The volatility modelling and bond fund price time series forecasting of VUB bank: Statistical approach

Dusan Marcek [1]

Abstract

The article describes the development and the use of the ARCH-GARCH (AutoRegressive Conditionally Heteroskedastic) type models. We provide information about the examination of the ARCH-GARCH models for the forecasting of the bond price time series provided by the VUB bank and compare of the forecast accuracy with the models based on the residuals analysis of the developed models.. We show some new aspects of time series analysis which may improve the forecasting ability of statistical models so that their can better capture the characteristics and dynamics of a particular time series and promise better goodness of fit.

Key words

Time series, classes ARCH-GARCH models, volatility, forecasting, forecast accuracy.

JEL Classification: C13, G32.

1. Introduction

This paper discusses and compares the forecasts from volatility models which are derived from statistical theory. The aim of the paper is to provide information about the examination of the ARCH-GARCH (AutoRegressive Conditionally Heteroskedastic) models for the forecasting of the volatility and bond price time series provided by the VUB bank and comparisons of the forecast accuracy with the models based on the residuals analysis of the developed models.

Volatility is an important factor in assets trading. By volatility we mean the conditional standard deviation of the underlying asset price. Volatility has many financial applications. For example volatility modelling provides a simple approach for calculating value at risk of a financial position in risk management. It also plays an important role in asset allocation under the mean-variance framework. To model the volatility for the commercial VUB bank of the Slovak Republic by means of high statistical theory, we developed several ARCH/GARCH models. In the basic ARCH/GARCH model economic shocks have no effects on conditional volatility. However, a stylized fact of financial volatility is that bad news (negative shocks) tend to have a larger impact on volatility than good news (positive shocks). For this reason we study also EGARCH/PGARCH (Exponential/Power) models which allow for leverage effects and exhaustive study of the underlying dynamics. The study was performed for the data available at http://www.vubam.sk/Default.aspx?CatID=40&fundId=4. The data is also listed at http://fria.fri.uniza.sk/files/data_VUB.

The paper is organized as follows. In Section 2 we briefly describe the basic ARCH-GARCH model and its variants: EGARCH, PGARCH models. In Section 3 we present the

[1] Dusan Marcek- University of Zilina, Department of Macro and Micro Economics, Univerzitna 1, 010 26, Zilina, Slovak Republic, e-mail: dusan.marcek@fri.uniza.sk;
Silesian University Opava, Faculty of Philosophy and Science, Czech Republic, e-mail: dusan.marcek@fpf.slu.

data, conduct some preliminary analysis of the time series and demonstrate the forecasting abilities of ARCH-GARCH modes of an application. In Section 4 other statistical tools for improving of the forecasting ability of ARCH-GARCH models are presented and we put an empirical comparison. Section 6 briefly concludes.

2. Some ARCH-GARCH Models for Financial Data

ARCH-GARCH models are designed to capture certain characteristics that are commonly associated with financial time series. They are among others: fat tails, volatility clustering, persistence, mean-reversion and leverage effect. As far as fat tails, it is well know that the distribution of many high frequency financial time series usually have fatter tails than a normal distribution. The phenomena of fatter tails is also called excess kurtosis. In addition, financial time series usually exhibit a characteristics known as volatility clustering in which large changes tend to follow large changes, and small changes tend to follow small changes. Volatility is often persistent, or has a long memory if the current level of volatility affects the future level for more time periods ahead. Although financial time series can exhibit excessive volatility from time to time, volatility will eventually settle down to a long run level. The leverage effect expresses the asymmetric impact of positive and negative changes in financial time series. It means that the negative shocks in price influence the volatility differently than the positive shocks at the same size. This effect appears as a form of negative correlation between the changes in prices and the changes in volatility.

The first model that provides a systematic framework for volatility modelling is the ARCH model of Engle (1982). Bollerslev (1986) proposes a useful extension of Engle´s ARCH model known as the generalized ARCH (GARCH) model for time sequence { y_t } in the following form

$$y_t = \nu_t \sqrt{h_t},$$
$$h_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i y_{t-i}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \tag{1}$$

where { v_t } is a sequence of iid random variables with zero mean and unit variance. α_i a β_i are the ARCH and GARCH parameters, h_t represent the conditional variance of time series conditional on all the information to time t -1, I_{t-1}.

In the literature several variants of basic GARCH model (1) has been derived. In the basic GARCH model (1) if only squared residuals ε_{t-i} enter the equation, the signs of the residuals or shocks have no effects on conditional volatility. However, a stylized fact of financial volatility is that bad news (negative shocks) tends to have a larger impact on volatility than good news (positive shocks). Nelson (1991) proposed the following exponential GARCH model abbreviated as EGARCH to allow for leverage effects in the form

$$\log h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{2}$$

Note if ε_{t-i} is positive or there is “good news”, the total effect of ε_{t-i} is $(1+\gamma_i)\varepsilon_{t-i}$. However contrary to the “good news”, i.e. if ε_{t-i} is negative or there is “bad news”, the total effect of ε_{t-i} is $(1-\gamma_i)|\varepsilon_{t-i}|$. Bad news can have a larger impact on the volatility. Then the value of γ_i would be expected to be negative (see Zivot and Wang (2005, p. 241)).

The basic GARCH model can be extended to allow for leverage effects. This is performed by treating the basic GARCH model as a special case of the power GARCH (PGARCH) model proposed by Ding, Granger and Engle (1993):

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i})^d + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^d \tag{3}$$

where d is a positive exponent, and γ_i denotes the coefficient of leverage effects (see Zivot and Wang (2005, p. 243)).

Another ARCH-GARCH models as the ARCH-GARCH regression and ARCH-GARCH mean model can be found in Marcek, (2001, p. 155 and 156).

3. An Application of ARCH-GARCH Models

We illustrate the ARCH-GARCH methodology on the developing a forecast model. The data is taken from the commercial VUB bank of the Slovak Republic and are available at http://www.vubam.sk/Default.aspx?CatID=40&fundId=4. The data is also listed at http://fria.fri.uniza.sk/files/data_VUB. The data consist of daily observations of the price time series for the bond fund of VUB (BPSVUB). The data was collected for the period May 7, 2004 to February 28, 2008 which provided of 954 observations (see Figure 1 and 4). To build a forecast model the sample period (training data set denoted A) for analysis $r_1, \ldots, r_{900}$ was defined, i.e. the period over which the forecasting model was developed and the ex post forecast period (validation data set denoted as E) $r_{901}, \ldots, r_{954}$ as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated.

Input selection is crucial importance to the successful development of an ARCH-GARCH model. Potential inputs were chosen based on traditional statistical analysis: these included the raw BPSVUB and lags thereof. The relevant lag structure of potential inputs was analysed using traditional statistical tools, i.e. using the autocorrelation function (ACF), partial autocorrelation function (PCF) and the Akaike/Bayesian information criterion (AIC/BIC): we looked to determine the maximum lag for which the PACF coefficient was statistically significant and the lag given the minimum AIC.

According to these criterions the ARMA(5) model was specified as

$$r_t = \xi + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + \phi_4 r_{t-4} + \phi_5 r_{t-5} + \varepsilon_t \tag{4}$$

where $\xi, \phi_1, \phi_2, \ldots, \phi_5$ are unknown parameters of the model, ε_t is independent random variable drawn from stable probability distribution with mean zero and variance σ_ε^2.

As we mentioned early, high frequency financial data, like our BPSVUB, reflect a stylized fact of changing variance over time. An appropriate model that would account for conditional heteroscedasticity should be able to remove possible nonlinear pattern in the data. Various procedures are available to test an existence of ARCH or GARCH. A commonly used test is the LM (Lagrange multiplier) test. The LM test assumes the null hypothesis H_0: $\alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$ that there is no ARCH. The LM statistics has an asymptotic χ^2 distribution with p degrees of freedom under the null hypothesis. For calculating the LM statistics see for example, Bollerslev (1986 Eqs. (27) and (28)). The LM test performed on the BPSVUB indicates presence of autoregressive conditional heteroscedasticity. For estimation the parameters of an ARCH or GARCH model the maximum likelihood procedure was used. The quantification of the model was performed by means software R2.6.0 at http://cran.r-project.org and resulted into the following mean equation:

$$r_t = 0.0000748 + 0.06628 r_{t-1} + 0.09557 r_{t-2} + 0.0275 r_{t-3} + 0.0528 r_{t-4} + 0.09795 r_{t-5} + e_t$$

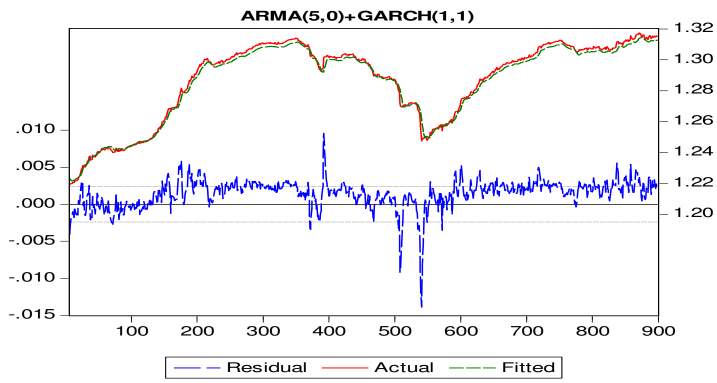
and variance equation

$$h_t = 1.958\ 10^{-8} + 0.1887\, e_{t-1}^2 + 0.8075\, h_{t-1} \qquad (5)$$

where e_t are estimated residuals of ε_t from Eq. (4).

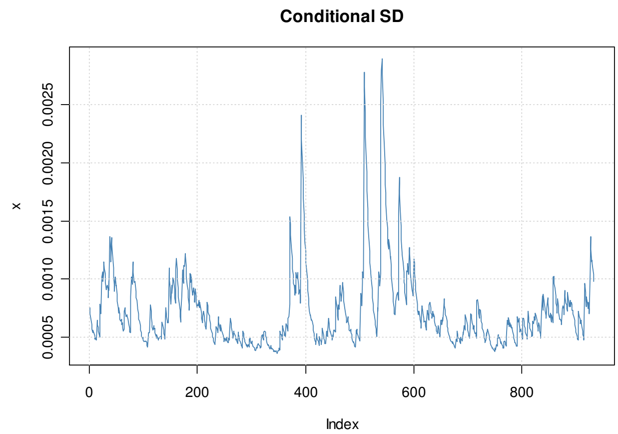
Finally to test for nonlinear patterns in price bond time series the fitted standardized residuals $\hat{\varepsilon}_t = e_t / \sqrt{h_t}$ were subjected to the BDS test. The BDS test (at dimensions N = 2, 3, and tolerance distances ε = 0.5, 1.0, 1.5, 2.0) finds no evidence of nonlinearity in standardized residuals of the BPSVUB. The fitted vs. actual values are graphically displayed by means software Eviews (http://www.eviews.com) in Figure 1 The volatility was estimated by means software R2.6.0 and is displayed in Figure 2.

Figure 1: Actual and fitted values of the VUB fund: ARMA(5,0)+GARCH(1,1)



ARCH-GARCH model (5). Residuals are at the bottom. Actual time series represents the solid line, the fitted vales represents the dotted line

Figure 2: The estimated volatility for ARMA(5,0)+GARCH(1,1) process, model (5)



4. Other statistical tools for improving of the forecasting ability of ARCH-GARCH models

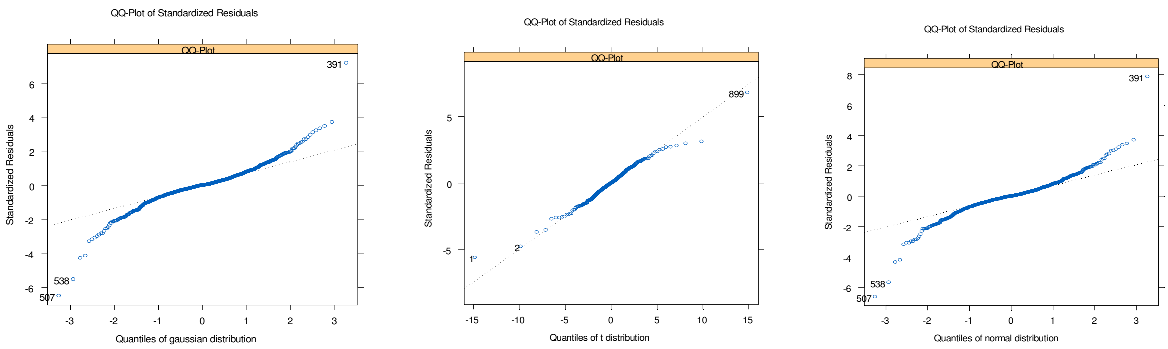
In many cases, the basic GARCH model with normal Gaussian error distribution (1) provides a reasonably good model for analyzing financial time series and estimating conditional volatility. However, there are some aspects of the model which can be improved so that it can better capture the characteristics and dynamics of a particular time series. For this purpose the Quantile-Quantile (QQ) plots are used. For example, the R system (http://cran.r-project.org/) assist in performing residual analysis (computes the Gaussian, studentised and generalized residuals with generalized error distribution – GED). In Figure 3 the QQ-plot reveals that the normality assumption of the residuals may not be appropriate. A comparison of QQ-plots in figure 3 shows that GED distribution promise better goodness of fit. This is confirmed by AIC and BIC criterions and Likelihood function displayed in Table 1. The GED error distribution provides the best fit because AIC and BIC criterions are the smallest.

Model	model.n (Gaussian)	model.t (studentised)	model.GED
AIC criterion	-10576	-10778	-10792
BIC criterion	-10533	-10730	-10744
Likelihood funct.	5297	5399	5406

Table 1: AIC, BIC and likelihood function for various types error distribution (model (4))

Finally, for catching the leverage effect, the model ARMA(5,0)+EGARCH(1,1) was estimated. The coefficient for leverage effect γ from equation (2) is statistical significant and equals -0.2099535, and it is negative which means that "bad news" have larger impact to volatility. If we compare the estimated volatility in Figure 2 with the VUB fund in Figure 1, we can see that in period of depression the "leverage effects" and the bad news cause the asymmetric jump in the volatility.

Figure 3: QQ-plot of Gaussian standard residuals (left), studentised (middle) and generalised(GED) (right)



As we mentioned above, the estimation of EGARCH and PGARCH models has showed the presence of leverage effects. The assumption of normal error distribution is also violated because the alternative error distributions provide better goodness of their fit. These findings indicate the chance of gaining better results in forecasting with using some of these models. Our suspicion was confirmed by computing the statistical summary measure of the model´s forecast RMSE. As we can see in Table 2 the smallest errors have just the GARCH with GED distribution.

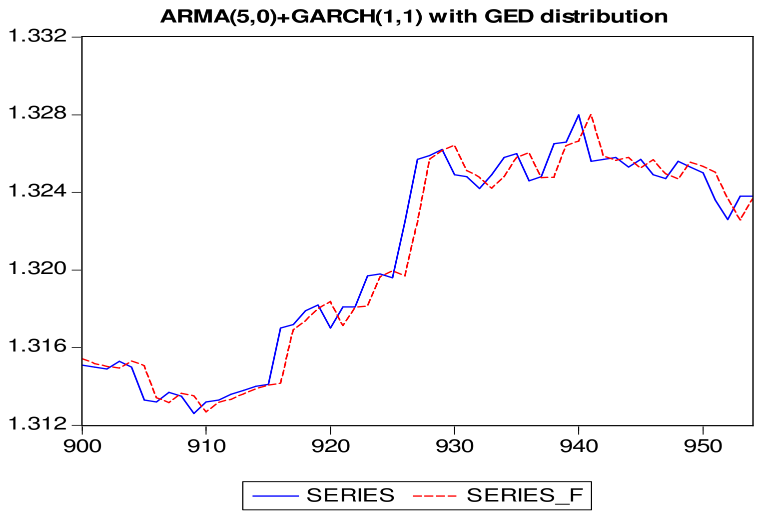
Model	AR(5)+ GARCH(1,1)	AR(5)+ EGARCH(1,1)	AR(5)+ PGARCH(1,1)

Distribution			
Gaussian	0.003461	0.001066	0.001064
t-distribution	0.002345	0.001064	0.001063
GED-distribution	0.001056	0.001063	0.001062

Table 2 Ex post forecast RMSEs for various extensions of GARCH models and granular RBF NN. See text for details.

After these findings we can make predictions for next 54 trading days using the model with the smallest RMSE, i. e. by the ARMA(5,0) + GARCH(1,1) with GED error distribution. These predictions are calculated by means software Eviews (http://www.eviews.com) and shoved in Figure 4.

Figure 4: Actual (solid) and forecast (dotted) values of the VUB fund



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Summary

Článek opisuje vývoj a aplikaci ARCH-GARCH modelů. Poskytuje návod na vývoj ARCH-GARCH modelů pro predikci dluhopisů VUB banky a porovnává predikční přesnost s modely rozšířené o analýzu rezíduí z vyvinutých modelů. Modely založené na analýze rezíduí jsou schopné lépe zachytit vývojovou dynamiku časových řad a tím vylepšují predikční schopnost.