separate calculation for each policy would be an undue burden on the insurance company, it may carry out the projection by grouping policies, provided that the grouping complies with all of the following requirements:

a) there are no significant differences in the nature and complexity of the risks underlying the policies that belong to the same group;

b) the grouping of policies does not misrepresent the risk underlying the policies and does not misstate their expenses;

c) the grouping of policies is likely to give approximately the same results for the best estimate calculation as a calculation on a per policy basis, in particular in relation to financial guarantees and contractual options included in the policies.

6 The remedies to limitations of the data

Where data does not comply with Data Quality requirements, insurance companies shall document appropriately the limitations of the data including a description of whether and how such limitations will be remedied and who will be responsible for that process.

The data, before adjustments to remedy limitations are made to it, shall be recorded and stored appropriately.

7 Appropriate use of approximations to calculate the best estimate

Where insurance and reinsurance undertakings have insufficient data of appropriate quality to apply a reliable actuarial method, they may use appropriate approximations to calculate the best estimate provided that all of the following requirements are met:

a) the insufficiency of data is not due to inadequate internal processes and procedures of collecting, storing or validating data used for the valuation of technical provisions;

b) the insufficiency of data cannot be remedied by the use of external data;

c) it would not be practicable for the undertaking to adjust the data to remedy the insufficiency.

External data can be used if they are more proper to use than internal data. The source of external data has to be known, as well as the assumptions and methods that were used to process the data. It is necessary to be able to identify the trends relating the external data and their changes in time. External data have to reflect the characteristics of own portfolio of insurance liabilities.

8 Adequacy and realism of assumptions

There are several tools how to get comfort for actuarial function with adequacy and realism of assumptions. First of all, there are experience studies – comparison of modelled variables against experience.

All assumptions should be documented in the assumptions catalogue, including their classification of materiality.

Experience studies should be performed on regular basis within assumptions review cycles, as stated in the catalogue.

The assumptions catalogue should include documentation of applied expert judgments and future management actions with their substantiation, estimated impact on technical provisions and trigger for their revaluation.

The date and result of assumptions review process should be documented, with substantial considerations for next review. The date of next review should be set at the end of last review.

For consideration of assumptions adequacy also the adequacy of assumptions grouping should be assessed. For that the triggers of assumptions clustering should be understood; for example, the actuarial function should understand what are the differentiating factors for lapse rates, it can be a level of guarantee included in the products or type of distributor etc. The number of assumptions groups should be set by considering the granularity against statistical significance of data.

When validating the assumptions, it is necessary for the actuarial function to understand the internal processes, their recent or planned changes, the future management actions, trends and volatility of risks.

The assumptions should be evaluated and used in compliance with the aim and techniques applied in actuarial models.

The aim of the experience studies is to compare the best estimate assumptions against experience, that may lead to update of best estimate. The process, methods and results of experience studies should be documented. If there are significant deviations between assumptions and experience these have to be explained. It should be understood if the deviations are caused by internal or external factors; or if they arise from volatility, the data or methods used. Actuarial function is responsible for the review and should give opinion on the adequacy of the process of assumptions review and suitability of experience studies.

The aim of the sensitivity analysis is to gain the indication of the sensitivity of technical provisions to change in assumptions. The detail of assumption monitoring should be set based on the results of sensitivity analysis.

9 Models and methodology

For the actuarial function it is important to get comfort with applied methodology and models. To achieve that, in the first step the valuation models and applied methodologies need to be clearly documented. The consistency of applied methodology with Directive, Delegated Act and Guidelines should be evaluated.

Actuarial function should review reports from model validations including details about key model limitations and applied expert judgments. Actuarial function should consider as well the method of homogeneous risk groups construction. Actuarial function shall also evaluate the operational controls of management and use of the model including controls of model changes during reporting period. In case auditor reports are available actuarial function can make use of them as well.

In case any nonstandard method was applied in calculation of technical provisions, actuarial function should point it out and describe and justify the selection of such method. It is important to point out any significant deviation of standards of actuarial praxis or use of simplifications.

Actuarial function should consider the appropriateness of model change process and reasons for all material model and methodology changes in last reporting period.

10 Risk margin

For the validation of technical provisions, the actuarial function has to consider the adequacy and reliability of risk margin as part of the technical provisions. The basis for risk margin calculation is the non-hedgable solvency capital requirement. Therefore, the actuarial function has to consider the calculation and future projections of non-hedgable solvency capital requirement. For the projections of future solvency capital requirements, the risk drivers are mostly used in praxis. For the validation of technical provisions, it is necessary to consider the appropriateness of used risk drivers. Risk drivers can be evaluated for example by full (or partial) calculation of (most significant) future solvency capital requirements.

11 Adequacy and reliability of technical provisions

Actuarial function can consider the technical provisions to be adequate and reliable in case it can be comfortable with the quality of data, appropriateness and realism of assumptions and appropriateness and applicability of actuarial and statistical methods and valuation models.

References

[1] Directive of European Parliament and of the Council 2009/138/ES

[2] Commission Delegated Regulation (EU) 2015/35 supplementing Directive 2009/138/EC of the European Parliament and of the Council.

[3] Páleš, M. (2016). *Aktuárstvo v režime Solventnosť II: s riešenými príkladmi v jazyku R.* Bratislava, Vydavateľstvo EKONÓM.

The contribution was supported by the grant VEGA, No. 1/0120/18, “Modern risk management tools in internal models of insurance companies in the context of the Solvency II Directive”

Technical indicators as a quantitative method of forecasting in the business economy

Andrea Kolková [1]

Abstract

The contribution defines the groups of quantitative forecasting methods used in the business economy, in particular their categorization and calculation procedure. Alternatively, the technical indicators that are commonly used to eliminate the risk of trades and forecasting on the financial markets, are used to predict. These methods are applied to selected examples of retail sales prediction. The results are evaluated and compared based on accuracy (RMSE, MAPE, MAE).

Key words

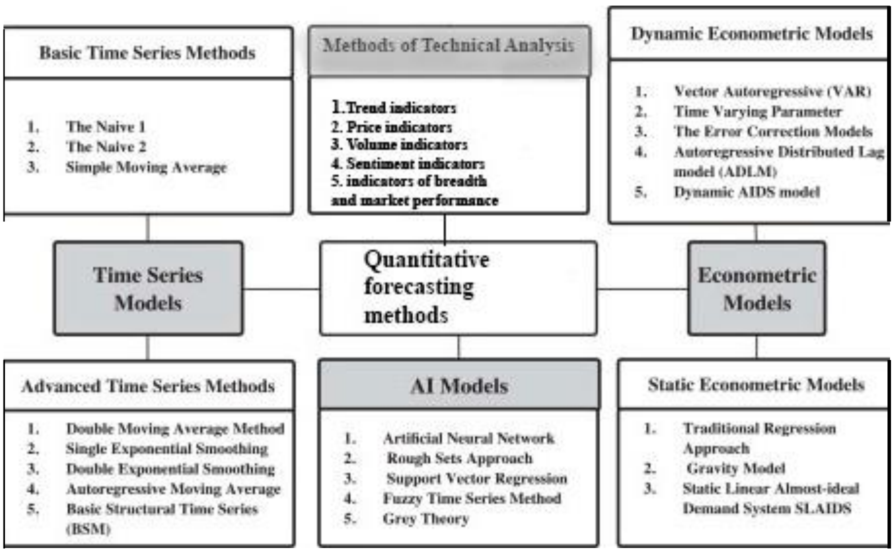
Forecasting, Technical indicators, Exponential smoothing models, Artificial neural network

JEL Classification: C53, G17, M21

1. Forecasting methods in business economy

Current trends in business and corporate management require correct and swift decision-making. The relevant decision is to be taken after a thorough quantitative analysis of not only the economic management of the current moment, but also the prediction of corporate values. Forecasting can be defined by Vincúr and Zajac (2007, p. 12) as a scientific discipline, whose object is to study the technical, scientific, economic and social factors and processes that affect the development of the objective reality of the world and which aims to create ideas - forecasts by its future state resulting from the interconnected effects of these factors and processes.

Figure 1 Categories of forecasting methods with technical indicator methods. Source: (Wang at all, 2018), own



[1] Ing. Andrea Kolková, Ph.D., Department of Business Administration, Faculty of Economics, VŠB – TU Ostrava, andrea.kolkova@vsb.cz.

The forecasting method is a whole lot. This article focuses only on quantitative methods of forecasting. The breakdown of quantitative methods in this article is shown in figure 1. The aim of the paper is to apply quantitative forecasting methods to predict sales in retail, with alternative use of indicators of technical analysis. The application will use methods from the basic and advanced time series methods, Artificial intelligence methods and already declared methods based on technical indicators.

1.1 Time series models

For the application, time series models were selected ETS model, Brown model, Holt-Winters model, ARIMA, BATS. ETS model (Exponential smoothing state space model) is defined in the article by Hyndman at all (2008). The relationship of the extended equation has a shape,

$$S_t = \alpha \sum_{i=0}^{t-1} (1-\alpha)^i y_{t-1} + (1-\alpha)^t S_0, \text{ where} \quad (1)$$

T is the length of the time series, y_{t-1} is the value of the time series, $\partial \in (0,1)$ is the equalization constant and S_0 je is the initial balancing value.

Multiple exponential smoothing by Brown (hereinafter Brown) defines a polynomial trend forecasts using multiple exponential averages to get another exponential settlement has gained exponential averages. Holt extended Brown's exponential smoothing adaptive estimation of the trend component of the new buffer constant β (Vincůr, 2007) according to,

$$\beta = \beta(S_t - S_{t-1}) + (1-\beta)\widehat{\beta_{1,t-1}}, \text{ where} \quad (2)$$

$S_t - S_{t-1} = \widehat{\beta_{0,t}} - \widehat{\beta_{0,t-1}}$ is the current state of the trend and $\widehat{\beta_{1,t-1}}$ is an adaptive estimate of the Trend Directive over time. β then the equalizing constant.

Damped trend methods were created as a response to the disadvantages Holt linear methods which exhibit a constant trend, however, empirical evidence suggests that this may lead to excessive prognosis, especially in the longer forecast horizon. Methods of damped trends then contain a parameter that dampens the trend on a straight line (Hyndman, 2018).

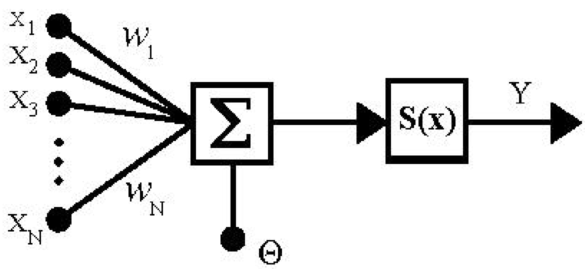
The BATS model is a generalization of traditional seasonal models, allowing more seasonal seasons (De Livera at all, 2011). Named the BATS model is actually an acronym for the Box-Cox model transformation, ARMA model, trend, and seasonal components. This must be accompanied by appropriate arguments ($\varpi, \phi, p, q, m_1, m_2 \ldots\ldots m_T$) for defining Cox - Box parameters and damping parameters p and q to express ARMA model parameters and seasonal periodicity is expressed by arguments $m_1, m_2 \ldots\ldots m_T$.

ARIMA (auto regressive integrated moving average) are based on the assumption that values of quantities can be compared to a random process. In a simplified way, it is possible to search for an internal structure of data and dependencies that generate data with acceptable accuracy (Marček, 2013).

1.2 Methods based on artificial neural networks

Artificial neural networks can be described as imperfect models of human brain thinking. From the biological interpretation of neuron function, a variant of mathematical neuron interpretation was compiled. Based on this interpretation, very simple models of artificial neural networks, linearly separated, first published by McCulloch (1943), emerged in the first half of the 20th century. Further development of the methods occurred after the emergence of multilayered networks. Model of artificial neuron shows the figure 2, where x_i are neural inputs, w_i are synaptic weights, Θ is the border, $S(x)$ is the transfer function of the neuron and Y is the output of the neuron.

Figure 2 Model of artificial neuron. Source: Jeanlagi – Draw by own force, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=3910972



Today, the theories of artificial neural networks are linked to computer artificial intelligence. Typical for them is that there is no algorithm of calculation, instead there is so-called neural network learning, where one can say that the neural network creates the algorithm itself.

1.3 Methods on base technical indicators

The basis of methods based on technical indicators is always mathematical statistics (Kolková, 2016). Sliding averages calculate the average value of the data in the width of its timeframe. After joining the rolling average of all days, we create a rolling average curve. Moving averages are now numerous. The base consists of Simple Moving Average (hereinafter SMA) and can be defined relationship,

$$SMA = \frac{\sum_1^N input}{n}, \text{ where} \quad (3)$$

N is the number of days for which the SMA is numbered. Exponential Moving Average (EMA hereinafter) is considered to be a better tool than the simple moving average because it gives more weight to current data and corresponds to the price changes more quickly than simple. It can be expressed by a relationship,

$$EMA = EMA_{-1} + K \cdot (input - EMA_{-1}), \text{ or} \quad (4)$$

$$EMA = K \cdot input + (1 - K) \cdot EMA_{-1}, \text{ where} \quad (5)$$

$$K = \frac{2}{N+1}. \quad (6)$$

2. Analysis of used data

The paper used the data of the Czech Statistical Office's data and development of retail sales by CZ-NACE. The calculations were used at constant prices to eliminate the impact of inflation and to ensure fair value. The data to be analyzed were from the time series of monthly values from January 2000 to April 2018. The data were analyzed in Table 1.

Table 1 Statistical description, source: own

	N	Minimum	Maximum	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
SC_45	220	30,00	127,50	72,9491	1,51588	22,48412	505,536
SC_47_1	220	57,50	122,80	92,6032	0,86576	12,84132	164,900
SC_47_2	220	87,90	158,00	114,1336	0,94624	14,03495	196,980
SC_47_3	220	67,00	120,40	95,8427	0,73900	10,96121	120,148
SC_47_4	220	7,10	213,30	56,3473	2,51981	37,37480	1396,876
SC_47_5	220	46,90	132,90	87,6891	1,32828	19,70152	388,150
SC_47_6	220	28,20	193,40	75,6541	1,84712	27,39723	750,608
SC_47_7	220	42,30	139,60	86,8227	1,26816	18,80984	353,810
SC_47_8	220	64,40	188,50	121,3591	1,59088	23,59654	556,797
SC_47_9	220	13,30	221,80	59,3455	2,58325	38,31575	1468,097

The individual data were described according to the CZ-NACE breakdown according to the logic given in Table 2.

Table 2 Definition data, source: own

CZ-NACE	
Trade, repair and maintenance of motor vehicles	45
Retail sale in non-specialized stores	47.1
Retail sale of food in specialized stores	47.2
Retail sale of automotive fuel	47.3
Retail sale of computer and communication equipment in specialized stores	47.4
Retail sale of household goods in specialized stores	47.5
Retail of goods for culture and recreation in specialized stores	47.6
Retail sale of other goods in specialized stores	47.7
Retail in stalls and markets	47.8
Retail trade services except stores, stalls and markets	47.9

The forecast was realized using the statistical program R using the TTR packages (Ulrich, 2017) and Forecast (Hyndman, 2008).

3. Results

To evaluate the suitability of particular methods of prognosis is first made ex post, which then were subsequently quantified measure of accuracy. The most accurate models were then included in the ex-ante prediction models. To analyze models were selected on the basis of the basic time series models, advanced time-series models based on technical analysis indicators and model from models of artificial intelligence.

3.1 Forecasting ex post

First, predictions were made using basic time series models, the ETS model, and advanced time series models such as the Holt's Winters model, the Brown model, the Damped model, the ARIMA models and the BATS models. The most accurate precision models were selected in the prediction. Models based on technical analysis indicators have been optimized and Table 3.1 lists the models with the highest degree of accuracy. The last of the analyzed models was the model from the group of artificial intelligence methods, namely the prognosis of time series using neural networks.

These models were applied to the ex post prediction of all selected retail sales. The results are shown in Table 3. It is clear that most models are best described using neural network models that show significantly better predictive accuracy than other models.

Table 3 Accuracy of ex-post forecasting methods, source: own

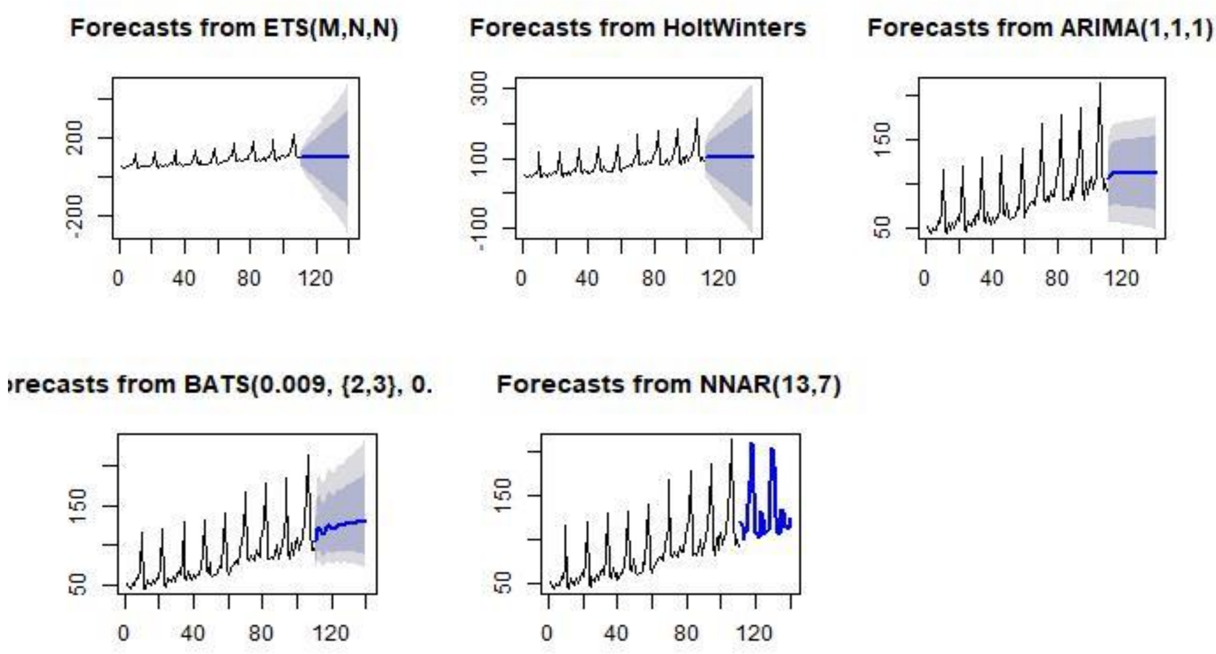
Accuracy	Model	Time series model						AI model	Technical Analysis	
		ETS	Holt Winters	Brown	Damped trend	ARIMA	BATS	NNm	EMA	SMA
RMSE		7,361793	8,738805	8,846565	8,608182	6,681145	6,260609	0,021823	8,255603	7,969358
MAPE	SC_45	10,903	10,173	10,462	10,154	8,755	8,529	0,030	9,006	8,654
MAE		5,902	7,031	7,169	7,036	4,849	4,676	0,016	6,524	6,243
RMSE		7,786169	8,026061	8,302775	8,002065	8,028956	7,761517	0,061662	8,244166	7,752346
MAPE	SC_47_1	6,101	5,970	6,201	5,871	6,882	6,118	0,052	6,503	5,478
MAE		5,212	5,505	5,621	5,424	5,905	5,291	0,045	5,747	5,240
RMSE		9,48137	8,445916	8,506071	8,464993	9,272786	9,470784	0,049265	9,193157	9,092309
MAPE	SC 47_2	5,808	5,483	5,573	5,447	5,640	5,826	0,016	5,943	5,958
MAE		7,050	6,220	6,317	6,151	6,860	7,073	0,019	6,812	6,855
RMSE		6,782829	6,62897	7,430883	6,618527	6,820796	6,500323	0,003609	7,755654	7,380513
MAPE	SC 47_3	6,061	5,576	5,924	5,464	5,976	6,035	0,003	6,383	6,090
MAE		5,352	5,179	5,545	5,087	5,274	5,382	0,002	6,160	5,848
RMSE		9,629049	18,64741	18,92513	18,67296	7,9778	7,751719	0,295766	19,32064	18,4086
MAPE	SC 47_4	18,210	19,305	21,448	18,797	17,486	14,354	1,145	15,699	15,002
MAE		5,067	11,178	11,476	10,834	4,914	4,652	0,201	10,503	10,134
RMSE		14,03425	12,68376	12,85011	12,71538	12,27475	11,35084	0,151849	13,0712	13,97761
MAPE	SC 47_5	11,674	10,805	11,402	10,814	11,679	9,745	0,144	10,736	10,994
MAE		8,397	9,080	9,371	9,095	8,937	7,641	0,103	9,766	8,844
RMSE		14,17794	16,51888	16,99755	16,55476	11,84129	11,6564	0,162439	16,45971	16,12532
MAPE	SC 47_6	15,765	13,021	12,773	12,728	13,183	10,037	0,205	10,028	10,005
MAE		9,053	10,327	10,156	10,195	7,854	6,448	0,106	9,119	8,931
RMSE		9,763775	10,60163	10,78604	10,62921	9,304033	9,381918	0,146463	10,6957	10,25862
MAPE	SC 47_7	9,600	8,931	9,357	8,939	9,051	8,898	0,128	8,100	8,274
MAE		7,265	7,720	7,944	7,724	6,797	6,782	0,097	7,363	7,531
RMSE		16,17364	17,84654	17,86364	17,77555	14,89657	15,16324	0,060098	17,39279	17,70689
MAPE	SC 47_8	9,460	11,656	11,837	11,587	8,494	8,774	0,033	11,174	11,096
MAE		12,429	13,169	13,283	13,068	11,331	11,629	0,042	12,758	12,870
RMSE		4,363067	13,99998	14,36196	14,03958	4,003016	3,91254	0,106574	17,09617	15,25755
MAPE	SC 47_9	9,867	11,580	11,946	11,630	9,123	8,841	0,292	12,454	12,458
MAE		3,175	7,442	7,969	7,470	2,863	2,783	0,071	9,453	8,790

For retail sales in the trade, repair and maintenance of motor vehicles, the BATS model is the second most accurate method. Retail in non-specialized stores is the neural network best described model based on technical analysis and SMA. Retailing in specialized stores, preferably on neural networks, describes the Damped Trend, as well as the retail sale of automotive fuel. The second place in the retail of computer and communication equipment in specialized stores is given by the BATS model, as well as the retailing of household goods in specialized stores and the retail of products for culture and sports in specialized stores. The retail of other commodities in a specialized store is best characterized by ARIMA models, as well as milling in stalls and markets. Retail outside stores, stalls and markets is most accurately predictable on neural networks using BATS models.

3.2 Forecasting ex ante

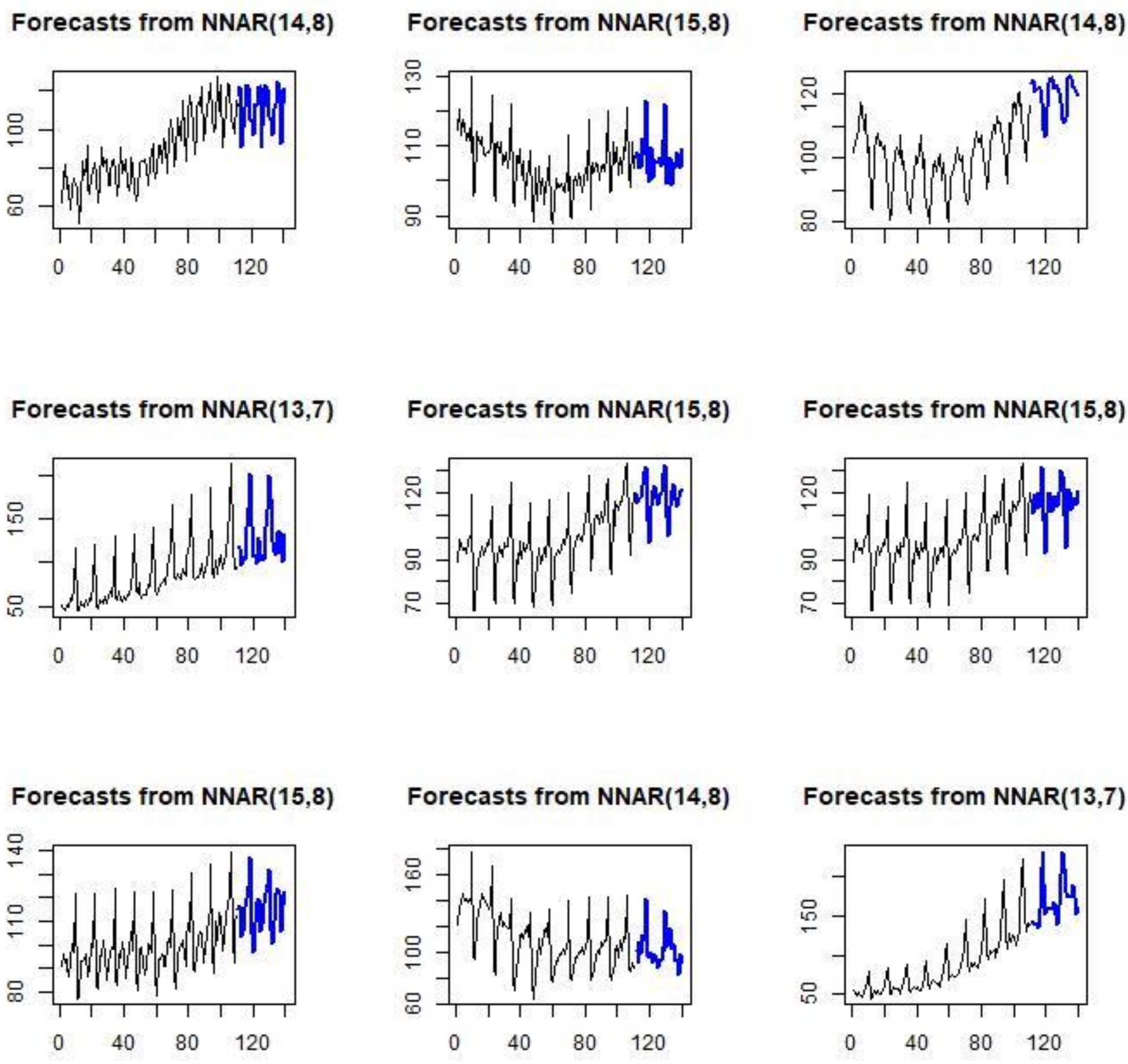
After selecting the appropriate models, an ex-ante prediction is performed. The forecast is realized for 30 periods ahead, i.e. until October 2020. For better understanding of the differences prediction models are first plotted prediction models as an example retail and communication equipment in specialized stores. These differences are shown in Figure 3. The graph shows differences in the individual predictions.

Figure 3 Forecasting ex ante in retail of computer and communication equipment in specialized stores, source: own



To fulfill the objective of the paper is also a prediction based on the most appropriate method selected in the prediction of ex post based on the selected rate accuracy. All retail sales are therefore predicted by the neural network model. The graphical results of the prediction are represented by Figure 4.

Figure 4 Forecasting ex ante from neural network, source: own



4. Conclusion

The aim of the paper was to apply quantitative forecasting methods to the prediction of sales in retail, with alternative use of indicators of technical analysis. Both the methods of the basic and advanced time series methods, as well as the methods based on artificial intelligence and alternatively methods based on technical indicators, which are not used in the business economy so far, have been used. The most accurate methods were based on artificial neural networks, which is in line with most of the interim research, but the results opposite, for example, Crone, Hibon and Nikolopoulos (2011) compared the basic time series method with neural network methods and the typical neural network forecast was in practice 4% less accurate. However, criticism of neural networks lies above all in their complexity and hence in demanding applications in corporate practice. Even in the academic sphere, the idea of simplicity of prognostic models for real practice is currently being developed (Zelner, 2001) or (Green and Armstrong, 2015). This problem could be eliminated using appropriate computing techniques, for example (Hyndman, 2008). If an AI method does not intend to apply for its complexity, technical indicators may also be an appropriate tool for predicting the business economy.

References

[1] DeLivera, A.M., Hyndman, R. J. and Snyder R. D. (2011). Forecasting time series with complex seasonal patterns using exponential smoothing. *Journal of the American statistical association*, 106(496), 1513-1527.

[2] Crone, S.F., Hibon, M., & Nikolopoulos, K. (2011). Advances in forecasting with neutral networks? Empirical evidence from the NN3 competition on time series prediction. International journal of Forecasting, 27, 635-660.

[3] Green, K. C. & Armstrong, J. S. Simple versus complex forecasting: The evidence. *Journal of Business Research,* 68 (8), 1678-1685. Retrieved from https://doi.org/10.1016/j.jbusres.2015.03.026.

[4] Hyndman R.J., & Athanasopoulos G. (2018). *Forecasting: principles and practice*. 2st edition. Open-access textbooks.

[5] Hyndman, R. and Khandakar, Y. (2008). Automatic time series forecasting: The forecast package for R. *Journal of Statistical Software*, 26(3).

[6] Hyndmand, R. J., Akram, Md. and Archibald, B. (2008). The admissible parameter space for exponential smoothing models. *Annals of statistical mathematics*, 60(2), 407-426.

[7] Jeanlagi (2018). *Draw by own force,* CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=3910972

[8] Kolková, A. (2016). Testing EMA Indicator for the Currency Pair EUR/USD. *International journal of entrepreneurial knowledge*. 2017(1).

[9] Marček, D. (2013). *Pravděpodobnostné modelovanie a soft computing v ekonomike*, SAEI, 18. Ostrava: VSB – TU Ostrava.

[10] McCulloch, W. and Pitts, W. (1943). A Logical Calculus of Ideas Immanent in Nervous Activity. *Bulletin of Mathematical Biophysics*. 5 (4). 115–133.

[11] Ulrich, J. (2017). *Functions and data to construct technical trading rules with R*. https://github.com/joshuaulrich/TTR

[12] Vincur, P., Zajac, Š. (2007). *Úvod do prognostiky*. Bratislava: Sprint.

[13] Wang, J. Luo, Y., Tang, L. and Ge P. (2018). Modeling a combined forecast algorithm based on sequence patterns and near characteristics: An application for tourism demand forecasting. *Chaos, Solitons and Fractals*. 108, 136-147.

[14] Zellner, A. (2001). Keep it sophisticatedly simple. In A. Zellner, H.A. Keuzenkamp, &M. McAleer (Eds.), Simplicity, Inference and Modelling: Keep it Sophisticatedly Simple (pp.242-262). Cambridge: Cambridge University Press.

Investing for Retirement with Focus on Pension Funds

Kateřina Kořená [1]

Abstract

In this paper author focuses on the assessment of the situation in the Czech Republic – the real possibilities how it is possible to prepare for a retirement.

Key words

Retirement, pension funds, transformed funds, participant funds, building savings, open-ended funds, bank accounts.

JEL Classification: G21

1. Introduction

The purpose of this paper is to explain and assess the situation in the Czech Republic – the real possibilities how it is possible to prepare for a retirement. The way to explain this situation is a description of the possible financial instruments and assessment of them – from financial point of view. The paper is based on a description of the current financial instruments used in the Czech financial market: pension fund instruments, saving buildings and open-ended investment funds and on an explanation of the using of these instruments. The final part deals with a recommendation for the future pensioners to reduce their risks connecting with the lowering of their income.

2. Pension Funds

In this part, we can explain the possibilities of a future pensioner. Firstly, it is necessary to mention that first part of the pensioner's income is a state pension. This means that at the age of a retirement a pensioner in the Czech Republic will receive this pension according to the amount of his previous salary and the length of his employment. Nowadays it is approximately from 40 to 50% of the average previous salary and it is possible to retire at 63 years (it is for 2018 and people born between 1953-1958 , from 2030 it is 65 years and every 5th year it will be discussed by government and can be changed). The claim to this state pension is after 34 years of the pension scheme (in 2019 it will be 35 years) and from 2010 the time of study it is not included in this obligatory time. This income in retirement bases on the 1st pillar of the Czech pension system and according to its functioning it is called pay-as-you-go system (PAYG). Secondly, we focus on financial instruments that are designated for retirement and therefore they are supported by the state in the form of state bonuses and reduction of the taxes – on the pension funds. According to the latest legislation, it is in the Czech Republic possible to distinguish two types of pension funds: transformed funds and newer participant funds as a part of the 3rd pillar of the Czech pension system (2nd pillar existed very shortly from 1.1. 2013 and was cancelled 31.12. 2015). The entrance to the transformed funds it is not possible now (from 1.1. 2013). Although in these pension funds it

[1] Ing. Kateřina Kořená, Ph.D., EkF VŠB-TUO, katerina.korena@vsb.cz.

is still allocated most of the invested money – see Table 1. The participant funds are now the only possibility of investing in the pension funds. From 1.1. 2016 these funds are open for children under 18 years, too. The total number of the participants in the 3rd pillar is decreasing, because the increase in number of participants in participation funds does not balance the decrease in the transformed funds – see Table 2.

Table 1: Total Invested Assets in the Pension Companies (in million CZK)

Pension company	Transformed Funds June 30 2014	Transformed Funds June 30 2018	Participation Funds June 30 2014	Participation Funds June 30 2018
Allianz	27 252	43 286	117	1 989
AXA	35 207	42 318	139	1 729
Conseq	5 649	7 454	21	1 673
Česká spořitelna	53 153	68 209	855	12 513
ČSOB	34 636	46 130	261	5 353
ING/NN	28 732	37 026	109	1 229
Komerční banka	37 955	50 808	358	4 626
Česká pojišťovna	76 102	98 617	658	7 553
In total	298 688	393 847	2 518	36 665

Source: The Association of Pension Companies of the Czech Republic

Table 2: Number of the Participants in the 3rd Pillar – Transformed and Participation Funds

Pension company	Transformed Funds June 30 2014	Transformed Funds June 30 2018	Participation Funds June 30 2014	Participation Funds June 30 2018
Allianz	462 556	416 141	7 609	44 812
AXA	398 955	305 666	7 690	33 740
Conseq	96 449	82 304	1095	32 243
Česká spořitelna	958 970	674 449	47 248	272 796
ČSOB	662 967	513 913	17 558	142 174
ING/NN	377 802	310 565	4 986	26 383
Komerční banka	520 871	415 924	32 401	130 371
Česká pojišťovna	1 238 940	969 713	39 458	201 422
In total	4 717 510	3 688 675	158 045	883 941

Source: The Association of Pension Companies of the Czech Republic

There are main differences between transformed and participant funds:

- Transformed funds are funds with guaranteed non-zero yields;
- You can withdraw ½ of your invested money after 15 years from transformed funds;
- In participant funds you can choose a more dynamic investment strategy;
- The yield from participant funds depends on this strategy and it is not guaranteed.

If we look at the performance of the pension funds during last years, it is clear that especially transformed funds are not able to overcome inflation because they naturally focus on bonds with low credit and exchange rate risk – see Table 3. Markedly in case of long term investing, they are not a suitable form of an investment – primarily in case that they are your only investment for your retirement. If the future pensioners rely just on the financial instrument they will receive money therein according of this performance = the value of this money can be lower in comparison with the value of this previous invested money because of inflation.

Table 3: Performance of Transformed Funds and Inflation from 2013 until 2017

Pension company	2013 Inflation 1,4	2014 Inflation 0,4	2015 Inflation 0,3	2016 Inflation 0,7	2017 Inflation 2,5
Allianz	1,60	1,64	1,38	1,03	0,41
AXA	2,29	1,46	1,10	1,03	0,76
Conseq	2,17	0,70	0,40	0,47	0,16
Česká spořitelna	1,30	1,42	0,85	0,68	0,51
ČSOB	1,70	1,40	1,20	0,70	0,63
ING/NN	1,41	1,13	0,88	0,66	0,69
Komerční banka	1,44	1,35	1,16	0,66	0,49
Česká pojišťovna	2,10	1,70	1,40	0,94	0,84

Source: The Association of Pension Companies of the Czech Republic

In case of participant funds, the most important is the choice of your strategy according of your age. This means that the younger you are you can afford to choose more dynamic investment strategy. In addition, from Table 4 we can see that there is a difference between conservative and dynamic strategy. Especially in case of Conseq Pension Company and its Global stock fund, it is evident that the strategy of this fund is more dynamic and based on equity investments.

Table 4 Performance of Participation Funds and Inflation from 2013 till 2017 (2 Types of Funds = Obligatory Conservative and the Most Dynamic Strategy of the Pension Company)

Pension company	2013 Inflation 1,4%	2014 Inflation 0,4%	2015 Inflation 0,3%	2016 Inflation 0,7%	2017 Inflation 2,5%
Allianz Conservative fund	1,44	1,31	1,22	-0,04	-1,63
Allianz Dynamic fund	1,76	2,78	4,98	1,51	4,07
AXA Conservative fund	0,49	2,67	0,59	0,02	-1,38
AXA Balanced fund	xxx	xxx	2,02	4,48	-0,1
Conseq Conservative fund	0.69	1,39	0,51	0,09	-0,90
Conseq Pension company Global stock fund	19,53	12,71	-0,08	10,81	9,96
Česká spořitelna Conservative fund	0,44	1,34	0,50	-0,05	-0,62
Česká spořitelna Dynamic fund	0,17	7,02	-1,04	7,42	8,2
ČSOB Conservative fund	0,94	2,68	1,35	0,00	-0,94
ČSOB Dynamic fund	0,66	3,28	7,66	5,36	5,53
ING/NN Conservative fund	0,27	0,61	1,15	0,21	-1,93
ING/NN Growth fund	xxx	xxx	-11,53	9,35	14,04
Komerční banka Conservative fund	0,36	1,33	0,36	-0,15	-1,57
Komerční banka Dynamic fund	2,16	5,81	1,60	0,87	7,51
Česká pojišťovna Conservative fund	2,46	0,87	0,97	-0,14	-0,30
Česká pojišťovna Dynamic fund	3,53	1,84	-0,32	6,36	8,20

Source: The Association of Pension Companies of the Czech Republic

3. Investing in OtherFinancial Instruments

In this part, we focus on the other possibilities of investing money in the Czech Republic – building savings and investing in funds and finally we compare it with bank accounts.

3.1 Building Savings

Concerning building savings - at the end of 2017 the building savings banks in the Czech Republic were overseeing 359 billion CZK of their clients' savings. During the last years the number of loans issued has stagnated, but the total value of the average loan is all the time increasing. Nowadays people are using building savings more often to fund the purchase of their houses or flats. Nevertheless, there is still a lot of investors using building savings only for investing their money. An advantage of this type of investing is closely connected especially with the length of time - see Table 5 and in Table 6 mentioned entries. We can state that the yield per year (internal rate of return) is decreasing according to the longer saving period, lower state support, lower interest rate, and the necessity of tax payment. It is clear that the building savings is presently not a suitable investment instrument for longer period because of its low yield. It can be used as a type of financial reserve but with very limited liquidity - it is not possible to withdraw money earlier than in 6 years.

Table 5: Yield of Building Savings and Length of Investing (Investing 1500 CZK Monthly + State Support 10%, June 30 2018)

Number of years	6	10	15	20	25	30
Yield (IRR)	3,7%	2,4%	1,9%	1,6%	1,4%	1,26

Source: Modrá pyramida, author

Table 6: Yield of Building Savings

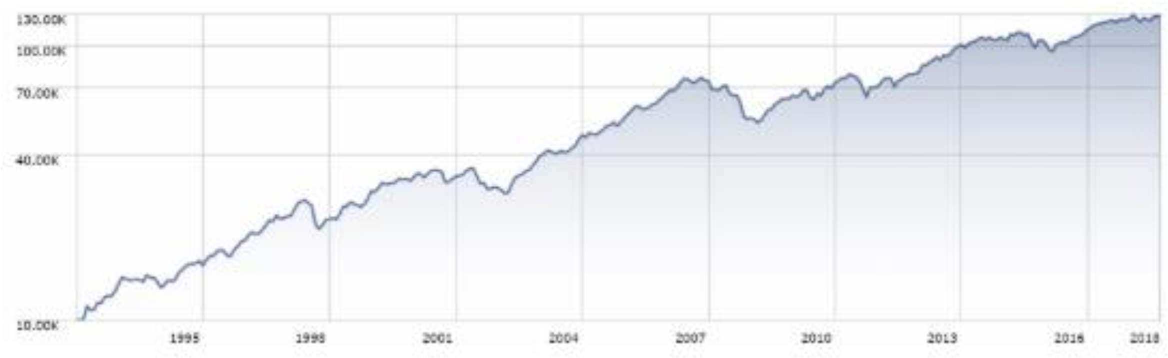
Investment per month in CZK	1 500	1 500	1 500	1 500	1 500	1 500	1 500	1 500
State support	25%	15%	15%	15%	10%	10%	10%	10%
Interest rate	3%	3%	3%	2%	2%	1%	1%	1%
Number of months (saving period)	60	60	72	72	72	72	72	74
Tax from interests 15%	No	no	no	no	no	no	yes	yes
Yield (IRR)	11,60%	8,45%	7,51%	6,55%	5,12%	4,15%	4,00%	4,32%

Source:Kateřina Kořená, Karel Kořený: Building Savings in the Czech Republic, 2015

3.2 Open-Ended Funds

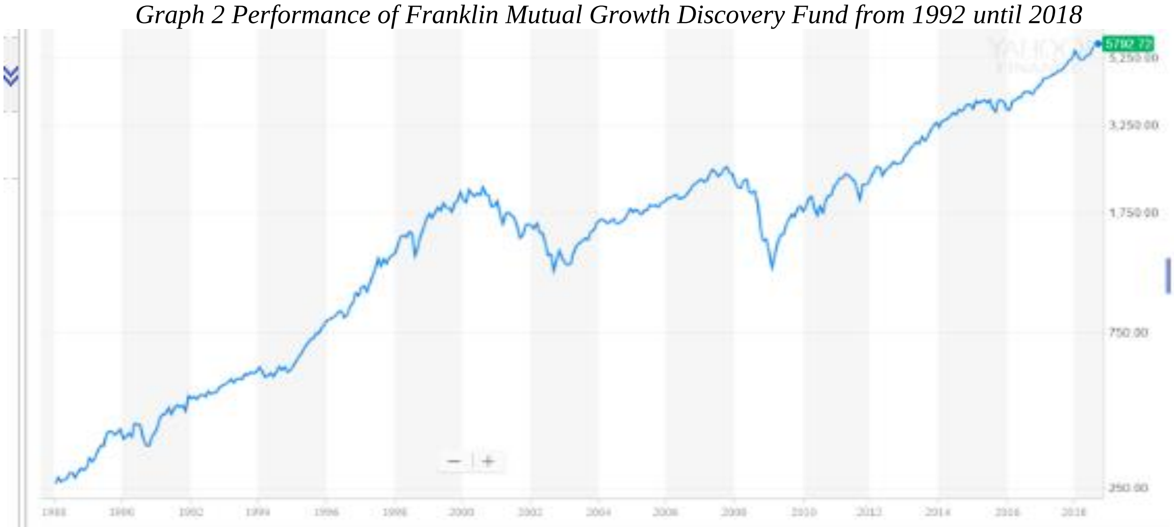
It is very difficult to estimate the result of investing for long-term period and therefore it is daring to state that this type of investing in open-ended funds is the best. Nevertheless, according to the performance of the global financial market and especially taking into account the performance of equity funds in past we can declare that it can be probable. In the Graph 1 as an example of the financial market we can see the performance of Standard & Poor Total Return Index from 1988 till 2018 (average yield 10,85 p.a.). The second Graph 2 describes the Franklin Mutual Growth Discovery Fund from 1992 till 2018 (average yield 10,34 p.a.).

Graph 1 Performance of Standard & Poor Total Return Index from 1988 until 2018



Source: Morningstar, www.morningstar.com

Graph 2 Performance of Franklin Mutual Growth Discovery Fund from 1992 until 2018



Source: Yahoo!Finance, www.finance.yahoo.com

3.3 Bank Accounts

If we look at the savings accounts of the banks in the Czech Republic, it is clear that especially in case that we want to invest higher sum of money the interest is generally very low. The higher interest is offered only by relatively new banks in the market because of their effort to achieve more clients. The established banks and in the same way the biggest banks in the market offer extremely low interest during the last 10 years. Although that the Czech central bank has started to increase this general level of interest rates the situation is changing very slowly. In addition, of course in comparison with the other financial instruments the level of interest in banks is always lower. Therefore, it is possible to state that invest money in these financial instruments for longer period is unreasonable.

Table 7: Savings Account Interest in the Czech Banks

Bank	Equa Bank	Air Bank	ČSOB	Česká spořitelna	Komerční banka
Savings Account Interest	1% till 200 000 CZK	1% till 250 000 CZK	0,21% till 250 000 CZK	0,21% till 200 000 CZK	0,06% till 10 mil. CZK
	0,2% from 200 000 CZK	0,2% from 250 000 CZK	0,01% from 250 000 CZK	0,01% from 200 000 CZK	0,05% from 10 mil. CZK

Source: Equa Bank, Air Bank, ČSOB, Česká spořitelna, Komerční banka (June 30 2018)

4. Conclusion

This paper is focused on the possibility of investing for retirement in the Czech Republic. The result from this paper and the main recommendation is following: it is not a good idea to rely only on the PAYG system state scheme and investing in the pension funds. The amount of the state pension in future is uncertain (the longer time horizon the worse we can predict it because of performance of the whole economy, future demographic situation and political factors). Concerning the investing in contemporary pension funds - their performance in transformed funds is very low and do not guarantee enough money for retirement. In case of new participant funds, it is very important to choose the proper investment strategy according to the time horizon – the portfolio of this fund for long term investing should consist mainly from stocks. Concerning building savings, this type of investing it is not suitable for longer period – the longer you invest in this financial instrument the lower the average yield is. The advantage of this instrument is connected with the situation in financial market and state policy, too. Nowadays the average yield is about 4 % (for 6 years) and this is relatively nice in comparison to the bank accounts and for this short period, but not in case of investing for your retirement. The author of this paper therefore recommends in case of long term investing to use equity instruments and due to the diversification stock funds. This recommendation does not mean that the pension funds and the building savings are not fitting for longer term investing at all but it means that the investing in these financial instruments is possible under the certain circumstances and e.g. can be a piece of conservative part of investors 'portfolio. Therefore, for final statement it is possible to express that every investor in the Czech Republic should primarily invest according to his financial plan. This financial plan must comply the goals of this investor and must be worked out properly based on his age and family and financial situation.

References

[1] Association of Pension Companies of the Czech Republic, www.apfcr.cz

[2] Czech National Bank, www.cnb.cz

[3] Association of Czech Building Savings Banks, www. acss.cz

[4] Building Savings Company Modrá pyramida, www.modrapyramida.cz

[5] Kateřina Kořená, Karel Kořený: Building Savings in the Czech Republic – in 10th International Scientific Conference Financial management of Firms and Financial Institutions, Ostrava VŠB-TU of Ostrava, Faculty of Economics, Department of Finance, 2015

[6] Yahoo!Finance, www.finance.yahoo.com

[7] Morningstar, www.morningstar.com

The dependence of macroeconomic indicators on sovereign rating

Jana Kotěšovcová, Jiří Mihola, Petr Wawrosz [1]

Abstract

Rating of a state constitutes a qualified evaluation of the state's reliability as a debtor, which informs investors investing in financial assets on important indicators. The research objective is, by means of a created regression and correlation model, to test the quality of the sovereign rating dependence on the macroeconomic indicators. In the correlation analysis, the accumulated correlation coefficient greater than 95% was calculated but at 100% multicollinearity. The high value of the total correlation coefficient was thus devalued by absolute multicollinearity. Given that in neither case was the general correlation coefficient higher than 80% at multicollinearity below 20%, it can be stated that the credit ratings are not explicable solely through economic characteristics, but in determining them also other effects were applied, for instance political and strategic.

Key words

Rating stamp, credit rating, rating methodology, macroeconomic indicators, sovereign rating

JEL Classification: L21, O12, O31, O33

1. Introduction

Sovereign rating evaluates the reliability of debtor, which informs investors on important indicators, such as public debt and its development and the overall condition and future prospects of the development of the state's economy. Rating thus significantly affects the overall view of investors of the country and of the willingness to invest there or not.

The aim of this paper is, by means of a regression and correlation model, to examine the quality of the sovereign rating dependence on the macroeconomic indicators. 17 indicators have been selected for 24 significant countries in order to apply multiple linear regression and correlation analysis. The aggregate correlation coefficient and multicollinearity will be calculated in the correlation analysis. The high value of the aggregate correlation coefficient could be devalued by the multicollinearity being too high. Those indicators will therefore be gradually selected on the basis of the correlation matrix that could be envisaged for the exclusion so that the value of the aggregate correlation coefficient R^2_j did not drop at the desired drop of multicollinearity M_{-j} at least below 50%, or was not below the highest correlation coefficient r_{iy}^2. In the above described way suitable macroeconomic indicators will gradually be sought for the exclusion from their group. This paper aims to find a set of indicators that will meet the above characteristics. If such a set of indicators is not found, it will give reason to presume that the credit ratings are not explicable solely by economic characteristics, but other factors, e.g. political or strategic, were applied significantly in the determination of those.

2. Current situation, data sources and algorithms used

2.1 The current view of the issue

[1] Ing. Jana Kotěšovcová jkotesovcova@volny.cz, Ing. Bc. Jiří Mihola, CSc. jiri.mihola@quick.cz, Mgr. Ing. Petr Wawrosz, Ph.D. petr.wawrosz@centrum.cz

The analysis of the rating and the selected macroeconomic indicators follows the research works of a number of international authors that is further develops. This is a rating area and its impact on financial markets, for example "The Relationship between credit default swap spreads, bond yields, and credit rating announcements" conducted by Hull, Predescu and White [10]. The research also follows the work of Budinský [3], "Relationship between sovereign ratings and CDS prices" which investigates the accuracy of the sovereign rating based on implicit rating that stems from the probability of a failure to pay debt, which is based on the credit default swap, or insurance to pay off bond (credit default swap). The research will focus on examining the dependence of rating on macroeconomic indicators. The most used was however the book "A century of sovereign ratings" by Gaillard [6]. Another publication that we worked with deals with the relations between the macroeconomic variables and the sovereign rating, specifically Canuto, Pereira, Porto [5]. The analysis of the sovereign rating factors is described by Alfonso [1], as well as by Cantor and Packer [4], Mellios and Blanc [12]. Other authors who examined the effect sovereign rating on credit default swap (CDS) are Blanco, Brennan and Marsh [2], Ismailescu and Kazemi and [11].

2.2 Data sources

After scrutinising information sources it was found out that the database that suits the purpose of this thesis is the World Development Indicators [14] database by World Bank which was primarily worked with. In addition to economic data, it was necessary to obtain data on the rating stamps for all three surveyed rating agencies Standard & Poor‘s, Moody‘s & Fitch in a long enough current time line. Data on the rating stamps granted by all three rating agencies were obtained from the data source countryeconomy.com [13].

Different criteria, such as population, size, GDP per capita can be used for the selection of the significance criteria of the selected countries. For the purpose of this thesis, the criterion of population was used, that is economically significant in terms of the current topic. 10 most populous countries in the world and the 14 most populous countries in Europe, i.e. a total of 24 countries, were selected for the comparison of the sovereign rating with selected macroeconomic indicators. Due to lack of data Pakistan, Nigeria and Bangladesh were excluded from the list of the most populous countries of the world, and the list was supplemented with other states according to population, namely Japan and the Philippines. Used were therefore the data from the following countries: China, India, USA, Indonesia, Brazil, Russia, Mexico, Japan, the Philippines, Vietnam, Japan, the Philippines, France, Great Britain, Italy, Spain, Ukraine, Poland, Romania, the Netherlands, Belgium , Greece, Czech Republic, Portugal and Hungary.

When selecting the macroeconomic indicators to calculate the rating, the World Development Indicators [14] database was used containing a total of 1,519 indicators. The methodology of all three international rating agencies and the available World Bank data were used in the selection. The indicators were selected to include all important macroeconomic areas, indebtedness, unemployment, inflation, GDP growth rate, export of merchandise and services, gross national product, the final consumption expenditure of households and government institutions, gross savings, interest payments, real interest rate, etc. 23 indicators were initially selected. These indicators were obtained for the period from the beginning of the new millennium, i.e. from 2000 to 2016, of the 24 most populous countries in the world.

After thorough consideration and for the sake of the data completeness the following 17 indicators were selected for the analysis: indicator x_1: total government debt (% of GDP), indicator x_2: unemployment rate (% of total workforce), indicator x_3: inflation, consumer prices (annually in %), indicator x_4: GDP growth rate in%, indicator x_5: GDP per capita (s. c. 2011 PPS, $), indicator x_6: government expenditures (% of GDP), indicator x_7: export of merchandise and services (% of GDP), indicator x_8: net exports of merchandise and services (% of GDP), indicator x_9: final consumption expenditure of households and government institutions (% of

GDP), indicator x_{10}: final consumption expenditure of government institutions (% of GDP), indicator x_{11}: gross national product (% of GDP), indicator x_{12}: gross savings (% of GDP), indicator x_{13}: imports of merchandise and services (% of GDP), indicator x_{14}: interest costs of servicing government debt (% of costs), indicator x_{15}: government revenues (+) / government expenses (-) as a% of GDP, indicator x_{16}: real interest rate (%), indicator x_{17}: foreign sales of merchandise and services (% of GDP).

The indicators were not selected according to their relevance to the rating degrees. This relevance will be analysed in the chapter "Analysis using regression and correlation model."

Although the selection of indicators did not come from their content, it can be compared e.g. to the work of Richard Cantor and Frank Packe [4]. A set of 17 indicators was used in this paper compared to 8 indicators in this publication. This work does not deal with soft indicators (economic development and history of defaults), which were used in the above referred to work of Cantor and Packe [4]. As results from the above mentioned, the set of the 17 indicators used can be considered representative in terms of the factual content.

2.3 Algorithms used

The algorithms used in this paper use the coefficient of determination R_j^2 of the dependence of the jth variable on other variables, which can be expressed by Garaj, Šujan [7] as:

$$R_j^2 = 1 - \frac{|\mathbf{R}|}{|\mathbf{R}_{-j}|} \quad (1)$$

wherein, $\mathbf{R}_{-j}$ is a minor of matrix R, which is the correlation determinant of the matrix of a set (z-1) of variables without variable x_j. Given that the determinant $|\mathbf{R}_{-j}$, same as the determinant $|\mathbf{R}|$ and a newly introduced indicator Φ^2 is always less than or at most equal to 1, it applies $|\mathbf{R}| \leq |\mathbf{R}_j|$ for j=1. The equality occurs when $|\mathbf{R}_{-j}|=1$.

We based the multicollinearity measurement on Hebák and Hustopecký [9], who state it as quotient of the aggregate tightness of the set of dependently variables and the entire system. The aggregate tightness is measured using the quantity 1 - | R |. This indicator has favourable properties and can be displayed together with the most important correlation characteristics. The aggregate tightness of the system can be expressed by a determinant of the correlation matrix by Hebák [8].

$$\Phi^2 = 1 - |\mathbf{R}| \quad (2)$$

It is analogy to the determination coefficient, belonging to a normalised orthogonal task. If the jth variable is a dependent variable, it means that independent variables are uncorrelated and the tightness indicator Φ^2_{-j} (defined as Φ^2 for the basic set, but after we have excluded the jth variable of it), becomes zero. Thus, we also got the interpretation of Φ^2 as a limit value of coordinate coefficients of determination for uncorrelated independent variables, i.e. at zero multicollinearity.

Multicollinearity is among the possible serious design problems of multiple linear regression models. Easy swinging of hyperplanes affects the yield of the regression parameter estimates to the impossibility of their economic interpretation, impedes the separation of individual factors, increases the sensitivity of response to the number of measurements. To work with a particular model, noting that the model is loaded with multicollinearity, and therefore determining the rate and size of it is better, is often not enough. From the analysis of different methods of multicollinearity measurements a hypothesis can be derived that the measurement of multicollinearity needs to capture the relativity of this concept. Using the above-defined term of tightness Φ^2, the indicator of multicollinearity M_j may be defined as the quotient of the tightness of the system not comprising and comprising a dependently variable by index j, i.e. by the relation:

$$M_j = \frac{\Phi^2_{-j}}{\Phi^2} \quad (3)$$

This way you can define M_j whenever $\Phi^2 \neq 0$. For $\Phi^2 = 0$ we'll have M_j=0 multicollinearity, so we can measure in the range from 0 to 1 or in % from 0 to 100%.

3 Analysis using regression and correlation model

3. 1 Procedure in the creation of regression and correlation model

Regression and correlation model is a system, which was introduced in this article for testing the dependencies between the credit rating (independent variable) and the selected macroeconomic indicators. The first step in creating a model was the selection of indicators, checking the data completeness and relevance, and also the transformation of some data using logarithms. The next step was testing the parameters using the Linear Regression Excel feature where the regression parameters for the regression function were found. This was followed by working out a correlation task which involved creation of a correlation matrix and a calculation of the aggregate correlation coefficient and determination of multicollinearity.

3.2 The analysis

17 indicators were selected out of the initial data described in Chapter 3 for the subsequent calculations. Those represent independent variables.

Table 1 *Results of the regression analysis for 17 indicators*

regresní parametry	b_8	b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0
odhad	-0,030	8,576	-0,006	0,467	0,039	-0,033	0,004	-0,078	12,459
chyba odhadu	0,027	6,902	0,004	0,121	0,020	0,031	0,011	0,049	10,791
R2j	0,95129	0,124							
F-test	8,544	7,000							
sr	2,105	0,108							

regresní parametry	b_{17}	b_{16}	b_{15}	b_{14}	b_{13}	b_{12}	b_{11}	b_{10}	b_9
odhad	-17,004	-0,018	-0,009	0,003	8,438	0,003	0,000	0,020	-0,676
chyba odhadu	14,440	0,014	0,019	0,012	7,626	0,009	0,000	0,015	0,529

Source: Own processing based on the World Development Indicators

The results of the regression analysis of 17 selected macroeconomic indicators are concentrated in Table 1. The first line contains estimates of the regression parameters b_1 to b_{17} and the estimate of absolute member b_0. The second line includes standard deviations of the stated regression parameters. The third line shows the value of the aggregate correlation coefficient R^2_j. Other figures represent the significance statistics. The aggregate correlation coefficient indicates that the regression grades were explained to 95.1% by the given indicators. Problematic are some low values of the regression parameters, such as b_{11}; b_{12} = 0.003; a_{14} = 0,003. It is likely that it will be possible to exclude these indicators from the calculation, but only after the correlation task is assessed. After verifying within the correlation analysis, the b_{11} indicator - the gross national product (% of GDP) - is a major candidate for elimination in the first round, i.e. for the reduction to 16 indicators.

The key basis for the correlation analysis is a complete correlation matrix containing all paired correlation coefficients of all pairs of variables, including the dependent variable, i.e. $\boldsymbol{r_{i,j}}$.

Table 2 *Correlation matrix for 17 indicators*

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	y
x_1	**1,000**	0,127	-0,051	-0,191	0,065	0,261	0,045	-0,451	0,473	0,296	0,451	-0,442	0,169	-0,006	-0,303	0,236	0,104	-0,190
x_2	0,127	**1,000**	-0,052	-0,672	0,314	0,222	-0,061	-0,260	0,480	0,251	0,260	-0,589	0,009	0,147	-0,380	0,412	-0,027	-0,073
x_3	-0,051	-0,052	**1,000**	0,403	-0,666	-0,264	-0,180	-0,246	-0,092	-0,528	0,246	0,124	-0,164	0,360	0,218	-0,437	-0,170	-0,681
x_4	-0,191	-0,672	0,403	**1,000**	-0,597	-0,459	0,022	0,250	-0,663	-0,504	-0,250	0,698	-0,071	0,009	0,610	-0,449	-0,023	-0,077
x_5	0,065	0,314	-0,666	-0,597	**1,000**	0,491	0,169	0,256	0,362	0,778	-0,256	-0,572	0,153	-0,426	-0,334	0,232	0,162	0,740
x_6	0,261	0,222	-0,264	-0,459	0,491	**1,000**	0,401	-0,055	0,322	0,602	0,055	-0,559	0,457	-0,323	-0,376	0,319	0,431	0,121
x_7	0,045	-0,061	-0,180	0,022	0,169	0,401	**1,000**	0,414	-0,161	0,375	-0,414	0,069	0,967	-0,563	0,228	0,241	0,992	0,059
x_8	-0,451	-0,260	-0,246	0,250	0,256	-0,055	0,414	**1,000**	-0,544	0,288	-1,000	0,344	0,192	-0,405	0,488	-0,348	0,310	0,434
x_9	0,473	0,480	-0,092	-0,663	0,362	0,322	-0,161	-0,544	**1,000**	0,230	0,544	-0,771	-0,013	0,138	-0,519	0,289	-0,090	-0,157
x_{10}	0,296	0,251	-0,528	-0,504	0,778	0,602	0,375	0,288	0,230	**1,000**	-0,288	-0,487	0,350	-0,390	-0,149	0,256	0,365	0,530
x_{11}	0,451	0,260	0,246	-0,250	-0,256	0,055	-0,414	-1,000	0,544	-0,288	**1,000**	-0,344	-0,192	0,405	-0,488	0,348	-0,310	-0,434
x_{12}	-0,442	-0,589	0,124	0,698	-0,572	-0,559	0,069	0,344	-0,771	-0,487	-0,344	**1,000**	-0,031	0,123	0,551	-0,340	0,020	-0,012
x_{13}	0,169	0,009	-0,164	-0,071	0,153	0,457	0,967	0,192	-0,013	0,350	-0,192	-0,031	**1,000**	-0,471	0,058	0,382	0,991	-0,019
x_{14}	-0,006	0,147	0,360	0,009	-0,426	-0,323	-0,563	-0,405	0,138	-0,390	0,405	0,123	-0,471	**1,000**	-0,208	0,074	-0,524	-0,354
x_{15}	-0,303	-0,380	0,218	0,610	-0,334	-0,376	0,228	0,488	-0,519	-0,149	-0,488	0,551	0,058	-0,208	**1,000**	-0,407	0,148	-0,047
x_{16}	0,236	0,412	-0,437	-0,449	0,232	0,319	0,241	-0,348	0,289	0,256	0,348	-0,340	0,382	0,074	-0,407	**1,000**	0,309	0,026
x_{17}	0,104	-0,027	-0,170	-0,023	0,162	0,431	0,992	0,310	-0,090	0,365	-0,310	0,020	0,991	-0,524	0,148	0,309	**1,000**	0,020
y	-0,190	-0,073	-0,681	-0,077	0,740	0,121	0,059	0,434	-0,157	0,530	-0,434	-0,012	-0,019	-0,354	-0,047	0,026	0,020	**1,000**

Source: Own processing based on the World Development Indicators

The correlation matrix representing Table 2 allows locating significantly correlated pairs of indicators, as well as the tightness of the link of dependent variable, i.e. the rating stamp (or its logarithm), and the individual explanatory variables. The tightness of the regression link between the dependent variable *y* and the individual explanatory variables does not absolutely exceed the absolute value 0.681, as shown in the last column of the correlation matrix, which occurs in the indicator x_3 inflation, consumer prices (annually in %). There are also low values, e.g. for indicator x_{12} gross savings (% of GDP), where the value of - 0.012 comes up. Low values of the paired correlation coefficients relative to the explained variable are in addition accompanied by high levels of certain paired correlation coefficients between the explanatory variables. In the correlation matrix, ten times higher value than 0.7 occurs between the paired correlation coefficients. We will get convinced of the significance of that finding in Table 3 containing four key variables of the correlation task.

Table 3 *Aggregate tightness and multicollinearity for 17 indicators*

$\|R_{-j}\|$ a $\|R\|$,	Φ^2_{-j} a Φ^2	M_j	R^2_j
0,00000000000000	1,00000000000000	1,0000000000000	0,951289
0,00000000000000	1,00000000000000		

Source: Own processing

The first column of Table 3 shows the values of the key determinants of the correlation matrix. In the second line, there is a determinant of the full correlation matrix, including the dependent variable |R|, while in the first line there is a subdeterminant of the correlation matrix of independent variables $|\mathbf{R}_{-j}|$. The values aggregated tightness $\boldsymbol{\Phi}^2_{-j}$ a $\boldsymbol{\Phi}^2$ are in the second column, the last but one column shows the values of multicollinearity $\mathbf{M_j}$, and in the last column there is a value of the aggregate correlation coefficient R^2_j. The formulas are given in chapter 2.3.

The values of the determinant and subdeterminant are virtually zero, with the result that the aggregate tightnesses are unit (therefore stated to 14 decimal places). The multicollinearity values are also one, which confirms the preliminary conclusion that the task features much more of the interrelationship between the explanatory variables than to the selected dependent variable, which is the average value of the credit rating. The aggregate tightness of 0.951289 is sufficiently high and is consistent with the value of the aggregate tightness indicated in Table 3. The results give the impression that we were able to gather enough information to explain the credit rating, since are determined in more than 95% in the described model. This gratifying result is however practically devalued by the extreme multicollinearity value equal to one. In these cases, it is usually proceeded to exchange some of the indicators or their reduction in order to improve the quality of the estimate of the regression parameters. Therefore individual indicators were gradually eliminated and possible drop in the aggregate regression coefficient controlled in the expected drop of multicollinearity.

The first eliminated indicator was x_{11} gross national product, which had a value of the regression parameter $b_{11} = 0.000$ equal to zero, and a high correlation coefficient to indicator x_9 final consumption expenditure of households and government institutions (% of GDP), which was 0.544. Thus we get new values of the aggregate key characteristics. Furthermore, other indicators were excluded in a similar manner until we reached the conclusions set out in Table 4.

Due to the unsatisfactory multicollinearity values, which are still too high, and with yet tolerable values of the aggregate correlation coefficient, we will continue in the same way in the gradual reduction of the indicators down to 2 indicators, until the aggregate correlation coefficient drops below the highest paired correlation coefficient of dependent variable, i.e. the rating grade to any of the explained variables. As shown in Table 4 "Correlation matrix for the 17 indicators," it is the value 0.681 that links the dependent variable (y) with indicator x_3 inflation, consumer prices (in% per annum). The gradual development of the aggregate characteristics and the excluded variables are shown in the below Table 4. We managed to get below the stated value 0.681 only after the exclusion of all indicators except for the last two, which are x_5 GDP per employee and $\mathbf{x_8}$ net exports of goods and services (% of GDP).

Development of aggregate characteristics in gradual exclusion of indicators from 17 to 2

Φ	Φ-j	M-j	R^2_j	number of indicators	Excluded indicator #
1,0000000000000	1,0000000000000	1,000000000000	0,951289	17	x
1.0000000000000	0.9999999999997	1.000000000000	0.951289	16	11
0.9999999996166	0.9999999933009	0.999999993684	0.942769	15	13
0.9999995499987	0.9999925476443	0.999992997642	0.939616	14	17
0.9999987227207	0.9999800717105	0.999981348966	0.935906	13	2
0.9999850935119	0.9997683221786	0.999783225435	0.935659	12	12
0.9999552721915	0.9993282758494	0.999372975613	0.933413	11	15
0.9998854184385	0.9985409272901	0.998655354780	0.921470	10	6
0.9996435289161	0.9954628289812	0.995817809235	0.921433	9	7
0.9982392268763	0.9838309253018	0.985566283926	0.891102	8	9
0.9961471204130	0.9690528177790	0.972800902519	0.875501	7	16
0.9840865875282	0.8782346863057	0.892436394760	0.869311	6	10
0.9736051024093	0.8191616940332	0.841369557335	0.854041	5	14
0.9567172643068	0.7594414059228	0.793799207202	0.820074	4	1
0.8267982901778	0.4858119514816	0.587582191755	0.663155	3	4
0.6372042800061	0.0657536978494	0.103190923088	0.611670	2	3

Source: Source: Own processing based on the World Development Indicators

As is clear from Table 4, the decreasing value of the aggregate correlation coefficient is accompanied by still high although decreasing multicollinearity. The majority of solutions have the collinearity close to 1. Multicollinearity of 0.84137 drops below the aggregate correlation coefficient 0.85404 only if there are 5 indicators, i.e. after the elimination of twelve indicators.

Only the last two cases for two or three indicators approximate to the desired zone with the aggregate correlation coefficient of at least above 0.5 and the multicollinearity at least below 0.5. This is an important signal to determine that the economic impacts on the credit rating are not as important as the political ones.

Conclusion

Sovereign rating measures the creditworthiness of individual states to meet their obligations. Identifying the main factors that affect sovereign rating is dealt with in the methodology of various international rating agencies. This paper aimed to examine the dependence of the development of the sovereign rating on the development of major macroeconomic indicators. Multiple linear regression and correlation analysis were used here as the methodology. When applying that method, in which time series were formed by individual countries, selection was made out of a large number of macroeconomic indicators and countries 17 indicators for 24 countries. Other indicators or countries were excluded, largely because of incomplete data. Using incomplete data would lead to distorted results. Statistical calculations would "perceive" missing values as zero. Multiple regression and correlation analysis gave the values of the regression coefficients, half of which had a lower value than 0.03 in the absolute value of (Table 1).

In the correlation analysis, aggregate correlation coefficient greater than 95% was found but at 100% multicollinearity (Table 3). The high value of the total correlation coefficient was thus devalued by absolute multicollinearity.

That's why indicators were gradually selected on the basis of the correlation matrix that could be envisaged for the exclusion so that the value of the aggregate correlation coefficient R^2_j did not drop at the desired drop of the multicollinearity M_{-j} below the desired 50%, or was not below the highest paired correlation coefficient r_{iy}^2 (see Table 2 for the correlation matrix). For this reason, suitable indicators were gradually selected for the exclusion from the set of the macroeconomic indicators. The desired result was achieved only after the elimination of 12 indicators (of the initial 17) when the multicollinearity of 84% dropped below the aggregate correlation coefficient of 85% (Table 4). However, multicollinearity of 10% was stated only at 2 indicators, when the aggregate correlation coefficient had a value of 61% (Table 4).

Given that in neither case was the general correlation coefficient higher than 80% at multicollinearity below 20%, it can be stated that the rating grades are not explicable solely through economic characteristics, but in determining them also other effects were applied, for instance political and strategic. That finding would not have been possible if only the value of the aggregate correlation coefficient R^2_j was worked, and the significant impact of multicollinearity M_{-j} was neglected. This means that rating assessment does not depend on the economic effects only, but also on other, e.g. political and strategic, effects. The research objective, to examine the quality of the sovereign rating dependence on the macroeconomic indicators by means of a created regression and correlation model, was fulfilled.

Dedication

The output has been created at the solving of the student project Improvement of Pension System as Intensification Factor of Economy (Zdokonalení penzijního systému jako intenzifikační faktor ekonomiky) using a special-purpose grant for a specific university research of the University of Finance and Administration (Vysoká škola finanční a správní).

References

[1] Afonso A. (2003) The determinants of Moody's sub-sovereign ratings, *Journal of Economics and Finance,* Springer.

[2] Blanco, R. Brennan, S. and Marsh I.W. (2005) The reaction of emerging market credit default swap spreads to sovereign credit rating changes, *The journal of Finance.*

[3] Budinský, P. (2015) Relationship between Sovereign Ratings and CDS Prices. In Dyczkowska, Kuździowicz. *Controlling and Knowledge.* Wroclaw: Publishing House of Wroclaw University of Economics, 2015. ISBN 978-83-7695-414-1.

[4] Cantor, R. and Packer F. (1996) *Determinants and Impact of Sovereign Credit Ratings.* Federal Reserve Bank of New York Economic Policy Review, [online]. [quot. 2010-10-15] Available at: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1028774

[5] Canuto, O., Pereira dos Santos, P.F. and P. C. de Porto. (2012) *Macroeconomics and Sovereign Risk Ratings.* Intl Econ. Comm. Policy. 03, 1250011.

[6] Gaillard N. *A century of sovereign ratings.* New York: Springer, 2012. ISBN 781461405238

[7] Garaj, V., Šujan, I. (1980) *Ekonometria*. Bratislava: Publishing House Alfa,

[8] Hebák, P. (2004) *Vícerozměrné statistické metody* (Multivariate Statistical Methodologies) 1. Praha: INFORMATORIUM, spol. s r.o., ISBN 80-7333-025-3

[9] Hebák, P, Hustopecký J. (1987) *Vícerozměrné statistické metody s aplikacemi*. (Multivariate Statistical Methodologies with Applications.) Praha: SNTL, Nakladatelství technické literatury Alfa (Publishers of Technical Literature Alfa)

[10] Hull, J. Predescu, M. and White A. (2004) The relationship between credit default swap spreads, bond yields, and credit rating announcements. *Journal of Banking & Finance,* Elsevier.

[11] Ismailescu, I. and H Kazemi H. (2010) The relationship between credit default swap spreads, bond yields, and credit rating announcements, *Journal of Banking & Finance,* Elsevier.

[12] Mellios, C. and Paget-Blanc E. (2006) An empirical analysis of the dynamic relation between investment-grade bonds and credit default swaps. *The European Journal of Finance.*

[13] Sovereigns Ratings List 2017 | countryeconomy.com. Economics | Macroeconomics | country macro data countryeconomy.com [online]. Available at: https://countryecono my.com/ratings [cit. 18. 10. 2017].

[14] World Development Indicators | DataBank. [online]. Copyright © [cit. 12. 10. 2017]. Available at: http://databank.worldbank.org/data/reports.aspx?source=world- develop ment-indicators

Solution of Emission Management Problem

Václav Kozmík, Martin Šmíd[1]

Abstract

Optimal covering of emissions stemming from random production is a multistage stochastic programming problem. Solving it in a usual way - by means of deterministic equivalent – is possible only given an unrealistic approximation of random parameters. There exists an efficient way of solving multistage problems – stochastic dual dynamic programming (SDDP); however, it requires the inter-stage independence of random parameters, which is not the case which our problem. In the paper, we discuss a modified version of SDDP, allowing for some form of interstage dependence.

Keywords

Multistage stochastic programming, Emission management, SDDP, time dependence.

JEL Classification: C44

1. Introduction

In order to reduce CO_2 emissions, EU, as well as some other countries, introduced the Cap-and-Trade system: Each year, companies are obliged to hand out emission allowances, one for each ton of CO_2 they emitted in the previous year. The allowances may be bought from the government in auctions, the companies may trade with them on various secondary markets; moreover, from protectionist reasons, governments grant some allowances to the companies for free. The allowances may be banked (saved for later). In addition to spots (the allowances themselves), various derivatives are traded on secondary markets, including futures and options. Thus, optimal covering of the emissions, amount of which is usually uncertain, is a complex problem.

[1]Institute of Information Theory and Automation, The Czech Academy of Sciences, Pod Vodárenskou věží 4, 182 08 Praha 8. Czech Republic. e-mail: smid@utia.cas.cz.
This work was supported by grant No. GA 16-01298S of the Czech Science Foundation.

2. Solution of the Optimal Emission Covering Problem

As the problem of optimal emission covering is dynamic with uncertain parameters, its mathematical description naturally leads to multi-stage stochastic programming problems, random parameters of which include at least the emitted amounts of CO_2 and the prices of the allowances spots and derivatives (see [9, 10]). Unfortunately, solution of such a problem by standard techniques is virtually impossible. Solving the problem by means of deterministic equivalent requires unrealistic approximations, otherwise it would lead to extremely huge equivalent problems. Dynamic programming, on the other hand, cannot be used due to an extensively rich state space.

Stochastic dual decomposition (SDDP) – a well known efficient technique for solving multi-stage problems – cannot be used, too, as it requires the stage-wise independence of the underlying random processes. This assumption, however, is not met by the emission prices, time series of which are are non-stationary. Although the non-stationarity can be circumvented by working with returns, see [3], such a trick is impossible within the emission management problem because there is anoother random parameter - the emitted amounts – involved.

There is, however, a way of reconciling the SDDP with dependence: using a hidden Markov model. In particular, we can assume the price process, to be a sum of a (non-homogeneous) Markov chain (e.g. one with a single initial state $\{0\}$, states $\{-1, 1\}$ at time one, $\{-2, 0, 2\}$ at time two, etc.) and an i.i.d. variable. Given this setting, we can use the modification of the SDDP for Markov Chains, described below, to solve the problem. Even though the (unconditional) distribution of the (log-)random parameter is not then a random walk as it would be if we modelled the price process standard way, it is similar to it (and may be made arbitrarily close by making the MC denser; this, however, would make the solution slower).

3. Stochastic dual dynamic programming with Markov Chains

Stochastic dual dynamic programming algorithm is very popular for solving multi-stage stochastic programs. On of the most common risk-averse formulations, nested CVaR model, presented for instance in [3] or [8] can we written in terms of dynamic programming equations as follows:

$$\begin{aligned} \min_{\boldsymbol{x}_1, u_1} \ & \boldsymbol{c}_1^\top \boldsymbol{x}_1 + \lambda_2 u_1 + \mathcal{Q}_2(\boldsymbol{x}_1, u_1) \\ \text{s.t. } & \boldsymbol{A}_1 \boldsymbol{x}_1 = \boldsymbol{b}_1 \\ & \boldsymbol{x}_1 \geq 0 \end{aligned} \tag{1}$$

with the recourse value $Q_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_t)$ at stage $t = 2, \ldots, T$ given by:

$$\begin{aligned} Q_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_t) = \min_{\boldsymbol{x}_t, u_t} \ & \boldsymbol{c}_t^\top \boldsymbol{x}_t + \lambda_{t+1} u_t + \mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t) \\ \text{s.t. } & \boldsymbol{A}_t \boldsymbol{x}_t = \boldsymbol{b}_t - \boldsymbol{B}_t \boldsymbol{x}_{t-1} \\ & \boldsymbol{x}_t \geq 0, \end{aligned} \tag{2}$$

where

$$\mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t) = \mathbb{E}\left[(1 - \lambda_{t+1})\, Q_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{t+1}) + \frac{\lambda_{t+1}}{\alpha_{t+1}} \left[Q_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{t+1}) - u_t \right]_+ \right]. \tag{3}$$

We use stochastic dual dynamic programming to solve, or rather approximately solve, equations above. SDDP does not operate directly on these equation. Instead, we first form a sample average approximation (SAA) of the model, and SDDP approximately solves that SAA. Thus in our context SDDP forms estimators by sampling within an empirical scenario tree. In the remainder of this article we restrict attention to solving that SAA via SDDP. See Shapiro [7] for a discussion of asymptotics of SAA for multi-stage problems, Philpott and Guan [6] for convergence properties of SDDP, and Chiralaksanakul and Morton [2] for procedures to assess the quality of an SDDP-based policy.

In most common applications of stochastic dual dynamic programming , we assume stage-wise independence of the underlying random process $\boldsymbol{\xi}_t,\ t = 2, \dots, T$. Following the idea of Philpott and Matos [5] we weaken here the assumption of stage-wise independence and suppose that the underlying random process depends on state of the system, which is a Markov chain. That said, we suppose that in the initial stage, our Markov chain is in the deterministic state denoted $s_1 \in S_1,\ |S_1| = 1$. For second stage, possible states are given by a set S_2 and transition is driven by transition matrix $T_{1,2}$. Generally, transition from stage t to $t + 1$ is driven by matrix $T_{t,t+1}$.

We assume that for each stage $t = 2, \dots, T$ and each state $s_t \in S_T$ there is a known (possibly continuous) distribution P_{t,s_t} of $\boldsymbol{\xi}_t$ and that we have a procedure to sample i.i.d. observations from this distribution. Using this procedure we obtain empirical distributions $\hat{P}_{t,s_t},\ t = 2, \dots, T,\ s_t \in S_T$. The scenarios generated by this procedure all have the same probabilities, but this is not required by the SDDP algorithm, which also applies to the case where the scenario probabilities differ.

We let $\hat{\Omega}_{t,s_t}$ denote the stage t sample space for state s_t, where $|\hat{\Omega}_{t,s_t}| = D_{t,s_t}$. All possible realizations in stage t are denoted $\hat{\Omega}_t,\ \hat{\Omega}_t = \bigcup_{s_t \in S_t} \hat{\Omega}_{t,s_t}$, with $|\hat{\Omega}_t| = D_t,\ D_t = \sum_{s_t \in S_t} D_{t,s_t}$. We use $j_t \in \hat{\Omega}_t$ to denote a stage t sample point, which we call a stage t scenario. For each sample point, we can determine its actual state in Markov chain, $s(j_t) \in S_t$. We define the mapping $a(j_t) : \hat{\Omega}_t \to \hat{\Omega}_{t-1}$, which specifies the unique stage $t-1$ ancestor for the stage t scenario j_t. Similarly, we use $\Delta(j_t, s_{t+1}) : \hat{\Omega}_t \to 2^{\hat{\Omega}_{t+1,s_{t+1}}}$ and $\Delta(j_t) = \bigcup_{s_{t+1} \in S_{t+1}} \Delta(j_t, s_{t+1})$ to denote the set of descendant nodes for j_t, where $|\Delta(j_t, s_{t+1})| = D_{t+1,s_{t+1}}$ and $|\Delta(j_t)| = D_{t+1}$. The empirical scenario tree therefore has stage t realizations denoted $\boldsymbol{\xi}_t^{j_t},\ j_t \in \hat{\Omega}_t$. At the last stage, we have $\boldsymbol{\xi}_T^{j_T},\ j_T \in \hat{\Omega}_T$, and each stage T scenario corresponds to a full path of observations through each stage of the scenario tree. That is, given j_T, we recursively have $j_{t-1} = a(j_t)$ for $t = T, T-1, \dots, 2$. For this reason and for notational simplicity, when possible, we suppress the stage T subscript and denote $j_T \in \hat{\Omega}_T$ by $j \in \hat{\Omega}$.

We emphasize using the same set of D_{t,s_t} observations at stage t for state s_t to form the descendant nodes of all N_{t-1} scenarios at stage $t-1$. This ensures the resulting empirical scenario tree has required independence properties. The SDDP algorithm does not apply, for example, to a scenario tree in which we instead use a separate, independent set of i.i.d. observations $\boldsymbol{\xi}_{t,s_t}^1, \dots, \boldsymbol{\xi}_{t,s_t}^{D_{t,s_t}}$ for each of the stage $t-1$ scenarios.

In order to apply SDDP, we split the recourse values in stage t by their corresponding states and arrive in the setup which resembles the classical stage-wise independent formulation. Let's denote the probability of transition from state s_t to s_{t+1} $p_{t,t+1}$, which is an element of the matrix $T_{t,t+1}$. The equations (2) and (3) now read as follows:

$$\begin{aligned} Q_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_{t,s_t}) = \min_{\boldsymbol{x}_t, u_t} \ & \boldsymbol{c}_t^\top \boldsymbol{x}_t + \lambda_{t+1} u_t + \mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t) \\ \text{s.t. } & \boldsymbol{A}_t \boldsymbol{x}_t = \boldsymbol{b}_t - \boldsymbol{B}_t \boldsymbol{x}_{t-1} \\ & \boldsymbol{x}_t \geq 0, \end{aligned} \tag{4}$$

where

$$\mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t) = \sum_{s_{t+1} \in S_{t+1}} p_{t,t+1} \mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t, s_{t+1}). \tag{5}$$

and

$$\mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t, s_{t+1}) = \mathbb{E}_{s_{t+1}} \left[(1 - \lambda_{t+1}) \, Q_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{t+1,s_{t+1}}) + \frac{\lambda_{t+1}}{\alpha_{t+1}} \left[Q_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{t+1,s_{t+1}}) - u_t \right]_+ \right]. \tag{6}$$

We give a brief description of the SDDP algorithm in order to give sufficient context for presenting our results. For further related details on SDDP, see [3], [4] and [8]. SDDP applies to the dynamic programming equations (1), (1) and (5). During a typical iteration of the SDDP algorithm, cuts have been accumulated at each stage. These represent a piecewise linear outer approximation of the expected future cost functions, $\mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t, s_{t+1})$, separately for each possible state. On a forward pass we sample a number of linear paths through the tree. As we solve a sequence of master programs (which we specify below) along these forward paths, the cuts that have been accumulated so far are used to form decisions at each stage. Solutions found along a forward path in this way form a policy, which does not anticipate the future. In fact, the solutions can be found at a node on a sample path via the stage t master program, even before we sample the random parameters at stage $t+1$. The sample mean of the costs incurred along all the forward sampled paths through the tree forms an estimator of the expected cost of the current policy, which is determined by the master programs.

In the backward pass of the algorithm, we add cuts to the collection defining the current approximation of the expected future cost function at each stage. We do this by solving subproblems at the descendant nodes of each node in the linear paths from the forward pass, except in the final stage, T. The cuts collected at any node in stage t apply to all the nodes in that stage, and hence we maintain a *single set of cuts* for each stage, however, separately for each future state s_{t+1}. We let $C_t t+1$ denote the number of cuts accumulated so far in stage t. This reduction is possible because of our Markov property assumption.

The following model (7) is based on equations above and acts as a master program for its stage $t+1$ descendant scenarios and acts as a subproblem for its stage $t-1$ ancestor:

$$\hat{Q}_t = \min_{\boldsymbol{x}_t, u_t, \theta_t} \quad \boldsymbol{c}_t^\top \boldsymbol{x}_t + \lambda_{t+1} u_t + \theta_t \tag{7a}$$

$$\text{s.t.} \quad \boldsymbol{A}_t \boldsymbol{x}_t = \boldsymbol{b}_t - \boldsymbol{B}_t \boldsymbol{x}_{t-1} \quad : \boldsymbol{\pi}_t \tag{7b}$$

$$\theta_t \geq \sum_{s_{t+1} \in S_{t+1}} p_{t,t+1} \theta_{t,s_{t+1}} \tag{7c}$$

$$\theta_{t,s_{t+1}} \geq \hat{\mathcal{Q}}^j_{t+1,s_{t+1}} + \left(\mathbf{g}^j_{t+1,s_{t+1}} \right)^\top \left[(\boldsymbol{x}_t, \, u_t) - \left(\boldsymbol{x}^j_t, \, u^j_t \right) \right], \quad \forall s_{t+1} \in S_{t+1}, j = 1, \ldots, C_{t,s_{t+1}} \tag{7d}$$

$$\boldsymbol{x}_t \geq 0 \tag{7e}$$

Decision variable θ_t in the objective function (7a), coupled with cut constraints in (7c) and (7d), forms the outer linearisation of the recourse function $\mathcal{Q}_{t+1}(\boldsymbol{x}_t, u_t)$ from model (4) and equations (5) – (6) . The structural and nonnegativity constraints in (7b) and (7e) simply repeat the same constraints from model (4). In the final stage T, we omit the cut constraints and the θ_T term.

As we indicate in constraint (7b), we use $\boldsymbol{\pi}_t$ to denote the dual vector associated with the structural constraints. As detailed in the articles [3] and [8], this dual vector is used to develop the cuts in the backward pass of the SDDP algorithm. For simplicity in stating the SDDP algorithm below, we assume we have known lower bounds L_t on the recourse functions.

Algorithm 1. *Stochastic dual dynamic programming algorithm*

1. *Let iteration* $k = 1$ *and append lower bounding cuts* $\theta_t \geq L_t$, $t = 1, \ldots, T-1$.
2. *Solve the stage* 1 *master program* ($t = 1$) *and obtain* $\boldsymbol{x}_1^k, u_1^k, \theta_1^k$.
   Let $\underline{z}_k = \boldsymbol{c}_1^\top \boldsymbol{x}_1^k + \lambda_2 u_1^k + \theta_1^k$.
3. *Forward pass: sample independent paths from* $\hat{\Omega}$ *and index them by* S^k. *Each path is sampled by first sampling the next stage state* s_{t+1} *by corresponding probability and then by sampling a scenario from* P_{t,s_t}

   For all $j \in S^k$ {
       For $t = 2, \ldots, T$ {
           Form and solve $\mathrm{sub}(j_t)$ *to obtain* $\left(\boldsymbol{x}_t^{j_t}\right)^k$ *and* $\left(u_t^{j_t}\right)^k$;
       }
   }

   Form the upper bound estimator $\overline{z}_k$ *with one of the estimators provided in [3].*
4. *If a stopping criterion, given* $\overline{z}_k$ *and* $\underline{z}_k$, *is satisfied then stop and output first stage solution* $\boldsymbol{x}_1 = \boldsymbol{x}_1^k$ *and lower bound* $\underline{z} = \underline{z}_k$, *otherwise continue to step 5.*
5. *Backward pass:*

   For $t = T-1, \ldots, 1$ {
       For all $j \in S^k$ {
           Determine the state in stage $t+1$ *in path* j: $s_{t+1} = s(j_{t+1})$
           For all descendant nodes $i_{t+1} \in \Delta(j_t, s_{t+1})$ {
               Form and solve $\mathrm{sub}(i_{t+1})$ *to obtain* $\hat{Q}_{t+1}^{i_{t+1}}$ *and* $\boldsymbol{\pi}_{t+1}^{i_{t+1}}$;
               Calculate subgradient (see [3], [8]);
           }
           Average optimal values and subgradients (see [3], [8]);
           Append the resulting cut to the collection (7d) for stage t;
       }
   }
6. *Let* $k = k+1$ *and goto step 2 with extended sets of cuts.*

See Bayraksan and Morton [1] for stopping rules that can be employed in step 4 and Philpott et al. [5] for an alternative upper bound evaluation procedure.

4. Conclusion

We outlined a way how to incorporate non-stationary random parameters into the (Markov) SDDP method. Clearly, there is much to do further, including a discussion about convergence of such algorithm, its accuracy, and numerical study. However, even without these, our paper may give an instruction for practical situations in which a multi-stage model with a non-stationary random parameter has to be solved.

References

[1] BAYRAKSAN, G. and MORTON, D. P. (2011): *A sequential sampling procedure for stochastic programming*, Operations Research 59, pp. 898–913.

[2] CHIRALAKSANAKUL, A. and MORTON, D. P. (2004): *Assessing policy quality in multi-stage stochastic programming*, The Stochastic Programming E-Print Series.

[3] KOZMÍK, V. and MORTON, D. (2013): *Risk-averse Stochastic Dual Dynamic Programming*, Optimization Online.

[4] PEREIRA, M. V. F. and PINTO, L. M. V. G. (1991): *Multi-stage stochastic optimization applied to energy planning*, Mathematical Programming 52, pp. 359–375.

[5] PHILPOTT, A. B., DE MATOS, V. L. and FINARDI, E. C. (2012): *On solving multi-stage stochastic programs with coherent risk measures*, Optimization Online.

[6] PHILPOTT, A. B. and GUAN, Z. (2008): *On the convergence of sampling-based methods for multi-stage stochastic linear programs*, Operations Research Letters 36, pp. 450–455.

[7] SHAPIRO, A. (2003): *Inference of statistical bounds for multistage stochastic programming problems*, Mathematical Methods of Operations Research 58, pp. 57–68.

[8] SHAPIRO, A. (2011): *Analysis of stochastic dual dynamic programming method*, European Journal of Operational Research 209, pp. 63–72.

[9] ŠMÍD, M and ZAPLETAL, F and HANČLOVÁ, J (2017): *Which Carbon Derivatives Are Applicable in Practise? A Case Study of a European Steel Industry*, Kybernetika 53, pp. 1071-1085.

[10] ZAPLETAL, F and ŠMÍD, M and KOPA, M (2018): *Multi-stage emissions management of a steel company*, to appear.

Risk of the Double International Income Taxation of the Foreign Operators of Digital Platforms and a Violation of the Bilateral Tax Treaties – case of Slovakia

Jana Kubicová[1], Kristína Jančovičová-Bognárová[2]

Abstract

This paper deals with fresh phenomenon existing in the business area: taxation of rewards earned by foreign operators of digital platforms for their mediation activities in matching supply and demand for services like accommodation or transport. The aim is to portray current state of taxation of rewards earned by companies that own and operate digital platforms. Attention is paid especially to the risk of the double international juridical taxation associated with uncoordinated actions introduced by some individual countries – here it is shown an example of the Slovak Republic. The paper concludes that individual uncoordinated actions like those taken by Slovak government might have good intentions but at the same time they result in risk of the double international juridical taxation and a violation of bilateral tax treaties.

Key words

digital platforms, taxation at source, permanent establishment, tax risk, corporate income tax

JEL Classification: H26, K2, K34, K42, M21

1. Introduction

Since the early 1990s, nations have been witnessing continuous and growing application of the information technology and internet in doing business. This phenomenon deeply affects national economies worldwide.

Currently implementation of digital business models gained remarkable economic and financial extents, digitizing the global economy, advancing rapidly and leaking into all areas of the economy. Approximately one third of the increase in industrial production is the result of the use of digital technologies. In 2006, it was just one of the top 20 companies in the world, with only 7 per cent of market capitalization. In 2017, it was up to 9 of the top 20 digital companies worldwide and had a 54 per cent share of world market capitalization. Between 2008 and 2016, e-commerce revenue grew by an average of 32 per cent per year, while the traditional retail sector averaged only one per cent per year (EC, 2017a).

Boom of the digital business models brought major challenges to the area of taxation and tax administration. They were mirrored and reported by expert literature (Nellen, 2015) and by scholarly literature (Bacache, 2015), (Bal, 2015), (Basu, 2016), (Li, 2003), (Raponi & O'Sullivan, 2015), (Spengel & Olbert, 2017), (Traversa & Ceci, 2015), (Vosloo, 2016), (IBFD, 2018).

The aim of this paper is to portray current state of taxation of rewards earned by companies that own and operate digital platforms. In particular the paper focuses on the problem of factual non-taxation of rewards earned by the foreign operators of digital platforms in source countries,

[1] Assoc. Prof., Ing. et Mgr. Jana Kubicová, PhD., MBA, University of Economics in Bratislava, Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic. e-mail: jana.kubicova@euba.sk
[2] Ing. Kristína Jančovičová-Bognárová, PhD. University of Economics in Bratislava, Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic. e-mail: kristina.jancovicova-bognarova@euba.sk

and discusses pioneering measures taken individually by the Slovak republic. Special attention is paid to the risk of double international juridical taxation that will most likely occur as a result of new tax provisions.

The paper proceeds as follows. Next section shows current situation in taxation of rewards of operators of digital platforms in the country of source. It also presents selected results of survey research taken by the European Commission concerning perception of income taxation of foreign operators of digital platforms. Then the third section presents analysis of the pioneering national tax law provisions taken by the Slovak Republic, and the section four discusses consequences of newly adopted measures, especially risk of double international taxation and violation of bilateral tax treaties. Finally, conclusion summarizes results of the research.

2. Current State of Taxation of Rewards Paid to Foreign Operators of Digital Platforms

2.1 The Status Quo: Non-Taxation of Rewards in the Country of Source

Digital platform is product of information technology and serve as intermediary instrument that helps to provide various kinds of services, among others accommodation or transport, two examples are Airbnb and Uber. They serve as a global virtual market place without borders, where suppliers offer services and inform about prices, and customers search for appropriate services. Digital platforms play brokering role in intercession between suppliers/providers of services and customers, who search for those services, helping them to match supply and demand for services, conclude contract, execute payment of full service price by buyer, and get receivables by seller of service for delivered service.

Digital platforms are operated by large companies that often reside in the USA or in the Netherlands. For their brokering role the operators of the digital platforms charge fees from providers of services. Fees charged by operators of digital platforms are subtracted from the full price a customer pays, thus provider of service receives the rest of full payment paid by customer.

Fees (e.g. 3%) are charged by operator of digital platform from the full payment (100%), and provider of service receives the rest of payment (e.g. 97%). Neither withholding tax is imposed on fees paid abroad nor is taxation of business of profit of permanent establishment applicable in the country of source. There are more details about tax law behind this phenomenon in the section 2.2. Non-taxation of fees paid from country of source to operators of digital platforms evokes further consequences.

The described *modus operandi* is friendly to customers and suppliers, and in addition, and to the contrast to the traditional “stone” agencies, it is global and operates without borders, thus suppliers and customers may benefit from all advantages global market size offers. However, alongside benefits, it creates several serious glitches. Let's describe them briefly.

- *Efficiency of taxation of income.* Efficiency of taxation of income and wealth associated with digital business models is newborn challenge to economists, designers of the tax system, politicians and tax administrations. Fees paid to operators of digital platforms represent abundant amount, however they are not subject to tax anywhere.
- *Fairness of taxation.* Non-taxation of fees in the country of source is perceived as unfair taxation especially by traditional "stone" companies as they must pay income taxes what in turn reduces their profit after tax.
- *Violation of fair competition*. Non-taxation of income in the country of source jeopardizes the competitive environment due to a significantly higher tax burden on traditional “stone” forms of business compared to companies that use digital platforms.

This undermines healthy competitive environment. The price competitiveness of traditional businesses is also under pressure as they are pushed to shift tax burden forward to customers.

- *Less than potential tax revenues*. There is the challenge to fill the gap between the potential and actual tax revenues of government budgets.
- *Free rider problem.* Consumption of services in the country where it is executed is surrounded by positive externalities that result from public finance expenditures. However, providers of services do not contribute fair amount to public finance.

The European Commission organised survey research with intention to gather opinions of taxpayers about taxation of digital economy. The questionnaire asks, among other also the following questions (labelled as Q). **Chyba! Nenalezen zdroj odkazů.** depicts detail about the responses.

Q1 To which extent do you agree with the following statements? : The current situation could push some Member States toward adopting uncoordinated measures that would lead to the fragmentation of the Single Market.

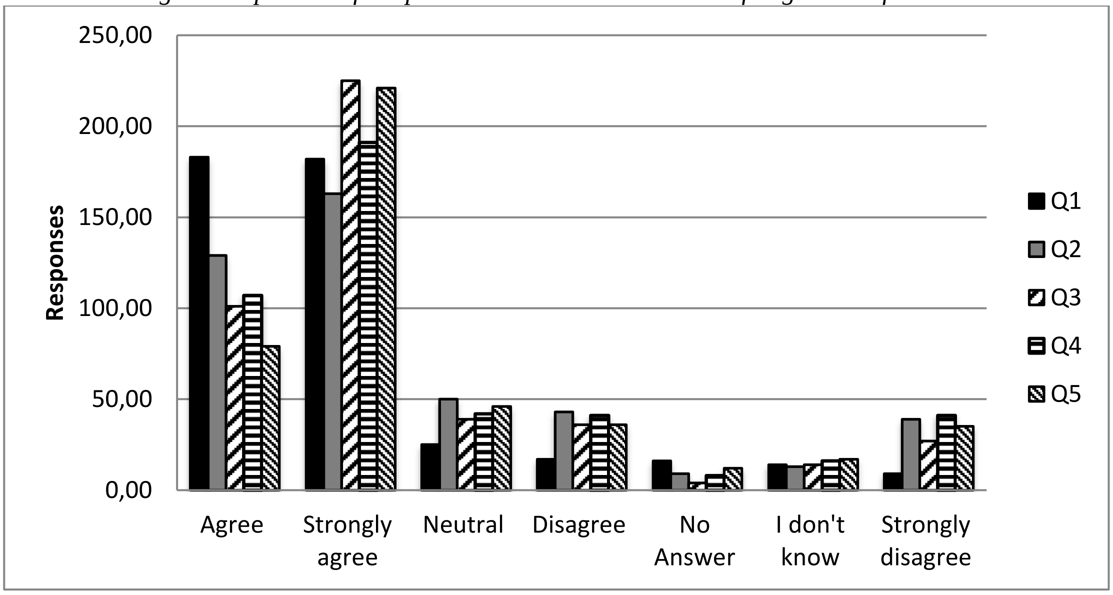
Q2 To which extent do you agree with the following statements? : The current international taxation rules do not allow for fair competition between traditional and digital companies.

Q3 To which extent do you agree with the following statements? : The current international taxation rules allow digital companies to benefit from certain tax regimes and push down their tax contributions.

Q4 To which extent do you agree with the following statements? : States are not able to collect taxes on the value that some digital companies create on their territory.

Q5 To which extent do you agree with the following statements? : Social fairness is impacted because some digital companies do not pay their fair share of taxes.

Figure 1 Opinions of Respondents on Income Taxation of Digital Platforms



Source: author´s own compilation based on EC data (EC, 2017e)

2.2 Legal Analysis: the Tax Law behind the Non-Taxation of Rewards Earned by the Foreign Operators of the Certain Types of Digital Platforms - the Gap

As it was said above, rewards earned by foreign operators of the certain types of digital platforms are taxed neither in the country where provider of services resides, nor in the country where service is executed. Note that these countries may be identical but it is not a rule.

As it is well known, in majority of countries all around the world taxes are proposed by governments and approved by Parliaments, and tax rules are introduced by legal acts. It implies that theory and policy of taxation is subject of interdisciplinary studies, mainly economics, finance and law. This paper is conceptual paper and employs analysis of tax law, international and national, with intention to uncover law provisions that lie behind the non-taxation of rewards earned by the foreign operators of the certain types of digital platforms. This subsection presents analysis of the tax law, international or national, which lies behind non-taxation of rewards earned by the foreign operators of the certain types of digital platforms.

There are following core elements of the national and international tax laws which lie behind the non-taxation of rewards earned by the foreign operators of the certain types of digital platforms:

1 According to the OECD Model Tax Convention (OECD MTC), and national bilateral tax treaties that are based on the OECD MTC, a permanent establishment is not defined based on the presence of digital platform operations performed on the territory of the host country.
2 According to the majority of national tax laws a digital platform does not constitute permanent establishment.
3 Rewards earned by the foreign operators of some digital platforms are not subject to any withholding tax anywhere - either in the country where provider of services resides or in the country where services are consumed or executed.

Ad 1. The *Article 7* of the OECD MTC states*: "*Profits of an enterprise of a Contracting State shall be taxable only in that State unless the enterprise carries on business in the other Contracting State through a permanent establishment situated therein. If the enterprise carries on business as aforesaid, the profits that are attributable to the permanent establishment … may be taxed in that other State." (OECD, 2017). It implies that the right to tax business profit belongs to the Contracting State where the company resides, unless a company has permanent establishment in the other Contracting State. Consequently, a definition of a permanent establishment is critical.

According to the *Article 5 of the OECD MTC: „*For the purposes of this Convention the term "permanent establishment" means a fixed place of business through which the business of an enterprise is wholly or partly carried." A list of situations when permanent establishment occurs includes: a place of management, a branch, an office, a factory, a workshop, a mine, an oil or gas weil, a quarry or any other place of extraction of natural resources. In addition, if time test is met, a construction permanent establishment occurs, and if certain conditions are met, the agency permanent establishment occurs. The key however is, that neither of listed situations include digital platform. It implies, that according to the OECD MTC usage of digital platform to assist provision of services in the country does not constitute permanent establishment.

Ad 2. When host country hosts business activities of an enterprise that resides in a non-Contracting State taxpayers are not eligible to apply treaty definition of permanent establishment. For this situations host countries incorporate definition of a permanent establishment into the *national* tax law. For example the Income Tax Act in the Slovakia defines permanent establishment in its section 16 para 2.

Ad 3. Rewards earned by the foreign operators of the certain types of digital platforms are not subject to the withholding tax in countries where services are executed.

To summarize, there is neither appropriate international nor national tax provisions that would enable taxation of rewards earned by the operators of the certain types of digital platforms who reside in abroad. As a result this rewards are not subject to income tax in a country of source. Next section describes remedies to non-taxation as developed and adopted by the Slovak Republic individually.

3. Remedies Taken by Individual Countries – example of Slovakia

From January 1, 2018 there has been in effect adjustment of the Income Tax Act effective in Slovakia (hereinafter ITA). In the Slovak tax law there are five new key tax provisions related to the taxation of rewards earned by the foreign operators of digital platforms. The system of new rules is built from the following elements:

1 ***Widened definition of a permanent establishment***. Adjusted definition of a permanent establishment incorporated into the ITA states:
   - „Recurrent mediation of transport and accommodation services is also considered to be a permanent activity in the territory of the Slovak Republic, including through a digital platform."

2 ***Widened list of incomes sourced on the territory of Slovakia.*** Rewards for mediation activities become to be deemed as income sourced from the territory of Slovakia. Section 16 letter e) point 10 of the ITA states:
   - Income from sources in the territory of the Slovak Republic of a taxpayer with limited tax liability is income from reimbursements from taxpayers with unlimited tax liability and from permanent establishments of taxpayers with limited tax liability, among others rewards for providing services of commercial, technical or other advice, for data processing, for marketing services, for management activity and for mediation activities, in the amount in which this remuneration is simultaneously recognized as tax expense under Section 19 ITA.

3 ***Withholding tax imposed on rewards for mediation activities*** paid to foreign operators of digital platforms. Section 43 para 3 ITA states:
   - The tax on income from sources in the territory of the Slovak Republic to taxpayers with limited tax liability except for the income of the permanent establishments registered in accordance with § 49a / 5 shall be deducted with a deduction in respect of income under § 16/1 / e-10 , i.e. rewards for mediation activities.

4 ***Registration obligation*** and notifying obligation. Section 49a ITA Coll. of Laws obliges persons to ask Tax Administration for registration for income tax purposes if:
   a) natural or juridical persons have licence to make business in Slovakia;
   b) if natural person becomes self-employed or starts to rent real estates in Slovakia;
   c) person is obliged to withhold tax and is has not been registered under points a) or b);
   d) natural or juridical person who has permanent establishment on the territory of Slovakia;
   e) If persons who shall ask Tax Administration of the Slovak Republic do not fulfil their obligation willingly, the ITA states that Tax Administration can register such persons *ex offo*.

5 ***Elimination of double international juridical taxation – new procedure***. Section 42 para 5 states:
   - „More detailed procedures for the application of the double taxation treaty in relation to the taxpayer under § 2 d) and e) shall be determined by the Ministry."

4. Discussion: Risk of the Double International Taxation and Violation of Bilateral Tax Treaties

The described system of new tax provisions incorporated into the ITA raises a serious tax risk - the risk of double international juridical taxation, a violation of bilateral tax treaties signed by Slovakia and risk that injured taxpayers will initiate international arbitrage to defend their rights guaranteed by the effective bilateral tax treaties.

If foreign operators of digital platforms were by the Tax Administration authority *ex offo* registered for income tax purposes and taxed, there would be a risk of violation of valid bilateral tax treaties signed by Slovakia. The reason is that no bilateral tax treaty signed between Slovakia and other Contracting States does incorporate adjusted definition of permanent establishment, i.e. permanent establishment based on digital platform operations in the host country. Here is the list of the underlying law provisions:

- Under **Article 3(2) of the Slovak-USA Tax Treaty Convention**: 'As regards the application of the Convention by a Contracting State *any term not defined therein* shall, unless the context otherwise requires or the competent authorities agree to a common meaning pursuant to the provisions of Article 26 (Mutual Agreement Procedure), have the meaning which it has under the laws of that State concerning the taxes to which the convention applies." Court of Justice of the European in case C-648/15 (Germany-Austria bilateral tax treaty) approved, that interpretation of *terms that are not defined in Convention* shall follow national laws. However, terms that are defined in the Convention cannot be interpreted according to the national laws. It suggests that Slovak national tax law is not applicable for the purpose of interpretation of permanent establishment, as this concept is appropriately defined in the Convention. Only if permanent establishment was not defined by Convention, it would be possible to use definition of permanent establishment from national tax law.
- Under **Article 31(1) of the Vienna Convention on the Law of Treaties** reads as follows: 'A treaty shall be interpreted in good faith in accordance with the ordinary meaning to be given to the terms of the treaty in their context and in the light of its object and purpose.'
- **Article 25(1) of the majority of bilateral tax treaties** signed by Slovakia provides that any person who considers him to have been prejudiced by taxation contrary to that convention may initiate a mutual agreement procedure between the competent authorities of the States Parties. Under Article 25(5) of the OECD MTC 'In the case of difficulties or doubts concerning the interpretation or the application of this convention, for which no solution can be found during a mutual agreement procedure between the competent authorities arranged in accordance with the preceding paragraphs of this article within a period of three years from the initiation of that procedure, the States Parties are obliged, at the request of the person referred to in paragraph 1, to submit the dispute to the arbitration. However, BTT between Slovakia and the USA has been effective since January 1993 and this provision is missing there.

Violation of bilateral tax treaties might not occur if there was dualistic international legal system, not monistic with prevalence of the international public law. For more details about monistic and dualistic legal systems see (Kubicová & Záhumenská, 2016), and (Koróncziová & Kačaljak, 2017). However, this is not the case of Slovakia as it has monistic legal system with prevalence of the international public law.

5. Conclusion

To summarise, current non-taxation of rewards earned by the foreign operators of certain types of digital platforms is perceived as unfair and international and national tax laws do not solve the problem properly. The individual separate actions of some countries, this paper presented case of Slovakia, are to the certain point beneficial as they make at least trial to solve problem and show possible intelligent solutions. The adverse side of entering such an individual pioneering path is that individual actions may nurture risk of a double international juridical taxation and violation of bilateral tax treaties.

To eliminate risk of double international juridical taxation a tricky tax provision was incorporated into the ITA. Section 42 para 5 states that: "More detailed procedures for the application of the double taxation treaty in relation to the taxpayer with unlimited and limited tax liability shall be determined by the Ministry. " In another words, elimination of double international juridical taxation can be made on the case-by-case basis. However, this provision neglects superiority of the bilateral tax treaties. Then a question springs into our minds: what are the bilateral tax treaties for in Slovakia?

Acknowledgments

This paper is 0,9 result of the research project supported under the scheme VEGA 1/0776/16 title "Taxation of Financial Sector and Harmonization Tendencies in the EU" funded by the Ministry of Education, Science, Research and Sport of the Slovak Republic and 0,1 result of the APVV-15-0322 project.

References

[1] Act No. 595/2003 Coll. Income Tax Act as amended by amendment 264/2017 Coll.

[2] Bacache, M., 2015. *Taxation and the Digital Economy: A Survey of Theoretical Models. Toulouse School of Economic.* [Online] https://ec.europa.eu/futurium/en/system/.../ficalite_du_numerique_9_mars_13_h.pdf

[3] Bal, A., 2015. Taxation of the Digital Economy. In: *International Tax Structures in the BEPS Era: An Analysis of Anti-Abuse Measures.* Amsterdam: IBFD.

[4] Budak, T., 2017. The Transformation of International Tax Regime: Digital Economy. *Inonu University Law Review InULR* 8 (2) , p. 297-329

[5] EC, 2017a. *Communication from the Commission to the European Parliament and the Council. A Fair and Efficient Tax System In The European Union for the Digital Single Market* , Brussels: European Commission.

[6] EC, 2017d. *Fair taxation of the digital economy. Questionnaire.* Brussels: European Commission.

[7] EC, 2017e. *Statistics: Fair Taxation of the Digital Economy.* Brussels: European Commission.

[8] Hongler, P. & Pistone, P., 2015. Blueprints for a New PE Nexus to Tax Business Income in the Era of the Digital Economy. *IBFD White Paper*, 20 January.

[9] IBFD, 2018. Navigating the Tax Challenges of the Digital Economy. *Bulletin for International Taxation (Special Issue)*, April.

[10] Koróncziová, A. & Kačaljak, M., 2017. GAAR as Tax Treaty Override - Slovak perspective. *DANUBE: Law and Economics Review*, 8(3), p. 139–155.

[11] Li, J., 2003. *International Taxation in the Age of Electronic Commerce: A Comparative Study*, Toronto: Canadian Tax Foundation.

[12] Nellen, A., 2015. Taxation and Today´s Digital Economy. *San José State University Papers*, pp. 27-37.

[13] OECD, 2015. *Addressing the Tax Challenges of the Digital Economy, Action 1 - 2015 Final Report.* Paris: OECD.

[14] OECD, 2017. *Model Tax Convention on Income and Capital. Condensed Version 2017.* Paris: OECD Publishing.

[15] OECD, 2018. *Tax Challenges Arising from Digitalisation - Interim Report 2018.* Paris: OECD.

[16] Raponi, D. & O´Sullivan, D., 2015. VAT and Taxation of the Digital Economy from the Perspective of the EU Policy Marker. In: *VAT and Digital Economy.* Alphen aan den Rijn: Wolters Kluwer Pub.

[17] SKDP, 2018. *Digitálne platformy čaká povinná registrácia na Finančnej správe.* [Online] http://danovecentrum.sk.

[18] Traversa, E. & Ceci, E. , E., 2015. VAT Fraud and the Digital Economy within the EU: Risks and Opportunities. In: *VAT and Digital Economy.* Alphen aan den Rijn: Wolters Kluwer Pub.

[19] UN, 1969. *Vienna Convention on the Law of Treaties of 23 May 1969.* New York: United Nations Treaty Series.

Do Firms' Loans and Visits of Tax Officials Relate to the Corruption Behaviour of Tax Officials?

Jana Kubicová[1], Bruno S. Sergi[2]

Abstract

Tax compliance of taxpayers is one of the major determinants of narrowing a gap between potential and real government budgets' tax revenues. There are two main foundations of taxpayers' non-compliance, one is tax evasion or avoidance behaviour, and another is corruption of tax officials who are expected to control taxpayers' compliance. Numerous empirical studies analysed possible reasons of taxpayer’s non-compliance and corruption of tax officials. This paper uses indicators published by Enterprise Surveys and search whether visits of tax officials, tax administration, tax rates and firms' bank loans are related to corruption behaviour of tax officials during their visits to entrepreneurs. To test association between above indicators, a correlation analysis and data for 146 countries are used. The results show that the more visits of tax officials the higher is an expectation of giving gifts to tax officials. To the contrast, firms' bank loans are statistically significantly negatively correlated with indicator of tax officials' corruption behaviour.

Key words

firms, bank loans, tax officials, corruption behaviour, correlation, Enterprise Survey indicators

JEL Classification: H26, K2, K34, K42, M21

1. Introduction

Tax compliance of taxpayers is one of the major determinants of narrowing a gap between potential and real tax revenues of government budgets. Various studies analysed possible reasons of taxpayer’s compliance, among them seminal works published by Allingham and Sandmo (Allingham & Sandmo, 1972), and Becker (Becker, 1968), (Becker & Stigler, 1974), recently Alm analysed what motivates tax compliance (Alm, 2017).

There are two main foundations of tax non-compliance, one is tax evasion or avoidance, and another is corruption of tax officials who are expected to control taxpayers' compliance. An empirical research about causes and consequences of corruption was published by Dimant and Tosato (Dimant & Tosato, 2017); the economics of corruption was studied by Rose-Ackerman (Rose-Ackerman, 1975), and Teichmann and Sergi studied business risk in bribery (Teichmann & Sergi, 2018).

Several studies paid attention more narrowly – to corruption in civil administration (Mabroor , 2005), and specifically to corruption in tax administration, for example Epaphra and Massawe (Epaphra & Massawe, 2017), ethics in tax administration (Visockaite & Birskyte, 2013), and the link between corruption, taxation and tax evasion was analysed by Alm and Liu (Alm & Liu, 2017). Escobari indicated imperfect detection of tax evasion in a corrupt tax administration (Escobari, 2012), Tjen and Evans studied causes and consequences of corruption in tax administration in Indonesia (Tjen & Evans, 2012). Numerous studies paid attention to factors determining corruption, for example Awasthi and Bayraktar studied whether tax simplification

[1] Assoc. Prof., Ing. et Mgr. Jana Kubicová, PhD., MBA, University of Economics in Bratislava, email: jana.kubicova@euba.sk
[2] Assoc. Prof., Bruno S. Sergi, PhD., Davis Centre for Russian and Eurasian Studies, Harvard University USA and University of Messina, email: bsergi@fas.harvard.edu

can help to lower tax corruption (Awasthi & Bayraktar, 2015). Other studies paid closer look at tax corruption and private sector development (Nguyen, et al., 2017).

To study corruption behaviour of tax officials, let's first define shortly corruption. "Corruption is the abuse of entrusted power for private gain." (Transparency International, 2018). Transparency International distinguishes between the following corrupt behaviours in the public sector: bribery, diversion of public funds, use of public office for private gain, nepotism in the civil service and State capture (Transparency International, 2017).

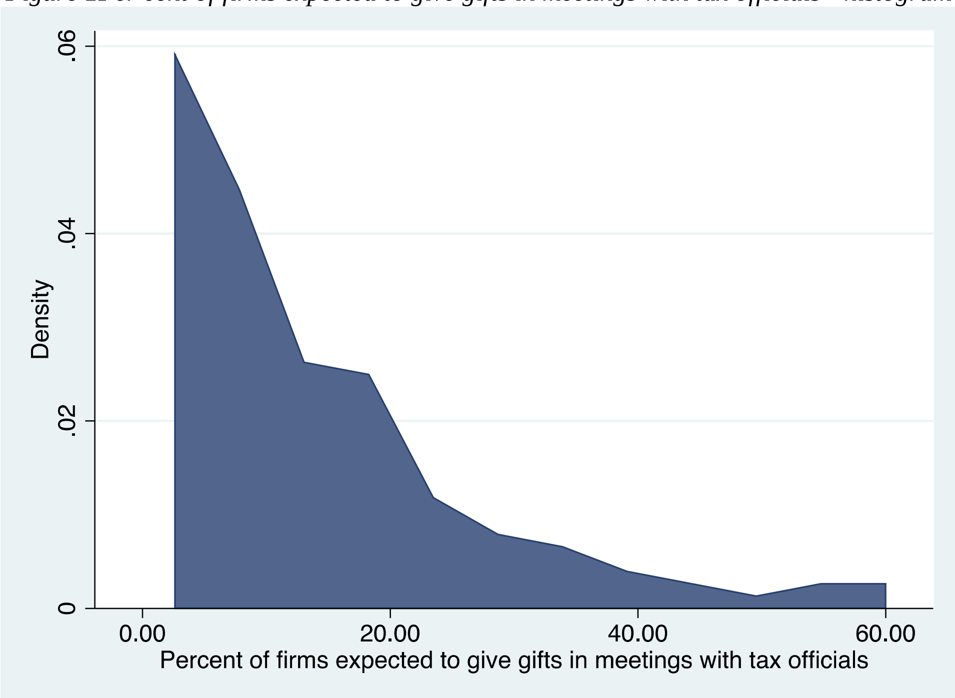
This paper pays attention to the corruption behaviour of tax officials, focusing on the usage of tax administration for private gain, specifically in the form of expecting gifts from visited taxpayers. It employs indicators published by the Enterprise Surveys and search for two possible factors of this kind of corruption behaviour in tax administration: first, number of visits of tax officials to entrepreneurs, and second firms' bank loans. The aim is to test whether visits of tax officials and firms' bank loans are associated with expectation of giving gifts to tax officials during their visits to entrepreneurs.

The paper is organised as follows. The next section formulates hypothesis and presents data. To test association between variables the correlation analysis was used. Third section presents results of the correlation analysis and discusses them. Finally, conclusion summarises results.

2. Hypothesis and Data

This paper focuses on the corruption behaviour of tax officials, in particular on expectation of firms giving gifts to tax officials in meetings with them. Figure 1 depicts density of per cent of firms expected to give gifts in meetings with tax officials. It is clear, that the problem of this kind of corrupted behaviour of tax officials is not negligible, in contrast, it represents serious problem. Leaving this problem untouched might have consequences, as this may lead to the intentionally lower corporate income tax payable by firms and consequently less than potential tax revenues received by general government budgets.

Figure 1Per cent of firms expected to give gifts in meetings with tax officials - histogram



Source: authors´ own work based on the Enterprise Survey 2018 (IFC, 2018)

The research in this paper focuses on the possible association between expectation of giving gifts by firms to tax officials in meeting them and indicators of number of visits of tax officials, perception of tax administration, tax rates, and firms' bank loans. The following hypotheses are formulated:

- *Hypothesis 1:* Per cent of firms visited or required to meet with tax officials is related to the per cent of firms expected to give gifts in meetings with tax officials;
- *Hypothesis 2*: Average number of visits or required meetings with tax officials is related to the per cent of firms expected to give gifts in meetings with tax officials;
- *Hypothesis 3*: Per cent of firms identifying tax administration as a major constraint is related to the per cent of firms expected to give gifts in meetings with tax officials;
- *Hypothesis 4*: Per cent of firms identifying tax rates as a major constraint is related to the per cent of firms expected to give gifts in meetings with tax officials;
- *Hypothesis 5*: Per cent of firms with a bank loan/line of credit is related to the per cent of firms expected to give gifts in meetings with tax officials;

To test above hypotheses the correlation analysis is applied, Pearson correlation coefficient is calculated per each pair of variables. Pearson’s *r* measures the linear relationship between two variables and its value describes the strength of the association between them. There can be positive, negative or no association. To determine statistically significant correlation it is detected significance at the 0,05 level (2-tailed) for each correlation coefficient.

To run correlation analysis there are used data published by the Entrepreneur Surveys edition, specifically indicators of expectation of giving gifts to tax officials in meeting them, visits of tax officials, perception of tax administration and tax rates by entrepreneurs, and percentage share of firms with bank loans. Number of observation per each indicator is 146, each observation represents one country in the world. Exact names of indicators and variables are listed in the Table 1, while a Table 2 presents the main descriptive statistics of each variable.

Table 1 Indicator Descriptions

Variable name	**Indicator***
gifts	per cent of firms expected to give gifts in meetings with tax officials
taxrates	per cent of firms identifying tax rates as a major constraint
taxadmn	per cent of firms identifying tax administration as a major constraint
visits	per cent of firms visited or required to meet with tax officials
visitsavgnbr	if there were visits, average number of visits or required meetings with tax officials
bkloans	per cent of firms with a bank loan/line of credit

*Legend: *Indicators are sourced from Enterprise Surveys* (IFC, 2018).
Source: authors´ own work

Table 2 Description of Variables

Variable name	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
gifts	146	12.98151	12.73667	0	62,6
visits	146	57.35342	21.37691	8.9	94.9
visitsavgnbr	146	2.761644	1.758575	1.1	18,8
taxrates	146	31.73219	17.84149	1.1	81.5
taxadm	146	63.20548	38.643	1	129
bkloans	146	33.18699	17.46847	2.7	79.6

Source: authors´ own computation

3. Results and Discussion

3.1 Results

This part presents results of the correlation analysis.

Table 3 shows Pearson coefficients and their significance on the 0,05 levels. Out of all pairs between **gifts** and the rest of variables, the results of a correlation analysis show statistically significant positive correlation between gifts and visits, gifts and average number of visits. As for perception of tax administration and tax rates as main constraints, both of them are positively correlated to the expectation to give gifts to tax officials in meetings, however the relationship is not statistically significant. Surprisingly, statistically significant is negative correlation between gifts and bank loans.

Table 3 Correlation matrix

variable	gifts	visits	visitsavgnbr	taxadmn	taxrates	bkloans
gifts	1,0					
visits	**0.3828*** *0,0000*	1,0				
visitsavgnbr	**0.2748*** *0.0008*	**0.3455*** *0,0000*	1,0			
taxadmn	0.1433 *0.0845*	0.1072 *0.1977*	0.0529 *0.5258*	1,0		
taxrates	-0.0118 *0.8875*	**0.2080*** *0.0118*	0.0338 *0.6850*	**0.2590*** *0.0016*	1,0	
bkloans	**-0.4573*** *0.0000*	**-0.3615*** *0.0000*	**-0.2092*** *0.0113*	-0.0823 *0.3234*	0.0294 *0.7243*	1,0

Source: authors´ own work

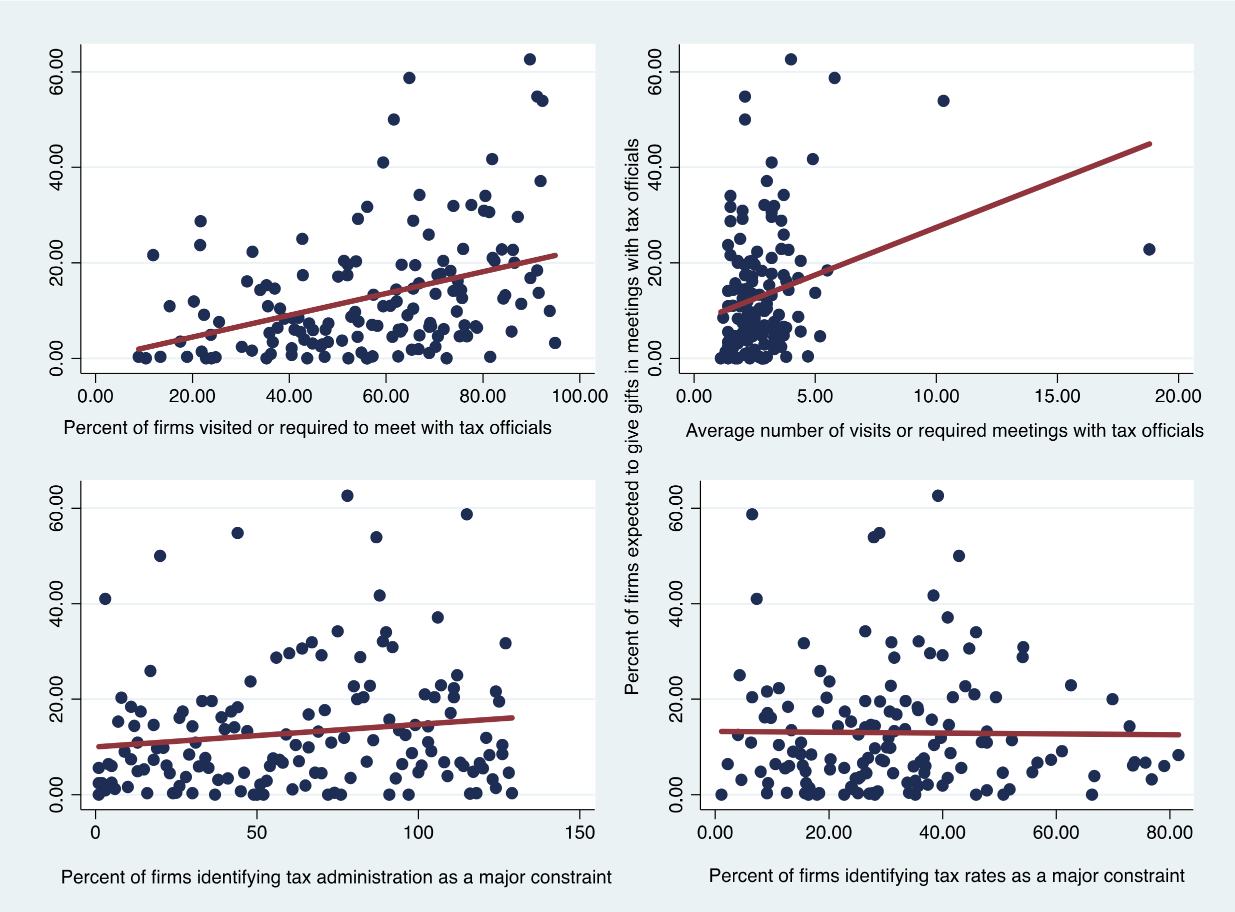
These results are discussed in two separate parts, as visits, average numbers of visits, perception of tax administration and tax rates have certain common ground. The relation between firms' bank loans and gifts is discussed solely 3.3. In interpreting and discussing above results authors take into account, that found correlations indicate two-way relationship, and cannot be interpreted as causal relationships.

3.2 Association of Visits of Tax Officials with Expected Bribery of Tax Officials

Figure 2 depicts details about relationships between expectation of giving gifts to tax officials in meetings them and a per cent of visited or met tax officials, a perception of tax administration and tax rates.

Strong positive correlation between visits or meetings of entrepreneurs with tax officials is not quite a big surprise. It asserts that personal meetings of entrepreneurs with tax officials provide more opportunities to give gifts to tax officials. This relation may be stronger when taxpayers may have intention to pay less tax than they should or if they want to avoid penalties for non-compliance. However, even if there is more space for giving gifts, there can be several techniques to prevent corruption behaviour of tax officials. The list can start with employing just highly moral and ethical persons to work in tax administration, continues with adoption of more non-personal communication between taxpayers and tax officials, by implementation of more electronic-based tools, and finally involving agents to make corrupted behaviour of tax officials scarier.

Figure 2 Table of scatterplots with trend lines for visits, average number of visits, tax rates, tax administration and gifts expected during visits



Source: author´s own compilation

The positive association between perceptions of tax administration as a main constrain for entrepreneurship and expectation of giving gifts to tax officials is not a matter to wonder too.

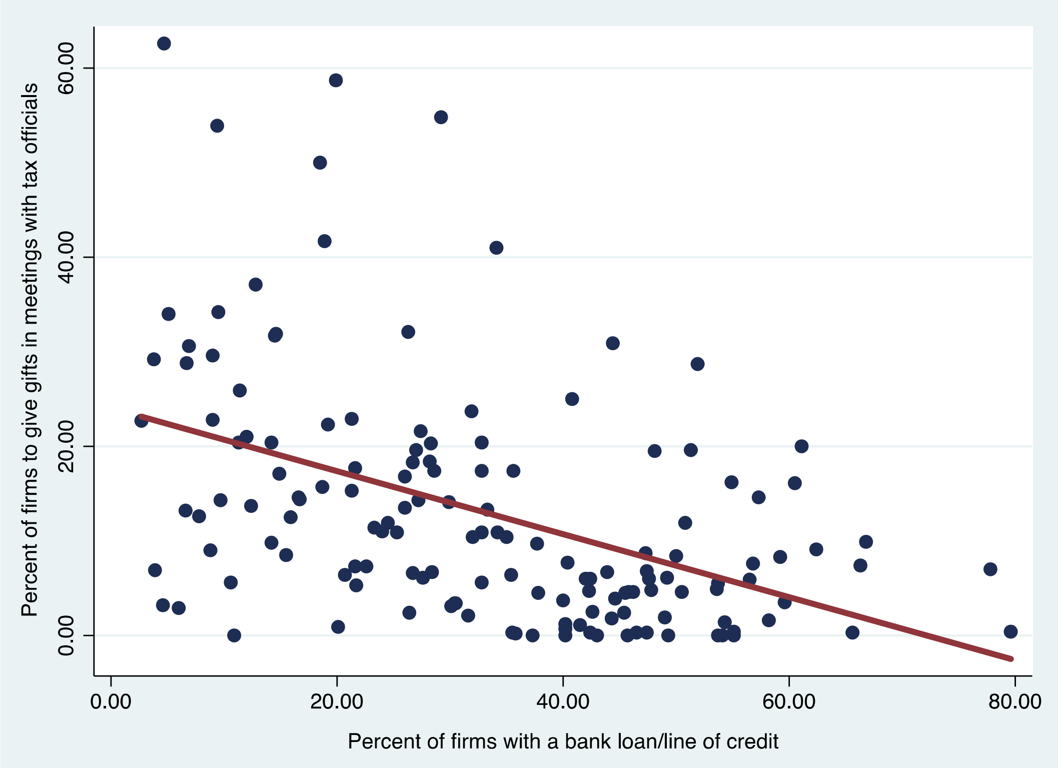
Finally, there is statistically non-significant positive relationship between perception of tax rates as a main constraint for firms and expectation of giving gifts to tax officials in meeting them. The more are tax rates perceived as a main constraint to entrepreneurs, the higher is an expectation of giving gifts to tax officials in meetings

3.3 Association of Firms' Bank Loans with Expected Bribery of Tax Officials

To our knowledge, bank loans of firms, or indebtedness of firms and its association to corrupted behaviour of tax officials has not been empirically studied yet. The results of correlation analysis shows, that there is statistically significant *negative* association between firms' bank loans and percentage share of firms to give gifts in meetings with tax officials.

A bit surprisingly, the higher is the percentage share of firms with a bank loan the lower is the percentage share of firms to give gifts in meetings with tax officials. It implies that the more indebted firms the fewer tendencies to bribery behaviour, and *vice versa*. The question is what might lie behind this phenomenon.

Figure 3 Firms with a bank loan and expected gifts to the tax officials



Source: author´s own work

One explanation, which springs to mind, could be that the more indebted firms do not have enough finance to provide gifts to tax officials. Another possible explanation could be, that the indebtedness creates tax costs in the form of interest deductible when computing corporate income tax base. Thus firms with debt might have lower corporate income tax base and thus the lower corporate income tax and if it is unpaid, then the lower tax debt and the lower penalty. It might imply that indebted firms that are visited by tax officials do not have enough motivation to corrupt tax officials who visit them to check their tax compliance. To explain this relationship further research is needed.

4. Conclusions

This paper presents results of research which studies relation between corruption behaviour of tax officials represented by expecting gifts from entrepreneurs during visits or meetings with entrepreneurs. It also presents research results of correlation analysis that tested relation between number of indebted firms and expectation of giving gifts to tax officials.

To test those relationships the correlation analysis was run on data from 146 countries published by Entrepreneur Surveys. The results show strong positive correlation between expecting gifts and visits or meetings of tax officials with entrepreneurs, positive but not statistically significant relationship with perception of tax administration indicator and perception of tax rates indicator. It was discovered strong negative correlation between firms' bank loans and expectation to give gifts to tax officials.

The limitation of research arises from applied research method - a correlation analysis. Should researchers, politicians or governments wanted more knowledge about causality between visits and expecting gifts, then other research method, such regression analysis must be applied. Challenging for the further research is also explanation of negative correlation between firms' bank loans and expecting gifts by tax officials.

Acknowledgments

This paper is 0,9 result of the research project supported under the scheme VEGA 1/0776/16 "Taxation of Financial Sector and Harmonization Tendencies in the EU" funded by the Ministry of Education, Science, Research and Sport of the Slovak Republic and 0,1 result of an APVV-15-0322 project.

References

[1] Allingham, M. G. & Sandmo, A., 1972. Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics,* 1(3-4), pp. 323-338.

[2] Alm, J., 2017. 'What Motivates Tax Compliance?. *Department of Economics Working Paper.*

[3] Alm, J. & Liu, Y., 2017. Corruption, taxation, and tax evasion. *eJournal of Tax Research, suppl. Special Edition: Tax and Corruption,* December, 15(2), pp. 161-189.

[4] Awasthi, R. & Bayraktar, N., 2015. Can tax simplification help lower tax corruption?. *Eurasian Economic Review,* 5(2), pp. 297-330.

[5] Becker, G. S., 1968. Crime and Punishment - An Economic Approach. *Journal of Political Economy,* 76(2), pp. 169-217.

[6] Becker, G. S. & Stigler, G. J., 1974. 'Law Enforcement, Malfeasance, and Compensation of Enforcers. *Journal of Legal Studies,* 3(1), pp. 1-18.

[7] Dimant, E. & Tosato, G., 2017. 'Causes and Consequences of Corruption: What Has Past Decade's Empirical Research Taught Us? A Survey. *Journal of Economic Surveys.*

[8] Epaphra, M. & Massawe, J., 2017. Corruption, governance and tax revenues in Africa. *Business and Economic Horizons,* 13(4), pp. 439-467.

[9] Escobari, D., 2012. Imperfect Detection of Tax Evasion in a Corrupt Tax Administration. *Public Organization Review,* 12(4), pp. 317-330..

[10] IFC, 2018. *Enterprise Surveys.* [Online] Available at: http://www.enterprisesurveys.org [Cit. 18 July 2018].

[11] Mabroor , M., 2005. Corruption in Civil Administration: Causes and Cures. *Humanomics,* 21(3/4), pp. 62-84.

[12] Nguyen, N. A., Doan, Q. H. & Tran-Nam, B., 2017. Tax corruption and private sector development in Vietnam. *eJournal of Tax Research, suppl. Special Edition: Tax and Corruption,* December, 15(2), pp. 290-311.

[13] Rose-Ackerman, S., 1975. The Economics of Corruption. *Journal of Public Economics,* 4(1), pp. 1-17.

[14] Ross , A. M. & McGee, R. W., 2012. Education Level and Ethical Attitude toward Tax Evasion: A Six-Country Study. *Journal of Legal, Ethical and Regulatory Issues,* 5(2), pp. 93-138.

[15] Teichmann, F. M. & Sergi, B. S., 2018. *Compliance in Multinational Organisations. Business Risks in Bribery, Money Laundering, Terrorism Financing and Sanctions.* Bingley: Emerald Publishing Limited.

[16] Tjen, C. & Evans, C., 2012. Causes and consequences of corruption in tax administration: An Indonesian case study. *eJournal of Tax Research, suppl. Special Edition: Tax and Corruption*, December, 15(2), pp. 243-261.

[17] Transparency International, 2017. *Corruption Perception Index 2017: Technical Methodology Note,* washington: Transparency International.

[18] Transparency International, 2018. *Anti-corruption Glossary*, Washington: Transparency International.

[19] Transparency International, 2018. *Corruption Perception Index,* s.l.: s.n.

[20] Visockaite, A. & Birskyte, L., 2013. Ethics in Tax Administration. *Socialiniu Mokslu Studijos*, 5(3).

The scope and quality of financial risk disclosures on the example of companies from the food sector quoted on the Warsaw Stock Exchange

Anna Kuzior, Małgorzata Rówińska [1]

Abstract

The paper presents and analyses the obligations of business entities in the financial risk disclosures resulting from the Accounting Act and IFRS. The financial statements of entities quoted on the Warsaw Stock Exchange in the food sector have been analysed. This sector comprises entities operating in Poland and abroad, and drawing up reports on the basis of the Accounting Act and IFRS. The analysis concerns the scope of disclosures relating to financial risk (mandatory/voluntary), their quality, presentation and degree of detail, which will allow to assess the comparability of the information presented.

Key words

financial risk, information disclosure

JEL Classification: D82, G14, G18, M41

1. Introduction

Making economic decisions regarding economic entities requires the availability of high quality, useful financial information. Such information is provided for external users, in particular investors and lenders, in the financial statement. On its basis, taking into account mainly the information presented in the statement of financial position, one forecasts future economic benefits and, on the other hand, their loss, usually related to future cash flows. Forecasting the future financial position of the entity, risk should also be taken into account, including the financial one, to which the entity is exposed. Legal regulations specific to economic entities on the preparation of financial statements stipulate the need to disclose information on financial risk, which usually takes the form of an appropriate note to the financial statement. The presented information on financial risk is extremely important for their stakeholders, especially capital providers. Both investors and lenders are interested in relatively detailed information on the types of risk that companies identify, the value of assets and liabilities exposed to a given risk, the risk management methods adopted. Such information should be presented by entities as part of financial reports, this obligation results from accounting regulations. Owning to a lack of a standardised financial risk report model, in practice, reporting entities undertake different approaches to the disclosure, scope and degree of detail. Therefore, it seems important to carry out research on the quality analysis of information disclosed on financial risk.

The usefulness of information on financial risk for users of reports means that it has been the subject of research conducted in various legal environments and based on information presented by entities of various nature (financial and non-financial) [Da Silva Jorge, Augusto, 2011; Drigă, 2012; Das, 2013; Lombardi, Coluccia, Russo, Solimene, 2015; Şenol, Karaca, Erdoğan, 2017]. Studies were carried out using various methods (e.g. manual or automated

[1] Anna Kuzior, PhD, Małgorzata Rówińska, PhD, associate professor, Department of Accounting, University of Economics in Katowice, anna.kuzior@ue.katowice.pl, malgorzata.rowinska@ue.katowice.pl

methods) and from many angles [Solomon, Muntean, 2012; Probohudono, Tower, Rusmin, 2013]. The research regarding the broadly understood quality of information dealt mainly with the tendency of entities to disclose information on financial risk [Campbell, Chen, Dhaliwal, Lu, Steele, 2014], determinants of disclosures, nature of disclosures (descriptive/numerical, good/bad). An interesting review of the literature on the issues of disclosure of information on financial risk and research conducted in this area is presented by Elshandidya, Shrivesb, Bamberc, Abrahamd [2018].

In this context, it seems interesting to assess the quality and quantity of information on financial risk disclosed by selected companies quoted on the Warsaw Stock Exchange. The purpose of the paper is therefore to conduct a critical analysis of the solutions adopted by companies in the selected sector (food sector) regarding the identification of types of financial risk and the financial risk information disclosures.

To this end, the authors adopted the method of analysis, in particular, of financial information provided by companies as well as accounting regulations (IFRS) and the literature. The methods of synthesis and deduction were also used.

2. Problem solution

In this study, the financial risk disclosures in the financial statements of companies from the food sector quoted on the Warsaw Stock Exchange for the period ended either on 30 June 2017 or 31 December 2017 (depending on the accounting year adopted by the entity) were analysed.

The sector covered by the analysis comprises 26 companies, including 21 Polish and 5 foreign ones. 23 companies prepare a report in accordance with International Financial Reporting Standards, 3 apply the Accounting Act. In case of capital groups, the consolidated financial statements were subject to review. In the first place, the note to the financial statement explicitly dealing with the disclosure of financial risk information was analysed, its content in terms of distinguished categories of risk, scope and nature of the information involved. In case of an incomplete content of information in the note, other parts of the report were examined whether they contain any information describing financial risk to which the entity is exposed. Against that background, an attempt was made to assess the companies' tendency to disclose information about financial risk and its comparability over space.

Each audited entity presented a separate section in the financial statement related to financial risk. From among the analysed entities, 20 companies posted a separate note regarding financial risk, usually entitled *"Goals and principles of financial risk management"* or *"Financial risk management"*, in the additional information to the financial statements. Only 1 company used a slightly different name of the note: *"Risk related to financial instruments"*. 4 entities presented information on financial risk in the note relating to financial instruments, 2 in the note on risk management (in one case, a sub-section on financial risk). In case of 25 companies, the notes were published together with other parts of financial statements, 1 company published this information in a separate file.

The information on financial risk presented by the entities may be divided into two main groups – the first is of introductory nature to the issue, the second characterises identified risk categories. The introductory information was presented by 20 entities, while the others merely characterised the identified risks. It was noted that the introductory part predominantly had a low volume (no more than a few sentences were presented by 17 entities), only three entities provided a more comprehensive commentary on risk essence, management and categories. Analysing this part of the note, it can be concluded that the introductory information covered the following areas:

- objectives and methods of risk management,
- identification of risk type,
- categories of financial assets exposed to risk,

– persons (bodies) responsible for risk management.

A quantitative summary of companies presenting individual areas related to risk management is shown in Table 1.

Table 1: Scope of disclosed introductory information on risk management in the analysed companies

Type of information	Number of companies	Proportion of companies presenting information in the surveyed group
Objectives and methods of risk management	16	62%
Identification of risk type	17	65%
Categories of financial assets exposed to risk	6	23%
Persons responsible for risk management	8	31%

Source: own elaboration based on companies' published financial statements

In the note on financial risk, entities distinguish and/or describe specific risk categories, disclose the objectives and principles of financial risk management. The analysis of the notes allowed to establish different approaches to distinguishing risks by the entities. They are presented in Table 2.

Table 2: Risk types identified by the companies

Risk type	Number of companies
Credit, liquidity, market	6
Credit, liquidity, exchange rate, interest rate	7
Credit, liquidity, exchange rate, interest rate, market	2
Credit, liquidity, exchange rate, interest rate, price	2
Credit, liquidity, market, operational	1
Credit, liquidity, price	1
Credit, liquidity	2
Credit, interest rate	1
Credit, exchange rate, interest rate, price	1
Credit, exchange rate, interest rate	2
Credit, , exchange rate, interest rate, price and other non-financial	1
Total	26

Source: own elaboration based on companies' published financial statements

It should be emphasised that all entities identified credit risk. Analysing this part of the notes, it should be pointed out that in each case the categories of risks indicated in the introductory part were exactly in line with the risks described in the second part of the note. It can be stated that while characterising financial risk and the goals of managing it, some entities indicated only the most important categories for themselves.

2.1. Analysis of disclosed information on credit risk

Each analysed financial statement described credit risk. However, the scope of information disclosed by individual companies was significantly different. Out of 26 surveyed companies, 24 defined the essence of credit risk, and 25 presented the rules for managing this risk applied in the entity. All entities that provided general information on credit risk referred to the risk related to receivables in the content of this information, 15 also disclosed information about cash and other financial assets. Irrespective of the introductory description, each company included information on credit risk related to receivables (with varied degrees of detail) in the note.

Generally, the companies indicated the methods adopted to mitigate this risk (24 companies), all of which presented the information using appropriate methods of counterparties' verification in terms of their credibility and establishing on this basis the terms and dates of payments, 22 companies stressed the adoption of a continuous repayment monitoring process, and 6 provided information about the insurance and collateral security used. In addition, the reports of 23 companies included information on the concentration of receivables or its absence. The regularity was observed whenever the problem of concentration of receivables concerned a given reporting entity, it indicated the proportion of the largest counterparties in the total amount of receivables.

10 of the analysed companies limited themselves only to describing the nature of credit risk and way of managing it, without presenting any information on value of assets exposed to credit risk. 16 companies provided information on the receivables expressed in value, of which only 5 companies provided details of maturities (usually using time frames: up to 1 month, from 1 to 3, from 3 to 6, from 6 to 12 and over 12 months). The remaining 11 entities disclosed the structure of receivables with appropriate amounts and, if applicable, their distribution, taking into account the repayment period up to and over 12 months. All entities that disclosed information about the amounts of receivables also indicated the amount of overdue receivables, including 7 entities which presented a detailed breakdown of overdue receivables by past due periods (using similar time frames as for not overdue receivables).

The companies also disclosed information about write-downs on receivables. 7 entities described the rules of making revaluation write-downs, and 14 entities disclosed general write-downs in the risk note.

It should be emphasized that 10 companies which omitted information expressed in value about the receivables in the financial risk note did not refer the report user to relevant notes prepared for receivables and write-downs, although all such notes were included in their financial statements.

In case of assets other than receivables exposed to credit risk, the disclosed information was much more limited. 15 entities characterised the credit risk of assets other than receivables, of which 12 disclosed the information on amounts involved. In particular, this information was disclosed by Ukrainian companies depositing cash in Ukrainian banks, where increased risk is associated with a disadvantageous political situation.

2.2. Analysis of disclosed information on liquidity risk

As it can be seen from Table 2, out of the surveyed entities 21 separated out information on liquidity risk and ways to limit it. In general, information about financial liquidity risk can be divided into descriptive, on the nature and management of liquidity risk, and expressed in value, within which the amounts of liabilities are disclosed together with the corresponding repayment dates. In case of 15 entities, information on liquidity risk started from defining it and presenting the nature of the problem. 14 entities provided general information on the liquidity risk management principles (of which only 9 indicated specific liquidity risk management methods – cash flow forecasting and the use of diversified sources of funding).

Expressed in value information on liabilities exposed to liquidity risk was presented by 17 entities. 9 disclosed information about repayment dates of specific amounts in the cross-section of detailed time frames (usually up to one month, in the period from 1 to 3, from 3 to 6, from 6 months to a year, over a year), 8 entities disclosed information on the amounts and dates of repayment of liabilities from periods not shorter than one year (usually up to a year, from one to three years, from 3 to 5 years). The entities, which did not disclose expressed in value information in the financial risk note, did not refer users of the report to the notes regarding liabilities (notes to the statement of financial position).

2.3. Analysis of disclosed information on market risk

In the financial statements, the companies (with the exception of one) disclosed information on market risk, although not all of them use such a risk specification. Out of the analysed companies, 9 literally indicate market risk (Table 2). More often, however, companies indicate specific 'variations' of market risk, in particular currency (exchange rate) risk and interest rate risk. As part of market risk, a small number of the companies also disclosed information on price risk (the risk of changes in prices of goods, materials). In addition, 2 of the analysed companies, describing the identified price risk, *de facto* described the risk of changes in exchange rates/prices, i.e. currency risk, and they have been included to such a group in this study. Table 3 presents the number of companies that disclose information on separate varieties of market risk.

Table 3: Varieties of market risk identified by the surveyed companies

Varieties of market risk	Number of companies disclosing risk information	Proportion of companies identifying risk in the surveyed group
Currency (exchange rate) risk	25	100%
Interest rate risk	18	72%
Price risk	10	40%

Source: own elaboration based on companies' published financial statements

The above table shows that within the framework of market risk all entities have identified currency risk. Each of the analysed companies is an exporter and/or importer of raw materials, goods and products, and therefore had open (long or short) currency positions, i.e. positions exposed to the risk of unfavourable exchange rate changes. Most of the analysed companies, as the target of currency risk management, have in fact indicated expected settlements in foreign currency (or currencies). It seems obvious, since companies identify currency risk, that they will also present information on the actions taken to mitigate this risk. According to the authors of this paper, such information is extremely important for stakeholders of financial statements. Unfortunately, analysing the companies' reports, it turned out that only some of them (12 companies) disclose information on the tools used to hedge against currency risk. Table 4 lists the currency risk management tools indicated by the companies.

Table 4: Currency risk management tools indicated by the companies

Currency risk management tool	Number of companies indicating the tool
Forward Rate Agreements	8
Currency options	5
Financial instruments (generally)	1
Internal instruments	1

Source: own elaboration based on companies' published financial statements

For the purpose of hedging against currency risk, the companies most often used derivatives (which is obvious). However, it can be noted that in this respect companies pointed to forward contracts more often. In the description of the adopted currency risk management methods, 5 companies also indicated currency options, but three of them declared that in the analysed period they did not conclude any option transactions. The reason indicated by the companies was significant losses incurred from previously concluded currency options. These companies decided to use forward contracts, but did not rule out the use of options in the future.

The basic tool for currency risk management, currently used by companies, as shown by the analysis, are forward contracts. According to the authors, they were rightly considered to be more secure than the currency options used in the past. It is puzzling that five companies in the financial statements indicated that they did not apply any hedging instruments against currency risk. In case of one of the companies (Otmuchów S.A.), this was justified because currency transactions did not materially affect the financial result. Other companies, in turn, declared a high share of transactions expressed in foreign currencies, but have not decided on any hedging operations (e.g. Grupa Kapitałowa Żywiec S.A., in which 85% of net assets is denominated in a foreign currency).

In addition to currency risk, as part of market risk, approximately 70% of companies pointed to interest rate risk. This risk refers mainly to long-term financial liabilities (in particular bank credits and loans). Similarly as in case of currency risk, not all companies indicated the objective of interest rate risk management. However, if they provided such information, it was a hedge of future interest payments. Of the 18 companies that indicated the threat of such a risk, only 5 companies presented a way to manage it. As a tool for interest rate risk management, these companies generally indicated the interest rate swap. In the analysed period, 2 companies did not apply such hedge due to the low risk level.

Identifying currency risk and/or interest rate risk requires companies to recognize groups of assets and liabilities at risk. In the analysed financial statements, companies generally included a description (characteristics) and amount of entries exposed to risk, though the place of this information in the financial statements and the degree of their detail deferrer. Moreover, as regards currency and interest rate risk, companies are required to perform a sensitivity analysis and present it as part of their financial statements. As part of the sensitivity analysis, the companies presented the impact of changes in the exchange rate or interest rate on the financial result and other comprehensive income. For the purpose of the sensitivity analysis, companies adopt the level of changes in the exchange rate and/or interest rate, and for these levels of changes (increases and decreases), they indicate the changes in the financial result and other comprehensive income. Almost all companies presented a sensitivity analysis in the analysed financial statements. 4 companies (foreign companies) did not include this information as part of the financial statements, they placed it in a separate report.

The third variation of market risk, as indicated by the entities, is price risk. 10 companies indicated the thread posed by price risk on their activity. By characterising price risk, the companies presented assets exposed to unfavourable price changes, primarily of products and raw materials. However, only three companies included methods of managing price risk in the financial statements. These are:

- diversification of suppliers' market,
- continuous market monitoring,
- purchase of raw materials directly from producers,
- entering into contracts with two suppliers,
- long-term sales planning of strategic raw materials
- prior price negotiations with consignees,
- increasing the stock of raw materials.

Regarding the hedge against price risk, 1 company stated that in the audited period it did not apply any collateral, while as many as 6 companies did not provide information on the hedging tools used.

3. Conclusions

The analysis of financial statements of companies from the food sector, quoted on the Warsaw Stock Exchange, allowed to indicate the following conclusions:

- All companies, as part of the annual report, posted separate notes (or sections) on financial risk, although a different amount of presented descriptive information can be seen.
- The types of risk identified by the companies are not homogeneous, in particular it concerns the approach to market risk.
- Not all companies provided expressed numerical information (regarding credit risk, liquidity risk and market risk) in the risk note. They expressed in value information on financial assets and liabilities in other places (separate notes), although they do not refer to these notes. A different degree of detail of information on items exposed to risk was also noted.
- The companies disclosed scant information on the applied methods of hedging against financial risk (taking into account its specific types).
- In principle, the companies did not exceed the mandatory disclosures resulting from the accounting law applicable to them.
- A greater tendency to disclose information was noticed in case of providing unfavourable information, which should be assessed positively from the point of view of the recipient of financial information.

According to the authors, observed regularities in the field of disclosing information about financial risk in financial statements indicate its differential quality and the lack of comparability of information from reports of various companies, which undoubtedly hinders the use of reports, especially by external stakeholders.

References

[1] Campbell J.L., Chen H., Dhaliwal D.S. Lu H., Steele L. B. (2014): The information content of mandatory risk factor disclosures in corporate filings. Rev Account Stud (2014) 19:396–455

[2] Da Silva Jorge M.J. , Augusto M.A.G. (2011): Financial risk exposures and risk management: evidence from European nonfinancial firms, Universidade Presbiteriana Mackenzie, Walter Bataglia (Ed.), V. 12, N. 5.

[3] Das S. (2013): Business and Financial Risk: a study on FMGc companies in India, Journal of Commerce & Accounting Research Volume 2 Issue 1 January.

[4] Drigă I. (2012): Financial risks analysis for a commercial bank in the Romanian banking system, Annales Universitatis Apulensis Series Oeconomica, 14(1).

[5] Elshandidya T., Shrivesb P.J., Bamberc M., Abrahamd S. (2018) : Risk reporting: A review of the literature and implications for future research Journal of Accounting Literature 40.

[6] Lombardi R., Coluccia D., Russo G., Solimene S. (2015) : Exploring financial risks from corporate disclosure: evidence from Italian listed companies, Published online: 6 November 2015 # Springer Science+Business Media New York.

[7] Probohudono A.N. , Tower G., Rusmin R. (2013): Risk disclosure during the global financial crisis. Social Responsibility Journal VOL. 9 NO. 1

[8] Şenol Z. Karaca S, Erdoğan S. (2017): The effects of financial risk management on firm's value: An empirical evidence from Borsa Istanbul Stock Exchange, Financial Studies – 4/2017.

[9] Solomon D.C, Muntean M. (2012): Assessment of financial risk in firm's profitability analysis. Economy, Transdisciplinarity Cognition , Vol. 15, Issue 2.

Evolutionary competition of behavioral rules in oligopoly games with memory

Fabio Lamantia[1], Davide Radi[2]

Abstract

Market competition models, especially with oligopolistic structure, represent a significant field of analysis, mainly for their practical applications. Because of limited information on the structure of the market, limited computation capabilities or bounded rationality, agents decide their strategies according to behavioral rules, which are dynamically chosen if more performing strategies are detected.
In this paper, we set up an evolutionary oligopoly with agents selecting between different behavioral rules to fix their production plans over time. We focus on two specific heuristics, namely Local Monopolistic Approximation and Gradient dynamics. We present a new extension of this setup by adding memory dynamics in the updating process, which regulates how strongly past profits signals drive the current selection of behavioral strategies.

Keywords

Cournot oligopolies; Bounded rationality; Adaptive adjustments; Evolutionary dynamics; Heterogeneous firms; Behavioral rules; Nonlinear dynamical systems.

JEL Classification: L13, D83, C61, C73

1. Introduction

Market competition models constitute a relevant field of analysis, both from a theoretical point of view and for practical applications. In a realistic framework, firms do not always necessarily play Nash equilibrium outcomes. This can occur because of agents' limited information on the structure of the market, limited computation capabilities or bounded rationality. For instance, whereas it is realistic enough to assume that firms know their own cost structure, it is very strong to assume that each firm possesses a global knowledge of market demand.

[1]Fabio Lamantia, Ph.D., Department of Finance, Faculty of Economics, VŠB-TU Ostrava, Sokolská 33, 701 21 Ostrava (Czech Republic), and Department of Economics, Statistics and Finance, University of Calabria (Italy). E-mail: fabio.lamantia@unical.it.
[2]Davide Radi Ph.D., Department of Finance, Faculty of Economics, VŠB-TU Ostrava, Sokolská 33, 701 21 Ostrava (Czech Republic), and Department of Economics and Management, University of Pisa (Italy). E-mail: davide.radi@unipi.it.
The research was supported by the ESF in Science without borders project, CZ.02.2.69/0.0/0.0/16-027/0008463, within the Operational Programme Research, Development and Education and by VSB-TU Ostrava under the SGS Project SP2018/34. The support is greatly acknowledged.

Consequently, firms employ different heuristics (or "behavioral rules") to take decisions over time in a context of strategic uncertainty.

In the economic literature, several kinds of such heuristics have been considered. Here we focus on two specific behavioral rules: i) Local Monopolistic Approximations (LMA), proposed in [2] and ii) Gradient dynamics, proposed in [1], see also [6] and [3]. The term *Local Monopolistic Approximation* summarizes its main features: LMA-firms assess a linear estimation for the future selling price by correctly guessing locally the slope of demand but neglecting the influence of market competitors on the selling price (monopolistic behavior). In other words, a generic LMA-firm believes that all the other competitors will produce at time $t+1$ the same quantity produced at time t. The term *Gradient dynamics*, instead, originates from the specific behavior of a boundedly rational oligopolist that, neglecting the functional form of its profits, decides its production plan in the direction of the gradient, which leads to an improvement of future profits. To avoid trivial scenarios, market demand is nonlinear of (unitary) isoelastic form, as often considered in similar models, see [15], [4], [14].

The distribution of behavioral rules among the players is not fixed but changes endogenously according to the perceived benefit of either strategy in terms of profit performances. This kind of evolutionary pressure in oligopolistic models has been employed in several related works, see, among others, [8], [10], [11], [13], [16], [12]. In particular, [5] provides an overview of evolutionary modeling of oligopoly games. Extending the work in [7], here through memory dynamics we allow the fitness measure of the evolutionary process to depend not only on the recent profit signals but also on past profits, although the latter have less importance in the comparisons.

The paper is organized as follows. Section 2 states the model by defining the behavioral rules under examination, namely the Gradient and the Local Monopolistic Approximation rules. Here, a dynamic memory term is introduced in the fitness function. Section 3 presents the four-dimensional map that provides a representation of the evolutionary dynamics of the different heuristics together with quantity and memory dynamics. Section 4 concludes.

2. The model

Let us consider an oligopoly model with $N \geq 2$ firms producing homogeneous goods. In the following, we denote by q_i the quantity produced by firm i with the following linear costs

$$C_i = c \cdot q_i + K_i \tag{1}$$

where $c > 0$ denotes the marginal cost (equal for all firms) and $K_i \geq 0$ represents the cost paid by firm i to acquire information on the market. Market price is given by an (unitary) isoelastic demand function

$$p = f(Q) = \frac{1}{Q} \tag{2}$$

where $Q = \sum_{i=1}^{N} q_i$ is the total industry output, with $Q > 0$.

Different behavioral rules are available to firms to set their production plans, delivered to the market at discrete time periods $t \in \mathbb{N}$. For the sake of simplicity, we develop the model by assuming that two kinds of players interact: *Gradient* (G-)players, producing $x(t)$ at time t, and *Local Monopolistic Approximation* (LMA-)players, producing $y(t)$ at time t.

By the action of the game, the fraction of players choosing either strategy changes over time as described below. In the following, we denote by $r(t) \in [0,1]$ the fraction of G-players and assume that the complementary fraction $1 - r(t)$ plays the LMA rule.

2.1 Gradient rule

Following this behavioral rule, a representative G-player, or G-firm, fixes its production at time t in the direction that maximizes its expected profits for time $t+1$. Consider current profits of a representative G-firm:

$$\pi_G(x(t), Q_{-1}(t)) = \left[\frac{1}{x(t) + Q_{-1}(t)} - c\right] x(t) - K_G \tag{3}$$

where

$$Q_{-1}(t) = (N-1)\{r(t)x(t) + [1 - r(t)]y(t)\} \tag{4}$$

is the aggregate production of its competitors and $K_G \geq 0$ in (3) is an 'information' cost of the G behavior. According to the gradient behavioral rule, a G-firm sets its next period quantity according to:

$$\begin{aligned} x(t+1) &= \max\left[0, x(t) + \lambda x(t)\,\tfrac{\partial \pi_G}{\partial x(t)}\right] \\ &= \max\left[0, x(t) + \lambda x(t)\left(\tfrac{Q_{-1}(t)}{(x(t)+Q_{-1}(t))^2} - c\right)\right] \\ &= \max\left[0, x(t) + \lambda x(t)\left(\tfrac{(N-1)[r(t)x(t)+(1-r(t))y(t)]}{(x(t)((N-1)r+1)+y(t)((N-1)(1-r)))^2} - c\right)\right] \end{aligned} \tag{5}$$

where $\lambda \geq 0$ is the speed of adjustment (assumed equal for all G-players).

2.2 Local Monopolistic Approximation Rule

Following the LMA behavioral rule a firm maximizes its next-period expected profit to determine the next-period output quantity. Specifically, a representative LMA-firm decides its production for time $t+1$, $y(t+1)$, by solving the following optimization problem:

$$\max_{y(t+1)\geq 0} \left[\widetilde{p}^e(t+1) - c\right] y(t+1) - K_L \tag{6}$$

where $K_L \geq 0$ is an 'information' cost of the LMA behavior and $\widetilde{p}^e(t+1)$ is the expected price, assessed through (see [2]):

$$\widetilde{p}^e(t+1) = p(t) + f'\left(Q^e_{-1}(t+1)\right)[y(t+1) - y(t)] \tag{7}$$

where $p(t)$ and $f(Q)$ are given in (2), and the expected quantity is $Q^e_{-1}(t+1) = Q_{-1}(t)$, being approximated by current aggregate production (4).

2.3 Memory

Differently from [7], here we consider a memory term in the fitness measure of the evolutionary process regulating the evolution of the fraction $r = r(t)$ of agents playing the G strategy. The evolutionary mechanism regulates how dynamically these fractions change according to the time average of the differences between past profits obtained by a representative LMA-firm and a G-firm:

$$\Delta\Pi(t) = (1-\omega)(\pi_L(t) - \pi_G(t)) + \omega\Delta\Pi(t-1) \tag{8}$$

In (8) $\omega \in [0,1)$ is a *memory* parameter, which measures the (geometric) decay rate of past profits, given at time t by:

$$\begin{cases} \pi_G(t) &= p(t)x(t) - (cx(t) + K_G) = \left(\frac{N-1}{NQ_{-1}(t)} - c\right)x(t) - K_G \\ \pi_L(t) &= p(t)y(t) - (cy(t) + K_L) = \left(\frac{N-1}{NQ_{-1}(t)} - c\right)y(t) - K_L \end{cases} \tag{9}$$

with $Q_{-1}(t)$ defined in (4).

Notice that when $\omega = 0$ in (8) the fitness measure depends only on the current gap between the profits obtained by the two heuristics. On the other hand, for $\omega \in (0,1)$ past profits also affect (8): an increment of ω entails more memory, that is a stronger influence of past profits on evolutionary selection, see [9] for details.

The distribution of weights in (8) is given by the set $\{(1-\omega)\omega^m, m \in \mathbb{N}\}$, so that more recent profits have a stronger influence on the evolutionary process than older profits.

3. Dynamics

As customary in this modeling framework, we consider a semi-symmetric setting, in which firms employing the same behavioral rule deliver the same quantity on the market and earn equal profits. These behavioral rules are described above and are given by (5) for a G-firm and by the maximizer of (6) for a LMA-firm and provide firms' production plans for time $t+1$.

Summing up, the coupling of quantity dynamics by G and LMA-firms with an exponential replicator for the evolution of the fraction of G-players, where fitness embeds a memory term, provides a dynamic representation of the system, given by the following four-dimensional map $T : A \subset \mathbb{R}^2_+ \times [0,1] \times \mathbb{R} \longrightarrow A$:

$$T: \begin{cases} x(t+1) = \max\left\{0, x(t) + \lambda x(t)\left(\frac{Q_{-1}(t)}{[x(t)+Q_{-1}(t)]^2} - c\right)\right\} \\ y(t+1) = \max\left\{0, (1-\alpha)y(t) + \frac{\alpha}{2}\left[y(t) + \frac{N}{N-1}Q_{-1}(t)\left(1 - c\frac{N}{N-1}Q_{-1}(t)\right)\right]\right\} \\ r(t+1) = \frac{r(t)}{r(t)+(1-r(t))e^{-\beta\Delta\Pi(t)}} \\ \Delta\Pi(t+1) = (1-\omega)[\pi_G(t+1) - \pi_L(t+1)] + \omega\Delta\Pi(t) \end{cases} \tag{10}$$

where $Q_{-1}(t)$ and $\pi_i(t)$, $i \in \{G, L\}$ are given, respectively, in (4) and (9). The additional parameters α and λ in (10) are the speeds of adjustment for LMA-firms and G-firms (see [3] for details), and β is the intensity of choice and measures how fast firms switch to the behavioral rule that offered the higher profits in the past.

The evolutionary oligopoly model (10) admits symmetric Nash equilibria as fixed points.

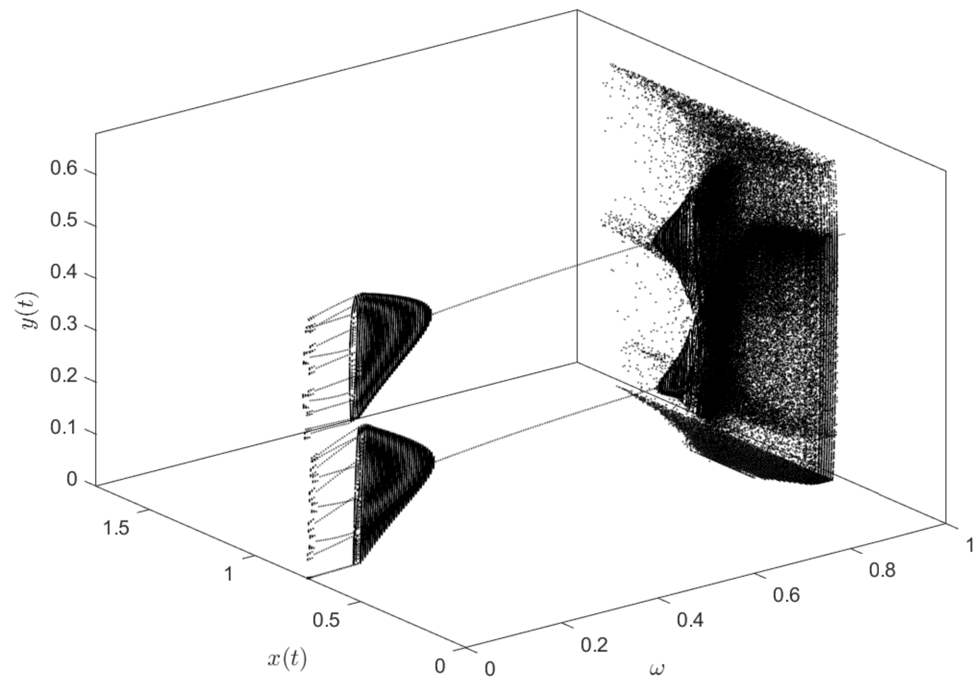


Figure 1: Bifurcation diagram showing the long-run dynamics in the subspace $x(t) - y(t)$ of the evolutionary oligopoly model (10) with respect to memory parameter $\omega \in [0, 1)$. Values of the parameters: $\lambda = 1$, $\alpha = 0.3$, $\beta = 22$, $c = 0.1$, $N = 20$, $K_G = 0.01$ and $K_L = 0$.

Theorem 1 *The subspaces of $\mathbb{R}^4$, denoted by E_0^+ and E_1^+ and given by*

$$E_0 = \left\{ \left(\frac{N-1}{cN^2}, \frac{N-1}{cN^2}, 0, \Delta\Pi \right) \middle| \Delta\Pi \in \mathbb{R} \right\} \quad \text{and} \quad E_1 = \left\{ \left(\frac{N-1}{cN^2}, \frac{N-1}{cN^2}, 1, \Delta\Pi \right) \middle| \Delta\Pi \in \mathbb{R} \right\}, \tag{11}$$

are fixed points (Nash equilibria) of the evolutionary oligopoly model (10) and $q^{CN} = \frac{N-1}{cN^2}$ is the symmetric Cournot-Nash output.

The global dynamics of the evolutionary oligopoly model (10) neglecting memory ($\omega = 0$) is investigated in detail in [7]. In particular, for $\omega = 0$ the model can admit a locally asymptotically stable 2-period cycle that can undergo a Neimark-Sacker bifurcation through which it loses stability and an attractor appears around it. An example of a stable periodic attractor can be observed for the following constellation of parameters:

$$(\lambda, \alpha, \beta, c, N, K_G, K_L) = (1, 0.3, 22, 0.1, 20, 0.01, 0) \tag{12}$$

For this market configuration, the Gradient rule stabilizes the market while the LMA rule destabilizes the market in the sense that an oligopoly populated by G-firms only learns to play the Cournot-Nash quantity in the long run. Contrary, an oligopoly populated by LMA-firms only fails to coordinate to play the Cournot-Nash quantity and converges towards a cyclical path. As specified above, the introduction of an evolutionary-type updating rule when $\omega = 0$ does not force firms to adopt the G-rule and to learn to produce the Cournot-Nash output. Here, we investigate if the introduction of a certain level of memory, i.e. fixing $\omega > 0$, allows firms to learn to play the Cournot-Nash output or at least to reduce the fluctuations in the output produced over time. The bifurcation diagram in Figure 1 shows that increasing the memory term ω, therefore accounting for the entire history of past profits when updating the behavioral rule to adopt, makes the G-rule more attractive and stabilizes the market. Specifically, from what it looks like a highly periodic or complex attractor observed for $\omega = 0$, the oligopoly coordinates towards a stable 2-period cycle when ω increases. However, when ω approaches the value one, i.e. firms weight historical profits more than recent performances, the updating mechanism leads firm to play the LMA-rule with more frequencies and this destabilizes the 2-cycle through what it looks like a Neimark-Saker bifurcation and an aperiodic attractor appears.

In summary, the numerical investigation seems to suggest that taking into account historical profits when updating the behavioral rule to adopt may help to stabilize the market, or at least it helps to learn to adopt the behavioral rule that allows to learn to play the Cournot-Nash quantity. However, when firms start to weight old historical profits more than recent ones, they update slowly towards the best performing strategy and this may destabilize the market as well. This is also observable in Figure 2, where the parameter are as in Figure 1 but the intensity of choice parameter β is higher and equal to 30. As we can observe, in this case the frenzy in selecting the most performing behavioral rule destabilizes the market but the memory term contributes to stabilize the market, although the stability of the 2-cycle cannot be recovered (it is unstable for each value of ω). However, also here a too large value of ω has a destabilizing effect rather than a stabilizing effect.

4. Conclusions

Dynamical models of strategic interaction in oligopolistic competition are a challenging field of research. Most oligopoly evolutionary models assume that only recent profit signals can drive the dynamic choice of correct heuristics. Here we have explored the influence of past profits memory on the stability properties of an oligopoly with nonlinear demand. In particular, we showed how the memory can have a stabilizing effect as long as the weight put on the oldest performances is not too high. In the case firms are anchored to old performances and put low weight to new ones, i.e. they are reluctant to follow recent profit signals in updating the behavioral rule to adopt, the memory term has a destabilizing effect. These evidences underline that the learning mechanism is only one important element that helps player to coordinate to play the Nash equilibrium. The performance index used to implement the learning mechanism is an equally important aspect.

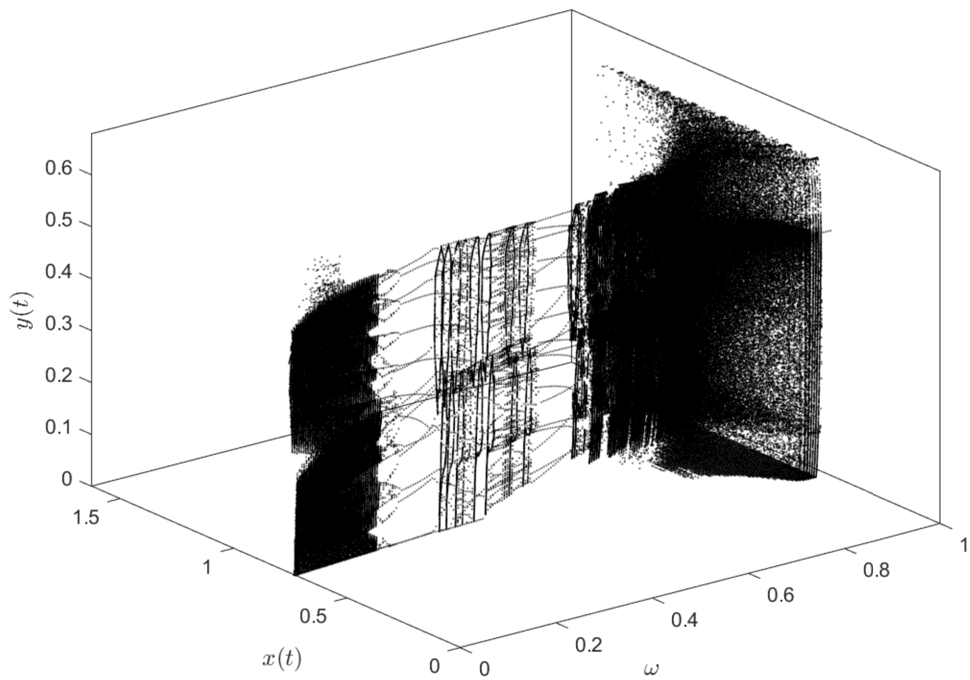


Figure 2: Bifurcation diagram showing the long-run dynamics in the subspace $x(t) - y(t)$ of the evolutionary oligopoly model (10) with respect to memory parameter $\omega \in [0, 1)$. Values of the parameters: $\lambda = 1$, $\alpha = 0.3$, $\beta = 30$, $c = 0.1$, $N = 20$, $K_G = 0.01$ and $K_L = 0$.

References

[1] Bischi, G.I. and Naimzada, A. (2000). Global Analysis of a Duopoly Game with Bounded Rationality, pp. 361–385. In *Advances in Dynamic Games and Applications*, Birkhauser.

[2] Bischi, G.I., Naimzada, A.K. and Sbragia, L. (2007). Oligopoly games with local monopolistic approximation. *Journal of Economic Behavior & Organization* 62(3): 371–388.

[3] Bischi, G.I., Chiarella, C., Kopel, M., and Szidarovszky, F. (2010). *Nonlinear Oligopolies: Stability and Bifurcations.* Springer-Verlag.

[4] Bischi, G.I., Lamantia, F., and Radi, D. (2015). An evolutionary Cournot model with limited market knowledge. *Journal of Economic Behavior & Organization* 116: 219–238

[5] Bischi, G.I., Lamantia, F., and Radi, D. (2018). Evolutionary oligopoly games with heterogeneous adaptive players. In *Handbook of Game Theory and Industrial Organization*, Volume I, Theory. Elgar.

[6] Cavalli, F., Naimzada, A., and Tramontana, F. (2015). Nonlinear dynamics and global analysis of a heterogeneous Cournot duopoly with a local monopolistic approach versus a gradient rule with endogenous reactivity. *Communications in Nonlinear Science and Numerical Simulation* 23(1-3):245–262.

[7] Cerboni Baiardi, L., Lamantia, F., and Radi, D. (2015). Evolutionary competition between boundedly rational behavioral rules in oligopoly games. *Chaos, Solitons & Fractals* 79:204–225.

[8] Droste, E., Hommes, C.H., and Tuinstra, J. (2002). Endogenous fluctuations under evolutionary pressure in Cournot competition. *Games and Economic Behavior* 40(2):232–269.

[9] Hommes, C., Kiseleva, T., Kuznetsov, Y., and Verbic, M. (2012). Is more memory in evolutionary selection (de)stabilizing? *Macroeconomic Dynamics* 16:335–357.

[10] Kopel, M., Lamantia, F., Szidarovszky, F. (2014). Evolutionary competition in a mixed market with socially concerned firms. *Journal of Economic Dynamics & Control* 48:394–409.

[11] Lamantia, F. and Radi, D. (2015). Exploitation of renewable resources with differentiated technologies: an evolutionary analysis. *Mathematics and Computers in Simulation* 108:155–174.

[12] Lamantia, F. and Radi, D. (2018). Evolutionary technology adoption in an oligopoly market with forward-looking firms. *CHAOS* 28(5): 055904.

[13] Lamantia, F., Negriu, A., and Tuinstra, J. (2018). Technology choice in an evolutionary oligopoly game. *Decisions in Economics and Finance* DOI https://doi.org/10.1007/s10203-018-0215-2.

[14] Lambertini, L. (2010). Oligopoly with hyperbolic demand: A differential game approach. *Journal of Optimization Theory and Applications* 145(1):108–119.

[15] Puu, T. (1991). Chaos in duopoly pricing. *Chaos, Solitons & Fractals* 1(6):573–581.

[16] Radi, D. (2017). Walrasian versus Cournot behavior in an oligopoly of bounded rational firms. *Journal of Evolutionary Economics* 27(5):933–961.

Parametric families for the Lorenz curve: an analysis of income distribution in the European countries

Tommaso Lando[1,2], Michaela Staníčková[2], Jiří Franek[2]

Abstract

We investigate differences and similarities of income distribution among European countries on the basis of the Lorenz curve (LC). By performing an empirical analysis, we compare different parametric models for the LC, namely, the power LC, the Lamè LC and a generalized bi-parametric version of the Lamè LC. In particular, besides focusing on the goodness-of-fit, we concern about the effectiveness of multi-parametric models in identifying situations of intersecting LCs.

Key words

Lorenz ordering, income inequality, disparity, stochastic dominance, European countries

JEL Classification: C44, E24, O52

1. Introduction

A fundamental issue facing the global economy is the widening poverty gap between the developed and less developed world and the widening distribution of income within countries and geographical regions. Impoverishment has several causes, including low income as a result of unemployment, under-employment, or low wage employment. It may also be caused by a failure of the government to provide a welfare safety net in the event of the above. Income can be earned from selling labour, including wages, which are the largest source of income, and salaries and commission, which represent a very small fraction of income in comparison with more developed economies. Some income is unearned, such as rents from land ownership and interest from lending money. These sources of income are less available in developing economies. Equity means fairness or evenness, and achieving it is considered an economic objective. Despite the general recognition of the desirability of fairness, it is often regarded as too normative a concept because it is difficult to define and measure. For most economists, equity relates to how fairly income and opportunity are distributed between different groups in a given society (Melecký, 2018; Staníčková, 2017).

The European Union (EU) faces many challenges. On the global stage, the EU has to speak with one voice to counter a plethora of political, military and economic crises. Internally, it needs to foster cohesion in spite of the many events that threaten the EU at its core. In this context, do social issues matter at all? If we look at the EU evolution over the past decades, substantial progress has been made in terms of building an internal market and an economic and monetary union, albeit not without problems, as the 2008 crisis has shown. It looks actually as if the EU and its Member States were mostly thinking in economic terms, hoping that economic solutions will fix all social problems at once. To negate the importance of social

[1] Dr. Tommaso Lando, University of Bergamo, via dei Caniana 2, Bergamo, Italy, tommaso.lando@unibg.it
[2] Dr. Tommaso Lando, Ing. Michaela Staníčková, Ph.D., Ing. Jiří Franek, Ph.D.
VŠB - TU of Ostrava, Faculty of Economics, Department of Finance, Department of European Integration, Department of Business Administration, Sokolská třída 33, 702 00 Ostrava 1, Czech Republic; tommaso.lando@vsb.cz, michaela.stanickova@vsb.cz, jiri.franek@vsb.cz

issues is to undermine the EU foundations (Allmendinger and Driesch, 2014). Many politicians and economists believe that economic growth replaces or diminishes the need for social policies (MacGregor Pelikánová and Beneš, 2017). The strict regulation of corporate non- financial reporting is not the best for good understanding and practice of corporate social responsibility (Pakšiová, 2016). However, the EU growth over the last decades has been accompanied by an increase in inequalities in many countries. Inequalities threaten social cohesion and they threaten growth.

If such concerns are correct, it is essential not only to build institutional structures for European social union but also to map social inequalities in the EU. The low growth performance in the EU over the recent decades has increased concerns regarding an increasing economic dispersion, income inequality at large, and social exclusion. Recent research works have stimulated fierce debate on inequality among academics and policymakers. The recent economic crisis revealed many of the weaknesses of the current European economic policy, not least at the level of its fiscal policy, monetary policy, industrial policy, and social policy, and its inability to address problems related to inequality and the competition fairness (MacGregor Pelikánová, 2017). Inequalities in the EU have been the object of extensive research over the last decade. Several factors can explain this widespread interest; especially the revival of growth theory (Romer, 1990; Aghion and Howitt, 1998) was contemporaneous with a growing empirical literature on economic convergence (Sala-i-Martin, 2006; Quah, 1997; Barro and Sala-i-Martin, 1995).

Therefore, inequality can be quantified by looking at the distribution of income or wealth. The distribution of wealth is likely to be much greater than income because wealth is built up over many decades, and for some families, over centuries. The distribution of income is relatively easy to measure - valuing wealth is more difficult. This is because wealth is often hidden from view, and because it changes its value over time.

The Lorenz curve (LC) is a fundamental tool for the graphical representation of income inequality. The LC evaluated in $p \in [0,1]$ gives the proportion of the total wealth corresponding to the $p100\%$ poorer part of the population. Clearly, perfect equality is represented by a LC with equation $L(p) = p$. If the LC of a distribution is higher than the LC of another distribution, we can rank such distributions with respect to the *Lorenz dominance* (LD). The LD criterion basically reflects the fact that one distribution may exhibit a lesser degree of inequality compared to the other. However, many empirical studies revealed that Lorenz curves often intersect in the practice, that is, it is not rare to find couples of distributions that cannot be ranked based on the LD. In such cases, we can compare the intersecting distributions by relying on weaker orders of inequality, for more information see Lando and Bertoli-Barsotti (2016).

Many different functional forms for the LC have been proposed in the literature. Some of them belong to mono-parametric families, which may be denoted as *ordered* families of LCs, in that the LD is fully characterized by the value of a unique parameter. This property definitely represents an advantage for such kinds of families, although multi-parametric families generally yield better performances in terms of goodness-of-fit. Moreover, by construction, ordered families do not permit LCs to cross, and this lack of flexibility may be inappropriate in a number of cases.

In this paper we study some different families of parametric LCs, namely, the power Lorenz curve (PLC), the Lamè class of LCs (LLC) (Sarabia et al., 2017), which actually consists of two slightly different formulas, and a generalized version of the Lamè curve (GLLC), that has been studied by Sarabia et al. (1999). Whilst the PLC and the LLC are mono-parametric ordered families, the GLLC depends on two parameters, where different parametric combinations may yield intersecting LCs. Therefore, the main objectives of this paper are:

– study the suitability of these different families to model income distribution, in terms of goodness-of-fit. Special attention is given to the advantages/disadvantages of mono-parametric models with respect to multi-parametric models;
– study the usefulness of multi-parametric families to describe non-dominated situations (i.e., cases of intersecting LCs). In particular, a multi-parametric model should be able to identify most of the crossing pairs of LCs.

To this purpose, we perform an empirical analysis of the LCs of 32 European countries in 2015. The data have been downloaded from Eurostat's database.

2. Methods

We recall that a preorder is a binary relation $\leq$ over a set S that is reflexive and transitive. In particular, observe that a preorder $\leq$ does not generally satisfy the antisymmetric property (that is, $a \leq b$ and $b \leq a$ does not necessarily imply$a = b$) and it is generally not total (that is, each pair a, b in S is not necessarily related by $\leq$).

Let F be a non-negative distribution with positive and finite expectation μ_F. The (generalized) inverse or quantile function of F is given by

$$F^{-1}(p) = \inf\{z: F(z) \geq p\} p \in (0,1),. \quad (1)$$

The Lorenz curve $L_F: [0,1] \to [0,1]$ is defined as follows (Gastwirth, 1971):

$$L_F(p) = \frac{1}{\mu_F} \int_0^p F^{-1}(t) dt \, , p \in (0,1). \quad (2)$$

We recall that the Gini index is given by twice the area between the Lorenz curve and the 45° line:

$$\Gamma(F) = 1 - 2 \int_0^1 L_F(t) dt. \quad (3)$$

Actually, for a given percentage p, $L_F(p)$ represents the percentage of "total" possessed by the low $100p\%$ part of the distribution. It is well known that the higher of two non-intersecting Lorenz curves can be obtained from the lower one by a sequence of income transfers from "richer" to "poorer" individuals. This criterion has been called "Pigou-Dalton condition". For this reason, in an economic framework, the higher of two non-intersecting LCs should be preferred, in that it shows less inequality compared with the lower one. This idea defines the LD, defined as follows.

Definition 1. We write $F \leq_L G$ if and only if $L_F(p) \geq L_G(p), \forall \, p \in (0,1)$. (4)

When the LD is not fulfilled, i.e. when LCs intersect, some weaker criteria can be used in order to obtain unambiguous rankings. Muliere and Scarsini (1989) and Aaberge (2009) suggest cumulating LCs from the left or from right: that is, attaching more weighting to low or top incomes. Such ambiguous situations are very important, in that is has been empirically shown that intersecting LCs is an extremely frequent situation in the practice.

Among the different functional forms that have been proposed to approximate the LCs of income distributions, we propose the following ones.

1) *The power LC:* The PLC is defined by the following formula:

$$L_P(p, a) = p^a, \; p \in (0,1), \; a \geq 1. \quad (5)$$

This basic model is clearly ordered with respect to the parameter a, since $L_P(p, a_1) \geq L_P(p, a_2)$ iff $a_1 \leq a_2$. For $a = 1$ we obtain the equality line.

2) *The Lamè LC:* The LLC is defined by the following two different formulas:

$$L_{L1}(p,a) = [1-(1-p)^a]^{1/a},\ p \in (0,1),\ a \in (0,1] \quad (6)$$

$$L_{L2}(p,a) = 1-(1-p^a)^{1/a},\ p \in (0,1),\ a \geq 1. \quad (7)$$

In both cases, for $a = 1$ we obtain the equality line. The LLC has been introduced by Henle et al. (2008) and more recently studied by Sarabia et al. (2017). Both curves are ordered with respect to the parameter a, in particular:

$$L_{L1}(p,a_1) \leq L_{L1}(p,a_2) \text{ iff } a_1 \leq a_2, \quad (8)$$

$$L_{L2}(p,a_1) \leq L_{L2}(p,a_2) \text{ iff } a_1 \geq a_2. \quad (9)$$

3) *The generalized Lamè LC:* The GLLC is defined by the following formula:

$$L_{GL}(p,a,b) = [1-(1-p)^a]^b,\ p \in (0,1),\ a \in (0,1], b \geq 1. \quad (10)$$

For $a = b = 1$ we obtain the equality line. Differently from L_P, L_{L1} and L_{L2}, this family is not ordered, in that different combinations of a and b may yield intersecting LCs.

In the next section, we compare the performance of these different parametric models in terms of goodness-of-fit, and we also analyze the effectiveness of the GLLC in identifying non-dominated cases.

3. Empirical analysis

Data have been retrieved online from Eurostat's website (the data have been studied also in Lando et al., 2017). In particular, Eurostat provides the "distributions of income by quantiles" with two options, in terms of income and living conditions indicator, namely: i) *top cut-off point*, which represents the income of the individual at the right end of the given quantile and; ii) *share of national equalized income*, which is the share of the total income which belongs to a given interval. Eurostat provides i) and ii) for the three quartiles, the four quintiles, the nine deciles and the first (and last) five percentiles. Unfortunately, we note that some countries present negative incomes in the first 2-3 percentiles. The presence of negative incomes hampers the applicability of properties 1, 2 and 3, discussed in the previous section. Moreover, smaller percentile values are generally less reliable and accurate. Then, we decided to consider the LCs starting with $p = 0.05$. Indeed, by properly cumulating the shares of national equalized income we can obtain the values of the LC for

$$p = 0,0.05,0.1,0.2,0.25,0.3,0.4,0.5,0.6,0.7,0.75,0.8,0.9,0.94,0.95,0.96,0.97,0.98,0.99,1,$$

that is, a LC with 18 nodes (excluding 0 and 1).

The first step of our analysis consists in analyzing the empirical LCs, computed from the observed data. For the year 2015, we obtained the LCs of 32 countries and compared each pair of LCs (i.e. 32×31/2 pairs) based on the LD relation. We find that the LD can rank only the 56% of the pairs, while the remainder 44% of the pairs present intersecting LCs (i.e. with one or more crossings). Thus, LD seems not to be an effective criterion in comparing LCs of European countries.

Then, as a second step, we estimated the parametric models studied in section 2 by fitting them to the 32 observed LC discussed above. For the sake of simplicity, we used the least squares method, which consists in finding the parametric values that minimize the square of the "distance" between the observed LC and the model considered. Such distance is actually a sum of quadratic differences, evaluated in all the 18 nodes of the observed LCs. By dividing this distance by the number of nodes (18) we obtain the *mean squared error* (MSE), which can be used as a measure of goodness-of-fit. We computed the average MSE over the 32 LCs, for all the models considered, and obtained the following results.

Table 1: Average MSE of the four models considered

Model	Av. MSE
PLC	0.2400
LLC(1)	0.0007
LLC(2)	0.0006
GLLC	0.0002

Source: own elaboration, 2018

The results show that the LLC (in both the two versions) provide an excellent performance in approximating the LC, whilst the PLC is not well fitting at all. The GLLC provides a smaller MSE compared to the LLCs (obviously) but the improvement is questionable, especially if we are just interested in the goodness-of-fit because the LLCs perform extremely well besides having just one parameter.

Now, it is interesting to see if the models considered can identify dominated or non-dominated LCs. As we discussed in section 2, all the ordered families cannot generate crossing LCs, by definition. Then, in the third step of our analysis, we focus just on the GLLC, and compute the number of times when observed LCs cross and estimated LCs also, vs. the times when LCs do not cross but the estimated ones do, and so on. In particular, we find that the GLLC is able to identify the LD in the 82% of the cases (the percentage is computed over the number of cases when the LD holds). On the other hand, the percentage of cases when intersecting estimated LCs correspond to intersecting observed LCs (computed over the set of intersecting observed LCs) is 77%.

Based on the getting results, as proof, right measurement is a powerful instrument for socio-economic progress, which is why efforts are constantly being made to improve their power and precision; wrong or imprecise measurement a source of hazard and even havoc (Turečková, 2015). The essential purpose of economic activity is the promotion of human development, welfare and well-being in a sustainable manner, and not growth for growth's sake, yet we lack effective measures to monitor progress toward these objectives. Advances in understanding, theory and measurement must necessarily proceed hand in hand. Measuring multiple dimensions of socioeconomic progress is indispensable to understanding its components, benchmarking success, and catalyzing improvement. What level have we reached in comparison to others? Are we doing well? Are we going in the right direction? Are we catching-up or lagging behind? Are we meeting benchmarks or are we missing them? Are we using our fair and sustainable share of resources or too much? Is a group of economies converging or not? Just to list a few. At the same time, we are surrounded by an abundance of indicators trying to provide answers to these questions, at different levels of sophistication, in many cases serving as a basis for evidence-based policy decisions. Such indicators often seek to measure much aggregated but also diffuse concepts, rich in value judgements but not always grounded in hard science. The most prominent examples we see are indicators of economic development and performance. In recent years these have been complemented by alternative progress and well-being measurements. These indicators are frequently presented in dashboards and scoreboards, as well as aggregated or model-based composite indicators or indices (CIs). In recent years, international organizations, think-tanks, and the social sciences have contributed to a dramatic expansion in the range of CIs indices measuring concepts such as human development, governance, or social capital. Therefore, a large number of composite indexes of economic and social well-being have been developed. Unfortunately, the methodological issues associated with CI construction have often been neglected or inadequately treated by index developers.

This can be part of further research, i.e. the orientation on income distribution representing by indices.

4. Conclusion

The level of social inequalities belongs to important indicators influencing the socio-economic development and other processes taking place in the social and economic realm. Facilitating rational income distribution and reducing poverty are mentioned among the main goals of public policy. It should be mentioned that such multidimensional phenomena as income disparity and poverty might be analyzed from many different perspectives, including the national and international, also within the EU. Striving for fairness in economic development is crucial in order for societies to be stable and citizens not to feel disenchanted. The economic crisis has put inequalities high on the political agenda and made this an issue of serious public concern. There is an increasing recognition that social policy can reduce inequality and poverty while simultaneously improving the economic functioning of the country as reflected in the idea of inclusive growth in the EU's Europe 2020 strategy, with references to a high-employment economy delivering economic, social, and territorial cohesion in which benefits of growth and jobs are widely shared. Inequality is a key problem facing the EU, and it has significant impacts not only on human well-being but also on economic performance. This study has tried to show that inequalities in the EU are not a recent phenomenon and that they have in general increased over recent times in most of the EU countries. As a general principle, it is important to note that many differences among people in the EU are created by society and systematically linked to life chances. The only way for the EU to meet these challenges is to not only strengthen economic growth policies through broad-based economic programme promoting marketization but also by resolutely pushing for the expansion of social aspects of the EU model (Allmendinger and Driesch, 2014).

The future design of European economic policy must then provide a framework in which the policy instruments essential for a monetary, fiscal, industrial, sectorial, and social policy consistent with full employment and a reduction in inequality play a more prominent role. Europe 2020 is a credible strategy of industrial policy for the future of Europe and has the merits of presenting clear actions, clear targets and a detailed measurement strategy to monitor implementation. Combatting inequality should be considered as an instrumental target for both sustainable and inclusive growth. European policymakers have a long to-do list to foster inclusive growth in Europe (Darvas and Wolff, 2016). In all the countries of the EU, the welfare state has come under intense scrutiny as a result of budgetary pressures and wider societal developments. European social policy responses need national and regional contextualization. Simultaneously, the EU needs a sense of common purpose and a common policy framework in support of national social policies. Its aim should be to create a virtuous circle whereby both pan-European cohesion and national cohesion are enhanced. Cohesion is about income and employment, but also about other dimensions of well-being.

The recent interest in inequality is thus simply the recognition of the centrality of the topic to economic theory, policy, and performance. The recent return of the topic of inequality has been triggered by important contributions to the empirical analysis of inequality (Galbraith, 2009), but these empirical analyses must be combined with an economic theory that is adequate to address the macroeconomic and microeconomic effects of inequality on social welfare. These problems are not always well diagnosed because the empirical measurement of inequality is often unable to take into account the geographical dimension of inequality, which is particularly complex in Europe. In this preliminary analysis, we were able to see some of the advantages/disadvantages of ordered families of LCs compared to bi-parametric families. Ordered families are easy to interpret and may provide an extremely accurate approximation: this is the case, for instance, of the Lamè class, which seems to be able to capture the shape of an LC, at least with regard to the dataset considered. Moreover, the unique parameter contains most of the information about the characteristics and the concentration patterns of the population, and can, therefore, be used as an index of inequality, just like the Gini index. In fact, the Gini index has a one-to-one correspondence with the parameter, in the case of mono-parametric families. On the other hand, the bi-parametric model that we focused on generalizes the Lamè class introducing an additional parameter. As an obvious consequence, the fit is enhanced, although this may be not sufficient to justify the use of a bi-parametric model. In this regard, the main advantage of a generalized model is the possibility of generating scenarios of crossing LCs, which happen to be quite frequent in practical situations. Indeed, our analysis shows that LCs of European countries cross almost half of the times so that it could be inappropriate to describe a partially-ordered set of LCs with a totally-ordered family. Differently, the GLLC address this issue by supporting most of the cases when LC do (or do not) cross. Future work will be aimed at extending this analysis to a larger number of parametric models for the LC.

Acknowledgements

The research was supported through the Czech Science Foundation (GACR) under project 17-23411Y and by VSB-TU Ostrava under the SGS Project SP2018/34.

References

[1] Aaberge, R. (2009). Ranking intersecting Lorenz curves. *Soc. Choice Welf* , 33, pp. 235-259.

[2] Aghion, P. and Howitt, P. (1998). *Endogenous growth theory*. Cambridge: MIT Press.

[3] Allmendinger, J. and Driesch, E. (2014). Social Inequalities in Europe: Facing the challenge. *Discussion Paper P 2014-005*. Berlin: Social Science Center.

[4] Barro, R. and Sala-i-Martin, X. (1995). *Economic growth*. New-York: McGraw Hill.

[5] Galbraith, J.K. (2009). Inequality, unemployment and growth: new measures for old controversies. *Journal of Economic Inequality*, 7(2), pp 189-206.

[6] Gastwirth, J.L. (1971). A general definition of the Lorenz curve. *Econometrica*, 39, pp. 1037-1039.

[7] Henle, J. M., Horton, N. J. and Jakus, S. J. (2008). *Modelling inequality with a single parameter. In Modeling income distributions and Lorenz curves* (pp. 255-269). New York: Springer.

[8] Lando, T., Bertoli-Barsotti, L. and Staníčková, M. (2017). Weak Orders for Intersecting Lorenz Curves: Analysis of income distribution in European countries. *Proceedings of the 35th International Conference on Mathematical Methods in Economics.* Ostrava: VŠB-TUO, pp. 402-407.

[9] Lando, T. and Bertoli-Barsotti, L. (2016). Weak orderings for intersecting Lorenz curves. *Metron*, 74(2), pp. 177-192.

[10] MacGregor Pelikánová, R. (2017). European Myriad of Approaches to Parasitic Commercial Practices. *Oeconomia Copernicana*, 8(2), pp. 167-180.

[11] M. (2017). Does the Full Harmonization of the Consumers´Protection Against Unfair Commercial Practices Via UCPD Fit in Europe 2020? *Czech Yearbook of International Law - CYIL*, 8, pp. 223-231.

[12] Melecký, L. (2018). The main achievements of the EU structural funds 2007–2013 in the EU member states: efficiency analysis of transport sector. *Equilibrium. Quarterly Journal of Economics and Economic Policy,* 13(2), pp. 285-306.

[13] Muliere P. and Scarsini M. (1989). A note on stochastic dominance and inequality measures. *Journal of Economic Theory*, 49(2), pp. 314-323.

[14] Pakšiová, R. (2016). Understanding of corporate social responsibility in large companies in Slovakia within the context of a sustainable development. In *Economic policy in the European Union member countries: proceedings of 14th international scientific conference*. Karviná: Silesian University in Opava, pp. 516-525.

[15] Quah, D. (1997). Empirics for growth and distribution stratification, polarization and convergence clubs. *Journal of Economic Growth*, 2(1), pp 27-59.

[16] Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5), pp. S71-S102.

[17] Sala-i-Martin, X. (2006). The world distribution of income: Falling poverty and… Convergence, Period. *The Quarterly Journal of Economics*, 121(2), pp- 351-397.

[18] Sarabia, J. M., Castillo, E. and Slottje, D. J. (1999). An ordered family of Lorenz curves. *Journal of Econometrics*, 91(1), pp. 43-60.

[19] Sarabia, J. M., Jordá, V. and Trueba, C. (2017). The Lamé class of Lorenz curves. *Communications in Statistics-Theory and Methods*, 46(11), pp. 5311-5326.

[20] Staníčková, M. (2017). Can the implementation of the Europe 2020 Strategy goals be efficient? The challenge for achieving social equality in the European Union. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 12(3), pp. 383-398.

[21] Turečková, K. (2015). Income Inequality by Method of Non-weighted Average Absolute Deviation: Case Study of Central and Eastern European Countries. *Equilibrium*. *Quarterly Journal of Economics and Economic Policy,* 10(4), pp. 99-110.

THE DEPARTMENT OF FINANCE IS SPONSORED BY