# Forecasting of Economic Quantities using Fuzzy Autoregressive Model and Fuzzy Neural Network

Dušan Marček [1]

**Abstract**

Most models for the time series of stock prices have centered on autoregressive (AR) processes. Traditionally, fundamental Box-Jenkins analysis have been the mainstream methodology used to develop time series models. We briefly describe developing a classical AR model for stock price forecasting. Then a fuzzy regression model is introduced. Following this description, an artificial fuzzy neural network based on B-spline member ship function is presented as an alternative to the stock prediction method based on AR models. Finally, we present our preliminary results and some further experiments that we performed.

**Key words**

Time series, signal processing, fuzzy regression model, lattice algorithms, linear programming, B-spline neural network.

**JEL Classification:** C13, G32.

## 1 Introduction

In [1] the stock price autoregressive (AR) models based on the Box-Jenkins methodology [2] were described. Although an AR model can reflect well the reality, these models are not suitable for situations where the quantities are not functionally related. In economics, finance and so on, there are however many situations where we must deal with uncertainties in a manner like humans, one may incorporate the concept of fuzzy sets into the statistical models. The fuzzy regression is another efficient approach for computing the parameter of the structure for an uncertain situation and for predicting of uncertain events following the decision.

The fuzzy regression models have been in use in analyses for many years. Lots of issues of journal Fuzzy Sets and Systems as well as many others have been articles whose analyses are based on the fuzzy regression models. From reviewing of these papers, it became clear that in economic applications the use of method is not on the same level as analyses using classical linear regression. Computers play an important role in fuzzy regression analyses and forecasting systems. The widespread use of the method is influenced by inclusion of fuzzy regression routines in major computer software packages and selection of appropriate forecasting procedure.

The primary objective of this paper is a focused introduction to the fuzzy regression model and its application to the analyses and forecasting from classical regression model of view. In Section 2, we briefly describe some basic notions of linear regression. Following this description in Section 3, we present evaluation of fuzzy linear regression model in the context of a practical application in comparison to the AR model. In Section 4 the B-spline neural network approach is applied. Empirical results are given in Section 5.

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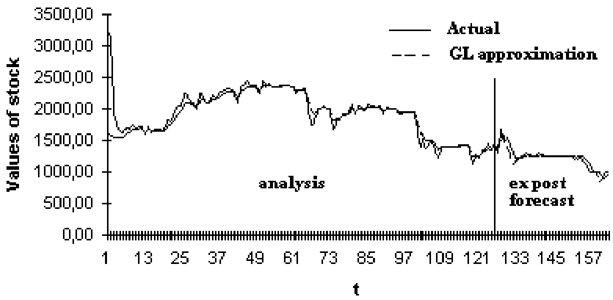
# 2 AR modelling

We give an example that illustrates one kind of possible results. We will regard these results as the referential values for the approach of fuzzy autoregressive and ANN modelling.

To illustrate the Box-Jenkins methodology, consider the stock price time readings of a typical company (say VAHOSTAV company). We would like to develop a time series model for this process so that a predictor for the process output can be developed. The data was collected for the period January 2, 1997 to December 31, 1997 which provided a total of 163 observations (see Fig. 1). The data set used in this paper (163 observations, stock price) is available in [3]. To build a forecast model the sample period for analysis $y_1$, ..., $y_{128}$ was defined, i.e. the period over which the forecasting model was developed and the ex post forecast period (validation data set), $y_{129}$, ..., $y_{163}$ as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated.

*Fig. 1: The data for VAHOSTAV stock prices (January 1997 - August 1997) and the values of the AR(7) model for VAHOSTAV stock prices estimated by GL algorithm*




After some experimentation, we have identified two models for this series. The first one (1) based on Box-Jenkins methodology [2] and the second one (2) based on signal processing (see [1]). In this paper we primarily concern with short-term forecasting, say one period in the future.

In order to study the regularities of this process, the observed time series is viewed as a realization of a stochastic process. To simplify our notes, we suppose that the times at which the observations are recorded are regularly spaced, which allows us to consider an index by taking only integer values. The process is denoted by $y_t = (y_t, \ t \in \mathrm{T})$, where the index set T is A for analysis data set or E for ex post forecast data set.

$$y_t = \xi + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t \qquad t = 1,2, \ldots , N-2 \tag{1}$$

$$y_t = -\sum_{k=1}^{7} a_k y_{t-k} + \varepsilon_t \qquad t = 1,2, \ldots , N - 7 \tag{2}$$

The final estimates of model parameters (1), (2) are obtained using OLS (Ordinary Last Square) and two adaptive filtering algorithms in signal processing [1]. The Gradient Lattice (GL) adaptive algorithm and Last Squares Latice (LSL) algorithm representing the parameter estimates of the predictors (1), (2) were used. In Tab. 1 the parameter estimates for model (2) and corresponding RMSE's are given. Fig. 1 shows the GL prediction results and actual values for stock price time series in both analysis and ex post forecast period.

# 3 Fuzzy autoregressive (FAR) modelling

Next, we examine the application of fuzzy linear regression model [6] to the stock price time readings used in (1) and (2). Recall that the models in (1) and (2) fit to the stock prices were the AR(2) and AR(7) processes. In the fuzzy regression model proposed by Tanaka et al. [10], the parameters are the fuzzy numbers. The regression function of such a fuzzy parameters can be modeled by the following equation

$$Y_t = A_0 * \varphi_0(x_{0t}) \oplus A_1 * \varphi_1(x_{1t}) \oplus, \ldots, \oplus A_k * \varphi_k(x_{kt}) = \mathbf{A}'\mathbf{x}_t \tag{3}$$

where $A_0, A_1, \ldots, A_k$ are fuzzy numbers, $\oplus$ and $*$ are fuzzy addition and fuzzy multiplication operators respectively, $Y_t$ is fuzzy subset of $y_t$. This kind of fuzzy modelling is known as fuzzy parameter extension.

| Model | Order | Est. proc | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_3$ | $\hat{a}_4$ | $\hat{a}_5$ | $\hat{a}_6$ | $\hat{a}_7$ | RMSE* |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 2 | OLS | 1.113 | -0.127 | | | $\xi = 26.639$ | | | 67.758 |
| (2) | 7 | GL | -0.7513 | -0.1701 | -0.0230 | -0.0128 | -0.0028 | -0.0472 | 0.0084 | 68.540 |
| (2) | 7 | LSL | -0.8941 | -0.6672 | 0.7346 | -0.2383 | 0.1805 | -0.5692 | 0.4470 | 94.570 |

*ex post forecast period

*Table 1: OLS, GL and LSL estimates of AR models*

The problem to find out fuzzy parameters gives the following linear programming solution [6]

$$\begin{aligned} &\min\ s = c_0 + c_1 + \ldots + c_k \\ &\text{subject to } c_j \geq 0 \\ \text{and} \quad & \\ &(h-1)\mathbf{c}'|\mathbf{x}| - (y_t - \mathbf{x}'\alpha) \geq 0 \\ &(1-h)\mathbf{c}'|\mathbf{x}| - (y_t - \mathbf{x}'\alpha) \geq 0 \\ &\qquad \text{for } t = 1, 2, \ldots, N \end{aligned} \tag{4}$$

where $c_j$ , j = 0, 1, ... , k is the width or spread around the center of the fuzzy number, $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_k)$ denotes vector of center of the fuzzy numbers for model parameters, $\mathbf{x}' = (x_0, x_1, \ldots, x_k)'$ denotes vector of regressor variables in (3), *h* is an inposed threshold $h \in$ [0, 1] (see [5]). A choice of the *h* value influences the widths $c_j$ of the fuzzy parameters. The *h* value expresses a measure of the fitting of the estimated fuzzy model (3) to the given data. The fuzzines of $\mathbf{c}' = (c_0, c_1, \ldots, c_k)$ of the parameters $\tilde{A}_0$, $\tilde{A}_1$, ... , $\tilde{A}_k$ for the models (1) and (2) are given in Tab. 2.

| h=0.5 $\widetilde{A}_k$ k: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Model AR(2) | | | | | | | | |
| Modal values($\alpha$) | 26.639 | 1.113 | -0.127 | | | | | |
| Spread (**c**) | 0 | 0 | 0.229008 | | | | | |
| | | | | | | | | |
| Model AR(7) | | | | | | | | |
| Modal values($\alpha$) | 45.930 | 1.085 | 0.0861 | -0.2531 | 0.0836 | -0.0057 | 0.2081 | -0.2281 |
| Spread (**c**) | 0 | 0 | 0 | 0 | 0.209587 | 0 | 0 | 0 |

*Table 2: The fuzzines of* $\mathbf{c}' = (c_0, c_1, \ldots, c_k)$ *of the parameters* $\widetilde{A}_0, \widetilde{A}_1, \ldots, \widetilde{A}_k$ *for the models (1) and (2)*

The forecast for future observation is generated successively through the Eq. (3) by replacing the functions of the independent variables ($\varphi_j(x_{jt})$, $j$ = 0, 1, ... , $k$ by observations $y_{t-j}$. Then the forecasting function of the fuzzy AR process is

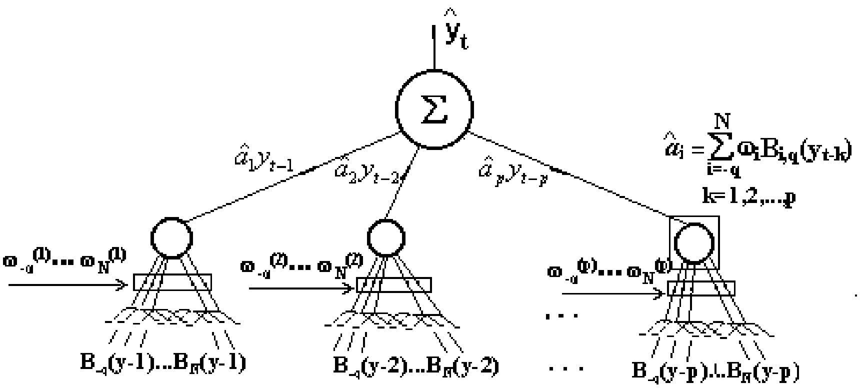
$$Y_{T+1}(T) = A_0 \oplus A_1 * y_T \oplus A_2 * y_{T-1} \oplus, \ldots, \oplus A_k * y_{T-k+1} \tag{5}$$

where $Y_{T+1}(T)$ is the forecast for period $T$+1 made at origin $T$. We observe that the forecasting procedure (5) produces forecast for one period ahead. As a new observation becomes available, we may set the new current period $T$+1 equal to $T$ and compute the next forecast again according to equation (5).

# 4 B-spline neural network approach

The concept of fuzzy neural network (FNN) can be approached from several different avenues. The one that we have used for stock price forecasts is described in [7], [3] and shown in Fig. 2. This figure shows the FNN with $p \times n$ input neurons (input layer), a single hidden layer with $p$ processing units (fuzzy neurons) and one output unit.

*Figure 2: The neuro fuzzy system architecture*




Input selection is of crucial importance to the successful development of FNN models. In models (1) and (2) potential inputs were chosen based on traditional statistical analysis: these included the raw stock price series and lags thereof. The relevant lag structure of potential inputs was analysed using traditional statistical tools: ACF, PACF and the MSE criterion. All the above techniques are in reality imprecise (we developed parsimonious models, that is, models which adequately describe the time series yet contain relatively few parameters, the theoretical ACF was estimated by the sample ACF, etc.). In fact we obtain a certain number of input values, but we are sure that these values are one of many other possible values. Thus, we will further suppose that the potential inputs, which were chosen based on statistical analysis, are fuzzy numbers characterized by a membership functions (the uncertainty is modeled as a possibility distribution) belonging to a class of bell shaped functions.

Inputs to the fuzzy neuron in hidden layer are fuzzy numbers denoted $B_{j,k,t}$, j = 1, 2, ..., p, $k$ identifies the order of the B-spline basis functions. They express the neural input signals in terms of their membership functions based on B-spline basis functions of the data. This concept is often called as B-spline FNN [7].

The learning algorithm is based on error signal. The neural network modifies the weights in synaptic connections with respect to the desired fuzzy system output $y_t$. The error of the fuzzy system, i.e., the difference between the fuzzy system forecast $\hat{y}_t$ and the actual value $y_t$ is analysed through the RMSE. Let $\hat{y}_t$ be a linear function

$$\hat{y}_t = \hat{a}_1 y_{t-1} + \hat{a}_2 y_{t-2} + \ \dots \ + \hat{a}_p y_{t-p} = \sum_{j=1}^{p} \ \sum_{t \in \mathrm{A}} \omega_{j,t} B_{j,k,t} y_{t-j} \tag{6}$$

where $\omega_{j,t}$ are the synaptic weights of input layer,

such that

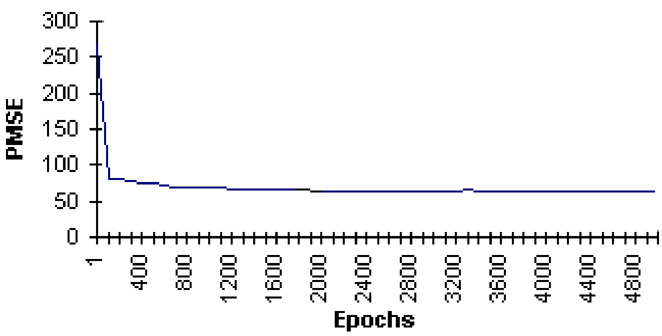
$$\sum_{t \in \mathrm{T}} (y_t - \hat{y}_t)^2 \le \sum_{t \in \mathrm{T}} [y(\mathbf{z}'_t \ ) - \hat{y}_t]^2 \tag{7}$$

for any other linear function $y(\mathbf{z}'_t \ )$. It is clear that such function can be derived by using the Normalized Back-Propagation (NBP) algorithm which minimizes the term of the left-hand side of the relation (6) by adjusting the weights $\omega_{j,t}$. The proof of the convergence of the NBP algorithm can be found in [4].

# 5 Empirical results

Our FNN was trained on the training data set. Periodically, during the training period, the RMSE of the FNN were measured not only on the training set but also on the validation set. The final FNN chosen for the stock price prediction is the one with the lowest error on the validation set. Note also, the training phase was finished after $5.10^3$ epochs, the best model being obtained after $2{,}3.\ 10^3$ epochs (see Fig. 3).

*Fig. 3 RMSE's - validation set*



The RMSE's of our predictor models are shown in Tab. 3. From this table can be seen that the basic (non fuzzy) artificial neural network architecture described in [6] does not support its use for daily frequencies. The initial results of the FNN forecasting model are clearly better.

| Model | RMSE* |
|---|---|
| AR(2) | 67.7 |
| Basic (non fuzzy) neural network (see [6]) | 67.2 |
| FNN | 63.5 |

* Validation set

*Table 3: RMSE's of our predictor models*

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