Risk estimation and backtesting at European FX rate markets

Odhad rizika ajeho zpětné testování na evropských měnových trzích

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Abstract

Financial markets are very sensitive to all kinds of risk. Immediately after any unexpected announcement the volatility of market returns is suddenly increased and market prices can potentially fall down. However, the announcement can influence prices of only some assets, while prices of others remain stable. It follows that a different risk type indicates a need for distinct methods of risk modelling, measuring and managing. The purpose of this paper is to identify if there is any similarity in risk estimation model performance across European FX rate market. We have documented that among the FX rates in study, the worst results were obtained for CHF. Moreover, there are important similarities in the occurrence of exceptions among Central European markets.

Key words

FX rate risk, backtesting, Lévy models

JEL Classification: C46, E37, G17, G24

1. Introduction

Financial markets are very sensitive to all kinds of risk. Immediately after any unexpected announcement the volatility of market returns is suddenly increased and market prices can potentially fall down. However, the announcement can influence prices of only some assets, while prices of others remain stable. The reason is that various assets are sensitive to distinct kinds of risk in a different way. It also follows that a different risk type indicates a need for distinct methods of risk modelling, measuring and managing. The purpose of this chapter is to identify – on the basis of a risk model performance, including backtesting procedure – if there is any similarity among particular European currencies or, to be more exact, their exchange rates with respect to Euro, and especially whether integration of new economies, such as the Czech Republic, Hungary or Poland, implies some similarities in risk estimation failures or if they still behave differently.

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In respect of that, we start with some basic foundations of market risk management, including a brief description of the backtesting procedure. Next, an advanced models in the form of a subordinated Lévy model[2] (VG model) is suggested as a useful tool to model the FX rate evolution and estimate the risk in terms of VaR more properly. In the analysis, these models are accompanied by a simplified approach to market risk modelling based on Brownian motion. Within the analysis, we consider seven European currencies that can be classified into three distinct groups – two old members of the EU that have, however, decided to stay out of the currency union, two well-developed European economies with close links to the EU economy, although staying outside, and three Central European countries that have joined the EU relatively recently. These economies represented by seven different currency exchange rates are accompanied by another two outside Europe. We first check if the suggested model works well ex post. After that, the risk is estimated ex ante on a moving window basis and observed exceptions are recorded. Besides standard Kupiec or Christofferson tests, we also analyze the mutual occurrence of exceptions for any two of the FX rates.

2. The concept of market risk management

An important part of market risk management of financial institutions is to estimate a short-term change in the portfolio value at a prespecified probability level, $\Delta V(\alpha, \Delta t1)$, i.e. a q-quantile of the portfolio returns distribution. Regulatory authorities generally require a financial institution to keep capital at least at the level of a worst-case ten-day loss with a significance of 0.01, $-\Delta V(0.01, \Delta 10)$, though the calculation is commonly based on a one-day risk multiplied by the square root of a given time length, $\sqrt{10}\Delta V(0.01,1)$. Hence, there is a 99% confidence that the incurred loss will not be higher than a quantity generally referred to as the Value at Risk (*VaR*).

For example, if the financial institution regularly keeps its capital level equal to the estimated one-day VaR at the probability of 1%, it will be able to cover potential losses on 99% of days each year. Or, from the other point of view, the capital will not suffice to cover losses on about two and half days every year.[3]

Alternatively, in the case of concern about the liquidity of the position, the horizon can be extended. This is the case, for example, with Solvency II requirements, which should be followed soon by all insurance companies and pension funds. Here, the VaR is calculated over a one year horizon at the probability level of 0.5%, since it is reasonable to assume that portfolios of such entities are rather long-term and rebalanced infrequently.

Although financial institutions are relatively unrestricted with respect to the type of the model they may use for VaR estimation, several qualitative as well as quantitative criteria must be fulfilled. One of them concerns backtesting, ie. how good the model is when applied to past data. Loosely speaking, applying a historical time series, i.e., the true evolution of market prices of a given financial instrument, the risk is estimated (ex ante) at time t for time $t + \Delta t$, and later compared with the true loss observed at time $t + \Delta t$ (ex post). This procedure is applied for a moving time window over the whole available data set.

[2] The most recent and complete monographs on the theory behind Lévy models and/or their potential application in finance are Applebaum (2004), Cont and Tankov (2010), Barndorff-Nielsen et al. (2001).
[3] Some basic tests are reviewed in Hull (2010) or Resti and Sironi (2007), although more in-depth analysis can be found in various papers, such as Berkowitz and O'Brien (2002), Pérignon and Smith (2010) or Berkowitz et al. (2011).

Within the backtesting procedure on a given time series of loss observations, $t = \{1, 2, \ldots, T\}$, two situations can arise – the loss is either higher than its estimation (VaR) or lower (obviously, the probability of the equality approaches zero for continuous variables). While the former case is denoted by 1 as an exception, the latter one is denoted by 0. On the zero-one sequence, it can be tested whether the number of ones (exceptions) corresponds with the assumption, ie. αn (with $n = T - 1 - m$, m is the length of data needed for initial estimation), whether the estimation is valid either unconditionally or conditionally, whether bunching is present, etc.[4]

Later, we will utilize two basic tests. First is the Kupiec test (Kupiec, 1995), which is derived from a relative amount of exceptions, i.e., whether its probability is different from the statistical point of view from the assumed probability. A given likelihood ratio on the basis of x^2 probability distribution with one degree of freedom is formulated as follows:

$$LR_{Unc} = \frac{\pi_{\exp}^{n_1}\left(1-\pi_{obs}\right)^{n_0}}{\pi_{\exp}^{n_1}\left(1-\pi_{obs}\right)^{n_0}},$$

where $\pi_{\exp}$ is the expected probability of an exception occurring, π_{obs} is the observed probability of an exception occurring, n_0 is the number of zeros and n_1 is the number of ones. However, the Kupiec test takes into account only the number of exceptions and is also referred to as the unconditional test.

By contrast, in order to assess whether the exceptions are distributed equally in time, i.e., without any dependency (autocorrelation), we should define the time lag first: in Christoffersen (1998) this is defined as the stage when an exception at one moment in time can significantly help to identify whether another exception will (or will not) follow on the subsequent day. Therefore, we should replace the original zero-one sequence by a new one, where 01, 00, 11 or 10 is recorded. Then we have the likelihood ratio as follows:

$$LR_{Con} = \frac{\pi_{obs}^{n_1}\left(1-\pi_{obs}\right)^{n_0}}{\pi_{01}^{n_{01}}\left(1-\pi_{01}\right)^{n_{00}}\pi_{11}^{n_{11}}\left(1-\pi_{11}\right)^{n_{10}}},$$

with $\pi_{ij} = \Pr\left(I_t = j \middle| I_{t-1} = i\right)$ and $\pi_{obs} = \dfrac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$. Obviously, these to test can be combined into one complex test.

3. Market risk modelling

Major financial institutions usually keep large portfolios of various financial instruments. Although the central limit theorem and the law of large numbers provide us with a justification to apply a model based on the normality of log-returns, ie. to ignore one-side fat tails, observations of fatal market crashes during recent decades indicate that the assumption of the normality of returns can lead to serious mistakes even for a large portfolio if it is not diversified well across all potential risk types.

Alternative one-dimensional models capturing both the empirically observed skewness and kurtosis are formulated either as various extensions of Gaussian distribution (Pareto or Student distribution, potentially also skewed, see Jondeau et al. (2006)) or as a mixture of distinct distributions. More importantly, during recent decades some authors have suggested using rather subordinated processes assigned to the general Lévy family of stochastic processes (see e.g. Cont and Tankov (2010) or Schoutens (2003) for a comprehensive review). We will now provide a definition of a generalized subordinated Lévy model, and briefly describe its main features and two selected cases of such models. However, we start with a formal definition of Lévy processes.

3.1 Lévy processes

Suppose a probability distribution that is infinitely divisible. Then, a stochastic process $\{X(t), t \in [0,T]\}$ is a Lévy process on [0,*T*], if (for $\tau \geq 0$):

1. it starts at zero: $X(0) = 0$,
2. its increments are independent: $X(t+\tau) - X(t)$ does not depend on $X(s)$, $s \leq t$,
3. its increments are stationary distributed: $X(t+\tau) - X(t) = X(\tau)$, in other words increments depend only on τ,
4. it is stochastically continuous: $\lim_{\tau \to 0} \Pr[X(t) > \varepsilon] = 0$ for $\varepsilon > 0$.

Note that for many Lévy models an infinite intensity of possibly very small jumps is an important feature (see also property 4).

Although it is more feasible to define a given Lévy process through its characteristic function, sometimes a probability density function and/or distribution function is needed. Any Lévy process *X* has to fulfil a specific characteristic function $\phi_X(u) = \mathrm{Exp}[\Phi_X(u)]$ with a characteristic exponent as follows (a Lévy-Khintchin formula):

$$\Phi_X(u) = i\gamma u - \tfrac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{\infty} \left[\exp(iux) - 1 - iuxI_{|x|<1}\right] \nu(dx).$$

Here, I_A is the indicator function (it gives one, if *A* holds and zero otherwise). It is apparent that for a given Lévy process *X*, we get a triplet of Lévy characteristics, $\{\lambda, \sigma, \nu\}$ that determines the character of the process. While the former two, $\gamma \in R$, $\sigma^2 \geq 0$, define the drift of the process (deterministic part) and its diffusion, the latter is a Lévy measure, $R \setminus \{0\}$. If it can be formulated as $\nu(dx) = u(x)dx$, it is a Lévy density, which is a measure similar to a probability measure in some sense – it need not be integrable and zero at origin. Note, that there exist two special cases of a Lévy process, a Wiener process, which does not jump at any time, and a Poisson process, which in turn does not contain a diffusion part.

3.2 Subordinated Lévy models

Subordinated Lévy models represent a rather nonstandard example of Lévy models. Nevertheless, such models allow one a rich applicability within various financial modelling issues, including credit risk or market risk modelling, which is given by their relative simplicity, including parameter estimation. Moreover, there is a very nice economic interpretation.

The basic idea that lies behind subordinated Lévy models is to evaluate a (geometric) Brownian motion, which used to be the most standard model applied in all branches of finance, not in a common time *t*, but rather in a stochastic time that can mimic a stochastic environment. In other words, we replace common time *t* by a suitable intrinsic process (*a subordinator*). From an economic point of view, such processes can be understood as a measure of economic activity, depending, e.g., on the occurrence of new information.

Denoting $Z(t;\sigma,\mu)$ as a Wiener process in dependency on time *t* with parameters $\mu = 1$ and $\sigma = \sqrt{dt}$, we can define a Lévy process $X(t;\theta,\vartheta)$ with drift θ and volatility ϑ driven by another Lévy process $l(t)$ with a unit mean and a variance specified by κ very simply – we replace *t* by $l(t)$. Thus

$$X(t) = \theta l(dt) + \vartheta Z(l_{dt}) = \theta l(dt) + \vartheta \varepsilon \sqrt{l(dt)}. \quad (1)$$

This relation can be interpreted in such a way that the increment of X within an infinitesimal time interval dt is of normal distribution with mean $\theta l(dt)$ and variance $\vartheta^2 l(dt)$. The mean of the driving process $l(dt)$ should be dt and its variance specified by κ will determine the *fat tails*. In turn, the mean of the overall process controls the asymmetry.

In order to model returns of financial assets, we need to rewrite (1) as follows

$$\mu dt + (\theta - dt)l(dt) + \vartheta Z(l_{dt}). \tag{2}$$

Here we deduce dt from θ in order to get zero mean (recall that dt is actually the mean of $l(dt)$) so that the desired expected return can be obtained easily by adding μdt. By contrast, in order to model the prices of financial assets we would need to put (2) into exponential:

$$S(t) = S(0)\exp[(\mu - \omega)dt + \theta l(dt) + \vartheta Z(l_{dt})],$$

where ω is a mean correcting parameter assuring that the expected future price will be matched.

3.3 Selected examples

Very useful subordinators are a *gamma process* from gamma distribution, $l(t) \equiv g(t;\kappa)$, leading to the *variance gamma model* (VG), and an *inverse Gaussian process* from inverse Gaussian distribution, $l(t) \equiv I(t;\kappa)$, leading to the *normal inverse Gaussian model* (NIG). In both cases, κ describes the variance of the subordinator. For more details on application examples, including estimation and simulation see eg. Tichý (2010).

4. Risk model evaluation

By its nature, a risk estimation is done ex-ante, ie. without any real knowledge about the things that are going to happen. It is therefore natural to check the model performance by a so called backtesting procedure. However, we first describe the data.

4.1 Data set description

For the purposes of our analysis we have collected daily FX rates of the British pound (GBP), Danish krone (DKK), Norwegian krone (NOK), Swiss franc (CHF), Czech koruna (CZK), Hungarian forint (HUF), Polish zloty (PLN), Japanese yen (JPY), and US dollar (USD), all with respect to EUR. This implies that the data set comprises four FX rates of countries whose economies should be well integrated with the Eurozone, although two of them have not joined the EU yet. Clearly, neither of them has decided to adopt the single European currency. Next, there are three Central European countries, which joined the EU relatively recently. Finally, we also have two currencies of relatively distant economies, but with important relations to the evolution of a global economy.

The time series starts on January 1, 2001. The last data taken are for December 31, 2010. Thus, the length of the series of log-returns is 2,520 observations. For each FX rate, basic descriptive statistics – mean, variance, standard deviation, skewness, and kurtosis – of daily log-returns (per annum, if applicable) were evaluated.

It is apparent that the mean (average drift over the time period we consider) varies substantially between –3.59% (USD) and 3.32% (CZK) p.a. We can also see that the EUR depreciated substantially with respect to CZK and CHF and slightly with respect to NOK. By contrast, from the point of view of the rest of the currencies, it was appreciating more (USD, GBP) or less (HUF, PLN, JPY).

Concerning the standard deviation, we can see that the most stable is DKK (0.46% p.a.), except for several deviations resulting in huge kurtosis; the other values of standard deviation are rather mild (5-10%) to medium (10-12%). The skewness of returns of particular FX rates is either slightly positive, insignificant, or more or less negative. However, we cannot identify any similarities among the FX rates assigned to any of the four groups (old EU countries, well developed European countries outside the EU, new EU countries and countries outside Europe). Similarly, no firm conclusion can be made in the case of kurtosis. Finally we should note that none of the returns distributions can be regarded to be Gaussian following the Jarque-Bera type tests.

Moreover, in figures in Appendix we provide the charts of the evolution of both daily FX rates and daily log-returns. It is evident, that several periods of very high instantaneous volatility are present for all FX rates, except DKK, or, in other words, extreme events that drive the measure of kurtosis. These can be observed mainly at the beginning of the time series and also about two years from the end (i.e., during recent financial crises).

4.2 Backtesting of single positions (modelling ex ante)

In the following text we will concentrate on the backtesting results of single positions in particular FX rates assuming three different significance levels (0.1%, 1% and 5%) for VaR estimation from the point of view of either the long position or the short position, which will be accompanied by the median. We will consider a standard market model (Brownian motion) and the VG model. In order to estimate the parameters of these models we will use returns over either 250 or 500 preceding days.

We start on the same day for both cases so that the total number of loss observations is always identical. In particular, we start on day $t = 521$ and use either 250 or 500 preceding returns to estimate the next day's VaR ($t + 1$) for particular significance levels. Next it is compared to the actual observed loss (return with minus sign) and either one or zero is recorded. This procedure is repeated on a moving window basis until we reach the last day, ie. we get exactly 2,000 observations of losses, which simplifies an ad-hoc comparison. For example, assuming 1% significance we should record a loss higher than the VaR on just 20 days.

The total number of exceptions recorded over the last eight years for particular models is documented in Table 3 (here, the results are based on a 250-day time window, which is in line with Basel II recommendations) and Table 4, where the series of 500 daily returns is used for model estimation. In particular columns we can see the results for various significances, while the rows depict results for given FX rates. In the last row we also state the assumed number of exceptions (Asmp.). If the number is provided in bold font, it means that the model is significant at 5% probability level (according to the Kupiec test). Similarly, if the number is emphasized by italics, the model is significant according to the Christoffersen test (see the next subsection).

Starting with the standard market model (BM), we can clearly see that the model mostly does not work well for any significance (for both positions, short and long) and all FX rates. Moreover, in both cases the best median backtesting is obtained for the two FX rates with lowest skewness (CZK, USD) or relatively low skewness (DKK) with respect to kurtosis. We can therefore assume that the inefficiency in median backtesting is caused by the lack of parameters of the BM.

Since the alternative model, VG, should allow us to fit the observed skewness well, the results should be much better, at least in the case of the median, since it should be resistant to both volatility and excess kurtosis. Sadly, the improvement is rather minor, although it seems that longer data series work better. However, since risk measurement is related to unwanted results we are much more interested in how the models work in tails. Starting with Table 8.6, one can notice important improvement over the simple BM model in both tails, left and right, and also for all significance levels. Obviously, for some FX rates the results are better than for the others. For example, GBP, CHF, and USD are quite problematic, CHF being the worst (the model is acceptable only in the median). By contrast, in the cases of DKK and CZK we can get very good results.

Concerning the short and long position, it seems that backtesting works better for the short position if the far tails are taken into account. However, in the case of a significance level 0.05, the short position models fall behind the long position ones. Finally, it can also be seen that the longer time window slightly improves the model's performance – naturally, using more returns for estimation allows us to get a better picture of the kurtosis, which is by definition a measure of rare events.

By contrast, a return volatility is a rather short-memory type measure. Finally, we thus try to combine two different window lengths for the estimation – while the first two moments will be estimated on a 250-day basis, in order to get the skewness and kurtosis we will use 500 days. Moreover, the underlying data will be normalized to get zero mean and unit variance over a one year window. This will allow us to eliminate potential error in the volatility estimation, but to get a clearer picture of the impact of rare events on exceptions at particular markets. Note also that the volatility of particular FX rates was significantly different – thus the normalization will also help us to overcome this feature.

These final results are included in Table 1. Obviously, since the BM does not allow us to take into account the higher moments of the distribution, we cannot observe any particular improvement. However, concerning the VG model, the progress in the performance in the tails is more than evident. We will therefore use this combination for further analysis of the similarities among particular FX rate markets.

Table 1 Backtesting results of single positions in FX rates according to BM (panel I), VG (panel II)); estimation based on 500/250 days

Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*
GBP	15	*41*	**118**	***992***	***114***	*32*	*11*
DKK	*6*	***24***	***94***	**1011**	***88***	***27***	*6*
NOK	*15*	44	**119**	**961**	***95***	***26***	*6*
CHF	*17*	*42*	*125*	***1014***	138	63	*27*
CZK	*9*	*35*	***90***	***1006***	**105**	*33*	*11*
HUF	*13*	*50*	***121***	*955*	***91***	*30*	*8*
PLN	*18*	45	**112**	***966***	*78*	*32*	*12*
JPY	*8*	***22***	***85***	***1043***	130	*61*	*20*
USD	10	30	***101***	***1017***	***114***	*35*	*14*
Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*
GBP	**3**	**21**	***108***	***1042***	127	***25***	***2***
DKK	**2**	***19***	*131*	**1005**	*122*	***19***	***2***
NOK	*6*	***23***	**107**	**1007**	***109***	***24***	***2***
CHF	6	*30*	153	***976***	137	*30*	6
CZK	**3**	***17***	***101***	***991***	**107**	***16***	***4***
HUF	**4**	***18***	**118**	*1044*	137	***24***	***4***
PLN	*7*	**24**	**112**	***1004***	***98***	***20***	7
JPY	**5**	***20***	***116***	***1010***	122	***29***	***5***
USD	**3**	**27**	***102***	***1011***	***117***	***27***	*6*
Asmp.	2	20	100	1000	100	20	2

4.3 Backtesting analysis

Let us return to Table 1 first. Initially, the model performance was assessed according to the Kupiec test, which is based only on the relative number of exceptions. It is evident that the subordinated Lévy model (VG) works very well for DKK or CZK and well for most of the other FX rates, except CHF, regardless of the group. However, a high quality model should moreover not lead to any bunching, i.e., clusters of exceptions during subsequent days.

Unfortunately, such testing requires relatively long data series and a higher number of observed exceptions. It is therefore not very suitable for far tails analysis, i.e., if we observe only two or three exceptions, any conclusion about potential bunching is not reliable. In our case, we can therefore test the exceptions series on bunching only for a 5% VaR and sometimes also 1% VaR. For some currencies, clusters can be identified when the long position is considered, for others they are present only in the case of the short position – results that imply model acceptance according to this type of test at the probability level of 0.05 are distinguished by italics in Table 1 (including the cases, when no conclusion can be drawn).

It would also be interesting to check what happens, if there is an exception in the risk estimation on a given FX rate – i.e., can the exceptions be observed on the same day for all FX rates? The answer can be found in Table 2, where the relative conditional exceptions of single position $VaR_{5\%}$ according to the VG model for the first/last 1,000 days are depicted. For example, if the risk model for GBP fails, then the probability that the model for NOK will also fails is either 0.13 or 0.23 for the first 1,000 and last 1,000 days, respectively. Note also that the probability of the reverse (the GBP model fails conditionally on a NOK model failure) is slightly different, which is obviously given by the different number of exceptions for both FX rates.

It is apparent that, assuming the VG model, there is a quite high probability of mutual failure for all three Central European currencies within both time periods (from 0.25 to 0.53) and also three countries outside the EU (CHF, JPY, USD), though NOK performs in a slightly different way. Concerning the differences between the first and last thousand days, i.e., it seems that the failures of the "new" economies (CZK, HUF, PLN) are more interconnected to the model failure of the more developed economies, such as GBP, DKK or NOK.

Table2 Conditional exceptions of single position $VaR_{5\%}$ according to VG model for first/last 1,000 days (probability)

First 1,000 days	GBP	DKK	NOK	CHF	CZK	HUF	PLN	JPY	USD
GBP	1.	0.05	0.13	0.06	0.07	0.06	0.11	0.19	0.21
DKK	0.07	1.	0.09	0.14	0.02	0.06	0.06	0.09	0.10
NOK	0.16	0.08	1.	0.13	0.02	0.03	0.09	0.15	0.14
CHF	0.09	0.17	0.17	1.	0.00	0.06	0.09	0.11	0.10
CZK	0.07	0.02	0.02	0.00	1.	0.28	0.32	0.15	0.05
HUF	0.09	0.07	0.04	0.06	0.44	1.	0.36	0.11	0.07
PLN	0.11	0.05	0.07	0.06	0.35	0.25	1.	0.06	0.10
JPY	0.20	0.07	0.13	0.07	0.16	0.07	0.06	1.	0.33
USD	0.20	0.07	0.11	0.06	0.05	0.04	0.09	0.30	1.
Last 1,000 days	**GBP**	**DKK**	**NOK**	**CHF**	**CZK**	**HUF**	**PLN**	**JPY**	**USD**
GBP	1.	0.07	0.23	0.08	0.17	0.12	0.15	0.11	0.29
DKK	0.06	1.	0.07	0.06	0.07	0.10	0.10	0.06	0.03
NOK	0.21	0.07	1.	0.08	0.15	0.22	0.25	0.04	0.13
CHF	0.10	0.09	0.11	1.	0.15	0.05	0.04	0.39	0.25
CZK	0.15	0.07	0.15	0.11	1.	0.29	0.33	0.08	0.10
HUF	0.10	0.11	0.21	0.04	0.29	1.	0.43	0.01	0.06
PLN	0.16	0.13	0.30	0.04	0.41	0.53	1.	0.03	0.05
JPY	0.12	0.07	0.05	0.33	0.10	0.02	0.03	1.	0.38
USD	0.27	0.04	0.13	0.19	0.10	0.07	0.04	0.34	1.

5. Conclusion

In this paper we have discussed the performance of subordinated Lévy models in risk estimation (in terms of VaR) for single positions in several FX rates with respect to EUR. We have analyzed the risk models separately within the ex post and ex ante modelling. It was shown, that if the input data are correctly defined there is almost no difference among particular FX rates. However, since real VaR estimation is done ex ante, without clear knowledge about the future evolution, the model may fail more (or fewer) times than expected.

Both models performed very well under both types of test (relative amount of exceptions and their independency in time) mainly for DKK and CZK, which might be interpreted as evidence that both FX rates are relatively well connected to EUR with good predictability about the volatility and ratio of extreme events. However, the worst results were, quite surprisingly, obtained for CHF, i.e., the simple model utilizing past data equally is not suitable for efficient risk estimation. These conclusions can be confirmed even if the original data series is split into several periods.

By contrast, the mutual dependency of exceptions shows that all three Central European countries are interconnected – if the model for one of the FX rates fails, it is quite probable that the model for the other two FX rates will fail too. This observation is an obvious implication of the way that the global market looks at these three countries – as more risky than the old EU countries and mutually interconnected.

The results may be interesting for financial policy evaluation and may also help financial institutions in the internal risk management process. An interesting extension of the analysis would be to model the FX rates as a portfolio and study the impact of particular FX rates.

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Summary

Financial markets are very sensitive to all kinds of risk. Immediately after any unexpected announcement the volatility of market returns is suddenly increased and market prices can potentially fall down. However, the announcement can influence prices of only some assets, while prices of others remain stable. It follows that a different risk type indicates a need for distinct methods of risk modelling, measuring and managing. The purpose of this paper is to identify if there is any similarity in risk estimation model performance across European FX rate market.

Table 3 Backtesting results of single positions in FX rates according to BM (panel I), VG (panel II); estimation based on 250 days

Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*
GBP	14	44	**118**	**988**	**117**	**29**	9
DKK	6	**25**	**93**	**1007**	**94**	**28**	9
NOK	19	46	120	**960**	**93**	**24**	6
CHF	16	43	124	**1012**	135	58	25
CZK	10	36	**91**	**1002**	**106**	31	10
HUF	14	52	120	952	**91**	32	7
PLN	16	45	**109**	**960**	78	31	11
JPY	8	**21**	**84**	1050	131	63	20
USD	10	32	**99**	**1010**	**115**	37	12
Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*
GBP	10	**29**	**109**	**1023**	124	**29**	**4**
DKK	**1**	**20**	**93**	**1007**	**95**	**19**	**4**
NOK	8	**26**	**107**	**1011**	**109**	**29**	**3**
CHF	12	39	142	**1009**	131	43	8
CZK	**2**	**21**	**98**	**992**	**106**	**18**	**4**
HUF	**5**	**24**	**116**	**1035**	136	33	7
PLN	8	**26**	**112**	**1006**	**97**	**22**	8
JPY	9	**25**	**114**	**996**	**116**	31	10
USD	**5**	32	**109**	**1004**	**114**	**25**	**3**
Asmp.	2	20	100	1000	100	20	2

Table 4 Backtesting results of single positions in FX rates according to BM (panel I), VG (panel II); estimation based on 500 days

Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*
GBP	19	53	**118**	**988**	**106**	34	13
DKK	6	**21**	**84**	**1014**	**82**	**22**	**4**
NOK	23	50	**117**	**958**	**97**	**27**	8
CHF	18	45	**119**	**1012**	138	59	20
CZK	16	38	**94**	**995**	**102**	39	11
HUF	19	54	124	**946**	**101**	34	9
PLN	22	46	**102**	**949**	78	32	16
JPY	10	**27**	**85**	**1042**	130	57	29
USD	9	34	**98**	**1005**	**104**	42	16
Quantile	*0.1%*	*1%*	*5%*	*50%*	*95%*	*99%*	*99.9%*

GBP	8	**26**	**110**	**1039**	130	**23**	6
DKK	**1**	**16**	**92**	**1002**	**86**	**16**	**3**
NOK	6	**24**	**111**	**1005**	**113**	**22**	**3**
CHF	6	36	145	**995**	132	35	9
CZK	**2**	**17**	**99**	**995**	**106**	**14**	**4**
HUF	**4**	**23**	122	1050	136	30	6
PLN	6	**25**	**101**	**1006**	**100**	**24**	7
JPY	**5**	**27**	**111**	**1009**	**110**	32	8
USD	6	**23**	**100**	**1003**	**101**	**27**	6
Asmp.	2	20	100	1000	100	20	2

Figure 1 Daily evolution of FX rates

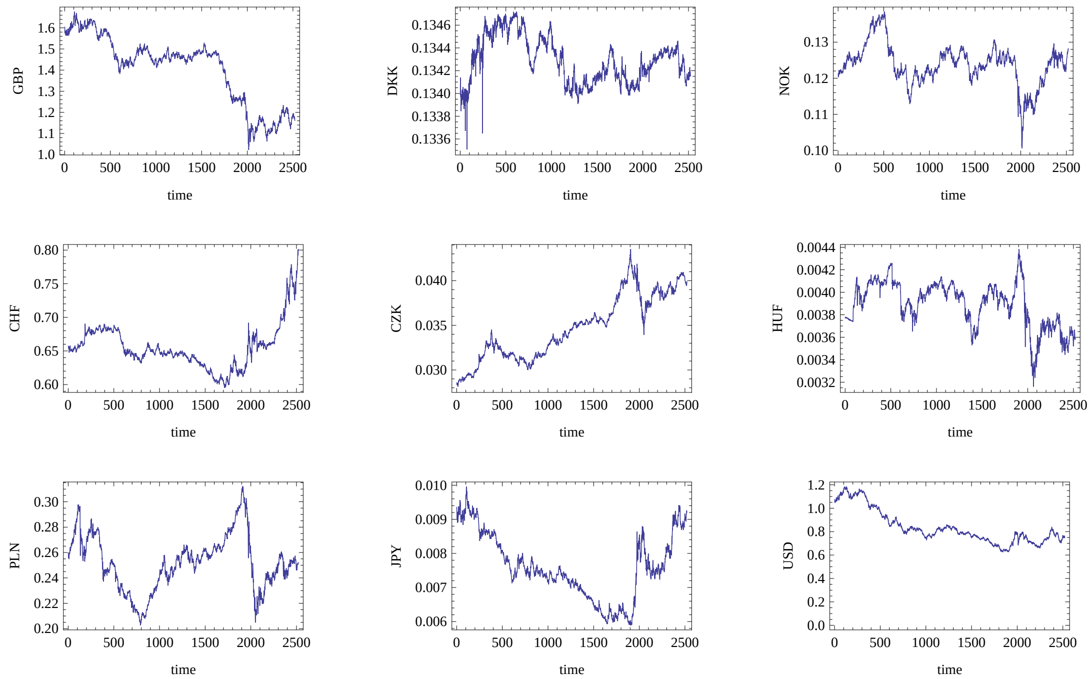

Figure 2 Daily log-returns of FX rates