Economical and Financial view of Ageing[1]

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Abstract

Classical methods of accounting provide distorted information about ageing, its dynamics and methods of influence. The problem is in depreciation which is done in historical prices and in most cases the depreciation is a function only of one parameter, i.e. time. The second important parameter should be a production.

The main objective of this work is to describe a valuation from the view of ageing and from the view of asset maintenance which slow down the ageing. Such a model provides better view of assets age-structure of the company. Its principle lies in the asset valuation in actual price level.

Keywords

Ageing, depreciation, maintenance, dual time, price level

1 Maintenance and Ageing Economy

Expense valuation of an asset is given by

$$C_{t+1} = [C_t(1+i) - o_{t+1} + m_{t+1}]^+, \tag{1}$$

where

t is time period in years since the acquisition,

C_{t+1} is valuation of asset in the year $t+1$, in the price level of the year $t+1$ (after taking inflation into account),

C_t is valuation of asset in the year t, in the price level of the year t (after taking inflation into account),

i is average growth price index in the given branch (written as a decimal number, i.e. 2.8% is 0.028)

o_{t+1} is accounting depreciation caused by the ageing in the price level of the year $t+1$ (after taking inflation into account),

m_{t+1} is reduction of ageing caused by invested resources of maintenance in the price level of the year $t+1$ (after taking inflation into account),

means, that if the number inside the brackets is a non-negative number then it is used this number, but if the number inside the brackets is a negative number, than it is used zero instead. Interpretation of this is that value of an asset could not be negative.

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Remark 1 Expression (1) means that asset valuation in the next year is this year value appreciated by the growth price index, depreciated by ageing (accounting depreciation) and appreciated by invested resources of maintenance.

Remark 2 Inflation in this paper should be understood as a change in the price level. This has several dimensions and effects (Growth price index, Producer Price Index (PPI), etc.).

Remark 3 Asset maintenance means such activities which maintain or modify asset state so that ageing is slowed down. It does not means common operating activities which have no influence on ageing, i.e. refuelling of gasoline or refilling operating materials such as the paper or the ink in a printer.

Solution of (1) is

$$C_t = (1+i)^t \left(C_0 - \sum_{j=1}^{t} \frac{o_j - m_j}{(1+i)^j} \right) \quad \text{for} \quad C_0 \geq \sum_{j=1}^{t} \frac{o_j - m_j}{(1+i)^j} \tag{2a}$$

till the depreciation is finished and

$$C_t = 0 \tag{2b}$$

after the depreciation is finished.

Solution given by (2a) and (2b) means that actual valuation is equal to the present value of the original cost adjusted by the present value of difference between ageing and asset maintenance.

In the mentioned simple model it is used the assumption that maintenance slow down ageing.

The term $\sum_{j=1}^{t} \frac{o_j - m_j}{(1+i)^j}$ is present value of ageing (depreciations o_j) adjusted by the maintenance m_j. As the discount rate there is used average growth price index in the given branch and as the *present* it is used the time of asset purchasing.

In the case when it is used straight-line depreciation with the zero scrap value, i.e.

$$o_j = (1+i)^j \frac{C_0}{T}, \tag{3}$$

where

T is normative time of economic-technical life,

$\frac{C_0}{T}$ is annual depreciation expense in the historical price, i.e. time-linear ageing,

$(1+i)^j$ is revaluating factor which shifts annual depreciation expense in the historical price to the price level of the j th year, and

$(1+i)^j \frac{C_0}{T}$

is annual depreciation expense in the price level of the j th year from the year of purchasing,

then it is obtained from (2a) and (2b)

$$C_t = (1+i)^t \left(C_0 \left(1 - \frac{t}{T}\right) + \sum_{j=1}^{t} \frac{m_j}{(1+i)^j} \right) \quad \text{for} \quad C_0 \left(1 - \frac{t}{T}\right) + \sum_{j=1}^{t} \frac{m_j}{(1+i)^j} \geq 0 \tag{4a}$$

$$C_t = 0 \quad \text{else.} \tag{4b}$$

For the model example (situation) it is possible to derive mean annual maintenance expenses of asset as $m_0 = \alpha C_0; \;\; 0 < \alpha < 1, \qquad m_j = \alpha C_0 (1+i)^j$.

Then

$$C_t = (1+i)^t C_0 \left(1 - \frac{t}{T} + t\alpha\right) \quad \text{for} \quad 1 - \frac{t}{T} + t\alpha \geq 0 \tag{5a}$$

$$C_t = 0 \quad \text{else,} \tag{5b}$$

where α is proportional part of the original cost (in the price level of the year of purchasing) which is in "average" annually expended for the asset maintenance. Due to this maintenance it is enlarged life-time. This enlarged time t_L is solution of the next equation

(6)

The mathematical solution of the previous equation is

$$t_L = \frac{T}{1 - \alpha T}. \tag{7}$$

This solution could not have economical and real meaning; therefore it is necessary to fulfil the next condition

$$t_L = \frac{T}{1 - \alpha T} > T, \tag{8}$$

i.e. it is obtained enlarged life-time t_L which is greater than the original life-time T . From the previous results is it obtained

$$0 < 1 - \alpha T < 1 \quad \Rightarrow \quad \alpha < \frac{1}{T}. \tag{8}$$

Therefore if straight-time depreciation is used then "average" annual expenses in historical prices have to be less than depreciation.

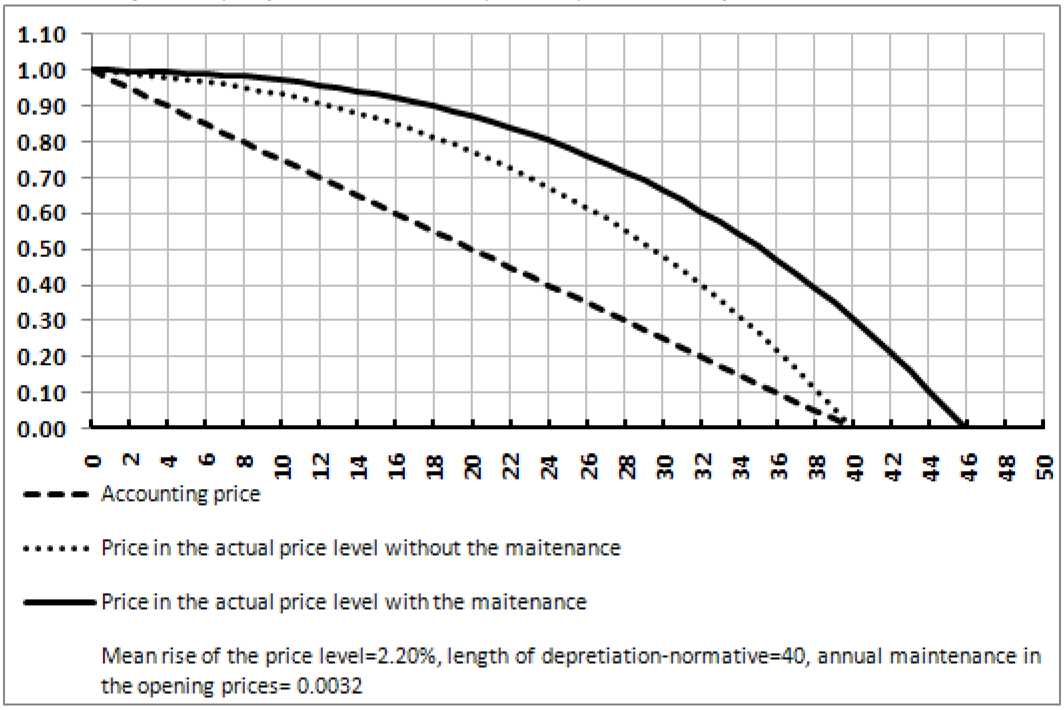
For $\alpha = \frac{1}{T}$ results from (5a) and (5b) that asset value (measured in the price level of the year of purchasing) is the same for all years and only its valuation will be changed as the price level is changing.

Remark 4 The previous part is based on the assumption that the value (valuation in the price level of the year of purchasing) does not rise in the time. Interesting and useful task is to find out $\alpha \in (0,1)$ so that the life-time is extended by $100\beta\%$ of the original normative life-time. Such α could be found as a solution of this equation

(9)

Model situation for unite purchasing cost, i.e. $C_0 = 1$, is demonstrated in the following figure.

Fig. 1: Graph of the valuation in dependency on the time for the case $C_0 = 1$



C_t is internal valuation of an asset in the company. This valuation needs to have a real-life interpretation, therefore it has to fulfil some special conditions. These could be different for each asset. One of these "market" property (i.e. condition) of the price is that price declines as the time flows by, i.e. older asset is usually sold for lower price than the original purchasing price. This can be rewritten as

$$C_t = (1+i)^t\left(C_0 - \sum_{j=1}^{t}\frac{o_j - m_j}{(1+i)^j}\right) \le C_{t-1} = (1+i)^{t-1}\left(C_0 - \sum_{j=1}^{t-1}\frac{o_j - m_j}{(1+i)^j}\right) \quad \text{for} \quad C_0 - \sum_{j=1}^{t-1}\frac{o_j - m_j}{(1+i)^j} \ge 0. \tag{10}$$

General solution of this equation is

$$i\left(C_0 - \sum_{j=1}^{t}\frac{o_j - m_j}{(1+i)^j}\right) \le \frac{o_t - m_t}{(1+i)^t} \quad \text{for} \quad C_0 - \sum_{j=1}^{t-1}\frac{o_j - m_j}{(1+i)^j} \ge 0. \tag{11}$$

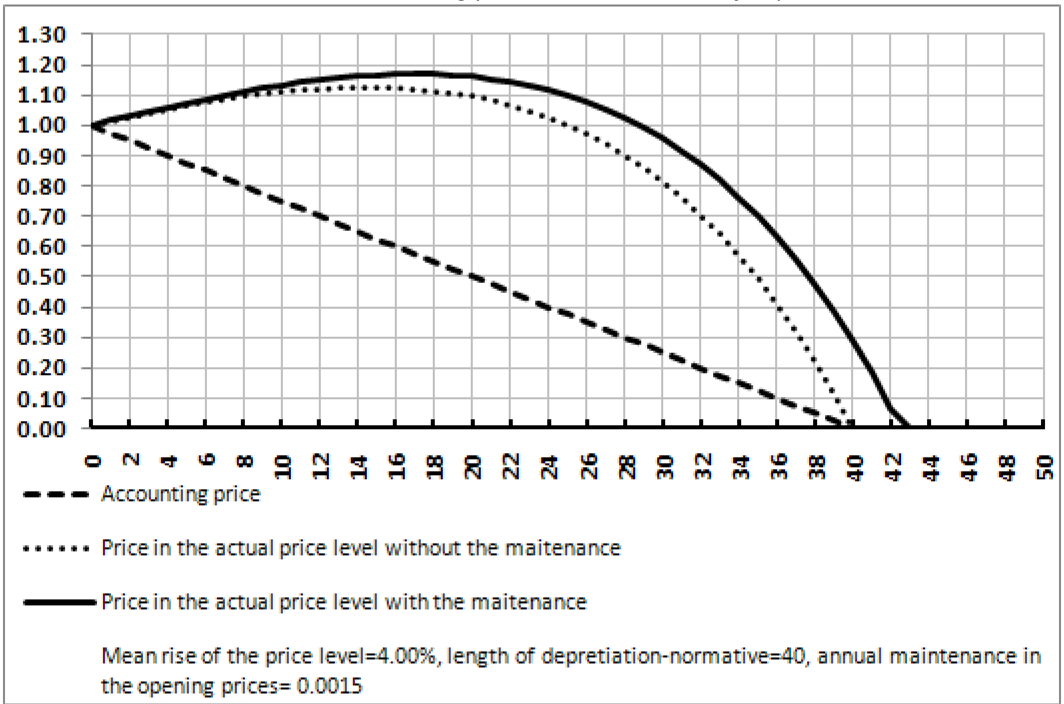
Special solution for linear ageing $o_j = (1+i)^j \frac{C_0}{T}$ and equal average maintenance $m_j = \alpha C_0 (1+i)^j$ leads to

$$\alpha \le \frac{1}{T} - \frac{i}{1+it} \quad \forall t \ge 1 \quad \Rightarrow \quad 0 \le \alpha \le \frac{1}{T} - \frac{i}{1+i}. \tag{12}$$

If this condition is fulfilled, then the solution is non-rising salvage value in the actual price level. In the economic sense this means that average annual maintenance expenses (expressed as a decimal number from unit purchasing number) that rise above $0 \le \frac{1}{T} - \frac{i}{1+i}$ in the price level of the year of purchasing are not possible to be exercisable.

If the condition $0 \le \frac{1}{T} - \frac{i}{1+i}$ is not fulfilled, it means that normative life-time T is inappropriately chosen with respect to changes of the price level. Because of this it is necessary to add condition for normative (accounting) life-time. In this special case it is $T \le 1 + \frac{1}{i}$. The situation where this condition is not fulfilled is show in the Figure 2.

Fig. 2: Graph of the valuation in dependency on the time for the case $C_0 = 1$, situation where it is wrongly used normative time of depreciation



2 Dual Time Problem

In the previous part there were studied the case, where the ageing is represented by the time as a physical quantity. For a lot of assets it is inappropriate model. In some cases it is better to use ageing represented by production (e.g. number of kilometres for cars). Then the ageing (depreciation) is expressed by

$$o_j = (1+i)^j \frac{p_j}{P} C_0, \tag{13}$$

where

p_j is total production in j th year in some units that are not influenced by the change of the price level,

P is normative total production of the whole asset life.

General expression given by (2a) and (2b) has in this case form

$$C_t = (1+i)^t \left(C_0 \left(1 - \sum_{j=1}^{t} \frac{p_j}{P} \right) + \sum_{j=1}^{t} \frac{m_j}{(1+i)^j} \right)$$
$$\text{for } C_0 \left(1 - \sum_{j=1}^{t} \frac{p_j}{P} \right) + \sum_{j=1}^{t} \frac{m_j}{(1+i)^j} \geq 0 \tag{14a}$$

till the depreciation is finished and

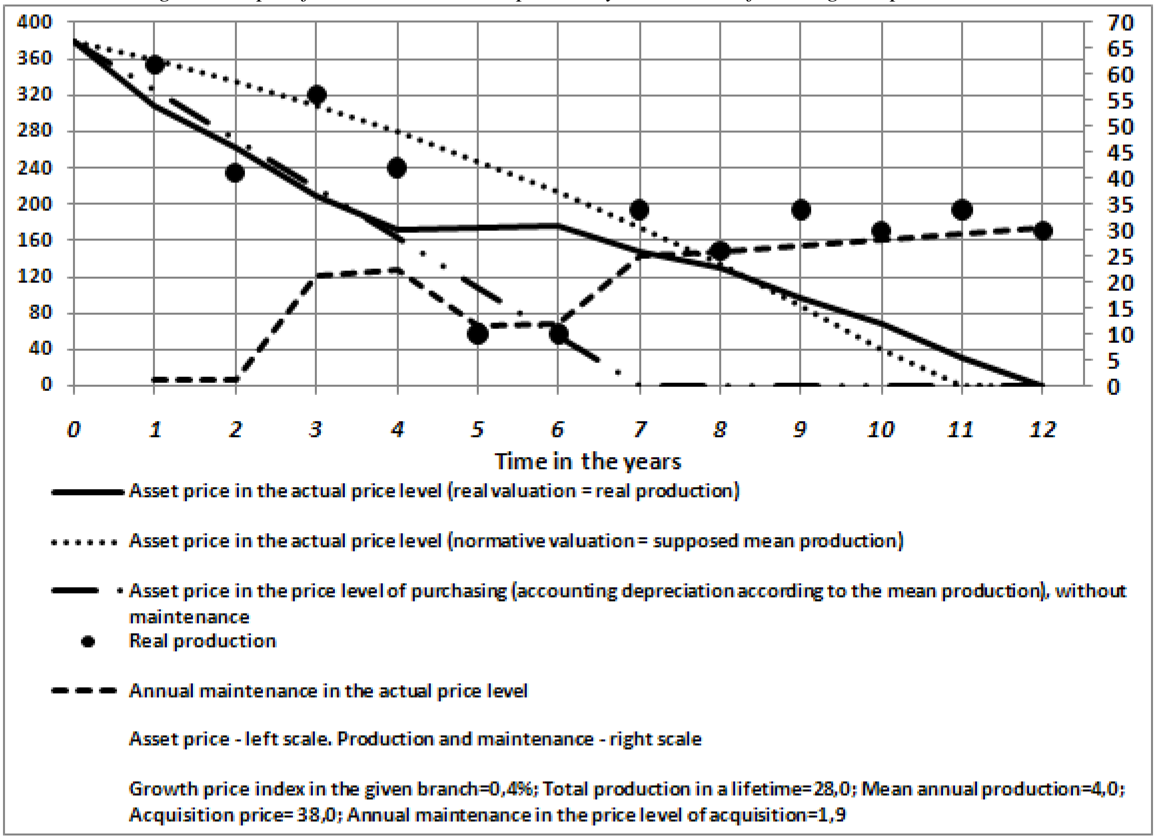
$$C_t = 0 \tag{14b}$$

after the depreciation is finished.

In the special case where $\frac{p_j}{P} = \frac{1}{T} \; \forall j$ i.e. the production is equally distributed then both models (time based and production based) are identical.

Remark 5 Ageing measured in the units of production can be in different units of measurement. Those units of measurement could be kilometres for vehicles; the number of machine pressed pieces for stamper; cumulated amount of the power that were turned off by power on-off switch, etc. All this means that unlike the time based measurement the production based measurement of the time can slow down or accelerate. Example of the model case is in Figure 3.

Fig. 3: Graph of the valuation in dependency on the time for the given parameters.



3 Next research

- Involvement of the production incomes into the model (i.e. not only cost view as it is presented in the paper) in those situations where it is possible.
- Procedures optimization of the maintenance costs, so that resources for maintenance are spending according to some criterion with economical interpretation.
- Models for groups of assets with different times of purchasing.
- Involving noise term, i.e. probability models of time and production to the next substantial failure. Decision criterion whether there should be repair or new purchase after substantial failure.

Literature

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Summary

Klasické účetní postupy poskytují o stárnutí, jeho dynamice a ovlivňování poměrně zkreslené informace. Jde především o to, že se odpisuje v historických cenách a často se

odpisuje jen časově, nikoliv produkčně. Cílem tohoto příspěvku je popsat vývoj ocenění daného aktiva z pohledu stárnutí a z pohledu péče o aktivum, která stárnutí zpomaluje. Takový model pak umožňuje lepší pohled na „věkovou“ strukturu aktiv v podniku. Jeho podstatou je výpočet ceny (ocenění) aktiva v aktuální cenové hladině.