Some possibilities of risk measuring for a small and rather homogenous currency portfolio

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Abstract

For financial institutions, foreign exchange (FX) rates commonly constitute the most important part of the market risk. In order to assess the risk of a position several models can be used. However, since real FX returns exhibit higher than normal kurtosis and since the very far tails of the distribution should also be measured, the Monte Carlo simulation of multidimensional Lévy processes seems to be the most efficient approach. In this paper we show how a simplifying multidimensional Lévy model can provide a substantial improvement over a more standard approach of the (geometric) Brownian motion, when VaR (AVaR) is calculated and the backtesting procedure is regarded to be the most important criterion. The improvement is apparent mainly for a rather small and homogenous portfolio with requirement for a high degree of confidence of risk covering.

Keywords

FX rate portfolio, multidimensional Lévy models, variance gamma model, normal inverse gaussian model, VaR, AVaR, backtesting.

1. Introduction

Foreign exchange rates market belongs to the most efficient and liquid segments of financial markets. Each relevant information is adapted into the market price immediately after its observation, which can lead to excess kurtosis of the probability distribution of returns and presence of jumps. It is given by the fact, that the information comes to the day light with various intensity. Moreover, since the information with either positive or negative price influence can be more frequent, also the skew of the returns can be significant.

Standard approaches to model the evolution of financial quantities (such as stock prices, interest rates, foreign exchange rates) are based on the Gaussian distribution (through the application of a Wiener process or a Brownian motion) with possible combination with the Poisson distribution (a pure jump process). More advanced models belong to the family of Lévy processes, processes with infinite activity of jumps. Up to now, there have been introduced various types of such models, with some of them defined as subordinated (geometric) Brownian motions. Hence, the standard clock time is replaced by a suitable stochastic process, obviously a nonegative one, to model the arrival of a new information.

From the riskmanagement perspective, it is important to measure the risk of overall position, ie how the portfolio value can change during time. Generally, the value is...
sensitive to the evolution of several distinct stochastic factors. As an implication, the dependency among particular sources of randomness must be taken into account, when the portfolio risk is estimated.

The intention of this paper is to make a further contribution to the theory and practice of portfolio modeling by multidimensional Lévy models (VG and NIG). In order to get the dependency structure, a simplified model based on a subordinator process that is unique for all assets is applied – for more details on other alternatives and previous research, see [10].

We proceed as follows. In Section 2, we start with the definition of two risk measures (VaR and AVaR) and the motivations for risk measuring. Furthermore, the procedure to validate the VaR model by backtesting approach is describe. After that, Section 3 is devoted to the Lévy models family. Next, in Section 4 we derive the simplified model for the dependency of subordinated Lévy models. Finally, in Section 5 we describe the data set of six distinct FX rate market quotes taken over last 8 years, each with respect to CZK (Česká koruna) and next, in Section 6 the results are obtained. More particularly, GBM, VG, and NIG models are applied to estimate a one day VaR and cVaR of three distinct portfolios by Monte Carlo simulation approach. The parameters of the models are regularly estimated on the basis of 1000 preceding business days for the last four years of the time series available.

2. VaR, AVaR, and Backtesting

Financial institutions measure the risk they bear with various purposes, since almost each stakeholder is interested either in their longterm performance or riskiness. Probably the most important purposes are the following: the regulators’ requirements on capital adequacy and the internal target levels on probability of surviving.

Regulators and supervisor authorities of financial sectors generally require any financial institution to be able to cover the (unexpected) loss at some probability level \( \alpha \) with its capital (with some simplification, the equity capital and subordinated debts),

\[ UL = \text{VaR}(\alpha) - EL. \] (1)

In other words, the amount of available capital must be adequate (ie capital adequacy) to the risk to which the entity is or can be exposed.

Considering the case of market risk, a zero mean is generally supposed. It is so that the VaR measure for ten-days\(^2\) is used to express the risk level. Note, however, that by supervisors of the financial industry it is generally allowed the entity to measure a one-day VaR and multiply it by the square root of respective time length (the interval of ten days). The model used to measure the risk should be well defined and functional, which is verified by the validation. Within this procedure, several steps can be distinguished (backtesting, stresstesting, etc.).

The intention of backtesting is to assure, that the model would provide good estimates of the risk if applied to the past. It works as follows.\(^3\) For a given data, the parameters of the model are estimated. Next, the risk measure is calculated for the next day. Finally, the true loss is compared to the precalculated risk. If the loss exceeds the risk measure

\(^2\)VaR\(_{p,t}\) is defined as the highest loss, which can be incurred with a given probability \( p \) over a time of length \( t \).

\(^3\)Although the supervisors require to calculate a ten-day loss at a given probability level, for backtesting purposes a one-day loss is used.
calculated previously, such a day must be marked as an exception. If we have applied
the backtesting procedure for \( n \) days and the risk is measured with a probability \( p \), there
might be only \((1 - p) \times 100 \times n\) exceptions. Say that we apply the backtesting for four
years (about 1000 of business days) with \( p = 0.99 \). It gives us that 10 exceptions can
occur. For more statistically based judgement of the model, a Kupiec’ \( LR \) statistic of \( \chi^2 \)
distribution with one degree of freedom can be used:

\[
LR = -2 \ln[(1 - \alpha)^{N - M} \alpha^M] + 2 \ln[(1 - M/N)^{N - M}(M/N)^M],
\]

(2)

where \( \alpha \) is the significance at which we measure the risk, \( N \) is the number of calculations
and \( M \) is the true number of exceptions.

From the point of view of internal purposes, the significance of the loss assumed in
Basel II should be revised – generally, it should be derived from the target risk level. For
example, if there is a target of AA rating, and this class provides the probability of default
\( PD \) within a given time length, the capital should allow the bank to cover any loss which
can occur with probability \( 1 - PD \).

However, \( VaR \) measure provides us with a one number. But it does not say, how much
we can loss if bad times occur. It is the reason, why one can prefer to adopt another risk
measure, denoted either as cVaR (conditional VaR, [17]), shortfall or AVaR (average VaR,
see eg [16]). Although the names are different, the calculation is the same – we take all
the loss above VaR and average them. This is the loss which can happen in bad times in
average.

3. Lévy models

The processes, which belong to the broadly defined Lévy-type models family,\(^4\) can be
characterized by independent and stationary increments. Another typical feature is a so
called stochastic continuity – the probability, that a jump will occur within a particular
time interval \( \tau \) is zero.

3.1 Formal definition

The key step within the definition of advanced Lévy models is to formulate a characteristic
function \( \phi \). Its use allow us to avoid several problems connected with the application
of probability distribution function of a random variable \( X \), \( F_X(x) \). The relationship
between the characteristic function of random variable \( X \), \( \phi_X(u) \), and its distribution
function, \( F_X(x) \), is given by Fourier-Stieltjes transformation:

\[
\phi_X(u) = \mathbb{E}[\exp(iuX)] = \int_{-\infty}^{\infty} \exp(iux) dF_X(x).
\]

(3)

For a characteristic function, it holds in general, that \( \phi(0) = 1 \) and \( |\phi(u)| \leq 1 \) for all
\( u \in \mathbb{R} \). It also holds, that a characteristic function always exists, is continuous and
determines the distribution function of a given probability distribution uniquely.

Suppose a probability distribution, which is infinitely divisible. Then, a Lévy process
is a stochastic process \( X(t) \) with zero origin and independent and stationary increments
defined for all such infinitely divisible distribution. Moreover, increments of such processes
over time interval \( \tau \geq 0 \), i.e. \( X_{t+\tau} - X_t \), has characteristic function \( (\phi(u))^\tau \).

\(^4\)For more on Lévy models see [4] or [1].
Cumulant of the characteristic function $\Phi(u) = \ln \phi(u)$ is denoted as a characteristic exponent and fulfills the Lévy-Khintchin formula:

$$
\Phi(u) = i\gamma u - \frac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{\infty} \left( \exp(iux) - 1 - iux\mathbb{1}_{|x|<1} \right) \nu(dx).
$$

(4)

Here $\gamma \in \mathbb{R}$, $\sigma^2 \geq 0$ and $\nu$ is a measure on $\mathbb{R}\backslash\{0\}$ with

$$
\int_{-\infty}^{\infty} \inf[1, x^2]\nu(dx) = \int_{-\infty}^{\infty} (1 \wedge x^2)\nu(dx) < \infty.
$$

(5)

For a given infinitely divisible distribution, we can define a so called triple of Lévy characteristics,

$$\{\gamma, \sigma^2, \nu(dx)\}.$$

The former two define the drift of the process (deterministic part) and its diffusion. The latter is a Lévy measure. If it can be formulated as $\nu(dx) = u(x)dx$, it is a Lévy density. It is similarly to the probability density, with the exception that it need not be integrable and zero at origin.

### 3.2 Subordinated exponential Lévy models

The admissible prices of financial assets are usually restricted only to positive values, so that exponential Lévy models, i.e. models with a Lévy process $X(t)$ in the exponential should be preferred. It gives us the following formula to describe the dynamic of an asset price $S(t)$:

$$
S(t) = S_0 e^{\mu t + X(t)}.
$$

(6)

Here, in the exponential part of the model the Lévy process $X(t)$ is accompanied by a deterministic drift term, $\mu$.

Many Lévy models commonly applied in Finance are formulated as a (geometric) Brownian motion driven by a particular intrinsic process (subordinator/subordinated process). From an economic point of view, such processes can be understood as a GBM within a (random) business time (it depends on economic activity, arrival of new information, etc.).

Denoting $Z(t; \sigma, \mu)$ as a Wiener process in dependency on time $t$ with parameters $\mu = 1$ and $\sigma = \sqrt{t}$, i.e. $Z_t = \varepsilon \sqrt{t}$, $\varepsilon \in \mathcal{N}(0; 1)$, we can define Brownian motion $X(t; \theta, \vartheta)$ with drift $\theta$ and volatility $\vartheta$ driven by another Lévy process $\ell(t)$ with a unit mean and a variance specified by $\nu$ simply when we replace $t$ by $\ell(t)$. Thus

$$
X_t = \theta \ell(t) + \vartheta \varepsilon \sqrt{\ell(t)},
$$

(7)

which can be rewritten as:

$$
X_t = \theta \ell(t) + \vartheta \varepsilon \sqrt{\ell(t)}.
$$

(8)

This relation can be interpreted in such a way that the increment of $dX$ within an infinitesimal time interval $dt$ is of normal distribution with mean $\theta \ell(dt)$ and variance $\vartheta^2 \ell(dt)$. The mean of the driving process $\ell(t)$ should be $dt$ and its variance will determine the fat tails. Similarly, the mean controls the asymmetry.

Very useful subordinators are a gamma process leading to the Variance gamma model (the variance is not given by standard time but by the so called gamma-time, hence the
Variance gamma model) and an Inverse Gaussian process leading to NIG model (Normal Inverse Gaussian model).  

Concerning the parameter estimation, we can get them e.g. by maximization of the likelihood function (on the basis of a Lévy process density and the set of real data set) or solving the equations for particular moments (on the basis of the characteristic function and the empirically estimated moments of the distribution). In Table 1 we show the latter approach for both, the VG and NIG models.

Table 1: Basic moments of VG and NIG distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>VG(g(t; ν); θ, θ)</th>
<th>NIG(I(t; ν); θ, θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>θ</td>
<td>θ</td>
</tr>
<tr>
<td>Variance</td>
<td>θν(3θ2 + 2νθ2)</td>
<td>3θν(θ2 + νθ2)−1/2</td>
</tr>
<tr>
<td>Skewness</td>
<td>3θν3θν(3θ2 + 2νθ2)</td>
<td>3θν3θν(3θ2 + 2νθ2)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3θν4θνθ2(3θ2 + 2νθ2)</td>
<td>3θν4θνθ2(3θ2 + 2νθ2)</td>
</tr>
</tbody>
</table>

4. Multidimensional Lévy models

The approaches to model the dependency structure of random terms differs due to the probability distribution we consider. Since the standard approach to market risk modeling is still based on the application of a (geometric) Brownian motion, i.e. the higher moments of the underlying distribution are ignored, Cholesky decomposition is sufficient. However, the subordinated Lévy models introduced in the preceding section are defined by means of two distinct distribution. This fact obviously does not allow us to apply the Cholesky decomposition if not equipped by other more advance tools. Below, we provide the theoretical analysis of multidimensional subordinated Lévy models.

Suppose that the evolution of a financial quantity can be described well only by the Lévy model (7). Consider two assets. Since each process consists of two random terms, the subordinator ℓ and the Wiener process ϵ, the covariance formula for two possibly dependent subordinated processes of the Lévy type, X1 and X2, is the following:

\[
\text{cov}[X_1, X_2] = \theta_1 \theta_2 \text{cov}[\ell_1, \ell_2] + \var_1 \var_2 \text{E}[\sqrt{\ell_1 \ell_2}] \text{E}[\epsilon_1 \epsilon_2].
\]

(9)

It is useful to derive also and its correlation counterpart:

\[
\text{cor}[X_1, X_2] = \frac{\text{cov}[X_1, X_2]}{\text{var}[X_1] \text{var}[X_2]} = \frac{\theta_1 \theta_2 \text{cov}[\ell_1, \ell_2] + \var_1 \var_2 \text{E}[\sqrt{\ell_1 \ell_2}] \text{E}[\epsilon_1 \epsilon_2]}{\var_1 \var_2 + \nu_1 \theta_2^2 + \nu_2 \theta_1^2}.
\]

(10)

Looking at either (9) or (10), we can see that the dependency between both processes can arise either through the dependency of subordinators ℓ or through the dependency of

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5For more details on Variance gamma model see e.g. Madan and Seneta [13] (for symmetric case) and Madan and Milne [12] and Madan et al. [14] (for asymmetric case). Similarly, Normal Inverse Gaussian (NIG) model is due to Barndorff-Nielsen [2] and [3] and its generalisation the Hyperbolic model was introduced in Eberlein and Keller [9].
Wiener processes – standard normal variables $\epsilon$ (recall that $\ell$ and $\epsilon$ should be mutually independent).

The dependency modeling of VG processes has been extensively studied during last years, see e.g. [11], [15], or [18]. We now formulate three models that seem to be the most intuitive. For simplicity, we will consider only two-dimensional case.

**Model 1** Suppose that the subordinator is identical for both assets and that the standard normal variables $\epsilon$ are independent. In such a case, the correlation formula (10) can be simplified as follows:

$$
corr[X_1, X_2] = \frac{\theta_1 \theta_2 \nu}{\sqrt{\vartheta_1^2 + \nu \vartheta_1^2} \sqrt{\vartheta_2^2 + \nu \vartheta_2^2}}.
$$  

(11)

At the first sight, it might be surprising that even if we have independent Wiener processes, the correlation is non-zero. However, we should be aware of the fact that the subordinators are identical so that the jumps arise at the same time-moments. Only these events create the dependency. In order to fit the model parameters to the empirically observed data, we should proceed due to the formulas of Table 1 to calculate the second to fourth moment of the distribution (variance, skewness and kurtosis). However, we should proceed in such a way that we arrive at only one parameter of the subordinator variance, $\nu$, comparing to $n$ parameters of drift, $\theta$, and volatility, $\vartheta$.

Moreover, the parametrization should be done so that also the correlation formula will hold. Considering two distinct assets, we have seven equations but only five parameters.

**Model 2** Consider again identical subordinator $\ell$. This time, however, the standard normal variables $\epsilon$ are dependent. This feature gives us the following formulation of the correlation:

$$
corr[\mathcal{VG}_1, \mathcal{VG}_2] = \frac{\theta_1 \theta_2 \nu + \vartheta_1 \vartheta_2 \rho}{\sqrt{\vartheta_1^2 + \nu \vartheta_1^2} \sqrt{\vartheta_2^2 + \nu \vartheta_2^2}}.
$$  

(12)

Here, parameter $\rho$ indicates the dependency of Wiener processes or, in other words, the correlation of standard normal variables. This allows us to apply the Cholesky decomposition in order to model the dependency structure of Wiener processes (due to the properties of the process, $\rho$ should have no effect on either variance, skewness or kurtosis). Comparing with Model 1, we have one parameter more for each pair of processes (correlation of Wiener components $\rho$).

A significant disadvantage of both models, Model 1 and Model 2, which is usually inconsistent with market observations is that the subordinator jumps (and the prices change) at only the same moments in time. Thus, it can not happen that only one asset price changes, while the other stay the same. This problem can be relaxed applying the following approach.

**Model 3** Let us assume the VG process. Due to the well known properties of gamma distribution, we can decompose the gamma variable $X$ as follows:

$$
X = G + I, \quad G \in \mathcal{G}[a/\nu, \nu], \ I \in \mathcal{G}[(1-a)/\nu, \nu] \implies X \in \mathcal{G}[1/\nu, \nu].
$$  

(13)

\[\text{In general, we have } 3n + 1 \text{ equations and } 2n + 1 \text{ parameters.}\]
Hence, the arrival of information is decomposed into information of general nature, $G$, affecting all assets, and idiosyncratic events, typical only for a particular asset, $I$. This gives us the following formulation of the correlation formula:

$$\text{cor}[\mathcal{V}_G^1, \mathcal{V}_G^2] = a \frac{\theta_1 \theta_2 \nu}{\sqrt{\theta_1^2 + \nu \theta_1^2} \sqrt{\theta_2^2 + \nu \theta_2^2}}$$  \hspace{1cm} (14)

**Model 4** This approach connects the overall distribution directly. A very efficient way is to utilize a suitable copula function. We analyze this topic in more details in related papers [19].

### 5. Description of FX rate data

The data set we consider comprises of daily effective FX rates for EUR, GBP, HUF, PLN, SKK, and USD with respect to CZK as published by the Czech National Bank, ie generally the market quotes at 2 p.m. We monitor the market data starting on January 1, 2000. The last quotes were taken on December 31, 2007. It follows that we dispose of a time series of 2014 observations for log-returns of six distinct FX rates. For each FX rate basic descriptive statistics – mean, standard deviation, variance, skewness and kurtosis – of daily log-returns (per annum, if applicable) were evaluated, see Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EUR</th>
<th>GBP</th>
<th>HUF</th>
<th>PLN</th>
<th>SKK</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td>$\mu$</td>
<td>$-0.038$</td>
<td>$-0.058$</td>
<td>$-0.038$</td>
<td>$-0.019$</td>
<td>$-0.009$</td>
</tr>
<tr>
<td><strong>variance</strong></td>
<td>$\sigma^2$</td>
<td>0.0028</td>
<td>0.0067</td>
<td>0.0062</td>
<td>0.010</td>
<td>0.0034</td>
</tr>
<tr>
<td><strong>standard deviation</strong></td>
<td>$\sigma$</td>
<td>0.053</td>
<td>0.082</td>
<td>0.079</td>
<td>0.100</td>
<td>0.058</td>
</tr>
<tr>
<td><strong>skewness</strong></td>
<td>$\kappa_3$</td>
<td>$-0.297$</td>
<td>$-0.411$</td>
<td>$-0.790$</td>
<td>$-0.533$</td>
<td>0.0646</td>
</tr>
<tr>
<td><strong>kurtosis</strong></td>
<td>$\kappa_4$</td>
<td>7.432</td>
<td>5.320</td>
<td>9.748</td>
<td>12.209</td>
<td>7.618</td>
</tr>
</tbody>
</table>

We can see that the mean returns p.a. (the drift over the whole length) varies substantially between $-1\%$ (SKK) and $-9\%$ (USD). The standard deviation of two FX rates is around $5\%$ (SKK, EUR), another two are close to $8\%$ (GBP, HUF) and the last two are slightly above $10\%$ (PLN, USD). Except the SKK rate, the skewness is significantly negative, the highest is for HUF ($-0.8$). By contrast, the highest kurtosis can be observed for the PLN rate (12), while the USD is not very far from the Gaussian. When testing if the distribution can be regarded to be the Gaussian, several tests of Jarque-Bera type can be used. Here, the hypothesis of normality must be strongly rejected for all FX rates, including the USD rate data.

In order to simplify further calculations, the portfolio construction and to concentrate our attention on the non-linearity and non-normality, we can normalize the vectors of returns to get standardized time series with zero mean and unit variance. Clearly, there will be no apparent effect for both, the skewness and kurtosis and the correlation matrix, which we provide bellow (actually, it will be identical with the covariance matrix).
Inspecting the matrix of linear correlation coefficients, we can see that correlations for all pairs of FX rates are between 0.25 and 0.66. Thus, the dependence seems to be positive, but not perfect. The highest values are obtained for the pairs (GBP, USD) and (EUR, SKK). The dependence for the pair (EUR, GBP) is also relatively high. By contrast, the other Central European FX rates (HUF and PLN) exhibit lower correlations.

6. Portfolio modeling

Consider three distinct portfolios with the composition as follows: \( \Pi_A = (0.5, 0, 0, 0, 0.5, 0) \), \( \Pi_B = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6) \), and \( \Pi_C(0, 0, 0, 0, 0.5, 0.5) \). It gives us that \( \Pi_A \) consists of two highly correlated FX rates, which are most probably sensitive to the arrival of the same information – the subordinator might be well unique. By contrast, \( \Pi_C \) includes two rather heterogenous FX rates. Finally, \( \Pi_B \) is set of all available rates with equal weights.

The modeling by Lévy processes allows us to fit also the higher moments of the distribution. However, it simultaneously make the estimation of the parameters more data demanding (kurtosis is commonly less stable than variance). We therefore use 4 preceding years to fit the model and estimate the risk (by MC simulation, assuming 500 000 trials) for the next day considering VG and NIG model, both with unique subordinator, and GBM. Each model is considered for three distinct significance levels, \( \alpha = \{0.01, 0.001, 0.0003\} \). Hence, the first level corresponds to the Basel II requirement, while the others are related to the Economic capital calculation for banks with good or very good ratings. Next, the estimated VaR is compared to the true loss incurred on a given day (Figure 1). If the VaR is exceeded, we get an exception day. This procedure is repeated over 4 years. The backtesting results are depicted in Table 3.

![Figure 1: True loss over the last four years for \( \Pi_A \) and \( \Pi_C \)](image-url)
Since the last confidence level is very high, the data we have should provide us one exception with probability $1/3$. Although in reality it happens once for all models, the results seem to be highly confident when tested e.g. by Kupiec’s statistic LR (2). Another interesting results are apparent from Figure 2. The left part depicts the true loss on the portfolio, on the right we can observe four curves of VaR evolution in time for $\alpha = 0.01$ (lower two) and $\alpha = 0.0003$. Since the results for VG and NIG are almost interchangeable, we produce only one of them (VG, gray curves) accompanied by the GBM (black curves).

![Figure 2: VaR and AVaR estimation, $\Pi_A$](image)

It is interesting to see that although the data are normalized (variance is still one and the average return is zero), the VaR and AVaR slightly decline for each model and probability. The reason might be the dependency among particular assets – since the linear correlation falls slightly, there was a bigger effect of diversification. Next, we observe that the VG and NIG models provide us with not very stable results. This might be give by (a) the changes in the skew and kurtosis for various intervals; and (b) the complexity of the model, ie many trials are needed to get a smooth results.

Next, we proceed to portfolio sensitive to rather heterogenous rates, SKK and USD. Although the unique subordinator model, in theory, should catch the dependency, the backtesting (Table 4) seems to be very good, much better than for GBM model, although the risk is slightly overestimated by both, VG and NIG for a lower significance level.

By inspection of Figure 3 it is apparent that the lower correlation implies higher diversification so that the VaR and AVaR are lower, too. Moreover, the distances between VaR and AVaR are not so high as for $\Pi_A$. Finally, the subordinated Lévy models give us sometimes even lower risk levels than GBM.

We have also analyzed portfolio of all assets. The results are not reported here, since...
there is no significant difference as compared to preceding tasks.

7. Conclusions

The presence of jumps in financial asset returns implies a need for power tools in order to calculate the risk measures and related capital requirements in an appropriate way. A very popular tool for modeling of higher moments of probability distribution can be subordinated Lévy models. However, there arise a problem if multidimensional issues should be solved.

In this paper we have tried to apply the backtesting procedure to the risk estimated by a simple multidimensional model (VG or NIG) with a unique-subordinator for three distinct portfolios. Although in related research on this topic we showed that such a model cannot fit all descriptive statistics of the set of variables in the right way, especially when heterogeneous assets are considered, here we show that for a given set of data (at least) the risk measures (VaR, AVaR) are well estimated.

To summarize the results we obtained under given assumptions, for the purpose of capital requirement to market risk modeling in the line with Basel II, ie with 99% confidence, GBM is a sufficient model. However, to measure the risk for the purposes of economic capital calculation, ie the risk of far tails, multidimensional Lévy models seem to be very useful, even if the uniqueness of the subordinator is supposed.
References


**Summary**

*Možnosti měření rizika malého a spíše homogenního měnového portfolia*