Application of immunization strategy in management of treasury bond portfolio based on the example of bond market in Poland

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Abstract

The aim of the research is to design a portfolio consisting of treasure bonds issued by Ministry of Finance in Poland and check to what extent it is resistant to changes of interest rates. Immunization strategy is to be used build such a portfolio that would be resistant to changes of interest rates and at the same time able to keep the required value. The research consists of basic parts: theoretical and empirical.

Key words:

capital market, bond market, treasure bonds, investment strategy in bonds, immunization of bond portfolio, bond market in Poland.

1 Introduction

When deciding to invest on the capital market, the investor faces a choice of instruments into which to invest his money. The 'careful' investor would rather invest into instruments of little investment risk like bonds.

Investing into bonds requires specifying aims, setting investment limits, choosing investment strategy to achieve set aims, and finally design a treasury bond portfolio. If the investor`s aim is to design a such a bond portfolio that would guarantee optimization of some index, he will use indexing strategy, if, however, the aim is to adapt income structure from bonds to paying off expected liabilities, the owner of bonds will apply strategy adjusting to money flows or portfolio immunization.

The aim of the research is to design a bond portfolio consisting of owing financial instruments as treasury bonds are and to check how such a portfolio reacts to changes of interest rates in order to achieve the aim strategy of portfolio immunization will be applied.

2 Treasury bonds – characteristics

A treasury bond – owing treasury note, in which the issuing body, being the debtor to the bond owner, takes the obligation to buy it out in due time and is most frequently described by terms like nominal value (value paid to the owner in due time from which interest is calculated; expresses amount of debt to the owner), buyout date (date of paying back bonds by the issuer), bond interest rate (coupon rate), interest payment rate – usually yearly or half-yearly [5].

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There are many kinds of bonds on the capital market – with regard to the issuing body like those issued by the State, company, municipal ones, bank bonds (rarely), and bonds with regard to interest on bonds like fixed interest bonds or changeable interest bonds and non-interest ones.

In bond analysis the basic operation is bond valuation, that is to assess its value at a given time. In order to assess the bond value (fixed and floating interest rate), the following formula was applied (at a yearly interest payback)

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}, \quad (1)$$

where :

P – bond value,

n – number of years for which bond was issued (number of periods to buyout date),

C_t – income (money flow) for having bonds during the period numbered t,

r – internal payback bond rate, describes value of market interest rate or income rate required by investor.

When interests are paid more frequently than once a year, the formula for assessing bonds is

$$P = \sum_{t=1}^{nm} \frac{C_t}{(1+\frac{r}{m})^t}, \quad (2)$$

where: m – number of paid interests in a year; others remain the same.

For bonds of floating interest, only interest values for the first interest period are known while the other interests can vary depending on changes of interest rates on the market. The value of the interest rates are expressed as reference rate[2] and a certain added value, so called margin [5; 6]. That is why approximate values are obtained when assessing floating interest bonds.

An important of role in assessing the value of bonds plays the income rate for buying out bonds presented here as *YTM*, which means interest rate expected by the investor who will buy treasury bond at market price, keep it till buyout date and reinvest interests at that rate [2; 5]. Formula (1) is used to calculate this value where interest rate r is constant at each period

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+YTM)^t}, \quad (3)$$

at notations as above and in situation when interests are paid once a year. If, however, interests are paid more frequently, the formula is amended like

$$P = \sum_{t=1}^{nm} \frac{C_t}{(1+\frac{YTM}{m})^t}, \quad (4)$$

where m means number of paid interests in a year. Other notations remain the same.

Value of *YTM* helps to evaluate the attractiveness of treasury bonds into which bondholder invests, in comparison with other ways of investing capital.

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[2] Reference rates are interest rates whose value can be changed depending on market rate changes; the basis for calculating these rates are usually rates taken from interbank market (for example WIBOR or EURIBOR). WIBOR – Warsaw Interbank Offered Rate, which means interest rates of interbank deposits in Warsaw. It is the interest at which banks grant credits to other banks; in other words it is the price of money on the Polish market reflecting the mean cost of money for a given day at a given period [www.nbp.pl].

3 Hazards related to treasury bond investing

All investments made on financial market bear some risk. Treasury bonds regarded as securities of little investment risk are "subject to", among others, credit risk whose main element is a possibility of not keeping obligations by the issuer (failing to pay term interests and nominal at due buyout date) and interest rate risk which manifests itself as price risk and reinvestment risk[3] [5].

Treasury bonds (subject of analysis) are regarded as credit risk free. However, they are not free from price risk that takes place when the investor is forced to sell bonds before buyout date (when market rates increase, price of bonds decrease) or reinvestment risk – it is assumed that obtained interest is reinvested according to income rate *YTM*, however, if the reinvestment rate (depending on market interest rates) is different, it may turn out that the profit from investing interest will be lower than assumed [3].

The basic interest rate risk measurement, especially bond price risk, is *duration*[4] which may be presented in a few ways[5], however, *effective duration* seems to be the most adequate and is expressed by the formula

$$ED = \frac{P_{-} - P_{+}}{2P(\Delta r)}, \tag{5}$$

where:

ED – effective duration,

P – bond value before change of income rate,

P_{-} – bond value in case of income rate decrease,

P_{+} – bond value in case of income rate increase,

Δr – change of income rate.

This measurement informs about approximate decrease (increase) of bond value where income rate increases (decreases) by one percent point.

An alternative risk measurement is Macaulay` s *duration*[6] expressed in formula

$$D = \frac{\sum_{t=1}^{n} \frac{t \cdot C_t}{(1+YTM)^t}}{P}, \tag{6}$$

where notations are as above. If interests are paid more frequently than once a year, the formula is

$$D = \frac{\sum_{t=1}^{nm} \frac{t \cdot C_t}{(1+\frac{YTM}{m})^t}}{P} \cdot \frac{1}{m}, \tag{7}$$

where:

m – number of interest periods in a year; the other notations as above.

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[3] In addition, pre-term buy out risk, currency risk, inflation risk, floating risk, risk of risk, income curve risk, event risk and task risk [3] are described.

[4] Another measurement of bond price risk (complementary and assessing more precisely change of bond price) is convexity.

[5] On the basis of [5; 6].

[6] Macaulay` s duration describes the moment at which effects caused by floatation of interest rates are balanced and the investment profit equals the assumed one (interest rate decrease causes an increase of bond value but decreases profit from interest reinvestment; however, when treasury bonds are held till buyout date, the profit from bonds will be lower than expected) [2].

In addition, sometimes is used the so-called modified *duration* which is expressed by formula

$$MD = \frac{D}{1+YTM}, \tag{8}$$

where the value is closer to effective duration than duration itself.

If you have a portfolio of bonds, you may fix duration of it and Macaulay` s duration of portfolio is expressed by formula

$$D_p = \sum_{i=1}^{n} w_i \cdot D_i , \tag{9}$$

where:

D_p – portfolio duration,
D_i – duration of i-bond,
n – number of different bonds in portfolio,
w_i – profit sharing of i-bond in portfolio.

4 Immunization of bond portfolio

There are several strategies of investing in bonds, however, the strategy of immunization is outstanding out of the most important ones like active, passive or adjusting. Immunization strategy is a combination of active strategy in which return rate from bond portfolio are generated on the basis of expectations concerning indexes of changes (like interest rate) determining values of described assets, and passive strategy consisting in designing such a portfolio which will obtain assumed in advance effectiveness [2; 3]. A classical immunization of portfolio means designing such an investment portfolio which would in given time guarantee substantial income regardless of interest rate values. Such a strategy is recommended in situations where the investor` s aim is to create a bond portfolio which will make it possible to meet an obligation at the moment of bond buyout (so-called single immunization) [4].

Price risk and reinvestment risk, being elements of interest rate risk, interact contrary to the final value of investment. At fluctuations of market interest rates there may be serious discrepancies between planned and obtained investment values. Immunization of treasury bond portfolio allows to endeavor to eliminate those discrepancies by obtaining a balance between the expected and obtained value [7]. The point is to obtain such a bond portfolio that is not very sensitive to interest rate risk [5].

In order to design an immunized bond portfolio (obtain assumed income from portfolio regardless of market interest rate values) two conditions have to be met: duration of built portfolio must equal the length of investment horizon and current investment value has to be equal to current value of future liabilities; equality of portfolio and length of investment horizon (necessary condition for effective application of immunization strategy) eliminates positive and negative effects of interest rate changes [4].

The above mentioned method assumes that if interest rates change, they change at the same degree and in the same direction. However, if this condition is not met, the portfolio works differently (best is when portfolio consists of bonds whose duration is close to portfolio duration) [7]. Portfolio duration changes due to interest rate fluctuation in investment horizon, and when at free structure of interest rates the change is caused by time. Despite that immunization of portfolio is possible when the portfolio is restructured in such a way that its duration is equal to time period left to termination of investment [4].

5 Treasury bonds in Poland

5.1 Empirical data

Empirical research was based on data taken from Ministry of Finance, NBP Broker, Warsaw Stock Market. In the month of August 2009 the following State Treasury bonds were on sale:

Kind	Code	Sale period	Interest	Issue price (in PLN)	Interest payment	Buyout date	Left duration time in years
2-year bonds of fixed interest	DOS0811	01.08.2009 - 31.08.2009	4.75% per year, fixed for 2 years	100.00	yearly interest capitalization	1-31 August 2011	2
3-year bonds of variable interest	TZ0812	01.08.2009 - 31.10.2009	variable interest[7], 4.17% in the 1st interest period	99.90	payout of half-yearly interest	1 August 2012	3
4-year indexed bonds	COI0813	01.08.2009 - 31.08.2009	variable interest, 5.75% in 1st yearly interest period	100.00	payout of yearly interest	1-31 August 2013	4
10-year pension indexed bonds	EDO0819	01.08.2009 - 31.08.2009	variable interest, 6.75%, in 1st yearly interest period	100.00	yearly interest capitalization	1-31 August 2019	10

Table 1: List of offered for sale treasury bonds in August 2009. Data from internet site www.obligacjeskarbowe.pl.

A bond portfolio consisting of presented above financial instruments will be below designed and next checked to what extent bond portfolio is resistant to interest rate fluctuation. To achieve that aim, strategy of immunization will be used. The portfolio will consist of two and three- year bonds.

The four-year bonds issued by State Treasury in Poland are indexed treasury bonds whose aim is to guarantee their owners real purchasing power of paid out interests and amount of money used to buy them. The interest rate and redemption price may depend on, for example, the price of gold or rate of inflation [www.obligacjeskarbowe.pl], that's why these bonds where ignored in designing bond portfolio. The ten-year pension indexed bonds were not taken into account either because of long time horizon and variable interest[8].

5.2 Valuation of treasury bonds

In case of a fixed interest two-year treasury bond in the 1st interest period, interest is calculated from nominal value of single obligation, and in the second period it is calculated from nominal value increased by interests after completing first interest period. Table 2 presents results of a two-year bond valuation for which it was assumed that r, being the required by investor income rate, equals consecutively 4%, 4.75 % and 5 %.

Years	Money flows	Discounted money flows for different values r		
		r = 4%	r = 4.75%	r = 5%
1	4.75	4.57	4.53	4.52
2	104.75	96.85	95.47	95.01
Sum	109.50	101.41	100	99.54

Table 2 : Valuation of two-year bond for different values of required income rate [*source: own research*].

[7] Variable interest based on WIBOR.
[8] It is with making a possible forecast error in long time horizon.

Fixed interest causes that the buyer already at the moment of purchasing knows value of interest they are to receive after two years of saving. In case of $r = 4.75\ \%$ the bond value equals its nominal value what means that the bond is sold at the nominal value.

In case of a three-year bond, it was assumed that the required by investor income rate is 4.17. However, it is a bond of variable interest therefore due to knowing interest rate only for the first interest period (6 months) the forecast for bond interest for consecutive interest periods was set. Reference rate values WIBOR and forecasts obtained on the basis of estimated autoregressive model[9] were used. Forecast was set basing on historical data taken from Feb.1998 to Aug. 2009 (WIBOR values for Feb. 1st and Aug. 1st from 1998 to 2009).

The autoregressive model is $y_t = 0.816157 y_{t-1} + 0.887174$. Table 3 presents historical data and forecast results.

t		WIBOR value	WIBOR forecast
1	February 1998	24	
2	August 1998	19	20.47
3	February 1999	13	16.39
4	August 1999	13	11.50
5	February 2000	17.5	11.50
6	August 2000	17.5	15.17
7	February 2001	18	15.17
8	August 2001	15.5	15.58
9	February 2002	10	13.54
10	August 2002	8.5	9.05
11	February 2003	6.5	7.82
12	August 2003	5.25	6.19
13	February 2004	5.25	5.17
14	August 2004	6	5.17
15	February 2005	6.5	5.78
16	August 2005	4.75	6.19
17	February 2006	4.25	4.76
18	August 2006	4	4.36
19	February 2007	4	4.15
20	August 2007	4.5	4.15
21	February 2008	5.25	4.56
22	August 2008	6	5.17
23	February 2009	4.25	5.78
24	August 2009	3.5	4.36
25	February 2010	forecast	3.74
26	August 2010	forecast	3.94
27	February 2011	forecast	4.10
28	August 2011	forecast	4.24
29	February 2012	forecast	4.35

Table 3 : WIBOR reference rate value from Feb. 1998 to Aug. 2009 including forecasts for Feb. 2010 to Feb. 2012 [*source: own research*].

Adjustment coefficient to this model for data is $R^2 = 0.860194$. Mean square forecast mistake equals 0.152544. On the basis of current interest and reference rate forecast, the value of three-year treasury bond of variable interest was calculated. It was assumed that the bond

[9] It was checked empirically that adjustment coefficient for autoregressive model compared to other considered models was closest to value 1.

interest equals the value of reference value plus yearly margin at the amount of 0.67 (as the difference between yearly interest of three-year bonds and reference rate value in Aug. 2009). Two variants were taken: income rate equal to interest during first interest period and income rate depending on WIBOR forecast. Results are presented in the following table.

Years	Money flows	Discounted money flows at fixed rate r = 4.17%	Discounted money flows at income rate depending on WIBOR forecast
0.5	2.082915000	2.040373	2.040373
1	2.204654888	2.115518	2.157052
1.5	2.304013750	2.165705	2.252074
2	2.385106181	2.196140	2.329490
2.5	2.451290336	2.210981	2.392582
3	102.4053070	90.47964	99.90000
Sum	113.8332872	101.2084	111.0006

Table 4 : Valuation of 3-year bond [*source: own research*].

Bond value certainly depends on generated forecasts, investor` s required income rate and set fixed margin value.

5.3 Setting selected bond risk measurements

In order to examine risk level, the following items were calculated:

- *YTM*, the income rate at the buyout period
- Effective duration
- Macaulay` s duration
- Modified duration

Results of calculation are presented in the table.

	Bond interest (in %)	YTM (in %)	Effective duration	Macaulay` s duration	Modified duration
2-year bond of market value P=101.41	4.75	4	1.879907	1.954963989	1.914963989
2-year bond of market value P=100	4.75	4.75	1.866370	1.954653938	1.907153938
2-year bond of market value P=99.54	4.75	5	1.861796	1.954550632	1.904550632
3-year bond of market value P=99.90	variable	4.65	2.756600	2.841847000	2.715573000

Table 5 : Selected bond risk measurements [*source: own research*].

In case of two-year fixed interest bonds whose interest equals r = 4.75%, the effective duration value is 1.86637 what means that when income rate changes by 1% point, bond value will change approximately by 1.86637%. For a fixed interest two-year bond r = 4.75%, Macaulay` s duration amounts to 1.95 what means that after one year and 11.5 months the investor will get back invested costs together with interests.

5.4 Application of immunization strategy

Process of immunizing bond portfolio consists in putting together its components in such a way that at a given time, it will be possible to gain a certain amount of money regardless of interest rate fluctuation on the market. In order to immunize the portfolio, money should be invested into bonds whose bond portfolio duration equals investment horizon and whose current liability value equals future liabilities.

Example

A portfolio consisting of two-year treasury bonds of fixed interest and nominal value at PLN 100 together with three-year bonds at changeable interest at PLN 99.90 will be designed. The investor` s aim is to obtain a portfolio of value at PLN 100,000 in two and a half year.

The equations are[10]

$$1.95\,x + 2.84\,y = 2.5$$
$$x + y = 1,$$

where x – stand for two-year bond share in portfolio, while y – three-year bonds.

Having solved it, the results are x = 0.382 and y = 0.618, which means that share of two-year bonds in portfolio is 38.2% and three-year ones is 61.8%. Current value of final investment is 100000/((1+0.0446/2)^5) = 89558.79 zloty at the assumption that half-year capitalization of income and for mean interest rate of 4.46% for two and three-year bonds. Taking into consideration that only total number of bonds may be bought, the investor should buy 343 two-year bonds and 544 three-year bonds. Therefore, the total cost of buying bonds would amount to 89644.6 zloty.

Treasury bond	Share in portfolio	Intended amount for buying bonds	Number of bonds
Two-year	0.382	34300	343
Three-year	0.618	55344.6	554

Table 6 : Structure of bond portfolio [*source: own research*].

The method of portfolio immunization allows to have a portfolio resistant to interest rate fluctuation, therefore protected against risk of price change and reinvestment risk. Let us check whether a portfolio consisting of fixed and variable interest rates confirms it. Let us assume that YTM increases by 1%, remains unchanged or drops by 1%. The changes of portfolio values are presented in Table 7.

	YTM		
	1% decrease	**No change**	**1% increase**
Two-year bond	38325.54784	38530.68257	38736.0675
Three-year bond	63158.21175	61898.87189	60678.0796
Portfolio value	101483.7596	100429.5545	99414.14709

Table 7 : Final values` of bond portfolio for different YTM scenarios [*source: own research*].

Differences result from number of made rounding in transformations preceding obtained results presented in Table 7. Portfolio value after 2.5 years is approximately equal to desired final value regardless of interest rate fluctuation. Therefore, we may assume that the designed protection of future payments is resistant to interest rate fluctuation.

6 Summary

Designing a portfolio consisting of treasury bonds, immunization strategy was put into use. This method allows to obtain a portfolio of fixed value irrespective of interest rate fluctuations. Although, for chosen interest rates the values of designed portfolio differ, it is a result of transformations making during calculations.

[10] The first equation derives from formula (9), the second one stems from the fact that the sum of shares in portfolio (in fraction form) equals one.

On the basis of presented work, the conclusion to be drawn is that immunization strategy may be applied both when designing portfolio consisting of fixed and variable interest rate bonds. However, in case of variable interest rate bonds there is a necessity of forecasting bond interest rates in consecutive interest periods. Certain risks can appear with selecting proper forecast method, accuracy of forecast and right choice of fixed margin. Therefore, designing a portfolio of variable interest rate is becoming connected with a bigger risk.

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Summary

The aim of the research is to design a portfolio consisting of treasure bonds issued by Ministry of Finance in Poland and check to what extent it is resistant to changes of interest rates. Immunization strategy is to be used build such a portfolio that would be resistant to changes of interest rates and at the same time able to keep the required value. The research consists of basic parts: theoretical and empirical.