Risk estimation for FX rates: basic backtesting techniques with some application

Tomáš Tichý

Abstract

The soundness of risk monitoring and measuring system is a key point for the reliability of financial institutions. One of the features of a reliable risk model is that it passes a backtesting procedure – a comparison of the one step ahead risk estimation and a true loss occurred on a given day – without any troubles. Within this paper, basic methods of backtesting procedure are reviewed, including their pros and cons. Next, several approaches are applied on a case of equally weighted normalized FX rate sensitive portfolio. It is documented that both applied tests can provide different recommendations for the model.

Keywords

FX rate portfolio, multidimensional Lévy models, variance gamma model, normal inverse gaussian model, VaR, AVaR, backtesting.

1. Introduction

Financial institutions play an inherent role within the economic system, since their existence allows an efficient transfer of funds, liquidity, maturity, and also risks. The efficiency is given by the fact that the markets (or the world, in general) are not ideal or perfect – doing business is more simple with huge amount of funds, information are not available for free, financial instruments are difficult to understand. However, financial institutions have at their disposal a huge amount of funds and many high-skilled employers so that the operations can be done quickly and effectively.

Although the well-managed financial institutions help to improve the conditions of the economic system, their faults can obviously result into the contrary. To prevent the failures of financial institutions and to increase the confidence into the financial system, some sort of regulation and subsequent supervision is necessary. One of its most important parts within the banking industry is to specify which model is qualified to measure the risk exposure. In this paper we concentrate ourselves at market risk models and their ability to estimate the risk exposure soundly. (For further discussion on risk management of financial institutions see eg. Hull (2010) or Resti and Sironi (2007).)

The ability of (market) risk models to estimate the risk exposure soundly is commonly assessed by the so called backtesting procedure. It works as follows: At time $t$ the risk (eg. in terms of $VaR$) is estimated for time $t + 1$ (say a next day). Later, it is compared wit
the true loss record. If the loss is higher than the estimation, such a day is referred to as the exception and denoted by 1. Otherwise we record 0. The procedure is repeated over a given time length (usually several years). The sequence of 0’s and 1’s should fulfill some statistical assumptions. Generally the most simple way is to compare the true number of exceptions to the assumption about them (Kupiec, 1995). The review of some further techniques can be found eg. in Berkowitz et al. (2010).

Recently, there have been published several papers dealing with the analysis of risk models via backtesting (preferably according to Kupiec’s test). While e.g. Alexander and Sheedy (2008) assumed Gaussian/Student/GARCH/Empirical models for a simple position and Rank (2007) analyzed similar multi-position models joined by several copula functions, in Tichý (2010a,b) the performance of ordinary elliptical copula Lévy driven models (e.g. skewness and kurtosis in the marginal distribution and potentially non-linearity, but still symmetry in the dependency was allowed) with/without respect to various time span to estimate the parameters were analyzed. By contrast, the main contribution of this paper is to compare the risk estimation efficiency according to basic backtesting procedures.

We proceed as follows. In the following section, the most important backtesting models are reviewed. Next, the multidimensional model for the portfolio evolution is defined in terms of marginal Lévy subordinated models joint together by ordinary copula functions. Finally, these models are applied for risk estimation of an equally weighted normalized FX rate-sensitive portfolio and their prediction power is evaluated according to selected backtesting approaches.

2. Backtesting approaches

Within the backtesting procedure, the ability of a given model to estimate the future loses is to be tested. In the context of the market risk, the backtesting procedure can be applied on models in the form of VaR, cVaR or even overall distribution of the losses. Loosely speaking, applying the historical data, ie. true evolution of market prices of financial instruments, the risk is estimated (ex ante) at time $t$ for time $t + \Delta t$, where $\Delta t$ is usually set to 1 business day, and compared with the true loss observed at time $t + \Delta t$ (ex post). This procedure is applied for moving time window over whole data set.

In line with the standards for bank supervision as defined within Basel II, let us assume that the risk is estimated for one day horizon, $\Delta t = 1$. Denote Value at Risk of a portfolio $X$ estimated at day $t$ for the next day $t + 1$ on a given significance level $\alpha$ as:

$$VaR_X(t, t + 1; \alpha)$$

and the true loss observed at $t + 1$ with respect to the preceding day $t$ as:

$$L_X(t, t + 1).$$

Within the backtesting procedure on a given time series $\{1, 2, \ldots, T\}$, two situations can arise – the loss is higher than its estimation or lower (from the stochastic point of view, the equality shouldn’t arise). While the former case is denoted by 1 as an exception, the latter one is denoted by zero:

$$I_X(t + 1, \alpha) = \begin{cases} 1 & \text{if } L_X(t, t + 1) > VaR_X(t, t + 1; \alpha) \\ 0 & \text{if } L_X(t, t + 1) \leq VaR_X(t, t + 1; \alpha). \end{cases}$$

(1)

On the sequence $\{I_X(t + 1, \alpha)\}_{t=1}^{T-1}$, where $m$ is a number of data (days) needed for the initial estimation, it can be tested whether the number of ones (exceptions) corresponds
with the assumption, ie. $\alpha \times n$ (where $n = T - 1 - m$), whether the estimation is valid either unconditionally or conditionally, whether bunching is present, etc.

Since the distribution of exceptions in time should be identical and independent (iid Bernoulli variables), we can generally assume, that the number of exceptions will be with probability $q$ within:

$$P[I_X(t+1, \alpha) \in (n\alpha) - Z_{(1-q)/2} \sqrt{n\alpha(1-\alpha)}, n\alpha + Z_{(1-q)/2} \sqrt{n\alpha(1-\alpha)}] = q,$$

where $Z_x$ denotes the value of the distribution function of binomial distribution for $x$.

2.1 Unconditional test of Kupiec, 1995

Kupiec test is derived from a relative amount of exceptions, ie. whether their number is from the statistical point of view different from the assumption. A given (likelihood ratio) on the basis of $\chi^2$ probability distribution with one degree of freedom is formulated as follows:

$$LR_{uc} = \left(1 - \alpha\right)^{N-m}\frac{\alpha^m}{(1-m/N)^{N-m}(m/N)^m}.$$  

Since $-2 \ln LR_{uc}$ is asymptotically distributed according to $\chi^2$ with one degree of freedom, we can make the following rearranging:

$$-2 \ln LR_{uc} = -2 \ln \left(1 - \alpha\right)^{N-m}\frac{\alpha^m}{(1-m/N)^{N-m}(m/N)^m} + 2 \ln \left(1 - \frac{m}{N}\right)^{N-m} - 2 \ln \left(\frac{m}{N}\right)^m$$

or

$$-2 \ln LR_{uc} = -2(N-m) \ln(1-\alpha) - 2m \ln \alpha + 2(N-m) \ln(1 - \frac{m}{N}) + 2m \ln \frac{m}{N}$$  

(5)

Here, $\alpha$ is expected probability of exceptions (given that $VaR(\alpha)$), $m$ is the true number of exceptions from $N$ observations, and therefore $\frac{m}{N}$ is the observed probability of exceptions.

The most common standard is to test the results on a probability level of 5% and thus, corresponding value of $\chi^2$ is 3.84. Hence, if (4) is higher than 3.84 (6.635 for $p = 1\%$ and 2.7 for $p = 10\%$), the model must be rejected through the inadequate risk estimation. Note also, that these results can be related to $k$ factor of Basel II.

Considering the Basel II recommendations, the backtesting is carried out at least for the period of one year. Hence, with 99% confidence level, ie. $\alpha = 0.01$, we can expect about 2.5 exceptions and the model can be accepted by the supervisor for up to 10 exceptions. However, Kupiec (1995) has showed, that even with 8 exceptions, which gives us $VaR(0.03)$ instead of $VaR(0.01)$, the probability of successful identification of incorrect models is only $\frac{2}{3}$. This drawback can be overcome by increasing the length of the data for several years.

2.2 Indepency of exceptions by Christoffersen, 1998

In order to assess, whether the exceptions are distributed equally in time, ie. without any dependency (autocorrelation), we should define the time lag first: in Christoffersen (1998) it is defined as the stage, when exception at one time moment can significantly help to identify whether another exception will (not) follow on the subsequent day. Therefore,
we replace the original sequence \( \{I_X(t+1, \alpha)\}_{t=1+m}^{T-1} \) by a new one, \( \{I_{IX}(t+1, \alpha)\}_{t=1+m}^{T-2} \), as follows:

\[
I_{IX}(t+1, \alpha) = \begin{cases} 
  a & \text{if } \{I_X(t), I_X(t+1)\} = \{0, 0\} \\
  b & \text{if } \{I_X(t), I_X(t+1)\} = \{0, 1\} \\
  c & \text{if } \{I_X(t), I_X(t+1)\} = \{1, 0\} \\
  d & \text{if } \{I_X(t), I_X(t+1)\} = \{1, 1\}
\end{cases}
\]

and subsequently calculate the number of pairs \( a, b, c \) and \( d \) in \( \{I_{IX}(t+1, \alpha)\}_{t=1+m}^{T-2} \), which implies the number of exceptions following the good estimate:

\[
m_0 = \frac{b}{a+b}
\]

and bad estimate, respectively:

\[
m_1 = \frac{d}{c+d}
\]

The statistics used in Christoffersen (1998) is:

\[
LR_{ind} = \frac{(1-m/N)^{N-m}m/N}{m_0(1-m_0)^{m_0}(1-m_1)^{m_1}}.
\]

Alternatively:

\[
-2 \ln LR_{ind} = -2(N-m) \ln(1-m/N) - 2m \ln m/N + 2d \ln m_1 + 2c \ln(1-m_1) + 2a \ln(1-m_0) + 2d \ln m_1 + 2c \ln(1-m_1).
\]

Also this statistics is asymptotically distributed according to \( \chi^2_2 \). However, this kind of testing can identify only the first-kind (Markovian) dependency. An alternative test was suggested by Christoferssen and Pelletier (2004) with intention to measure (and test) the time among particular exceptions.

### 2.3 Conditional test

A combination of preceding tests is a so called conditional risk coverage:

\[
LR_{cc} = \frac{(1-\alpha)^{N-m}m/N}{m_0(1-m_0)^{m_0}(1-m_1)^{m_1}}.
\]

where \(-2 \ln LR_{cc}\) is distributed according to \( \chi^2_2 \), which gives us:

\[
LR_{cc} = LR_{uc} \times LR_{ind}.
\]

Although this test takes into account two different features of the risk model, it makes an average. Hence, it is difficult to identify the error sources and in case of a very good quality of independency of exceptions, their incorrect number can be overlooked.

### 2.4 Other tests

Obviously, there exist many other tests focusing on other properties, such as Berkowitz et al. (2006) working with martingale difference hypothesis, regression tests, test with more significance levels, such as Crnkovic and Drachman (1997), Diebold et al. (1998) as well as Berkowitz (2001) or Giacomini and Komunjer (2005) or tests for expected shortfalls (cVaR’s) or loss function tests, such the ones in Lopez (1999):

\[
L(VaR_t(\alpha); x_{t+1}) = \begin{cases} 
  1 + (x_{t+1} - VaR_t(\alpha))^2 & \text{if } x_{t+1} \leq -VaR_t(\alpha) \\
  0 & \text{if } x_{t+1} > -VaR_t(\alpha)
\end{cases}
\]

\[
L(VaR_t(\alpha); x_{t+1}) = \begin{cases} 
  1 + (x_{t+1} - VaR_t(\alpha))^2 & \text{if } x_{t+1} \leq -VaR_t(\alpha) \\
  0 & \text{if } x_{t+1} > -VaR_t(\alpha)
\end{cases}
\]
3. Copula Lévy models for portfolio risk estimation

In order to assess the risk of a portfolio, i.e. unexpected changes in its value, a joint probability distribution of all relevant drivers of random evolution should be estimated, though marginal distributions and a suitable tool to express the dependency among particular factors can be estimated separately.\(^3\)

Actually, such a decomposition can be of great value since joint probability distribution generally presumes identical margins, at least at elementary levels. By contrast, choosing e.g. copula functions to rebuild independent marginal distributions into dependent structure gives us a great portion of freedom when estimating the marginal probability distribution.

3.1 Marginal distribution by subordinated Lévy processes

The major task of financial model building is to allow one to fit also extreme evolution of market prices. It is a matter of fact that returns at financial markets are neither symmetrically distributed nor without exceed peaks (or heavy tails) over time, which is in contradiction with Gaussian distribution. A very feasible way to fit both skewness (non-symmetry) and kurtosis (heavy tails) is to apply a subordinated Lévy model, a rather non-standard definition of Lévy models as time changed Brownian motions, which goes back to Mandelbrot and Taylor (1967) and Clark (1973).

Generally, a Lévy process is a stochastic process, which is zero at origin, its path in time is right-continuous with left limits and its main property is that it is of independent and stationary increments. Another common feature is a so called stochastic continuity. Moreover, the related probability distribution must be infinitely divisible. The crucial theorem is the Lévy-Khintchine formula, i.e. the characteristic function of any Lévy process takes the following form:

\[
\Phi(u) = i\gamma u - \frac{1}{2} \sigma^2 u^2 + \int_{-\infty}^{\infty} \left( \exp(iux) - 1 - iux\mathbb{1}_{|x|<1} \right) \nu(dx)
\]  

(14)

with the triplet of Lévy characteristics, that fully determine the process:

\[\{\gamma, \sigma^2, \nu(dx)\}\].

The former two define the drift of the process (deterministic part) and its diffusion. The latter is a Lévy measure. If it can be formulated as \(\nu(dx) = u(x)dx\), it is a Lévy density. It is similarly to the probability density, with the exception that it need not be integrable and zero at origin. The first focus at Lévy models with jumps goes back to 1930’s. Some of the recent and complete monographs on the theory behind and/or application of Lévy models are e.g. Cont and Tankov (2004) and Bertoin (1998).

Define a stochastic process \(Z(t; \mu, \sigma)\), which is a Wiener process, as long as \(\mu = 1\) and \(\sigma = \sqrt{t}\), its increment within infinitesimal time length \(dt\) can be expressed as:

\[dZ = \epsilon \sqrt{dt}, \quad \epsilon \in \mathcal{N}[0, 1],\]

(15)

where \(\mathcal{N}[0, 1]\) denotes Gaussian distribution with zero mean and unit variance. Then, a subordinated Lévy model can be defined as a Brownian motion\(^4\) driven by another Lévy

\(^3\)This section is rewritten Section 2 of [22].

\(^4\)For our purposes a Brownian motion is a Wiener process without any premise on \(\mu\) and \(\sigma\).
Table 1: Comparison of selected models

<table>
<thead>
<tr>
<th>Model</th>
<th>skewness</th>
<th>kurtosis</th>
<th>intrinsic time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownian motion</td>
<td>0</td>
<td>0</td>
<td>equivalent to $t$</td>
</tr>
<tr>
<td>variance gamma $VG(\ell(t); \theta, \vartheta)$</td>
<td>nonzero if $\theta \neq 0$; its sign is determined by the sign of $\theta$</td>
<td>presumably excess</td>
<td>$\ell(t) \in G[a,b]$</td>
</tr>
<tr>
<td>normal inverse Gaussian $NIG(\ell(t); \theta, \vartheta)$</td>
<td>nonzero if $\theta \neq 0$; its sign is determined by the sign of $\theta$</td>
<td>presumably excess</td>
<td>$\ell(t) \in IG[a,b]$</td>
</tr>
</tbody>
</table>

process $\ell(t)$ with unit mean and positive variance $\kappa$. The only restriction for such a driving process is that it is non-decreasing on a given interval and has bounded variation.

Hence, we replace standard time $t$ in

$$X(t; \mu, \sigma) = \mu dt + \sigma Z(t)$$

by its function $\ell(t)$:

$$X(\ell(t); \theta, \vartheta) = \theta \ell(t) + \vartheta Z(\ell(t)) = \theta \ell(t) + \vartheta \epsilon \sqrt{\ell(t)}.$$  

Due to its simplicity (tempred stable subordinators with known density function in the closed form), the most suitable models seem to be either the variance gamma model – the overall process is driven by gamma process from gamma distribution with shape $a$ and scale $b$ depending solely on variance $\kappa$, $G[a,b]$, or normal inverse Gaussian model – the subordinator is defined by inverse Gaussian model based on inverse Gaussian distribution, $IG[a,b]$.

3.2 Dependency modeling by copula approach

A useful tool of dependency modeling are the copula functions, i.e. the projection of the dependency among particular distribution functions into $[0,1]$, \( C : [0,1]^n \rightarrow [0,1] \) on $\mathbb{R}^n$, $n \in \{2,3,\ldots\}$.  

Actually, any copula function can be regarded as a multidimensional distribution function with marginals in the form of standardized uniform distribution.

For simplicity assume two potentially dependent random variables with marginal distribution functions $F_X, F_Y$ and joint distribution function $F_{X,Y}$. Then, following the Sklar’s theorem:

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)).$$

If both $F_X, F_Y$ are continuous a copula function $C$ is unique. Sklar’s theorem implies also an inverse relation,

$$C(u,v) = F_{X,Y}(F_X^{-1}(u), F_Y^{-1}(v)).$$

5In this paper, we restricted ourselves to ordinary copula functions. Basic reference for the theory of copula functions is Nelsen (2006), while Rank (2007) and Cherubini et al. (2004) target mainly on the application issues in finance. Alternatively, Lévy processes can be coupled on the basis of Lévy measures by Lévy copula functions.
Formulation (19) above should be understood such that the joint distribution function gives us two distinct information: (i) marginal distribution of random variables, (ii) dependency function of distributions. Hence, while the former is given by \( F_X(x) \) and \( F_Y(y) \), a copula function specifies the dependency, nothing less, nothing more. That is, only when we put both information together, we have sufficient knowledge about the pair of random variables \( X, Y \).

### 3.3 Parameter estimation

There exist three main approaches to parameter estimation for copula function based dependency modeling: exact maximum likelihood method (EMLM), inference for margins (IFM), and canonical maximum likelihood (CML). While for the former all parameters are estimated within one step, which might be very time consuming, mainly for high dimensional problems or complicated marginal distributions, the latter two methods are based on estimating the parameters for the marginal distribution and parameters for the copula function separately. While assuming IFM, marginal distributions are estimated in the first step and the copula function in the second one, for CML instead of parametric margins empirical distributions are used. On more details see any of the empirically oriented literature such as Cherubini et al. (2004). In this paper we will assume IFM approach.

### 4. Comparative results

In order to apply the back-tests defined above, we will assume a portfolio equally sensitive to six normalized (zero mean, unite variance) FX rates (EUR, GBP, HUF, PLN, SKK, USD) with respect to CZK. The risk will be estimated by one-day VaR day-by-day over five years (2004–2008) for distinct significance levels for both long and short positions, more particularly, we have \( \alpha \) equal to 0.001 (very high investment grade within economic capital measuring), 0.005 (equivalent to the significance within the solvency capital requirement), 0.01 (capital requirement for market risk within Basel II), 0.05, 0.1, 0.15 (minimal capital requirement within Solvency II), 0.2, and the same for the right tail. The estimated risk is compared with the true loss recorded for a given day. If the risk (loss) is higher than its estimation, we mark this day as an exception.

For all the models, the 0-1 sequence is obtained. First, on the basis of the total number of exceptions, the values of Kupiec test are obtained. Then, related \( p \)-value of \( \chi^2 \) distribution is calculated. Moreover, Christoffersen approach to test the independency of
exceptions is applied and once again, the p-values are obtained. Since in Tichý (2010b) model(4, 4) and model(1, 4) were identified as the most favorable ones, the complete results are provided only for them (see Table 2 and 2).

Table 2: Values of Kupiec/Christofersen test over 2004-2008, model(4,4)

<table>
<thead>
<tr>
<th>Assumption</th>
<th>GBM-G</th>
<th>VG-G</th>
<th>NIG-G</th>
<th>GBM-St</th>
<th>VG-St</th>
<th>NIG-St</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR (0.001)</td>
<td>1.3</td>
<td>0.00/–</td>
<td>0.05/–</td>
<td>0.01/–</td>
<td>0.00/–</td>
<td>0.05/–</td>
</tr>
<tr>
<td>VaR (0.005)</td>
<td>6.3</td>
<td>0.00/0.22</td>
<td>0.09/–</td>
<td>0.16/–</td>
<td>0.00/–</td>
<td>0.31/–</td>
</tr>
<tr>
<td>VaR (0.01)</td>
<td>12.7</td>
<td>0.22/0.52</td>
<td>0.22/–</td>
<td>0.18/–</td>
<td>0.03/0.36</td>
<td>0.24/–</td>
</tr>
<tr>
<td>VaR (0.05)</td>
<td>63.4</td>
<td>0.16/0.08</td>
<td>0.41/0.01</td>
<td>0.27/0.03</td>
<td>0.18/0.03</td>
<td>0.11/0.01</td>
</tr>
<tr>
<td>VaR (0.10)</td>
<td>127</td>
<td>0.02/0.00</td>
<td>0.52/0.00</td>
<td>0.58/0.00</td>
<td>0.27/0.00</td>
<td>0.62/0.00</td>
</tr>
<tr>
<td>VaR (0.15)</td>
<td>190</td>
<td>0.03/0.00</td>
<td>0.60/0.03</td>
<td>0.38/0.02</td>
<td>0.17/0.01</td>
<td>0.70/0.06</td>
</tr>
<tr>
<td>VaR (0.20)</td>
<td>253</td>
<td>0.14/0.14</td>
<td>0.22/0.15</td>
<td>0.14/0.15</td>
<td>0.50/0.47</td>
<td>0.55/0.33</td>
</tr>
<tr>
<td>median</td>
<td>634</td>
<td>0.00/0.96</td>
<td>0.04/0.88</td>
<td>0.02/0.96</td>
<td>0.18/0.78</td>
<td>0.21/0.78</td>
</tr>
<tr>
<td>shortVaR (0.80)</td>
<td>253</td>
<td>0.00/0.05</td>
<td>0.04/0.05</td>
<td>0.02/0.08</td>
<td>0.02/0.08</td>
<td>0.50/0.02</td>
</tr>
<tr>
<td>shortVaR (0.85)</td>
<td>190</td>
<td>0.00/0.03</td>
<td>0.03/0.02</td>
<td>0.02/0.02</td>
<td>0.01/0.03</td>
<td>0.20/0.01</td>
</tr>
<tr>
<td>shortVaR (0.90)</td>
<td>127</td>
<td>0.03/0.00</td>
<td>0.36/0.00</td>
<td>0.31/0.00</td>
<td>0.13/0.00</td>
<td>0.62/0.01</td>
</tr>
<tr>
<td>shortVaR (0.95)</td>
<td>63.4</td>
<td>0.57/0.00</td>
<td>0.86/0.01</td>
<td>0.93/0.00</td>
<td>0.66/0.00</td>
<td>0.83/0.01</td>
</tr>
<tr>
<td>shortVaR (0.99)</td>
<td>127</td>
<td>0.00/0.45</td>
<td>0.03/0.33</td>
<td>0.03/0.33</td>
<td>0.00/0.45</td>
<td>0.16/0.24</td>
</tr>
<tr>
<td>shortVaR (0.999)</td>
<td>6.3</td>
<td>0.00/–</td>
<td>0.05/–</td>
<td>0.02/–</td>
<td>0.00/–</td>
<td>0.09/–</td>
</tr>
</tbody>
</table>

From the results it is apparent, that model(1, 4) slightly overcomes model(4, 4) according to both tests. Next, due to Kupiec test, the Student copula models work clearly better than Gaussian models. However, both Lévy models considered here, overcome simple GBM-St model, even if VG or NIG models are coupled by Gaussian copula function. By contrast, the test of Christofersen focuses only on the equality of the distribution of exceptions in time, but not their number. Thus, it only evaluate, how fast the models are in the reaction on fundamental risk increases. From both tables it is obvious that the models are generally very poor.
5. Conclusions

The presence of jumps and unexpected decreases (increases) in price provide very challenging task on any risk model. A common approach to evaluate the ability of the model to estimate the risk soundly, is known as backtesting. The models can be tested from several point of view. In this paper we accompanied the basic test of Kupiec, which is based only on the number of exceptions, by testing their independency in time. Even if the models suggested eg. in Tichý (2010b) seemed to be promising, the testing of the independency of exceptions shows that some semi-parametrical extension to these models might be useful.

References

Summary

Odhad rizika pro forex: zaklady techniky backtestingu s aplikací

Trh se zahraničními měnami představuje pravděpodobně nejlikvidnější součást finančního trhu. Za účelem určení rizika pozice lze využít několik modelů. Vzhledem k tomu, že reálné výnosy měnových kurzů vykazují vyšší než normální špičatost a jelikož je v rámci riskmanagementu potřebné pracovat i s velmi vzdálenými konci pravděpodobnostního rozdělení, zdá se být optimálním postupem využití simulace Monte Carlo více-rozměrných Lévyho procesu. V tomto článku je studováno, jakým způsobem může Lévyho modely na bázi oridinarních kopula funkci rispét ke kvalitě modelu odhadu rizika K posouzení je aplikován backtesting.