Modeling Mortgage Loss Distribution

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Abstrakt

The credit risk, a risk of counterparty default, was the first risk captured by risk management systems in banks. In our paper, we will show on an example of mortgage delinquency rates that a normal distribution can be outperformed in description of losses and that in some cases, the assumption that losses follow a normal distribution, can be very dangerous. Especially during volatile periods comparable to the current crisis, the normal distribution can underestimate tail losses. This imperfection can be corrected by assuming an alternative, e.g. generalized hyperbolic distribution for credit losses.

Klíčová slova

Credit Risk, Mortgage, Delinquency Rate, Generalizad Hyperbolic Distribution, Normal Distribution

1 Introduction

The credit risk, a risk of counterparty default, was the first risk captured by risk management systems in banks. Several decades after first risk management systems were introduced, the credit risk still remains the risk that has the most attention. The credit risk accompanies each business but is most visible in the financial sector. The business of the whole financial sector is based on a reallocation of capital sources. A traditional bank accepts deposits from the population and provides financing (loans) to individuals and companies that demand capital. One of the biggest risks arising from financial operations is the risk of counterparty default, commonly known as a "credit risk". Leaving unmanaged, the credit risk would, with a high probability, result in a crash of a bank.

In our paper, we will focus on the credit risk quantification methodology. We will demonstrate that the current regulatory standards for credit risk management are at least not perfect, despite the fact that the regulatory framework for credit risk measurement is more developed than systems for measuring other risks, e.g. market risks or operational risk. The current financial regulation was developed and maintained by European supervisory institutions (Basel Committee on Banking Supervision, CEBS – Committee of European Banking Supervisors) and its standards are summarized in the Second Basel Accord ("Basel II", Bank for International Settlement, 2006), a document describing principles that should be applied in risk management at minimum. Basel II precisely defines methods for management and measurement of credit risk. Especially measurement methods, derived from theoretical models (Vasicek, 1987), are of a high interest and a majority of space is dedicated to them. The Basel II is implemented into the European law by the "CRD – Capital Requirements Directive" (European Comission, 2006).

Credit risk is in the Basel II regulated more than other types of risks, for whose Basel II leaves more independency. Basel II allows only two possible methods how to measure credit

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risk – standardized approach (STA) and internal rating based approach (IRB) (for more details on these two methods see Bank for International Settlement, 2006). The main difference between STA and IRB is that, under IRB, banks are required to use internal measures for both the quality of the deal (measured by the counterparty's "probability of default – PD") and the quality of the deal's collateral (measured by "loss given default – LGD"). The counterparty's probability of default is a chance that the counterparty will default (or, in other words, fail to pay back its liabilities earlier than 90 days past due) in the upcoming 12 moths. The LGD is an estimate of how much of already defaulted amount will lose a bank in reality (after the collection process is finished). The LGD variable takes into account recoveries from the default, i.e. an amount that a creditor is able to collect back from the debtor after the debtor defaults. These recoveries mainly come from collateral sales and from bankruptcy proceedings.

PDs and LGDs are two major and common measures for a deal quality and basic parameters for credit risk measurement. PD is usually obtained by one of the following methods: from a scoring model (Moody's KMV, JP Morgan CreditMetrics, etc…), from a Merton-based distance-to-default model (mainly used for commercial loans; Merton, 1973 and 1974) or as a long-term stable average of past 90+ delinquencies2. LGD can be modeled as a function of collateral value. This paper treats LGD as fixed and won't describe techniques of its modeling. This simplification will allow us to focus more on the PD and explain the behavior of PDs more in detail.

Once we obtain PDs and LGDs, we are able to calculate an average loss. The average loss is a mean measure of the credit risk and is a sufficiently exact measure of credit risk on a long-term horizon. The problem is that losses on a portfolio occur with a certain probability distribution that is positively skewed. Thus to protect against credit risk, a bank has to decide on a level of probability, on which it would still be reasonable to protect itself against losses. The regulatory level of probability is 99.9%. This level may seem a bit excessive because it can be interpreted in the way that banks should protect themselves against a loss that occurs once in a thousand years. The fact is that such a far tail in the loss distribution was chosen because we lack the data – nobody has got even a hundred year long loss history.

The quantification of a 99.9% loss is usually calculated by a Value-at-Risk type model (Anthony Saunders, 2002 or Fredrik Andersson, January 2001). IRB approach is a type of Value-at-Risk model and approximates the loss distribution with the standardized normal distribution.

In this paper, we will introduce a new approach for measuring credit risk. This approach can be classed with the Value-at-Risk models and from the IRB method is different by assuming that losses follow a class of Generalized Hyperbolic Distributions. In a general form, the new approach can be used to measure credit risk of many types of products – i.e. consumer loans, mortgages, overdraft facilities, commercial loans with a lot of variance in collateral, exposures to sovereign counterparties and governments, etc. To test our model, we will demonstrate its goodness-of-fit on a nationwide mortgage portfolio. Moreover, we will compare our results with the IRB approach, prove that the credit risk quantification method based on the normal distribution is not very exact and comment on what difficulties can come when the assumption of normality turns out to be inappropriate.

The paper is organized as follows. After the introduction we will describe usual credit risk quantification methods and Basel II embedded requirements in detail. Then we will derive a new method of measuring credit risk, based on the class of Generalized Hyperbolic

[2] Delinquency is often defined as a delay in installment payments – e.g. 90+ delinquencies can be interpreted as a delay in payments of more than 90 days.

Distributions and Value-at-Risk methodology. The last part will focus on the data description and results of numerical calculations. We will show that the class of generalized hyperbolic distributions can capture the credit risk more accurately than the IRB approach from Basel II. At the end we summarize our findings and bring recommendations for further research.

2 Credit risk measurement methodology

The Basel II document is organized into three separate pillars. The first pillar requires banks to quantify credit risk, operational risk and market risk by a method approved with the supervisor. For credit risk there are two possible quantification methods: "The Standardized Approach" (STA), which is more basic, and "The Internal Rating Based Approach" (IRB). Both methods are based on quantification of risk-weighted assets for each individual exposure. The biggest difference between these two methods is that the STA uses a fixed percentage of risk-weighted assets to quantify a largest loss that could occur on the regulatory (99.9%) level of probability, whether the IRB method uses deal-based risk measures PD and LGD to obtain a mean loss and then the loss distribution to get the largest possible loss on the regulatory probability level. Loss itself is defined as an amount that is really lost when a default occurs. Default is a delay in payments due more than 90 days (90+ delinquencies).

There is no PD or LGD feature in the STA method and thus the method is relatively inaccurate. On the other hand, the advantage of this method is its simplicity. The IRB approach is more accurate but relatively difficult to maintain. A bank using the IRB method has to develop its own scoring and rating models to estimate PDs and LGDs. These parameters are then used to define each separate exposure3. An average loss that could occur in following 12 months is calculated as follows:

$$EL = PD \cdot LGD \cdot EAD, \qquad \text{(i)}$$

where EAD is the exposure-at-default and EL is an abbreviation for "Expected loss". We will borrow the EL calculation to obtain the first moment of our loss distribution. The difference is that we will hold both LGD and EAD fixed at 1 so the expected loss is only derived from the PD. This simplification doesn't mean a loss in generality. The LGD and EAD can be calculated as an EAD weighted average in the case of LGD and a sum over the portfolio in the case of EAD. Thus, our economic loss is equal to PD, showing the percentage of all accounts that defaulted.

EL is an average loss that would occur each year and thus is something that banks count into their loan pricing models. It necessarily has to be covered by ordinary banking fees and/or interest rate payments. However, EL is the "mean loss" and thus is unable to catch any volatility in losses. To protect themselves against the loss volatility, banks should hold capital to cover maximum loss that could occur on the regulatory probability level at minimum. To capture the variability in credit losses over time and to calculate the needed quantile of the loss distribution, we need a second moment of the loss distribution, the standard deviation at minimum.

On the deal level, the standard deviation calculation can be derived from the properties of the default. The default is a binary variable – it either occurs (with a probability equal to PD) or not (with a probability equal to (1-PD)). The loss occurs with the same probability as the default but is usually lower than the defaulted amount (due to the fact, that the bank sells it's

[3] Exposure is a usual expression for the balance on a separate account that is currently exposed to a default. We will adopt this expression and use it in the rest of our paper.

collateral and collects partly the defaulted amount – this is, in fact, LGD) and thus follows a binomial distribution. We can calculate standard deviation of a loss by substituting into the formula for binomial distribution standard deviation. The final formula is:

$$ULC = EAD \cdot LGD \cdot \sqrt{PD(1-PD)} \quad \text{(ii)}$$

In the formula above we call the standard deviation of a loss "ULC" or "unexpected loss contribution", because the ULC show us the uncertainty of the loss. However, on the portfolio level, the standard deviation calculation is not so straightforward. Deals are correlated among each other. We have a complicated correlation structure that is usually unknown and thus we don't even know how individual deals in our portfolio are interacting.

The second pillar of Basel II (Pillar 2) requires banks (in addition to Pillar 1 calculations) to develop their own models to quantify all identified risks that a bank could face in the upcoming year. The idea behind is that banks know much better their risk profiles much better and thus are able to calculate capital needed to cover unexpected losses more accurately than using Pillar 1 methods. The final capital that a bank should hold to cover its credit risk is the maximum of the following two capital requirements: the regulatory capital requirement calculated by an approved Pillar 1 (i.e. STA or IRB) method and the internal capital requirement calculated by the bank's own model. The Internal capital requirement is often called "economic capital".

In our research we suggest a model that could be used as an internal bank's model to calculate the economic capital requirement under Pillar 2. The main idea of our model is to use a different loss distribution, which would be able to capture loss development better than the normal distribution suggested by the IRB approach.

3 The economic capital model

There are two possible ways how to assess economic capital. First is the so called bottom-up approach, when the stock of economic capital requirement is calculated for each exposure and then aggregated using a correlation structure (this method is used in IRB approach). The second possibility is to calculate economic capital by a "top-down" approach, which in fact means, that the capital requirement is calculated for the whole bank without any concern in individual exposures. We will use the top-down approach because our data are collected from the whole US economy and don't have the granularity needed for the bottom-up approach.

The first step is to choose the right loss distribution. Our model is based on the class of generalized hyperbolic distributions, first introduced in (O.E. Barndorff-Nielsen, 1985). The advantage of this class of distributions is that it is general enough to describe a fat-tailed data. It was shown (Eberlein, 2001, 2002, 2004) that the class of generalized hyperbolic distributions is able to capture the variability in financial data in a better way than the normal distribution, which is being used by the IRB approach. Generalized hyperbolic distributions were already used in an asset (and option) pricing formula (Rejman, 1997; Eberlein, 2001 or Chorro, 2008), for the Value-at-Risk calculation of the market risk (Eberlein, 2002; Eberlein, 1995 or Wenbo Hu) and in a Merton-based distance-to-default model to estimate PDs in the banking portfolio of commercial customers (e.g. Oezkan, 2002). We will show that the class of generalized hyperbolic distributions can be used for the approximation of a loss distribution for the retail banking portfolio with a focus on mortgage book. The crucial assumption of our model is that credit losses from mortgages follow a generalized hyperbolic distribution over time.

The next step in the economic capital calculation is to calculate the difference between the mean of the loss distribution and its certain percentile (equals to the probability level used for the economic capital calculation). We will use the 99.9% percentile of the loss distribution because of the simplicity of consequent comparison to the IRB method. The mean of the loss distribution is calculated using (i) and the fact that LGD and EAD are fixed at 1.

3.1 The class of generalized hyperbolic distribution

The class of generalized hyperbolic distributions is a special, quite young class of distributions. It is defined by the following Lebesque density:

$$gh(x;\lambda,\alpha,\beta,\delta,\mu) = a(\lambda,\alpha,\beta,\delta)(\delta^2+(x-\mu)^2)^{\frac{\lambda-0,5}{2}} \times K_{\lambda-0,5}(\alpha\sqrt{((\delta^2+(x-\mu)^2)})\exp(\beta(x-\mu)) \quad \text{(iii)},$$

where

$$a(\lambda,\alpha,\beta,\delta) = \frac{(\alpha^2-\beta^2)^{0,5\lambda}}{\sqrt{2\pi}\cdot\alpha^{(\lambda-0,5)}\delta^{\lambda}K_{\lambda}(\delta\sqrt{\alpha^2-\beta^2})} \quad \text{(iv)}$$

and K_λ is a Bessel function of the third kind (or modified Bessel function – for more details on Bessel functions see Abramowitz, 1968). The GH distribution class is a mean-variance mixture of Normal and Generalized Inverse Gaussian (GIG) distributions. Both Normal and GIG distributions are thus subclasses of Generalized Hyperbolic Distributions. μ and δ are scale and location parameters, resp. Parameter β is a skewness parameter and a transformed parameter $\bar{\alpha}=\alpha\delta$ determines kurtosis. The last parameter λ is a determination of distribution subclass. There can be found several alternative parameterizations in the literature using transformed parameters to obtain scale- and location-invariant parameters. This is a useful feature that will help us with the economic capital allocation to individual exposures. For the moment generating function and for more details on the class of generalized hyperbolic distributions, see Appendix.

Because the class of generalized hyperbolic distributions was historically used for different purposes in economics as well as in physics, one can find several alternative parameterizations in the literature. In order to avoid any confusion, we list most common parameterizations. These are:

$$\zeta = \delta\sqrt{\alpha^2-\beta^2},\ \rho = \frac{\beta}{\alpha} \quad \text{(vi)}$$

$$\xi = (1+\zeta)^{-0,5}, \qquad \chi = \xi\rho \quad \text{(vii)}$$

$$\bar{\alpha} = \alpha\delta,\ \bar{\beta} = \beta\delta \quad \text{(viii)}$$

Main reason for using alternative parameterizations is to obtain a location- and scale-invariant shape of the moment generating function (see Appendix).

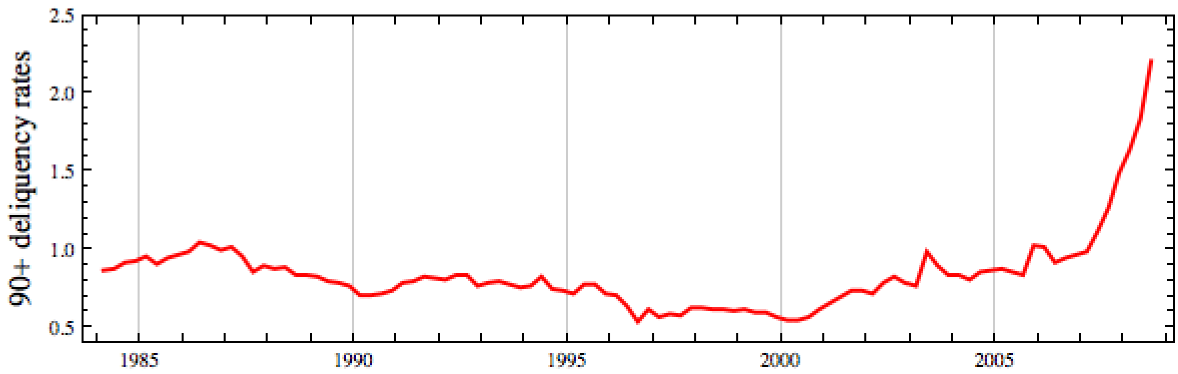
4 Data and results

4.1 Data description

To verify whether a Value-at-Risk based model built on the class of generalized hyperbolic distributions is able to better describe the behavior of mortgage losses, we will use a data for the US mortgage market. The dataset consists of quarterly observations of 90+ delinquency rates (more than 90 days past due accounts) collected by the U.S. Department of Housing and

Urban Development and the Mortgage Bankers Association4. The dataset begins with the first quarter of 1984 and ends with the third quarter of 2008 and represents the most recent dataset available at the time. The development of U.S. mortgage delinquency rates is illustrated in the Figure 1. We observe an unprecedentedly huge increase in delinquency rates beginning with the second quarter of 2007.

Figure 1: Development of US mortgage delinquency rates



We have tested the dataset for autocorrelation with the Ljung-Box Q statistic and our results show that, on the 95% probability level, the dataset is strongly autocorretaled and non-stationary (with the gamma coefficient equal to 1.05 and significant on the 95% probability level). Moreover, we performed the Augmented Dickey-Fuller test for a unit root process and we can't reject on the 95% probability level the null hypothesis that the dataset follows a unit-root process. Thus we have decided to use logarithmic returns and the new dataset is defined by:

$$del_t = \ln\left(\frac{DEL_t}{DEL_{t-1}}\right), \quad \text{(xi)}$$

where DEL are delinquencies in the original dataset. The Ljung-Box Q statistic now shows that we can't reject the null hypothesis of no autocorrelation in the delt dataset. Logarithmic changes can be at the end easily converted back to the original values by:

$$DEL_t = DEL_{t-1} \cdot \exp(del_t) \quad \text{(xii)}$$

4.2 Results

We fitted several distributions to the sample delt and sorted them by the chi-square goodness-of-fit statistic. The compared distributions were LogLogistic, Logistic, LogNormal, PearsonV, Inverse Gaussian, Normal, Gamma, Extreme Value, Stable and the Class of Generalized Hyperbolic distributions. The distributions were fitted to the delt dataset by maximizing the log-likelihood function, constructed from the distribution density function. In the set of compared distributions, we were particularly interested in the goodness-of-fit of the Class of Generalized Hyperbolic Distributions and their comparison to other distributions. The log-likelihood function in the case of the Class of Generalized Hyperbolic Distributions has got the following form:

[4] The Mortgage Bankers Association is the largest US society, representing the US real estate market with over 2,400 members (banks, mortgage brokers, mortgage companies, life insurance companies, etc…).

$$L(\lambda,\alpha,\beta,\delta,\mu) = \log a(\lambda,\alpha,\beta,\delta) + \left(\frac{\lambda}{2}-\frac{1}{4}\right)\sum_{i=1}^{n} log(\delta^2+(x_i-\mu)^2) + \sum_{i=1}^{n}\left(\log\left(K_{\lambda-0,5}\left(\alpha\sqrt{\delta^2+(x_i-\mu)^2}\right)+\beta(x_i-\mu)\right)\right) \quad \text{(xiii)}$$

For the fitting procedure we have used the R package "ghyp", which uses a different parameterization of the distribution density. The second step is to test the hypothesis that the empirical dataset comes from the tested distribution. We used the chi-square goodness-of-fit test in the form:

$$\chi^2 = \sum_{i=1}^{k}(O_i - E_i)^2 / E_i \quad , \quad \text{(xiv)}$$

where Oi is the observed frequency in the i-th bin, Ei is the frequency implied by the tested distribution and k is the number of bins. The test statistic follows the chi-square distribution with (k – c) degrees of freedom, where c is the number of estimated parameters. In the following table there are sorted distributions together with p-values of the chi-square statistic. The chi-square statistic will be used to decide on whether we can or cannot reject the null hypothesis that the dataset delt is drawn from the tested distribution. In order to reflect different number of parameters for tested distributions, we have divided obtained Chi-square statistics by number of degrees of freedom (number of parameters in the tested distribution).

We have used a different statistic to compare all tested distributions and sort them by their goodness-of-fit: the Anderson-Darling statistic. This statistic is a measure of the distance between the original sample and a tested distribution. The advantage of Anderson-Darling (compared to e.g. Kolmogorov-Smirnov or Chi-square test) is that the statistic is able to capture the bias in tails. The following table summarizes our results and is sorted by the Chi-square statistic:

Distribution	Wasserstein Metric	Anderson Darling Distance	Chi-square Statistic	P-value of Chi-square	Reject/Cannot Reject on the 90% probability level
Generalized Hyperbolic	0.005	0.19	4.98	0.55	Cannot Reject
Stable	0.007	0.26	9.91	0.27	Cannot Reject
LogLogistic	0.006	0.42	9.98	0.44	Cannot Reject
Logistic	0.007	0.59	11.10	0.35	Cannot Reject
PearsonV	0.009	0.90	12.45	0.26	Cannot Reject
LogNormal	0.009	0.91	12.45	0.26	Cannot Reject
Inverse Gaussian	0.009	0.93	14.69	0.14	Cannot Reject
Normal	0.011	1.33	15.14	0.13	Cannot Reject
Gamma	0.009	0.95	16.49	0.09	Reject
Extreme Value	0.015	1.68	18.29	0.05	Reject

Table 1: Comparison of goodness-of-fit of tested distributions

According to the Table 1, both Chi-square and Anderson-Darling tests show that the best fit has got the Generalized Hyperbolic Distribution (GHD). The Figure 1 shows graphically the difference between chosen distributions. We were particularly interested in the GHD and Stable Distribution because these two distributions show much better fit than other distributions, according to the Anderson-Darling test. From the Figure 1 we can see that the

GHD is able to describe both the skewness and the kurtosis of the dataset. We compared GHD, Stable, Normal, Lognormal and Logistic distributions (Normal and Lognormal because of their role in Pillar 1 calculations and Logistic because it has shown a good performance). The Figure 2 shows the difference in tail behavior of GHD, Stable, Normal, Lognormal and Logistic distributions and points out the gap in the right tail between GHD and Stable on one side (both heavy-tailed) and remaining distributions on the other side.

Figure 2: Compared histograms: distributions vs. del_t dataset

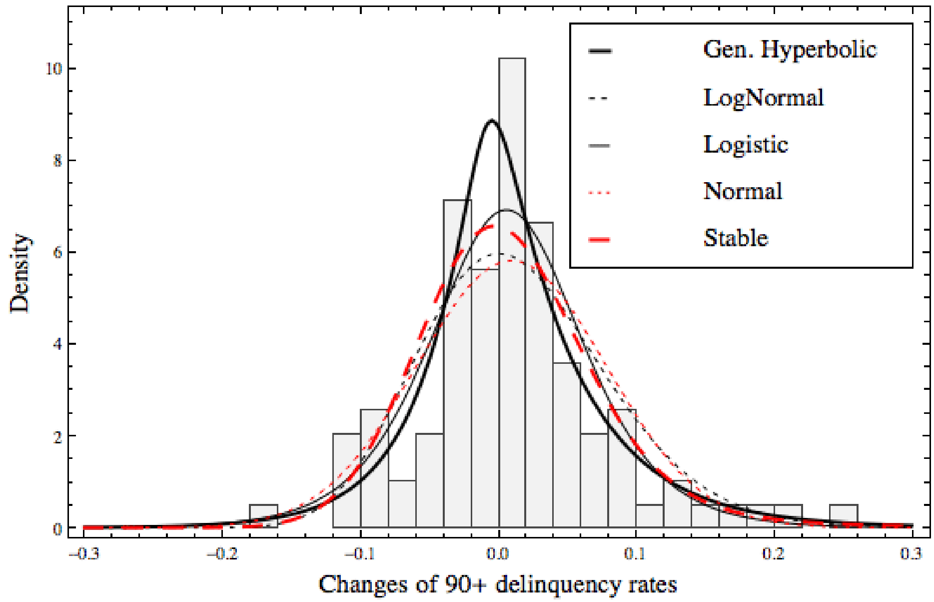
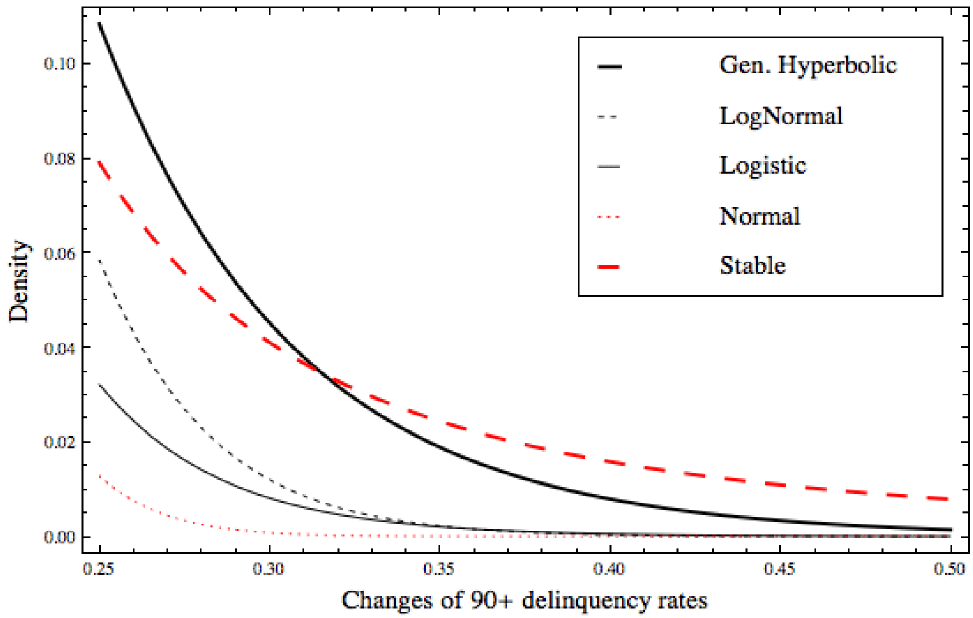


Figure 3: Comparison of the right-tale behavior



Our calculations show that the Class of Generalized Hyperbolic distributions is able to describe the behavior of delinquencies much better than other distributions widely used for risk assessment (Normal, Lognormal, Logistic, Gamma). This fact can have a large impact to economic capital requirement as the Class of Generalized Hyperbolic Distributions is heavy-tailed and thus would imply a larger stock of capital for covering a certain percentile delinquency. We will now demonstrate that the difference between economic capital requirements calculated under the assumption that mortgage losses follow a Generalized Hyperbolic Distribution and under the Basel II IRB method (assuming normal distribution).

4.3 Economic capital on a one-year horizon: implications for the crisis

The IRB formula, defined in the Pillar 1 of the Basel II Accord, assumes that losses follow a normal distribution. We have shown that this assumption is at-least not perfect and that an alternative, in our case generalized hyperbolic distribution, is able to capture the volatility in delinquencies much better. The problem of the normal distribution is the tail behavior and because the regulatory capital requirement is calculated on the 99.9% probability level, this disadvantage may lead to serious mistakes in the assessment of capital need. To show the difference between the regulatory capital requirement (calculated by the IRB method) and an economic capital requirement calculated by our model, we will perform the economic capital requirement calculations on the 99.9% probability level. Moreover, we will calculate the mean value of delinquency rate for both distributions to show the mean expected delinquency rate on a one-year horizon.

The problem is that we have estimated both normal and generalized hyperbolic distributions on a quarterly dataset and we need to transfer obtained quarter changes to yearly figures. This is not a problem for the normal distribution because under the normal distribution, elements follow a random walk and the convolution is quite simple to calculate. For the GHD, elements follow a Lévy motion and thus we would need to calculate a second convolution of the GHD. This calculation is rather more difficult and therefore we have decided to use simulations to obtain yearly figures.

To evaluate the distribution performance, we will calculate mean values of the delinquency rate predicted by both generalized hyperbolic and normal distributions on a one year horizon. Predicted values will be compared with the original dataset and the mean squared error of both predictions will be calculated. Our results are summarized in the following table. Moreover, the "99.9% quantile failure" column shows how many times was the actual delinquency rate higher than the 99.9th quantile of the prediction. The MSE is very similar in both cases, which indicates that both distributions were very close in predicting the mean value. However, there is a difference in tails. Due to the 99.9% quantile failure, normal distribution is much less capable of capturing the behavior. From 95 observations, the actual delinquency rate was three times larger than the 99.9th quantile of the normal distribution. This did not happen in the case of GHD.

Distribution	**Mean Squared Error**	**99.9% quantile failure**
GHD	0.02823	0
Normal	0.02873	3x (Q1, Q2 and Q3 2008)

Table 2: GDH vs. Normal distribution MSE and 99.9% quantile failure

Moreover, we will compare the distribution performance on recent delinquencies. In the last four quarters of the original dataset (Q3 2007 – Q3 2008) the delinquency rate increased dramatically (from 1.26% in Q3 2007 up to 2.2% in Q3 2008). In the table below, the delinquency mean value and 99.9% quantile, implied by both normal and GHD distributions, are summarized. Compared with the 2.2% delinquency rate of the third quarter of 2008, we see that the normal distribution even on the 99.9% level of probability failed to capture the high increase in delinquencies observed. This could lead to under-capitalization of the mortgage part of the US banking sector and serious problems with funding.

Quantile	GHD Q3 2008 implied delinquency rate	Normal distr. Q3 2008 implied delinquency rate	Absolute difference GHD vs. Normal
Mean Value	1.30%	1.28%	0.02 pp
99.9%	2.266%	1.96%	0.31 pp

Table 3: GDH vs. Normal distribution Q3 2008 implied delinquency rate (pp=percentage points)

The GHD implied delinquency rate on the 99.9% level of probability is higher than the observed value. This fact can be interpreted as a success in derivation of capital needed to cover unexpected credit losses.

5 Conclusion

We have compared several distributions performance in credit risk quantification. For this purpose, we have used quarterly dataset of mortgage delinquency rates form the US financial market. Especially two classes of distributions, stable and generalized hyperbolic, showed much better performance, measured by Wasserstein and Anderson-Darling metrics. From these two, the class of generalized hyperbolic distributions was slightly better in describing the used dataset.

The current banking regulation, summarized and formalized in The Second Basel Accord (Basel II) uses the normal distribution for the credit risk assessment. In the loss distribution, the mean value (expected loss) should be covered by banking fees and interest and the difference between the mean value and the 99.9th quantile (unexpected loss) should be covered by a stock of capital. We have compared the predicted stock of capital that a bank would need to cover unexpected losses on the 99.9% level of probability.

Our results show that the normal distribution wasn't able, even on the 99.9% level of probability, to capture the change in delinquency rate, whereas the generalized hyperbolic distribution predicted such a stock of capital, which was sufficient to cover even a recession high increase in delinquency rate. Therefore, using the class of generalized hyperbolic distributions is more suitable to measure credit risk for a mortgage portfolio.

We have proved that using the normal distribution to quantify credit risk is an assumption that could be easily over-performed by choosing a different, alternative distribution, such as the class of generalized hyperbolic distributions. There are still several questions that need to be answered before the Class of Generalized Hyperbolic Distributions can be used for credit risk assessment. More empirical studies have to be performed to proof the goodness-of-fit of the Class of Generalized Hyperbolic Distributions. The next suggestion is to add an LGD feature to the calculation to obtain a general credit risk model.

Acknowledgements

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Appendix

The moment generating function for the Class of Generalized Hyperbolic distributions is of the form:

$$M(u) = e^{u\mu}\left(\frac{\alpha^2-\beta^2}{\alpha^2-(\beta+u)^2}\right)^{\lambda/2}\frac{K_\lambda(\delta\sqrt{\alpha^2-(\beta+u)^2}}{K_\lambda(\alpha^2-\beta^2)}, \qquad \text{(v)}$$

where u denotes the moment. For the first moment, the formula simplifies to:

$$M(1) = E(x) = \mu + \frac{\beta\delta}{\sqrt{\alpha^2-\beta^2}}\frac{K_{\lambda+1}\left(\delta\sqrt{\alpha^2-\beta^2}\right)}{K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)}, \qquad \text{(vi)}$$

Whether the second moment is calculated in (technically) more difficult way:

$$M(2) = Var(x) = \delta^2\left(\frac{K_{\lambda+1}\left(\delta\sqrt{\alpha^2-\beta^2}\right)}{\left(\delta\sqrt{\alpha^2-\beta^2}\right)K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)}\right) + $$
$$+\frac{(\beta\delta)^2}{\alpha^2-\beta^2}\left(\frac{K_{\lambda+2}\left(\delta\sqrt{\alpha^2-\beta^2}\right)}{K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)} - \left(\frac{K_{\lambda+1}\left(\delta\sqrt{\alpha^2-\beta^2}\right)}{K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)}\right)^2\right) \qquad \text{(vii)}$$

By substituting from the equations (vi) into (v) we obtain much simpler expression for the first and second moment of the class of generalized hyperbolic distributions. Following equations express the first and the second moment of the class of generalized hyperbolic distributions in their scale- and location- invariant shape:

$$M(1) = E(x) = \mu + \frac{\beta\delta}{\sqrt{\alpha^2-\beta^2}}\frac{K_{\lambda+1}(\zeta)}{K_\lambda(\zeta)}, \qquad \text{(ix)}$$

$$M(2) = Var(x) = \delta^2\left(\left(\frac{K_{\lambda+1}(\zeta)}{\zeta K_\lambda(\zeta)}\right) + \frac{\beta^2}{\alpha^2-\beta^2}\left(\frac{K_{\lambda+2}(\zeta)}{K_\lambda(\zeta)} - \left(\frac{K_{\lambda+1}(\zeta)}{K_\lambda(\zeta)}\right)^2\right)\right) \qquad \text{(x)}$$

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Summary

Kreditní riziko, neboli riziko selhání protistrany, se stalo prvním rizikem, které bylo podchyceno systémy řízení rizik v bankách. V našem článku ukážeme na příkladě hypotečních delikvencí, že normální rozdělení může být v popisu kreditních ztrát překonáno a že v některých případech může být předpoklad normálního rozdělení kreditních ztrát dokonce značně nebezpečný. Především v průběhu volatilitních období, porovnatelných se současnou ekonomickou krizí podceňuje normální rozdělení kreditní ztráty ve chvostech. Tato nedokonalost může být upravena tím, že budeme předpokládat alternativní, v našem případě zobecněné hyperbolické rozdělení kreditních ztrát.