Estimating Volatilities by the GARCH and the EWMA model of PetroChina and TCL in the Stock Exchange Market of China

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Abstract

Volatility is an important parameter for financial risk management and it is applied in many issues such as option pricing, portfolio optimization, VaR methodology and hedging, thus the forecasting of volatility or variance can be regarded as a problem of financial modeling. In this paper, it will estimate the volatility use the historical approach and applying the GRACH model and the EWMA model in the same stock data of PetroChina and TCL on the Shanghai and Shenzhen Stock Exchange Market of China, it will use the result of mean square error to shows which is better model to calculation of assets.

Key words

Volatility, EWMA model, GARCH model, maximum likelihood methods, mean square error, VaR

JEL Classification: C13, C53.

1. Introduction

In this paper will description of the risk factors used in the VaR analysis. The analysis requires various levels of assumptions. It will start with the historical data, and get to know how well the models allow the risk manager to model and measure portfolio risk. The model's GARCH and EWMA are time-variation in risk, summarizing the main approaches will be summary in the theory section of paper then make example to calculation.

2. Estimating Volatility

Define σ_t as the volatility of a market variable on day *t*, as estimated at the end of day *t-1*. The square of the volatility on day t σ_t^2 is the variance rate.

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time.

$$R_{i,t} = \ln(\frac{P_{i,t}}{P_{i,t}-1}) \text{ for i=1,2,…,t.} \tag{1.1}$$

Where, t+1: Number of observations,

$P_{i,t}$: Stock price at end of ith interval (i=0,1,…,t).

Because $P_i = P_{i-1}e^{R_i}$, R_i is the continuously compounded return in the ith interval. The usual estimate, P, of the standard deviation of the σ_i 's is given by

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$$\sigma_i = \sqrt{\frac{1}{N-1}\sum_{t=1}^{n}[R_{i,t} - E(R_i)]^2} = \sqrt{\sigma_i^2} \ , \tag{1.2}$$

Where $E(R_i)$ is the mean of the $R_{i,t}$'s.

$$\ln\frac{P_T}{P_0} \sim \phi[(R_{i,t} - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\,], \tag{1.3}$$

Following that σ itself can be estimated as σ^*.

$$\sigma^* = \frac{P}{\sqrt{\tau}}, \tag{1.4}$$

The standard error of this estimate can be shown to be approximately $\sigma^* / \sqrt{2t}$.

Suppose that the value of the market variable at the end of day i is P_i. The variable R_i is defined as the continuously compounded return during day i.

$$R_i = \ln\frac{P_i}{P_i - 1}, \tag{1.5}$$

An unbiased estimate of the variance rate per day, σ_t^2 using the most recent *n* observations on the R_i is

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{N}[R_{t-1} - E(R_i)^2], \tag{1.6}$$

Where $E(R_i)$ is the mean of the R_i 's:

$$E(R_i) = \frac{1}{N}\sum_{i=1}^{N} R_{t,i} \ , \tag{1.7}$$

For the purposes of calculating Value at Risk, the formula in equation (1.7) is usually changed in a number of ways R_i, $E(R_i)$, *N-1*.

Where R_i is defined as the proportional change in the market variable between the end of day i-1 and the end of day i so that

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}}, \tag{1.8}$$

$E(R_i)$ is assumed to be zero and N-1 is replaced by t.

The formula for variance rate becomes

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N} R_{t-i}^2 \ , \tag{1.9}$$

2.1 The EWMA Model

Variances are modeled using the Exponentially Weighted Moving Average (EWMA) forecast. The EWMA model is a particular case of the model in follows equation (1.10).

$$\sigma_t^2 = \sum_{i=1}^{m} \alpha_i R_{t-i}^2 \ , \tag{1.10}$$

Where the weights, α_i, decrease exponentially as move back through time. Specifically, $\alpha_{i+1} = \lambda\alpha_i$ where λ is a constant between zero and one.

It turns out that this weighting scheme leads to a particularly simple formula for updating volatility estimates. The formula is

$$\sigma_t^2 = \lambda\sigma_{r-1}^2 + (1-\lambda)R_{r-1}^2, \tag{1.11}$$

The estimate, σ_n, of the volatility for day n is calculated from σ_{t-1} and R_{t-1}.

To understand why equation (1.11) corresponds to weights that decrease exponentially, it substitute for σ_{n-1}^2 to get

$$\sigma_t^2 = \lambda[\lambda\sigma_{t-2}^2 + (1-\lambda)R_{t-2}^2] + (1-\lambda)R_{t-1}^2, \tag{1.12}$$

Or $\sigma_t^2 = (1-\lambda)(R_{t-1}^2 + \lambda R_{t-2}^2) + \lambda^2\sigma_{t-2}^2$, (1.13)

Substituting in a similar way for σ_{t-2}^2 gives

$$\sigma_t^2 = (1-\lambda)(R_{t-1}^2 + \lambda R_{t-2}^2 + \lambda^2 R_{t-3}^2) + \lambda^3\sigma_{t-3}^2, \tag{1.14}$$

Continuing in this way will see that

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{N}\lambda^{i-1}R_{t-i}^2 + \lambda^N\sigma_0^2, \tag{1.15}$$

For a large m, the term $\lambda^N\sigma_0^2$ is sufficiently small to be ignored so that equation (1.11) is the same as equation (1.10) with $\alpha_i = (1-\lambda)\lambda^{i-1}$. The weights for the R_i's decline at rate λ as move back through time. Each weight is λ times the previous weight.

2.2 The GARCH Model

The Generalized Autoregressive Heteroskedastic (GARCH) model is developed by Engle (1982) and Bollerslev (1986). The difference between the GARCH model and the EWMA model is analogous to the difference between equation (1.10) and following equation (1.12).

$$\sigma_t^2 = \lambda\omega + \sum_{i=1}^{N}\alpha_i R_{t-i}^2, \tag{1.12}$$

In GARCH model, σ_t^2 is calculated from a long-run average variance rate, as well as from σ_{t-1} and R_{n-1}. The equation for GARCH is

$$\sigma_t^2 = \gamma\sigma_t^2 + \alpha R_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{1.13}$$

Where, α is the weight assigned to R_{t-1}^2, and β is the weight assigned to σ_{t-1}^2. Because the weights must sum to one:

$$\gamma + \alpha + \beta = 1, \tag{1.14}$$

The EWMA model is a particular case of GARCH where $\gamma = 0$, $\alpha = 1-\gamma$, and $\beta = \gamma$.

Setting $\omega = \gamma V$, the GARCH model in following equation,

$$\sigma_t^2 = \omega + \alpha R_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{1.15}$$

This is the form of the model that is usually used for the purposes of estimating the parameters. Once ω, α, β have been estimated, it can calculate γ as $1-\alpha-\beta$. The long-term variance can then be calculated as ω/γ. For a stable GARCH process it requires $\alpha+\beta<1$. Otherwise the weight applied to the long-term variance is negative.

2.3 Maximum Likelihood Methods and Mean Square Error

Maximum likelihood methods involve choosing values for the parameters that maximize the change of the data occurring. The probability density of the m observations occurring in the order that they are observed is

$$L(\theta) = \prod_{i=1}^{N}[\frac{1}{\sqrt{2\pi v}}\exp(\frac{-R_i^2}{2\sigma})], \tag{1.16}$$

Using the maximum likelihood method, the best estimate of v is the value that maximizes this expression. Maximizing an expression is equivalent to maximizing the logarithm of the expression. Taking logarithms of the expression in equation (1.16) and ignoring constant multiplicative factors, it can be seen that will maximize

$$L(\theta) = \sum_{i=1}^{N} [-\ln(v) - \frac{R_i^2}{\sigma}]$$

or

$$L(\theta) = -T\ln(v) - \sum_{i=1}^{N} \frac{R_i^2}{v}, \quad (1.17)$$

Differentiating this expression with respect to v and setting the result equation to zero, the maximum likelihood estimate of v is $\frac{1}{T}\sum_{i=1}^{N} R_i^2$.

Maximum likelihood methods are usually used to estimate parameters in GARCH and similar models from historical data. These methods involve using an iterative procedure to determine the parameter values that maximize the chance or likelihood that the historical data will occur.

The mean square error is the function between the actual data and the fitted values of the date. Let $y = (y_1,..., y_n)'$ denote the fit date, θ is the estimate data, the function $f(\theta \mid x) = (f_1,..., f_n)'$ is the function to fit *y*, which depends on the parameters θ and possibly other data x. The estimation problem is to find the parameters θ so that each f_i is as close as possible to y_i, for $i = 1,2,..., n$. Following equation is the error function.

$$\varepsilon(\theta \mid x) = y - f(\theta \mid x), \quad (1.18)$$

The general least squares objective for making $f(\theta \mid x)$ as close as possible to y is

$$L(\theta) = [\varepsilon(\theta \mid x)'\varepsilon(\theta \mid x)] = \min(\theta), \quad h(\theta) \le 0, \quad (1.19)$$

Where $h(\theta)$ are the constraints on the values of the parameters. To requires is get minimize the sum of the squared errors between the observed and fitted values.

3. Volatility Forecasting by the GARCH and the EWMA model

3.1 Methodology of Volatility Forecasting by the GARCH and the EWMA model

The GARCH model for one-period forecasting is defined by

$$y_{t+1} = \hat{y}_{t+1} + \varepsilon_{t+1}, \quad (2.1)$$

$$\sigma_{t+1,t}^2 = \omega + \alpha \cdot \varepsilon_t^2 + \beta \cdot \sigma_{t,t-1}^2, \quad (2.2)$$

Where $\omega, \alpha, \beta \ge 0$ and simultaneously $\alpha + \beta < 1$, y_{t+1} is the observed value of the forecasted financial quantity, $\hat{y}_{t+1}$ is its estimate at time t. In the linear case $\hat{y}_{t+1} = a + b \cdot x_t$, where x_t is an observable independent variable. Further, ε_{t+1} is the error of the forecast, ε_t^2 and $\sigma_{t+1,t}^2$ define the observed and the forecasted variance, $[\omega, \alpha, \beta, a, b] = \vec{\theta}$ are estimated parameters.

There exists a modification to the GARCH model, by which it is sufficient to estimate just the two parameters of equation (2.2), α and β. This is the case if $\omega = V \cdot (1 - \alpha - \beta)$, where V is the long term variance. This modification is referred to as the variance targeting.

GARCH model can be generally defined by

$$\sigma_{t,t-1}^2 = \omega + \sum_{i=1}^{p} \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \cdot \sigma_{t-j,t-j-1}^2, \quad (2.3)$$

Problem 1: Mathematical formulation of forecasting the variance by the GARCH model

Objective function

$$L(\omega,\alpha,\beta)=\sum_t z_t \rightarrow \max$$

Constraints

$$\alpha+\beta<1, \quad \text{(P1)}$$

$$\omega,\alpha,\beta\geq 0, \quad \text{(P2)}$$

$$z_t=-\ln\sigma^2_{t,t-1}-\frac{R_t^2}{\sigma^2_{t,t-1}}, \quad \text{(R1)}$$

$$\sigma^2_{t+1.t}=\omega+\alpha\cdot R_t^2+\beta\cdot\sigma^2_{t,t-1}, \quad \text{(R2)}$$

Forecast over 4 periods

$$\sigma^2_{t+4,t}=\omega\cdot\sum_{k=0}^{3}(\alpha+\beta)^k+(\alpha+\beta)^3\cdot(\alpha\cdot R_t^2+\beta\cdot\sigma_t^2), \quad \text{(R3)}$$

A special case of the GARCH model is the single-parametric EWMA model, where $\omega=0,\alpha=1-\lambda,\beta=\lambda$ and so $0\leq\lambda<1$, λ is usually called the decay factor.

$$\sigma^2_{t+1,t}=(1-\lambda)\cdot\varepsilon_t^2+\lambda\cdot\sigma^2_{t,t-1}, \quad (2.4)$$

The parameters of the model can be estimated either by the maximum likelihood methods or by the minimizing the criterion of the root mean square error (RMSE) as a mathematical programming problem

$$RMSE=\sqrt{\frac{1}{t}\cdot\sum_t z_t}\ , \quad (2.5)$$

Where $z_t=\varepsilon_t^2-\sigma^2_{t,t-1}$.

Problem 2: Mathematical formulation of forecasting the volatility by the EWMA model

Objective function

$$RMSE=\sqrt{\frac{1}{T}\sum_{t=2}^{T}z_t^2}\rightarrow \min$$

Constraints

$$0\leq\lambda<1, \quad \text{(P1)}$$

$$z_t=R^2_{i,t}-\sigma^2_{i,t,t-1}(\lambda)\ \text{for variance}, \quad \text{(R1)}$$

$$z_t=R_{i,t}\cdot R_{j,t}-\sigma_{ij,t,t-1}(\lambda)=R_{ij,t}-\sigma_{ij,t,t-1}(\lambda)\ \text{for covariance}, \quad \text{(R2)}$$

$$\sigma^2_{i,t+1,t}(\lambda)=\lambda\cdot\sigma^2_{i,t,t-1}(\lambda)+(1+\lambda)\cdot R^2_{i,t}, \quad \text{(R3)}$$

$$\sigma_{i,t+1,t}(\lambda)=\sqrt{\sigma^2_{i,t+1,t}(\lambda)}, \quad \text{(R4)}$$

$$\sigma_{ij,t+1,t}(\lambda)=\lambda\cdot\sigma_{ij,t,t-1}(\lambda)+(1-\lambda)\cdot R_{i,t}\cdot R_{j,t}=\lambda\cdot\sigma_{ij,t,t-1}(\lambda)+(1-\lambda)\cdot R_{ij,t}, \quad \text{(R5)}$$

4. Calculation of Volatility Forecasting by the GARCH and the EWMA model

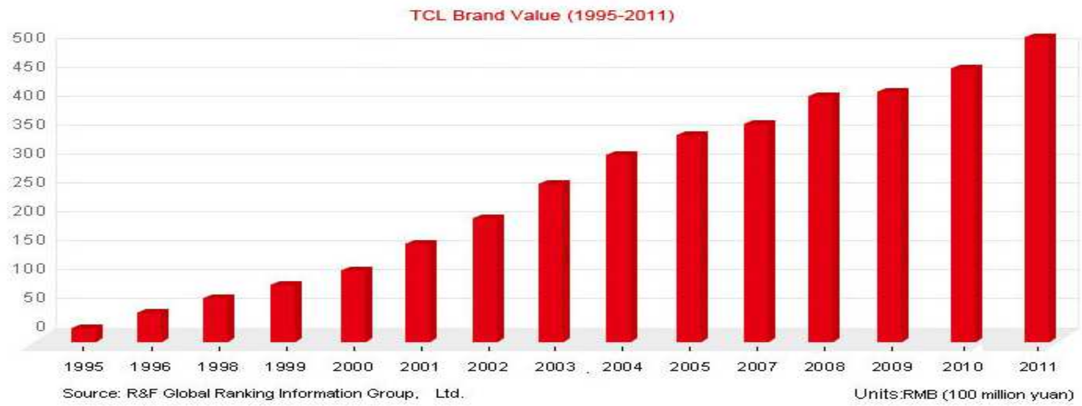
4.1 Forecasting the Variance by the GARCH model

Consider time series of historical returns for asset A1 (stock of PetroChina on Shanghai stock exchange market) and asset A2 (stock of TCL on Shenzhen stock exchange market) R_t in the Chinese stock market. The mean value of returns does not significantly differ from zero, all random samples are supposed to be normally distributed. Following table 2.2 is prices trend PetroChina vs. TCL (2008-2012).

PetroChina Company Limited (601857.ss) is mainly engaged in the production and sale of oil and gas related products. The Company's main businesses include the exploration, development, production and sale of crude oil and natural gas; refining of crude oil and petroleum products, production and sale of basic and derivative petrochemical products and other chemical products; sale of refined oil products and trading business; the transportation of natural gas, crude oil and refined oil, as well as the sale of natural gas. The Company's major products include gasoline, diesel, kerosene, synthetic resin, synthetic fiber raw materials and polymers, synthetic rubber and urea. The Company is involved in the retail of refined oil and the international trading of petroleum and natural gas, and the construction and operation of oil and gas pipelines. As of December 31, 2011, the Company operated 19,362 of gas stations, produced 87.15 million tons and sold 146 million tons of gasoline, diesel and kerosene.

Founded in 1981, TCL is one of the largest consumer electronics enterprises in China with a global presence. TCL Corporation has three listed companies: TCL Corporation (SZ.000100), TCL Multimedia (HK.1070) and TCL Communication (HK.2618). Currently, TCL Corporation has set up four business units – TCL Multimedia Holdings, TCL Communication Holdings, China Star Optoelectronics Technology and TCL Home Appliances Group, as well as six business groups – System Technology Unit, Techne Group, Emerging Business Group, Investment Group, Highly Information Industry and Real Estate Group. In 2011, the brand value of TCL had exceeded RMB 50.118 billion (USD 7.59billion), rising from 70th with RMB 690 million in 1995 to the No. 1 TV brand in China today. The elevation of this brand value represents TCL's transformation into a truly global brand. Following table 2.1 is TCL brand value from 1995 to 2011.

Table 2.1 TCL brand value (1995-2011)



Picture 2.2 Prices trend PetroChina vs. TCL (2008-2012)

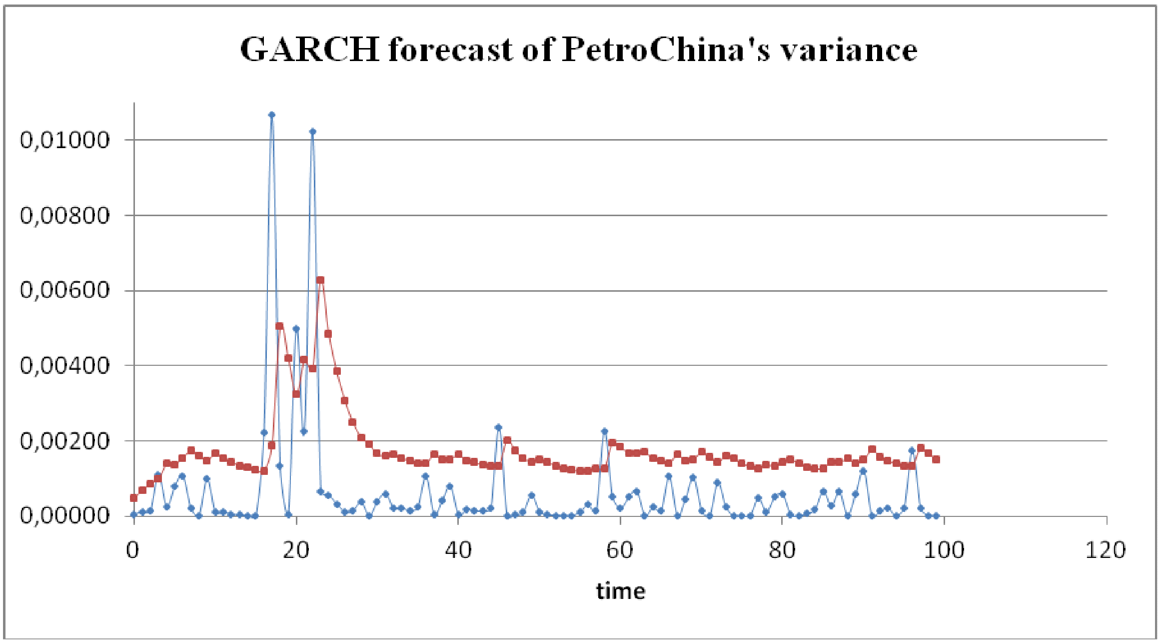
Computing the forecasting variance of returns is using GARCH model according to the mathematical formulation of problem 1. First, calculating the observed and the forecasted variance is according to the problem 1's equation (R2). Following table is about the GARCH forecast.

Table 2.3 GARCH forecast

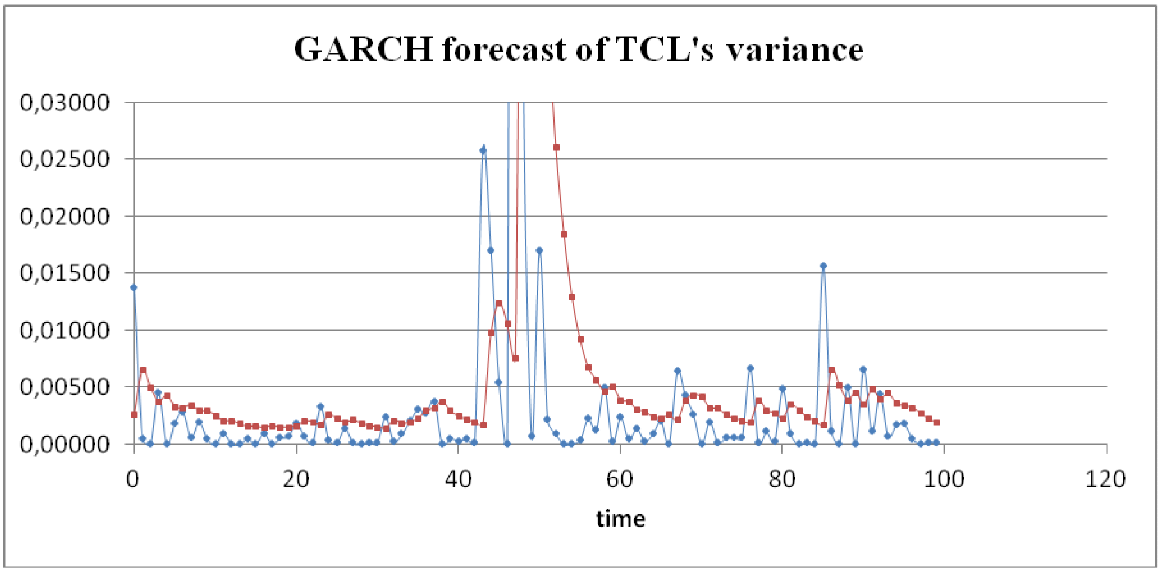
Time	PetroChina					TCL				
t	R_t^2	$\sigma_{t+1,t}^2$	z_t	$\sigma_{t+4,t}^2$	ε_t	R_t^2	$\sigma_{t+1,t}^2$	z_t	$\sigma_{t+4,t}^2$	ε_t
1	0.00006	0.00048	7.52767	-	0.01380	0.00268	0.76636	-	0.01113	0.01380
2	0.00011	0.00070	7.10729	0.00051	0.00055	0.00658	4.94090	0.00466	-0.00603	0.00055
3	0.00016	0.00087	6.86734	0.00064	0.00001	0.00502	5.29406	0.00342	-0.00501	0.00001
4	0.00110	0.00100	5.80516	0.00103	0.00451	0.00378	4.38518	0.00401	0.00073	0.00451
5	0.00025	0.00139	6.40028	0.00103	0.00001	0.00437	5.43176	0.00298	-0.00436	0.00001

Second, setting up the Excel Solver and calculating the parameters by the GARCH model over one period then over 4 periods. Using the maximizing the likelihood method get the results of PetroChina and TCL are 609.374915 and 461.67934, the mean square error of PetroChina and TCL are 0.0000025006 and 0.0006286. Following Pictures are about the GARCH forecast of variance and covariance.

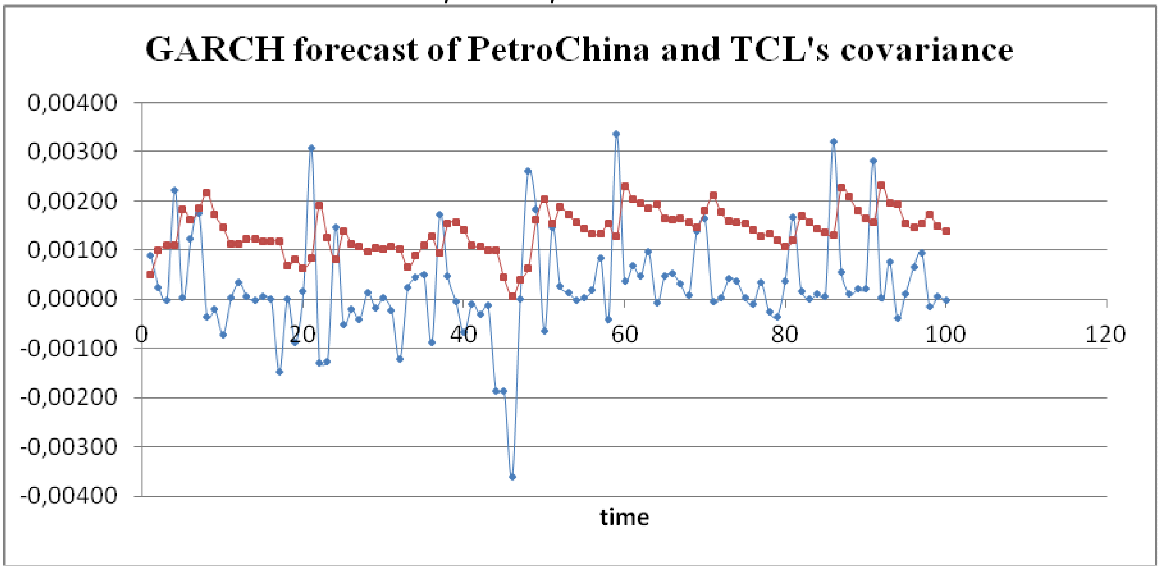
Picture 2.1 GARCH forecast of PetroChina's variance



Picture 2.2 GARCH forecast of TCL's variance



Picture 2.3 GARCH forecast of PetroChina and TCL's covariance



4.2 Forecasting the Volatility by the EWMA model

Consider time series of weekly historical returns for two assets A1 (stock of PetroChina) and A2 (stock of TCL). After verification it supposes that the mean value of returns is zero. Computing the forecasting the volatility and covariance of both assets using the EWMA model according to the mathematical formulation of problem 2.

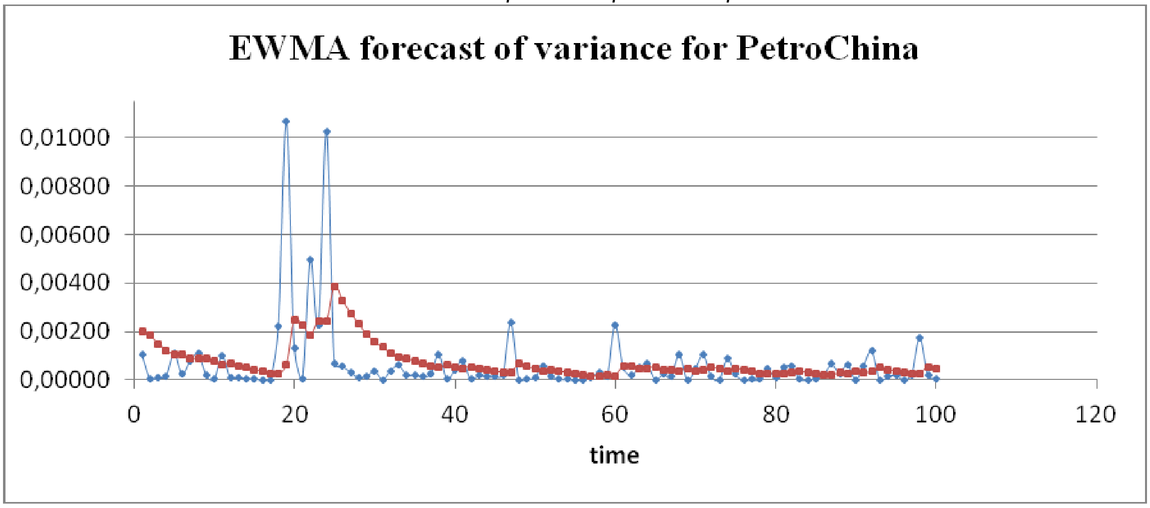
First, according the problem 2's equation (R1)-(R5) to calculation for $R_{i,t}^2, R_{ij,t}, \sigma_{i,t+1,t}^2(\lambda)$, and $\sigma_{ij,t+1,t}(\lambda)$. Following table is about EWMA forecast.

Table 2.4 EWMA forecast

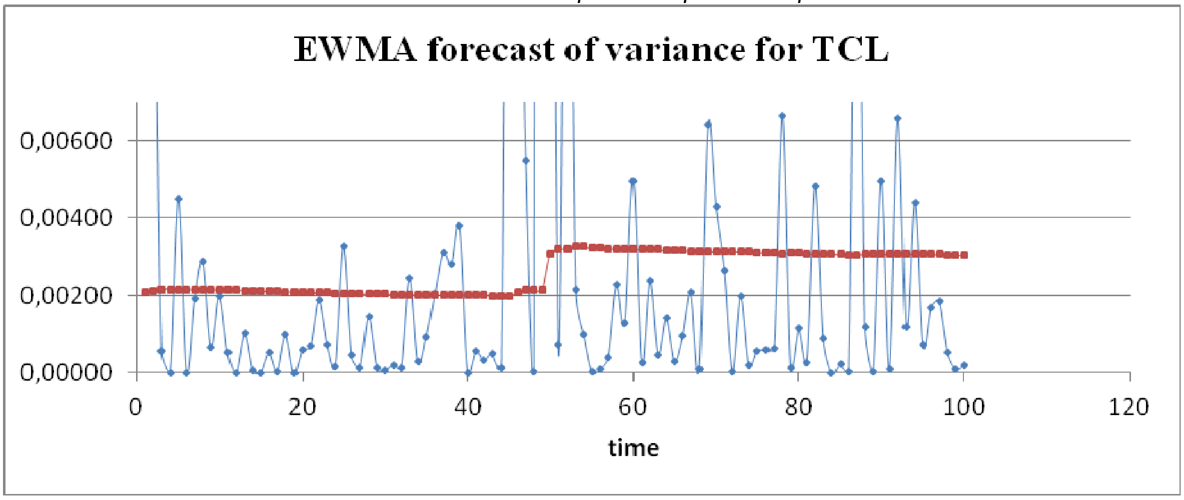
Time	PetroChina				TCL				PetroChina, TCL		
t	R_t^2	$\sigma_{t,t-1}^2$	$\sigma_{t,t-1}$	ε_t	R_t^2	$\sigma_{t,t-1}^2$	$\sigma_{t,t-1}$	ε_t	$R_{A1}R_{A2}$	$\sigma_{A1A2t,t-1}$	ε_t
1	0.00105	0.00200	0.04469	-0.00095	0.01150	0.00207	0.04553	0.00942	0.00348	0.00093	0.00255
2	0.00006	0.00182	0.04266	-0.00176	0.01380	0.00211	0.04594	0.01169	0.00090	0.00117	-0.00027
3	0.00011	0.00149	0.03860	-0.00138	0.00055	0.00216	0.04646	-0.00161	0.00024	0.00114	-0.00090
4	0.00016	0.00123	0.03508	-0.00107	0.00001	0.00215	0.04639	-0.00215	-0.00003	0.00106	-0.00109
5	0.00110	0.00103	0.03208	0.00007	0.00451	0.00214	0.04629	0.00237	0.00223	0.00095	0.00128

Second, setting up the Excel Solver and calculate the parameters of the EWMA model for the variances of both assets and their covariance. The variance of PetroChina and TCL are 0.001574359 and 0.02386, the covariance of PetroChian and TCL is 0.001090191. The $0 \le \lambda < 1$, the result of λ that λ_A is 0.000002471, λ_B is 0.00056, λ_{AB} is 0.001090191. If the results is 1 means the homoscedasticity is constant variance. Following pictures are about the EWMA forecast of variance and covariance. The mean square error of PetroChina and TCL are 0.000002471 and 0.00056.

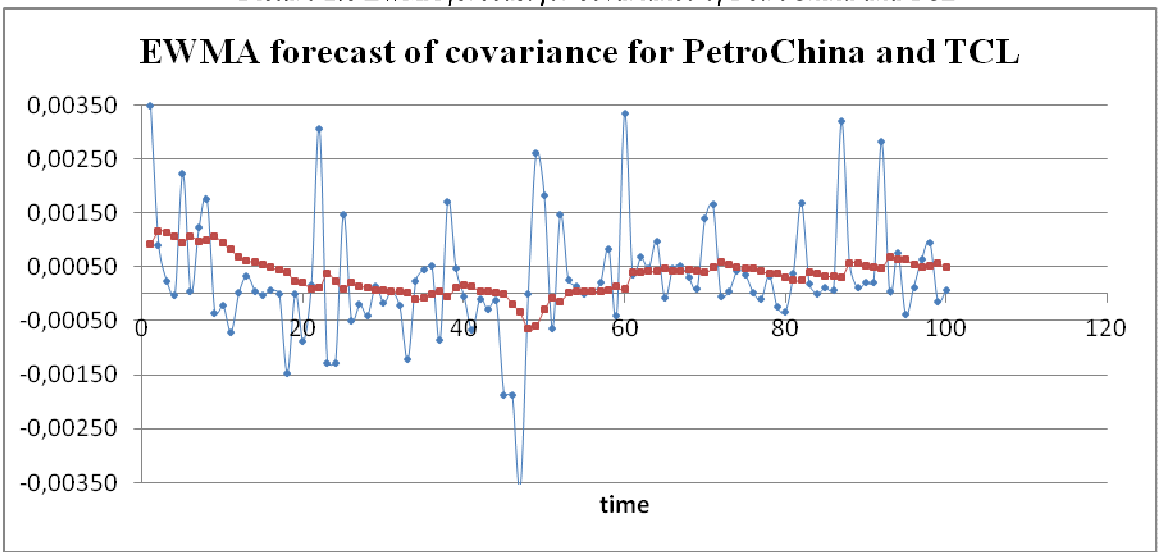
Picture 2.4 EWMA forecast of variance for PetroChina



Picture 2.5 EWMA forecast of variance for TCL

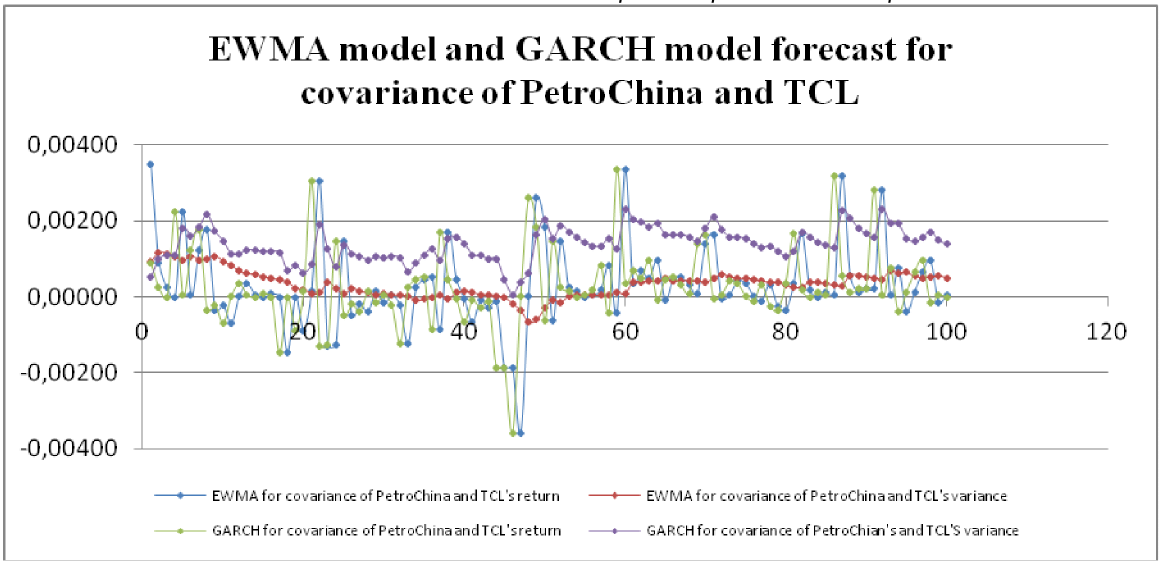


Picture 2.6 EWMA forecast for covariance of PetroChina and TCL



The characteristic property of the GARCH model is the slower reversion to the observed variance after greater shocks. The GARCH model is better compared with the EWMA model for short-term forecasting. However, the potential differences are rather eliminated for longer time periods so that results are more similar. Following picture is EWMA and GARCH model forecast for covariance of PetroChina and TCL.

Picture 2.7 EWMA model and GARCH model forecast for covariance of PetroChina and TCL



5. Conclusion

In practice, the volatility of asset, like the asset's price, is a stochastic variable. Unlike the asset price, it is not directly observable. In the EWMA model and the GARCH model, the weights assigned to observations decrease exponentially as the observations become older. The GARCH model differs from the EWMA model in that some weigh is also assigned to the long-run average variance rate. Bothe the EWMA and GARCH models have structures that enable forecasts of the future level of variance rate to be produced relatively easily. In the paper, it's trying to shows different of two models, which use the same data of assets to calculation according of the EWMA and GARCH model.

Firstly in the section 1 and section 2, it's about theoretical estimating the volatility. Then in the following section are the models of EWMA and GARCH and the maximum likelihood methods and mean square error, it's about the theory of mathematical foundation. In the section of 3 is methodology part, in this section it use the same real stock data of PectroChian on Shanghai stock exchange market and TCL on Shenzhen stock exchange market to calculation of the EWMA model and the GARCH model. During the calculation, compare the result of mean square error get to know the GARCH model is better than EWMA model in both the case of PetroChina and TCL.

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