Fuzzy Forecasts of Financial Series

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Abstract

Risk investment management and risk assessment are interrelated concepts. The risk may vary in its forms where measurable factors act alongside immeasurable factors whose role can by no means be underappreciated. The risk management models can significantly grow in their effectiveness provided that both measurable and immeasurable factors are taken into account. The models that can easily adapt formalized ways of all-type risk factor application are linguistic models. This paper concentrates on the linguistic models and their application in relation to the investment process.

Key words
fuzzy sets, fuzzy numbers, linguistic variables, fuzzy knowledge base, risk investment management

1 Introduction

The capital investment or capital transformation designed for the purpose of achieving a certain goal constitutes an important aspect of the capital market. Assuming the fact that capital market players act rationally and follow all well-established market rules, it would seem safe to say that the asset allocation has been determined and monetary flows fully defined. However, such an assumption proves wrong since each action of the capital market makes its reaction the scope of which may at times be difficult to predict or outright unpredictable. Therefore, it seems quite reasonable to say that indefiniteness is an immanent feature of the capital investment. The fact that a pre-defined investment goal will always be achieved can by no means be taken for granted.

In the subject literature (Tarczynski (2003)) two concepts have been outlined and they are measurable uncertainty, i.e. risk, and immeasurable uncertainty, i.e. uncertainty per se. We can talk about measurable uncertainty if and only if the following conditions are observable:

- investment result to be achieved in the future is ambiguous and future states of environment can be identified;
- probability distribution in relation to the future environmental states has been identified.

The afore-mentioned measurable uncertainty conditions generate knowledge of probability characteristics related to investment process, which altogether is a crucial determiner of the investment risk management.

The knowledge of investment process essentials is a result of the objective knowledge on the states of environment and market mechanisms. This particular knowledge is usually based on empirical research (historical data) as well as pre-defined conceptions related to market mechanisms (e.g. financial instrument prices are modeled by the Brown theory). The full awareness of the probability characteristics acting as an indispensable factor of the investment process and the assumption that it is there is not always true. This argument can be supported by the evidence of numerous losses of various investors (Barings Bank— GBP 800 million loss in 1995, Nat West— GBP 85 million loss in 1997). It is widely known that the objective

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knowledge on the investment process exists in correlation with the subjective knowledge (expert knowledge) which, as long as it is used correctly, can help specify the investment process. The theory of probability works as formal means of transforming the objective knowledge whereas the fuzzy set theory makes it possible to formalize the expert knowledge (fuzzy knowledge base). Further on in this paper, some useful methodology of transforming fuzzy knowledge base for the purpose of the investment process forecast will be discussed.

2 Fuzzy knowledge base

The fuzzy set theory makes it possible to formalize expert knowledge. The theory key concepts are: fuzzy sets, fuzzy numbers, linguistic variables. These particular concepts are defined within a certain universe of discourse \( X \) by a membership function.

The fuzzy set \( A \) we define as follows (Zadeh (1968)):

\[
A = \{ (x, \mu_A(x)) : x \in X, \mu_A(x) \in \{0, 1\} \} \quad (2.1)
\]

where: \( \mu_A : X \rightarrow \{0, 1\} \) is a membership function of the membership of an element of the universe of discourse \( X \) to set \( A \).

Let us assume that \( A \) is a set of high stock returns and the space \( X \) denotes all possible shares in the capital market. If one share has a noticeably high rate of return then its degree of membership to set \( A \) is 1. Consequently, if no growing tendency is observed in terms of the stock rate of return, the membership grade of such rate equals 0. The membership grade of an interim share to set \( A \) is a number in the range of \((0,1)\) which gets automatically closer to 1 once its characteristics get closer to the above mentioned set. In other words, the higher the rate of return, the closer the share gets to 1.

The set \( A \) is a fuzzy set fully characterized by the membership function.\(^2\)

The accurate attribution of the membership grade to a set by the element of the universe of discourse discussed above is quite difficult. This operation is mostly subjective and contextualized.

It is important to keep in mind that the results of our discussions here as based on the theory of fuzzy sets are determined by the adequate defining of the membership function. In practice, the membership function is defined by means of either statistical survey method or an expert method. The latter is a method where an expert marks out general parameters of the membership function and the parameters of the function of a certain category are subsequently outlined by test and trial. More specifics on the nature of the membership function as well as algebra of fuzzy sets may be found in A. Lachwa’s work (2001).

Fuzziness and probability which are phenomena of different nature and form may occur next to each other. According to Zadeh, a fuzzy random event is a fuzzy set defined within the domain of elementary events, measurable in Borel’s terms.

The probability of the fuzzy random event \( A \) can be depicted in the form of the following equation:

\[
P(A) = \sum_\omega p(\omega) \mu_A(\omega) \quad (2.2)
\]

where: \( p(\omega) \) is the probability of the elementary event \( \omega \).

Fuzzy numbers, which are another important concept, can be defined as follows:

\(^2\) The characteristic function which takes only two values 0 or 1 is a special type of a membership function.
A fuzzy number is a normalized, convex fuzzy set outlined within the domain of real numbers $\mathbb{R}$ whose membership function is segmentally continuous. Specifically, the LR fuzzy number is the fuzzy set $A$ defined within the domain of real numbers expressed as the following membership function:

$$
\mu_A(x) = \begin{cases} 
L \left( \frac{m-x}{\alpha} \right) & \text{for } x < m \\
1 & \text{for } x = m \\
R \left( \frac{x-m}{\beta} \right) & \text{for } x > m
\end{cases}
$$

(2.3)

where:

- $L(.)$ – increasing function
- $R(.)$ – decreasing function
- $\alpha, \beta$ – positive parameters

If $X$ is a rate of return of a share, $m$ – desirable value of the rate of return and $\alpha=\beta$ is equal to the standard deviation of the rate of return then the fuzzy number (2.3) represents the variability of the stock returns.

Linguistic variables are marked out by fuzzy sets, e.g. low, medium, high. We say that the stock rate of return is low, medium, high, which means that it is compatible with a certain range of real numbers where these numbers reflect the variability of the stock rate of return. The membership of an element to the fuzzy set (membership function) that specifies linguistic variables reflects the range of possibilities of this particular function. The above mentioned concepts can serve as tools to build fuzzy knowledge base for the investment process.

Let us assume that the investment process discussed thus far $X_t \in X \subset \mathbb{R}$, $t=1,2,\ldots$ is the Markov process, i.e. the process which observes the following rule:

$$
P(\frac{X_t}{X_{t-1}, X_{t-2}, \ldots X_{t-k}}) = P(\frac{X_t}{X_{t-1}})
$$

(2.4)

Let us assume that:

$$
X = \bigcup_{i=1}^{k} a_i
$$

(2.5)

where $a_i$ are disjoint subsets of the universe of discourse $X$. Furthermore, let us suppose that the probability distribution of the investment process is known:

$$
p_{ij}(x_{t-1}, x_t) = P(x_{t-1} \in a_i, x_t \in a_j)
$$

(2.6)

$i,j = 1,\ldots,k$

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3 For the definition of a normalized, convex fuzzy set see A. Lachwa (2001).

4 The distribution of the investment process is outlined empirically on the basis of implementing the process $X_t$. 

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The (2.6) is used in order to determine boundary distribution of the process
\[ p_i (x_{t-1}) = \sum_j p_{ij} (x_{t-1}, x_t) \]  
(2.7)
as well as its conditional distributions
\[ p_{ij} (x_t', x_{t-1}) = \frac{p_{ij} (x_{t-1}, x_t)}{p_i (x_{t-1})} \]  
(2.8)

The investment process will be defined here by linguistic categories as described below:
within the universe of discourse \( X \) we outline \( l \) fuzzy states \( A_1, A_2, \ldots, A_l \) which represent the degree of the investment process (linguistic variables);
we specify membership functions \( \mu_{Aj} (x_t \in a_i) = \mu_{Aj} (a_i) \) that meet the condition of \( \sum_j \mu_{Aj} (a_i) = 1 \)

The probability of the fuzzy state \( A_j \) taking place as in (2.2) we can mark out with the aid of the following formula:
\[ P (x_{t-1} \text{ is } A_j) = \sum_i p_i (x_{t-1}) \mu_{Aj} (a_i) = P_{Aj} (x_{t-1}) \]  
(2.9)

The probability of \( A_i, A_j \) occurring jointly in relation to the investment process in \( t-1 \) and \( t \) we can define mathematically:
\[ P (x_{t-1} \text{ is } A_i, x_t \text{ is } A_j) = \sum_r \sum_s p_{rs} (x_{t-1}, x_t) \mu_{Al} (a_r) \mu_{Aj} (a_s) \]  
(2.10)

The formulas (2.9) and (2.10) make up the fuzzy knowledge base of the investment process.

3 Linguistic model of the investment process

Linguistic models are used for fuzzy forecasts of financial series. The following holds for the linguistic model:
\[ R^{(i)} : \{ \text{IF (antecedent) THEN (consequent)} \} \quad i = 1, 2, \ldots, p \]

where the antecedent describes a set of conditions whereas the consequent makes a conclusion (Helendoorn, Driankov (1997)).

To specify the linguistic model it is necessary to determine input variables (antecedent) as well as output variables (consequent), both of which are usually linguistic variables. Importantly enough, at this stage it is essential to determine fuzzy sets of these particular linguistic variables as well as, even more importantly, to outline their membership function.

The MIMO (multiple input—multiple output) model consists of the fuzzy rules of the following type:
\[ R^{(i)} : \{ w_i \text{ IF } (x_1 \text{ is } A_1^{i} \ldots i x_n \text{ is } A_n^{i}) \text{ THEN } (y_1 \text{ is } B_1^{i} \ldots i y_m \text{ is } B_m^{i}) \} \quad i = 1, \ldots, p \]  
(3.1)
where: \( w_i \) – weight of the rule
\[
x = (x_1 ... x_n) \rightarrow \text{input variable, } x \in X \subseteq \mathbb{R}^n
\]
\[A^i_1 ... A^i_n\] – linguistic values of the input variable
\[
y = (y_1 ... y_m) \rightarrow \text{output variable, } y \in Y \subseteq \mathbb{R}^m
\]
\[B^i_1 ... B^i_m\] – linguistic values of the output variable

If inputs and outputs are independent variables then the MIMO model can be transformed into the set of the SISO models (single input—single output). The fuzzy rules in the SISO model are as follows:

\[
R^{i} \{ w_i \text{ IF } (x \text{ is } A^i_i) \text{ THEN } (y \text{ is } M^i_i)\} \tag{3.2}
\]

where: \( i \) – number of the fuzzy rule associated with the linguistic value \( A^i_i \) of the variable \( x \)

\( M^i_i \) – structure of the consequent containing linguistic variable and weight

The structure \( M^i_i \) can take the following form:

\[
y \text{ is } B^i_1 \text{ with weight } w_{i1}
\]
\[
\text{also } y \text{ is } B^i_2 \text{ with weight } w_{i2}
\]
\[
\text{also } y \text{ is } B^i_m \text{ with weight } w_{im}
\]

The weights \( w_{ij} \) specify the linguistic model significantly; they are denoted either statistically or by experts.

The model (3.2)\(^5\) can be used for fuzzy forecasts of the investment process. In this case, we replace the weight \( w_i \) with the probability \( P_{A^i_i} (x_{t-1}) \) and define the weights \( w_{ij} \) by the use of the conditional probabilities \( P_{A^i_i/A^j_j} (x_t/x_{t-1}) \). The input variable is the value of the investment process in \( t-1 \) whereas the output variable is the value of the process in \( t \). The fuzzy sets \( A^i_i \) and \( B^i_i \) are identical to the fuzzy states of the process as it was discussed in the second part of this paper.

4 Conclusions

The capital market generates a significant number of measurable as well as immeasurable signals on a daily basis. When transformed rationally, these particular signals contribute to successful outcomes of the capital investment. The investment process should therefore be supported by certain formalized methods which would combine the objective knowledge as well as the expert knowledge. The methodology outlined in this paper may be adapted as one of the possible tools to achieve this particular goal.

REFERENCES


\(^5\) This particular model has been used for the forecasts of Euro rates (Walaszek Babiszewska (2005)).
Summary

This paper concentrates on the methodology of constructing fuzzy knowledge base of the investment process with the support of the fuzzy set theory. Fuzzy sets, fuzzy numbers and linguistic variables have been used here as indispensable tools to build the linguistic models which help formalize both objective and expert knowledge. These particular models can be applied for forecasting purposes such as fuzzy investment process prediction.