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CONTENTS

Impact of COVID-19 Crises on Accounting Entities Providing Accommodation Services in Slovakia

Miriama Blahušiaková [1]

Abstract

Tourism is one of the most affected sectors by crisis. Due to mass trip cancellations, suspension of national and international flights or postponing of events, COVID-19 crisis has significantly changed the tourism industry in global aspect. The paper focuses on analysis of the impact of COVID-19 on tourism sector in the Slovak Republic, with focus on hotel industry, where there has been a rapid decrease in sales and profit or loss in 2020 in comparison with 2019 reported.

Key words

COVID-19, sales, profit or loss, financial situation, accommodation facilities

JEL Classification: M48, J38, M54

1. Introduction

In 2020 the whole world was attacked by COVID-19 pandemic that first appeared in 2019 in Wuhan (Abdulamir & Hafidh, 2020; Ait Addi et al., 2020; Aljofan & Gaipov, 2020). As a result of the rapidly spreading outbreak that grew into a global pandemic, and in an effort to protect health, several countries all over the world, including the Slovak Republic, have adopted various restrictive measures. In the Slovak Republic, services, schools, boarders were closed, movement of persons was limited, and the economic and social life was attenuated. Work has switched to the online environment. If the type of the work allowed it, employers recommended employees to stay and work from their homes (home office). There also has been an extensive reduction in business. No one expected that the whole world would be in crisis as a result of the pandemic. In June 2020, the Ministry of Finance of the Slovak Republic published a forecast (MFSR, 2020) with the prediction that the Slovak economy would fall by 9.8% in 2020 due to the global pandemic, despite the fact that in February 2020 the economy was forecast to grow by 2.2%. Due to a stronger third quarter, the economy has closed with a decline of 5.2%, which is a better result than the original forecasts (SOSR, 2021).

In an effort to eliminate the negative effects of the crisis, countries have adopted various measures to support the declining economy. Several countries have implemented the so-called "Kurzarbeit" to compensate accounting entities for losses caused by the crisis.

In the Slovak Republic the Act No. 67/2020 Coll. on Certain Emergency Financial Measures in Relation to the Spread of the Dangerous Contagious Human Disease COVID-19 was adopted on April 2, 2020 (hereafter referred to as "lex korona"). This act has already been amended several times. It has regulated emergency measures in the field of taxation, customs and accounting, in particular tax administration, motor vehicle tax, administrative fees, accounting income tax, the area of the financial market and the area of budgetary rules. As a result of the lex korona Act, deadlines for submitting financial statements, for filling tax returns, for payment of taxes, etc. have been postponed.

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Since the end of April 2020, due to the improving epidemiological situation in the Slovak Republic, gradual release of measures began, which was divided into four phases. Despite the fact that after the summer season it was evident that the economic and social life returned back to normal, in autumn the epidemic situation began dramatically getting worse and the outbreak turned into the second wave of pandemic, which again led to dramatic restrictive measures in Slovakia that were much stricter than in the first wave of the outbreak.

The pandemic has caused a substantial drop in the demand for travelling that affected mainly tourism sector. Due to mass trip cancellations, suspension of national and international flights or postponing of events, the COVID-19 crises has significantly changed the tourism industry (Cohen, 2020; Fraccascia & Alvarado, 2020; Gallego et al., 2020). In order to cope with the unprecedented crisis (Sharma et al., 2020), hotels have had to devise a number of impromptu innovations to safeguard health and safety of all parties involved, and in the process restore consumer process in the lodging industry.

As previous world crises proved, the crises (Ivanov & Stavrinoudis, 2018) significantly affected the hotel industry having a negative impact on tourism destinations. A decreased demand for tourism results in lower occupancy and revenues in the hotel industry and, in consequence, the reduction of employment and deterioration of living conditions of local communities (Pawlicz, 2012). In this case (Napierała, Leśniewska-Napierała, & Burski, 2020) hoteliers are forced to work together with local authorities and destination management organizations.

According to Kullová (2020) the restrictive measures related to the COVID-19 pandemic have significantly affected the so-called HORECA segment that includes hotels, restaurants and cafes in Slovakia, too. The legislative measures caused a rapid drop in sales from March 2020 to June 2020 by a quarter in this segment in Slovakia. According to data from Bisnode Company, the first wave of pandemic has swept away about a thousand restaurants, fast food and pubs. The most vulnerable were smaller entities. Entities that managed the first wave successfully, have recorded an average decrease in sales of 40 to 50% (Kekely, 2020).

At the beginning of the second wave of the pandemic (after the end of the summer season), the first sector to experience a catastrophic drop in sales were travel agencies, followed by hotels and restaurants (Ižip, 2020). Up to 60 % of companies in the gastronomy sector reported a decrease in sales in comparison with 2019, while only 32% of companies reported attendance comparable to summer 2019. The estimated decrease in sales in accommodation facilities was up to 40% compared to the summer season 2019. In the hotel and catering sector, more than 20,000 employees were dismissed in Slovakia in 2020. According to data from the Ministry of Transport and Construction of the Slovak Republic (hereafter referred to as "MTCSR") which covers tourism, the number of foreign visitors fell by more than half in 2020, and the number of nights the foreigners spent in our hotels was lower than in 2019. The summer season mitigated the decline in visits, when the number of domestic visitors increased significantly, mainly due to the fear of travelling abroad. However, they could not completely make up for the missing foreign clientele. The worst situation was in city hotels, where the average occupancy did not reach even 20% of their capacity and the decrease in overnight stays declined by up to 70% compared to 2019 (Kekely, 2020).

Wang & Ritchie (2010) emphasize that hotel industry particularly demands effective crises management, as it is one of the most vulnerable sectors hit by the crises. Crises caused by SARS epidemic, bird flu, terrorists attacks and global financial crises of the years 2008-2009 not only led to a decline in hotel revenues but also to a higher risk of investments in the hotel industry (Chen, 2011). In this case, state support is crucial due to high probability of a massive closure in the hotel industry (Chien & Law, 2003). The state authorities (Kubcikova, Kirimhan, & Li, 2019) should understand the role of both tourism and the hotel industry in the destinations and reconsider state tourism policies towards sustainability to respond to future crises more efficiently. The government and state involvement is also required for recovery and development of the sector in the future. Pondering the way forward (Rastegar, Higgins-Desbiolles, & Ruhanen, 2020), using the opportunity for a major transformation requires restorative action to design more responsible, ethical and sustainable forms of tourism.

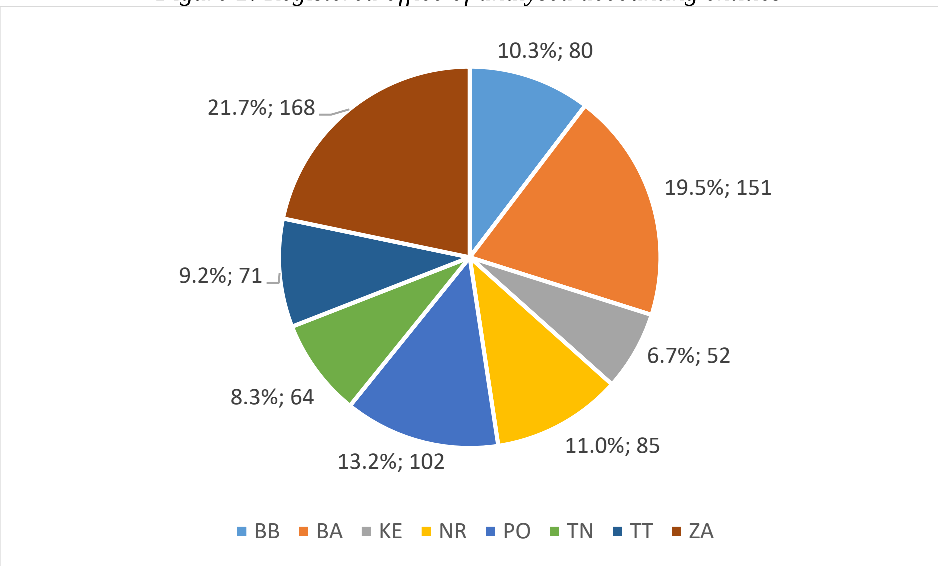
2. Aim and Methodology

Our paper focuses on the analysis of the impact of the crisis connected with COVID-19 pandemic on the tourism sector, with emphasis on hotel accommodation, tourist accommodation, and any similar accommodation. An accommodation facility is according to Antalová (2017) a hotel, a botel, a boarding house, an apartment house, a tourist hostel, a cottage settlement, a campsite, or private accommodation. The aim of the paper is to analyse the impact of the negative consequences of the crisis related to the COVID-19 pandemic on the financial position of the accounting entities providing hotel and accommodation services in 2020 in comparison with 2019.

The financial statements of accounting entities operating in the tourism sector with focus on hotel accommodation, tourist accommodation, and any similar accommodation for the accounting period 2020 were selected from the website www.finstat.sk. Out of the total number of 1,077 entities, 773 accounting entities providing accommodation services have been selected and subjected to a more detailed analysis. We have excluded those accounting entities that were in liquidation or entered into liquidation in 2020, and accounting entities that had not presented any sales in their financial statements neither in 2019, nor in 2020.

Out of the analysed 773 accounting entities, 80 entities had their headquarter in the Banská Bystrica (BB) Region, 151 entities in the Bratislava (BA) Region, 52 entities in the Košice (KE) Region, 85 entities in the Nitra (NR) Region, 102 entities in the Prešov (PO) Region, 64 entities in the Trenčín (TN) Region, 71 entities in the Trnava (TT) Region, and 168 entities in the Žilina (ZA) Region (Figure 1).

Figure 1: Registered office of analysed accounting entities



Source: Own research based on the analysis of accounting entities

The analysed accounting entities were divided into three categories according to the number of employees. There were 488 entities in the category of 0-9 employees, 91 entities in the category of 10 – 49 employees, and 12 entities in the category of more than 50 employees. Up to 182 entities have not stated the number of employees in their financial statements.

Software SAS Enterprise Guide and SAS programming language have been used for our analysis, through which multiple comparisons, interval estimates, and predictions of probability have been applied. All results were interpreted qualitatively.

3. Analysis of the Impact of Restrictive Measures on Accounting Entities´ Financial Position

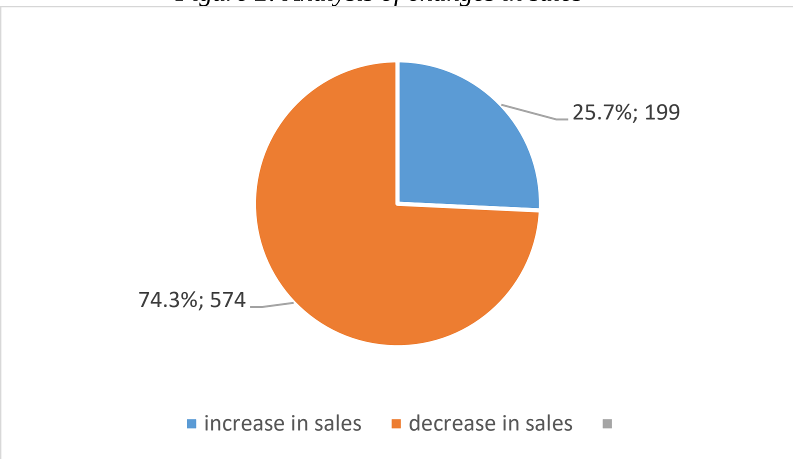
We have analysed changes in sales and profit or loss in selected accounting entities providing accommodation services in 2020 in comparison with 2019.

3.1 Analysis of the Impact of Restrictive Measures on Sales

The analysis of the presented financial statements revealed that within the analysed category of accounting entities, there was an overall decrease in sales of € 133,954,676, which represents a year-to-year decrease of 35.4% compared to the previous accounting period.

Up to 574 entities reported a decrease in sales, while 199 entities reported an increase in sales for accounting period 2020 in comparison with accounting period 2019 (Figure 2).

Figure 2: Analysis of changes in sales



Source: Own research based on analysis of financial statements

Accounting entities with a decrease in sales in 2020 in comparison with 2019 reported sales by € 140,436,150 lower in 2020 compared to 2019. The drop represents an average decrease in sales of € 244,662 per accounting entity.

Out of the accounting entities that had reported a decrease in sales, 56 entities (9.8%) had their headquarters in the BB Region, 114 (19.9%) in the BA Region, 40 (7.0%) in the KE Region, 68 (11.8%) in the NR Region, 74 (12.9%) in the PO Region, 53 (9.2%) in the TN Region, 58 (10.1%) in the TT Region, and 111 (19.3%) in the ZA Region (Table 1 column percentages "Decrease"). The analysis proved that most accounting entities with a decrease in sales, had their headquarters in the BA Region. There was a drop in sales in almost 83% of entities providing hotel and accommodation services in the TN Region, in 81.7% of entities in the TT Region, and in 80% of entities providing hotel and accommodation services in the NR Region (Table 1 row percentages "Decrease"). Out of the total number of 574 accounting entities that reported a decrease in sales, up to 194 entities (33.8%) reported a decline of more than 50 percent in sales.

As stated before, despite the bad pandemic situation, 199 entities had reported an increase in sales in 2020 in comparison with 2019. The total sum of increased sales was € 6,481,474 that represents an average increase in sales € 32,570 per accounting entity. Out of the accounting entities that had reported an increase in sales, 24 (12.1%) entities had their headquarters in the BB Region, 37 (18.6%) in the BA Region, 12 (6.1%) in the KE Region, 17 (8.5%) in the NR Region, 28 (14.1%) in the PO Region, 11 (5.5%) in the TN Region, 13 (6.5%) in the TT Region, and 57 (28.6%) in the ZA Region (Table 1 column percentages "Increase").

Table 1: Analysis of changes in sales based on the place of business

*cell percentages **row percentages ***column percentages	Changes in sales		
Place of business - region	**Decrease**	**Increase**	**Total**
BB	**56** (*7.2%) (**70%) (***9.8%)	**24** (*3.2%) (**30.0%) (***12.1%)	**80 (*10.4%)**
BA	**114** (*14.7%) (**75.5%) (***19.9%)	**37** (*4.8%) (**24.5%) (***18.6%)	**151 (*19.5%)**
KE	**40** (*5.2%) (**76.9%) (***7.0%)	**12** (*1.5%) (**23.1%) (***6.1%)	**52 (*6.7%)**
NR	**68** (*8.8%) (**80.0%) (***11.8%)	**17** (*2.2%) (**20.0%) (***8.5%)	**85 (*11.0%)**
PO	**74** (*9.6%) (**72.5%) (***12.9%)	**28** (*3.6%) (**27.5%) (***14.1%)	**102 (*13.2%)**
TN	**53** (*6.9%) (**82.8%) (***9.2%)	**11** (*1.4%) (**17.2%) (***5.5%)	**64 (*8.3%)**
TT	**58** (*7.5%) (**81.7%) (***10.1%)	**13** (*1.7%) (**18.3%) (***6.5%)	**71 (*9.2%)**
ZA	**111** (*14.4%) (**66.1%) (***19.3%)	**57** (*7.3%) (**33.9%) (***28.6%)	**168 (21.7%)**
Total	**574 (*74.3%)**	**199 (*25.7%)**	**773**

Source: Own processing in SAS Enterprise Guide based on information from financial statements

We have found positive correlation between changes in sales and place of business (Table 2) where there is a statistically significant correlation at significance level 0.1 (p-value = 0.070).

Table 2: Assessment of the association between changes in sales and place of business

Statistic	DF	Value	Prob
Chi-Square	7	13.0864	0.0700
Likelihood Ratio Chi-Square	7	13.1817	0.0678
Phi Coefficient		0.1301	
Contingency Coefficient		0.1290	
Cramer's V		0.1301	

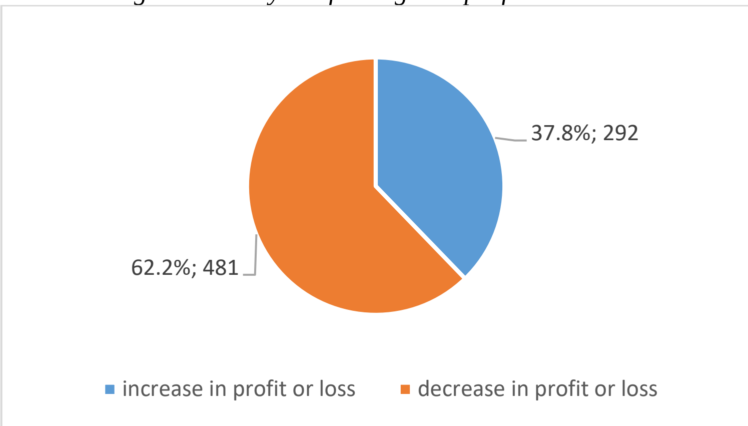
Source: Own processing in SAS Enterprise Guide based on own research

Performing a more detailed analysis of the accounting entities that had reported a decrease in sales, we have investigated that up to 360 entities belong to smaller accounting entities that employ less than 10 employees (that represents 73.8% of this entities' size category); 85 entities (93.4%) employ 10 to 49 employees, and 12 entities (100%) employ more than 50 employees. The remaining entities with a drop in sales in 2020 compared to 2019 had not stated the number of employees in their financial statements.

3.2 Analysis of the Impact of Restrictive Measures on Profit or Loss

The selected accounting entities were further subjected to the analysis of changes in profit or loss. The analysis revealed that within the analysed category of accounting entities, there was an overall decrease in profit or loss of € 54,991,395 that represents a year-to-year decrease of 416.1% in comparison with the previous accounting period. Out of 773 of accounting entities (Figure 3), 481 entities (representing 62.2% of the analysed entities) reported a decrease in profit or loss, and 292 entities (representing 37.8% of the analysed entities) reported an increase in profit or loss.

Figure 3: Analysis of changes in profit or loss



Source: Own research based on analysis of financial statements

The profit or loss in the 481 accounting entities that reported a decrease in profit or loss in 2020 in comparison with 2019, decreased in total amount of € 59,547,809. The profit or loss in the 292 accounting entities that reported an increase in profit or loss in 2020 in comparison with 2019 increased by a total amount of € 4,556,414. In this context, it is important to pay attention to the fact that the total profit or loss in 2020, in those accounting entities that reported an increase in the profit or loss in 2020 compared to 2019, was a loss of € 402,653.

Out of the 574 accounting entities with a decrease in sales, 422 (73.5%) entities also reported a decline in profit or loss, whilst 152 (26.5%) entities reported an increase in profit or loss in the accounting period despite the fall in sales in 2020 compared to the accounting period 2019 (Table 3 column percentages "Decrease in sales").

Out of the 199 accounting entities with a rise in sales in 2020 compared to 2019, 140 (70.4%) entities also reported an increase in profit or loss, whilst 59 (29.6%) entities reported a drop in profit or loss (Table 3 column percentages "Increase in sales").

Table 3: Analysis of changes in profit or loss based on changes in sales

*cell percentages **row percentages ***column percentages	**Changes in sales**		
Changes in profit or loss	**Decrease in sales**	**Increase in sales**	**Total**
Decrease in profit or loss	**422** (*54.6%) (**87.7%) (***73,5%)	**59** (7.6%) (**12.3%) (***29,6%)	**481 (*62.2%)**
Increase in profit or loss	**152** (*19.7%) (**52.1%) (***26,5%)	**140** (*18.1%) (**47.9%) (***70,4%)	**292 (*37.8%)**
Total	**574 (*74.3%)**	**199 (*25.7%)**	**773**

Source: Own processing in SAS Enterprise Guide based on information from financial statements

There is a statistically significant correlation between changes in sales and changes in profit or loss (Table 4) at any level of significance (p-value < 0.0001).

Table 4: Assessment of the association between changes in sales and changes in profit or loss

Statistic	DF	Value	Prob
Chi-Square	1	120.9957	<.0001
Likelihood Ratio Chi-Square	1	119.4225	<.0001
Continuity Adj. Chi-Square	1	119.1365	<.0001
Phi Coefficient		0.3956	
Contingency Coefficient		0.3679	
Cramer's V		0.3956	

Source: Own processing in SAS Enterprise Guide based on own research

The analysis of changes in profit or loss depending on the region of the registered office (headquarter) of the accounting entity is in Table 5.

Table 5: Analysis of changes in profit or loss depending on the place of business

*cell percentages **row percentages ***column percentages	**Changes in profit or loss**		
Place of business - region	**Decrease**	**Increase**	**Total**
BB	**42** (*5.4%) (**52,5%) (***8.7%)	**38** (*4.9%) (**47,5%) (***13%)	**80 (*10.3%)**
BA	**102** (*13.2%) (**67,5%) (***21.2%)	**49** (*6.3%) (**32,5%) (***16.8%)	**151 (*19.5%)**
KE	**35** (*4.5%) (**67,3%) (***7.3%)	**17** (*2.2%) (**32,7%) (***5.8%)	**52 (*6.7%)**
NR	53 (*6.9%) (**62,4%) (***11.0%)	**32** (*4.2%) (**37,6%) (***11.0%)	**85 (*11.1%)**
PO	**61** (*7.9%) (**59,8%) (***12.7%)	**41** (5.3%) (**40,2%) (***14.0%)	**102 (*13.2%)**
TN	**42** (*5.4%) (**65,6%) (***8.7%)	**22** (*2.9%) (**34,4%) (***7.5%)	**64 (*8.3%)**
TT	**50** (*6.5%) (**70,4%) (***10.4%)	**21** (*2.7%) (**29,6%) (***7.2%)	**71 (*9.2%)**
ZA	**96** (*12.4%) (**57,1%) (20.0%)	**72** (*9.3%) (**42,9%) (***24.7%)	**168 (*21.7%)**
Total	**481 (*62.2%)**	**292 (*37.8%)**	**773**

Source: Own processing in SAS Enterprise Guide based on information from financial statements

Out of the total number of accounting entities operating in the analysed sector, in which the profit or loss decreased in 2020 compared to 2019, up to 21.2% of companies were located in the BA Region, and almost 20.0% of companies were located in the ZA Region (Table 5 column percentages "Decrease"). Up to 70.4% of companies located in the TT Region operating in the hotel accommodation sector reported a decrease in profit or loss in 2020 compared to 2019. It is followed by the BA Region, where the profit or loss fell in 67.5% of entities. The KE Region is ranked third, where the profit or loss decreased in 67.3% of entities operating in the analysed sector in the given region (Table 5 row percentages "Decrease"). Statistically significant correlation was not obtained between changes in profit or loss and the place of business.

3.3 The Government Aid for the Tourism Sector

In October 2020 the government approved aid for the domestic tourism sector in the amount of 100 million euros, and in April 2021 approved an increase in aid of a further 120 million euros to be redistributed to catering and accommodation facilities, travel agencies, aqua parks, swimming pools, tourist guides, botanical and zoological gardens, museums for public, tourist attractions, etc., whose sales fell by at least 40% or more in comparison with 2019 (MTCSR, 2021). The financial contribution should be from 4 to 10%, depending on the decrease in sales. The eligible period for drawing this assistance is from April 1, 2020 to December 31, 2020, with the first stage of applications being form April 1, 2020 to October 30, 2020. The second stage of applications is from November 1, 2020 to March 31, 2021. The maximum amount of de minimis aid to a single entity may not exceed € 200,000 during the period covering the current fiscal year and the two preceding fiscal years. From July 1, 2021, the state aid project will depend on the individual stages of the Covid Automat.

4. Conclusions

Due to restrictive measures connected with the COVID-19 pandemic and due to attenuation of economic and social life, many entrepreneurs faced a crisis that led to limitation and closure of business activities. The most affected sector was the tourism sector, where sales decreased by more than 40% in 2020 in comparison with 2019.

The government aid aimed at mitigating the negative effects of the crisis was very embarrassing, chaotic, with unnecessary bureaucratic constraints at the beginning. As a result of that, many entities did not even try to ask for help.

In our paper we focused on the analysis of the impact of COVID-19 crisis on hotel and accommodation facilities, where we investigated a decrease in sales and profit or loss in 2020 compared to the previous accounting period. More than 74% of entities operating in this sector reported lower sales in 2020 in comparison with 2019; most of them were located in the BA Region and in the ZA Region. A decrease in sales of more than 50% was reported by more than one third of entities operating in this sector. The year-to-year decline in profit or loss in the analysed accounting entities was even more significant, when we noted a decrease of up to more than 416% compared to the previous accounting period.

Hotels and accommodation facilities make only a part of accounting units in the tourism sector. It would be interesting to analyse sales and profit or loss in other areas of this sector. We believe that state aid, which is available for this sector of business, will help to start a stagnant business and revive tourism.

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Development of national insurance markets of V4 countries

Martina Borovcová [1]

Abstract

The aim of the article is to analyse, compare and evaluate the situation and development in the national insurance markets of the V4 countries in the period 2009–2019. The indicators used to compare national insurance markets are total gross premiums, density, penetration, market share, gross claims payments and loss ratio. Their values for the observed period are obtained from the OECD database. Subsequently, national markets are assessed on the basis of selected indicators using multi-criteria analysis methods. The article applies, and previously described, the Scoring method, respectively a modification of this method, the Method of allocating 100 point and the Weighted Sum Approach method. The findings reveal the final assessment of the national insurance markets of the V4 countries and their development.

Key words

Insurance market, V4 countries, indicators, Scoring method, Weighted Sum Approach method,

JEL Classification: C02,C4, G2, G11

1. Introduction

The insurance market is a place of conflict between the demand and supply of insurance. The existence and functioning of the insurance market is related to the functioning of the entire economy of a given state, of which the insurance market is a part. As a result, it is possible to perceive differences in individual national insurance markets. The article therefore aims to compare selected national insurance markets using selected indicators and evaluate their level and development. The indicators used for this comparison are total gross premium, density, penetration, market share, gross claims payments and loss ratio.

Total gross premiums (TGP), defined as the total insurance premiums in the reporting country, are a major indicator of the importance of the insurance industry in the economy of the country. The TGP is used to determine the rankings of insurance companies within the national insurance market, the order of individual national insurance markets within economic groupings, etc. The value of the TGP related to the various bases may be a more relevant indicator of the assessment of the insurance market of a given economy. Such an indicator can be, for example, national insurance market share, insurance density, or insurance penetration. Market share in OECD (MAS) is defined as a country's national insurance market compared with the OECD insurance market total. It measures the importance of the national insurance market as an index using the OECD insurance market total as base. Density (DEN) is used as an indicator for the development of insurance within a country and is calculated as ratio of total insurance premiums to whole population of a given country. Penetration (PEN) is used as an indicator of insurance sector development within a country and is calculated as the ratio of

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total insurance premiums to gross domestic product in a given year. Penetration rate indicates the level of development of insurance sector in a country.

If, on the one hand, indicators are evaluated on the basis of the insurance company's income, ie premiums, then, on the other hand, attention should be paid to indicators on the basis of core expenses, ie insurance benefits. This can be ensured by the gross claims payments and loss ratio. Gross claims payments (GCP) comprise all payments in respect of the financial year including reinsurance. A claim is a demand made by the insured, or the insured's beneficiary, for payment of the benefits provided by the insurance contract or for coverage of an incurred loss. In the insurance industry, a loss ratio (LOR) is one indicator of how financially stable an insurance company is. It's the ratio of losses paid to premiums earned. That is, a comparison of how much the company spent settling claims and how much it earned from paying customers.

The basic principles of multicriteria decision-making will be used to assess the level and development of national insurance markets, which is one of the options mentioned (Fotr, Dědina, Hrůzová, 2010), (Brožová, Houška, Šubrt, 2014), (Ramík 1999), (Raju, Kumar, 2014). Thus, the preference of indicators will be determined first, using the selected method of direct determination of weights, and then the Weighted Sum Approach method, which they describe (Ishizaka, Nemery, 2013), (Doumpos, Zopounidis, 2014). The aim of the article is to analyze, compare and evaluate the development and situation of national insurance markets of the V4 countries.

2. Methodology

In order to fulfill the set goal and perform analysis, evaluation and subsequent comparison of the national insurance markets of the V4 countries, it is necessary to use multi-criteria evaluation methods. First of all, it is necessary to determine the weights of the criteria, ie the indicators for assessing the level of the insurance market. Subsequently, the variants, ie the individual national insurance markets, will be compared with each other so that it is possible to compare and evaluate them.

The method of direct determination of criteria weights is used to determine the weights of criteria. This is a modified Scoring method, a Method of allocating 100 points. The weighted sum method is used to compare variants. These methods are briefly described in the following text.

The modified Scoring method, the Method of allocation 100 point, is based on assigning points to individual criteria based on the decision-maker's preferences. It awards the highest number of points to the primary goal and assigns the number to the rest at its discretion. Subsequently, the difference between the main and all secondary targets is determined. Then we only start from the knowledge of certain differences, and we can set a suitable combination of possible scenarios. The Method of allocating 100 points is very closely connected with this described scale. It consists in dividing all one hundred points without any residue between the criteria in accordance with their significance. Here, too, the link applies, the more points the criterion has, the more it is essential for the contracting authority. The phase of determining the weights of the criteria follows. The procedure is that the decision-maker assigns a weight of 1 to the least important criterion and then must determine how many times the penultimate criterion of the preferential order is more significant than the last one. This procedure is only repeated with the third and fourth criteria, of course also in the order from the end. Only in the last step, when we get to the most important criterion, we compare it in relation to the last one. The result is significance coefficients, or non-standard weights. These can still be converted to standard weights using equation (1). The numerator is written in the number of points of the selected criterion and in the denominator the sum of the number of divided points, which can be seen in the formula entered below with an explanation. It is important that the sum of the values of the standard weights is equal to one.

$$v_i = \frac{f_i}{\sum_{i=1}^{n} f_i}, \tag{1}$$

where v_i is the standard weight of the i-th criterion, f_i is number of preferences of the i-th criterion, n specifies the number of criteria.

2.1 Weighted Sum Method

The Weighted Sum Method requires cardinal information, a criterion matrix Y and a vector of criteria weights. It constructs an overall rating for each variant, so it can be used both to find one of the most advantageous variants and to arrange the variants from best to worst. The weighted sum method is a special case of the utility function method. It is based on the principle of maximizing utility. If the variant a_1 reaches a certain value y_{ij} according to the criterion j, it thus brings the user a benefit which can be expressed by means of a linear utility function. The total utility of a variant is expressed by the weighted sum of the values of the partial utility functions

$$u(a_i) = \sum_{i=1}^{m} v_j u_j(y_{ij}), \tag{2}$$

where u_j are the partial functions of the utility of the individual criteria and v_j are the weights of the criteria. The procedure of the weighted sum method is given by the following steps.

We convert minimization criteria to maximization criteria, for example, according to the relation (3) and we thus receive an evaluation for each variant by how much it is better than the worst variant according to the relevant criterion. For simplicity, we will always denote the transformed criterion matrix Y. This adjustment is not necessary, it serves to simplify the next step.

$$y_{ij} = \max_{i=1,\dots,m} \left(y_{ij}\right) - y_{ij}. \tag{3}$$

Then we determine the ideal variant H with evaluation ($h_1, \dots, h_m$) and the basal variant D with evaluation ($d_1, \dots, d_n$). Next, we create a standardized criterion matrix R, the elements of which we obtain using formula (4). The matrix R already represents a matrix of values of the utility function from the i-th variant according to the j-th criterion, because the elements of this matrix are linearly transformed criterion values such that $r_{ij} \in \langle 0; 1 \rangle$. Then the basal variant corresponds to a value of zero and the ideal variant to a value of one.

$$r_{ij} = \frac{y_{ij} - d_j}{h_j - d_j} \tag{4}$$

For individual variants, we calculate the aggregate utility function according to formula (5). We then sort the variants in descending order according to the values of $u(a_i)$ and consider the required number of variants with the highest values as a solution to the problem.

$$u(a_i) = \sum_{j=1}^{n} v_j r_j. \tag{5}$$

3. Data

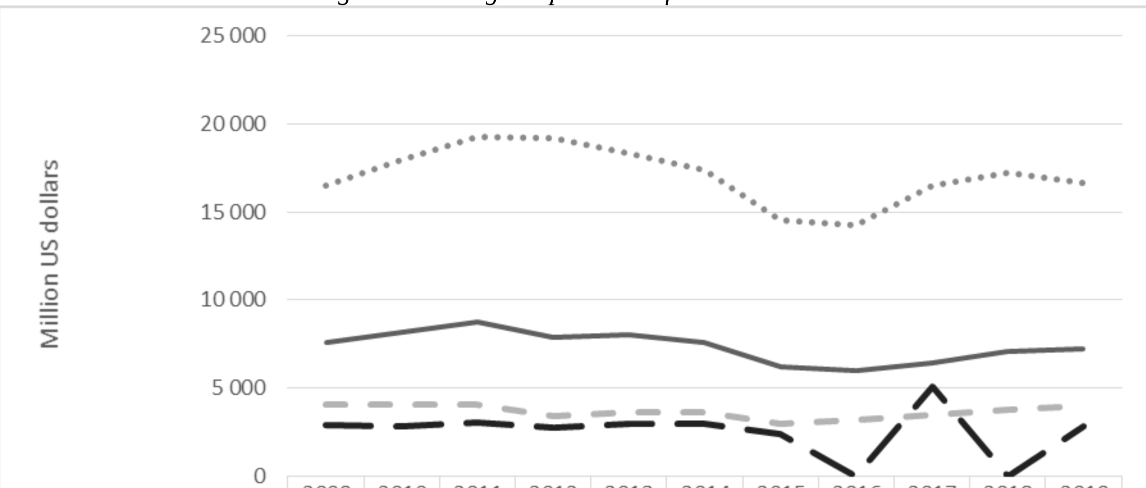
Data from the OECD database, available on their websites, are used to assess the level and development of the national insurance markets of the V4 countries in 2009–2019.

As a consequence of the implementation of the new OECD Global Insurance Statistics' framework, there is a break in series between 2008 and 2009 regarding life and non-life business data where composite insurance undertakings exist. Up until 2008, the insurance business is broken down between life and non-life business. As of 2009, the insurance business is broken down between the business of pure life, pure non-life and composite undertakings and composite undertakings' business is further broken down between life and non-life business. Some countries do not allow for insurance undertakings to be active in both life and non-life insurance business and therefore composite insurance undertakings do not exist in these countries. In other countries (e.g., Austria, Belgium, Hungary, Italy, Mexico, Portugal, Spain) however, the share of employment in composite insurance undertakings accounts for more than half of the whole domestic insurance sector. Therefore, to have comparable data across years for life business data (resp. non-life), one has to sum up the life (resp. non-life) business of pure life (resp. non-life) undertakings and the life (resp. non-life) business of composite undertakings as of 2009.

In 2016, a new supervisory framework for insurance and reinsurance companies was implemented in the European Union (EU) and the European Economic Area (EEA). This framework includes harmonised reporting requirements across EU/EEA countries. These requirements led to changes in national data collection, which may hamper the data comparability with the ones collected before 2016 for some countries which only collect data with the Solvency II framework.

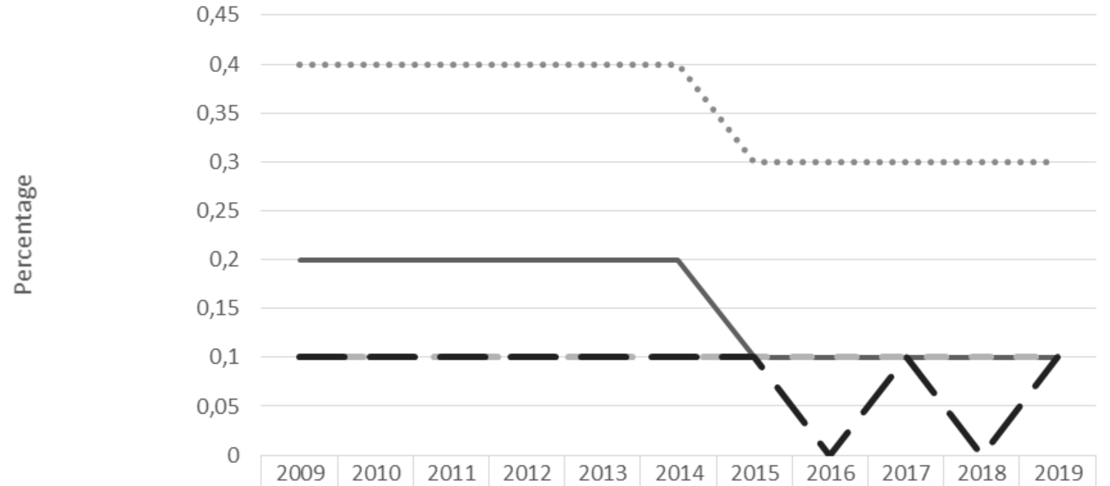
The following figures show the development of the indicators TGP, MAS, DEN, PEN, GCP and LOR.

Figure 1: Total gross premium of V4 countries



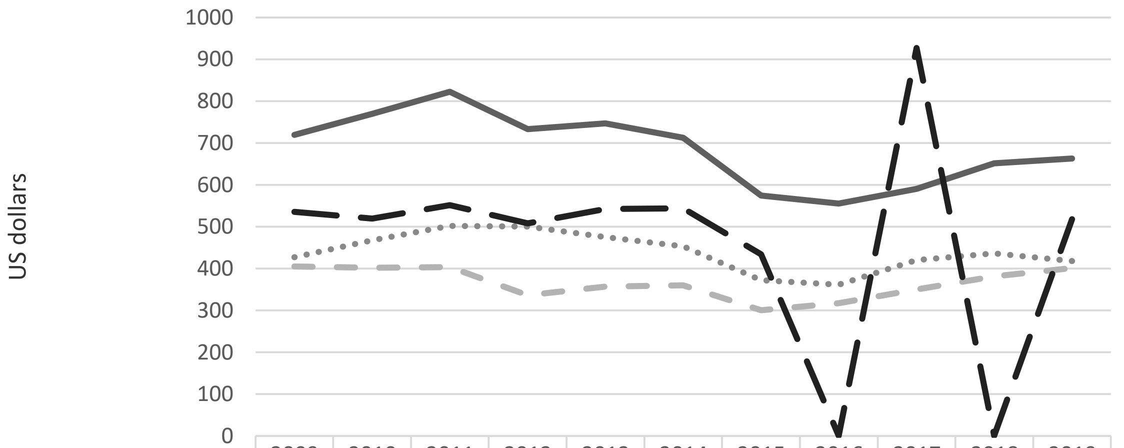
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Czech Republic	7 562	8 167	8 764	7 846	7998	7608	6173	6002	6402	7052	7214
Hungary	4081	4053	4054	3377	3582	3584	2991	3146	3484	3767	3955
Poland	16475	17963	19289	19231	18307	17412	14539	14213	16495	17211	16627
Slovak Republic	2899	2816	2998	2772	2959	2986	2400	0	5047	0	2823

Figure 2: National market share of V4 countries



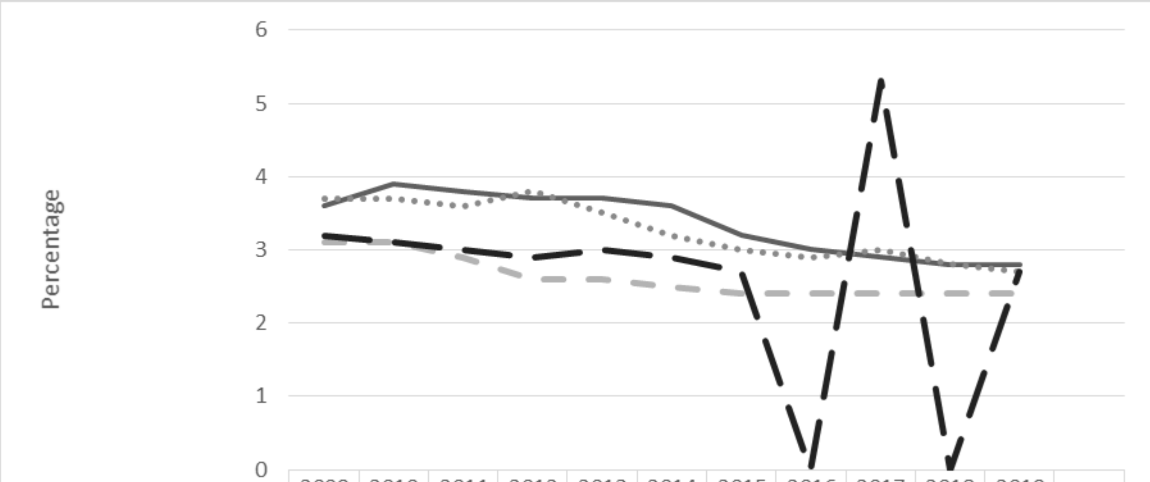
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Czech Republic	0,2	0,2	0,2	0,2	0,2	0,2	0,1	0,1	0,1	0,1	0,1
Hungary	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
Poland	0,4	0,4	0,4	0,4	0,4	0,4	0,3	0,3	0,3	0,3	0,3
Slovak Republic	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0	0,1	0	0,1

Figure 3: Density of V4 countries



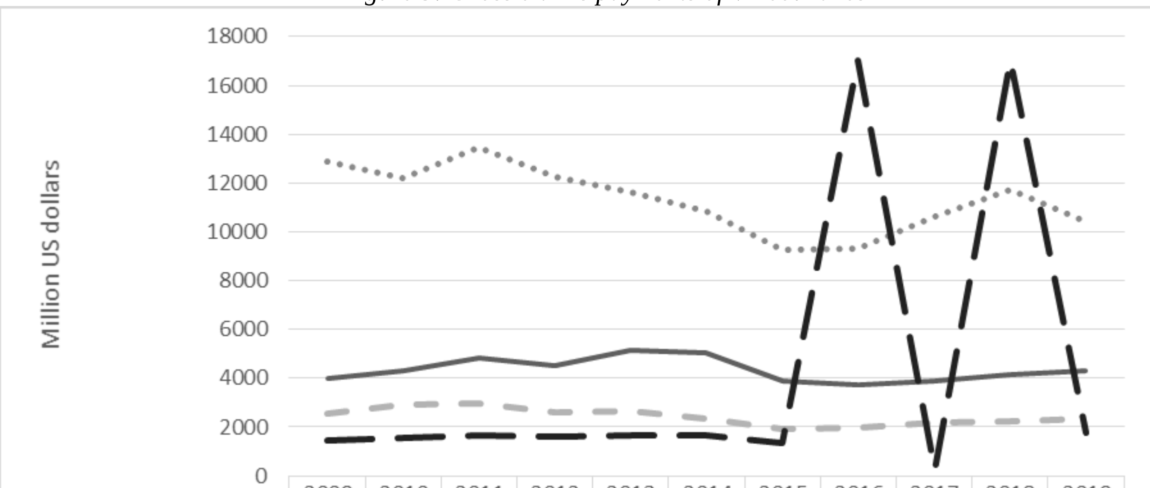
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Czech Rpublic	719	770	822	733	747	712	574	555	590	651	663
Hungary	405	402	403	336	357	360	300	317	350	381	401
Poland	427	467	502	500	475	453	372	361	420	436	418
Slovak Republic	535	519	551	508	542	544	434	0	927	0	518

Figure 4: Penetration of V4 countries



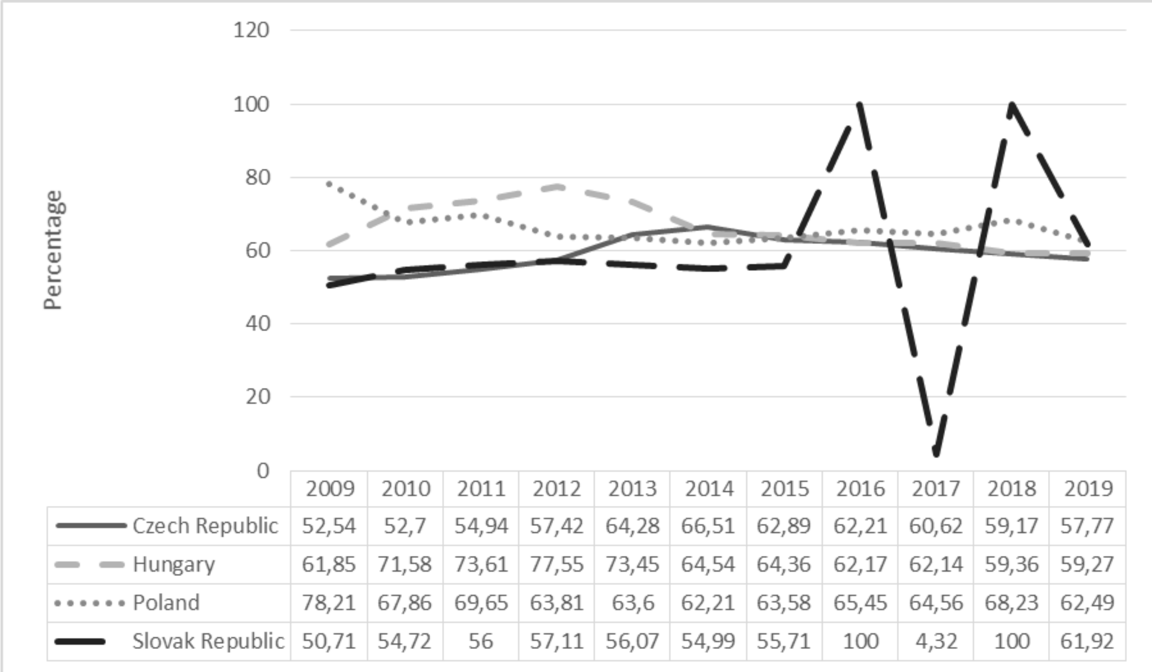
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Czech Republic	3,6	3,9	3,8	3,7	3,7	3,6	3,2	3	2,9	2,8	2,8
Hungary	3,1	3,1	2,9	2,6	2,6	2,5	2,4	2,4	2,4	2,4	2,4
Poland	3,7	3,7	3,6	3,8	3,5	3,2	3	2,9	3	2,8	2,7
Slovak Republic	3,2	3,1	3	2,9	3	2,9	2,7	0	5,3	0	2,7

Figure 5: Gross claims payments of V4 countries



	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Czech Republic	3973	4304	4815	4505	5141	5060	3882	3734	3881	4173	4312
Hungary	2524	2901	2984	2619	2631	2313	1925	1956	2165	2236	2344
Poland	12887	12189	13435	12272	11643	10832	9244	9302	10649	11743	10391
Slovak Republic	1470	1541	1679	1583	1659	1642	1337	17000	218	17000	1748

Figure 6: Loss ratio of V4 countries



4. Results

The national insurance markets of the V4 countries are evaluated using selected indicators from the above. Relativized indicators, which can be used to evaluate and compare the markets of different economies, seem to be suitable for evaluation. The following are used for evaluation and comparison: the insurance density indicator, the loss ratio indicator and the insurance penetration indicator. The analysed period is the years 2009–2019.

Using formula (1), the weights of the indicators are first determined.

Table 1: Determination of criteria weights

Indicator	Density	Loss ratio	Penetration	Total
Score	45	35	20	100
Weight v_i	0,45	0,35	0,20	1,00

After calculating the weights, the level of national insurance markets of the V4 countries and their development are analyzed and evaluated. First, the ideal value of H_j and the basal value of D_j are determined for each indicator and for individual years. Subsequently, the utility values are calculated, which are used to compare individual national insurance markets and their development. Formulas (3), (4) and (5) are used for this.

Table 2: Evaluation of national insurance markets of V4 countries according to the density indicator

Year	H_j	D_j	$r_{ij} \cdot v_j$			
			Czech Republic	Hungary	Poland	Slovak Republic
2009	719	405	0,45	0	0,031529	0,186306
2010	770	402	0,45	0	0,079484	0,143071
2011	822	403	0,45	0	0,106325	0,15895
2012	733	336	0,45	0	0,185894	0,194962
2013	747	357	0,45	0	0,136154	0,213462

2014	712	360	0,45	0	0,118892	0,235227
2015	574	300	0,45	0	0,118248	0,220073
2016	555	0	0,45	0,257027	0,292703	0
2017	927	350	0,187175	0	0,054593	0,45
2018	651	0	0,45	0,263364	0,301382	0
2019	663	401	0,45	0	0,029198	0,200954

Table 3: Evaluation of national insurance markets of V4 countries according to the loss ratio indicator

Year	H_j	D_j	$r_{ij} \cdot v_j$			
			Czech Republic	Hungary	Poland	Slovak Republic
2009	50,71	78,21	0,326709	0,208218	0	0,35
2010	52,7	71,58	0,35	0	0,068962	0,312553
2011	54,94	73,61	0,35	0	0,074237	0,330129
2012	57,11	77,55	0,344692	0	0,235274	0,35
2013	56,07	73,45	0,184666	0	0,19836	0,35
2014	54,99	66,51	0	0,059852	0,130642	0,35
2015	55,71	64,36	0,05948	0	0,031561	0,35
2016	62,17	100	0,34963	0,35	0,319654	0
2017	4,32	64,56	0,022892	0,01406	0	0,35
2018	59,17	100	0,35	0,348371	0,272337	0
2019	57,77	62,49	0,35	0,238771	0	0,042267

Table 4: Evaluation of national insurance markets of V4 countries according to the penetration indicator

Year	H_j	D_j	$r_{ij} \cdot v_j$			
			Czech Republic	Hungary	Poland	Slovak Republic
2009	3,7	3,1	0,166667	0	0,2	0,033333
2010	3,9	3,1	0,2	0	0,15	0
2011	3,8	2,9	0,2	0	0,155556	0,022222
2012	3,8	2,6	0,183333	0	0,2	0,05
2013	3,7	2,6	0,2	0	0,163636	0,072727
2014	3,6	2,5	0,2	0	0,127273	0,072727
2015	3,2	2,4	0,2	0	0,15	0,075
2016	3	0	0,2	0,16	0,193333	0
2017	5,3	2,4	0,034483	0	0,041379	0,2
2018	2,8	0	0,2	0,171429	0,2	0
2019	2,8	2,4	0,2	0	0,15	0,15

Table 5: Evaluation of national insurance markets of V4 countries

Year	Czech Republic	Hungary	Poland	Slovak Republic
2009	0,943376	0,208218	0,231529	0,569639
2010	1	0	0,298446	0,455624
2011	1	0	0,336117	0,511301
2012	0,978025	0	0,621168	0,594962
2013	0,834666	0	0,49815	0,636189
2014	0,65	0,059852	0,376807	0,657955
2015	0,70948	0	0,299809	0,645073

2016	0,99963	0,767027	0,80569	0
2017	0,24455	0,01406	0,095972	1
2018	1	0,783164	0,773719	0
2019	1	0,238771	0,179198	0,393221
Total	9,359726	2,071093	4,516605	5,463963

From the values calculated above, it is possible to evaluate the development of the national insurance market in the Czech Republic as the best. The national insurance markets of the Slovak Republic and Poland reach lower values. The lowest values are reached by the Hungarian insurance market.

5. Conclusions

The aim of the article was to analyze, compare and evaluate the situation and development in the national insurance markets of the V4 countries in 2009–2019. The indicators used to compare national insurance markets were TGP, DEN, PEN, MAS, GCP and LOR. Their values for the observed period were obtained from the OECD database. Subsequently, national markets were assessed on the basis of selected indicators using methods of multi-criteria analysis.

By applying a modified Scoring method, the Method of allocating 100 points, the preference of selected indicators was determined. The most important indicator with the highest preference was the density indicator with a weight of 0.45. A weight of 0.35 was calculated for the loss ratio indicator and the lowest preference was determined for the penetration indicator, 0.2. Based on the application of the Weighted Sum Method, the development on the national insurance market in the Czech Republic can be evaluated as the best, the development in Hungary appears to be the worst, according to the calculated values.

Acknowledgments

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Portfolio making under unstable uncertainty: moving mean-semivariance model

Adam Borovička [1]

Abstract

Portfolio making is not an easy problem due to its complexity embedded in an environment of uncertainty which, moreover, is usually not stable over time. To make a robust investment portfolio, the unstable uncertainty should be considered. Difficult conditions require the use of a suitable supportive decision-making tool. Advanced version of a well-known mean-variance model called mean-semivariance model seems as a convenient means. This model accepts non-normally distributed returns with the possibility of investment diversification through semi(co)variances non-penalizing a upside return volatility. Uncertainty instability is integrated into this model by a proposed dynamization of return and risk through the 'moving' form of mean and semivariance. Both characteristics are enumerated in all overlapping subperiods from which moving means and semi(co)variances are calculated. The developed model is applied to make a portfolio from the stocks traded on the Czech Stock Exchange. To demonstrate the benefits of a dynamized model, the result is compared with a mean-semivariance model.

Key words

Moving mean-semivariance, portfolio, stock, unstable uncertainty

JEL Classification: C44, C61, G11

1. Introduction

The topic of portfolio making is still alive. No wonder, because more and more people are deciding what to do with their free funds. Nowadays, there are many possibilities. The capital market is a complex world with not easily predictable development. In such an environment, the investor usually needs to lean on some decision support mechanism. The proposal of such a supportive tool that would take into account the most important investment criteria and the typical instability on the capital market will also be the subject of this article.

One of the most popular tools for a portfolio making is a mean-variance model proposed by H. Markowitz (Markowitz 1952; 1959). Its popularity is accelerated by the ability to accept two most important investment characteristics – return and risk. Moreover, the concept is easy to use in practice. However, this model also faces criticism. Swisher and Kasten (2005) and many others (including Markowitz) discuss and criticize the concept of variance that assumes the same investor attitude to upside and downside return volatility. Penalizing upside variability is not reasonable for the investor. The second application drawback can be an assumption of the normally distributed returns. Many empirical studies, e.g. Simkowitz and Beedles (1978) or Borovička (2020) confirm an asymmetric and skewed distribution. To eliminate both drawbacks, the variance concept is improved to the semivariance form (Markowitz, 1991) that has been significantly discussed by many other authors (e.g. Balzer,

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1994; Ballestero, 2005). Thus, mean-semivariance model seems to be very application proficient. Characteristics benefits motivate me to further improve this concept.

The key issue is the (in)stability of the observed investment characteristics over time. The instability can be caused by often changing uncertainty on the capital market. To be closer to the investment reality, the return and risk should be understood as a dynamic element considering their instability. Therefore, I propose to include this variability (instability) through the 'moving' concept. This approach is to dynamize the process using a moving average. The observed historical period is divided into a few shorter parts shifting by one-time subperiod. Based on the partial mean and semi(co)variances from these time-overlapping periods, the portfolio return and risk are calculated via simple or weighted averaging. The portfolio is then more robust and effectively diversified. Last, this approach is a less data complexity and easy to use in practice.

The main aim of this article is to improve the well-known mean-semivariance model to include the unstable uncertainty, or the variability of return and risk over time. For this purpose, a dynamic version of mean-semivariance model called moving mean-semivariance model is proposed. The second aim is to demonstrate the application benefits of the developed model on the capital market. The portfolios of the stocks traded on the Czech Stock Exchange RM-System are made. This segment of the Czech stock exchange market is chosen for its significant accessibility to a wide range of investors. The compositions of the portfolios are analyzed and compared with the portfolios selected via a 'static' mean-semivariance model. This analysis is supported by plotting the effective frontiers.

The rest of the article is structured as follows. Section 2 designs a moving mean-semivariance model. Section 3 performs an empirical research on the Czech capital market. Section 4 summarizes the contribution of the paper and outlines some ideas for future research.

2. Moving mean-semivariance model

The contribution of this section is a description of the proposed moving mean-semivariance model. The main emphasis is on a calculation of moving mean and semi(co)variances of returns of the assets and market portfolio.

2.1 Moving mean for assets and market portfolio return

Let q define the number of equally long time periods with m observations of returns for n assets. The asset return can be determined not only on the basis of the price differences over time, but can also include a dividend or coupon payment. The (expected) return of the i-th asset in the t-th period is calculated as a following mean

$$\overline{r}_i^{\,t} = \frac{\sum_{k=1}^{m} r_{ik}^t}{m} \qquad i = 1, 2, ..., n, t = 1, 2, ..., q, \tag{1}$$

where $r_{ik}^t, i = 1, 2, ..., n, k = 1, 2, ..., m, t = 1, 2, ..., q,$ represents the k-th observation of a return of the i-th asset in the t-th period. The difference between the beginning, or end, of two consecutive periods is constant throughout the observed history. Neighbouring periods overlap between the beginning of one period and the end of the previous one. Then the (expected) return of the i-th asset as moving mean can be expressed in a simple, or weighted form as follows

$$r_i = \frac{1}{q}\sum_{t=1}^{q}\overline{r}_i^{\,t}\ \text{, or}\ r_i = \sum_{t=1}^{q} w_t \overline{r}_i^{\,t} \qquad i = 1,2,...,n, \tag{2}$$

where $w_t, t = 1,2,...,q,$ denotes the weight of the *t*-th period. The weights are standardized, so the following holds $\sum_{t=1}^{q} w_t = 1.$ The weights reflect an importance of the subperiods. For instance, a stronger effect of last subperiods to the development at the beginning of the investment can be considered. Then the influence declines over time. Therefore, values of the weights can have a declining trend towards the past. The concepts for setting the weights can be varied. The weights can be determined by means of some supportive quantitative tool, e.g. the scoring method (Fiala, 2013).

To calculate the semi(co)variances, the market portfolio and its characteristics must be determined. Let $\mathbf{r_{mp}^t} = (r_{mp_1}^t, r_{mp_2}^t, ..., r_{mp_m}^t)^T$ be a vector of *m* historical returns of the market portfolio in *t*-th period. The market portfolio can be formulated through a naive strategy. The portfolio then contains all *n* selected assets with the same proportions. Then the *k*-th return of market portfolio in the *t*-th subperiod is calculated as follows

$$r_{mp_k}^t = \frac{1}{n}\sum_{i=1}^{n} r_{ik}^t \qquad k = 1,2,...,m, t = 1,2,...,q. \tag{3}$$

The (expected) return of market portfolio in the *t*-th subperiod is computed as follows

$$r_{mp}^t = \frac{1}{m}\sum_{k=1}^{m} r_{mp_k}^t \qquad t = 1,2,...,q. \tag{4}$$

2.2 Moving semi(co)variances of returns

The covariance of return of the *i*-th and *j*-th asset in the *t*-th subperiod is calculated as

$$\sigma_{ij}^t = \frac{\sum_{k=1}^{m}(r_{ik}^t - \overline{r}_i^{\,t})(r_{jk}^t - \overline{r}_j^{\,t})}{m} \qquad i, j = 1,2,...,n, t = 1,2,...,q, \tag{5}$$

where r_{ik}^t, or r_{jk}^t, $i, j = 1,2,...,n, k = 1,2,...,m, t = 1,2,...,q,$ represents the *k*-th observation of the *i*-th, or *j*-th asset in the *t*-th period. Then the moving covariance of return of the *i*-th and *j*-th asset can be calculated as a simple, or weighted moving average

$$\sigma_{ij} = \frac{1}{q}\sum_{t=1}^{q}\sigma_{ij}^t \text{, or}\ \sigma_{ij} = \sum_{t=1}^{q} w_t \sigma_{ij}^t \qquad i, j = 1,2,...,n. \tag{6}$$

Further, let $v^t(\mathbf{r_{mp}^t} > r_{mp}^t)$ denote a semivariance above the mean (expected) return for a market portfolio in the *t*-th subperiod. Then the following for each subperiod can be calculated

$$v^t(\mathbf{r_{mp}^t} > r_{mp}^t) = \frac{1}{m}\sum_{k=1}^{m}\left[\max(r_{mp_k}^t - r_{mp}^t, 0)\right]^2 \qquad t = 1,2,...,q. \tag{7}$$

Now we can formulate for the *t*-th subperiod the matrix $\mathbf{B^t} = (b_{ij}^t)$ with the generic elements calculated as follows

$$b_{ij}^t = \beta_i^t \beta_j^t v^t(\mathbf{r_{mp}^t} > r_{mp}^t) \qquad i, j = 1,2,...,n, t = 1,2,...,q. \tag{8}$$

Sharpe's ratio β_i^t, or β_j^t for the *i*-th, or *j*-th asset is specified as

$$\begin{aligned} \beta_i^t &= \frac{\sigma_{imp}^t}{\sigma_{mp}^{t^2}} \qquad i = 1, 2, ..., n, t = 1, 2, ..., q \\ \beta_j^t &= \frac{\sigma_{jmp}^t}{\sigma_{mp}^{t^2}} \qquad j = 1, 2, ..., n, t = 1, 2, ..., q \end{aligned} , \tag{9}$$

where σ_{imp}^t, or σ_{jmp}^t denotes a covariance of return of the *i*-th, or *j*-th asset and return of the market portfolio, $\sigma_{mp}^{t^2}$ represents a variance of the market portfolio return.

Let $\mathbf{V^t} = (\sigma_{ij}^t)$ denote the matrix of covariances and variances in the *t*-th subperiod. Then, the matrix of semivariances and semicovariances for *t*-th subperiod can be specified as $\mathbf{V_s^t} = \mathbf{V^t} - \mathbf{B^t}$, whose element is computed as $\sigma_{ij_s}^t = \sigma_{ij}^t - b_{ij}^t, i, j = 1, 2, ..., n, t = 1, 2, ..., q.$

Finally, the moving semi(co)variance of return of the *i*-th and *j*-th asset can be calculated as simple, or weighted moving average

$$\sigma_{ij_s} = \frac{1}{q} \sum_{t=1}^{q} \sigma_{ij_s}^t, \text{or } \sigma_{ij_s} = \sum_{t=1}^{q} w_t \sigma_{ij_s}^t \qquad i, j = 1, 2, ..., n. \tag{10}$$

2.3 Model with the 'moving' characteristics

After data collection and calculation of the characteristics, a moving mean-semivariance model can be formulated. The model is derived from the formulation made by Ballestero (2005) that has been tested in a number of applications (e.g. Pla-Santamaria and Bravo, 2013). The model is formulated as minimizing a portfolio risk expressed by moving semivariance under the minimum required level of return as follows

$$\begin{aligned} &\min \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij_s} x_i x_j \\ &\sum_{i=1}^{n} r_i x_i \geq r' \\ &\sum_{i=1}^{n} x_i = 1 \\ &x_i \geq 0 \qquad i = 1, 2, ..., n \end{aligned} , \tag{11}$$

where x_i, or x_j, $i, j = 1, 2, ..., n,$ is a variable represents a proportion of the *i*-th, or *j*-th asset in the portfolio, r' denotes a minimum required level of a portfolio return (reference return), $r_i, i = 1, 2, ..., n,$ is a return of *i*-th asset expressed as moving mean and σ_{ij_s} is semicovariance of return of the asset *i* and *j* also computed using the proposed 'moving' concept. In necessary, the model can be extended by additional conditions, e.g. minimum/maximum proportion of one asset in the portfolio. These conditions can be grouped into the set *X*. We must keep in mind that the additional conditions can modify the classical shape of the effective frontier.

How to determine a minimum required level of return? This process can be inspired by a minimum and maximum possible portfolio return. Let $r_{\min} = \sum_{i=1}^{n} r_i x_i^{\min}$ and $r_{\max} = \sum_{i=1}^{n} r_i x_i^{\max}$ be a minimum and maximum attainable portfolio return actually representing its basal and ideal value on the set of all necessary conditions for a real-life portfolio making problem (conditions from model (11) and any additional conditions situated in the set X). For $\mathbf{x}^{\mathbf{min}} = (x_1^{\min},...,x_n^{\min})^T$ and $\mathbf{x}^{\mathbf{max}} = (x_1^{\max},...,x_n^{\max})^T$, the following must hold

$$\begin{array}{ll} \mathbf{x}^{\mathbf{min}} = \arg\min \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij_s} x_i x_j & \mathbf{x}^{\mathbf{max}} = \arg\max \sum_{i=1}^{n} r_i x_i \\ \sum_{i=1}^{n} x_i = 1 \qquad , & \sum_{i=1}^{n} x_i = 1 \qquad . \\ x_i \geq 0 \quad i = 1,2,...,n & x_i \geq 0 \quad i = 1,2,...,n \\ x_i \in X \quad i = 1,2,...,n & x_i \in X \quad i = 1,2,...,n \end{array} \tag{12}$$

The basal value of return is reasonably determined on the basis of the lowest possible level of risk. Now, the reference return can be specified as follows

$$r' = r_{\min} + \lambda(r_{\max} - r_{\min}), \tag{13}$$

where parameter $\lambda \in \langle 0,1 \rangle$ actually represents a rate of risk aversion in the context of 'an unwritten rule' of the capital market 'higher return-higher risk'. The risk aversion is escalating by approaching zero. Of course, the reference return level can be determined in another (subjective) way on the interval $\langle r_{\min}, r_{\max} \rangle$.

If all incoming data is ready, the model (11) can be solved. Since the objective function is nonlinear we can assume finding only a local optimum. This depends on the shape of the set of feasible solutions. The same situation occurs with model (12) on the left side. The second model is linear, so a global optimum is found.

3. Making a portfolio of the stocks traded on the RM-System

In the empirical research, we will focus on the usual investment case. It is an investor in the productive age who has (regularly) a smaller amount of free funds. The purpose of such an investment may be rather in the horizon of a longer-term consumption, for example to protect financially the retirement age. Such an investor would rather be satisfied with a smaller, but 'surer' return. Many investors in this category are not very experienced. Portfolio management will be rather passive. The composition of the investment portfolio will not be changed much. For a less experienced investor, the portfolio should not contain non-traditional products (e.g. financial derivates). The number of investment products should be limited to a reasonable level (approximately five) for better orientation of an inexperienced investor. The portfolio is often made in collaboration with an investment counsel.

3.1 Data collection, calculation of investment characteristics

A very friendly investment option for 'smaller' investors can be found in a more modest Czech stock exchange called RM-System. Standardized trading units are smaller. The investor can focus on the traditional investment instrument – stock. The offer of traded companies issuing stocks is wide. The ease of trading is supported by a user-friendly online trading system EasyClick (see more RM-System, 2021a) which can be used by the investor himself or a person close to him (e.g. investment counsel, more experienced colleague, more capable friend, etc.).

Based on the investment strategy specified above, a longer-time history is selected for a calculation of the investment characteristics. This period is set from 2011 to 2019, representing the price falls and ups interspersed by calmer times. Last (pandemic) period can be used for a short-term evaluation of the portfolio performance. Thirteen stocks traded on

RM-System have a sufficient history to be included in the empirical analysis. These are stocks of the following companies: ČEZ, Erste Group Bank, Exxon Mobil, Intel, Komerční banka, McDonald's, Microsoft, Nokia, Orco, O2 C.R., Philip Morris, VIG and Volkswagen.

Stock prices from the last trading day of each month in the selected period are downloaded from the web of RM-System (2021b), as well as the dividends in each year from the web of Miras Lébl (2021) and associated web sites. The monthly return includes a proportional part of the dividend released in a particular year. To reflect an unstable uncertainty, a nine-year period is divided into five overlapping five-year subperiods gradually shifted by one year, from 2011-2015 to 2015-2019. In each subperiod, the mean and semi(co)variances of the stocks are calculated from 60 observations through formulas (1) and (5). Then the moving means and semi(co)variances are computed via simple and weighted average (2) and (6). The weights are determined on the basis of the idea of more significant influence of recent development. The calculation of the weights is based on subjective discretion reflecting personal (analytic) investment experiences supported by the scoring method using the scoring interval $\langle 1,10 \rangle$ for evaluation. Then the weights of the subperiods are determined at the value of 0.1 (score 2) for period 2011-2015, 0.1 (score 2) for 2012-2016, 0.2 (score 4) for 2013-2017, 0.25 (score 5) for 2014-2018 and 0.35 (score 7) for 2015-2019. Both characteristics represented by mean and semi(co)variances are shown in the following table (Table 1).

Table 1: Moving means and semi(co)variances [in %]

σ_{ij_s}	ČEZ	Erste	Exxon	Intel	KB	McD	Micro	Nokia	Orco	O2	PM	VIG	VW
ČEZ	27.92	7.43	-0.10	4.72	10.83	-0.33	3.01	7.41	-13.39	4.28	-2.51	4.27	8.63
Erste	7.74	52.83	-1.33	-2.66	8.34	-0.37	5.86	-2.04	-9.82	0.38	-0.97	12.76	14.03
Exxon	-0.01	-1.33	22.18	11.39	-10.33	6.05	9.86	5.28	-10.27	-9.97	-0.52	4.39	8.89
Intel	4.83	-2.66	11.39	33.70	-9.95	7.81	18.53	6.51	-17.94	0.13	0.72	2.35	5.77
KB	10.98	8.34	-10.33	-9.95	109.41	0.25	-9.23	8.05	7.05	1.07	2.49	-6.53	-5.37
McD	-0.28	-0.37	6.05	7.81	0.25	18.87	7.63	8.78	-18.04	-0.34	2.55	3.55	0.86
Micro	3.17	5.86	9.86	18.53	-9.23	7.63	31.48	-0.02	-30.05	7.04	-2.11	3.63	6.79
Nokia	7.80	-2.04	5.28	6.51	8.05	8.78	-0.02	85.85	-36.97	-6.64	2.00	2.17	-5.43
Orco	-12.95	-9.82	-10.27	-17.94	7.05	-18.04	-30.05	-36.97	515.97	-41.91	-4.38	2.98	-9.29
O2	4.53	0.38	-9.97	0.13	1.07	-0.34	7.04	-6.64	-41.91	157.10	6.36	3.31	-13.39
PM	-2.51	-0.97	-0.52	0.72	2.49	2.55	-2.11	2.00	-4.38	6.36	13.83	-0.46	-5.84
VIG	4.50	12.76	4.39	2.35	-6.53	3.55	3.63	2.17	2.98	3.31	-0.46	29.05	17.25
VW	8.89	14.03	8.89	5.77	-5.37	0.86	6.79	-5.43	-9.29	-13.39	-5.84	17.25	57.80
r_i	0.39	1.03	0.28	1.37	0.22	1.54	2.23	0.19	0.1	1.11	1.24	-0.3	0.39

Note: Erste ~ Erste Group Bank, Exxon ~ Exxon Mobile, KB ~ Komerční banka, McD ~ McDonald's, Micro ~ Microsoft, O2 ~ O2 C.R., PM ~ Philip Morris, VW ~ Volkswagen.

3.2 Portfolio making

To make a portfolio, the model (11) is extended by the condition ensuring the limit on the number of stocks in the portfolio. As mentioned above when making a portfolio, the maximum number of stocks may be (approximately) five. On the other side, an elementary diversity in a portfolio composition should be maintained. It means that the portfolios with only one, or possibly two stocks will not be allowed. Therefore, the minimum, or maximum proportion of one stock is set at 20%, or 40%. To integrate this requirement in the portfolio making process, adequate constraints are added through the set *X* specified above. Then the following are formulated

$$\begin{aligned} &0.2y_i \le x_i \le 0.4y_i \qquad i = 1, 2, \ldots, n \\ &y_i \in \{0,1\} \qquad i = 1, 2, \ldots, n \end{aligned}, \tag{14}$$

where $x_i, i = 1, 2, \ldots, n,$ denotes the proportion of the *i*-th stock (in the order from Table 1, i.e. from $i = 1 \square$ ČEZ to $i = 14 \square$ Volkswagen. The binary variable $y_i, i = 1, 2, \ldots, n,$ is used to express the presence $(y_i = 1)$ or absence $(y_i = 0)$ of the *i*-th stock in the portfolio.

Now, the model (11) extended by (14) can be formulated as follows

$$\begin{aligned} &\min \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij_s} x_i x_j \\ &\sum_{i=1}^{n} r_i x_i \ge 1.26 \\ &\sum_{i=1}^{n} x_i = 1 \\ &0.2y_i \le x_i \le 0.4y_i \qquad i = 1, 2, \ldots, m \\ &x_i \ge 0 \qquad i = 1, 2, \ldots, n \\ &y_i \in \{0,1\} \qquad i = 1, 2, \ldots, m \end{aligned} . \tag{15}$$

The reference level of portfolio return r' is determined in the spirit of more risk-averse long-term investment strategy. Therefore, the parameter λ from (12) is stated lower than 0.5. The reference return is calculated as $r' = 0.92 + 0.4(1.78 - 0.92) \square 1.26.$ The basal and ideal return is determined through (12).

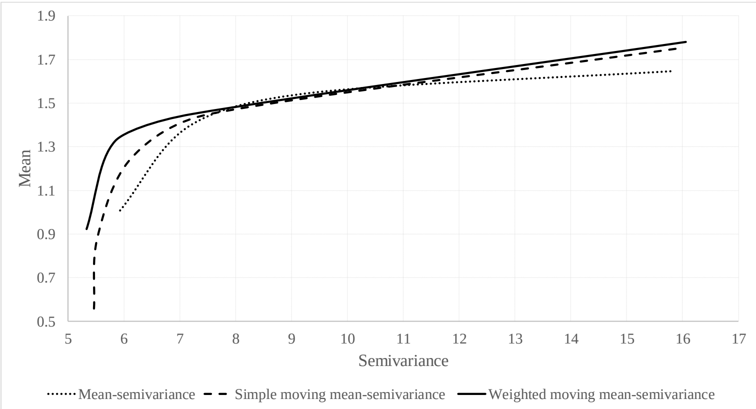
Thanks to the additional constraints for the limiting proportion of one stock, it is not surprising that an effective portfolio (with minimized risk) can provide a return higher than the minimum required r'. In our case, the solution of the model (15) represents the portfolio with the return 1.33% and risk 5.85% in the following composition: 20% ČEZ, 20% McDonald's, 20% Microsoft and 40% Philip Morris. The highest possible proportion of Philip Morris stock could be expected. This stock provides almost the same (expected) return as the reference level. Further, this stock has the great diversification skills. Its semivariance, hence semicovariances of returns with other funds are very low. At first glance, the inclusion of the stock of the energy company ČEZ may be a small surprise, thanks to its lower return. However, the strength of this asset lies in a risk reduction, especially in combination with the stocks of Microsoft and McDonald's providing the highest returns. It is therefore natural that these two stocks will strengthen at the expense of the others with a demand for a higher return of the portfolio (under a decreasing risk aversion). On the other side, some stocks do not have a chance to get into the portfolio. An example for all is the stock issued by the developer company Orco. This stock provides a very low (expected) return, which is also significantly unstable over time indicated by a huge semivariance.

3.3 Discussion on the application impacts of the proposed concept

First, it must be stated that the idea of semivariance has proved its worth in investment practice. Penalizing only negative return deviations is reasonable. The empirical benefit of a semivariance concept is underlined by often non-normally distributed returns. A minor disadvantage can then be a potentially slightly more complex application. However, as empirical research confirms, this is no problem with today's computer technology and software (optimization tool LINGO and MS Excel in our case). Finally, it is worth mentioning that the composition of the market portfolio can significantly affect the result. It is therefore necessary to determine the market portfolio prudently.

We will now focus on the application, or empirical impacts of the proposed dynamized version of the mean-semivariance model. Let's first compare the results of original mean-semivariance and moving mean-semivariance model using a simple and weighted moving average. This comparison will be performed via the (approximate) efficient frontiers made by three notified models including the same conditions of the performed case study. The condition limiting the number of stocks in the portfolio causes 'non-standard' shape of these functions. It means that higher return does not always have to be burdened by a higher risk. Then the curves are not monotonically increasing. To simplify the comparison, the curves are smoothed as shown in Figure 1. It should be noted that the approximation is very slight.

Figure 1: Smoothed effective frontiers via three various form of a mean-semivariance model



The portfolios made using all three models contain similar stocks (6-7 companies) in various proportions, which is ultimately confirmed by similarly evolving approximate efficient frontiers. The effective frontier of a weighted mean-semivariance model mostly lies at the highest, which evokes a slightly higher return at a particular level of risk. This fact, representing more effective diversification, is mainly supported by the last period 2015-2019 providing a very positive stock performance which is clearly given the highest weight. The expected more significant impact of the recent subperiod on the immediate future development was correct, as shown by a positive development on the capital market in the period from January 2020 to May 2021. Although this evaluation period for a longer-term investment strategy is short, the positive performance of the selected portfolio (remind in Section 3.2) in this period indicates that the proposed moving mean-semivariance model helps to make a reasonable and robust investment decision. The idea of a dynamization is proving to be a step in the right direction. However, be careful, the value of the investment at the end of the investment period will always show the profitability of the decision.

4. Conclusion

The article deals with a portfolio making under unstable uncertainty manifesting in the variability of the investment return and risk over time. For this purpose, the moving mean-semivariance model is proposed. A dynamized version thus gets closer to the reality of often changing market conditions through the return and risk formulated as moving mean and semivariance. The investment decision is then more robust and diversified as demonstrated by the empirical study on the Czech Stock Exchange RM-System. The compositions of the stock portfolios made by static and dynamic mean-semivariance model are different which confirms the usefulness and convenience of a developed dynamization. The (smoothed) efficient frontiers are displayed and their dissimilarity is analyzed. Moving mean-semivariance model shows its application power thanks to more effective reflection of market conditions. This statement is supported by the practically convenient acceptance of an asymmetric, skewed distribution of returns and penalizing only a downside return volatility. The proposed model is proving to be very beneficial in the world of investment decisions.

For further research, the weights of particular overlapping subperiods could be deeply analyzed because their settings could significantly affect the results. The second interesting idea is to compare the model with its fuzzy form also ensuring the uncertainty instability, designed by Borovička (2021), from an algorithmic and empirical point of view.

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Risk Spillover Effect Analysis by Applying Selected Models

Miroslav Čulík, Petr Gurný, Lun Gao, Xincheng Zhang [1]

Abstract

The goal of this paper is the application of the extreme value theory, Copula function and conditional value-at-risk method. Particularly, EVT-Copula-CoVaR model is constructed and combined with the Copula function to analyze the dynamic correlation between the price of gold and the world's major stock markets. On the basis of the proposed model and results, the conditional value-at-risk (CoVaR) and the marginal risk spillover effect (ΔCoVaR) measures are used to analyze the impact of gold prices on the world's major stock markets. The empirical results show that the fluctuation of gold price has a certain risk spillover effect on the world's major stock markets.

Key words

risk spillover, volatility effect, extreme value theory, VaR, CoVaR, Copula function

JEL Codes: C51, G15, G17

1. Introduction

In the international financial market, risks and benefits exist at the same time. Some investors are more aggressive and are more inclined to a higher expected rate of return of investment behaviors and therefore actively pursue risks. Some investors are more conservative and expect to avoid risks to reduce losses. This requires a clearer definition of risks, so as to improve the ability of risk management and enhance the reliability of decision-making. With the development of basic sciences such as mathematics and statistics, and the improvement of information science and technology, risk decision-making has gradually integrated into relevant theories to help investors make analytical decisions. Researchers can establish appropriate and study the inherent characteristics of financial data through mathematical analysis. The most famous model among them is the value at risk (VaR) measurement model launched by JP Morgan, which represents the maximum possible value of a financial investment portfolio's loss in a certain period of time under a given probability level. It quantifies market risks with greater uncertainty through given standards, so that the risks of different financial asset portfolios can be quantified and compared in a unified standard type.

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Economic globalization inevitably increases the closeness of economic relations between different countries and regions. With the gradual improvement of the global capital market opening level, the risks contained in the market are no longer limited to a single market. Risks will gradually spread in different markets. The risks of a single industry or market will most likely spread to other industries and markets through interconnected market channels, eventually leading to systemic financial risks. The contagion and spread of this risk among different industry markets is called risk spillover effect. In the 2008 global financial crisis that caused huge losses caused by the subprime mortgage crisis, the world underestimated the scale of risk spillovers in the subprime loan and its derivative securities markets, and lacked effective methods to measure extreme risks and spillover effects. The VaR model commonly used by financial institutions only captures risk probability events within a given confidence interval (usually 95% or 99%), which obviously underestimates extreme risks (that is, risk events that originate from outside the given confidence interval). The impact and analysis are only limited to the risk control of a single financial institution or the financial market itself, but the impact of the risk event on the entire financial system is not considered, resulting in the quantified financial risk being far lower than the actual Level. At the same time, traditional financial theories generally assume that the distribution of financial asset returns obeys a normal distribution. However, with the deepening of research in recent years, a large number of empirical data analysis shows that the distribution of financial asset returns often has the characteristics of leptokurtic, fat tail and skewness. Based on the above situation, it can be seen that the traditional value-at-risk model has greater limitations. The difficulties encountered in market risk control and risk quantification are mainly based on the selection of the specific distribution characteristics of the fat tail part of the return sequence, the selection of the confidence interval based on the specific distribution and the construction of extreme risks and spillover effects measurement models.

As the weather vane of the world economy, gold and the stock market play an important role in the world financial market system. As a "quasi currency", gold occupies an important position in the fiscal reserves of all countries in the world. Traditionally, gold is often regarded as a hedge against inflation. Investors will treat gold as a part of their investment portfolio to diversify risks. Therefore, in the long run, contrary to the stock market, the price of gold has an obvious negative correlation with the world economic situation. In order to effectively reduce the systemic risks caused by local risk spillovers, it is necessary to consider the effective quantification of the financial risk spillover effects under extreme conditions. That is, under the conditions of an open economy, quantify the interdependence between different types of financial markets in different countries and regions and the risk spillover effects caused by the interdependence.

The goal of this paper is the application of the extreme value theory, Copula function and conditional value-at-risk method. Particularly, EVT-Copula-CoVaR model is constructed and combined with the Copula function to analyze the dynamic correlation between the price of gold and the world's major stock markets. On the basis of the proposed model and results, the conditional value-at-risk (CoVaR) and the marginal risk spillover effect (ΔCoVaR) measures are used to analyze the impact of gold prices on the world's major stock markets.

2. Research overview

The mutual influence of risks between different markets has always been a hot area for scholars to study. The gold market is known as the weather vane of the world economy due to the characteristics of gold preservation. The fluctuation of the stock market and the fluctuation of gold prices have a strong correlation. The volatility of the stock market and the gold market brought about by every economic crisis has brought about the transfer of wealth and the capital flow of the international capital market, resulting in the risk of different markets changing with the volatility. Research on the risk spillover effects of the gold market on the stock market will help financial investors construct a reasonable investment portfolio based on market feedback to hedge market risks. At the same time, the research on financial market risk spillover also has certain significance for financial market supervision and the formulation of macroeconomic and monetary policies. Basher and Sadorsky (2016) pointed out in their research that gold is similar to bonds and other assets, and has a better effect in hedging stock market risks. This means that there is a certain correlation between gold price fluctuations and stock market fluctuations. Similarly, Raza et al. (2016) studied the relationship between the commodity futures market and the stock market of emerging countries. They confirmed that the volatility of the commodity futures market has a negative impact on the stock markets.

In view of the disadvantages of traditional value-at-risk models in measuring extreme tail risks, Adrian and Brunnermeier (2008, 2016) proposed the use of Conditional Value at Risk (CoVaR) methods to quantify risk spillovers between different institutions or markets. The CoVaR method makes up for the shortcomings of tools such as the indirect measurement of risk only by variance. Especially for the practical application of risk management, its ideas and methods are qualitative changes. Girardi and Ergun (2013) conducted a more in-depth study on the definition of CoVaR. Focusing on the more serious crisis events in the tail distribution, the multi-dimensional GARCH model is used to simulate the widespread risk spillover effects among financial institutions. Subsequently, many scholars used the CoVaR method proposed by Adrian and Brunnermeier (2016) to study risk spillover effects. Bernal et al. (2014) introduced ΔCoVaR to measure systemic risk, and assess the degree of influence on systemic risk when problems occur in different financial sectors, such as banking and insurance. The empirical results show that between 2004 and 2012, the financial sector such as banks and insurance companies in the Eurozone has a relatively greater impact on systemic risk than other financial service sectors, and the specific banking industry has a greater impact on systemic risk than the insurance industry. Castro and Ferrari (2014) used a sample of 26 large European banks to empirically analyze the contribution of CoVaR in measuring financial institutions' systemic risks. Copula function theory was first proposed by Sklar (1959). Its essence is to describe the marginal distribution of the joint distribution of random variables and the correlation structure between variables. Embrechts et al. (1999) first applied the Copula function to analyze the financial market risk. They believed that the dependency index derived by the Copula function was consistent with the actual situation of the financial market. In recent years, its application in the financial industry and insurance industry has gradually expanded. The application of Extreme Value Theory (EVT) used to be mainly based on climate and hydrology. In recent years, its application in the financial industry and insurance industry has gradually expanded. For the extreme value theory, the core problem is how to model the extreme events that occur scientifically and reasonably. The current literature does not conduct separate research on extreme value theory and financial market risk, but uses extreme value theory to cooperate with more different risk models to achieve the purpose of comprehensive research.

The current research on financial market risk is mostly based on the combination of quantile regression method and GARCH family model to measure VaR value, and there are not many literatures on the combination of Copula function and extreme value theory to calculate VaR. Most of the CoVaR literature only focuses on the research on the risk transmission path and direction, but rarely involves the intensity of risk spillover, which is not conducive to a comprehensive and in-depth understanding of risks and affects risk supervision. Based on the advantages and disadvantages of the above models, this article will use extreme value theory combined with the

Copula function to construct a spillover effect for quantifying risk to study the contagion of the gold market to stock market risk.

3. Model introduction

3.1 Conditional Value at Risk

Adrian and Brunnermeier (2008) were first who proposed the CoVaR (Conditional Value-at-Risk) method. This method is a risk measurement method proposed on the basis of VaR to measure the risk between financial institutions, which helps to quantify the systemic risks of financial institutions and the risks with other financial institutions. The most significant difference between the VaR method and the CoVaR method is that the CoVaR value can measure the risk spillover effect of one financial institution on another financial institution. Recalling the definition of VaR mentioned above, if a given financial institution i is at the rate of return r_t^i and the confidence level p, then VaR_{1-p}^i can be expressed as:

$$Pr\left(r_t^i \leq VaR_{1-p}^i\right) = 1 - p. \tag{1}$$

($\mathrm{VaR}_{1-\mathrm{p}}^{\mathrm{i}}$ is usually a negative value, but in actual application, it is generally expressed as a positive value). VaR is a risk assessment of a single financial asset and cannot reflect the degree of risk spillover between financial markets or assets. Adrian and Brunnermeier (2008) proposed the concept of CoVaR on the basis of VaR. It represents the value of risk faced by financial asset i when financial asset j is at a risk level. Therefore, $CoVaR_{1-p}^{i/j}$ reflects the conditional risk of financial asset i to financial asset j, which can be expressed as:

$$Pr(r_t^i \leq CoVaR_{1-p}^{i/j} \middle| r_t^i = VaR_{1-p}^j) = 1 - p \tag{2}$$

It can be seen from the above formula that the essence of $CoVaR_{1-p}^{i/j}$ is a condition VaR, which measures the total risk of financial asset i, including the risk value of financial asset i itself and the risk spillover effect of financial asset j. CoVaR reflects conditional risk and infectious spillover risk, see Brayek et al. (2015). It is mainly aimed at measuring the risk of extreme events under extreme tail probabilities. It is a conditional concept that can be used to capture the effect of risk spillover. In order to evaluate the risk spillover effect of financial asset j on financial asset i, this study defines $\Delta CoVaR_{1-p}^{i/j}$, the specific formula is as follows:

$$\Delta CoVaR_{1-p}^{i/j} = CoVaR_{1-p}^{i/j} - VaR_{1-p}^i \tag{3}$$

Considering that the VaR of different financial assets has relatively large differences and $\Delta CoVaR_{1-p}^{i/j}$ can only indicate the size of the risk spillover effect, it is necessary to standardize $\Delta CoVaR_{1-p}^{i/j}$ in order to reflect the strength of the spillover effect of financial assets:

$$\%CoVaR_{1-p}^{i/j} = \frac{\Delta CoVaR_{1-p}^{i/j}}{VaR_{1-p}^i} \times 100\% \tag{4}$$

3.2 Extreme value theory

Extreme value theory deals with extreme situations of risk. It has the ability to estimate beyond sample data and can accurately describe the tail distribution. In a statistical sense, extreme values refer to maximum and minimum values. Although the extreme values in some data sets do not have a large gap with other data, there are still extreme values in this data set.

The extreme value theory mainly includes two types of models, namely the traditional block maxima method (BMM) model and the Peak Over Threshold (POT) model. The difference between these two types of extreme values lies in the selection of data. The POT model has a predetermined threshold. When there is data that exceeds the threshold, it will be acquired and formed into a new group, using a new sequence to model; the BMM model is different, the initial data will be classified, and then the maximum value in each group will be obtained to form a new group, and the new group will be used for modeling. But both models work with extreme data in the tail, rather than analyzing the overall distribution.

The BMM model adopts maximum likelihood estimation and probability weighted moment estimation method. In its use, a large amount of sample data is often needed to model the maximum value after the block. Due to the limited acquisition of tail data, this method has great application difficulties in practice. Therefore, Peak Over Threshold (POT) model is used more in practice.

POT is a key branch of extreme value theory. Its main feature is to model all observations in the sample that exceed a sufficiently large threshold. The form is simple, easy to calculate, and has a wide range of applications. The POT model has two types of methods: one is the semiparametric method based on Hill-type estimator; the other is the full parameter method based on the generalized Pareto distribution.

The semiparametric method is when the shape parameter of the tail distribution $\xi > 0$, use the Hill-type estimator to estimate the tail index $\alpha = 1/\xi$. If $\xi > 0$, $L(x)$ is a slow changing function, if and only if $\bar{F}(x) = 1 - F(x) = x^{1/\xi}L(x)$, then $F \in MDA(H)$. Suppose that $X_1, X_2, \ldots, X_{n\_}$, n is a sample from the population distribution $F(X)$, and its order statistic is $X_{(n)} \geq \ldots \geq X_{(n-k)} \geq \ldots \geq X_{(2)} \geq X_{(1)}$, where $X_{(n-k)}$ is a larger observation value, and there are k sample points greater than $X_{(n-k)}$. Hill (1975) gave an estimate of α:

$$\hat{\alpha} = [\frac{1}{k}\sum_{i=n-k+1}^{n}(lnX_i - lnX_{n-k})]^{-1}, 2 \leq k \leq n \tag{5}$$

It can be seen from the above formula that $\hat{\alpha}$ depends on the sample points greater than a certain threshold $X_{(n-k)}$. Therefore, how to choose $X_{(n-k)}$ is the key to correctly estimate $\hat{\alpha}$. There are methods such as excess expectation function graph, Hill graph, Du Mouchel 10% principle, etc. Take Hill chart as an example, in practical applications, Hill chart is used to determine k, that is, $X_{(n-k)}$. Take $k, (k = 2, \ldots, n)$ as the abscissa and $\hat{\alpha}$ as the ordinate to draw the plot, and select the data corresponding to the coordinate k of the starting point of the stable region of the tail index in the Hill graph $X_{(n-k)}$ serves as the threshold u.

Different from the semiparametric method, the full-parametric method uses the generalized Pareto distribution to simulate the tail distribution that exceeds the threshold, and then estimate its shape parameter ξ. It is assumed that the excess values are mutually independent and obey the generalized Pareto distribution; the time when the excess value occurs obeys the Poisson distribution; at the same time, the excess value and the generation time of the excess value are independent of each other. Assuming that $F(X)$ is the distribution function of financial asset loss, and assuming that u is a sufficiently large threshold, then $Y = X - \mu$ is called the excess loss, and its distribution function can be recorded as:

$$F_u(y) = P(X - u \leq |X > u), \;\; 0 \leq y \leq X_F - u \tag{6}$$

Among them, $X_F = sum\{X_F \in R: F(X) < 1\} \leq \infty$ is the right endpoint of $F(X)$. The out-of-limit distribution function represents the probability that the loss exceeds the threshold. The larger y value gives the loss that exceeds the threshold. The multiplication formula is defined as (Wang et al. 2018):

$$F_u(y) = \frac{F(y+u)-F(U)}{1-F(u)} \tag{7}$$

By simplifying the above formula, we can get the final distribution function of financial asset loss:

$$F(x) = F(y+u) = F_u(y)(1-F(u)) + F(u), x > u \tag{8}$$

According to the previous derivation and according to the Fisher-Tippett theorem, it can be shown that if the distribution of the maximum value sequence is known to converge, its limit distribution can be transformed into a generalized extreme value distribution $(H_{\xi,\mu,\sigma(x)})$ with a specific value of the parameter α, μ, δ. In addition, according to the results of Balkem, Haan (1974) and Pickands (1975), if F belongs to the maximum attractive field of H, then the generalized Pareto distribution is the limit distribution of the over-limit distribution, that is, exist $X \sim F, \xi \in R, F \in MDA(H)$, if and only if there is a certain positive measure function $\beta(u)$, the following limit theorem exists:

$$\lim_{u \to X_F} \underset{0 \leq Y \leq X_F - \mu}{\mathrm{SUP}} \left| F_u(y) - G_{\xi,\beta(u)}(y) \right| = 0 \tag{9}$$

where $G_{\xi,\beta(u)}(y)$ is a generalized Pareto distribution. The above formula shows that for a sufficiently large threshold, the overrun distribution function can be approximated by the generalized Pareto distribution. The generalized Pareto distribution is defined as:

$$F_u(y) \approx G_{\xi,\beta}(y) = \begin{cases} 1-(1+\xi\frac{y}{\beta})^{-1/\xi}, \xi \neq 0 \\ 1-e^{-y/\beta},\ \xi = 0 \end{cases} \tag{10}$$

where ξ is the shape parameter, β is the scale parameter, and $\beta > 0$, when $\xi > 0$, $x \geq 0$; when $\xi < 0$, $0 \leq x \leq -\beta/\xi$. When $\xi > 0$, the generalized Pareto distribution corresponds to the thick-tailed ordinary Pareto distribution, which is the most relevant to risk measurement; When $\xi = 0$, it corresponds to an exponential distribution; when $\xi < 0$, it corresponds to a short-tailed distribution, such as a uniform distribution. The parameter ξ and β is unknown and need to be estimated based on excess loss data.

There are many methods to estimate ξ and β, such as maximum likelihood estimation, moment estimation method and so on. The maximum likelihood estimation method is the most commonly used method. Suppose it is taken from the sample data $X_1, X_2, \ldots, X_n$ whose population distribution is F. The sample points larger than the threshold are recorded as $\hat{X}_1, \hat{X}_2, \ldots, \hat{X}_n$, and there are N_u sample points in total. Calculate the over-limit value $y_i = \hat{X}_j - u$. From the definition of the generalized Pareto distribution above, the density function of the generalized Pareto distribution can be obtained:

$$g'_{\xi,\sigma}(y) = \frac{1}{\beta}(1+\frac{\xi}{\beta}y)^{-(1+1/\xi)} \tag{11}$$

among them, when $\xi > 0$, $y \geq 0$; when $\xi < 0$, $0 \leq x \leq -\beta/\xi$, and its log likelihood function:

$$l(\xi,\beta;y) = -N_u ln\beta - \left(1+\frac{1}{\beta}\right)\sum_{i=1}^{N_u} ln(1+\frac{\xi}{\beta}y_i) \tag{12}$$

Under the premise of the likelihood function, the likelihood equation can be derived:

$$\begin{cases} \frac{\partial l}{\partial \beta} = -\frac{N_u}{\beta} + (1+\beta)\sum_{i=1}^{N_u}\frac{y_i}{\beta(\beta+\xi y_i)} \\ \frac{\partial l}{\partial \xi} = \frac{1}{\xi^2}\sum_{i=1}^{N_u} ln(1+\frac{\xi}{\beta}y_i) - (1+\beta)\sum_{i=1}^{N_u}\frac{y_i}{\beta(\beta+\xi y_i)} \end{cases} \tag{13}$$

Let the above two equations be equal to zero, the maximum likelihood estimates of the parameters ξ and β can be obtained.

3.3 Copula function definition and related theorems

Copula function is actually a kind of function cluster that connects joint distribution functions with their respective marginal distribution functions. It was first proposed by Sklar (1959). With the development of modern information technology, it began to be applied to the financial field in the late 1990s.

Definition: The Copula function is a connection function that connects the joint distribution function of d random vectors with their respective edge distribution functions. If a function C satisfies (McNeil et al. 2005):

$C: [0,1]^d \to [0,1]$,

$C(u_1, u_2, \dots, u_d)$ increases monotonically with respect to u_i， $i \in \{1,2,..,d\}$,

To all $u_i \in [0,1], i \in \{1,2,\dots,d]$, exist $C(1,\dots,1,u_i,1,\dots,1) = u_i$,

To all$(a_1,\dots,a_d),(b_1,\dots,b_d) \in [0,1]^d$ and $a_i \le b_i$ exist:

$\sum_{i_1=1}^{2}\dots\sum_{i_2=1}^{2}(-1)^{i_1+\dots+i_d}C(u_{1i_1},\dots,u_{di_d}) \ge 0$ where $u_{j1} = a_j, u_{j2} = b_j, j \in \{1,2,\dots,d\}$.

Theorem: (Sklar, 1959) If there are d random variables, $F(x_1,\dots,x_d)$ is the $d-ary$ joint distribution function with marginal distribution function $F_1(x_1),\dots,F_d(x_d)$, then there is a Copula function $C(u_1,u_2,\dots,u_d)$ that makes the following equation true:

$$F(x_1,\dots,x_d) = C(F_1(x_1),\dots,F_d(x_d)) \tag{14}$$

If $F_1(x_1),\dots,F_d(x_d)$ is a continuous function, then $C(u_1,u_2,\dots,u_d)$ is uniquely determined; On the contrary, if $F_1(x_1),\dots,F_d(x_d)$ is a univariate distribution function, then $C(u_1,u_2,\dots,u_d)$ is a Copula function. On this basis, $F(x_1,\dots,x_d)$ determined by $F(x_1,\dots,x_d) = C(F_1(x_1),\dots,F_d(x_d))$ is a $d-ary$ joint distribution function with edge distribution $F_1(x_1),\dots,F_d(x_d)$.

In addition, the Copula function has the following properties:

$C(u_1,u_2,\dots,u_d)$ is monotonous and non-decreasing with respect to each variable,

$C(u_1,u_2,\dots,0,\dots,u_d) = 0, C(1,\dots,1,u_i,1,\dots,1) = u_i$,

To any $u_i, v_i \in [0,1](i = 1,2,..,d)$， has $|C(u_1,u_2,\dots,u_d) - C(v_1,v_2,\dots,v_d)| \le \sum_{i=1}^{d}|u_i - v_i|$

Make $C^-(u_1,u_2,\dots,u_d) = max(\sum_{i=1}^{d}u_i - N + 1,0)$, $C^+(u_1,u_2,\dots,u_d) = min(u_1,u_2,\dots,u_d)$, then for any $u_i \in [0,1](i = 1,2,\dots,d)$， there is: $C^-(u_1,u_2,\dots,u_d) \le C(u_1,u_2,\dots,u_d) \le C^+(u_1,u_2,\dots,u_d)$. Denoted as$C^- \prec C \prec C^+$. Call C^- and C^+ as the lower and upper bounds of Frechet, respectively， where $d \ge 2$,C^+ is a $d-ary$ Copula function, but when $d > 2$, C^- is not a Copula function.

If $U_i \sim U(0,1), (i = 1,2,\dots,d)$ are independent of each other, then $C(u_1,u_2,\dots,u_d) = \prod_{i=1}^{d}u_i$.

Before actually using copula theory, it is necessary to estimate the unknown parameters. Assuming that the joint distribution function of the random vector (X, Y) is $F(X, Y)$, the joint density function is $f(x, y)$, the marginal distribution functions are $F_1(X)$ and $F_2(Y)$, respectively, the marginal density functions are $f_1(x)$ and$f_2(y)$, and the density function of the corresponding Copula function $C(u, v)$ is $c(u, v)$, Then there are:

$$f(x, y) = c(F_1(X), F_2(Y))f_1(x)f_2(y) \tag{15}$$

where $c(u, v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$, The above formula shows that a joint density function $f(x, y)$ can be decomposed into two parts, one part $c(u, v)$ becomes the density function of the corresponding Copula function C(u,v), which reflects the dependence structure of random variables X and Y; the other part is the product $f_1(x)f_2(y)$ of the edge density function the related theories are proposed by Sahamkhadam et al. (2018). Assuming $\{(X_i, Y_i), i = 1,2, \ldots, n\}$ is a sample of a random vector (X, Y), then its log-likelihood function is:

$$l = \sum_{i=1}^{n} lnc(F_1(X_i), F_2(Y_i) + \sum_{i=1}^{n}[lnf_1(X_i) + lnf_2(Y_i)] \tag{16}$$

It can be seen from the above formula that as long as the edge distribution function and edge density function of the random vector (X, Y) are known, the relevant parameters can be obtained on this basis.

After clarifying the marginal distribution and selecting the optimal Copula function, the spillover effect of financial assets is calculated according to the definition of CoVaR. The specific process can be described in the following way. First, suppose there is $(U, V) \sim C$, where C is the copula function $F_{i,j}(x^i, x^j)$ represents the distribution function of the joint distribution, U represents the marginal distribution function $F_i(x^i)$ of financial assets X^i, and V represents the marginal distribution function $F_j(x^j)$ of financial assets X^j. The corresponding density functions are $f_{i,j}(x^i, x^j)$, $f_i(x^i)$ and $f_j(x^j)$. Thus, the form of the conditional distribution density function of financial assets X^i is as follows:

$$f_{i|j}(x^i|x^j) = \frac{f_{i,j}(x^i, x^j)}{f_j(x^j)} \tag{17}$$

According to Sklar's theorem, the above formula can be derived as:

$$f_{i|j}(x^i|x^j) = c(f_i(x^i), f_j(x^j))f_i(x^i) \tag{18}$$

The distribution function of the above formula is:

$$F_{i|j}(x^i|x^j) = \int_{-\infty}^{x^i} c(F_i(x^i), F_j(x^j))f_i(x^i)dx^i \tag{19}$$

The marginal distribution functions $F_i(x^i)$ and $F_j(x^j)$ in the above formula can be obtained by the Beta-skew-t-EGARCH-EVT model previous, the density function $f_i(x^i)$ is the derivative of the marginal distribution function $F_i(x^i)$, and c is the density function of the optimal Copula function selected above. According to Mainik (2012), $CoVaR_{1-p}^{i/j}$ can be expressed as:

$$CoVaR_{1-p}^{i/j} = F_{i|j}^{-1}(1 - p|VaR_{1-p}^{j}) \tag{20}$$

In the above formula, $\mathrm{F}_{\mathrm{i|j}}^{-1}$ is the inverse function of $\mathrm{F}_{\mathrm{i|j}}$. Under normal circumstances, its analytical solution is difficult to find, so it is converted into the following expression for solution:

$$\int_{-\infty}^{x^i} c(F_i(x^i), F_j(x^j)) f_i(x^i) dx^i = 1 - p \tag{21}$$

That is, the solution x^i of the above formula is $CoVaR_{1-p}^{i/j}$.

4. Empirical analysis

In order to study the dynamic correlation and risk spillover effects between the gold market and the stock markets of different countries and regions, this paper selects the daily data of the gold spot price rate of return and the rate of return of major international stock market indexes (Standard & Poor's Index (S&P500), Nasdaq Index (Nasdaq), Dow Jones Index (Dow Jones) London FTSE100 Index (FTSE), Paris CAC Index (CAC40), Germany DAX40 Index (DAX40), Nikkei 225 Index (N225) , Hong Kong Hang Seng Index (HSI), Shanghai Composite Index (SHZ), Shenzhen Component Index (SHE)) as the research samples, the data source is Yahoo Finance. Considering that a longer data interval can better reflect the spillover effects between different markets, the collection period is from November 15th, 2005 to December 15th, 2020. The paper takes the first-order logarithmic difference of the gold spot price and stock market index during the sampling period to calculate the daily return rate, and multiplies the return rate result by 100 to reduce the error, that is:

$$R_t = \ln(p_t / p_{t-1}) \times 100$$

After eliminating invalid data, the descriptive statistical results of the data shown in the following table are calculated: The skewness coefficient of the gold price return sequence and the return sequence of each stock market index is close to the skewness coefficient 0 corresponding to the normal distribution, and the kurtosis coefficient is much larger than 3 corresponding to the normal distribution. At the same time, the Jarque-Bera test result of the return rate sequence shows that the probability value p is 0, that is, the gold price return rate sequence and the stock market index return rate sequence are significantly different from the normal distribution at the 5% significance level. Therefore, it can be preliminarily judged that neither the gold price return rate sequence nor the stock market index returns rate sequence obeys the normal distribution.

Table 1 Descriptive statistics of the data

	Mean	Max	Min	StD	SKEW	KURT	J-B test(P-value)
Gold	0.0350	11.9691	-11.7970	1.5315	-0.0818	12.6194	0
HSI	0.0158	13.4068	-13.5820	1.4841	-0.0344	8.9301	0
SZSE	0.0451	9.1615	-9.7500	1.8630	-0.5630	3.0551	0
SSE	0.0308	9.0343	-9.2562	1.6145	-0.6448	4.6782	0
Nikkei225	0.0171	13.2346	-12.1110	1.5040	-0.4525	7.7674	0
Nasdaq	0.0458	11.1594	-13.1492	1.3783	-0.4945	9.3497	0
CAC40	0.0051	10.5946	-13.0983	1.4180	-0.2710	8.1759	0
DAX40	0.0248	10.7975	-13.0549	1.3894	-0.2373	8.2059	0
DowJones	0.0271	10.7643	-13.8418	1.2239	-0.4947	16.1341	0
S&P500	0.0287	10.9572	-12.7652	1.2766	-0.5613	13.6087	0
FTSE100	0.0048	9.3842	-11.5124	1.2016	-0.4094	9.8378	0

In the following research, in order to conduct a deeper study on the return rate of gold price and the return rate sequence of each stock market index, the article made a Q-Q plot corresponding to each return rate sequence. Due to space limitations, only the Q-Q plot of the data HIS is shown here. It can be clearly seen from the figure below that the upper and lower tails of the HIS yield deviate significantly from the normal distribution and have significant fat tail characteristics. Similarly, the Q-Q plot test on other return sequence data selected in this article has also obtained similar conclusions. Combining the kurtosis values of each return sequence in the above table, it can be concluded that on the selected data, the gold price return rate and the return rate data of each stock market index generally have significant kurtosis and fat tail characteristics.

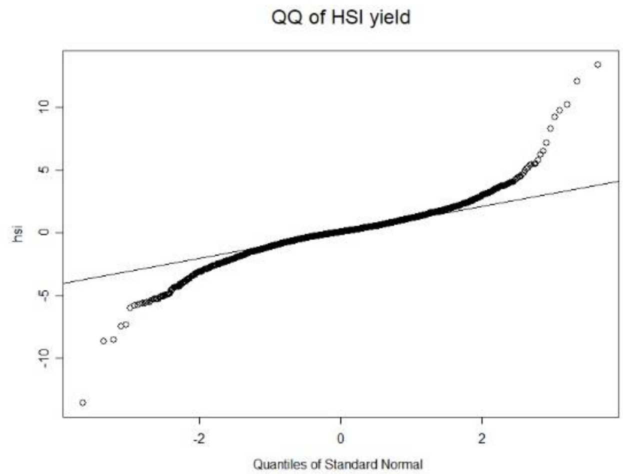

Figure 1 QQ plot of HSI sequence

According to the relevant information of extreme value theory introduced above, the generalized Pareto distribution under the frame of extreme value theory can better fit the tail distribution of the return sequence. This article will use Du Mouchel's 10% threshold selection criteria to determine the upper and lower tail thresholds of each rate of return. After obtaining the upper and lower tail thresholds, the generalized Pareto distribution is used to fit the selected upper and lower tails, and the empirical distribution is used to fit the intermediate data between the upper and lower tails. The scale parameter $\beta(u)$ and shape parameter ξ of the corresponding generalized Pareto distribution are estimated by the maximum likelihood estimation method. Still taking the data HIS as an example, the figure below shows the GPD distribution fitting diagnosis chart based on the data HIS. As shown in the figure, most of the points are concentrated near the distribution curve (including the over-threshold distribution curve and the tail distribution curve). Only a few points deviate, which does not affect the fitting effect. The model calculation results show that the model fits the data well. The same results can be obtained by performing the same fitting on other data series and testing their effects. Because of space reasons, the article will not repeat them here.

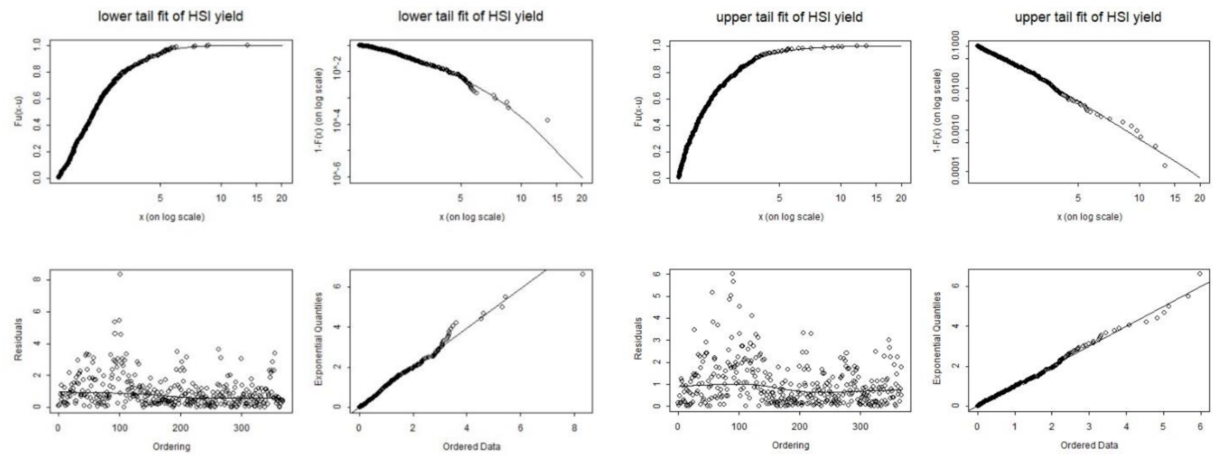

Figure 2 GPD distribution fitting diagnosis of return rate of HSI series (upper and lower tail)

This paper uses extreme value theory to model the gold price return rate and the main stock market index return rate. After determining its marginal distribution, the Copula function is used to characterize the dependent structure relationship of the sample data sequence. Substituting the estimated value of the parameter into the above-mentioned conditional distribution density function formula can obtain the marginal distribution function of the return rate of each stock market index. After determining the marginal distribution of the stock index return rate of the selected stock market, the Copula function can be used to capture the correlation structure of each stock index return rate sequence and the gold price return rate sequence. Using the principle of maximum likelihood function value Loglike and minimum AIC, BIC, HB, select the best fitting function from commonly used Copula functions, take the copula function fitting results of gold price return rate and HIS index return rate as an example:

```
> compare.copulaFit(cop.gumbel.fit, cop.joe.fit,cop.frank.fit,cop.kimeldorf.sampson.fit,cop.tawn
                     loglike        AIC        BIC         HQ
          cop.gumbel.fit  9.677186 -17.354373 -11.139096 -15.142382
             cop.joe.fit  6.949974 -11.899947  -5.684670  -9.687956
           cop.frank.fit  5.415838  -8.831676  -2.616399  -6.619685
cop.kimeldorf.sampson.fit 13.195655 -24.391310 -18.176034 -22.179320
            cop.tawn.fit 17.171355 -28.342710  -9.696879 -21.706737
             cop.bb1.fit 15.478975 -26.957951 -14.527397 -22.533969
             cop.bb3.fit 49.406472 -94.812944 -82.382390 -90.388962
             cop.bb4.fit 13.230301 -22.460602 -10.030048 -18.036620
             cop.bb5.fit  9.677186 -15.354373  -2.923819 -10.930391
             cop.bb6.fit  6.949974  -9.899947   2.530607  -5.475965
             cop.bb7.fit 16.055912 -28.111824 -15.681270 -23.687842
          cop.normal.fit  5.204585  -8.409169  -2.193892  -6.197178
             cop.bb2.fit 13.237495 -22.474989 -10.044435 -18.051007
> |
```

Figure 3 Fitting results of each copula function

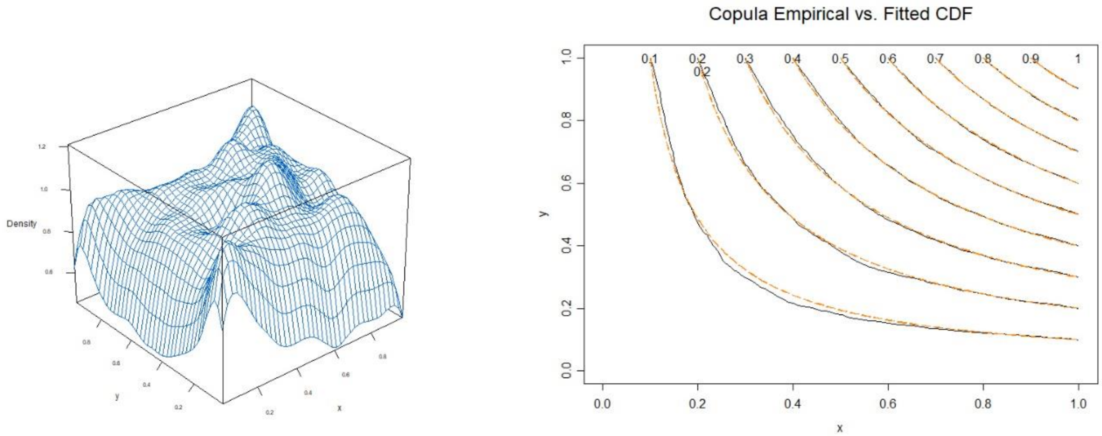

Figure 4 Density and Fit result of BB3 Copula

According to the above results, the optimal Copula function is selected as BB3 Copula and on this basis, the estimated parameters are substituted into the BB3 Copula function, and the BB3 Copula distribution function is obtained as follows:

$$C(u,v) = exp\{-[\delta^{-1}ln(e^{\delta(-ln(u)^{\theta}} + e^{\delta(-ln(v)^{\theta}} - 1]^{\frac{1}{\theta}}\}, \theta > 1, \delta > 0$$

From the distribution function of the BB3 Copula function, the density function can be obtained, so that the density function can be estimated with maximum likelihood, and the Kendall τ correlation coefficient and the upper and lower tail correlation coefficients of the gold price return rate and the return rate of each stock market index can be calculated. The results are shown in the following table:

Table 2 Fitting results of stock index return rate and gold's dependence structure

Data sequence	Optimal Copula	θ	δ	Kendall τ	Lower tail coefficient	Upper tail coefficient
HSI	BB3 Copula	1.0763	0.0932	0.1130	0.0956	0.0960
SZSE	BB3 Copula	1.0651	0.0669	0.0920	0.1957	0.0830
SSE	BB3 Copula	1.0714	0.0710	0.0991	0.0912	0.0903
Nikkei225	BB3 Copula	1.0559	0.0494	0.0761	0.0724	0.0721
Nasdaq	BB3 Copula	1.0707	0.0606	0.0939	0.0876	0.0895
CAC40	BB3 Copula	1.0626	0.0689	0.0907	0.0847	0.0800
DAX40	BB3 Copula	1.0677	0.0727	0.0967	0.0930	0.0860
DowJones	BB3 Copula	1.0670	0.0573	0.0892	0.0791	0.0851
S&P500	BB3 Copula	1.0753	0.0682	0.1012	0.1179	0.0948
FTSE100	BB3 Copula	1.0850	0.1082	0.1265	0.0970	0.1057

The correlation between the gold market and each stock market is characterized by the BB3 Copula function, and the Kendall τ correlation coefficients are all positive, indicating that the markets have a strong positive dependence and the risk of risk spillover may be high. The lower tail coefficients are all greater than zero, indicating that the risk spillover effects between the data samples are all positive, and the spillover effects are different. This may be related to the size of the stock market and the maturity of the local financial market and market preferences. So far, this article has established the Copula dependency structure function of the gold price return rate sequence and other stock market index return rates. In order to compare the strength of the spillover effect of the gold market risk on the stock market, we use the method mentioned above to calculate the CoVaR, ΔCoVaR and %CoVaR of each stock index return sequence under the 5% significant level when gold market is at risk. The results are shown in the table.

Table 3 Result of VaR, CoVaR, ΔCoVaR and %CoVaR

Data sequence	VaR	CoVaR	ΔCoVaR	%CoVaR
HSI	-2.3341	-2.3804	0.0462	1.9807%
SZSE	-3.0678	-3.3377	0.2699	8.7974%
SSE	-2.6463	-2.7008	0.0545	2.0590%
Nikkei225	-2.3444	-2.3653	0.0210	0.8939%
Nasdaq	-2.1645	-2.2117	0.0472	2.1800%
CAC40	-2.2170	-2.2444	0.0274	1.2368%
DAX40	-2.2170	-2.2145	0.0025	0.1139%
DowJones	-1.8191	-1.8593	0.0402	2.2103%
S&P500	-1.9182	-1.8593	0.0589	3.0719%
FTSE100	-1.8422	-1.8939	0.0517	2.8073%

It can be seen from the results of the above table that under a given confidence interval, SZSE has the largest value at risk and Dow Jones has the smallest value at risk. The risk price of the Asian stock market is significantly higher than that of Europe and the United States. From the perspective of risk spillover effects, conditional value-at-risk is greater than value-at-risk, and there is an obvious positive spillover effect. From the perspective of risk spillover intensity, which is %CoVaR, the highest spillover intensity is 8.7974% of SZSE, and the smallest is 0.1139% of DAX. From the perspective of market division, the market spillover intensity of the Shanghai Stock Exchange Index, the Shenzhen Stock Exchange Index, the Hong Kong Stock Exchange and the stock exchanges in the United States and the United Kingdom is significantly greater than that of the French, German and Japanese markets. This may be because the stock markets of China and the United States are larger than other markets and have greater capital liquidity. It can be seen that in actual risk management, usually only consider the value at risk which is easy to underestimate the risks faced by the market. Taking the risk overflow of gold to SZSE as an example, the actual value at risk, that is, the conditional value at risk, is significantly greater than the theoretical value at risk. And this is only a risk overflow caused by a change in the gold market. It can be seen that the traditional value-at-risk model seriously underestimates the risk, which is likely to bring great uncertainty in risk management.

Finally, this paper draws on the model validity test method and the specific form is: According to the marginal distribution model and the optimal Copula function calculated above, randomly generated 50000 sets of (X^i, X^j) values. Then, Randomly select 200 values closest to$VaR_q^j$from the randomly generated X^j values, and at the same time obtain the 200 X^i values corresponding toX^j, mark it as M, so that the ratio of the number of data less than $CoVaR_{1-p}^{i|j}$ in M to the total number of data in X is the obtained test value. The test values of the article data results are shown in the following table:

Table 4 Result of validity test

HSI	SZSE	SSE	Nikkei225	Nasdaq	CAC40	DAX40	DowJones	S&P500	FTSE100
5.60%	4.50%	4.80%	5.50%	4.90%	5.10%	5.50%	4.70%	4.80%	5.10%

The posterior test results show that the EVT-Copula-CoVaR model fits the correlation structure between the gold market and the stock market better. Financial institutions and regulatory authorities can use this model method to effectively evaluate the direction and intensity of risk spillovers when risk events occur in other financial institutions (or financial markets), and further improve risk management decision-making capabilities.

5. Conclusion

Through the above research, it has been found that the EVT-Copula model can effectively fit the relevant structure of financial markets under extreme market conditions. This article combines the analysis characteristics of the two models to construct the EVT-Copula-CoVaR model. The generalized Pareto distribution is used to fit the upper and lower tails of each stock market index return sequence, while the data in the middle of the upper and lower tails of the stock index return sequence is fitted with an empirical distribution. According to the Kendall τ correlation coefficient and the upper and lower tail correlation coefficients (mainly focusing on the lower tail correlation coefficient), qualitative analysis of the risk spillover effects of gold price fluctuations on major stock indexes. The analysis based on this model shows that the gold market has a certain risk spillover effect on the world's major stock markets. Model diagnosis and posterior testing show that the model method can effectively measure the risk spillover of a single financial institution (or financial market), which is beneficial to financial institutions, investors and financial regulatory authorities to track changes in systemic risks in a timely manner.

Acknowledgements

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Sectors' product prediction under structural shocks by input-output analysis

Dana Dluhošová, Zdeněk Zmeškal [1]

Abstract

The demand and supply shocks are nowadays the effects of coronavirus phenomena. It influences ways of sectors production prediction used in financial planning, valuation and investment decision. Input-output analysis can be applied. The objective of the paper is the sectors' production prediction under demand and supply economic shocks. The applied methodology is described, and an example of industries sectors production prediction in the Czech economy under economic shocks is calculated. The applied conception was verified.

Keywords

Input-output analysis, prediction, sectors, economic shocks

JEL Classification: C53, C67, D57, L16

1. Introduction

Prediction of sector production is a crucial problem in financial planning, companies valuation, and investment decision-making. Index and proportional methods can be used in stable periods. However, the approach application is complex in demand or supply shocks. These situations happened in economic development nowadays. Coronavirus 2019 occurrence has led to government confine measures causing a demand shock and supply shocks. A sharp demand decrease gives demand shock due to coronavirus in several sectors. Supply shocks are given by insufficient input product delivery of a few sectors caused by the rapid opening economy.

The input-output (Leontief) analysis makes a rational and better production prediction under economic shocks. Two balanced equations can be applied: production (usage) output distribution balance and production (sources) input balance.

The paper's objective is to apply input-output analysis to sectors' production prediction under demand and supply shocks of the Czech economy. The first part is devoted to the input-output method description and data sources. The next part includes the calculation of production prediction under demand and supply shocks in the Czech economy.

2. Methodology and data sources description

We suppose the application of two balance equations.

The output production equation is the following,

$$A \cdot x + y + p = x, \tag{1}$$

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where A is a matrix of unit (direct) consumption coefficients, $a_{ij} = x_{ij} / x_j$, x is sectors' production vector, y is domestic final production vector, p is export production vector, $c = A \cdot x$ is inner system production vector.

The input production equation is formulated as follows,

$$\hat{X} \cdot A^T \cdot e + \hat{X} \cdot \hat{V}^T \cdot e + \hat{X} \cdot \hat{Q}^T \cdot e = \hat{X} \cdot e^T \equiv x, \tag{2}$$

where $\hat{X}$ is a diagonal matrix of vector x, A^T is transpose matrix, e is a unit vector, $\hat{V}$ is a diagonal matrix of unit value-added vector s_v, $\hat{Q}$ is a diagonal matrix of unit exclusive import vector s_q. Consumption of system production vector is $m = \hat{X} \cdot A^T \cdot e$, consumption of value-added vector $v = \hat{X} \cdot \hat{V}^T \cdot e$, consumption of import vector $q = \hat{X} \cdot \hat{Q}^T \cdot e$.

The unit (direct) consumption and distribution vectors are calculated: unit final production vector $y_y = \hat{Y} \cdot \hat{X}^{-1} \cdot e$, unit export production vector $y_p = \hat{P} \cdot \hat{X}^{-1} \cdot e$, unit import consumption vector, $s_q = \hat{Q} \cdot \hat{X}^{-1} \cdot e$, unit value-added consumption vector $s_v = \hat{V} \cdot \hat{X}^{-1} \cdot e$.

The production prediction with a demand shock is derived from (1),

$$x_1 = \left(E - A\right)^{-1} \cdot \left(y_1 + p_1\right), \tag{3}$$

where $B = \left(E - A\right)^{-1}$ is a so-called matrix of complex consumption, E is a diagonal matrix of the unit vector, index 1 means prediction.

The supply shock of input sources is derived consumption of minimal sources (value-added, import). For instance, the supply impact of import is given as follows,

$$x_1 = \hat{S}_q \cdot \hat{Q}_1 \cdot e, \quad \text{here} \quad \hat{Q}_1 = E \cdot q_1 \tag{4}$$

Data of input-output analysis of the Czech economy are taken over from the Czech statistical office Czech Statistical Office | CZSO. The input-output tables are published every five years. The last one is for 2015, in SIOT Supply and use tables Supply and use tables (czso.cz). Applied industry-by-industry tables are in the file, SIOToxo2015s_en.xls. The tables are made for NACE units.

Time-series of sectors chosen indicators are in the section, Indicators of production accounts and generation of income accounts time series, Indicators of production accounts and generation of income accounts time series (czso.cz). Particular time series data are in section ables by industry (A88). Production output data are in Sectors' Production output Output (current prices) (czso.cz), TB0001P1a_EN.xlsx. Value added time series are in Sectors' Gross value-added output Gross value added (current prices) (czso.cz), TB0001B1Ga_EN.xlsx.

The input-output data for the 2019 year are calculated by aggregation of NACE to industry, from input-output table year 2015 and modification by fundamental aggregate values (production, value-added) of 2019year.

3. The Czech economy sectors' production prediction under demand and supply shocks

The objective of the example is the impact calculation of demand and supply shocks on sectors' production. We suppose to know data for the 2019 year with the prediction for the 2020 year, categorised by industry. Furthermore, proportional export and value-added is supposed.

The output equation (3) is for final domestic shock y_1 is modified,

$$x_1 = \left(E - A - \hat{Y}_p\right)^{-1} \cdot \left(y_1\right), \tag{5}$$

subsequently production items c_1, p_1 and consumption items m_1, v_1 and q_1 are calculated.

The input equation (4) for import shock q_1 is computed due to (4),

$$x_1 = \hat{Y}_q \cdot \hat{Q}_1 \cdot e,$$

then particular consumption items m_1, v_1 are calculated. Further items y_1, p_1 and c_1 are calculated as follows,

$$r_1 = y_1 + p_1 = (E - A)^{-1} \cdot (x_1), \tag{6}$$

vectors y_1, p_1 are calculated by proportion by this way, $y_1 = \hat{R}_1 \cdot \left(y_y / \left(y_y + y_p \right) \right)$ and $p_1 = \hat{R}_1 \cdot \left(y_p / \left(y_y + y_p \right) \right)$, and $\hat{R}_1 = E \cdot r_1$.

3.1 Input data

NACE data are aggregated into 18 industry sectors: A Agriculture, forestry and fishing; B Mining and quarrying; C Manufacturing; D Electricity, gas, steam and air conditioning supply; E Water supply; sewerage, waste management and remediation activities; F Construction; G Wholesale and retail trade; repair of motor vehicles and motorcycles; H Transportation and storage; I Accommodation and food service activities; J Information and communication; K Financial and insurance activities; L Real estate activities; M Professional, scientific and technical activities; N Administrative and support service activities; O Public administration and defence; compulsory social security; P Education; Q Human health and social work activities; R Arts, entertainment and recreation; S Other service activities.

Investigated is demand shock, caused by the pandemic situation and government measures to decrease final domestic production in sector I by 20%, sector H by 60% and increase sector J by 20%. A supply shock is given by the lack of input import production of sector C by 30% and F by 20%. Table 1 illustrates the final domestic production and import consumption prediction after predicted shocks.

Table 1: Vectors of domestic final production and import consumption after shocks [mil CZK]

Sectors	y_1	q_1
A	79464	77956
B	2203	144817
C	1266786	2235305
D	112528	55758
E	33800	15946
F	441151	11833
G	364612	28145
H	145363	93087
I	61528	67455
J	293973	67609
K	95365	29072
L	600017	-8921
M	58526	77489
N	102575	75102
O	405426	21872
P	270944	5890
Q	363997	11108
R	89444	7786
S	60553	8021
Total	4848255	3025331

Input data o input-output table for the 2019 year presents Table 2. Unit direct coefficients are calculated, y_y, y_p, s_v, s_q and A, see Table 3 and Table 4.

Table 2: Vectors of input-output items 2019 [mil CZK]

Sectors	x	c	y	p	m	v	q
A	352062	213783	79464	58814	166335	107771	77956
B	204765	191455	2203	11107	29836	30112	144817
C	7982899	3319287	1266786	3396826	3479737	1309868	3193293
D	438127	289195	112528	36405	241437	140932	55758
E	147785	85472	33800	28513	82762	49076	15946
F	969815	501759	441151	26905	660088	294935	14791
G	1163092	721455	364612	77024	562914	572033	28145
H	824950	432969	181704	210277	442221	289642	93087
I	296819	63441	153819	79559	121368	107996	67455
J	655533	273127	244978	137428	257432	330492	67609
K	421517	306732	95365	19419	181060	211384	29072
L	874158	268004	600017	6137	398189	484890	-8921
M	706441	550003	58526	97912	354336	274617	77489
N	383175	231715	102575	48885	201291	106782	75102
O	456418	42115	405426	8877	124027	310519	21872
P	314087	31589	270944	11553	63710	244486	5890
Q	396356	20284	363997	12075	134658	250591	11108
R	132338	35403	89444	7491	68444	56108	7786
S	106964	31493	60553	14918	39438	59505	8021
Total	16827299	7609283	4927892	4290124	7609283	5231739	3986277

Table 3: Vectors of direct unit coefficients

Sectors	y_y	y_p	s_v	s_q
A	0,2257	0,1671	0,3061	0,2214
B	0,0108	0,0542	0,1471	0,7072
C	0,1587	0,4255	0,1641	0,4000
D	0,2568	0,0831	0,3217	0,1273
E	0,2287	0,1929	0,3321	0,1079
F	0,4549	0,0277	0,3041	0,0153
G	0,3135	0,0662	0,4918	0,0242
H	0,2203	0,2549	0,3511	0,1128
I	0,5182	0,2680	0,3638	0,2273
J	0,3737	0,2096	0,5042	0,1031
K	0,2262	0,0461	0,5015	0,0690
L	0,6864	0,0070	0,5547	-0,0102
M	0,0828	0,1386	0,3887	0,1097
N	0,2677	0,1276	0,2787	0,1960
O	0,8883	0,0194	0,6803	0,0479
P	0,8626	0,0368	0,7784	0,0188
Q	0,9184	0,0305	0,6322	0,0280
R	0,6759	0,0566	0,4240	0,0588
S	0,5661	0,1395	0,5563	0,0750

Table 4: Matrix A

Sectors	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
A	0,1021	0,0010	0,0184	0,0080	0,0025	0,0007	0,0087	0,0006	0,0342	0,0002	0,0001	0,0006	0,0014	0,0050	0,0002	0,0004	0,0016	0,0082	0,0013
B	0,0022	0,0124	0,0137	0,1445	0,0048	0,0081	0,0009	0,0016	0,0004	0,0002	0,0002	0,0010	0,0028	0,0012	0,0008	0,0009	0,0004	0,0007	0,0006
C	0,2161	0,0591	0,3012	0,0949	0,1675	0,1747	0,1229	0,0963	0,1417	0,0750	0,0288	0,0837	0,0807	0,0718	0,0305	0,0417	0,1552	0,0716	0,0888
D	0,0085	0,0125	0,0112	0,1852	0,0192	0,0063	0,0095	0,0125	0,0129	0,0048	0,0080	0,0317	0,0069	0,0135	0,0269	0,0272	0,0232	0,0186	0,0178
E	0,0041	0,0014	0,0037	0,0101	0,1358	0,0038	0,0028	0,0016	0,0071	0,0008	0,0008	0,0077	0,0020	0,0098	0,0037	0,0043	0,0058	0,0037	0,0038
F	0,0128	0,0038	0,0040	0,0175	0,0197	0,2804	0,0116	0,0216	0,0136	0,0031	0,0074	0,0896	0,0345	0,0090	0,0469	0,0120	0,0165	0,0193	0,0101
G	0,0557	0,0117	0,0434	0,0237	0,0548	0,0317	0,1088	0,0463	0,0382	0,0294	0,0189	0,0194	0,0495	0,0576	0,0112	0,0104	0,0285	0,0274	0,0323
H	0,0162	0,0290	0,0109	0,0167	0,0125	0,0101	0,0372	0,2521	0,0061	0,0116	0,0229	0,0033	0,0088	0,0553	0,0196	0,0036	0,0053	0,0104	0,0148
I	0,0019	0,0005	0,0008	0,0017	0,0024	0,0008	0,0058	0,0075	0,0155	0,0021	0,0012	0,0046	0,0071	0,0389	0,0061	0,0063	0,0096	0,0105	0,0078
J	0,0037	0,0006	0,0038	0,0057	0,0047	0,0027	0,0182	0,0097	0,0126	0,1938	0,0491	0,0059	0,0279	0,0110	0,0250	0,0172	0,0060	0,0237	0,0308
K	0,0148	0,0024	0,0039	0,0145	0,0178	0,0096	0,0230	0,0222	0,0067	0,0073	0,2056	0,0808	0,0228	0,0263	0,0069	0,0070	0,0119	0,0230	0,0305
L	0,0045	0,0008	0,0027	0,0064	0,0404	0,0239	0,0521	0,0131	0,0611	0,0207	0,0255	0,0530	0,0278	0,0141	0,0142	0,0194	0,0163	0,0443	0,0237
M	0,0199	0,0051	0,0117	0,0121	0,0428	0,1134	0,0526	0,0146	0,0204	0,0244	0,0305	0,0366	0,2066	0,0321	0,0289	0,0151	0,0129	0,0316	0,0158
N	0,0040	0,0036	0,0042	0,0068	0,0164	0,0128	0,0206	0,0178	0,0220	0,0137	0,0159	0,0242	0,0130	0,1675	0,0260	0,0064	0,0080	0,0314	0,0112
O	0,0040	0,0016	0,0008	0,0016	0,0146	0,0003	0,0025	0,0124	0,0018	0,0016	0,0047	0,0020	0,0019	0,0035	0,0163	0,0010	0,0023	0,0049	0,0036
P	0,0009	0,0001	0,0005	0,0011	0,0009	0,0003	0,0016	0,0020	0,0014	0,0016	0,0040	0,0037	0,0041	0,0041	0,0067	0,0258	0,0013	0,0022	0,0029
Q	0,0003	0,0000	0,0003	0,0002	0,0010	0,0002	0,0008	0,0007	0,0013	0,0005	0,0010	0,0017	0,0010	0,0012	0,0010	0,0005	0,0261	0,0032	0,0019
R	0,0002	0,0000	0,0001	0,0001	0,0013	0,0001	0,0005	0,0002	0,0067	0,0004	0,0003	0,0045	0,0012	0,0012	0,0005	0,0006	0,0009	0,1726	0,0208
S	0,0004	0,0000	0,0006	0,0004	0,0008	0,0005	0,0038	0,0033	0,0052	0,0013	0,0047	0,0014	0,0016	0,0024	0,0003	0,0029	0,0079	0,0097	0,0504

3.2 Results and discussion

The impact of both shocks in relative values is presented in Table 5 and Table 6.

Table 5: Demand shock results

Sectors	output items				input items		
	x_1	c_1	y_1	p_1	m_1	v_1	q_1
A	-2,63%	-3,61%	0,00%	-2,63%	-2,63%	-2,63%	-2,63%
B	-1,42%	-1,44%	0,00%	-1,42%	-1,42%	-1,42%	-1,42%
C	-1,36%	-1,88%	0,00%	-1,36%	-1,36%	-1,36%	-1,36%
D	-1,42%	-1,97%	0,00%	-1,42%	-1,42%	-1,42%	-1,42%
E	-1,79%	-2,50%	0,00%	-1,79%	-1,79%	-1,79%	-1,79%
F	-0,84%	-1,58%	0,00%	-0,84%	-0,84%	-0,84%	-0,84%
G	-1,41%	-2,13%	0,00%	-1,41%	-1,41%	-1,41%	-1,41%
H	-9,64%	-5,29%	-20,00%	-9,64%	-9,64%	-9,64%	-9,64%
I	-43,92%	-4,92%	-60,00%	-43,92%	-43,92%	-43,92%	-43,92%
J	11,49%	3,86%	20,00%	11,49%	11,49%	11,49%	11,49%
K	-1,45%	-1,90%	0,00%	-1,45%	-1,45%	-1,45%	-1,45%
L	-1,17%	-3,80%	0,00%	-1,17%	-1,17%	-1,17%	-1,17%
M	-1,37%	-1,51%	0,00%	-1,37%	-1,37%	-1,37%	-1,37%
N	-1,80%	-2,60%	0,00%	-1,80%	-1,80%	-1,80%	-1,80%
O	-0,32%	-3,45%	0,00%	-0,32%	-0,32%	-0,32%	-0,32%
P	-0,16%	-1,50%	0,00%	-0,16%	-0,16%	-0,16%	-0,16%
Q	-0,08%	-1,50%	0,00%	-0,08%	-0,08%	-0,08%	-0,08%
R	-0,96%	-3,39%	0,00%	-0,96%	-0,96%	-0,96%	-0,96%
S	-1,24%	-3,63%	0,00%	-1,24%	-1,24%	-1,24%	-1,24%

Table 6: Supply shock results

Sectors	input items				output items		
	x_1	m_1	v_1	q_1	c_1	y_1	p_1
A	0,00%	0,00%	0,00%	0,00%	-20,63%	31,90%	31,90%
B	0,00%	0,00%	0,00%	0,00%	-17,93%	257,96%	257,96%
C	-30,00%	-30,00%	-30,00%	-30,00%	-22,75%	-35,16%	-35,16%
D	0,00%	0,00%	0,00%	0,00%	-9,73%	18,90%	18,90%
E	0,00%	0,00%	0,00%	0,00%	-11,34%	15,56%	15,56%
F	-20,00%	-20,00%	-20,00%	-20,00%	-12,75%	-27,78%	-27,78%
G	0,00%	0,00%	0,00%	0,00%	-15,25%	24,91%	24,91%
H	0,00%	0,00%	0,00%	0,00%	-6,46%	7,13%	7,13%
I	0,00%	0,00%	0,00%	0,00%	-3,39%	0,92%	0,92%
J	0,00%	0,00%	0,00%	0,00%	-3,54%	2,53%	2,53%
K	0,00%	0,00%	0,00%	0,00%	-3,63%	9,71%	9,71%
L	0,00%	0,00%	0,00%	0,00%	-4,15%	1,83%	1,83%
M	0,00%	0,00%	0,00%	0,00%	-9,08%	31,91%	31,91%
N	0,00%	0,00%	0,00%	0,00%	-5,45%	8,34%	8,34%
O	0,00%	0,00%	0,00%	0,00%	-4,67%	0,47%	0,47%
P	0,00%	0,00%	0,00%	0,00%	-3,78%	0,42%	0,42%
Q	0,00%	0,00%	0,00%	0,00%	-4,28%	0,23%	0,23%
R	0,00%	0,00%	0,00%	0,00%	-0,74%	0,27%	0,27%
S	0,00%	0,00%	0,00%	0,00%	-5,06%	2,11%	2,11%

Demand shocks of three sectors close to the Czech Republic's situation (H Transportation and storage; I Accommodation and food service activities; J Information and communication) substantially influenced their production (-9,64%, -43,92%, 11,49%). The production of other sectors decreased by 1,5% and are mutually influenced. The sectors inner distribution and export are also changed because of the mutual relationship of the sectors. Input sources consumption and value-added are also decreased in all sectors, and level is proportional to production decrease.

The supply shocks are given by a decrease of the exclusive import of two sectors, Manufacturing (C) and Construction (F), by 30% and 20%. This situation is similar to pandemic influence in the Czech economy because of the lack of electronic parts for the automotive industry and construction materiál inventories. The production is proportional to sectors import, and analogically the value-added. Interesting is that sectors inner product distribution, final domestic production, and export are decreased in a very variable way. So mutual relationship among sectors is sufficient for outputs of sectors.

4. Conclusion

The paper was devoted to predicting sectors production under demand and supply shock caused by pandemic phenomena influencing economic and social life. The input-output methodological instrument was described and analysed. Distribution (output) and consumption (input) balanced equations were formulated commonly and modifiable. Reasonable prediction of sectors production reflecting pandemic effect in the Czech Republic in industrial categorisation was made. Results confirmed the suitability of applying described methodological concept for economic shocks.

The economy is understood as a complex, mutually connected system, and input-output analysis can be a suitable instrument for modelling the situation introduced. The more precise production prediction can be successfully used in financial planning, valuation and investment decision of sectors and companies.

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[5] The Czech statistical office Czech Statistical Office | CZSO.

[6] SIOT Supply and use tables Supply and use tables (czso.cz).

[7] Indicators of production accounts and generation of income accounts time series, Indicators of production accounts and generation of income accounts time series (czso.cz).

[8] Production output data are in Sectors' Production output Output (current prices) (czso.cz)

[9] Value added time series are in Sectors' Gross value-added output Gross value added (current prices) (czso.cz)

Open problems about flexible consumers in eletricity markets

Ruth Domínguez[1]

Abstract

To attain a sustainable energy system, two elements are fundamental: renewable sources and energy efficiency. Renewable sources require a large installed capacity and a high flexibility level of the power system to face the variability of the energy sources. Small consumers are promoted to take part in the green energy transition to contribute to the energy efficienty by installing solar panels for self-generation, reducing their consumption, adapting part of their electricity consumption to the system needs, etc. Therefore, one of the main issues in this area of research is to analyse how the participation of flexible consumers may influence the outcomes of other electricity market agents and the market itself. Different decision-making problems must be considered and many uncertain parameters are involved in this type of models: the electricity prices, the renewable power availability, the electricity demand, etc.

Keywords

Decision-making Problems, Electricity Markets, Flexible Consumers, Retailer, Stochastic Programming

JEL Classification: C61, D47, Q41

1. Introduction

Climate change is a major threat humanity is facing, whose origin resides in burning fossil fuels. To mitigate the effects of climate change and turn the economic system into a sustainable one, the European Green Deal targets at a climate neutral continent by 2050. Reaching this goal passes through two main issues: to develop a circular economy such that the resources are efficiently managed and to transform the energy system that leads to net zero carbon emissions. In the energy system, the electricity generation, distribution, and consumption play a key role. On one hand, the electricity sector is responsible of at least 30% of the greenhouse gas (GHG) emissions. On the other hand, the versatility of this form of energy and the possibility of generating electricity using renewable sources make the electricity be the main driver to the energy system decarbonization.

In the "Clean energy for all Europeans" package (so-called "Winter Package") published by the European Commission in 2018, the use of renewable energies, the energy efficiency, and the participation of the consumers in the energy transition are highly encouraged. The development of solar technologies, batteries and smart grids allow small consumers to reduce their energy consumption from the grid, while at the same time they can provide auxiliary services to the system in exchange of economic benefits. In this way, flexible consumers are able to manage their electricity consumption and contribute to the green energy transition. However, to allow the participation of new agents, the electricity markets also need to implement deep transformations. In this package, the European Commission also establishes as

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an important goal to keep on integrating the electricity market among all European countries. The modifications in the electricity markets must consider the variability of renewable sources since achieving renewable-dominated electric energy systems is crucial to mitigate the climate change effects. Intermittent renewable sources, such as wind and solar power, introduce high uncertainty in the system operation because of their variability and their lack of controllability. Clearly, this is a highly complex problem, which requires to be faced from a threefold viewpoint: Technical, Economical, and Financial.

Considering this context, it is important to analyze how the participation of flexible consumers influences the outcomes of the electricity markets. A flexible consumer is considered as a consumer of electricity that can adjust its consumption to some extent according to the system needs or electricity prices. A flexible consumer may be a residential consumer that has available solar photovoltaic panels, batteries, an electric vehicle, or simply is decided to manage its electricity consumption according to the price signals. Other type of flexible consumers could be industries, businesses, farms, etc. that are able to manage their electricity consumption and may also have electricity generation units. These types of activities are framed in the Demand Side Management (DSM).

Small consumers can purchase their electricity needs through a retailer or joining an association of small consumers and participate directly in the market. The original business of an electricity retailer (ER) was just to sell electricity to clients and purchase the required energy in the available electricity markets. However, in the current market framework the ER must face new challenges to optimize their profit, i.e., to efficiently manage both the clients' consumption and the clients' production while dealing with the uncertainty of the electricity prices. Similarly, a cooperative of small consumers willing to participate in the electricity markets must face the uncertainty related to the prices and their own consumption and possibly renewable generation. Those represent interesting decision-making problems that can be formulated using stochastic optimization or other methodologies based on establishing a relation between the uncertain parameters and the decision variables.

The DSM and the Demand Response (DR) tools have been widely studied in the last years. A list of representative papers in this field follows. In Behboodi et al. (2016) the price of the demand elasticity as a function of the wind penetration is analyzed. In Asensio, Muñoz-Delgado and Contreras (2017) a bi-level approach is proposed to determine the expansion decisions in distribution networks and renewable power considering the DSM and the uncertainty of the demand and the renewable production. Anjo et al. (2018) presents an evaluation of the impact of the DSM on the capacity expansion of the power system in Portugal. No uncertainty is considered and the proposed model is solved using a commercial software. The authors conclude that the DSM allows to reduce the capacity to be built to reach a low carbon system. In Dvorkin (2018), an equilibrium problem to determine the investment decisions in storage capacity considering the presence of DSM in a competitive market is presented. The results measure how much the DSM, the investment costs and the storage efficiency influence the investment decisions in storage capacity. On the other hand, Zappa, Junginger and von den Broek (2019) provides a wide analysis of the technical and economic feasibility of achieving a fully European power system. A commercial software is used to carry out the numerical analyses. Lyncha et al. (2019) presents an equilibrium model with a demand response aggregator to analyze the influence of the DSM on the capacity markets. Energy, reserve capacity and investment decisions are considered. In this line, Domínguez and Carrión (2019) proposed a model that allows to determine the investment decisions in generating and storage capacity considering the constraints related to the DSM. Secondly, a model representing the daily operation of the electricity market is used to analyze the system operation. One of the conclusions obtained is that the presence of storage units highly influences the participation of consumers through DR.

The following papers are more related with the electricity supply for consumers. In Carrión et al. (2007), the authors propose a stochastic optimization problem to determine the supply strategy of a large consumer consider the CVaR as risk measure. Carrión, Gotzes and Schultz (2009) tackles the problem of a retailer that supplies electricity to some consumers and participates in the future and daily markets. Stochastic dominance is applied to define the best participation in the markets considering benchmarks acceptable for the retailer. In Conejo, Morales and Baringo (2011), it is presented an optimization model to determine the real-time demand response of a consumer according to the price signals. Robust optimization is applied to model the price uncertainty. Zugno et al. (2013) proposes a bi-level model for retailers' participation in markets with demand response considering the Stackelberg relationship between retailers and consumers. Nojavan, Zare and Mohammadi-Ivatloo (2017) proposes a robust approach to define the prices to be offered by the retailer to the consumers. In Münzel et al. (2019) it is analyzed how the financial incentives influence electric vehicles sales. These are just examples among the multiple papers tackling the problem of electricity retailers and consumers considering a DSM framework.

Even though the technical literature about the participation of consumers and retailers in electricity markets is wide, the new framework characterized by reaching net-zero GHG emission electricity generation opens multiple questions that are still to be analyzed. This work aims at summarizing some of these problems, providing a sort of to-do list for researchers in the area.

2. Context, open problems and some hints

2.1 Context

The decision framework describing the context of the problems that must be tackled reads as follows:

1. It is assumed that the electric energy system where consumers and retailers acquire their electricity needs comprises a large capacity of intermittent renewable power plants, such as wind, solar, and run-of-river hydro units. With intermittent renewable power we refer to the electricity generation coming from renewable sources whose availability is variable along the day and the year and is difficult to predict in the middle- and short-term. This electricity generation introduces high uncertainty on the system operation and on the market agent's participation due to its influence on both the market prices and the generation availability.
2. Smart grids are assumed to be available such that there is a bi-directional communication between the producers/system operator and the consumers.
3. It is assumed that DSM technologies are available, as well as the fact that consumers have the possibility to respond to the price signals and to install renewable capacity for self-consumption.
4. The market is considered to be operated by an entity whose role consists of managing the consumer bids and the producer offers and, in the last extent, to determine the electricity price in a certain period of time.
5. Renewable producers have the possibility to sign a Power Purchase Agreement (PPA) with an electricity retailer (ER), with a large consumer or a cooperative of small consumers.

2.2 Open problems

Considering the above, we can identify at least 5 main problems that are important to address.

Problem 1. To model the participation of an ER in the electricity markets considering a context with DSM and high intermittent renewable production.

Problem 2. To model the characteristics of different flexible consumers and their participation in the electricity markets, especially in the auxiliary services markets.

Problem 3. To apply stochastic dominance constraints to determine the best bidding strategy of an ER, a large consumer, or an association of consumers considering the characteristics of flexible consumers.

Problem 4. To model the decision-making problems involving ER or large consumers under uncertainty applying an approach based on linear decision rules.

Problem 5. To analyze the economic outcomes of the ER and the consumers along the year considering different DSM frameworks.

2.3 Some hints

To address the problems above, we believe that there exist several tools that can allow to efficiently solve the decision problems faced by electricity retailers and consumers considering uncertainty sources. These tools are based on the formulation and resolution of problems of mathematical programming and allow to analyze the economic outcomes obtained by the market participants.

Stochastic optimization, which is based on characterizing the random parameters considering their probability distributions, see Birge and Louveaux (1997). This technique allows to generate discrete values representing the probability distributions of the random parameters, as well as incorporating risk measures such as the CVaR or the stochastic dominance. Conejo, Carrión and Morales (2010) widely describes how to model problems of electricity markets under uncertainty.

Stochastic dominance relations, which allow to establish a ranking between distributions, see Levy (2006). This is different from other techniques that compare only a finite set of moments of the distribution, e.g. the expected value, the variance, the kurtosis, the skewness, etc., because stochastic dominance relations consider the entire distributions, which is particularly interesting when it is important to account for all the quantiles of the distributions. There exist already some works that apply stochastic dominance in capacity expansion problems, see e.g. Domínguez et al. (2021). Still, this technique has not been widely applied in electricity market problems, and hence, its application to the problems described above will represent a relevant contribution.

Decision rules, which assume a dependency between the decision variables and the random parameters Kuhn, Wiesemann and Georghiou (2011). Note that to include a precise representation of the random parameters using a set of scenarios leads to very large problems that may be computational intractable. In those cases, it may be convenient to apply a decision rule approach to represent the random parameters. Domínguez, Carrión and Conejo (2021) present a comparison among different approaches dealing with this issue.

Decomposition techniques, which allow to efficiently solve large-scale problems. Due to the size of the problem when a large number of scenarios or constraints are included, decomposition techniques can be applied such that the original problem can be decomposed in smaller ones easier to solve. A detailed description of decomposition techniques can be found in Conejo et al. (2006).

Clustering algorithms, used to select a precise representation of the random input data. To achieve computational tractability, clustering algorithms, as those proposed in Baringo and Conejo (2013) and Domínguez and Vitali (2021), are commonly used in power systems problems with high penetration of renewable sources due to their ability to generate a set of representative periods of the operation variability.

Bilevel programming, which is appropriate for modelling problems where there is a decision maker (leader) that optimizes its objective function (first level problem) considering that another decision maker (follower) will react optimizing its own objective function (second level problem), see Dempe (2002). This modelling approach can be very adequate to determine the bidding strategy of a retailer or an association of consumers that participate in the electricity markets.

3. Conclusion

In this work, we provide an extended literature review for problems related to the decision-making problems of electricity consumers and retailers. We describe the framework in which these problems may be solved and some mathematical tools and methodologies useful to model problems with multiple uncertainty sources. There are multiple open problems that scholars are nowadays tackling and in this paper a list of some of those problems is provided. Finally, we provide some ideas about the mathematical instruments that could be the most effective in formulating, solving and analyzing these problems.

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Rate of Growth Convergence in Two-stage DCF Model[1]

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Abstract

In this paper, we investigate the possibilities of the rate of growth convergence in two stages DCF model. The goal of the paper is to violate simplified and unrealistic assumption of jump change of the rate of growth between first and second stages. We compare the difference in the resulted values while using of classical two stages model and model with chosen growth rate function (linear, quadratic, logarithmic and exponential function) in first stage. We also demonstrate the importance of the convenient growth rate function depending on the length of the first stage.

Key words

Assets valuation, two-stage DCF models, growth rate modelling

JEL Classification: G12; G32

1. Introduction

Assets valuation is one of the basic aspects of financial markets, which always have paramount importance to investors. Thera are a lot of different approaches, which can be used for assets valuation. In this paper, we will deal with discounted cash flow (DCF) method.[3] The basic model in this area was firstly proposed by Gordon (1962). This model was so called one-stage model and it was based on constant dividend (free cash flow, FCF) growth rate. This model is pretty simple and straightforward, since it requires only estimates of two parameters (FCF growth rate and discount rate). Nevertheless, it also has some serious limitations. It is suitable only for companies with a long run stable FCF. Moreover, the assumption of constant growth rate of the FCF forever is not realistic. A lot of models were introduced to relax this simplified assumption. As a main work in this area we can consider the research of Malkiel (1963), where two-stage model was introduced. This model has first n years of extraordinary growth followed by a stable growth forever. It means, that value of asset can be obtained as the sum of first stages present values plus a discounted value of the general Gordon growth model at year n. This model is suitable for valuing companies, which expect to have higher (or lower) a growth rate in initial (first) stage than normal (long term, sustainable) growth rate. This can occur because of a specific investment or patent right, that can result in higher profit, or just because of a stage of a company's life cycle. Limit of this model is in the sharp drop (rise) of a growth rate from the first to second (stable) stage. To

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[3] Originally referred as dividend discount model (Gordon, 1962). We will use the term DCF model in the paper, as it is more general.

avoid this limitation, Fuller and Hsia (1984) propose an H-model, which works with a linear decline of the growth rate. This model was of fundamental importance especially in the precomputing era, as it reduced the number of parameters while maintaining the analytical solution. However, the result of the model was only an approximation. We can also mention here a three-stages DCF models, which can estimate an asset's value more precisely, however it requires a larger number of inputs. An empirical comparison of the basic Gordon model and its variations can be found in Sorensen and Williamson (1985). Conclusions of this work suggest that the increased complexity of the model improves the results (the 3-stage model shows the best results). Some other extensions were proposed by Brooks and Helms (1990), who propose an N-stages model with quarterly FCF and fractional period, Barsky and De Long (1993), who propose a model with permanent growth calculated as geometric average of past FCF changes, or Donaldson and Kamstra (1996), who worked with a Monte Carlo simulation and numerical integration of random joint process of FCF growth and discount rate. Extended reviews of the basic DCF model and its extensions can be found in Kamstra (2003) and Damodaran (2012).

In this paper, we will work with basic two-stage DCF model. We will start from the basic model with sharp drop (rise) in FCF growth rate and then we will use some chosen function (linear, quadratic, logarithmic and exponential) to model a more realistic evolution of the growth rate in first stage of asset life. Then we will analyse an effect of the length of the first stage in estimating the value using a basic and more realistic development of the growth rate.

2. Two-stage DCF model methodology

We will start with the basic one-stage valuation DCF model proposed by Gordon (1962):

$$V=\sum_{t=1}^{\infty}\frac{FCF\cdot\left(1+g\right)^{t-1}}{\left(1+R\right)^{t}}=\frac{FCF}{R-g}, \tag{1}$$

where V is the value of an asset, t is particular period, g is constant growth rate and R is discount rate.[4]

Assuming that the expected lifetime of an asset can be divided into two phases according to the FCF growth rate, the value of an asset can generally be estimated as:

$$V=FCF_0\cdot\left\{\sum_{t=1}^{T}\frac{\prod_{i=1}^{t}\left(1+g\left(i\right)\right)}{\left(1+R\right)^{t}}+\frac{\left[\prod_{i=1}^{T}\left(1+g\left(i\right)\right)\right]\cdot\left(1+g_B\right)}{\left(R-g_B\right)\cdot\left(1+R\right)^{T}}\right\}, \tag{2}$$

where FCF_0 is initial free cash flow in time 0, T is length of the first stage, i is particular period in the first stage, $g\left(i\right)$ is a growth rate function in first stage and g_B is sustainable growth rate in second stage.

Next, we will consider different $g\left(i\right)$ in first stage: *i)* $g\left(i\right)$ is a constant function, *ii)* $g\left(i\right)$ is a linear function, *iii)* $g\left(i\right)$ is a quadratic function, *iv)* $g\left(i\right)$ is a logarithmic function, *v)* $g\left(i\right)$ is an exponential function.

[4] For simplification, we will assume that R is constant and lower than g for the whole paper.

2.1 Constant function

Assuming that $g(i) = g_A$, where g_A is a constant growth rate in the first stage, we can rewrite (2) as:

$$V = FCF_0 \cdot \left[\sum_{t=1}^{T} \left(\frac{1+g_A}{1+R} \right)^t + \frac{(1+g_A)^T \cdot (1+g_B)}{(R-g_B)\cdot(1+R)^T} \right] \tag{3}$$

and after some rearrangements we can get Malkiel (1963) two-stage valuation formula:

$$V = FCF_0 \cdot \left(\frac{1+g_A}{R-g_A} \right) \cdot \left[1 - \left(\frac{1+g_A}{1+R} \right)^{T-1} \cdot \frac{(g_A - g_B)}{(R-g_B)} \right]. \tag{4}$$

Recall that in (4) we have a jump change between g_A and g_B in time T, which is an unrealistic assumption in real world.

2.2 Linear function

If we assume linear function $g(i) = ai + b$, then after estimating the parameters[5], we get $g(i) = \frac{g_B - g_A}{T} \cdot i + g_A$ and substituting into the (2):

$$V = FCF_0 \cdot \left\{ \sum_{t=1}^{T} \frac{\prod_{i=1}^{t} \left(1 + \frac{g_B - g_A}{T} \cdot i + g_A \right)}{(1+R)^t} + \frac{\left[\prod_{i=1}^{T} \left(1 + \frac{g_B - g_A}{T} \cdot i + g_A \right) \right] \cdot (1+g_B)}{(R-g_B)\cdot(1+R)^T} \right\}. \tag{5}$$

2.3 Quadratic function

If we assume quadratic function $g(i) = ai^2 + bi + c$, then after estimating parameters, we get $g(i) = i \cdot \left(\frac{g_B - g_A}{T} \right) \cdot \left[1 - k \cdot \left(\frac{i}{T} - 1 \right) \right] + g_A$ for $k \in \langle -1;1 \rangle$, where $k \in (0;1\rangle$ means convex shape of function and $k \in \langle -1;0)$ means concave function (and $k = 0$ leads to the linear function, obviously). After substituting $g(i)$ into the (2), we obtain following valuation formula:

$$V = FCF_0 \cdot \left\{ \sum_{t=1}^{T} \frac{\prod_{i=1}^{t} \left\{ 1 + i \cdot \left(\frac{g_B - g_A}{T} \right) \cdot \left[1 - k \cdot \left(\frac{i}{T} - 1 \right) \right] + g_A \right\}}{(1+R)^t} + \right.$$

$$\left. + \frac{\left\{ \prod_{i=1}^{T} \left\{ 1 + i \cdot \left(\frac{g_B - g_A}{T} \right) \cdot \left[1 - k \cdot \left(\frac{i}{T} - 1 \right) \right] + g_A \right\} \right\} \cdot (1+g_B)}{(R-g_B)\cdot(1+R)^T} \right\}. \tag{6}$$

[5] For $A[0; g_A]$ and $B[T; g_B]$.

2.4 Logarithmic function

If we assume logarithmic function $g(i)=a\cdot\ln(i+b)+c$, then after estimating parameters, we get $g(i)=\frac{g_B-g_A}{\ln\left(\frac{T}{b}+1\right)}\cdot\ln\left(\frac{i}{b}+1\right)+g_A$ for $b\in(0;\infty\rangle$. $g(i)$ it is a convex function and higher value of b leads closer to the linear function. After substituting $g(i)$ into the (2), we obtain following valuation formula:

$$V=FCF_0\cdot\left\{\sum_{t=1}^{T}\frac{\prod_{i=1}^{t}\left[1+\frac{g_B-g_A}{\ln\left(\frac{T}{b}+1\right)}\cdot\ln\left(\frac{i}{b}+1\right)+g_A\right]}{(1+R)^t}+\frac{\left\{\prod_{i=1}^{T}\left[1+\frac{g_B-g_A}{\ln\left(\frac{T}{b}+1\right)}\cdot\ln\left(\frac{i}{b}+1\right)+g_A\right]\right\}\cdot(1+g_B)}{(R-g_B)\cdot(1+R)^T}\right\} \tag{7}$$

2.5 Exponential function

If we assume exponential function $g(i)=a\cdot e^i+b$, then after estimating parameters, we get $g(i)=\frac{g_B-g_A}{e^T-1}\cdot\left(e^i-1\right)+g_A$. After substituting $g(i)$ into the (2), we obtain following valuation formula:

$$V=FCF_0\cdot\left\{\sum_{t=1}^{T}\frac{\prod_{i=1}^{t}\left[1+\frac{g_B-g_A}{e^T-1}\cdot\left(e^i-1\right)+g_A\right]}{(1+R)^t}+\frac{\left\{\prod_{i=1}^{T}\left[1+\frac{g_B-g_A}{e^T-1}\cdot\left(e^i-1\right)+g_A\right]\right\}\cdot(1+g_B)}{(R-g_B)\cdot(1+R)^T}\right\} \tag{8}$$

3. Illustrative applications

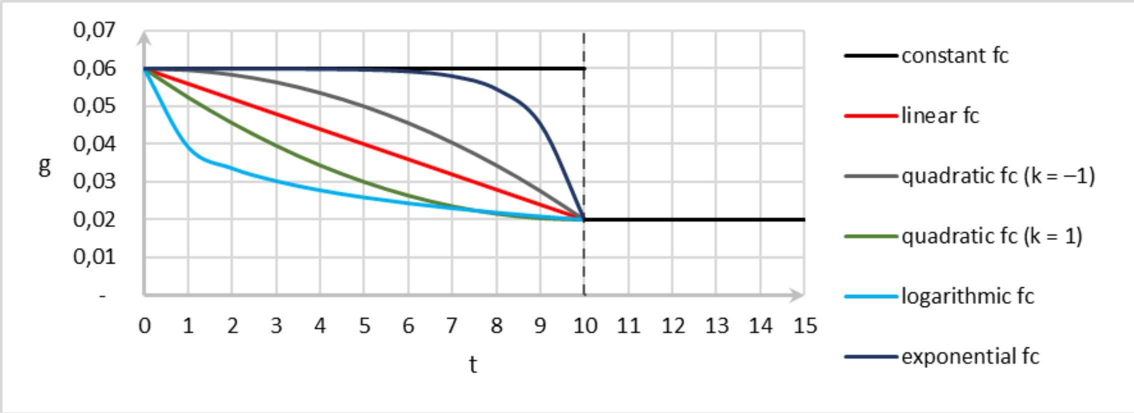
We will demonstrate different approaches to modeling FCF growth rates in the first stage using an illustrative example. As is clear from (4) to (8), in the case of a model with a functional development of the FCF growth rate, the determination of value is given by five parameters: FCF_0, g_A, g_B, T and R. The input data for the illustrative example are recorded in Table 1.

Table 1: Input data

g_A	g_B	FCF_0	R	T
0.06	0.02	100	0.1	10

We also have to set a variable parameter for quadratic and logarithmic function. We will use $k=-1$ and $k=1$ for quadratic function (to capture both convex and concave growth rate development) and $b=0.1$ for logarithmic function. By selecting the appropriate function and level of the variable parameters, it is possible to capture how long the asset (firm) is able to maintain the initial growth rate in the first phase, which is evident in Figure 1. It is obvious that the use of the logarithmic/exponential function is suitable if we assume the approximation of both growth rates in a relatively short time interval.

Figure 1: FCF growth rate development for given functions



The value of the asset (company) depending on the FCF growth rate function is shown in Table 2.

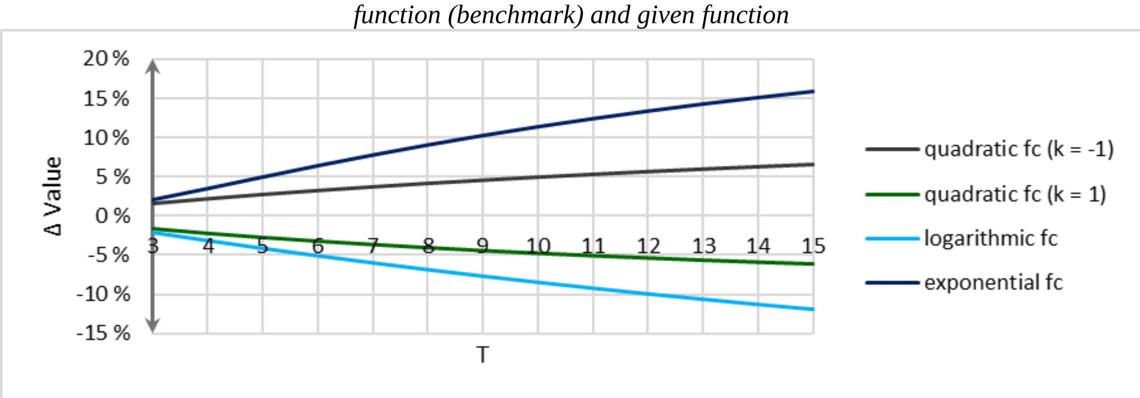
Table 2: Value of asset (company) for given functions

function	*equation*	*value of asset*
constant	(4)	1701
linear	(5)	1474
quadratic (*k=-1*)	(6)	1548
quadratic (*k=1*)	(6)	1405
logarithmic	(7)	1349
exponential	(8)	1642

It is clear from Table 2 that the use of a simplified two-stage valuation model with a constant development of the FCF growth rate in the first stage can lead to significant deviations in the estimated value.

Next, a sensitivity analysis for different length of the first stage is performed (see Figure 2). The value of the asset (company) estimated for a given length of the first stage and the corresponding growth rate function is always compared with the value of the asset obtained from the model with a linear function. The aim of this analysis is to determine whether it is sufficient to use a linear function to model the FCF growth rate when estimating the value of an asset (company), or whether it is desirable to use more complex functions with the assumed specific development of the growth rate.

Figure 2: Influence of the length of the first stage on the deviation between the value estimated using linear function (benchmark) and given function



It is clear from the results that in the case of the quadratic function, the values of the company always deviate by about 5%. When using logarithmic and exponential function, it can be observed that if the length of first stage is up to 6 years, then deviations of up to 5% are achieved. Deviations are more pronounced for longer length of the first stage, especially for more than 10 years, which, however, is not very common in practice. We can therefore conclude that the use of more complex functions plays a more important role, especially in the assumption of a specific development of the growth rate, which deepens with the increasing length of the first stage.

4. Conclusion

In the paper, possibilities of the FCF growth rate modelling within first stage of two-stage DCF method was developed and investigated. We focused on the violation of the simplified and unrealistic assumption of sharp drop (rise) of the growth rate between first and second stages, which is often used in the basic two-stage valuation model. We used a different type of function for modelling the FCF growth rate in the first stage: linear, quadratic, logarithmic and exponential, particularly. We found out that the use of a simplified two-stage valuation model with a constant development of the FCF growth rate in the first stage can lead to significant deviations in the estimated value. We also concluded that the use of more complex functions plays a more important role, especially in the assumption of a specific development of the growth rate, which deepens with the increasing length of the first stage.

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Intragroup Transactions and their Disclosure – a Case of Czech Developer Enterprises

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Abstract

The current business environment is generally characterised by the growing power and importance of multinational corporations, or holdings as such. This obviously entails a series of related questions and problems – including those relating to taxes. This area also poses challenges for research. The aim of this article is, on the basis of publicly accessible documents, to determine what types of transactions with related parties are carried out by entities engaged in development activity. The research made is based on a qualitative research based on the content analysis of the relevant documents. This article as such focuses on a partial set of problems, being the evaluation of the availability of publicly accessible information for the identification of transactions and also for the performance of research relating to the types of transactions carried out by development companies (namely members of the Asociace developerů, z.s (Developers Association)). With regards to the first of these, it may be concluded that publicly accessible information in the Czech Republic is very far from the standard of that submitted to the financial authorities in terms of its scope and form. In the context of the latter we may conclude that key transactions include the provision of credit financial instruments, services and leases, which is undoubtedly related to the specific position and role of development companies within the holding.

Key words

intragroup transactions, real estate, disclosure, tax management, financial statements

JEL Classification: H25, L74, M40

Introduction

It is clear that the issue of transfer prices is a highly topical and, in many respects, a problematic theme, because the volume of transactions between related entities have dramatically increased (Pistone, et al., 2019). In a way, this is also declared by the creator of the most widespread, most comprehensive and globally accepted standards – i.e., the OECD (2017). The topicality and considerable number of problematic aspects associated with transfer prices are also evidenced by the growing number of publications not only in specialised peer-reviewed journals and professional monographs abroad (for a summary, see e.g., Brychta et al., 2020). Transfer prices, which are used to value transactions (domestic and foreign) between related entities for tax purposes, are classed as areas of tax risks that are subject to tax audits (for some aspects see, for instance, Melnyk, 2017). At the same time, it is worth mentioning that the issue of transfer pricing is a complex and complicated one that can be viewed from many perspectives (Padhi, 2019). There are undoubtedly many types of transactions realized between related entities. The most common transactions between related

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entities seem to be sale of goods, the sale of fixed assets, the provision of licences, credit, loans, managerial or administrative services, the lease of production facilities and real estate rental (Vincencová, 2016). Special attention is also given to transactions where trademark providers and shared service centres are residents of low taxation countries (Vincencová, 2016).

The types of transactions and their taxonomy from the perspective of the types of entities and the transactions typical for those entities are doubtless worth exploring. In the conditions of the Czech Republic, potential sources of information in this respect are the financial statements and report on relations. Another potential source of valuable information is the annexe to the tax return, although that is intended solely for the financial authorities.

Financial statements are governed by Act No. 563/1991 Coll., on Accounting, as amended ("AA") and consist of the balance sheet, the profit and loss statement and also the notes (Section 18 (1) AA). Certain accounting entities are obliged to have them verified by an auditor. This obligation is based on the assessment of three criteria, i.e., the total value of the assets, the net annual turnover and the average recalculated headcount (Section 20 (3) AA). The content of entrepreneurs' financial statements is governed by Decree No. 500/2002 Coll., a decree that implements certain provisions of Act No. 563/1991 Coll., on Accounting, as amended, for accounting entities that are entrepreneurs using the double-entry bookkeeping system, as amended (the "Decree"). The most importance source of information as regards this topic are the notes to the financial statements. This contains information on short-term liabilities, liabilities to associates, loans, credit provided, collateral and other types of performance. The notes also include data not stated in the balance sheet, such as contingent liabilities, guarantees provided, etc. (Section 39 of the Decree).

Small and micro accounting entities, which are obliged to have their financial statements verified by an auditor, as well as medium-sized and large accounting entities, present information on transactions concluded between the accounting entity and a related party, and which were not concluded under normal market conditions (Section 39a and Section 39b of the Decree). They give the volume of the transactions, the nature of relations with the related party and other information essential to understanding the financial situation. Information can be grouped, with the exception of cases where separate information is essential in order to understand the impact of the transactions on the financial situation. The information stated includes total revenues with related entities, total purchases from related entities, liabilities to related entities, the relationship to the company, liabilities not stated in the balance sheet (guarantees provided, pledged assets, liabilities associated with the construction or purchase of property, liabilities to third parties, etc.).

One specific document is the **report on relations**, covered by Act No. 90/2012 Coll., on Business Corporations, as amended ("BCA"). This document contains information about the structure of relations in the group, the role of the controlled entity in the structure, and the methods and means of control. It also contains a list of selected negotiations on mutual contracts between related parties (Section 82 (2) BCA). Since 2021, the report does not provide information subject to protection or confidentiality, such as information violating bank secrets or information leading to market abuse (Section 85 (5) BCA). However, the report must state that this information is missing. With regards to information classed as trade secrets, a reasonable degree of generalisation is given (Section 85 (6) BCA). One new provision from 2021 is the verification of the report on relations by an auditor. This applies only to entities that are obliged to compile an annual report (Section 83 (4) BCA), or have their financial statements verified by an auditor.

One specific and, due to the non-public nature of the tax administration, publicly-inaccessible source of information is **the annexe to the tax return**, which identifies related parties, completed transactions and their value. The annexe to the tax return, unlike the data published in the report, also contains specific types of transactions (Nejvyšší kontrolní úřad, 2018). The obligation to file the annexe depends on the size of the enterprise, not the volume of transactions with the related party (Nejvyšší kontrolní úřad, 2018). Four areas of transactions are reported; for more details, see Table 1.

Table 1: Section of Annexe to Tax Return form

Area	Comprising
Section A	Purchase / sale of fixed assets, purchase / sale of stocks, revenues from the sale of own products and services.
Section B	Services, licence fees (incl. software), interest, leases and other transactions.
Section C	Credit financial instruments, shares in profit and other equity components. Provision/receipt of gratuitous performance, use of cash-pooling, financial and bank guarantees provided/received.
Section D	Long-term/short-term receivables and liabilities without credit financial instruments.

Source: compiled by author using (Finanční správa, 2021).

1. Aim and Methods of Research

The aim of this article is, on the basis of publicly accessible documents, to determine what types of transactions with related parties are carried out by entities engaged in development activity, which is a highly specific and, in functional terms, a very important and specific activity. The subject of the research comprised published financial statements and reports on relations relating to the period of 2018. The research focused on entities that are members of Asociace developerů, z.s., which as of 30 April 2021 comprised 33 members. Members that had not published the relevant documents were excluded. The final list thus includes a total of 30 entities researched. The content analysis method was used, which can be applied to various types of text data (Krippendorff, 2004). The benefits of this research can be seen on several levels. The first level is that it broadens knowledge of the identification of transactions carried out between the related parties of development companies. The other level is the creation of a basis for related research into tax management in development entities. The motivation behind this research and another of its results is to draw attention to what types of information contained in public documents are not, or might not be, available to normal users.

The criteria for the research, given in Table 1, correspond to the individual sections of the annexe to the tax return. The use of this classification is appropriate, as the information contained therein is used by the financial authorities partly to analyse risks and to subsequently plan tax audits. This approach is then also relevant in terms of the classification within the framework of tax management in a development company – it enables the more effective management and elimination of the related tax risks. Transaction Services, stated in section B, have been further broken down into other subgroups, these being information technology services, human resources services, marketing services, legal services, accounting and administrative services, technical services, quality control services and other services. So-called intragroup services with low added value are broken down in a similar manner (JTFP, 2011).

Entities were classified using the classification according to the accounting rules based on the size of the accounting entity, i.e., micro, small, medium-sized and large accounting entities. The categories are determined by assessing the same criteria as those applicable to the obligation to complete the annexe to the tax return or have financial statements verified by an auditor. The variables in question are the total value of assets ("A"), total annual net turnover ("T") and the average recalculated headcount ("H"). For the purposes of this article the criteria were assessed solely for the 2018 accounting period with the aim of providing an overview of the size of the accounting entities and their obligations relating to the financial statements. Information on the fulfilment of the criteria, the legal form of the business and the audited financial statements can be found in Table 2.

Table 2: Characteristics of the researched set (2018)

Companies	Number of entities	Legal form		Monitored indicators		
		joint-stock	limited liability	A > CZK 40 million	T > CZK 80 million	H > 50
With audited financial statements	15	5	10	15	10	4
Without audited financial statements	15	3	12	7	2	0

Source: compiled by authors while using (Ministerstvo spravedlnosti, 2021).

The first step was to define the information contained in the financial statements and in the report on relations, to determine the scope of the information published. Each document was then analysed, and the number of published documents and the quality of the information obtained were evaluated.

After the aforementioned determination of the methodology, the next section contains a summary of the results, after which there is a section presenting a discussion and the conclusion.

2. Results of the Research

2.1 Categories of accounting entities in the sample set researched

For the performance in the field of the development of building projects, the figure is typically A > CZK 40 million (22 out of 30), which reflects the financial demands of creating the ownership structures typical for this field of business. Criterion T is based on the number of projects implemented during the accounting period, and thus it may logically be expected that it will be far more variable than criterion A. In 2018, the criterion T > CZK 80 million was fulfilled by fewer than half of the companies (12 out of 30). This sector is characterised by the fact that the production factors most used are capital, land and highly skilled labour (Kalová and Brychta, 2021). The production factor of labour, on the other hand, is minimal (Kalová and Brychta, 2021). This is also reflected by the fact that criterion H > 50 was fulfilled by only a few companies (4 out of 30).

The predominant form of business through a limited liability company (22 out of 30), unlike a joint-stock company (8 out of 30), may seem to be more advantageous. It may lead to savings on the costs of the audit of the accounting entity. For this obligation to apply, a limited liability company must fulfil 2 out of the 3 conditions in question, whereas a joint-stock company is obliged to apply if it to fulfils just one of those conditions. As stated above, A > CZK 40 million was reported by a substantial proportion of the companies. The fulfilment of the criterion A > CZK 40 million also obliges the company to file the annexe to the tax return in relation to transactions paid to related parties. It may therefore be assumed that most of the companies compile this document for the financial authorities.

In terms of the size of the accounting entities, small accounting entities (19 out of 30) predominate. However, the set also includes micro (4 out of 30), medium-sized (5 out of 30) and large accounting entities (2 out of 30). The companies published their audited (15 out of 30) and unaudited financial statements (15 out of 30) in the public register.

2.2 Obligations of accounting entities in relation to the financial statements and annual report

The categorisation of an accounting entity determines the scope of its financial statements (Section 3a of the Decree), the quantity of documents it publishes (Section 21a AA) and the obligation to have its financial statements audited (Section 20 AA). The financial statements of micro accounting entities are not subject to an audit. Their financial statements may be prepared in summarised form and they are not obliged to publish a profit and loss statement. The same also applies for small unaudited accounting entities. Medium-sized and large accounting entities are subject to a compulsory audit and prepare their financial statements in full (Section 18 (4) AA). All controlled entities have the obligation to prepare and publish the report on relations (Section 82 BCA). However, it was not the purpose of this research to determine whether all the entities in question are considered to be controlled entities.

A comparison of the documents revealed qualitative differences; see Table 3 for more details. The balance sheet, profit and loss statement, including the notes, contain only limited information. Here we often find only summarised transaction values and minimal details of specific transaction types. The balance sheet, for instance, gives information on long-term financial assets (shares, loans and credit), payables to shareholders, and receivables and liabilities to controlled or controlling entities. The profit and loss statement contains information on revenues from shares, costs incurred on shares sold, and income and expense interest. Small accounting entities and micro accounting entities that are not obliged to have their financial statements verified by an auditor do not have to publish a profit and loss statement. In many cases, the report on relations contains merely the titles of concluded contracts. The title of a contract does not necessarily relate to the transaction type or does not necessarily contain all transactions. From just the title of a contract, it is not possible to determine whether the enterprise provided or used a given service. Companies which are audited in all documents provided more detailed and better-quality information.

The user must therefore seek out information across all these documents. The individual documents are more or less detailed, depending on whether the financial statements are audited or unaudited. The annexe to the tax return, in contrast, is brief, concise, clear and far more comprehensible. All the information is given clearly in one place and in a single document. The financial authorities therefore obtain clear and purposeful information, unlike other accounting users, who have to spend time and effort seeking out this information in the financial statements and report on relations, and certain information must be inferred by the users.

Table 3: Qualitative differences of documents in terms of the scope of information on transactions with related parties

Document	Information		
	related parties	transaction type	Value of transaction.
Balance sheet	NO	YES conditional	Only aggregate amounts for all related parties.
Profit and loss statement	NO	YES conditional	Only aggregate amounts for all related parties.
Notes	YES	YES	More detailed information is stated in the

			audited financial statements.
Report on relations	YES	YES conditional	Mostly not stated, reference to standard terms, bank or trade secrets.
Annexe to tax return	YES	YES	Detailed amounts for each related party.

Source: compiled by authors

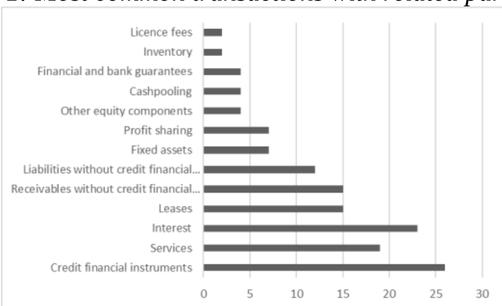
2.3 Quantity of published documents and quality of information

In the Czech Republic, enterprises publish their financial statements and report on relations in the collection of documents of the registration court with which the given accounting entity is registered. This registry is in electronic form and freely accessible to all users. An annual report, which includes the financial statements, was published by all the audited companies (15 out of 15). The report on relations was published by 14 of the 15 audited companies. Most of the companies that are not obliged to have their financial statements verified by an auditor did not publish the report on relations (9 out of 15); this was published by just six entities. The profit and loss statement was not published by 1 out of 30 entities. The information contained in the documents provided an overview of transactions with related parties. As the auditor verifies whether the financial statements give a true and fair view of the accounts, the audited financial statements should guarantee better quality information than unaudited financial statements.

2.4 Types of transactions with related parties

The most common transactions carried out by development entities with related parties include credit financial instruments (26 out of 30), services (19 out of 30), interest (23 out of 30) and leases (15 out of 30); for more details, see Figure 1.

Figure 1: Most common transactions with related parties



Source: compiled by authors

2.4.1 Section A – fixed assets and inventories

Companies engaged in development activity own shares in business corporations for the purpose of securing financing for the group, for the acquisition of real estate property or for the provision of shared services. Transactions relating to the purchase/sale of fixed assets occurred in 7 of the 30 entities and the purchase/sale of stocks in 2 of the 30 companies. In most cases, these involved the purchase/sale of equity and shares, the purchase/sale of land or the establishment of easements. Inventory items included work in progress (unfinished construction projects, the costs of long-term contracts concluded in relation to the provision of engineering and managerial services, commissions from brokerage agreements, etc.).

2.4.2 Section B – services, licence fees, interest, lease and other transactions

This section involved transactions carried out and received, or both types of cases are included. The most common were services provided (19 out of 30), interest paid (23 out of 30) and lease fees charged (15 out of 30). These are therefore very common transactions, which are reported by more than half of the entities reviewed. For many entities, the rental of property, apartments and commercial space comprises their main business activity. Licence fees were paid in connection with the use of trademarks and software (2 out of 30). Interest relates to the transactions specified in the following section, and as they were not reported by all the entities it may be assumed that credit financial instruments were provided without financial consideration.

2.4.3 Section C – credit financial instruments, shares in profit, other equity components, gratuitous performance, cashpooling, financial and bank guarantees

The implementation of development projects is highly demanding in terms of capital structure. It seems that the use of credit financial instruments as a part of a developer's activities is an immanent type of transaction (26 out of 30). Companies also boost equity through supplements outside their share capital, especially into other capital funds (4 out of 30). With ownership of equity interests and shares in companies, shares in profit are paid out (7 out of 30). Some companies are part of group cash-pooling (4 out of 30) and make the most of the advantages arising from the joint sharing of available funds. The provision of financial and bank guarantees (4 out of 30) is also significant. It is common to provide financial guarantees to bank loans, when the sponsor undertakes to pay any additional costs associated with building construction.

2.4.4 Section D – liabilities and receivables without credit financial instruments

Given the fact that shared services are very often used between related entities, the financial statements of companies as of the last day of the accounting period state both receivables (15 out of 30) and liabilities (12 out of 30). Accounts payable not related to credit financial instruments were also monitored. None of the companies stated that these concerned overdue liabilities or receivables.

2.5 Shared services in groups of enterprises

Most entities make the most of the advantages of shared services provided within the group. Information technology services (2 out of 30) included the use of applications and software. Human resources services (2 out of 30) were used primarily by staffing administration. Market services (4 out of 30) particularly included the use of trademarks, sales support and business cooperation. Legal services (3 out of 30) also included tax consultancy. Bookkeeping, payroll accounting, administration, audit accounting and economic consultancy were predominant in terms of accounting and administrative services (8 out of 30). The most common category, however, was that of other services (18 out of 30). This particularly includes managerial services, project management, facility management, asset management, development management, leasing management, property management, the management and lease of property, rental of property and equipment, mediation of services, servicing services, consultancy services, professional assistance, construction work, development activity, investor activity, provision of development-related services, construction coordination, preparation of documentation for zoning decisions and building permits.

3. Discussion and conclusion

Companies engaged in development activity are parts of holding structures, while members of the Asociace developerů are predominantly companies that are under foreign control (Kalová and Brychta, 2018). Given the fact that the total assets of most entities exceed CZK 40 million and those entities are part of business groups, it may be assumed that they are obliged to provide the financial authorities with the annexe to the tax return on transactions with related parties. Their data on transactions and related parties are used to analyse potential high-risk entities for tax audit purposes. In their tax management, companies must focus on an issue that is highly topical at present, i.e., transfer prices and how they are set in accordance with the arm's length principle. An especially topical issue arises in connection with this, i.e., the determination of correct transfer prices in the case of financial transactions, for which, incidentally, the OECD (2020) has created separate material. Also undoubtedly worthy of further attention is the examination of the individual types of transaction and their essence for the purpose of the taxonomy of development entities in connection with their positions (roles) in the holding and the types of transaction carried out. Following the results of the previous research (Brychta and Kalová, 2018) one can estimate that there will be certain platforms both in the types and scopes of the transactions realized between entities depending on the position of the subject in the holding structure and on the maturity stage of the development project(s). To validate this presumption another research aiming at more extensive sample of development companies shall follow.

The research so far made indicate that companies engaged in development activity own shares in business corporations for the purpose of securing financing for the group, for the acquisition of property or for the provision of shared services. It may be assumed that certain services may be categorised as services with low added value. This type of service, by nature, does not generate high added value (JTFP, 2011) and it is appropriate to cover them using so-called safe harbours (Solilová, 2013), which simplifies compliance, reduces costs, allows the sharing of administrative resources and more (Solilová 2013). However, the question arises as to what extent the use of safe harbours can be combined with "classic" procedures.

To sum up, we may also conclude that the financial statements and report on relations do not conveniently provide users with information on types of transactions between related entities. In this regard, it would certainly be appropriate and expedient if they corresponded more (or completely) with the content of the annexe to the tax return (for details of which, see above). This conclusion can be seen generally valid, because the rules are generally valid for all types of the subjects meeting above stated criteria not matter the area in which they operate. The validity of this conclusion is not determined by the sample of the entities under investigation.

Acknowledgment

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Risk minimization using distortion risk measures via linear programming

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Abstract

The paper deals with risk minimizing portfolios using distortion risk measures. First, the paper presents new formulation of coherent distortion risk measures via linear programming under assumption of equiprobable realizations of random returns. Second, the formulation is employed in the portfolio selection problem where a coherent distortion risk measure is minimized. Finally, in the empirical application, the sensitivity analysis with respect to a parameter of risk aversion of these risk minimizing portfolios is presented for a special type of distortion function – proportional hazard transform. The analysis includes in-sample and out-of-sample performance of the optimal portfolios with a focus on return and risk.

Keywords

Distortion risk measures, portfolio selection, linear programming

JEL Classification: D81, G11

1. Introduction

Investors typically seek for investments with high mean returns. Starting from a seminal paper of Markowitz (1952), a risk measuring and controlling has become another important issue in decision making. The risk of the investment can be modelled in various ways using e.g. risk measures, deviation functions or utility functions. Combining risk minimization with mean maximization leads to mean-risk portfolio selection problems which could be formulated in various different ways. If the investor wants only to minimize risk of the portfolio the portfolio optimization problems simplifies a lot when using risk measures. Especially, coherent risk measures (see Artzner et al. 1999) are the most popular ones because of their convexity.

In this paper, we investigate a special type of risk measures called distortion risk measures. These measures are very flexible in capturing the risk preferences of the decision maker. They are generated by a distortion function and cumulative distribution function of the returns. We present a new representation of distortion risk measures under assumption of discrete distribution of returns with equiprobable realizations. The main advantage of that is easy and straightforward implementation in mean-risk models which turn to be linear programs. One of the attractive properties of coherent distortion risk measures (generated by concave distortion function) is its consistency with the second order stochastic dominance. The basics of stochastic dominance go back to 1960th, see Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970). However, as demonstrated in e.g. Moriggia et al. (2019), Kabasinskas et al. (2020) or Kopa et al. (2021), it is still an useful tool in the decision making or portfolio optimization.

In the empirical part of the paper, we apply the new representation of distortion measures to the risk minimizing portfolio selection problem using data on 10 representative industry portfolios from Kenneth French library. The goal of the empirical study is to present the

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sensitivity analysis of this portfolio selection problem considering Proportional hazard transform (PHT) as the distortion function.

The remainder of this paper is structured as follows. Section 2 presents the basics of distortion measures and new representation. Section 3 formulates the risk minimizing portfolio selection problem using the representation. Section 4 presents the in-sample and out-of-sample results of the problem, including sensitivity analysis with respect to the parameter of PHT . The paper is summarized and concluded in Section 5.

2. Distortion risk measures

Let random variable $X \in \mathcal{X}$ represent a random loss of investment (portfolio) and let $F_X(x)$ be its cumulative distribution function. Then a risk measure is a functional of $X \in \mathcal{X}$ which assigns a real number to $X \in \mathcal{X}$. At present, the most popular risk measures are Value at Risk (VaR) and Conditional Value at Risk (CVaR). Distortion risk measures could be seen as their generalizations:

Definition 1 [Dhaene et al. 2012]
*Suppose that $g: [0,1] \to [0,1]$ is a non-decreasing function such that $g(0) = 0$ and $g(1) = 1$ (also known as the **distortion function**). Then, the **distortion risk measure** associated with the distortion function g is defined as*

$$\rho_g(X) = -\int_{-\infty}^{0} [1 - g(1 - F_X(x))]dx + \int_{0}^{\infty} g(1 - F_X(x))dx,$$

provided that at least one of the integrals is finite.

In this paper, we focus only on the coherent distortion risk measures.

Lemma 1 [Wirch and Hardy 1999, Sereda et al. 2010]
The distortion risk measure $\rho_g(X)$ is coherent if and only if g is a concave distortion function.

As discussed in Kopa and Zelman (2021), the most commonly used distortion measures are VaR and CVaR and those with the following distortion measures:

- **Proportional Hazard transform (PHT):** $g(x) = x^{1/\gamma}, \quad x \in [0,1], \gamma \geq 1.$ (1)
- The **Wang transform**: $g_\lambda(x) = \Phi(\Phi^{-1}(x) + \lambda)$ for $x \in [0,1], \lambda \geq 0$, where Φ is the standard normal distribution function.
- The **MINVAR** distortion function; $g(x) = 1 - (1 - x)^{1+\lambda}$ for $x \in [0,1], \lambda \geq 0$.
- The **MINMAXVAR** distortion function: $g(x) = 1 - (1 - x^{\frac{1}{1+\lambda}})^{1+\lambda}$ for $x \in [0,1], \lambda \geq 0$.

Since now, let us assume that random loss variable $X \in \mathcal{X}$ has a discrete distribution with equiprobable realizations x_i, $i \in \{1, \dots, m\}$: $P(X = x_i) = \frac{1}{m}$. Moreover, assume that the realizations are ordered from the smallest to the largest one, that is: $x_1 \leq x_2 \leq \cdots \leq x_m$. Then Kopa and Zelman (2021) proved that:

$$\rho_g(X) = x_1 + \sum_{i=1}^{m-1} (x_{i+1} - x_i) g\left(1 - \frac{i}{m}\right).$$

Now, let $x_0 = 0$ and then

$$\rho_g(X) = \sum_{i=0}^{m-1} (x_{i+1} - x_i) g\left(1 - \frac{i}{m}\right)$$

because $g(1) = 1$, see Definition 1. Using Abel summation lemma we get:

$$\rho_g(X) = x_m g\left(1 - \frac{m-1}{m}\right) - x_0 g(1) - \sum_{i=1}^{m-1} x_i \left(g\left(1 - \frac{i}{m}\right) - g\left(1 - \frac{i-1}{m}\right)\right)$$

and since the second term on right hand side is zero and $g\left(1 - \frac{m}{m}\right) = 0$ we may simplify it to:

$$\rho_g(X) = \sum_{i=1}^{m} x_i \left(g\left(1 - \frac{i-1}{m}\right) - g\left(1 - \frac{i}{m}\right)\right)$$

Finally, let $G_i = g\left(1 - \frac{i-1}{m}\right) - g\left(1 - \frac{i}{m}\right)$. Then

$$\rho_g(X) = \sum_{i=1}^{m} x_i G_i \tag{2}$$

Lemma 2:
If $g(x)$ is a concave function than $G_i \leq G_{i+1}$.

Proof: Since $g(x)$ is a concave function one directly has:

$$g\left(1 - \frac{i}{m}\right) \geq \frac{g\left(1 - \frac{i-1}{m}\right) + g\left(1 - \frac{i+1}{m}\right)}{2}$$

and: consequently:

$$g\left(1 - \frac{i}{m}\right) - g\left(1 - \frac{i+1}{m}\right) \geq g\left(1 - \frac{i-1}{m}\right) - g\left(1 - \frac{i}{m}\right)$$

what completes the proof.
Now we are ready to present the new represenatin of coherent distortion measures via linear programming where we use the properties of double stochastic matrices.

Theorem 1:
Let $g(x)$ be a concave function and let y_j be a realization of random loss $X \in \mathcal{X}$ in time period j, such that $P(X = y_j) = \frac{1}{m}$, j=1,...,m. Then

$$\rho_g(X) = \max_{z,\boldsymbol{W}} \sum_{i=1}^{m} z_i G_i \tag{3}$$
$$\text{s.t.} - z_i + \sum_{j=1}^{m} w_{ij} y_j = 0,\ i=1\ldots m$$

$$\sum_{j=1}^{m} w_{ij} = 1,\ i=1\ldots m$$
$$\sum_{i=1}^{m} w_{ij} = 1,\ j=1\ldots m$$
$$w_{ij} \geq 0,\ i,j=1\ldots m.$$

Proof:
Since $G_i \leq G_{i+1}$, see Lemma 2, optimal value of z_m equals to the largest loss, that is x_m. This is due to the fact that the objective is maximization and thanks to properties of double stochastic matrix $W = \{w_{ij}\}_{i,j}$. Similarly, for the optimal values of all other z_i we have: $z_i^* = x_i$, i=1…m, what completes the proof.

Despite of the fact that (3) is linear program and this LP represenation could be used in portfolio selection problems directly, we will rather proceed with a dual expression. The reason is that terms "$w_{ij}y_j$" would make the portfolio selection problem non-linear and non-convex, so computationally more demanding than LP problems.

Theorem 2:
Let $g(x)$ be a concave function and let y_j be a realization of random loss $X \in \mathcal{X}$ in time period j, such that $P(X = y_j) = \frac{1}{m}$, j=1,…,m. Then

$$\rho_g(X) = \min_{\boldsymbol{c},\boldsymbol{d}} \sum_{j=1}^{m} \mathrm{c}_j + \sum_{i=1}^{m} d_i \qquad (4)$$
$$\text{s.t. } -G_i y_j + \mathrm{c}_j + d_i \geq 0, \quad i,j = 1,..,m.$$

Proof:
The dual problém to (3) is:

$$\rho_g(X) = \min_{\boldsymbol{c},\boldsymbol{d}} \sum_{j=1}^{m} \mathrm{c}_j + \sum_{i=1}^{m} d_i$$
$$\text{s.t. } b_i y_j + \mathrm{c}_j + d_i \geq 0, \quad i,j = 1,..,m \qquad [w_{ij}]$$
$$-b_i = G_i \qquad [z_i]$$

and substituting last equations into inequalities eliminates variables b_i and completes the proof.

The advantage of (4) is that values y_j are not multiplicated by any variable what enables to formulate the risk minimizing portfolio selection problem as linear program what we demonstrate in the next session.

3. Distortion risk measures in portfolio selection problems

Let us consider a random vector $\mathbf{r} = (\mathrm{r}_1, \mathrm{r}_2, \ldots, \mathrm{r}_\mathrm{N})$ of returns of N assets in m equiprobable scenarios. The returns of the assets for the various scenarios are collected in matrix:

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^\mathrm{m} \end{pmatrix}$$

where $\mathbf{x}^\mathrm{t} = (\mathrm{x}_1^\mathrm{t}, \mathrm{x}_2^\mathrm{t}, \ldots, \mathrm{x}_\mathrm{N}^\mathrm{t})$ is the t-th row of matrix **X**. A vector of portfolio weights is denoted by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_\mathrm{N})'$. In this paper, we exclude short sales, that is, the set of all feasible portfolios Λ can be characterized as follows:

$$\Lambda = \{\boldsymbol{\lambda} \in R^N \,|\, \textstyle\sum_{n=1}^{N} \lambda_n = 1, \quad \lambda_n \geq 0, \; n = 1,2, \ldots, N\}.$$

In this notation, the loss of portfolio $\boldsymbol{\lambda}$ is - $\mathbf{r}^{\mathrm{T}}\boldsymbol{\lambda}$ with equiprobable realizations: $-\mathbf{x}^{\mathrm{t}}\boldsymbol{\lambda}, \mathrm{t} = 1,..,m$ which play the role of vector losses y_j in Section 2. Therefore, employing (4), the distortion risk minimizng problem is formulated as follows:

$$\rho_g(X) = \min_{\boldsymbol{c},\boldsymbol{d},\boldsymbol{\lambda}} \sum_{j=1}^{m} \mathrm{c}_j + \sum_{i=1}^{m} d_i \tag{5}$$
$$\text{s.t. } G_i \mathbf{x}^{\mathrm{j}}\boldsymbol{\lambda} + \mathrm{c}_j + d_i \geq 0, \quad i, j = 1,..,m$$
$$\boldsymbol{\lambda} \in \Lambda.$$

4. Empirical study

4.1 Data description

We consider ten industry representative portfolios from the Kenneth French library as the base assets in our empirical study. The data of daily returns are divided into two parts: in-sample period (1.1.2018 – 31.12.2020) and out-of-sample period (1.1.2021 – 30.6.2021). Descriptive statistics of both datasets are summarized in Table 1 and Table 2.

Table 1: Basic descriptive statistics of daily returns (in %) – in sample period

	mean	st. dev.	min	max	skewness	kurtosis
NoDur	0.029	1.288	-9.870	7.450	-0.757	14.139
Durbl	0.163	2.287	-14.430	15.030	-0.334	8.049
Manuf	0.044	1.603	-11.110	10.830	-0.515	11.515
Enrgy	-0.039	2.473	-19.730	16.000	-0.301	12.272
HiTec	0.114	1.740	-13.180	10.690	-0.501	9.730
Telcm	0.052	1.352	-9.080	9.060	-0.530	10.975
Shops	0.087	1.390	-10.610	7.050	-0.721	10.126
Hlth	0.061	1.363	-9.740	6.980	-0.410	8.313
Utils	0.037	1.545	-11.610	11.760	-0.051	18.257
Other	0.045	1.723	-13.380	12.240	-0.563	14.600

Table 2: Basic descriptive statistics of daily returns (in %) – out-of-sample period

	mean	st. dev.	min	max	skewness	kurtosis
NoDur	0.072	0.761	-1.980	2.420	-0.043	0.620
Durbl	0.093	2.633	-6.380	11.510	0.574	2.175
Manuf	0.123	1.036	-2.840	2.680	-0.137	0.438
Enrgy	0.345	2.063	-5.030	4.860	-0.046	-0.371
HiTec	0.129	1.343	-3.510	3.560	-0.258	0.289
Telcm	0.042	0.950	-3.100	2.530	-0.200	1.226
Shops	0.096	0.876	-2.840	2.260	-0.512	1.010
Hlth	0.076	0.872	-2.910	1.870	-0.424	0.444
Utils	0.053	0.947	-2.750	2.650	-0.340	0.618
Other	0.148	1.065	-3.100	3.190	-0.071	0.500

4.2 Results

As the distortion function we consider only PHT but with several parameters γ. To fulfil the assumptions of Theorem 2, we consider only $\gamma \geq 1$. Note that if $\gamma = 1$ then the distortion measure is expected value and, hence, the optimal portfolio invest everything in the second asset which is the most profitable one. The optimal portfolios of (5) for in-sample period and for considered parameters γ are summarized in Table 3.

Table 3: Compositions of optimal portfolios – in sample period

γ	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
1		1								
1.5	0.143					0.17	0.336	0.25	0.101	
2	0.31					0.221	0.161	0.308		
3	0.319					0.352		0.329		
4	0.313					0.474		0.213		
5	0.258					0.596		0.146		
10						0.91		0.09		

We can see that as the risk aversion expressed by parameter γ increases the portfolio is more concentrated in least risky assets. Moreover, for very large values of parameter γ, (almost) everything is invested in the asset with the highest minimal return. This is due to the fact that investors with extremely large risk aversions want to hedge against the worst realizations – smallest returns.

Finally, we present in Table 4 mean returns and distortion risk measures of the optimal portfolios from Table 3. For the sake of comparison, we express the risk using PHT distortion measure with $\gamma = 2$.

Table 4: Risk – return performance (in %) of optimal portfolios – in sample & out-of-sample period

in sample \ γ	1	1.5	2	3	4	5	10
Mean return	0.1631	0.0614	0.0535	0.0478	0.0468	0.0474	0.0527
Risk	1.8063	1.1001	1.0919	1.0958	1.1008	1.1083	1.1383
out of sample \ γ	1	1.5	2	3	4	5	10
Mean return	0.0927	0.0743	0.0707	0.0630	0.0588	0.0549	0.0454
Risk	1.5835	0.4787	0.4687	0.4856	0.5034	0.5305	0.6285

5. Conclusions

In this paper we deal with portfolio selection problems where the only objective is to minimize the risk of the portfolio. The risk is measured by distortion risk measures which could be seen as generalizations of Value-at-Risk and Conditional Value-at-Risk. A special attention is paid to distortion measures which are generated by concave distortion functions because these risk measures are coherent. Firstly, we derive a new representation of the coherent distortion functions in the form of linear program. Secondly, we apply it to the risk minimizing portfolio selection problems. Finally, we demonstrate the technique in the empirical study where we limit our attention on the PHT distortion functions and associated distortion measures.

In the empirical study, we have analysed the risk-return performance (in-sample and out-of-sample) of the risk minimizing portfolios for various values of parameter γ which can be seen as a parameter of the risk aversion. The higher the parameter is, the stronger the risk aversion is considered. Therefore, as this parameter increases the mean return of the optimal portfolio decreases. However, it is not true for all values of γ, becasue when it is very large, the risk measure is too much conservative – only the smallest return is important. As the consequence, the mean return may increase with increasing γ for large values of γ. Interesingly, this was observed only in the in-sample period but not in the out-of-sample period, where mean return always decreased with increasing γ. It is no surprise that in-sample and out-of-sample risk measured by PHT distortion measure with $\gamma = 2$ is minimal for the optimal portfolio of (5) when considering PHT with $\gamma = 2$.

The new representation of the coherent distortion measures could be attractive also in the multi-stage portfolio optimization problems such as in e.g. Vitali et al. (2017), Zapletal et al. (2020) or Kopa and Rusý (2021).

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Assessment of Factors Influencing Final Corporate Income Tax of Construction Sector in the Czech Republic

Karolina Lisztwanová, Iveta Ratmanová [1]

Abstract

Corporate income tax is a kind of direct tax, which influences not only profitability of individual subjects but is able to encourage taxpayers to change their behaviour. The total tax liability is an indicator describing results of these decisions. Details of it are clear from individual components of determination of the total tax liability. Methodology of pyramidal decomposition can be used for their assessment and analysis of variances is be used for assessment of influences of individual sub-indicators. Created general decomposition of the total tax liability is applied on individual conditions of the construction sector during period between years 2005-2019. According to the details of calculations the influence of individual sub-indicators was stated their ranking including.

Key words

Corporate income tax, total tax liability, pyramidal decomposition, functional method, construction sector

JEL Classification: H20, K34, H25

1. Introduction

Tax policy is integral part of fiscal policy. Details of tax policy are clear from structure of tax systems of individual countries. As far as definition of tax, tax is not precisely defined in the Czech legal system, but there is much to be found in literature. "Tax is defined as a mandatory, non-refundable, statutory payment to the public budget. It is a non-purpose and non-equivalent payment. The tax is repeated regularly at intervals (e.g. annual income tax payment) or is irregular and paid for certain circumstances (e.g. each time the property is transferred)". (Kubátová, 2018)

Taxes can be sorted according to various criteria such as taxpayers` income, object of tax, purpose of taxes, etc. With regard to the tax object income tax, property tax and consumption tax can be identified. (Musgrave, Musgrave, 1994) The income tax may be levied on income of individuals or corporates. Regarding corporates income tax is imposed by a tax jurisdiction on the income or capital of corporations or analogous legal entities. The tax object is the earnings (profits) or difference between incomes and expenses of individual entities. (Lisztwanová, Ratmanová, 2020)

A corporate tax is a questionable tax, as there are opinions that this tax is not economic justification. The argument for this view is that corporate profits will eventually become

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personal incomes of individual people and will be finally taxed by personal income tax. However, this tax exists still in most countries.

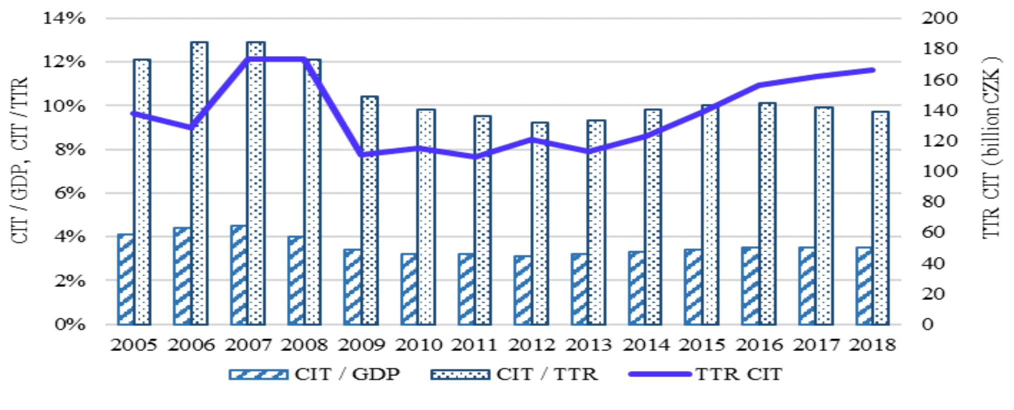
The corporate tax has many shortcomings from an economic point of view. For example, that companies transfer tax to other entities, and for this reason, corporate tax may not be paid effectively by the owners of the corporations. Furthermore, it is difficult to define corporate taxable profit and therefore this tax is not neutral. Especially multinational companies they have the ability to manipulate their tax base and shift their profits thanks to tax differentials through the use of transfer pricing mechanism, royalties, inter-company transactions, etc. The main shortcoming of corporate taxation is a problem of double taxation of dividends as well. (Radvan et al., 2014)

Nevertheless, the positive aspects of corporate tax exist. Corporates pay by this way for the use of public services and infrastructure in the country where they are based and do business and therefore income tax is considered as a payment for these services. Furthermore, this tax compensates for the limited liability of legal entities for their liabilities, because a legal entity is liable only up to the amount of the subscribed property, unlike a natural person who is liable for its liabilities with all its assets. (Široký et al., 2016)

In spite of the above-mentioned facts, corporate income tax is integral part of tax system of almost all countries from all over the world and has impact on decisions of individual corporates as all other taxes. The authors of tax policy are aware of this fact and it influences individual aspects of corporate income tax. Individual steps in determining corporates` tax liability offer to use allowances, credits or deductions, whereas an effort to support or to eliminate certain activities is observed.

When it comes to data of the Czech Republic, details about it are publicly available from Finanční správa. The following figure 1 shows details about the role of the corporate income tax in the Czech tax system. Regarding data of it, it is clear that total tax revenues coming from corporate income tax was affected by global economic crisis started in the year 2008, nevertheless increasing is observed during last years of selected period. The ratio between CIT revenues and total tax revenues developed by the same way as the total tax revenues. Declining was followed by acceleration, but growth rate was not as strong as in case of the total tax revenues. This fact is caused by construction elements of this ratio and confirm the position of the corporate income tax in the Czech tax system. The last ratio is corporate income tax quota. The development of its values is close to the ratio CIT/TTR.

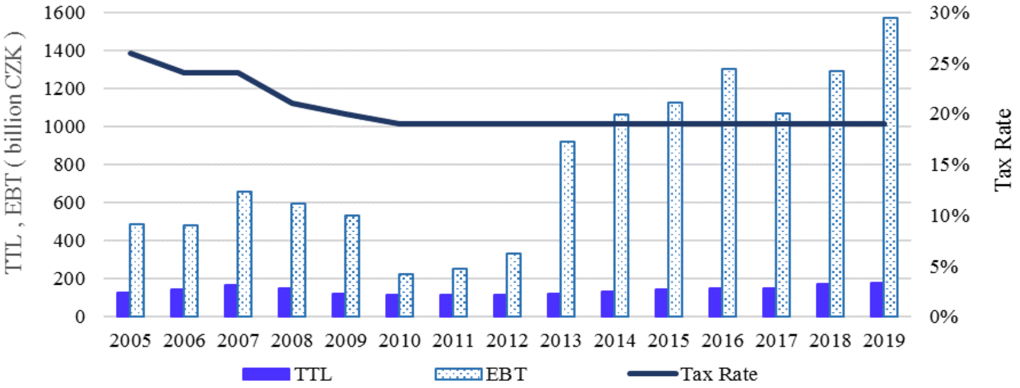
Figure 1: The development of TTR CIT, CIT/ TTR and tax quota of all sectors during observed period



Source: Authors` processing according to data of the European Commission and of Finanční správa

The following figure 2 shows relationship between total tax liability and tax rate during observed period. The effort to support economic activities by decreasing of the statutory tax rate is clear from data. It was reduced from the value 25 % in the beginning of selected period to 19 % in the year 2010. Comparing it with the development trend of the total tax liability of all entities it can be confirmed that decreasing of the tax rate must not always cause decreasing of the total tax liability. The last value that is clear from the figure 2 is earnings before tax. It can be simply described as the value of accounting profit of enterprises. With regard on real data it is clear that its development was more turbulent compared to the development of the total tax liability.

Figure 2: The development of TTL, EBT and STR of all sectors during observed periods



Source: Authors` processing according to data of Finanční správa

As it has been already mentioned, the corporate income tax liability can be influenced by many factors. The assessment of these real impacts of individual aspects on the total tax liability has been already made by Lisztwanová, Ratmanová (2017) for all sectors and by Lisztwanová, Ratmanová (2018) for finance and insurance sector and by Lisztwanová, Ratmanová (2020) for manufacturing sector. The goal of the paper is to assess the impact of tax-deductible costs, changes of tax rate, items reducing tax base and tax reliefs on the final tax liability of taxpayers of construction sector. The pyramidal decomposition of the total tax liability and analysis of variances will be used for assessing influence of individual items on the top indicator in the selected period between years 2005-2019 with public data of Finanční správa.

2. Selected Information about Construction Sector

In the Czech Republic, as in other economically developed countries, construction is one of the main pillars ensuring the development of the economy. Construction accounts for about 7% of the entire production economy of the Czech Republic, employs more than 8% of the total number of employed people and is able to absorb significant proportion of workers, even with lower or different qualifications. (MPO, 2019)

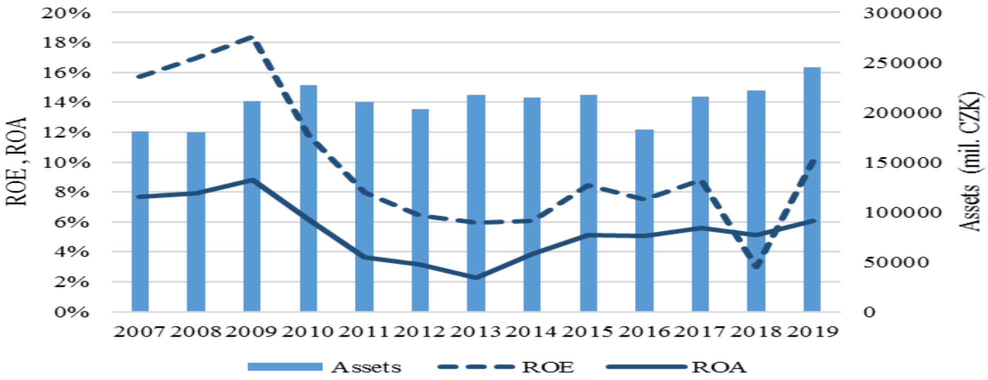
It is a sector that is fully comparable in significance to other major economic sectors, such as are energetic sector, manufacturing sector or transport, etc. A significant specificity of the construction industry is an important share of investments from public funds. The sector was one of the hardest hit by the financial and economic crisis, which began in 2008, not only in the Czech Republic, but also throughout the EU. At the same time, they face structural problems in the sector, such as a shortage of skilled workers, low attractiveness for young people due to working conditions or limited ability to innovate. (MPO, 2013)

Investment construction and construction are one of the basic pillars of economic development, among other things due to their high multiplier effect. Support for the development of industrial zones, inputs from foreign investors, developers` intentions in the area of housing construction and considerable funds from EU funds, which flow mainly into infrastructure, contribute to the re-development of the construction industry. (MPO, 2016)

Construction production peaked at the beginning of the period under review in 2008, after declining in following years it increased in 2015, when construction production was subsidized by subsidies from European funds. However, it has not yet reached level of the year 2008. Construction production peaked at the beginning of the period under review in 2008, after declining in others increased in 2015, when construction production was subsidized by subsidies from European funds, and also in recent two years. (MPO Stavebnictví České republiky, 2019)

The development of construction sector during selected period is obvious from following figure 3 which provide information about power of the sector via the value of assets, return of assets and return of equity. The observed data shows declining of both ratios with gradual increase of their values. In case of the ROE, important fall is observed in the year 2018. It was influenced first of all by increasing of the tax liability. Corporate tax decreased profit more than in previous years. Moreover, the companies began to finance their activities more with external sources (MPO, 2018).

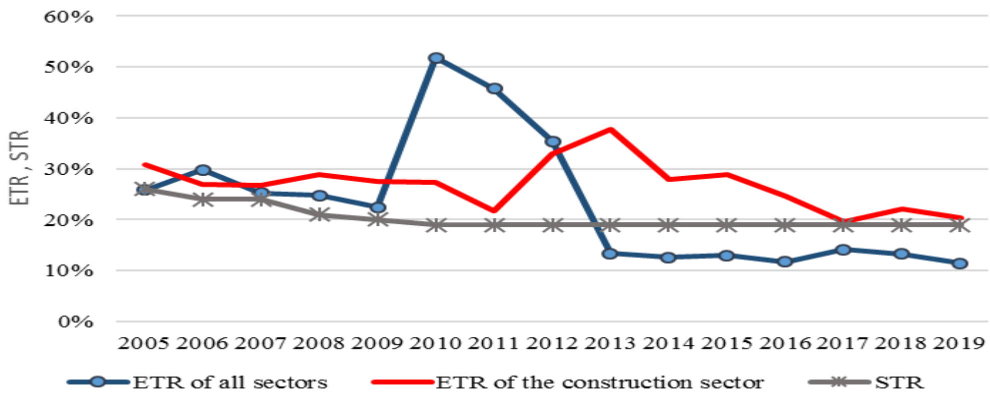
Figure 3: The development of assets, ROE and ROA of the construction sector



Source: Authors` processing according to the data of MPO, Analytické materiály a statistiky

The following figure 4 provides information about effective tax rate of corporate income tax of construction sector compared with the data of all sectors. Effective tax rate can be identified as a ratio between the total tax liability and earnings before tax. Its value is not influenced only by the statutory tax rate but by all allowances, credits and all adjustments of accounting profit to taxable profit.

Figure 4: ETR of all sectors, ETR of the construction sector and statutory tax rate



Source: Authors` processing according to data of Finanční správa

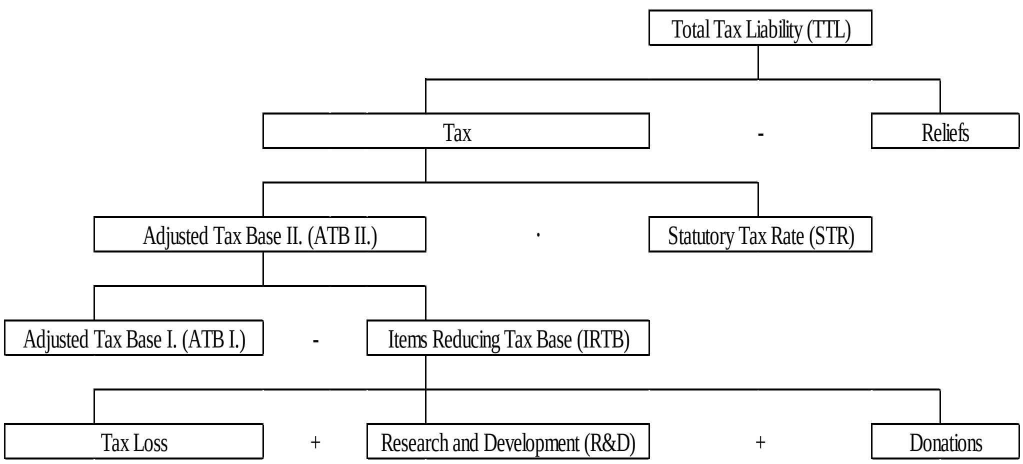
Comparing effective tax rate of construction sector with data of all sectors, different development is clear. Unlike overall data the effective tax rate of construction sector exceeded the value of statutory tax rate. The impossibility of using all methods to reduce the tax base or all tools to decline the tax probably caused this situation.

3. Methodology

In case of direct taxes it is necessary to compare actual financial results and direct tax burden. (Kuznetsova, Krzikallova, Kuznetsov, 2017), (Moravec et al., 2019). The total tax obligation of individual corporate is derived from the value of accounting profit. It is main important characteristic of corporate income tax. For calculation of the total tax is necessary to know not only accounting profit but moreover is necessary to respect individual rules of tax legislation. Generally said, accounting profit or loss must be transferred to the tax base. It means that first of all accounting revenues and incomes, all expenses and costs must be assessed in view of tax purposes. Exempted incomes should be excluded as well as non-tax costs and expenses to identify the tax base. Subsequently, the tax base is further adjusted by deductions (allowances) to determine the value of the adjusted tax base. The last aspect that may affect the final tax liability is tax credit as the item decreasing calculated tax. This general procedure can be identified in tax liability calculations all over the world. These individual steps of the determination of the total tax obligation is way, which is used by fiscal policy to support desirable activities and such as a way for stabilizing or stimulating of economy. With regard on details of the Czech tax legislation items reducing tax base comprise tax loss of previous periods, science and research expenses, expenditure on vocational education and gratuitous transaction on specific purpose. Investment incentives and employment of disabled people are tax credits.

As it has been already mentioned, the calculation of the total tax liability is the complex process with several important variables and the influence of them is not always the same. Following figure 5 provides overview about the total tax liability determination. The main approach is based on decomposition of the total tax liability as the top indicator. The top indicator can be decomposed into sub-indicators and this way it is possible to proceed up to the level of those sub-indicators that can no longer be decomposed.

Figure 5: Pyramidal decomposition of total tax liability



Source: Authors` processing according The Act no. 586/1992 Coll., on Income Taxes

Respecting details of figure 5, it is clear that top indicator - total tax liability - can be expressed as a decomposition of the individual sub-indicators. The final tax liability is influenced by individual adjustments of it through adjusted tax base I., tax loss, research and development expenses, donations and reliefs. Because of it they are set as individual sub-indicators the statutory tax rate including. With regard to the details of decomposition, there are multiplicative and additive relationship among indicators. By this way, there is no problem to identify indicator with the biggest and the lowest impact on the selected top indicator (Lisztwanová, Ratmanová, 2018). According to Dluhošová (2004), Dluhošová et al. (2010) and Zmeškal et al. (2013) for mathematical expression of influence of individual indicators on changes of top indicator in case of multiplicative relationship it has been used the functional method.

4. Assessment of Influence of Individual Indicators

Data of table 1 shows details of the total tax liability and selected sub-indicators. As it is clear, year-to-year changes of the total tax liability declare increasing and decreasing of it during selected period. The impact of the financial crisis is obvious in the massive decline in the year 2008 continuing in following years. With regard the data of figure 3 negative aspects of crisis came through falling of ROE and ROA ratios as well. Enhancement of the total tax liability can be observed from the year 2013 to the year 2016. The year 2017 brought declining of it for the last time during observed period followed by growth.

When it comes to sub-indicators, development of ATB I. copies development of the total tax liability. Year-on-year decline lasting until year 2012 and growth was observed in following years with short interruption of this development in the years 2016 and 2017.
Important increasing of tax loss as following sub-indicator can be watched from the year 2009 to year 2011 what is understandable given the companies` efforts to offset the loss from previous years in their tax liability.

The changes of research and development deductions cannot be described as something with the specific trend of development. Perhaps only in recent years (2017-2019) there is a certain increase. The similar fact can be stated in the case of donations as the following kind of deduction. The changes of the reliefs, which directly decrease the tax liability, were significant in the years 2006, 2007, 2009, 2014, 2015, 2018 and 2019, when their year-to year change exceeded the limit value 100 %.

Table 1: Year-to-year changes of selected items within period 2005–2019 in % of the construction sector

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
TTL	100.0	113.5	117.8	87.8	90.8	93.6	72.0	86.1	106.3	104.4	144.0	100.2	93.6	129.2	123.9
STR	100.0	92.3	100.0	87.5	95.2	95.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
ATB I.	100.0	124.3	115.8	98.6	99.1	97.8	75.6	84.8	107.7	107.2	139.9	97.2	95.5	125.8	122.6
Tax Loss	100.0	128.5	81.4	82.0	106.8	121.8	108.5	95.4	126.1	133.3	120.0	75.0	113.1	100.9	109.6
R&D	100.0	1 973.8	234.1	25.5	381.9	55.5	193.7	70.8	90.2	67.7	99.4	98.4	112.3	104.9	153.8
Donations	100.0	127.1	106.3	110.9	98.2	99.9	79.0	86.0	91.1	109.8	120.4	115.4	105.2	118.4	100.7
Reliefs	100.0	137.5	121.3	90.9	257.3	61.1	96.1	26.0	89.3	116.5	104.2	99.9	89.4	116.0	101.2

Source: Authors` calculation according to data of Finanční správa

The following table 2 describes the identified changes from different perspective. The values of the year 2005 are declared as basic data and the data of all following years are assessed in the light of these values. By this way it is clear that the total tax liability exceeds the value of the year 2005 in the year 2018. Interesting fact is clear from data about the development of the tax loss. The tax loss was used in all years and its value was always higher than in 2005, except years 2007 and 2008. The following fact, which should be mentioned, is the development of research expenses as the item reducing tax base. The massive increase of it is clear from real data. It means that the development of the construction sector is connected with investment activity in this area. Intelligent building components, robotic production equipment or the digitization of the construction industry itself is thus reflected in the tax area.

Table 2: The changes of selected items within period 2005–2019 in % of the construction sector

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
TTL	100.0	113.5	133.7	117.4	106.6	99.7	71.8	61.8	65.7	68.6	98.8	98.9	92.6	119.6	148.3
STR	100.0	92.3	92.3	80.8	76.9	73.1	73.1	73.1	73.1	73.1	73.1	73.1	73.1	73.1	73.1
ATB I.	100.0	124.3	143.9	141.9	140.5	137.5	103.9	88.1	94.9	101.8	142.5	138.5	132.3	166.4	204.1
Tax Loss	100.0	128.5	104.6	85.8	91.7	111.7	121.1	115.6	145.8	194.4	233.2	174.8	197.7	199.5	218.6
R&D	100.0	1 973.8	4 620.9	1 176.2	4 491.2	2 490.9	4 823.7	3 416.1	3 081.2	2 085.8	2 072.9	2 040.4	2 290.8	2 402.1	3 695.0
Donations	100.0	127.1	135.1	149.8	147.1	147.0	116.2	99.9	91.0	99.9	120.2	138.7	145.8	172.6	173.9
Reliefs	100.0	137.5	166.8	151.7	390.4	238.7	229.4	59.6	53.2	62.0	64.6	64.5	57.7	66.9	67.7

Source: Authors` calculation according to data of Finanční správa

According to the pyramidal decomposition of the total tax liability of construction sector following facts were found. As can be seen, table 3 shows the values of the year-to-year changes of the total tax liability of construction sector. Generally said the development of it can be described by very simply way. Periods with decrease of the total tax liability were followed by periods with increasing of it. It has been already mentioned in the comments of the table 1, that massive declining of the total tax liability was observed during the years 2008-2013. The crisis not only affected performance but also had an impact on the tax liability. Detail sub-indicators, which were identified in the pyramidal decomposition mentioned in the figure 5 can help to understand better what happened and which of the sub-indicators most affected identified changes. The following rule applies. The higher the value of the indicator is, the more this sub-indicator contributed to the development of the total tax liability. With regard the details of the table 3, it may happen that the value of change is zero. According to it, it seems that there is not any impact on the top indicator. Nevertheless, in comparing it with the power of influence of the rest indicators even a zero value of influence plays a role. It is lower than the highest positive value and at the same time higher than the lowest negative value of the influence of other sub-indicators.

Table 3: Power of influences of individual indicators of the construction sector, absolute changes in mil. CZK

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
Reliefs	26	20	-10	163	-104	-6	-116	-4	6	2	-0.05	-5	6	0.55
STR	-541	0	-1057	-349	-340	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ATB I.	1595	1235	-122	-72	-156	-1673	-787	338	343	2028	-197	-311	1703	1878
Tax Loss	-135	109	80	-23	-74	-34	20	-109	-175	-140	210	-82	-6	-69
R&D	-46	-63	76	-67	38	-44	26	6	19	0.24	0.61	-5	-2	-24
Donations	-8	-2	-4	0.69	0.03	7	4	2	-2	-5	-4	-2	-6	-0.30
Δ TTL	839	1259	-1016	-673	-428	-1737	-621	242	179	1882	9	-395	1682	1784

Source: Authors` calculation according to data of Finanční správa

Following table 4 provides detail information about influences of the sub-indicators on the total tax liability as the top indicator expressed as percentage value. Thanks to this fact-findings show the power of influence of them. Every change of the total tax liability means one unit, which the value is 100 %. The influences of the individual sub-indicators are positive and negative and total sum of their influences is 100 %. By this way is clear which sub-indicator contributed to the total change of the tax liability as a positive factor and which of them as a negative factor.

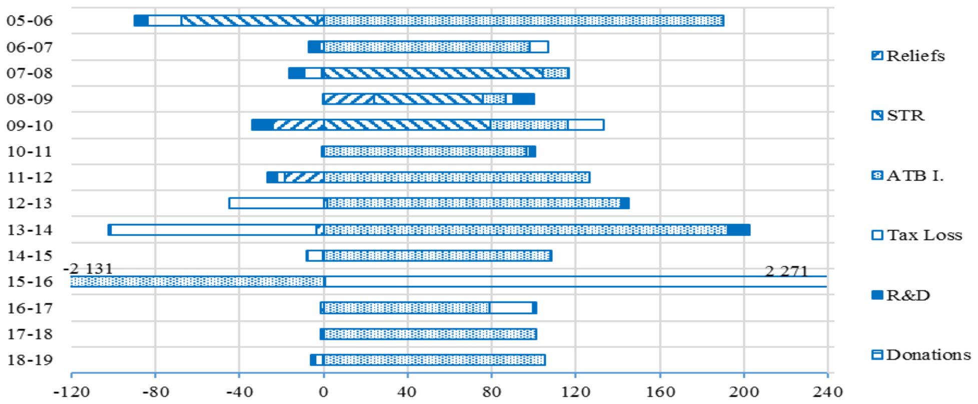
Table 4: Power of influences of individual indicators of the construction sector, percentage value

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
Reliefs	-3.1	-1.6	-1.0	24.2	-24.2	-0.4	-18.7	1.8	-3.4	-0.1	0.6	-1.2	-0.4	0.0
STR	-64.4	0.0	104.0	51.8	79.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ATB I.	190.1	98.1	12.0	10.8	36.4	96.3	126.7	139.6	192.0	107.8	-2 131.3	78.7	101.2	105.3
Tax Loss	-16.1	8.6	-7.9	3.4	17.3	2.0	-3.2	-44.9	-98.0	-7.4	2 271.1	20.8	-0.4	-3.9
R&D	-5.5	-5.0	-7.5	10.0	-9.0	2.5	-4.2	2.6	10.4	0.0	6.6	1.2	-0.1	-1.4
Donations	-1.0	-0.2	0.4	-0.1	0.0	-0.4	-0.6	0.9	-1.2	-0.3	-47.0	0.4	-0.4	0.0
Δ TTL	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Source: Authors` calculation according to data of Finanční správa

With regard the data of the table 4 is obvious that sub-indicator with the highest positive impact is the ATB I. Moreover, it can be identified as sub-indicator with the lowest impact on the total tax liability. The following figure 6 provides a graphical preview of the detected effects.

Figure 6: The power of influence of sub-indicators of pyramidal decomposition of the total tax liability in the case of construction sector, percentage value



Source: Authors` calculation according to data of Finanční správa

According to the kind of processing is obvious at first glance that the majority of indicators had a positive effect on the peak indicator.

5. Conclusion

The previous tables 3 and 4 informed about influences of the individual sub-indicators on the development of the total tax liability. Respecting a fact that characteristics of a certain value can be explained by the changes of its individual elements it can be assumed that the total tax liability during period between years 2005 and 2019 was influenced by changes of the selected sub-indicators. Respecting methodology for assessment of power of their influence mentioned in chapter 3, the table 5 depicts identified order of influence of the individual sub-indicators. The lowest value of order of individual sub-indicator declares the strongest impact on the changes of the total tax liability.

Table 5: Power of influences of individual indicators of the construction sector, percentage value

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
Reliefs	3	5	4	2	6	5	6	3	5	4	3	6	4	4
STR	6	3	1	1	1	4	2	5	3	3	4	5	2	2
ATB I.	1	1	2	3	2	1	1	1	1	1	6	1	1	1
Tax Loss	5	2	6	5	3	3	4	6	6	6	1	2	6	6
R&D	4	6	5	4	5	2	5	2	2	2	2	3	3	5
Donations	2	4	3	6	4	6	3	4	4	5	5	4	5	3

Source: Authors` calculation according to data of Finanční správa

If the order of influences of individual indicators is known, it is possible to determine the overall order of indicators in the monitored period. With regarding the details mentioned in the table 5, there is no doubt that sub-indicator ATB I. influenced development of the total tax liability the most. In spite of the fact that influence of the statutory tax rate were mostly identified as zero value, its influence was enough significant to come second. As it has been already commented the research and development activities gained importance during last fifteen years in the construction sector as well. The donations followed by reliefs were less important. Surprising is to some extent the lowest impact of the tax loss comparing it with the rest of the indicators.

Table 6: Ranking of indicators` impacts of the construction sector

	Construction Sector
Reliefs	5
STR	2
ATB I.	1
Tax Loss	6
R&D	3
Donations	4

Source: Authors` calculation according to data of Finanční správa

As conclusion can be stated that the development of the total tax liability of the construction sector was identified (see table 6). The individual sub-indicators affecting the total tax liability were determined. It could be assumed that the sub-indicator ATB I. had the highest impact of the changes of the tax liability. This fact was confirmed by processing the input data. Somewhat surprising was the significant impact of the change in the tax rate, which was ultimately higher than the impact of the changes in the tax loss.

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Socioemotional wealth importance and financial performance of family firms: A conceptual model and preliminary results

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Abstract

This paper aims to develop and present evidence for a conceptual model of how one of the unique attributes of family firms – the pursuit of non-economic goals, i.e., the importance of socioemotional wealth – is related to their financial performance. Based on a literature review, we suggest the existence of a causal chain in which socioemotional wealth importance positively affects financial performance directly and indirectly through the strength of family social capital (FSC) and reduced interpersonal conflict. In turn, stronger FSC and reduced family conflict will also contribute to financial performance. The theoretical assumptions are supported by correlative results from a study conducted among 21 family business managers who are members of the controlling family.

Key words

Financial performance, socioemotional wealth importance, family social capital, family firm

JEL Classification: M10, M14

1. Introduction

The family firm is one of the most common organisational forms in the world. Family businesses are considered to follow a unique set of economic and non-economic goals, which include transgenerational succession, care for the firm's reputation, provision of employment for family members, among others (Berrone et al., 2012; Debicki et al., 2016). The resource-based view suggests that the strength of family ties can become a source of significant competitive advantage (Arregle et al., 2007). On the other hand, the existence of family ties can also turn into a weakness. Family firms are also susceptible to conflicts that have hardly any parallel in non-family firms, such as marital discord or competition among brothers and sisters (Kubíček & Machek, 2020).

This paper aims to theoretically explain how the focus on non-economic goals, which we refer to as socioemotional wealth importance (SEWi), affects the financial performance of family firms. We address the direct effect and indirect effects in which SEWi affects financial performance through family conflict and family social capital.

Socioemotional wealth (SEW) has become one of the paradigms of family business research. It can be defined as the utility derived from the fulfilment of non-financial goals, which satisfies the needs of family members that control the business (Gomez-Mejia et al., 2007). SEWi can be understood as the extent to which family firms place importance on

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SEW. Recently, Debicki et al. (2016) suggested that SEWi can be approached through three dimensions: family prominence, family continuity, and family enrichment.

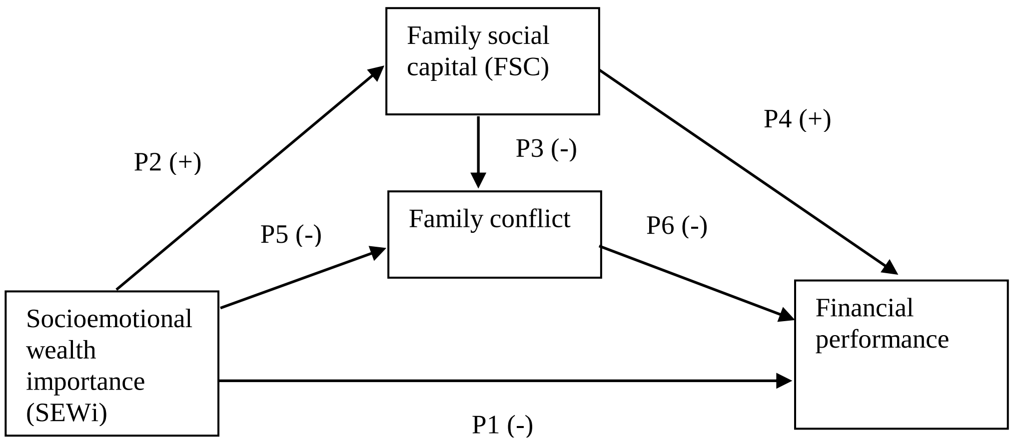
The second major construct considered in this study, social capital (SC), is generally defined as the actual or potential benefits resulting from durable networks and relationships (Bourdieu, 1986). The family business literature further developed the family social capital, which can be understood as the social capital developed among family members (Arregle et al., 2007).

Finally, in this study, we consider the performance effects of family conflict. In controlling families, disagreement between members of the controlling family can occur, which are mostly considered to be harmful to performance (Kubíček & Machek, 2020). The family business literature also uses the terms "affective conflict" or "relationship conflict" to refer to personal incompatibilities and disputes about non-task matters (Jehn & Bendersky, 2003).

2. Model development

This section presents the theoretical arguments for the conceptual model of how socioemotional wealth importance affects financial performance. The expected relationships are displayed in Figure 1.

Figure 1: The conceptual model



The relationship between socioemotional wealth and firm performance is still not well understood. As reported by Debicki et al. (2016), "it has often been shown that the pursuit of SEW can negatively impact financial performance". The reason is that the emphasis on non-financial goals will simply reduce the efforts to increase financial performance; family owners will focus on other, non-financial goals. In highly family-centric firms, nepotism and agency conflicts and possible lack of managerial ability (due to the preference of family members over more qualified non-family directors) can harm family firm performance (Miller & Le Bretton-Miller, 2014). Thus, we also posit that:

Proposition 1: Socioemotional wealth importance is negatively related to financial performance.

There is no doubt that the importance of non-economic goals and social capital (SC) are interrelated. In Berrone et al.'s (2012) conceptualisation of socioemotional wealth (known under the acronym FIBER), binding social ties are one of the key SEW dimensions. Likewise, Debicki et al.'s (2016) SEWi scale consists of the accumulation and conservation of social capital, described as "the importance of benefiting from the social relationships develop through the business and vice-versa". Despite the measurement scales assuming the existence of SEW-SC relationship, the research has been silent to date. A direct positive effect of SEW on social capital was confirmed only by Hernández and Jiménez (2014). On the other hand, Haynes et al. (2015) argue that overemphasis on SEW (in the form of overestimating the family members' abilities and resources) may hinder the existence of external networks and cause a weaker external social capital. A recent study argues that family social capital influences organisational social capital through SEW and human capital (Llanos-Contreras et al., 2021), indicating the existence of an indirect effect. Since the research is relatively undeveloped, the relationship needs to be validated. The preference of non-economic goals, the attempts to maintain family control, and succession aspirations contribute to close family relationships, family harmony and cohesiveness (i.e., family SC). Therefore, we expect that:

Proposition 2: Socioemotional wealth importance is positively related to family social capital.

The concept of social capital brings several advantages to family-operated companies. Family ownership and employment, representing one of the specifics of family firms, imply the existence of closed and strong networks that are based on kinship ties. Members in a closed network (i.e., the family members working in the family firm) develop social capital consisting of trust, shared vision, and norms within the social group. Social capital promotes harmony, cohesiveness (Bendig et al., 2020), shared goals, shared culture, and mutual understanding, which further contribute to better intragroup cooperation (Cruz et al., 2012). Strong family ties, trust, and mutual understanding are likely to create an open culture with a reduced level of dysfunctional conflicts and disagreements. Hence, we suggest that:

Proposition 3: Family social capital is negatively related to family conflict.

Unlike the emphasis on socioemotional wealth, family SC can improve the performance of a family firm (e.g., Miller et al., 2009; Kansikas & Murphy, 2011; Sanchez-Famoso et al., 2015; Herrero, 2018). Family members exhibit strong and durable relationships with each other, which further translates into mutual trust, shared vision, and collective goals. Family members and their behaviour in the firm can create a trustworthy and open culture with collaborative dialogue between family members and other non-family employees (Sorenson et al., 2009). In such a climate, social capital enables access to information and resources (Burt, 1992), promotes growth through good relationships with employees and customers (Marjanski et al., 2019) and increases financial performance through knowledge integration and better information sharing (Kansikas & Murphy, 2011). Moreover, social capital is positively related to innovations (Llach & Nordqvist, 2012; Sanchez-Famoso et al., 2014; Shi et al., 2015). A trustworthy work climate, information sharing, good relationships with customers and employees foster growth and innovation, which are the antecedents of the overall firm's performance. Therefore, based on the existing literature, we expect that:

Proposition 4: Family social capital is positively related to financial performance.

There is little evidence on the relationship between socioemotional wealth importance and family conflict. Family conflict is likely to undermine the transgenerational succession process (Filser et al., 2013). Likewise, affective detachment of family members, which is due to family conflict, will weaken the social ties among family members and identification of family members with the firm. All these factors suggest that family conflict has high socioemotional costs. In the only study we found that mentions the relationship between SEW and conflict, Morgan and Gómez-Mejía (2014) expect that "as means of preserving socioemotional wealth, family owners may seek to minimise negative emotions and maximise positive emotions in the present". Therefore, it can be expected that to protect their socioemotional wealth, families which place a high emphasis on SEW will do their best to prevent and avoid conflicts among family members. In other words:

Proposition 5: Socioemotional wealth importance is negatively related to family conflict.

Dysfunctional conflict is associated with poor interpersonal relationships and negative emotions in family firms and has mostly adverse effects (Kubíček & Machek, 2020). In family firms, family conflict is related to job and career dissatisfaction and turnover intentions (Beehr et al., 1997) and dissatisfaction with the quality of life (Danes et al., 2000). All these factors lead to the conclusion that at the organisational level, family conflict is associated with poor firm performance (Eddleston & Kellermanns, 2007). Thus, it can also be expected that within the scope of this paper, it holds that:

Proposition 6: Family conflict is negatively related to financial performance.

3. Empirical support: Preliminary results

To present evidence of the theoretical model developed in the previous section, we conducted a preliminary survey among family business managers. In the following section, we elaborate on the methods and results.

3.1 Material and methods

In most countries in the world, there is no official database of family firms, and on top of that, the definitions of family firms are far from being standardised. In our research, we adopted the definition of self-identification as a family firm (De Massis et al., 2012); the respondents should describe their firm as a "family business" to qualify for the survey. A CAWI method, employing an online survey prepared in Qualtrics, was employed. The survey was carried out among 21 family business managers who are members of the controlling family. The target group is relatively hard-to-reach; thus, convenience sampling, based on the authors' personal contacts to the representatives of the firms, was the only option. On the other hand, convenience sampling corresponds to the current practices in family business research (e.g., Jocic et al., 2021). Because of the sensitivity of the survey topics (especially the topic of family conflict), whenever possible, we emphasised that the survey is anonymous, and the respondents were guaranteed confidence (we collected no identification data on the person or firm being surveyed).

Regarding the measures, we relied on validated scales from the family business literature. In each case, the values of the individual scales' items were averaged to obtain a composite score. To measure *socioemotional wealth importance*, we employed the nine-item scale of Debicki et al. (2016), which captures three dimensions of SEWi: family prominence, family continuity, and family enrichment. The respondents were asked to indicate on a 5-point Likert scale the importance of maintaining family reputation, family unity, family values, recognition of the family in community, happiness of family members, family harmony or preservation of family dynasty. Examples of questions are: "Indicate how important are the following goals regarding the strategy of your firm" followed by "Recognition of the family in the domestic community for generous actions of the firm" or "Preservation of family dynasty in the business". *Family social capital* was measured using the nine-item ISC-IB scale of Carr et al. (2011). The respondents indicated the degree to which they agreed with statements such as "Family members who work in this firm engage in honest communication with one another", "Family members who work in this firm have confidence in one another", or "Family members who work in this firm are committed to the goals of this firm". To measure *family conflict*, we used the four-item scale of Paskewitz and Beck (2017) that measures relationship conflict among family members. Again, the respondents indicated on a 5-point Likert scale their agreement with statements such as "People often get angry while working in our family firm" or "There is much personal animosity among family members in our firm". Finally, *financial performance* was measured by three questions (Cooper & Artz, 1995) related to the satisfaction with net profit growth, market share, and firm sales relative to the rivals of the firm. An example is the statement "Relative to your rivals, how satisfied are you with your current performance in terms of net profit growth?".

3.2 Pairwise correlation among model variables

Since this paper presents preliminary results of the proposed model, we used pairwise Pearson correlation coefficients to uncover potential relations of interest and relationships for further examination. The Pearson's correlation coefficient (commonly denoted by r) quantifies the linear relationship between two variables in a single metric ranging from -1 (perfect negative correlation) to +1 (perfect positive correlation). Low values of the p-value (commonly denoted as p) indicate a statistical significance of the coefficients (Fenton & Neil, 2012). Table 1 displays the basic descriptive statistics (means and standard deviations) and correlative results. The Cronbach alpha (denoted by α), measuring the internal consistency between items of individual scales, is more than acceptable for all the scales (i.e., greater than 0.8).

Surprisingly, family firms seem to tend to be very optimistic about their social capital, suggesting that they perceive the existence of good interpersonal ties. Nevertheless, the pairwise correlations suggest that the expected relationships presented in the previous section have empirical foundations. Specifically, there seems to be a positive relationship between socioemotional wealth importance and firm performance ($r = 0.41$, $p < 0.1$), which does not support our first proposition. However, we observed a positive correlation between SEWi and family social capital ($r = 0.23$), in favour of the second proposition. Family social capital seems to be negatively correlated with family conflict ($r = –0.21$), thus supporting the third proposition. The fourth proposition postulates that family social capital is positively related to firm performance, which was also supported ($r = 0.25$). We observed that SEWi was negatively correlated with family conflict (r = –0.42, $p < 0.1$) and family conflict seems to be negatively related to performance ($r = –0.22$), supporting the fifth and sixth propositions.

Table 1: Pairwise Pearson correlations and descriptives (N = 21)

Variable	M	SD	α	1	2	3	4
1. SEWi	3.56	0.92	0.87	1.00			
2. FSC	4.22	0.54	0.90	0.23	1.00		
3. Family conflict	2.20	0.86	0.88	–0.42*	–0.21	1.00	
4. Performance	3.43	0.68	0.80	0.41*	0.25	–0.22	1.00

Note: M = mean, SD = standard deviation, α = Cronbach's alpha
* - significant at the 0.1 level

4. Discussion and conclusion

We found partial support for all propositions except for the first proposition P1. The importance of socioemotional wealth seems to increase the strength of family social capital and reduce family conflict, which has an indirect positive effect on financial performance. Proposition 1 predicted that SEWi would be negatively associated with firm performance. The findings can be explained by several reasons. First, we do not measure the objective financial performance but the satisfaction with it. Firms which do not place a high emphasis on financial goals are likely to be satisfied with a moderate financial performance that corresponds to the aspiration level. Second, Debicki et al. (2016) suggest that the link between SEWi and financial performance can be more complex. Recent studies presented a link between SEWi and organisational behaviours that are likely to yield financial returns, especially in the longer term: diversification, environmental performance, R&D investment, and innovativeness (Debicki et al., 2016). One of the key dimensions of SEWi, family prominence (i.e., the care for family and business reputation), is positively related to financial performance in many research samples worldwide; firms that carry the family name are more profitable than other firms (Machek et al., 2019).

One of the contributions of this paper is the hypothesis that socioemotional wealth importance can be positively related to financial performance. The idea that the focus on non-economic goals can be beneficial to the firm is not new, but it is generally expected that the benefits are likely to materialise in the long term. Our results suggest that there can also be short-term gains. We also contribute to the topic of conflicts in family firms, which is still underdeveloped, and the family business and conflict literature are not communicating enough (Kubíček & Machek, 2020).

For sure, this paper is not without limitations. First, convenience sampling, which is still a common practice in family business research, is an obvious limitation. Second, the sample size is small. Because of both limitations, the results should be strictly understood as exploratory, consistent with the conceptual purpose of this paper. To make statistical inference about the model relationships, future research will need to employ a larger sample size. Future studies could also investigate in detail the interplay between the individual components of the construct. For instance, family social capital comprises the structural, cognitive, and relational dimensions, which can have different financial performance outcomes. Likewise, family conflict can be dysfunctional (relationship conflict) or productive (cognitive conflict), depending on the nature and subject of disagreements.

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Do Trading Rules Influence the Market Risk Capital Requirement During a Crisis Period? Evidence from the UK and US Markets

David Neděla [1]

Abstract

This paper aims to analyse the impact of several technical analysis indicators and the stochastic dominance approach in the portfolio decision process in various markets during different crisis periods capturing different market conditions. The impact is mainly examined for the needs of market risk capital requirement analysis based on Basel III market risk capital requirement. To determine the weights of individual assets, maximizing performance ratio portfolio model is suggested. Two strategies for implementing technical analysis rules and the stochastic dominance rule in the portfolio creation process are considered. Strategy 1 aims to eliminate the whole systemic risk of the market with the alternative of investing in a risk-free asset. The second strategy focused on the use of assets that meet particular alarm rules. The results show that the use of strategy 1 to find systemic risk during the crisis reduced the risk of a portfolio with a similar level of profitability. When examining the impact on the market risk of the capital requirement, it was not possible to find a rule that would outperform the simple model in both areas, portfolio performance and MCMR levels.

Key words

Technical analysis; stochastic dominance; portfolio optimization; capital requirement

JEL Classification: G11, G15, G20

1. Introduction

Investors use two basic ways of investing to analyse the stock market in order to make more money, namely, the Fundamental Approach (FA) and Technical Analysis (TA). While analysts who use FA seek to determine a company's intrinsic value based on overall economy, earnings per share, cash flow, and many other financial metrics, TA uses market data analysis (close price, volume, etc.) to evaluate stock market prices. The TA is based on the assumption that past prices and volumes could signal future price developments, and thus generate signals to buy or sell assets. The effectiveness of TA is discussed by many researchers and academics, such as Taylor (2014) or Kouaissah et al. (2020). Already at the end of the twentieth century, Brock et al. (1992) applied two very well-known TA indicators on the Dow Jones Industrial Index for a period of 90 years and concluded that “results are consistent with technical rules having predictive power. Kouaissah et al. (2019) provided theoretical foundations for using moving average (MA) rules in the stock market. In contrast, some of the literature was against the usefulness of technical analysis due to mixed empirical results such as Park et al. (2007) summarized that between 95 studies, 56, 20, and 19 studies find positive, negative, and mixed results with respect to technical trading strategies.

Recently, the stochastic dominance (SD) approach has been highly preferred by researchers, which can also be used for comparing portfolio; see McNamara (1998) or Kopa et al. (2008).

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The SD approach allows us to compare different random variables by their distribution function, such as asset returns or different risk indicators. SD rules of the order of one to four are particularly interesting because they cumulatively impose standard assumptions of risk aversion, prudence, and restraint, which are necessary conditions for standard risk aversion; see Kimball (1993). SD has seen considerable theoretical development and empirical application in various areas of economics, finance, and statistics in recent decades; see Post (2003). In McNamara (1998), the SD approach is applied in the asset selection process.

As the risk in financial markets increases due to the effect of the financial crisis, the pressure on capital requirements on financial institutions that conduct frequent trading increased. At the end of the twentieth century, the banking sector must comply with the Basel regulations provided by the Basel Committee on Banking Supervision (BCBS); see BCBS (1995). After the financial crisis in 2008, the BCBS revised the market risk framework; see BCBS (2009). The revised framework was a part of the new Basel III framework; see BCBS (2010).

The area of analysis trading rules impact to banks capital requirement has not been analysed in depth, therefore, the motivation for this analysis is to depict how the impact of the rules may be reflected in the risk management of banks during unstable market conditions.

The paper aims at analysing the impact of technical indicator rules and the stochastic dominance rule for asset selection in the portfolio decision process with respect to market risk capital requirement in UK and US markets during two investment horizons capturing different market conditions.

2. Methodology

In this section, the methodology applied for empirical analysis is characterized and formulated. Firstly, we will focus on trading rules based on indicators of TA and the SD approach. The portfolio performance measures and the selected model are then formulated. Finally, the calculation of the market risk capital requirement is determined according to the Basel III approach.

2.1 Trading Rules

Given the stated objective of the paper, let us first define possible trading rules. The technical analysis area includes many variously derived indicators, therefore, only several frequently applied indicators are selected; see Mills (1997) or Kouaissah et al. (2019). One of the most popular tools, or simple momentum indicators, used in TA is the moving average (MA). The equation of $MA_{T,n}(x)$ calculation is following:

$$MA_{T,n}(x) = \frac{\sum_{i=0}^{n-1} x_{T-1}}{n} \tag{1}$$

where x_T represents the asset price at time T and n is the selected length of period. In a decision process, the comparison of moving averages with a long N and short n period is used.

The next selected indicator is the exponential moving average (EMA). This indicator is based on a weighted moving average where the highest emphasis is placed on current prices. The formula for calculating $EMA_{T,n}(x)$ for the weighted factor $k = \frac{2}{n+1}$ is following:

$$EMA_{T,n}(x) = EMA_{T-1,n}(x) + k \cdot \left[x_T - EMA_{T-1,n}(x)\right] \tag{2}$$

When the EMA is calculated for the first time, the EMA is the same as the simple MA calculated by formula (1).

Consequently, we can determine signals for investing (buy/hold or sell) as simple rules. If MA (as EMA) is considered as a technical analysis indicator, the signals are defined as follows:

$$Signal\begin{cases} MA_{T,n}(x) \geq MA_{T,N}(x) & buy/hold \\ MA_{T,n}(x) < MA_{T,N}(x) & sell \end{cases} \tag{3}$$

The EMA is needed for the calculation of the moving average convergence divergence trend oscillator (MACD), which is one of the simplest and most effective momentum indicators available; see Metghalchi et al. (2019). The MACD equation is defined by subtracting the long-term EMA (N periods) from the short-term EMA (n periods). Mathematical expression is

$$MACD_{T,n,N}(x) = EMA_{T,n}(x) - EMA_{T,N}(x) \tag{4}$$

In the decision process, a signal curve, usually $EMA_{T,9}(x)$ or the zero-horizontal line, is necessary for the calculation. In a situation where the MACD, defined in Equation (4), is used to determine the buy and sell signals, the rule is as follows:

$$Signal\begin{cases} MACD_{T,n,N}(x) > EMA_{T,9}(x) \wedge MACD_{T-1,n,N}(x) \leq EMA_{T-1,9}(x) & buy/hold \\ MACD_{T,n,N}(x) < EMA_{T,9}(x) \wedge MACD_{T-1,n,N}(x) \geq EMA_{T-1,9}(x) & sell \end{cases} \tag{5}$$

The last indicator selected from the technical analysis is the relative strength index (RSI), which is a momentum indicator that measures the magnitude of recent price changes to evaluate overbought or oversold conditions at the current price of a share or other asset. The RSI value oscillates or moves up and down between 0 and 100. First, an up-change (UC) and a down-change (DC) are calculated for each day; see Anderson et al. (2015). We can define a relative strength (RS) as the ratio between the n-day EMA of the UC time series and the n-day EMA of the DC time series. Analysts usually consider 14 or 9 days EMA, and the formula is as $RS_n(x) = \frac{EMA_n(UC)}{EMA_n(DC)}$, where $EMA_n(x)$ is calculated by Equation (2). Thereafter, the RSI is calculated as follows:

$$RSI_n(x) = \frac{100}{1+RS_n(x)} \tag{6}$$

The trading signals can be set as follows:

$$Signal\begin{cases} RSI_n(x) \leq 65 & buy/hold \\ otherwise & sell \end{cases} \tag{7}$$

Trading rule based on stochastic ordering is the final selected approach. The FSD can be defined as follows: a random variable A first order stochastically dominates a random variable B, expressed as $A \succ_{FSD} B$, if for any z applies $\Pr(A > z) \geq \Pr(B > z)$, where $\forall z \in \mathbb{R}$ and there is at least one z for which a strong inequality applies. Furthermore, if the cumulative distribution function is defined as $F_A(x) \leq \int_{-\infty}^{x} f(z)\mathrm{d}z,$ the random variable A second order stochastically dominates the random variable B, expressed as $A \succ_{SSD} B,$ if for any x applies $\int_{-\infty}^{x} F_A(z)dz \leq \int_{-\infty}^{x} F_B(z)dz,$ where there is at least one x for which a strong inequality applies. Asset selection rule based on SSD approach can be defined as: assets that SSD dominate at least one asset and concurrently are not SSD dominated by other assets are selected. Mathematically:

$$Signal\begin{cases} x_i \succ_{SSD} x_j \wedge x_i \text{is } SSD \text{ nondominated, where } i \neq j & buy/hold \\ otherwise & sell \end{cases} \tag{8}$$

2.2 Portfolio Model and Performance Measures

Let $\mathbf{x} = [x_1, x_2, \dots, x_z]$ is a vector of asset weights and $\mathbf{r} = [r_1, r_2, \dots, r_z]$ is a vector of gross returns, the expected return of the portfolio is an equal weighted average of the asset's expected return formulated as $E(\mathbf{x'r})$. Variance of a portfolio σ_p^2 is defined as $\mathbf{x'Qx}$, where $\mathbf{Q}$ is a covariance matrix of assets. The standard deviation is determined as $\sigma_p = \sqrt{\sigma_p^2}$ and semivariance below the mean is expressed as $SV_p = E((\mathbf{r}_m - \mathbf{x'r})_+^2)$, where the function

$(\lambda)_+^2 = (\max(\lambda, 0))^2$ and $\mathbf{r}_m = \frac{1}{z}\sum_{i=1}^{n} r_i$. The riskiness is $VaR_\alpha(X) = F_X^{(-1)}(\alpha) = -\min\{x|\Pr(X \leq x) \geq \alpha\}$, where X is a random return variable and F_X is its distribution function, then $F_X(\mu) = \Pr(X \leq \mu)$, see Rockafellar et al. (2002). The equation for $CVaR_\alpha(X)$ calculation is $CVaR_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_\alpha \mathrm{d}\alpha$.

Performance ratios are more appropriate for examination, due to the combination of the expected excess return of the asset and its risk. The Sharpe ratio (SR) is one of the most widely used ratios; see Rachev et al. (2008) or Sharpe (1994). The calculation is $SR = \frac{E(\mathbf{x}'\mathbf{r}-\mathbf{r}_f)}{(\mathbf{x}'\mathbf{Q}\mathbf{x})^{\frac{1}{2}}}$, where $\mathbf{r}_f$ is riskless return or benchmark return. The subsequent ratio used to measure portfolio performance is the Sortino ratio (SoR), where the standard deviation as a measure of risk is replaced by the downward or negative deviation, formally defined as $SoR = \frac{E(\mathbf{x}'\mathbf{r}-\mathbf{r}_f)}{E((\mathbf{r}_f-\mathbf{x}'\mathbf{r})_+^2)^{\frac{1}{2}}}$; see Sortino et al. (1994). Another ratio that measures the excess return of the portfolio with maximum drawdown is called the Calmar Ratio (CalR), calculated as $CalR = \frac{E(\mathbf{x}'\mathbf{r}-\mathbf{r}_f)}{\max_{t=1,\dots,T} dd_t(\mathbf{x}'\mathbf{r})}$, where $dd_t(\mathbf{x}'\mathbf{r}) = \max_{s=1,\dots,t} w_s(\mathbf{x}'\mathbf{r}) - w_t(\mathbf{x}'\mathbf{r})$ provided for the calculation $w_s(\mathbf{x}'\mathbf{r}) = \sum_{s=1}^{t} \mathbf{x}'\mathbf{r}_s - \mathbf{r}_{ft}$.

Due to many performance measures of the portfolio being identified, it is as well possible to optimize the portfolio based on these indicators; see Rachev et al. (2008). In general, this problem can be written as the following equation:

$$\begin{gathered} \max \Gamma(\boldsymbol{x}'\boldsymbol{r}) \\ \boldsymbol{x}'\boldsymbol{e} = 1 \\ x_i \geq 0 \; ; \; i = 1, \dots, z \end{gathered} \tag{9}$$

where $\Gamma(\boldsymbol{x}'\boldsymbol{r})$ represents one of the performance measures (such as $SR, SorR, STARR$, or CR).

2.3 Basel III Market Risk Capital Requirement

In the case of BCBS, the standardised calculation of the daily market risk capital requirement (MRCR) is determined based on the higher level of either the VaR of the previous day or the 60-day average VaR multiplied by a correction factor with a 10-day holding period and a 99% confidence interval. The formulation of CR can be determined as $MRCR_t = \max\left\{VaR_{t-1}, (3 + k_t)\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}\right\}$, where VaR_{t-1} is the daily VaR estimated based on the previous day and k_t is the multiplication factor. For more information on the penalty zones determining k_t under the Basel II, see Jiménez-Martín et al. (2009). In the recent BCBS paper, they proposed a transition from 99% VaR to CVaR with a confidence level of $\alpha = 97.5$ %. The new CR is calculated as follows:

$$MRCR_t = \max\left\{CVaR_{t-1}, (3 + k_t)\frac{1}{60}\sum_{i=1}^{60} CVaR_{t-i}\right\} \tag{10}$$

3. Empirical Procedure with Results

For analysis purposes, the data set is contained by daily adjusted close prices of stocks included in the FTSE 100 and NASDAQ-100 indices traded on the UK and US stock markets, downloaded from the Yahoo Finance website[2]. The analysis is carried out in two different time periods containing the financial crisis period, i.e., 1.1.2006 – 31.12.2010 and the Covid-19 crisis period, i.e., 1.1.2016 – 30.6.2021. The investment itself starts in the second year because of the

[2] https://finance.yahoo.com

one-year rolling window approach for determining asset weights. Due to the application of the maximizing performance ratio portfolio approach or for an alternative investment instrument, the risk-free rate is needed, therefore, 3-month government bond returns of individual countries are used, downloaded from the Investing.com website[3]. In all indices, several time series of stock are not included due to an incomplete data set.

The empirical procedure of trading rule implication is divided into four steps. In the first step, the supplementary matrix $M_{T,i}$ during the investment period T of particular assets x_i, where $i \in \langle 1, z \rangle$, is created. The values in $M_{T,i}$ are found throughout the investment horizon according to the alarm rules of the SSD and TA indicators by equations (3), (5), (7), and (8). Due to the requirement of two length intervals that are necessary in MA and EMA, several alternatives of (n, N) are substituted. Recommended values (12,26,9) are used for the MACD indicator. To recall, according to trading rule (8), only non-dominated are selected using the SSD approach. The calculation of the values obtained in the matrix is as follows:

$$M_{T,i} \begin{cases} 1 & if\ buy/hold\ signal\ in\ time\ t\ applies \\ 0 & if\ sell\ signal\ in\ time\ t\ applies \end{cases} \tag{11}$$

After assembling the matrix, we can proceed to the second step, including the application of one of two investment strategies. If applying strategy 1 (S1), the trading rules are used to predict the interval of systemic risk during the investment horizon; see Giacometti et al. (2015) or Kouaissah et al. (2019). If the proportion of assets satisfying rule $M_T = \frac{1}{z}\sum_{i=1}^{z} \mathrm{I}_{[M_{T,i}=1]} \geq \omega$, where ω is the threshold parameter of the systemic risk, it is assumed that the probability of systemic risk loss in the market is low. Otherwise, the investment in a risk-free asset is preferred. Kouaissah et al. (2020) mentioned that the amount of the threshold value ω depends on the investor. In this paper, a value of 25% is set for TA rules and a value of 11 % is set for the SSD rule. The distinction between the parameter values is given by the properties of the indicators. Strategy 2 (S2) differs from the previous one in that the investor should only invest in specific assets that meet the rules in a given interval. The rules that must be met are the same as for S1. Furthermore, asset weights in a portfolio are determined according to the selected portfolio models defined in Equation (9). Weights are calculated on the basis of the one-year rolling window. In the final step, the portfolio final value W_T and performance indicators are calculated. The procedure for calculating the W_T is as follows:

$$W_{t+1} = \begin{cases} (W_t - t_c)(1 + x' r_{t+1}) & if\ buy/hold\ signal\ in\ time\ t\ applies \\ (W_t - t_c)(1 + r_{f_{t+1}}) & otherwise \end{cases} \tag{12}$$

where t_c are transaction costs set proportionally as 20 basis points. In the portfolio decision-making process, a 20-day rebalancing interval is considered. For simplicity, the initial investment in the portfolio W_0 is set as 1 currency unit. It is also assumed that the value of the weight is not limited, and a short sale of shares is prohibited, i.e. $x_i \geq 0$ and $x_i \in \langle 0,1 \rangle$.

At the beginning of the empirical procedure, the most suitable portfolio model that maximizes one of the performance ratios is selected for further purposes. The resulting portfolio performance values are shown in Table 1.

As can be seen in Table 1, the most appropriate model is maxCalR as both the final value of the portfolio W_T and the portfolio performance indicators are the highest for all markets and investment periods. For this reason, this model will be applied during the subsequent analysis.

Since it is necessary to determine the parameters n and N for the needs of the MA and EMA rules, a sensitivity analysis of these parameters is performed. Values of $n \in \langle 5{,}100 \rangle$ $N \in \langle 5{,}250 \rangle$ with differences of 5 were chosen. For each combination of strategy, market, and period, the 3 most suitable combinations are selected according to the achieved value of SR,

[3] https://www.investing.com

while the individual trading rule is combined with the maxCalR model (9). Thereafter, the portfolio characteristics of individual portfolios are determined by applying the relevant trading rules while a specific strategy is being considered. The results of these portfolios are presented in the Table 2.

Table 1: Results of portfolio models without trading rules

Model	W_T	$E(\mathbf{x'r})$	σ_p	$VaR_{0.05}$	SR	$STARR$
FTSE 100 during investment period 1.1.2007 – 31.12.2010						
maxSR	1.1745	0.0002	0.0152	0.0245	0.0089	0.0036
maxSorR	1.4128	0.0003	0.0163	0.0275	0.0194	0.0080
maxCalR	1.9581	0.0007	0.0210	0.0322	0.0304	0.0136
maxSTARR	1.6876	0.0005	0.0174	0.0291	0.0283	0.0116
FTSE 100 during investment period 1.1.2017 – 30.6.2021						
maxSR	1.2066	0.0002	0.0135	0.0194	0.0156	0.0065
maxSorR	1.0807	0.0001	0.0139	0.0207	0.0061	0.0025
maxCalR	2.0914	0.0008	0.0163	0.0241	0.0511	0.0212
maxSTARR	1.0209	0.0000	0.0137	0.0211	0.0015	0.0006
NASDAQ-100 during investment period 1.1.2007 – 31.12.2010						
maxSR	1.3850	0.0003	0.0198	0.0317	0.0161	0.0068
maxSorR	1.4203	0.0003	0.0206	0.0319	0.0166	0.0069
maxCalR	1.6547	0.0005	0.0239	0.0359	0.0207	0.0090
maxSTARR	1.5634	0.0004	0.0208	0.0325	0.0211	0.0088
NASDAQ-100 during investment period 1.1.2017 – 30.6.2021						
maxSR	2.6849	0.0011	0.0185	0.0293	0.0606	0.0240
maxSorR	2.9617	0.0012	0.0195	0.0283	0.0632	0.0252
maxCalR	4.2177	0.0016	0.0243	0.0385	0.0672	0.0269
maxSTARR	2.7767	0.0012	0.0190	0.0315	0.0611	0.0249

Table 2: Results of portfolio models with trading rules

S1							S2						
Rule	W_T	$E(\mathbf{x'r})$	σ_p	$VaR_{0.05}$	SR	$STARR$	Rule	W_T	$E(\mathbf{x'r})$	σ_p	$VaR_{0.05}$	SR	$STARR$
FTSE 100 during investment period 1.1.2007 – 31.12.2010													
MA(50,250)	3.0952	0.0011	0.0168	0.0232	0.0650	0.0299	MA(15,130)	3.0015	0.0011	0.0194	0.0274	0.0546	0.0262
MA(65,220)	3.0952	0.0011	0.0168	0.0232	0.0650	0.0299	MA(20,130)	2.9112	0.0011	0.0197	0.0295	0.0523	0.0246
MA(65,210)	3.0952	0.0011	0.0168	0.0232	0.0650	0.0299	MA(70,160)	3.0466	0.0011	0.0203	0.0280	0.0529	0.0255
EMA(70,135)	3.1425	0.0011	0.0169	0.0234	0.0656	0.0303	EMA(40,55)	2.6988	0.0010	0.0195	0.0274	0.0489	0.0235
EMA(90,120)	3.1425	0.0011	0.0169	0.0234	0.0656	0.0303	EMA(30,35)	2.7714	0.0010	0.0201	0.0295	0.0488	0.0235
EMA(85,130)	3.1425	0.0011	0.0169	0.0234	0.0656	0.0303	EMA(25,40)	2.8534	0.0010	0.0207	0.0295	0.0488	0.0231
MACD	1.5522	0.0004	0.0166	0.0229	0.0247	0.0102	MACD	1.3650	0.0003	0.0201	0.0301	0.0141	0.0059
RSI	1.9581	0.0007	0.0210	0.0322	0.0304	0.0136	RSI	1.7173	0.0005	0.0222	0.0339	0.0230	0.0105
SSD	1.8891	0.0006	0.0207	0.0310	0.0292	0.0129	SSD	2.0846	0.0007	0.0201	0.0260	0.0348	0.0169
FTSE 100 during investment period 1.1.2017 – 30.6.2021													
MA(10,30)	2.0733	0.0008	0.0145	0.0205	0.0566	0.0244	MA(10,15)	2.4721	0.0010	0.0162	0.0215	0.0631	0.0283
MA(15,25)	2.0852	0.0008	0.0148	0.0216	0.0561	0.0236	MA(85,90)	2.1855	0.0009	0.0174	0.0233	0.0506	0.0233
MA(15,35)	2.0258	0.0008	0.0144	0.0200	0.0552	0.0236	MA(50,90)	2.1310	0.0009	0.0178	0.0243	0.0479	0.0230
EMA(90,245)	2.0914	0.0008	0.0163	0.0241	0.0511	0.0212	EMA(50,145)	1.9762	0.0008	0.0154	0.0230	0.0497	0.0208
EMA(100,250)	2.0914	0.0008	0.0163	0.0241	0.0511	0.0212	EMA(60,135)	1.8898	0.0007	0.0152	0.0229	0.0471	0.0196
EMA(90,250)	2.0914	0.0008	0.0163	0.0241	0.0511	0.0212	EMA(50,150)	1.8606	0.0007	0.0153	0.0231	0.0457	0.0189
MACD	1.8771	0.0007	0.0140	0.0190	0.0506	0.0213	MACD	1.5980	0.0005	0.0151	0.0215	0.0350	0.0134
RSI	2.0914	0.0008	0.0163	0.0241	0.0511	0.0212	RSI	1.8826	0.0007	0.0165	0.0225	0.0432	0.0196
SSD	2.3107	0.0009	0.0134	0.0187	0.0707	0.0305	SSD	1.2206	0.0002	0.0180	0.0247	0.0124	0.0052
NASDAQ-100 during investment period 1.1.2007 – 31.12.2010													
MA(25,40)	3.4903	0.0012	0.0200	0.0305	0.0617	0.0280	MA(5,180)	3.0835	0.0011	0.0224	0.0340	0.0495	0.0232
MA(30,40)	3.4903	0.0012	0.0200	0.0305	0.0617	0.0280	MA(85,245)	2.9408	0.0011	0.0231	0.0353	0.0460	0.0212
MA(25,45)	3.4903	0.0012	0.0200	0.0305	0.0617	0.0280	MA(10,225)	2.8142	0.0010	0.0222	0.0347	0.0460	0.0216
EMA(15,130)	2.3472	0.0008	0.0201	0.0305	0.0418	0.0179	EMA(5,160)	2.6773	0.0010	0.0221	0.0335	0.0440	0.0207
EMA(20,140)	2.3472	0.0008	0.0201	0.0305	0.0418	0.0179	EMA(35,155)	2.6736	0.0010	0.0222	0.0357	0.0438	0.0204
EMA(15,160)	2.3472	0.0008	0.0201	0.0305	0.0418	0.0179	EMA(40,150)	2.6373	0.0010	0.0220	0.0351	0.0435	0.0202
MACD	1.3928	0.0003	0.0181	0.0298	0.0179	0.0073	MACD	1.5021	0.0004	0.0232	0.0360	0.0172	0.0074
RSI	1.6547	0.0005	0.0239	0.0359	0.0207	0.0090	RSI	1.6762	0.0005	0.0235	0.0356	0.0216	0.0096
SSD	1.4273	0.0004	0.0233	0.0358	0.0150	0.0063	SSD	1.4811	0.0004	0.0245	0.0395	0.0157	0.0068
NASDAQ-100 during investment period 1.1.2017 – 30.6.2021													
MA(85.120)	4.4232	0.0017	0.0242	0.0385	0.0696	0.0278	MA(5,20)	4.8374	0.0018	0.0236	0.0313	0.0759	0.0304
MA(90,135)	4.4232	0.0017	0.0242	0.0385	0.0696	0.0278	MA(100,190)	5.5942	0.0020	0.0266	0.0411	0.0735	0.0299
MA(100,130)	4.4232	0.0017	0.0242	0.0385	0.0696	0.0278	MA(30,220)	4.6480	0.0017	0.0241	0.0372	0.0725	0.0288
EMA(5,25)	3.6431	0.0015	0.0206	0.0352	0.0713	0.0282	EMA(70,140)	4.8420	0.0018	0.0250	0.0386	0.0716	0.0288
EMA(5,20)	3.4658	0.0014	0.0205	0.0352	0.0687	0.0271	EMA(95,150)	4.4094	0.0017	0.0239	0.0381	0.0706	0.0286
EMA(45,120)	4.2177	0.0016	0.0243	0.0385	0.0672	0.0269	EMA(70,135)	4.6643	0.0017	0.0248	0.0377	0.0706	0.0280
MACD	3.9452	0.0016	0.0205	0.0325	0.0761	0.0304	MACD	3.3407	0.0014	0.0209	0.0338	0.0654	0.0268
RSI	4.2177	0.0016	0.0243	0.0385	0.0672	0.0269	RSI	2.9249	0.0012	0.0235	0.0379	0.0517	0.0203
SSD	3.4690	0.0014	0.0222	0.0368	0.0636	0.0253	SSD	2.2570	0.0009	0.0252	0.0406	0.0367	0.0148

From Table 2, it can be observed that in most cases of the application of the MA and EMA indicators with S1, the values found for the best three combinations are identical. In the second strategy (S2), due to a different amount of invested assets, the characteristics of the portfolio are not the same. The results show that it is with these two indicators that the investor is able to surpass the results of the models without trading rules. When the remaining rules are included in the portfolio strategy, the results usually do not reach advantageous values compared to the simple model, with the exception of e.g., SSD rule with S1 on the UK market during the period of the Covid 19 crisis. From the data shown, it is apparent that the advantage of S1 with MA or EMA is the reduction in portfolio risk (expressed as σ_p or $VaR_{0.05}$) at an approximate or higher rate of return, which has a positive effect on portfolio performance indicators (SR or $STARR$). On the contrary, S2 shows an increase in profitability at approximately the same level of risk, which has the same effect on the performance. The difference in results between these strategies can be explained by eliminating systemic risk, including an alternative risk-free investment. If we compare the increase in W_T values when including S1 or S2 in both crises, we can observe that the effect was more pronounced during the financial crisis. The duration of the crisis, together with the portfolio rebalancing interval, will have some effect on this, and we should also be aware of short-term rapid impact of the Covid 19 crisis on the world financial markets.

In the final step of empirical analysis, the main purpose of the paper is examined, specifically, whether the use of a strategy with a trading rule has a positive effect on the level of MRCR value. The calculation is performed using Equation (10) and individual curves are plotted in the following Figures 1 and 2 for S1 and Figures 3 and 4 while S2 is used.

Figure 1: MRCR on UK (left) market and US (right) market during period 2008–2010 while S1 is applied

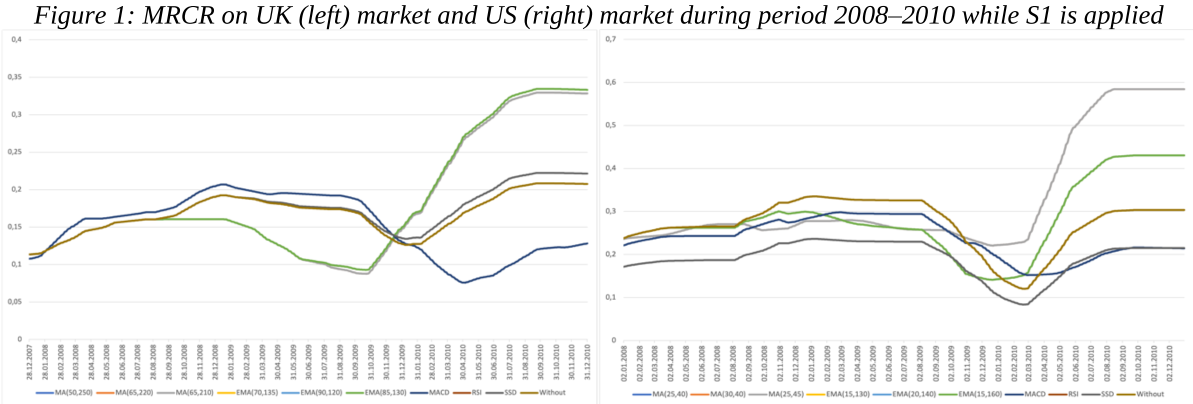
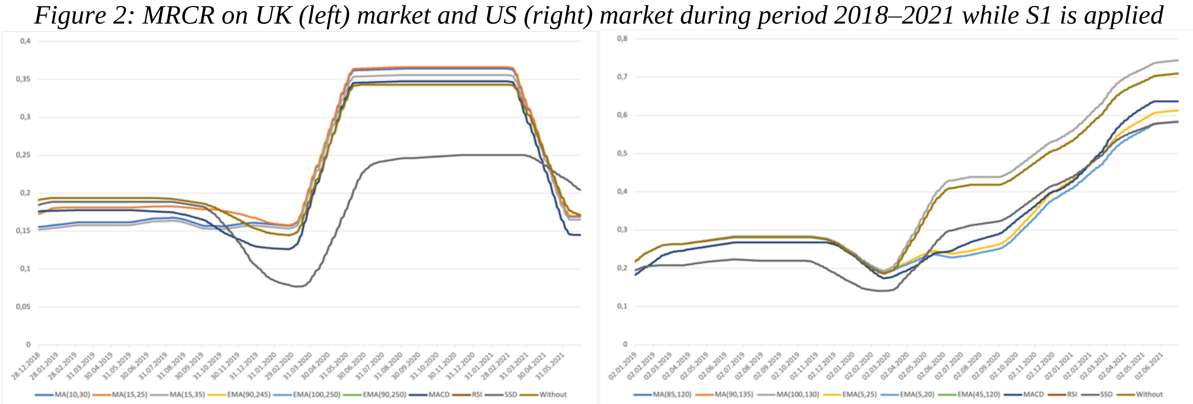


Figure 2: MRCR on UK (left) market and US (right) market during period 2018–2021 while S1 is applied



We can see from Figure 1 that during the financial crisis, the MA and EMA curves (light grey and green) decreased compared to a simple portfolio model (brown curves), but immediately after the crisis at the beginning of the 2010 year, the situation completely changes in both markets. The expected impact of the MCMR being lower during the crisis is slightly achieved, but this is partly influenced by trading signals to purchase risk or risk-free assets before the crisis. For banking institutions, it would be advantageous for the MCMR rate to be below the level of a simple model at a time when crisis data are already included in the calculation. Given the fact that the riskiness of the portfolio when considering this strategy was clearly lower, a significant post-crisis increase of MRCR is surprising. One reasonable explanation for this situation is determining MRCR on the historical data basis. Looking at the values of the vertical axis, we find a greater repercussion on the US market. A similar situation is evident from Figure 2, where the differences between the individual curves are very slight, only in the US market we can observe a lower MCMR rate for EMA rules, but the W_T and risk of these portfolios is lower. In contrast, favorable portfolio characteristics as well as MCMR values are generated using the SSD rule.

Figure 3: MRCR on UK (left) market and US (right) market during period 2008–2010 while S2 is applied

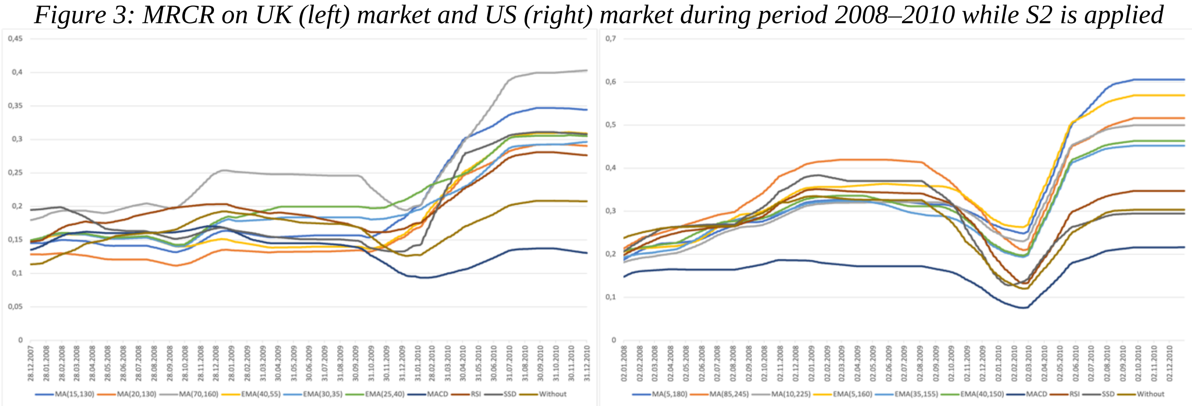
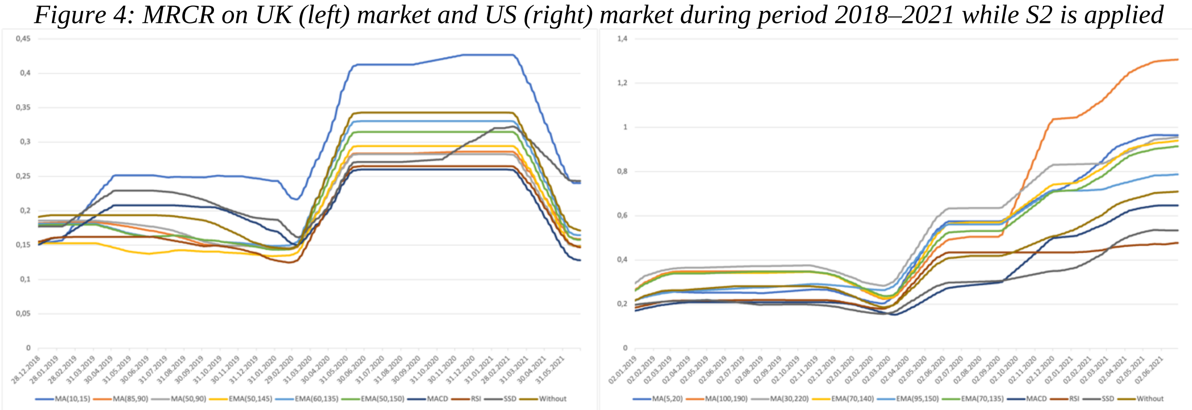


Figure 4: MRCR on UK (left) market and US (right) market during period 2018–2021 while S2 is applied



If a second strategy how to use trading rules for selecting investment assets is included, the conclusion is similar to the previous Figures 1 and 2. In most cases, all the recommended trading rules from the previous analysis examined from the portfolio characteristics are inappropriate from the MCMR point of view compared to the simple portfolio model without including trading rules. The exception is in the UK market in Figure 4, where the MA (85,90) and MA (50,90) rules are below the level of the brown curve, basically indicating a kind of benchmark throughout the period. In the remaining market conditions in Figures 3 and 4, it is not possible to find a rule that would surpass the general model in both areas, portfolio performance and MCMR levels.

4. Conclusion

This paper provided new perspectives on the combination of technical analysis and stochastic dominance trading rules on the development of market risk capital requirement from the perspective of a banking institution. The object of this paper was to analyse the impact of technical indicator rules and the stochastic dominance rule for asset selection in two portfolio creation strategies with respect to the market risk capital requirement. To make the analysis more representative, the UK and US markets during two investment horizons affected by the crisis were considered.

From empirical results, it can be observed that in most cases of the application of the MA and EMA indicators together with particular strategies provides the highest portfolio performance in period of crisis. The results showed the advantage of S1 with MA or EMA due to the reduction of portfolio risk while the rate of return is higher or at least remains the same. On the contrary, while S2 were applied, an increase in profitability was achieved at approximately the same level of risk, which had the same effect on performance. When examining the impact of the capital requirement on market risk, in most market situations, no rule overcomes the general model in both portfolio performance and MRCR levels. There was also observed a significant post-crisis increase of MRCR.

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Evaluation of Classification Ability of Multiclass Rating Models

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Abstract

The paper aims to develop rating models using discriminant and logistic regression analysis methods and evaluate their classification accuracy based on the classification tables and ROC analysis. The estimated models represent classification rules for assigning the rating. Since we consider five rating groups in the empirical study, we face multiclass classification models' evaluation. The classification accuracy is typically assessed by the ROC analysis, for example, in scoring models. However, in the multiclass model, we face a more complex problem. The main purpose of this paper is to recommend and present an approach, which is suitable for this kind of problem. The overall results of this paper suggest that although the classification tables serve as a good indication of classification accuracy, one should pay attention to ROC analysis which provides more detailed information about the model.

Keywords

AUC, classification, discriminant analysis, logistic regression, rating, ROC curve

JEL Classification: C52, G24, G33

1. Introduction

This study is focused on the problem of rating modelling based on discriminant and logistic regression analysis methods. However, the main goal of this paper is not to deal in detail with individual models and the influence of financial variables on ratings. Instead, in this article, we will focus on verifying the classification capability of developed models. The estimated models represent classification rules for assigning the rating. Since we consider five rating groups in the application study, we face multiclass predictive models' evaluation. The ROC analysis typically assesses the predictive accuracy; however, we face a more complex problem in the multiclass model. The main goal of this paper is to recommend and present an approach, which is suitable for this kind of problem.

The rating plays the role of assessing the degree of credit risk. Therefore, it is necessary to have data containing ratings for selected companies or instruments to estimate rating models that will replace this role in a certain way. However, obtaining comprehensive data, including both the rating and other characteristics of individual entities, is limited, as it is usually not publicly available. Therefore, in the application study, rating models will be created based on the so-called MORE Rating, similar to global agencies' rating. The primary purpose of rating models is to provide a corresponding rating assignment as an alternative to the agency rating. Generally, the models offer rating which is solely based on publically available financial information. By such means, it is possible to assess unquoted companies, which is significant for countries with low numbers of companies with agency ratings. It is easy to use the models; one must put the right figures into the model and interpret results. Some credit agencies focus

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almost exclusively on quantitative data, which they incorporate into a mathematical model. Thus, a well-estimated model partially fulfils the role of rating agencies. While rating agencies emphasise that financial and non-financial factors matter in predicting default and bond rating, the academic literature has focused primarily on economic indicators' ability to predict ratings. However, recent studies have shown that other factors, such as the state and perspective of the overall economy or the industry characteristics, may improve rating models' predictive ability. The credit rating models are typically based on historical performance and the credit risk event, for example, default, bankruptcy or the change of credit rating assessment.

Some research of bond rating dates back to the fifties of the twentieth century, for example, Hickman (1958) or Fisher (1959). A regression analysis became one of the most used methods to estimate rating in this period. An alternative approach to predict bond ratings is the multiple discriminant analysis introduced, for example, by Pinches and Mingo (1973), Ang and Patel (1975), Altman and Katz (1976) and Belkaoui (1980). Subsequent research compared particular statistical methods; e.g. Kaplan and Urwitz (1979) compare ordered probit analysis with ordinary least square regression, Wingler and Watts (1980) compare ordered probit analysis with multiple discriminant analysis. Recent studies come from the theoretical framework mentioned above and extend statistical methods for new non-conservative approaches such as neural networks (Dutta and Shekhar, 1988; Surkan and Singleton, 1990). Waagepetersen (2010) assesses the relationship between quantitative models and expert rating evaluation. More recently, Altman, Sabato and Wilson (2010) focus on the importance of non-financial information within risk management.

The following structure corresponds to the main aim of this paper. The next section briefly describes the statistical methods by which we will derive rating models. Due to the paper capacity, attention is paid only to the main concepts, with the appropriate references to the literature. Since the main task of the paper is to present and apply the method of multiclass model's evaluation, in the next part of the second chapter, we will focus mainly on the essence of classification tables and ROC curves. Finally, the main part of the paper, which is the application and assessment of models, can be found in the third chapter. As mentioned above, attention is paid to evaluating the classification ability of models using classification tables and ROC analysis. The main results and recommendations are then summarised in conclusion.

2. Overview of methodology

Rating models will be developed based on the methods of discriminant analysis and logistic regression. Both approaches are shortly described further in the text, emphasising the literature that deals with these approaches in detail. Since the paper focuses on assessing the classification ability of multiclass models, this chapter describes the essence of classification tables and the basics of ROC analysis.

2.1 Description of methods for model estimation

Discriminant analysis is a standard statistical method used to separate groups and thus a suitable method for credit scoring or bond rating modelling. The analysis can be used for two primary objectives: first, the description of group separation; second, predicting or allocating observations to groups. Huberty and Olejnik (2006) distinguish between descriptive discriminant analysis (DDA) and predictive discriminant analysis (PDA). The purpose of DDA is usually the study of comparison among a certain number of groups for each of which we have several outcome variable scores. However, suppose the single set of response variables are used as predictors, and there is a single grouping variable. In that case, the primary purpose is to analyse how well group membership of analysis units may be predicted using PDA.

Correspondingly, Rencher (2002) differentiates between discriminant and classification functions. Discriminant functions separate groups, while classification functions assign individual units to one or more groups. In group separation, linear functions of variables describe the differences between two or more groups. The main objective is to identify the relative contribution of p variables to split. The latter problem is focused on the prediction or allocation of observations to groups, which is a common goal of discriminant analysis. A prediction rule then consists of a set of linear combinations of predictors, where the number of combinations reflects the number of groups. Discriminant functions are linear combinations of variables that best separate groups, for example, the k groups of multivariate observations. The description of discriminant analysis and methods can be found, for instance, in Rencher (2002), Manly (2004), Huberty and Olejnik (2006), Tabachnik and Fidell (2007), Harrell (2010 or Hair et al. (2014).

Logistic regression is a rather different approach to discriminant analysis. In finance, logistic regression is mostly used in its bivariate context. However, it can be easily modified for the outcome variable with more than two possible values. The common problem where logistic regression can be applied is the prediction of default. Most bankruptcy models are based on scoring methodology, where there are two possible values of the outcome variable, for example, default and non-default. Multinomial logistic regression must be applied when exploring relationships among rating and firms' indicators since there are more than two dependent variable categories. In this case, the number of categories comes from the number of rating groups. The simplest case is when there are just two rating categories: investment and speculative grades. Then, the outcome rating is dichotomous or binary. Univariate or multiple logistic regressions can be used to estimate the prediction model. There is a vast literature on logistic regression methods, for example, Hosmer, Lemeshow and Sturdivant (2013), Menard (2010), Harrel (2010), or Tabachnik and Fidell (2007).

Generally, if we intend to describe the relationship between an outcome (dependent) variable and a set of independent (predictor or explanatory) variables, logistic regression is a suitable method. Hosmer et al. (2013) distinguish among several types of logistic regression models according to the number of variables used in the model, for example:

- Binary (dichotomous) models – the model with a single variable, the multiple logistic regression model,
- polychotomous models – the multinomial logistic regression model, the ordinal logistic regression model.

2.2 Classification tables and ROC analysis

The estimated models represent a classification rule which is used to assign objects into classes. Thus, we need to know how effectively this classification rule works, preferably using the validation sample. It means that the available data are split into two datasets: an experimental sample (training) used for constructing the rule and the validation sample used for assessing the performance. There are other ways to split the data, for example, the leave-one-out method, when only one data point is put in the validation sample and others in the experimental sample, or the bootstrap methods.

To understand how the performance is measured, we need to specify some terms required for further analysis. According to Krzanowski and Hand (2009), a classification rule yields a score $s(\boldsymbol{X})$ for each object. It will result in distribution $p(s|\mathrm{P})$ for objects in the positive group, P, and distribution $p(s|\mathrm{N})$ for objects in the negative group, N. Then, the classifications are given by comparing the scores with a threshold, *T*. Krzanowski and Hand (2009) claim that if we can find a threshold $T = t$ such that all members of class P have scores that are all greater than t, and all members of class N have scores all less or equal to t, we attain the perfect classification. However, the two sets of scores typically overlap to some extent, and perfect classification is impossible. In this case, performance is measured by the extent to which scores for objects in class P tend to take large values, and scores for objects in class N tend to take small values. The methods used for these measurements are based on the two-by-two classification table resulting from cross-classifying the true class of each object by its predicted class. Krzanowski and Hand (2009) state that the proportions of the validation set that fall in this table's cells are empirical realisations of the joint probabilities $p(s>t,P), p(s>t,N), p(s\le t,P), p(s\le t,N)$. Then, different ways of summarising these four joint probabilities yield various measures of classification performance. Generally, we can use the misclassification or error rate measure, which is the probability of a class N object having a score greater than t or a class P object having a score less than t. The misclassification rate is a widely used criterion; however, it weights the two kinds of classification (class N misclassified as P, and vice versa) equally important.

We use the following two conditional probabilities and one marginal probability in the evaluation of classification ability (Krzanowski and Hand, 2009):

- The false positive rate (fp) – the probability that an object from class N yields a score greater than $t: p(s>t|\mathrm{N})$,
- the true positive rate (tp) – the probability that an object from class P yields a score greater than $t: p(s>t|\mathrm{P})$,
- the marginal probability that an object belongs to class $\mathrm{P}: p(\mathrm{P})$.

Next, we use two complementary conditional rates and one complementary marginal probability (Krzanowski and Hand, 2009):

- The true negative rate (tn), $p(s\le t|\mathrm{N})$ - the proportion of class N objects which are correctly classified as class N, equal to $1-fp$,
- the false negative rate (fn), $p(s\le t|\mathrm{P})$ - the proportion of class N objects which are correctly classified as class N, equal to $1-fp$,
- the marginal probability that an object belongs to class $N: p(\mathrm{N})=1-p(\mathrm{P})$.

The true positive rate is typically called the Sensitivity (Se), and the true negative rate is the Specificity (Sp). The rates described above are all conditional probabilities of having a particular predicted class given the true class. As Krzanowski and Hand (2009) point out, there are obvious relationships between the various conditional, marginal, and joint probabilities. For example, the misclassification rate e of a classification rule can be expressed as a weighted sum of the true positive and false positive rate:

$$e=(1-tp)\cdot p(\mathrm{P})+fp\cdot p(\mathrm{N}). \tag{1.1}$$

Since the true positive and negative rates of a classification rule are complementary, they are typically used together as joint performance measures. Generally, the true positive rate increases as t decreases, while the true negative rate decreases with a lower t. Thus, we can find the misclassification rate as the value of t which leads to the overall minimum of the weighted sum e in the formula (1.1). Krzanowski and Hand (2009) suggest another way to determine the threshold by choosing the maximum $tp-fp$, or $tp+tn-1$ (Sensitivity + Specifity – 1). The maximum value is called the Youden index (YI).

Generally, the performance measures are based on comparing the distributions of the scores for the positive and negative populations. A good classification rule tends to produce high scores for the positive population and low scores for the negative population. The larger the extent to which these distributions differ, the better the classifier. The graphical depiction of both two distributions is presented by the ROC (Receiver Operating Characteristic) curve.

The method based on the ROC curve is a commonly used way for assessing the performance of classification rules. First, the graph shows the true positive rate (tp) on the vertical axis and the false positive rate (fp) on the horizontal axis, as the classification threshold t varies. Then, the misclassification rate is the minimum distance between the curve and the upper left corner of the square containing the ROC plot (Krzanowski and Hand, 2009).

If we develop a classification rule for more than two classes, we face a more complex problem. For example, the rating models assign objects into several rating categories. In this case, we combine multiple ROC curves and use different approaches for assessing the performance. Chen (2009) suggests treating the situation by a series of two-class analyses. There are two main approaches to how the ROC analyses can be achieved:

- Assuming k classes, we produce k different ROC curves by considering each class in turn as population P and the union of all other classes as population N,
- we have all $k(k-1)$ distinct pairwise-class ROC curves.

Both approaches are suitable for summary statistics, such as the AUC (Area Under the Curve). In the case of perfect separation of P and N, AUC is the area under the upper borders of the ROC (the area of a square of side one, so the upper bound is 1). In random allocation, AUC is the area under the chance diagonal (the area of a triangle whose base and height are equal to 1, so the lower band is 0.5). Based on Krzanowski and Hand (2009), the AUC can be generally expressed as

$$AUC = \int_0^1 y(x)dx. \qquad (1.2)$$

The AUC can be defined as the average positive rate, taken uniformly over all possible false positive rates in the range (0,1). A frequently used interpretation of AUC is that it is a probability that the classifier will allocate a higher score to a randomly chosen individual from population P than it will to a randomly and independently chosen individual from population N.

3. Empirical study

The main objective of the empirical study is to develop models for rating prediction based on commonly used financial variables. However, in this paper, the main focus is on assessing the classification ability, as described in Chapter 2.2. The application aims to estimate the models using logistic regression and discriminant analysis. In addition, we focus on the sample selection effect. Therefore, the primary purpose of this study is to determine the impact of the sample selection approach and used method and compare and evaluate estimated models.

3.1 Dataset and procedure

This study analyses corporate credit rating from eight countries from Central and Eastern Europe (CEE): the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia. The models are estimated based on MORE Rating[2] (the Multi-Objective Rating Evaluation). The dataset includes 1249 very large industrial companies from the mining, manufacturing, and construction sectors for 2002-2007. We have observations available for each company according to individual years, which gives 6646 cases with an assigned rating for all years. For the estimation, the dataset is divided into an experimental and validation sample.

[2] MORE ratings classify companies similarly as rating agencies (Bureau van Dijk Electronic Publishing, 2008). The MORE rating is calculated using a unique model that references the company's financial data to create an indication of the company's financial risk level.

The partial aim of this study is to compare the models developed based on two approaches of data splitting, referred to as 'random' and 'non-random'. Table 1 summarises both ways of dataset split, random and non-random (numbers in brackets). Firstly, all companies and years' observations are randomly divided into experimental (83%) and validation (17%) samples. In the second way of splitting, the experimental sample (84.3%) contains observations from the first five years (2002-2006), and the validation sample (15.7%) includes data from the year 2007. Although the validation sample does not have observations in the outer categories B and AA, it will be used for validation. Therefore, it reflects the actual rating assessment of the firms in this particular year. Compared to the first approach, this division is given by the year of observation, and for this study, the dataset will be considered non-random.

Table 1: Experimental and validation sample

Rating	Code	Experimental sample		Validation sample	
		Number of observations	**Percentage**	**Number of observations**	**Percentage**
B	1	277[i] (368)[ii]	5.51 (6.57)	91 (0)	5.62 (0)
BB	2	893 (1183)	17.76 (21.12)	290 (644)	17.92 (61.63)
BBB	3	2353 (2428)	46.80 (43.35)	719 (312)	44.44 (29.86)
A	4	1212 (1314)	24.11 (23.46)	414 (89)	25.59 (8.52)
AA	5	292 (308)	5.83 (5.5)	104 (0)	6.43 (0)
Total		5028 (5601)	100 (100)	1618 (1045)	100 (100)

i – random, ii – non-random

We consider ten financial variables as predictors with a potential effect on rating: logarithm of total assets (*lnta*), return on assets (*roa*), return on equity (*roe*), equity to total assets (*eqta*), logarithm of interest coverage (*lnintcov*), logarithm of liquidity ratio (*lnliqr*), logarithm of cash flow (*lncf*), logarithm of current ratio (*lncurr*), logarithm of long-term debt to total assets (*lnltdta*) and the percentage of ebitda to total debt (*ebitdar*). Using the discriminant (LDA), and multinomial (MLR) and ordinal (OLR) logistic regression analysis methods, we develop six rating models according to the number of classification classes and used dataset (Table 2). We consider five rating categories as dependent variables and ten financial variables as independent predictors in all models.

Table 2: Summary of rating models

Model	Method	Dataset
Model 1 Model 2	LDA	Non-random Random
Model 3 Model 4	MLR	Non-random Random
Model 5 Model 6	OLR	Non-random Random

3.2 Classification tables

The main classification characteristics of the estimated models are summarised in Table 3. The criteria for comparison and ranking the models are the following classification tables (confusion matrices):

- The resubstitution classification table is obtained by classifying the observations used to build the discriminant model, referred to as Class. (ES).
- The classification table, which is based on the estimation ability of the validation sample, referred to as Class. (VS).

Table 3: Classification characteristics

Model	Method	Sample	No. of obs. (ES)	Class. (ES)	No. of obs. (VS)	Class. (VS)
1	LDA	Non-random	3518	0.8511	792	0.8775
2	LDA	Random	3274	0.8525	1036	0.8600
3	MLR	Non-random	3518	0.8846	792	0.8801
4	MLR	Random	3274	0.8861	1036	0.8851
5	OLR	Non-random	3518	0.8469	792	0.8649
6	OLR	Random	3274	0.8494	1036	0.8600

The overall classification ability of all models is similar; however, multinomial logistic models indicate the highest classification accuracy. As shown in Table 3, the models show good classification ability when using the experimental sample and the validation sample. The detailed classification tables are summarised in Annex 1. Although the individual models show slight differences in the classification accuracy of the individual classes, they are overall very similar. To conclude the classification accuracy of estimated models, we will focus on the ROC analysis in the next part.

3.3 ROC analysis

Our application developed classification rules for five rating classes, which is a more complex problem than the binary task. To assess the performance of estimated models, we will combine multiple ROC curves. As mentioned in Chapter 2.2, there are two main approaches how to evaluate the models:

- Assuming 5 classes, we produce 5 different ROC curves by considering each class in turn as population P and the union of all other classes as population N. Thus, this approach requires creating 5 ROC curves for each model, totally 30 ROC curves for all models.
- If we are concerned about the pairwise-class ROC curves, then we have to produce $5(5-1)=20$ROC curves for each model, a totally 120 ROC curves in our application.

Based on the above data, it is clear that the ROC analysis for more than two-class models is much more complex and laborious compared to the two-class model evaluation. Therefore, in the ROC analysis, we will focus on the first approach. For the sake of scope and clarity, only some partial results will be presented further in this paper.

Since we found that the random MLR model shows the best results based on the previous analysis using classification tables. We will compile and interpret ROC curves for this model in more detail. As we evaluate one MLR model (Model 4), we will construct five ROC curves using the first approach. The constructed ROC curves are presented in Annex 2. We see that the model shows a very good ability to classify rating groups 3 and 4. However, the other graphs show us a somewhat non-standard shape of the ROC curve, which even falls below the diagonal level. The model's weakness is rating 5, while the classification of ratings 1 and 2 is slightly better. However, if we assess the ability to classify according to AUC, the values indicate an overall good classification accuracy. A certain explanation for the worse classification ability of rating groups 5, respectively 1 and 2, is the number of observations with this rating in the data sample. It can be assumed that with a higher and more even number of observations in individual groups, the classification ability of the model could also increase.

Due to the complexity of comparing a multiclass model using ROC curves, we will now focus on the AUC criterion. Table 4 shows the data on AUC and standard error for each model, according to individual categories. Model 4, with the highest overall classification accuracy, performs an increased ability to predict rating groups 4 and 3. Although the AUC is relatively high for other rating groups, the standard error is also higher, which reduces the overall quality of the model. For comparison, we can look at Model 1. Although this model has a slightly lower overall classification ability (see Table 3), on the other hand, the model shows a more uniform ability to predict individual groups.

Table 4: AUC

Model	Cat 1	Cat 2	Cat 3	Cat 4	Cat 5
1	0.9282 (1.5e+03)	0.9660 (58.5217)	0.9615 (0.0522)	0.9651 (29.1885)	0.9896 (10.3059)
2	0.9847 (189.2545)	0.9783 (24.4359)	0.9369 (33.4701)	0.7618 (264.3357)	0.9485 (101.3124)
3	0.8204 (89.1037)	0.9734 (15.2719)	0.9567 (31.1444)	0.6304 (206.4606)	0.9685 (72.1311)
4	0.9440 (92.6120)	0.9278 (82.9925)	0.9511 (40.4607)	0.9666 (21.8489)	0.7828 (137.5765)
5	0.3975 (9.0887)	0.8878 (0.0075)	0.9458 (18.1759)	0.9369 (147.9278)	0.9500 (84.4514)
6	0.7044 (77.6807)	0.9567 (97.2729)	0.9304 (17.9168)	0.9583 (16.1070)	0.9747 (18.1306)

4. Conclusion

The basic way of evaluating the classification ability is the use of classification tables. In this case, the models should be applied to data other than the ones from which they were derived. A suitable way is to divide the dataset into experimental and validation samples. Thus, we can determine whether the model can correctly assign objects not used in the analysis. Based on a comparison of the overall classification capability of the models, it is clear that the quality of the models is very similar, and their classification capabilities do not differ significantly. When using a validation sample, all models show a classification accuracy above 85%, a very high number considering they classify five rating groups. The advantage of classification tables is a general overview of how a given model can classify individual objects. The disadvantage, however, is that this is only basic information, which should be supplemented by further analysis.

Various methods are used to assess the classification ability in more detail. ROC curves and comparisons of the AUC of individual models were used in this paper. As shown in this article, ROC analysis in multiclass models is a relatively complex and laborious matter. There are two primary approaches to how to proceed. In this paper, we compiled as many ROC curves as there are categories for a given model. Then, we compare each category with the other classes as a whole. This paper showed five ROC curves for the model whose overall classification capability appeared to be the highest. In a more detailed analysis of this model, we found that although the classification accuracy is relatively high, it varies among rating groups. Based on the presented results, it is clear that evaluating the qualification capability of multiclass models is a complex problem and needs to be given sufficient attention.

Acknowledgement

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Annexes

Annex 1: Classification tables (validation sample)

Model 1

actrat_10y	classification 2	3	4	5	Total
3	28	451	21	0	500
	100.00	94.75	9.33	0.00	63.13
4	0	25	193	12	230
	0.00	5.25	85.78	19.05	29.04
5	0	0	11	51	62
	0.00	0.00	4.89	80.95	7.83
Total	28	476	225	63	792
	100.00	100.00	100.00	100.00	100.00

Model 2

actrat	classification 1	2	3	4	5	Total
1	11	18	0	0	0	29
	64.71	11.69	0.00	0.00	0.00	2.80
2	6	130	33	0	0	169
	35.29	84.42	5.87	0.00	0.00	16.31
3	0	6	487	26	0	519
	0.00	3.90	86.65	10.92	0.00	50.10
4	0	0	42	208	10	260
	0.00	0.00	7.47	87.39	15.38	25.10
5	0	0	0	4	55	59
	0.00	0.00	0.00	1.68	84.62	5.69
Total	17	154	562	238	65	1,036
	100.00	100.00	100.00	100.00	100.00	100.00

Model 3

actrat_10y miss	predrat_10y 2	3	4	5	Total
3	32	449	19	0	500
	100.00	95.33	8.44	0.00	63.13
4	0	22	196	12	230
	0.00	4.67	87.11	18.75	29.04
5	0	0	10	52	62
	0.00	0.00	4.44	81.25	7.83
Total	32	471	225	64	792
	100.00	100.00	100.00	100.00	100.00

Model 4

actrat_miss	predrat_10 1	2	3	4	5	Total
1	24	5	0	0	0	29
	82.76	3.18	0.00	0.00	0.00	2.80
2	5	141	23	0	0	169
	17.24	89.81	4.21	0.00	0.00	16.31
3	0	11	484	24	0	519
	0.00	7.01	88.64	9.80	0.00	50.10
4	0	0	39	215	6	260
	0.00	0.00	7.14	87.76	10.17	25.10
5	0	0	0	6	53	59
	0.00	0.00	0.00	2.45	89.83	5.69
Total	29	157	546	245	59	1,036
	100.00	100.00	100.00	100.00	100.00	100.00

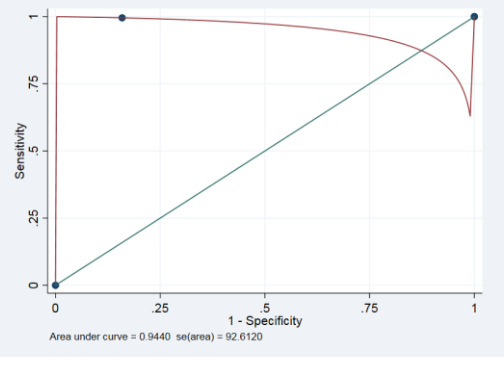
Model 5

actrat_10oy	predrat_10oy 2	3	4	5	Total
3	32	445	23	0	500
	100.00	94.28	9.87	0.00	63.13
4	0	27	194	9	230
	0.00	5.72	83.26	16.36	29.04
5	0	0	16	46	62
	0.00	0.00	6.87	83.64	7.83
Total	32	472	233	55	792
	100.00	100.00	100.00	100.00	100.00

actrat_10o	predrat_10o 1	2	3	4	5	Total
1	21	8	0	0	0	29
	80.77	5.06	0.00	0.00	0.00	2.80
2	5	134	30	0	0	169
	19.23	84.81	5.48	0.00	0.00	16.31
3	0	16	477	26	0	519
	0.00	10.13	87.20	10.57	0.00	50.10
4	0	0	40	210	10	260
	0.00	0.00	7.31	85.37	16.95	25.10
5	0	0	0	10	49	59
	0.00	0.00	0.00	4.07	83.05	5.69
Total	26	158	547	246	59	1,036
	100.00	100.00	100.00	100.00	100.00	100.00

Annex 2: ROC curves (Model 4)

Cat1



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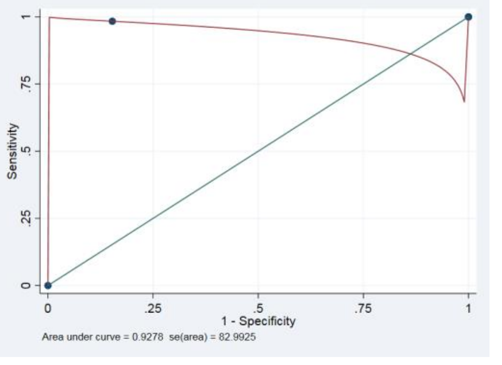

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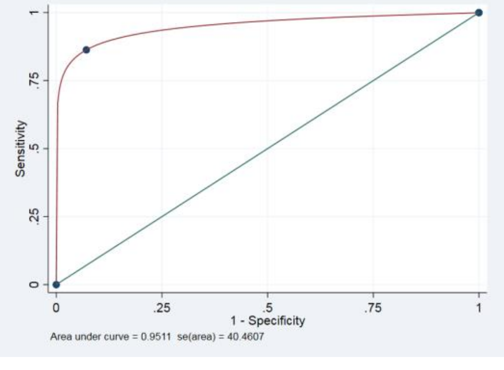

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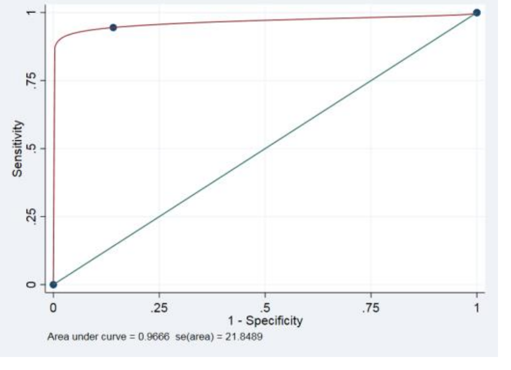

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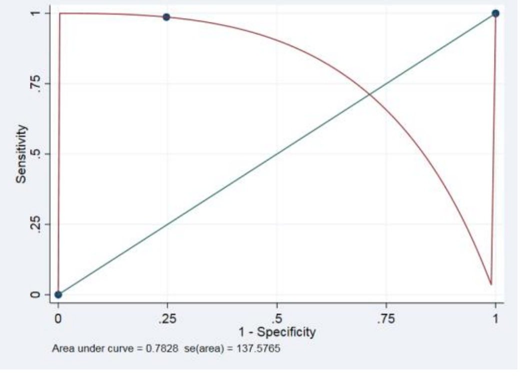

Impact of Covid-19 on the Credit Risk of a Portfolio of Debt Assets

Josef Novotný[1], Kateřina Kořena [2]

Abstract

The topic of the conference paper is determination of credit risk for debt assets portfolio for the time period before Covid-19 and for the period after Covid-19. The main objective of the conference paper is to determine the value of economic capital of a portfolio including ten selected debt assets by using the CreditMetrics™ model for the time period before Covid-19 and for the period after Covid-19 and compare the calculated value of economic capital for the analyzed periods..

Key words

CreditMetrics™, economic capital, credit risk, recovery rate, value at risk, asset value model, Monte Carlo simulation

JEL Classification: G21, G24, G28.

1. Introduction

Credit risk represents the potential loss if the counterparty (borrower) is unable to meet its obligations both on time and in full. Credit risk is the most significant risk in banking and failure to manage this risk has led to bank failures many times. As a result of Covid-19, there have been restrictions on the movement of people within countries, with negative effects on the functioning of economies. The economies of the countries went into deep recession, which had a negative impact on the financial situation of companies and thus on their ability to repay their debts.

The aim of the paper is to calculate the value of economic capital for a constructed portfolio of ten debt instruments for the period before Covid-19 and for the period after Covid-19 using the CreditMetrics methodology and to compare the value of economic capital.

2. Description of the CreditMetrics™ methodology

The model was developed by J.P. Morgan in 1977 as a mark-to-market model and allows to describe a portfolio of financial assets using VAR methodology. The essence of this methodology is to convert all risks into a common denominator, the change in the value of a portfolio of debt assets (as a result of a change in rating, there is a change in the credit margin that translates into the discount rate that directly affects the present value of the debt asset).

The basis of the model is a transition matrix that gives the probability of moving from one rating category to another.

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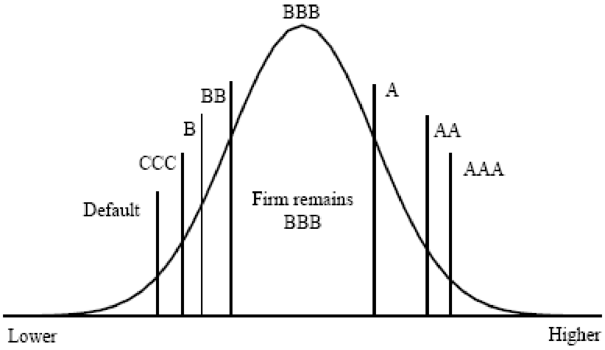
The VAR represents the maximum possible losses at a given confidence level (usually 99% but more frequently 99.5% or even 99.9%,) over a given time interval and can be interpreted in two different ways.

1) At a given significance level α, the losses on a portfolio of debt assets ($-\Delta\tilde{\prod}$) over a given time interval will be greater than a predetermined value of loss (*VAR*), e.g. there is only a 1% probability that the loss will be greater than a predetermined value of XZ Euro), this can be expressed by the relation,
$$\Pr\left(-\Delta\tilde{\prod} \geq VAR\right) = \alpha\,. \tag{1}$$
2) At a given significance level α, the profit on the portfolio of debt assets ($\Delta\tilde{\prod}$) over a given time interval will be less than a predetermined profit level (-VAR), this statement can be written as follows,
$$\Pr\left(\Delta\tilde{\prod} \leq -VAR\right) = \alpha\,. \tag{2}$$

The VAR calculation can be performed using the analytical method or the Monte Carlo simulation method, which is based on a large number of simulations of the evolution of the value of the asset portfolio. The essence of the model is to determine the probability distribution of the asset portfolio value increment (*(*$\Delta\tilde{\prod}$*)* at a given confidence level α. The asset portfolio value increment can be written using the following formula,
$$\Delta\tilde{\prod} = \tilde{V}_P^T - V_P^t = \sum_n \tilde{V}_{n,j,T} \cdot x_n - \sum_n V_{n,i,t} \cdot x_n\,, \tag{3}$$
where $\tilde{V}_P^T (V_P^t)$ is the initial (predicted) value of the portfolio, $V_{n,i,t}$ is the value and x_n is the quantity of the n-th asset in the i-th rating category in the asset portfolio. $\tilde{V}_{n,j,T}$ represents the value of the n-th asset in the j-th rating category at the end of a predetermined time horizon *T*. The time horizon is generally one year. The value of an asset is based on the rating grade that the asset has at the end of the time horizon.

In the CreditMetrics methodology, the underlying process of developing the value of an asset (debt instrument) $\tilde{V}_{n,j,T}$, is based on the Asset Value Model, which is based on an option-theoretic approach. According to this theory, the value of a firm is a random variable with some distribution. If this asset value should fall so significantly that it is less than the amount of outstanding financial liabilities (the value of the firm falls below the "bankruptcy threshold"), then the firm will be unable to meet its debts to creditors and will default. However, this does not mean that the probability of default must be estimated on the basis of the variability of the value of the firm. Firm value volatility is used to quantify the probability of joint rating changes. In modelling the rating of a firm in a portfolio, the market value of the firm is used as a reflection of market share prices. Then, if the value of the firm exceeds a certain level, the rating will change. This is illustrated in Figure 1.

Figure. 1: Asset value model and thresholds



Zdroj: : CreditMetricsTM – Technical document

Assuming that the firm's value thresholds are known, it is necessary to model the change in the firm's value in order to describe the evolution of the rating. The change in firm value is represented by the asset turnover *r*, assumed for modelling purposes to be $r \sim N(0,1)$. Assuming that *r* has a normal distribution, the probability of occurrence of each event can be calculated. The calculation of the thresholds is shown in Table 1 below.

Table. 1: Calculation of thresholds for each rating level

Rating	Probability corresponding to the Asset value model
AAA	$1-\Phi(Z_{AA}/\sigma)$
AA	$\Phi(Z_{AA}/\sigma) - \Phi(Z_{A}/\sigma)$
A	$\Phi(Z_{A}/\sigma) - \Phi(Z_{BBB}/\sigma)$
BBB	$\Phi(Z_{BBB}/\sigma) - \Phi(Z_{BB}/\sigma)$
BB	$\Phi(Z_{BB}/\sigma) - \Phi(Z_{B}/\sigma)$
B	$\Phi(Z_{B}/\sigma) - \Phi(Z_{CCC}/\sigma)$
CCC	$\Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma)$
Default	$\Phi(Z_{Def}/\sigma)$

Source: : CreditMetricsTM – Technical document

The asset thresholds (Z_{AA}, Z_{A}, Z_{BBB},.......Z_{D}) between each rating category are determined using the normalised normal distribution function Φ (0;1) and the transition matrix.

When simulating the value of the portfolio, correlations between individual borrowers need to be determined. CreditMetrics determines correlations indirectly, based on a set of indices that calculate correlations between borrowers. First, correlation matrices between the industry indices are created using the industry indices of each country and then the borrowers are assigned to each industry (to each index), including their share (w^{odv}) of their activity in the industry. The development of the return on equity of a firm operating in one industry can be written using the following equation,

$$r^{A} = w^{odv} \cdot r^{odv} + w^{spec} \cdot r^{spec}, \qquad (4)$$

where r^{A} is the stock return of a given firm, w^{odv} is the coefficient of determination and w^{spec} is the proportion of revenue that is firm-specific, r^{odv} represents the portion of returns explained by the sector index, and r^{spec} is a weight characterising the firm's specific return Since standardized returns are assumed (the normalized variance of the firm is σ2 = 1), then we can deduce w^{spec} as follows:

$$w^{spec} = \sqrt{1-\left(w^{odv}\right)^{2}} \; . \qquad (5)$$

The calculation of correlations between firms can be solved using matrices. The correlation matrix of each index is called the C matrix. Not only do the weights for the individual indices enter into the calculation, but also the specific components, so it is necessary to create an auxiliary matrix $\overline{C}$ *(m+n,m+n)* that includes both, which can be written as follows,

$$\overline{C} = \left[\begin{array}{ccc|ccc} & & & 0 & \dots & 0 \\ & C & & \vdots & \ddots & \vdots \\ & & & 0 & \cdots & 0 \\ \hline 0 & \dots & 0 & & & \\ \vdots & \ddots & \vdots & & E & \\ 0 & \cdots & 0 & & & \end{array}\right]. \tag{6}$$

The upper left part of the matrix represents the correlations between the individual indices, the lower right (inverse) matrix represents the correlations between the specific components of individual firms that are equal to one (ones on the diagonal) and independent of the specific components of other firms (other values of zero). The rest of the matrix is occupied by zeros, reflecting the absence of correlations between the specific components and the indices. Next, it is necessary to construct a matrix W(m+n,n), where the columns represent individual firms and the rows represent industry weights and firm specific turnovers, this matrix can be written as follows,

$$W = \left[\begin{array}{ccc} w_{11} & \cdots & w_{1n} \\ \vdots & \cdots & \vdots \\ \vdots & \cdots & \vdots \\ w_{m1} & \cdots & w_{mn} \\ \hline w_{k1}^{spec} & \cdots & 0 \\ 0 & \cdots & w_{kn}^{spec} \end{array}\right]. \tag{7}$$

The correlation matrix of individual firms' returns *A(n,n)* is given by the following relation,

$$A = W^T \cdot \overline{C} \cdot W, \tag{8}$$

where W^T represents the transposed matrix *W*.

When simulating the return on assets (credit instruments), it is necessary to take into account correlations between individual borrowers, for this purpose the Cholesky algorithm is used. Firstly, the correlation matrix *A* has to be decomposed into a lower triangular matrix *A** using the Cholesky decomposition, this decomposition is given by the following relations::

$$a_{ii} = \sqrt{\left(s_{ii} - \sum_{k=1}^{i-1} a \right)}, \tag{9}$$

$$a_{ij} = \frac{1}{a_{ii}} \left(s_{ij} - \sum_{k=1}^{i-1} a_{ik} \cdot a_{jk} \right), \tag{10}$$

where *s* symbolizes the elements of the original correlation matrix A, then *a* represents the elements of the lower triangular matrix *A**. The matrix of correlated scenarios *Z* can be written as follows,

$$Z = A^* \cdot Y, \tag{11}$$

where *Y* represents the generated matrix with independent variables with standard normal distribution.

To determine the probability distribution of a portfolio increment, individual assets should be valued at both the initial decision date *t* and the end date of the predetermined time horizon

T, the value of the asset being based on its rating at that date. The value of an asset at time *T* when assigned to the i-th rating category is given by,

$$\tilde{V}_T^i = \sum_{T+n}^{T+n} \frac{CF_{T+n}}{\left(1+{}_t f_{T,T+n}^i\right)^n}, \tag{12}$$

where CF_{T+n} are the cash flows arising from the asset, ${}_t f_{T,T+n}^i$, is the forward rate determined at time t for the interval $T,T+n$ based on forward yield curves for individual ratings. The forward rate for the i-th rating is given by,

$$f_n^i = \left(1+f_n^F\right) \cdot \left\{ \frac{1 - RR \cdot \sum_{j=1}^{n} \frac{p_j^i - p_{j-1}^i}{\left(1+f_j^F\right)^j}}{1-p_n^i} \right\}^{1/n} - 1, \tag{13}$$

where RR is the expected recovery rate, usually determined from historical data, p_n^i is the probability of default over n years in the i-th rating category, f_n^F is the one-year risk-free rate, given by,

$$f_n^F = \frac{\left(1+f_n\right)^n}{\left(1-f_{n-1}\right)^{n-1}} - 1+, \tag{14}$$

where f_n is the forward rate (e.g. LIBOR, PRIBOR, EUROLIBOR, IRS – interest rate swap etc).

Using the probability distribution of portfolio gains, economic capital can be defined as the difference between the VAR at a given significance level and the mean of the losses, which can be written as,

$$EK = VaR_\alpha - E\left(-\Delta\tilde{\Pi}\right). \tag{15}$$

3. Credit risk determination using CreditMetrics for Covid-19 period

This section calculates the value of economic capital for the pre-Covid-19 period and for the period affected by the Covid-19 pandemic. For the period before Covid-19, the input data for that period is used and the calculations are realized at 2. 1. 2020. Meanwhile, for the Covid-19 period, the data used already takes into account the effects of the covide crisis and the calculations are made to 2. 1. 2021. The calculations are performed using the CreditMetrics methodology described in the previous section.

3.1 Input data

The calculation of economic capital is performed on a portfolio consisting of 10 bonds traded on the Frankfurt Stock Exchange (FSE) with a nominal value of EUR 10 million, with bonds of each company having a nominal value of EUR 1 million. The important characteristics of the bond portfolio are set out in the following *Table. 2.*

Table. 2: Basic characteristics of the bond portfolio

Name	Rating	Maturity	Nominal value	Cupon	pcs.
NIKE	AA-	03/2030	2 000,00 €	2,85%	500
General Electric	BBB+	09/2029	1 000,00 €	4,13%	1000
Daimler AG	BBB+	08/2028	1 000,00 €	1,13%	1000
Tencent	A+	02/2025	200 000,00 €	3,80%	5
BMW	A+	01/2023	1 000,00 €	2,38%	1000
Deutsche Wohnen SE	A-	08/2028	1 000,00 €	1,68%	1000
Deutsche Telekom AG	BBB	07/2023	10 000,00 €	2,25%	100
Oracle Corp.	A	03/2026	2 000,00 €	1,65%	500
Deutsche Bank	BBB+	02/2025	1 000,00 €	2,75%	1000
Sanofi	AA	04/2024	100 000,00 €	0,63%	10

Source:: Frankfurt Stock Exchange (FSE)

To calculate the present value of the bonds, we need to use risk-free rates (*fnF*), which in this case were calculated from the interest rate swap (IRS) rates from Erste Group. The calculated risk-free rates are captured in the following Table 3.

Table. 3: Interest rate swap (IRS) and calculated forward rates

Rok	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
IRS	-0,20%	-0,18%	-0,12%	-0,01%	0,08%	0,19%	0,30%	0,42%	0,52%	0,63%	0,72%
fnF	-0,20%	-0,17%	0,02%	0,31%	0,43%	0,75%	0,95%	1,27%	1,35%	1,64%	1,60%

Source:: Erste Group

To calculate the present value of the bonds, you also need to know recovery rate. For the calculation of economic capital for the period from 2.1.2020, data as of the end of 2019 were used. For the calculation of economic capital for the period from 2.1.2021, data as of the end of 2020 were used. Recovery rates for each period are shown in the following Table 4.

Table. 4: Average recovery rates for European senior debt

Year	Estimated recovery rate
2019 (4Q)	57 %
2020 (4Q)	55 %

Source: S&P Global Ratings - Leveraged Finance: European Leveraged Finance And Recovery

The Covid-19 crisis had an impact on the economies of individual countries and thus on the financial situation of companies. The result was an increase in the probability of default, which is captured in the following. Table 5.

Table. 5: Corporate Annual Default Rates By Rating Category

Year	AAA	AAA	AAA	BBB	BB	B	CCC/C
2019 (4Q)	0,00	0,00	0,00	0,11	0,00	1,49	29,76
2020 (4Q)	0,00	0,00	0,00	0,00	0,93	3,52	47,48

Source: S&P Global Ratings - 2020 Annual Global Corporate Default And Rating Transition Study

3.2 V Calculating economic capital using CreditMetrics

To derive the covariance and correlation matrix, we first need to calculate the returns. Then, using the forward yield curves that are based on the transition matrix, the bond values for each rating level will be determined. The transition matrix will also be used to derive the transition limits between each rating grade. Subsequently, a Monte Carlo simulation will be carried out to generate 25,000 random returns for each bond. The correlated returns will be determined as the product of these random returns and the Cholesky matrix. Based on the transition limits, each yield will be assigned a rating category and each bond will be assigned its respective value according to this rating. The value of the entire portfolio will be determined as the sum of the values of the individual bonds. In the following Table 6, the resulting portfolio values for each period are shown.

Table. 6: Resulting debt portfolio values for 2020 and 2021 in €

	2020			2021		
	Value at initial rating	**Expected value**	**Expected loss**	**Value at initial rating**	**Expected value**	**Expected loss**
NIKE	1 133 704	1 133 520	184	1 120 165	1 119 988	177
General Electric	1 232 849	1 231 781	1 067	1 234 306	1 233 073	1 234
Daimler AG	976 606	975 814	792	983 774	982 709	1 065
Tencent	1 180 493	1 180 220	272	1 166 988	1 166 729	259
BMW	1 082 154	1 081 941	213	1 070 375	1 070 184	191
Deutsche Wohnen SE	1 027 344	1 026 980	364	1 029 863	1 029 456	407
Deutsche Telekom AG	1 072 600	1 068 798	3 802	1 062 918	1 058 441	4 477
Oracle Corp.	1 045 681	1 045 533	148	1 050 933	1 050 825	108
Deutsche Bank	1 113 781	1 112 923	857	2 087 509	2 085 677	1 832
Sanofi	1 009 731	1 009 647	83	1 012 688	1 012 619	69
Portfolio	**10 874 942**	**10 867 158**	**7 784**	**11 819 519**	**11 809 701**	**9 818**

The results show that in the Covid-19 period there was a slight increase in the value of the portfolio and also an increase in the value of the expected loss.

Table 7 below shows the portfolio values for different significance levels, on the basis of which the value of economic capital is then calculated.

Table 7: Portfolio value for different significance levels for two time periods in €

	2020		2021	
alpha	**Portfolio value**	**VaR (€)**	**Portfolio value**	**VaR (€)**
0,1	10 210 564	-664 377	10 930 567	-888 951
0,5	10 439 968	-434 974	11 244 993	-574 525
1	10 567 030	-307 912	11 285 180	-534 338

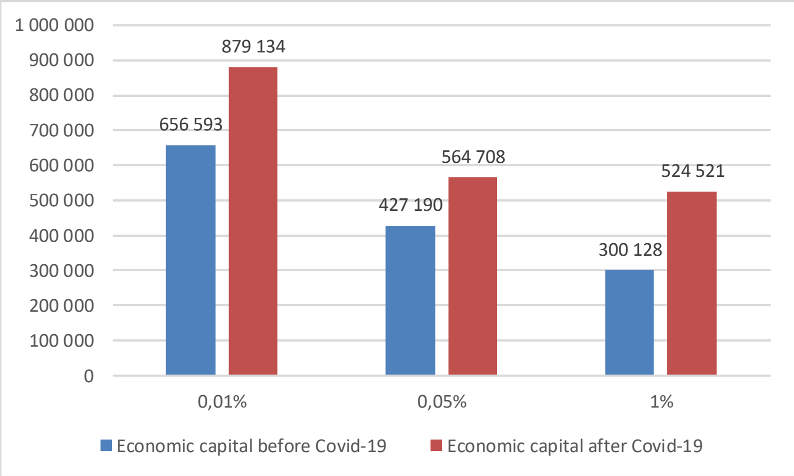
On the basis of the results obtained, it can be said that the value of the portfolios in the period before the Covid-19 pandemic will not fall below € 10 210 564 with a probability of 99,9 % (0.01% significance level) and the size of the loss will not exceed € 664 377, while in the following period (the period affected by the Covid-19 pandemic) the value of the portfolio will not fall below € 10 930 567 with a probability of 99,9 % (0.01% significance level) and the size of the loss will not exceed € 888 951.

The value of economic capital is calculated as the value of the loss (VaR) at a given significance level less the value of the expected loss. The value of economic capital is shown in Table 8 and Chart 1 below.

Table. 8: Value of economics capital for different significance levels in €

	2020		2021	
VaR v %	Expected loss	Economic capital	Expected loss	Economic capital
0,01	7 784	656 593	9 818	879 134
0,05		427 190		564 708
1		300 128		524 521

Chart 1: Value of economic capital at the 0,01%, 0,05% and 0,1% confidence levels for two time periods in €



The results show that there was a slight increase in the value of the debt portfolio in the second period (the period affected by the Covid-19 pandemic). Portfolio value at initial rating increased by € 934 759 (an increase of about 8.5%). The value of the expected loss increased by € 2 034, from € 7 784 to € 9 818 (an increase of 26%). The value of economic capital at the 0.01% significance level increased by € 222 540, from € 656 593 to € 879 134 (an increase about 34%). The value of economic capital at the 0.05% significance level increased by € 137 517, from € 427 190 to € 564 708 (an increase about 33%).

4. Conclusion

The results show that in the second period (the period affected by the Covid-19 pandemic) there was both a significant increase in the value of the expected loss and a significant increase in the value of economic capital.

The Covid-19 pandemic had a negative impact on the financial situation of firms, leading to an increase in the probability of default, as well as a decrease in the recovery rate, leading to an increase in the value of economic capital. While very highly rating debt assets had minimal impact from Covid_19, the impact of Covid_19 on the value of economic capital increased as credit ratings decreased.

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Uncertainty Effects of European Integration

Daniel Pastorek1

Abstract

We developed a new monthly index of European integration uncertainty based on a textual analysis of major European newspapers in 1986 – 2020 (more than 20 million newspaper articles). We find that our index seems to correspond to the real-world events stressing the European integration process. Our index is correlated with the economic policy uncertainty index but not entirely. Our index differs in occasional spikes and empathizes on real-world events differently (e.g., in time of financial crisis or UK referendum). Using vector autoregression framework, our results suggest that European integration uncertainty is likely to have a real impact on a number of financial integration and structure indicators.

Key words:

uncertainty, integration, European Union, media news

JEL Classifications: D80, F36

1. Introduction

The European integration process is a result of the interaction of interests, which outcomes are not a priori clear to be positive or negative. The economic integration is an integral part of this process with visible results achieved, begin the European Payments Union and the European Coal and Steel Community to a common currency. Nonetheless, it is clear from the financial markets, that we are far from perfectly integrated markets and importantly, this level of convergence varies over time. That's because integration is still an ongoing process and its outcomes are often challenged by sets of real-world events. These events are often preceded by uncertainty about the effects of the integration. On the one side, there are various integration approaches through national representatives look through in building common project or finding a solution, which may end up in paralysis the functioning of the EU (from 'empty chair' crisis to blocking of EU treaty by the UK in 2011). On the other side, there are uncertain economic impacts of integration outcomes. For example, as in the case of deployment of the euro and its excepted immense but unfulfilled effect on increased international trade (Rose, 2000), still ongoing debates on the benefits and costs of the euro or actual uncertain economic development of the UK after Brexit or the EU itself.

In this paper, we assume that uncertainty about European integration affects the behavior of economic agents, which has an impact on market forces. To measure this uncertainty, we develop the monthly uncertainty index of the European integration based on a textual analysis of major European newspapers in 1986 – 2020 (more than 20 million newspaper articles). We examine the effects of our new uncertainty index for a number of integration and structure variables within a vector autoregression framework.

Significant relation of the newspapers text quantification and economic variables suggest e.g., Garcia (2013) using a fraction of negative/positive words in relation with stock market

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returns. Husted et. al (2019) developed a news-based index to capture the monetary uncertainty of the Federal Reserve using a similar approach as the economic policy-uncertainty index (EPU) by Baker et. al. (2016). Index of geopolitical risk by Caldara and Iacoviello (2018) and others such as index of American partisan conflict (Azzimonti, 2018). Most of these types of studies were originally for autarkic economies like the United States (even some European variants were developed e.g., EPU for EU), but importantly and to prior our knowledge, none of these indexes is concerning uncertainty and European integration.

Various methods were proposed to measure uncertainty in the euro area. Disagreement among professional forecasters or stock market volatility (VSTOXX) is a traditional proxy, but it has several known drawbacks. Jurado et. al. (2015) argues that disagreements in the forecasts have limited numbers of expectations and not certainly capturing expectations of the economy as a whole, problems of known systematic biases and results are likely to reflect rather differences in opinions than uncertainty. Market volatility indexes are designed as a proxy for financial uncertainty and substantial variations may be associated rather with risk-aversion than uncertainty (Bekart et. al. 2013). Other approaches deriving uncertainty from macro-econometric models (Carriero et. al., 2017; Jurado et. al., 2015; Mumtaz, 2018).

We differ from authors who used newspapers text quantification framework because we focus explicitly on articles relevant to uncertainty and the European integration. Current indexes like widely used EPU have been originally designed for an autarkic United States economy. The European variant of EPU is compilated from the national sub-indexes, the same as our new index. However, while the economic-policy uncertainty naturally differs more across the states of the European Union, integration is a common process. This can be seen from our data as well as with comparison with other national uncertainty sub-indexes. Although measuring the European uncertainty is interesting per se, this exercise helps us to 1) provided empirical evidence for ongoing debates about the European integration challenges, 2) examine the consequences of European uncertainty on (dis)integration, which is important especially since market fragmentation remains higher now than before the crisis, 3) examine the sources of uncertainty in process of European integration.

Our index seems to correspond to the real-world events stressing the European integration process. As we were expecting, the most dramatic spike was measured in time of the UK referendum. An interesting finding is that levels of uncertainty persist significantly higher above average even after and at historically the highest values.

The results suggest that European uncertainty have an economic impact. Our analysis using vector autoregression (VAR) models shows that uncertainty is likely to have a real impact on cross-border activities within the Euro area. We find that a greater uncertainty translates into higher cross-border activities but with the reduced use of collateral.

The paper is structured as follows. Section 2 present the construction of our uncertainty index of the European integration. Section 3 presents the effects of uncertainty on integration and structure variables. Section 4 concludes and finally appendix with additional details on our index and VAR models.

2. Constructing an Uncertainty Index of European Integration

This section is structured as follows. First, we present the data use to construct our European integration uncertainty index. Second, we present the results of aggregate data in 1986:1 to 2020:12 and discuss its changes over time. Finally, we compare our index with the existing economic-policy uncertainty index.

2. 1. Data

We use the data from 2 newspapers which represent wide market share for every of 5 selected countries: Germany (*Handelsblatt* and *Frankfurter Allgemeine Zeitung*), the United Kingdom (*The Times* and *The Financial Times*), Italy (*Corriere Della Sera* and *La Repubblica*), Spain (*El Mundo* and *El Pais*) and France (*Le Monde* and *Le Figaro*).[2]

We extract the data from the digital newspaper's archives of Factiva, ProQuest and for Germany using each own archives. To generate the European uncertainty, we use automatized textual search for a triple combination of following keywords: "Europe" or "EU" or "European"; and "integration" or "enlargement" or "Brexit" or "Grexit"; and "concern/s" or "problem/s" or "doubt/s" or "uncertain/ty".[3]

We search for the frequency of articles meeting the condition of keywords combinations and following Baker et. al (2016), then normalize and sum over 10 newspapers. The aggregate European uncertainty index (EUI) is defined as

$$EUI_{i,t} = \frac{\frac{1}{P}\left(\sum_{p=1}^{p} \frac{X_{i.p,t}}{stdev(X_{i,p,t})}\right)}{\frac{1}{T}\left(\sum_{t=1}^{T} \frac{1}{P}\left(\sum_{p=1}^{p} \frac{X_{i.p,t}}{stdev(X_{i,p,t})}\right)\right)} x\ 100 \qquad (1)$$

where

$$X_{i,p,t} = \frac{n_{i.p,t}}{N_{p,t}} \qquad (2)$$

n represents a number of articles meeting the condition of keyword combination *i*, *p* stands for a concrete newspaper with $p = 1, \ldots, P$, where *P* is a total number of newspapers. *t* denotes the used time interval with $i = 1, \ldots, T$ and *N* is the total number of articles published in time *t* and newspaper *n*, without condition *i*. Therefore, expression (2) represents the normalized frequency of articles.

Our index construction is based on aggregating data with monthly frequency from January 1986 to December 2020. We work with more than 20 million published newspaper articles over time. Note that not all mentioned newspapers have been available from the beginning of our sample period (see Appendix – Table A1).

2. 2. European Integration Uncertainty Index

We present the European uncertainty index in 1986:1 – 2020:12 in Figure 1. We find that began our sample period to the late 1990s the European uncertainty has been typically below the long-term average. Subsequently, until the mid-2000s, uncertainty was slightly above average with occasional spikes such as in a period of the Constitutional Treaty rejection in French and Netherland in May/June 2005. Except for the uncertain period of the European debt crisis peaks, a considerable shift in long-term average started shortly after the Greek referendum in July 2015.

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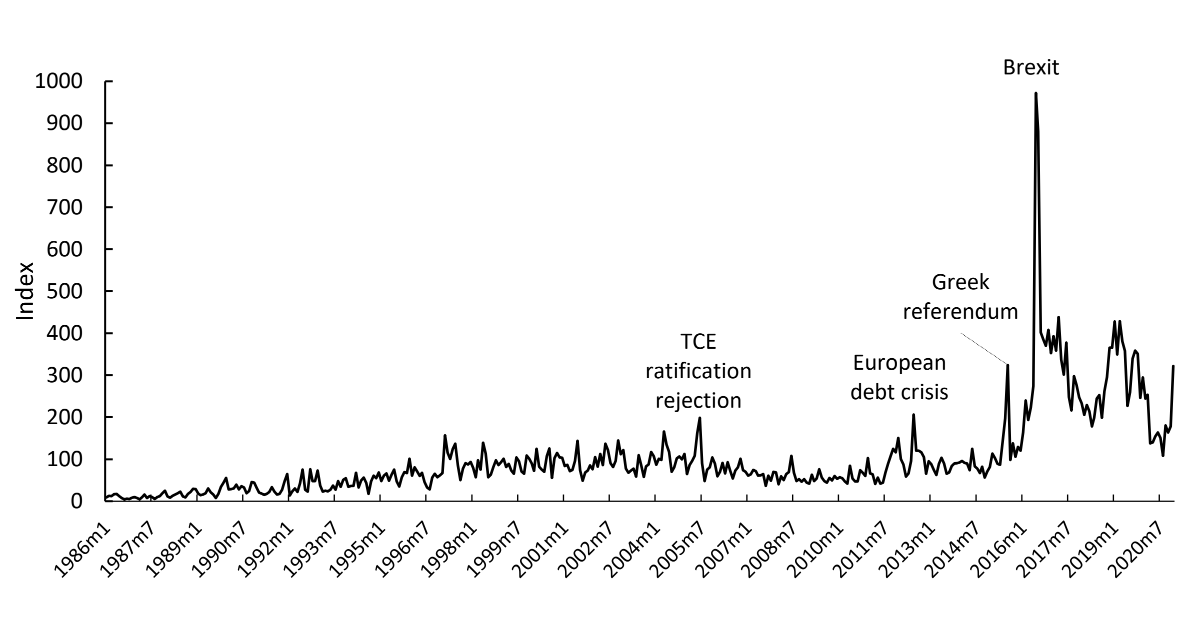
2 The selection of countries is based on the availability of data. The representing countries are Europe's five consistently largest economies from 1980 until 2021.

3 The selection of words is chosen to refer to Europe, integration, and uncertainty. The words "Brexit" and "Grexit" have been added as they represent differences in language usage over time. Note that searched words are the same in every language, but may contain different language variants (see Appendix – Table A1).

Figure 1: European Integration Uncertainty Index



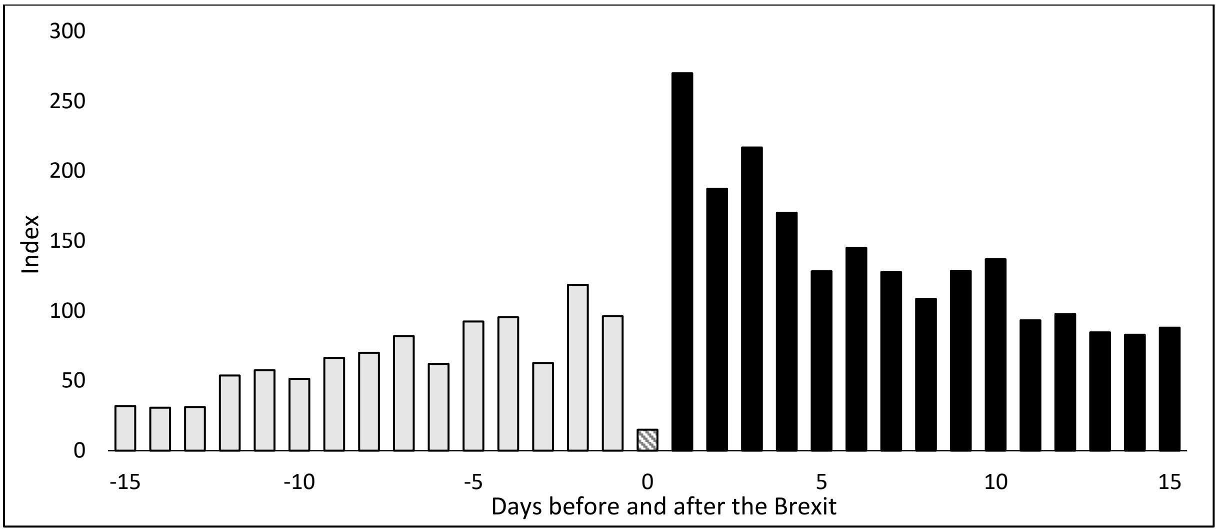
Notes: The European Integration uncertainty index is compiled from 2 main newspapers for every selected country. These are: Germany, the United Kingdom, Italy, Spain, and France. The newspapers articles must contain a triple combination of the following keywords: "Europe" or "EU" or "European"; and "integration" or "enlargement" or "Brexit" or "Grexit"; and "concern/s" or "problem/s" or "doubt/s" or "uncertain/ty". The index is at the monthly normalized frequency from 1986:1 to 2020:12 with a mean of 100.

The highest spike in index occurred after the United Kingdom membership referendum or "Brexit" in June 2016. The index values persist significantly higher above average even after. In line with our findings, Hobolt (2016) argue that the UK referendum is likely to be the most significant event in the EU's history and cannot be dismissed as just a sign of British exceptionalism. The explanation of persisting high values could be explained by prolonging and unprecedent negotiations between the EU and UK. However, some authors point to the possibility of disintegration or the so-called domino effect due to Brexit (Schimmelfennig, 2018; Hobolt, 2016).

As a robustness check, we generate indexes only by national newspapers. We compare the total of 5 different indexes and resulting indexes in Figure A1 and Table A1. We find that all indexes are strongly positively correlated for a whole period. Although after Brexit there is higher dispersion in level between national indexes, the pattern in data remains the same and does not contradict our observations.

In addition, to review the newspaper articles focusing on the uncertainty of the European integration, we provide daily data around the major spike in our index, i.e., Brexit. We choose 15 days before and after the event. We present the resulting index in Figure A2. The index construction and newspaper selection remain the same with a mean of 100. With the referendum approaching, the level of uncertainty started increasing. After the results of the referendum on 24 June 2016 and following spike, uncertainty began to decline slowly. However, the average values remain higher after this event.

Figure 2: European Integration Uncertainty before and after the Brexit: Daily Frequency



Notes: The index is compiled from 2 main newspapers for every selected country. These are: Germany, the United Kingdom, Italy, Spain, and France. The newspapers articles must contain a triple combination of the following keywords: "Europe" or "EU" or "European"; and "integration" or "enlargement" or "Brexit" or "Grexit"; and "concern/s" or "problem/s" or "doubt/s" or "uncertain/ty". The index is provided at the daily frequency, 15 days before and after the result of the UK referendum (the announcement of referendum results is detonated as day 0 in the figure. The index is scaled with a mean of 100.

2. 3. European Integration Uncertainty vs Economic Policy Uncertainty

We present the comparison of our European integration uncertainty index with economic policy uncertainty (EPU) for EU by Baker et. al. (2016) in 1997:1 to 2020:12 in Figure 2.[4]

The economic policy uncertainty index using the same text quantification framework, but we differ from the authors at a key point. We are focusing on articles relevant to uncertainty and European integration. The original EPU index was designed for the autarkic United States economy, which differ from the uncomplete European Union. While the economic-policy uncertainty naturally differs more across the states of the European Union, integration is a common process. The EPU variant for EU is compilated from national sub-indexes and we are comparing datasets for the same countries as in our selection.

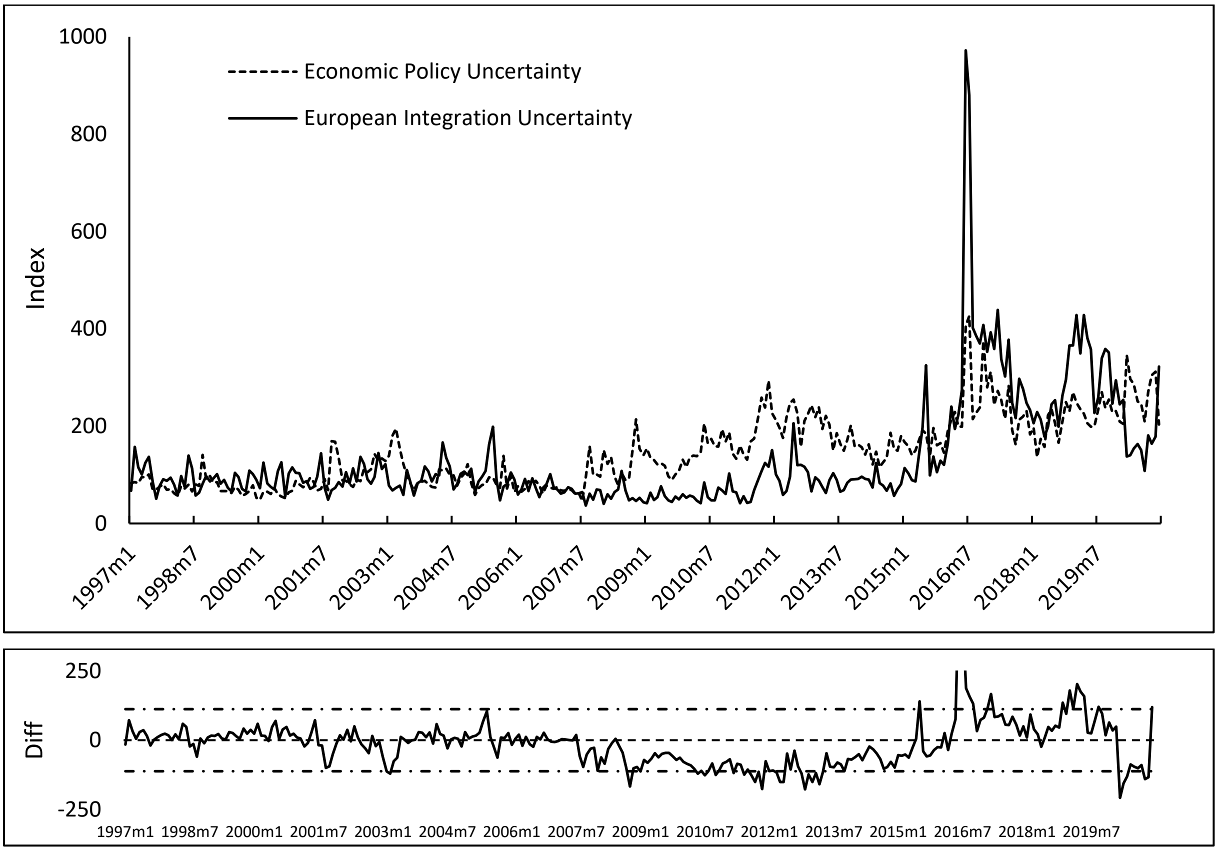
We find that our index is positively correlated with the Baker et. al. (2016) uncertainty index, but not fully and differ. To the outburst of the global financial crisis, our indexes do not seem to vary much. However, from this point, dispersion in our indexes is visible. While economic-policy uncertainty increased above the long-term average, the European integration uncertainty remains below, with exception of spikes in a period of the European debt crisis. This dispersion in indexes lasts until the beginning of 2016. After that, an evident increase in levels is in both indexes. Besides the differences in levels over time, the indexes differ in occasional spikes and empathize with real-world events differently.

The result of the comparison is interesting but not surprising at all. Very after the subprime mortgage crisis, the world was facing a financial crisis on a global scale with uncertain macroeconomic consequences. As of early 2010, these consequences mutated from banking crisis to sovereign debt crisis threatening the credibility of the euro currency (Overbeek, 2012), with a peak between 2010 and 2012. Some similarities and correlation of indexes are expecting as the

[4] We are comparing indexes since 1997:1 because more data have been available since that year. The data for economic policy uncertainty by Baker et. al. (2016) is from their website policyuncertainty.com.

different types of uncertainty are interrelated. In the case of differences, indexes emphasize real world-events variously in depends on the nature of indexes. As an example, in the case of Brexit.

Figure 3: Comparison of European Integration Uncertainty and Economic Policy Uncertainty



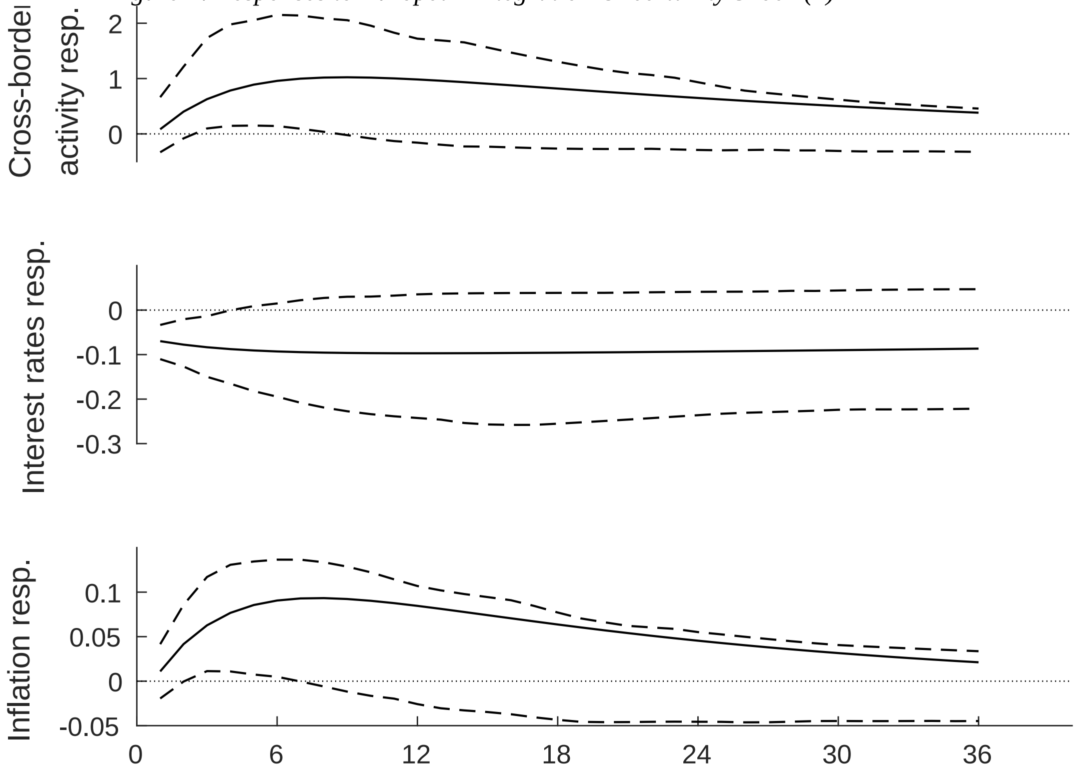
Notes: The European Integration uncertainty index is compiled from 2 main newspapers for every selected country. These are: Germany, the United Kingdom, Italy, Spain, and France. The newspapers articles must contain a triple combination of the following keywords: "Europe" or "EU" or "European"; and "integration" or "enlargement" or "Brexit" or "Grexit"; and "concern/s" or "problem/s" or "doubt/s" or "uncertain/ty". The economic policy uncertainty index is from Baker et. al. (2016). The indexes are at the monthly normalized frequency from 1997:1 to 2020:12 with a mean of 100. The lower figure represents differences between European integration uncertainty and economic policy uncertainty. The dashed upper and lower bands represent the value of one standard deviation of our European integration uncertainty index.

3. Uncertainty effects of integration and cross-border activities

Several studies have tried to examine the driving factors in the global increase of cross-border activities and the role of uncertainty. VIX as a proxy for a measure of global uncertainty has been suggested to be the important push factor of international capital flows (Bruno and Shin, 2014; Cerruti et. al., 2017; Passari and Rey, 2015). Further studies are focused on examining cross-country heterogeneity in uncertainty and its effects on international capital flows (Julio and Yook, 2015; Choi and Furceri, 2019; Biswas and Wei, 2021) and only a few studies are particularly focused on the Euro area (e. g., Schmit and Zwick, 2015).

Economic integration is an integral part of European integration. To the best of our knowledge, these studies do not investigate uncertainty stemming directly from the European integration process. Therefore, we examine the economic consequences of the European integration uncertainty for a number of integration and structure variables. We estimate the vector autoregression (VAR) models for the European economy defined as follows: European integration uncertainty, integration variable, interest rates, inflation. As a proxy for uncertainty, we use a log of our newly developed index, the integration variable represents the share of cross-border activity in TARGET2 for the first model and in the second model, we use the percentage use of cross-border collateral in the Eurosystem monetary policy operations. For interest rates, we are following common practice in literature, and we use shadow short term interest rates for the euro area and inflation is measured as HICP.[5] We report responses of our integration variables, interest rates and inflation to one standard deviation the European integration uncertainty shock with 90% confidence bands.

Figure 4: Responses to European Integration Uncertainty Shock (1)



Notes: Impulse response functions of one standard deviation to the European integration uncertainty index with 90% confidence bands. The vector autoregression model covers monthly data from 2008:1 to 2020:12.

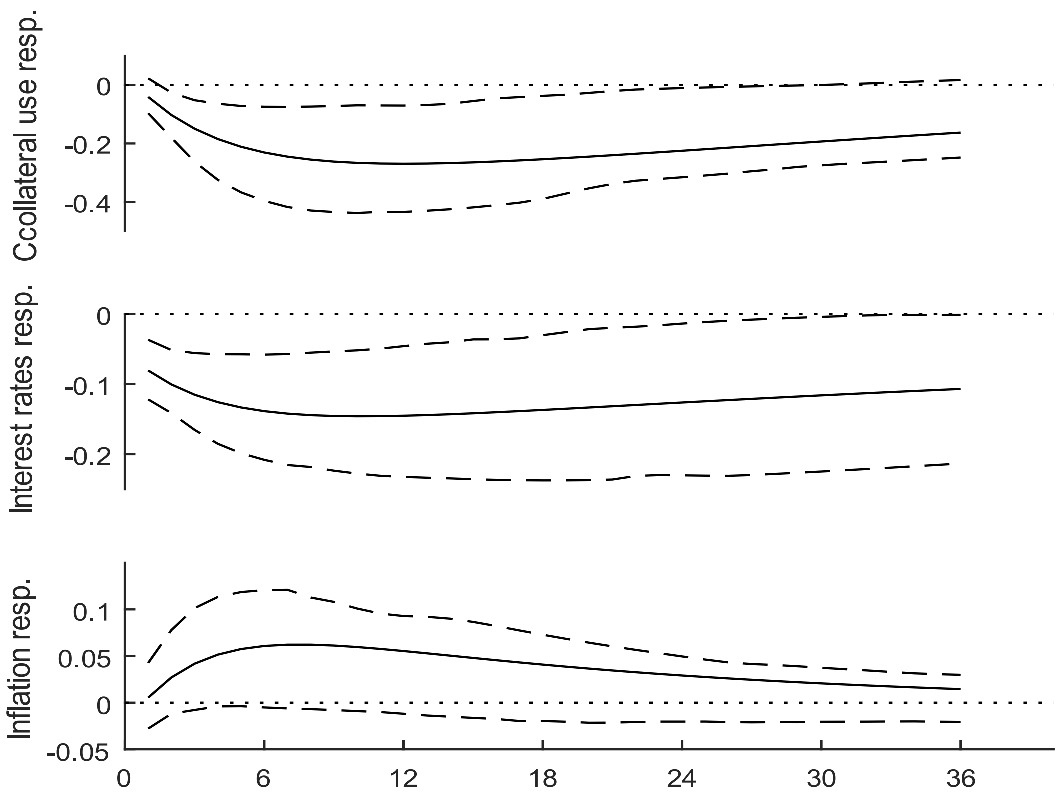
The impulse response function shows that one standard deviation shock to the European uncertainty index increased cross-border activity and becomes statistically significant after 3 months after the shock (Figure 3). The effect lasts for approximately 9 months after. In the case of the use of cross-border collateral, shock cause a decrease with statistical significance after 2 months (Figure 4). The effect lasts for approximately up to 2 years. Our results also show that

[5] Shadow interest rates (SSR) aims to measure the accommodation in monetary policy when the short rate is at the zero-lower bound (Meinem and Roehe, (2017)). The data of indicators of financial integration and structure in the euro area (cross-border activity and use of collateral) and inflation are from www.ecb.europa.eu websites.

interest rates loosening after unexpected uncertainty shock and indicate the follow ing response of an increase in inflation.

The results seem to be consistent with the literature concerning uncertainty and fluctuations in capital flows. When uncertainty shock occurs, total cross-border claims increases because banks are likely to rebalance their portfolios (Choi and Furceri, 2019). The fact of increased cross-border activities and decline in the use of collateral may be possibly explained due to this portfolio rebalancing. If banks credit supply would be driven by fundamental factors (seeking for higher returns) one would expect an increase in collateral requirement. Still, these results confirm the economic and statistical significance of our index on a number of integration and structure variables.

Figure 5 Responses to European Integration Uncertainty Shock (2)



Notes: Impulse response functions of one standard deviation to the European integration uncertainty index with 90% confidence bands. The vector autoregression model covers monthly data from 2008:8 to 2020:12.

4. Conclusions

We developed a proxy for European integration uncertainty based on textual analysis of more than 20 million articles from 10 major EU newspapers from January 1986 – December 2020. Our index seems to correspond to the real-world events stressing the European integration process. The results are in line with other authors suggesting that the United Kingdon referendum is likely to be the most significant event in the EU's history. These results also correspond with the overall assessment of a challenging last decade of the European Union. This empirical evidence supports ongoing debates and current literature about the European integration challenges. Also, the index allows examining the sources and magnitude of uncertainty in process of European integration over time. We then find that our index is positively correlated with economic-policy uncertainty (EPU), but not fully and differ. The significant difference is in the time of the outburst of the global financial crisis when the European integration uncertainty remains below the long-term average. Visible spikes in this selected period occur only after the banking crisis transformed into a sovereign debt crisis.

Using our newly developed measure of European integration uncertainty, we examine its effects on a number of financial integration and structure indicators. With the use of VAR models, we find that this uncertainty increased cross-border activity within the Euro area, but reduce the use of cross-border collateral. The fact of increased cross-border activities and a decline in the use of collateral suggests banks portfolio rebalancing. Our results also show that interest rates loosening after unexpected uncertainty shock and indicate the following response of an increase in inflation. However, these results open potential for further research.

Overall, our index seems to be a relevant proxy for the European integration uncertainty. Importantly, the results show the significance of uncertainty stemming from European integration on a number of integration and structure variables within the Euro area.

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Appendix

Table A 1

Germany (1986-, monthly). Newspapers: Handelsblatt (1986) and Frankfurter Allgemeine Zeitung (1993). Database: using each newspapers own archives. Key words: integration or erweiterung or Brexit or Grexit and Europa or EU or europäisch and unsicher or unsicherheit or problem or probleme or besorgnis or sorge or sorgen or bedenked or Zweifel
United Kingdom (1986, - monthly). Newspapers: the Times of London (1986) and Financial Times (1986). Database: Factiva. Key words: integration or enlargement or Brexit or Grexit a Europe or EU or European and uncertain or uncertainty or concern or concerns or problem or problems or doubt or doubts
Spain (1995, - monthly). Newspapers: El Mundo (1995) and El Pais (2001). Database: Factiva. Key words: integración or amplición or Brexit or Grexit and Europea or Europeo or Europa or EU and incierta or incierto or incertidumbre or preucupación or preocupaciones or problema or problemas or duda or dudas
France (2001, - monthly). Newspapers: Le Monde (2011) a Le Figaro (2001). Database: Factiva for Le Monde, ProQuest for Le Figaro. Key words: l´inégration or élargissement or Brexit or Grexit and L´Europe or EU or europééne or européen and incertaine or incertain or incertitude or préoccupation or soucis or problème or problèmes or inquietude or doute or les doutes
Italy (1997, - monthly). Newspapers: Corriere Della Sera (1997) and La Repubblica (2005). Database: Factiva. Key words: integrazione or allargamento or Brexit or Grexit and Europa or EU or Europea or Europeo and incerta or incerto or incertezza or preoccupazione or preoccupazionni or problema or i problem or dubbi or dubbio

Figure A 1 Uncertainty indexes by national newspapers

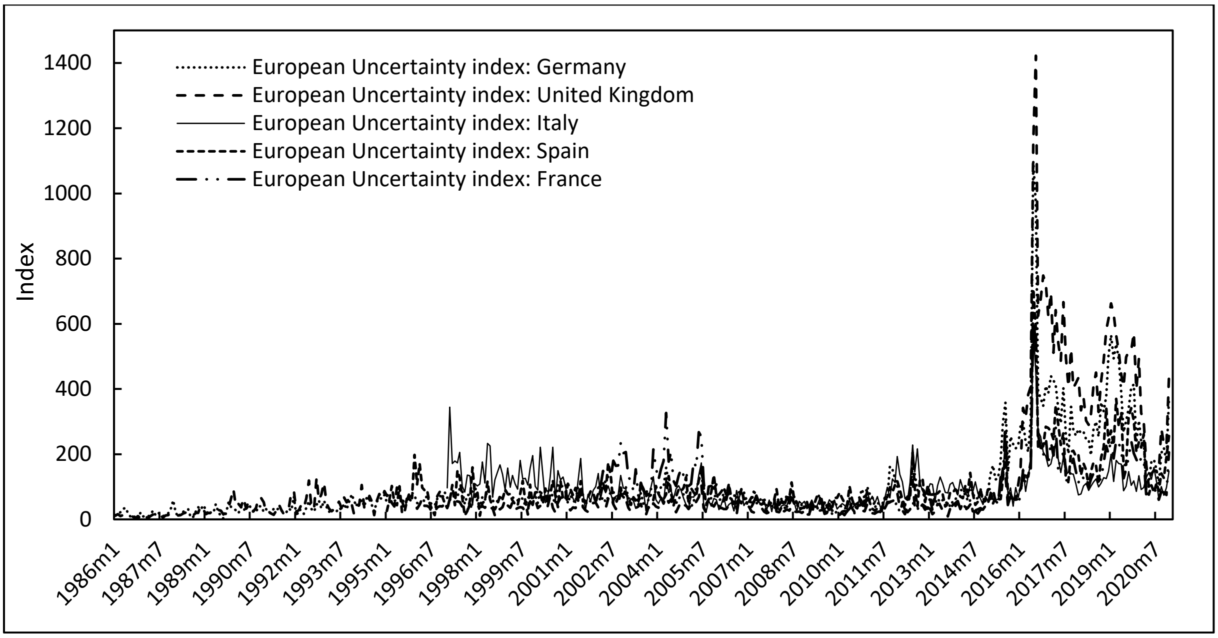

Table A 2 European Integration Uncertainty Sub-indexes: Correlations

European Integration Uncertainty:	Germany	United Kingdom	Italy	Spain	France
Germany	1				
United Kingdom	0.94*** (0.000)	1			
Italy	0.81*** (0.000)	0.75*** (0.000)	1		
Spain	0.92*** (0.000)	0.90*** (0.000)	0.80*** (0.000)	1	
France	0.78*** (0.000)	0.75*** (0.000)	0.73*** (0.000)	0.80*** (0.000)	1

Portfolio selection during the crises

Francesca Pavanati, Sergio Ortobelli Lozza[1]

Abstract

In this paper, we propose a Mean-Variance analysis, with the aim of analyzing the different investment choices in the European Stock Market and the ambitious goal of comparing the effects on the Stock Market during periods of crises. To do this, five countries with the largest capitalization in Europe were chosen: Italy, Germany, France, Spain and United Kingdom. For each of the five Stock Markets, we propose an ex-post analysis based on the mean-variance optimal choices during the last two decades. Therefore, for each stock market, we examine the *ex-post* optimal mean-variance investments evaluating which sectors were suffering during the subprime crisis, the sovereign credit risk crisis and the covid 19 crisis.

Key words:

Portfolio choice, market crises, mean-variance analysis, European stock market.

JEL Classification: G11, G01

1. Introduction

On January 30, 2020, following the reporting by China of a cluster of cases of pneumonia of unknown etiology in the city of Wuhan, the World Health Organization declared a public health emergency of international interest following the outbreak of Coronavirus (Covid-19) in China. The following day, considering the particularly widespread nature of the epidemic, several European governments declared a state of emergency and implemented the first measures to contain the contagion throughout their country (see Casagrande et al. (2020), Agnoletti et al. (2020)). The strongly uncertainty regarding the global economic outlook triggered strong turbolence on the stock markets which, on a global level, was reflected in large falls in prices and an increase in volatily (see Ruzzi and Rubi (2020)). The impact differed across geographical areas and sectors, depending on exposure to the pandemic and the effects of lockdown measures. Share price trends in the Eurozone, in particular, fell below the level recorded at the beginning of 2007 and price volatilty also increased significantly. Portfolio choices have changed radically in 2020 even if this crisis presents several differences respect to the previous ones (see, among others, Bertocchi et al. (2013), Biglova et al. (2014)). For this reason, in this paper, we want to examine the differences on optimal choices during some crises of the last decades. In particular, we propose a Mean-Variance analysis of the five European Stock Markets with the largest capitalization in Europe: Italy, Germany, France, Spain and the United Kingdom. It's considered appropriate to point out that the timeframe of our analysis is characterized by several financial crises: the Argentina crisis of 2001, the dot-com crisis of 2001, the subprime crisis of 2007-2009, the European sovereign debt crisis of 2010-2011, the Chinese currency crisis of 2014-2015 and, finally, the recent Covid-19 crisis. Specifically, this paper considers three crises: the 2007-2009 subpime crisis, the 2010-2011 European sovereign debt crisis and the beginning of Covid-19 crisis.

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In Section 2 we propose the mean-variance empirical analysis and we discuss the observed differences of the optimal choices. In the final section we summarize the results.

2. The empirical analysis

In this paper, we deal the portfolio selection problem in a mean-variance framework for five European stock markets. We consider the portfolio of returns $x'R$, where $x' = [x_1, \dots, x_n]$ is the vector of percentages of wealth, and $R = [R_1, \dots, R_n]'$ is the vector of returns. We generally assume that no short sales are allowed (i.e., $x_i \geq 0$ for any *i=1,…n*). In particular, at the time t, the return of i-th asset obtained over the period $(t-1, t)$ is given by $R_{i,t}=\frac{P_{i,t}-P_{i,t-1}+A_{i,t}}{P_{i,t-1}}$ where $P_{i,t}$ is the price of the i-th security at time t and $A_{i,t}$ denotes the cash movement from owning the i-th security over the period $(t-1, t)$.

According to the mean variance portfolio selection model in the empirical analysis we fit the efficient frontier solving the following problem for different values of the mean *m*:

$$\begin{gathered} min_x \; x'Qx \\ E(x'R) = m \\ \sum_{i=1}^{n} x_i = 1; \quad x_i \geq 0; i = 1, \dots, n \end{gathered} \quad , \tag{1}$$

where Q is the variance covariance matrix and *m* is a fixed mean belonging to the interval $[\underline{m}, \overline{m}]$ ($\underline{m}$ is the mean of the global minimum variance portfolio and $\overline{m}$ is the mean of the asset with the greates mean).

2.1 The Dataset

The dataset used in this analysis has been extracted from *Datastream*. The analysis is placed temporally over a period of about 21 years: from November 1999 till November 2020. We use a window of four years (1000 trading days) to fit every month the mean-variance efficient frontier for each of the five European stock markets we consider: United Kingdoom, Germany, France, Italy and Spain. In particular, for each country the assets are shared respect to their sector (Datastream Classification) as it follows:

1) 425 stocks for the the UK stock market that are shared in 12 sectors (automotive, construction, electronic, financial, food, industrial, insurance, consumer goods, pharmaceutical, technology, support services and mining)
2) 520 stocks for the German stock market, shared in 10 sectors (automotive, chemical, healthcare, industrial, pharmaceutical, technology, consumer goods, electronic, energy and telecommunications)
3) 397 stocks for the France stock market, shared in 10 sectors (automotive, healthcare, industrial, insurance, mining, consumer goods, pharmaceutical, construction, electronic and food)
4) 113 stocks for the Italian stock market, shared in 12 sectors (banking, technology, electronic, financial, industrial, insurance, tourism, construction, telecommunications, media and manufacturing)
5) 69 stocks for the Spain stock market, shared in 9 sectors (construction, energy, food, industrial, mining, real estate, financial, pharmaceutical and consumer goods)

2.2 The optimal choice procedure and discussion

For each efficient frontier we compute forty mean-variance optimal portfolios: starting from the global minimum variance portfolio till the maximum mean asset. According to Papp et al. (2005) and Kondor et al. (2007), we use a Principal Component Analysis (PCA) to reduce the dimensionality of the problem, since the number of assets is large for each stock market. In practice, for each stock market, at the k-th recalibration (that applies every month, i.e., every 21 trading days), we evaluate the following steps:

Step 1: We apply the PCA to the correlation matrix of the returns. Doing so, we identify the factors that explain at least the 60% of the variability. Then, we approximate the returns regressing them on these few principal components (with an OLS estimator).
Step 2: We solve problem (1) for 40 fixed mean *m*, fitting the efficient frontier.
Step 3: For the s-th optimal portfolio (s=1,…,40 corresponding to the s-th position of the efficient frontier), we compute the ex-post wealth as it follows:

$$\boldsymbol{W}_{\boldsymbol{t_{k+1},s}} = (\boldsymbol{W}_{\boldsymbol{t_k,s}})\boldsymbol{z}_{\boldsymbol{t_{k+1},s}}^{(\boldsymbol{ex-post})}$$

where $\mathbf{W}_{\mathbf{t_k,s}}$ is the ex-post wealth obtained at time $\mathrm{t_k}$ from the s-th optimal portfolio of the efficient frontier and $\mathrm{z}_{\mathrm{t_{k+1},s}}^{(\mathrm{ex-post})}$ is the s-th portfolio ex-post gross return obtained during the period $[\mathrm{t_k}, \mathrm{t_{k+1}}]$.
Step 4: For each optimal portfolio of the efficient frontier, we evaluate the sectors in which we invest (according to the analysis proposed by Kouaissah and Ortobelli (2020)).
Step 5: We repeat the previous steps for all observations and for each stock market.

From this analysis we can report the results in several tables and graphs. Of course, we cannot report all of them in this paper and thus we summarize the results reporting only the case of the German stock market that is the best performing one.

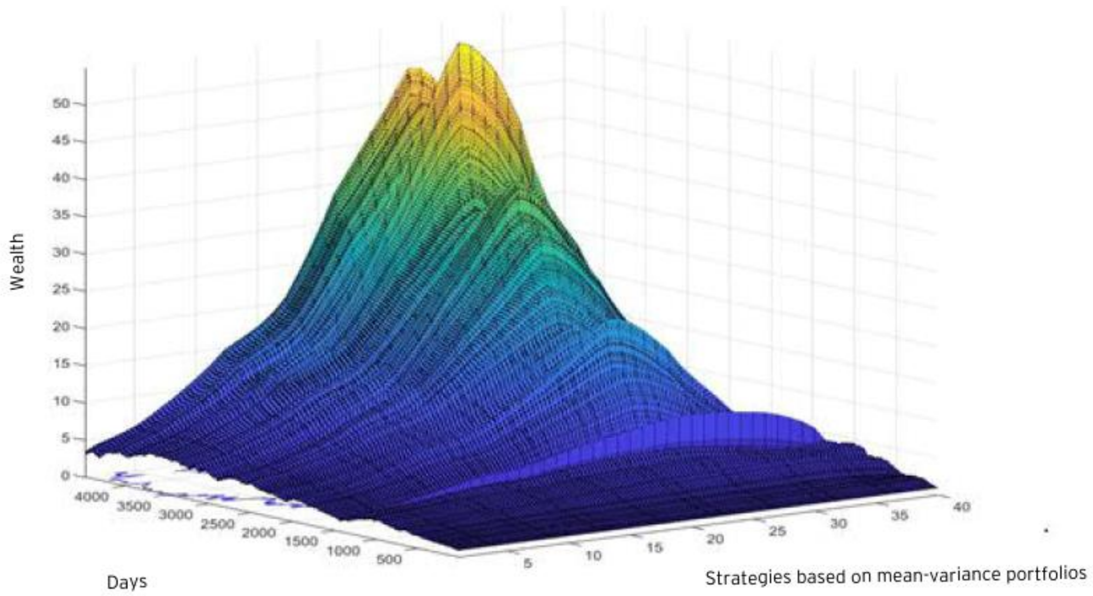

Figure 1: *Ex-post wealth for optimal mean-variance portfolios of Germany*

In particular, Figure 1 represents the ex-post wealth obtained for forty strategies deriving by optimal mean-variance portfolios. The first strategy corresponds to the most conservative choice of the global minimum variance portfolio, that is the lest risky choice of the efficient frontier, and often less profitable. In the case of the German stock market, the 33rd strategy presents the maximum ex-post final wealth and, finally, the 40th strategy corresponds to the choice of mean maximizer, that is also the most risky choice in the mean-variance framework.

Next, graphs represent which sectors were profitable or distressed during the period of crises. Referring to the example of Germany, the following graphs show investments in the various sectors during the subprime crisis (see Figure 2), the European sovereign credit risk crisis (see Figure 3) and the Covid-19 crisis (see Figure 4).

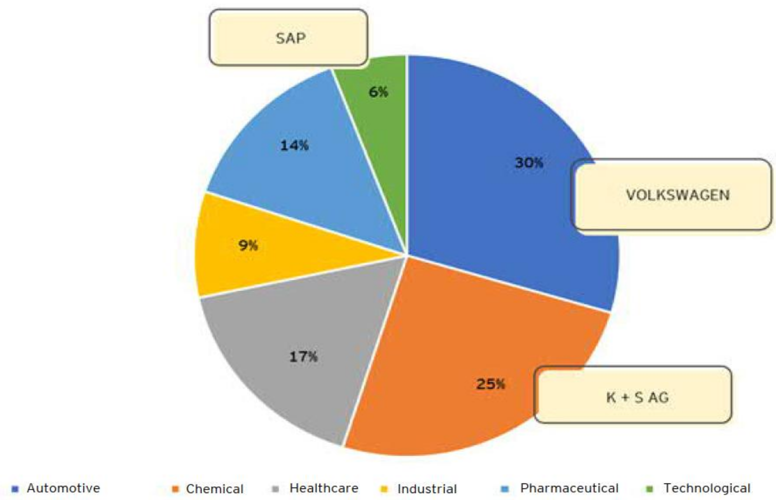

Figure 2: *Investments, in %, made in various sectors in Germany during the subprime crisis*

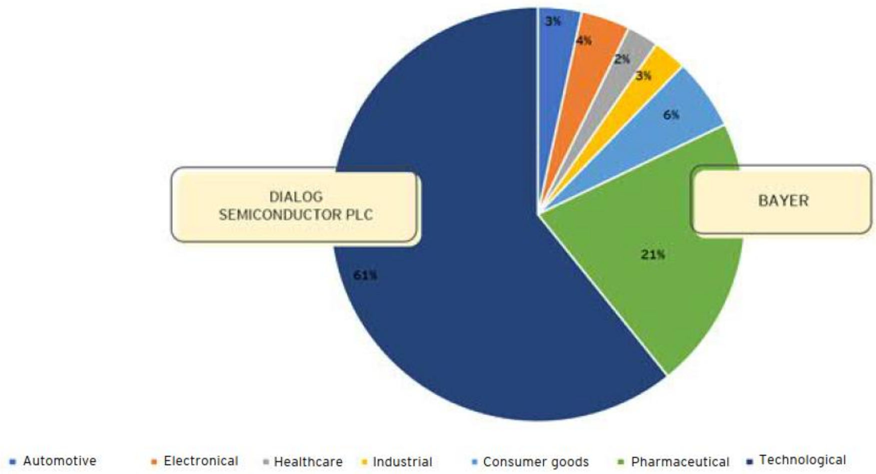

Figure 3: *Investments, in %, made in various sectors in Germany during the European sovereign credit risk crisis*

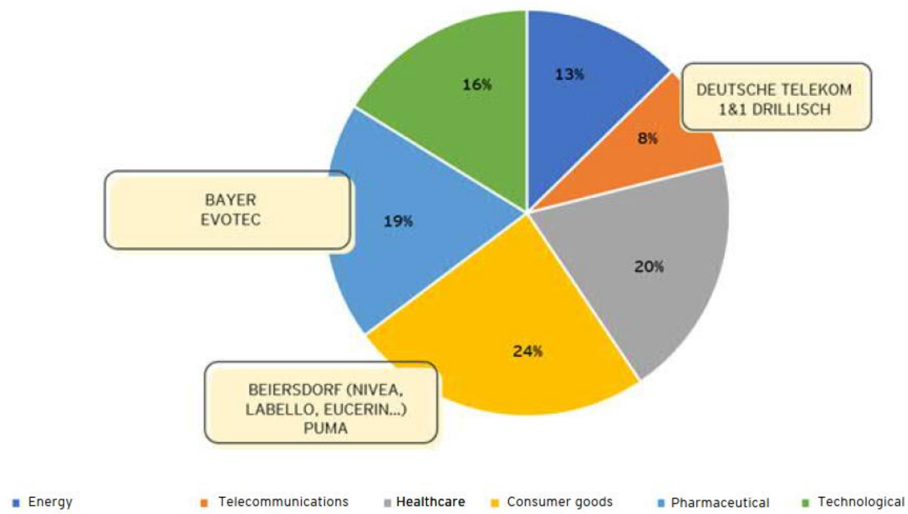

Figure 4: *Investments, in %, made in various sectors in Germany during the Covid-19 crisis*

From Figure 2 we deduce that the chemical, automotive and only in minor part (6%) the technology sectors are the "mean-variance optimal" sectors during the subprime crises in the German stock market. Figure 3 suggests that the technology sector (61%) chemical sector (21%) and only in minor part automotive (3%), electronic (4%), healthcare (2%), industrial (3%) sectors are the principal "optimal" sectors for the German stock market during the European sovereign credit risk crisis. Figure 4 reports the percentage of optimal choices during the Covid 19 market crisis for the German market. In particular, we observe that the optimal mean variance choices were distributed as it follows: the 24% in Consumer goods, 19% in Pharmaceutical, 20% in Healthcare, 16% in Technological, 13% in Energy and 8% in Telecommunication sectors.

2.3 The results

We focus our attention on the optimal choices during three systemic crises. Each crisis has different timing, methods and consequences and on the basis of the analysis carried out, commonalities and differences have emerged, depending on the analyzed country. Specifically, the banking sector was particularly weakened following the subprime crisis in all five countries. On the other hand, the sectors in which the greatest investment was made are different. Specifically, in Italy there was greater investment in the tourism sector, while in Germany, France, Spain and the United Kingdom the automotive sector enjoyed greater investment during the subprime crisis. Regarding the European sovereign debt crisis, the sector most affected were different depending on the country under consideration: Italy weakened in tourism; France suffered reductions in investment in the construction sector, together with the United Kingdom; Spain suffered losses in the food sector and Germany saw reductions in investment in the healthcare, industrial and automotive sectors. However, during the Eruopean sovereign debt crisis, investment increased in certain sectors: technology in Italy and Germany, pharmaceuticals in France and Germany, and industry in Spain and the United Kingdom. Overall, therefore, during the European sovereign debt crisis, taking into consideration only the five countries analyzed in this study, the pharmaceutical and industrial sectors suffered the least. Finally, regarding the Covid-19 crisis, the sectors that have inevitably been characterized by greater investment are healthcare and pharmaceuticals (especially in Germany, France, Spain and the United Kingdom). The sectors that, on the other hand, have particularly suffered are consumer goods, telecommunications and finance. These results are briefly summarized in the following Table 1. In particular, we evidence with X the sectors where we observe a large reduction of investments, while we denote with V the sectors where are substantially maintained the investments.

Table 1: *Brief summary of the main investments during the crises (for sectors in five stock markets).*

SUBPRIME	**Italy**	**Germany**	**France**	**Spain**	**UK**
Automotive sector	X	V	V	V	V
Tourism sector	V	X	X	X	X
Banking sector	X	X	X	X	X
CREDIT RISK	**Italy**	**Germany**	**France**	**Spain**	**UK**
Technology sector	V	V	X	X	X
Pharmaceutical sector	X	V	V	X	X
Industrial sector	X	X	X	V	V
Construction sector	X	X	V	X	V
Food sector	X	X	X	V	X
COVID 19	**Italy**	**Germany**	**France**	**Spain**	**UK**
Healthcare sector	V	V	V	V	V

Pharmaceutical sector	V	V	V	V	V
Consumer goods sector	X	V	X	X	X

3. Concluding remarks

The paper examines the optimal portfolio choices in five European stock markets during three crises of the last two decades. In particular, we evaluate optimal mean-variance choices during the subprime crisis, the sovereign credit risk crisis and the recent (not finished) Covid 19 crises. Even if each country reacts differently to a crisis and each crisis is characterized by different timescales, methos and effects, we observe a major common behavior during the subprime cirsis and Covid 19 crisis while there are greater differeces during the sovereign credit risk crisis. This difference is not very surpricing taking into account that some European countries were involved directly in the credit risk crisis.

Aknowledgments

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Analysis and Prediction of EVA of the Manufacturing industry in the Czech Republic

Barbora Ptáčková , Dagmar Richtarová[1]

Abstract

This paper is devoted to the analysis and prediction of the financial performance of the Manufacturing industry of the Czech Republic. Financial performance is evaluated according to Economic Value Added. The aim of the paper is to propose pyramidal decomposition of EVA of the Manufacturing industry and its prediction for each quarter of one year based on the results. First, the financial performance of the Manufacturing industry of the Czech Republic is evaluated according to Economic Value Added. Then, pyramidal decomposition of EVA is proposed. On the basis of the component ratios of Economic Value Added prediction for each quarter for one year is performed. In the end, results are commented and discussed.

Key words

Financial performance, Economic Value Added, Manufacturing industry, simulation of financial ratios.

JEL Classification: G30, M0, O12, C6

1. Introduction

Nowadays, economic environment is ever - changing and together with those changes there are also changes in the companies operating in this environment. Financial performance and financial health of the companies is determined by the ability to generate value added. It means profitability of whole business, the return on the invested capital and the return of inputs. One of the main goals of the financial decision-making is to increase financial performance of companies, respectively of the industry in which companies operate. It is usually solved by many authors, see Copeland (2005), Vernimmen (2005), Brealey (2014). Domestic authors dealing with financial performance of a company or industry are Mařík (2005), Neumaierová (2005), Dluhošová (2010) or Kislingerová (2010) and Dluhošová, Ptáčková, Richtarová (2018). Generally, financial performance of a company is understood as the ability of companies to create some value added.

Another important task of the financial management is to determine, how the financial performance of a company is influenced. One of the ways is to apply pyramidal decomposition method together with analysis of deviations. Then, it is possible to determine the interactions among the indicators and quantify the influence of component indicators on the base indicator, Dluhošová (2010).

In financial decision making the allocation of money is under conditions of uncertainty. Prediction of financial indicators is one of the techniques for replicating uncertain process and evaluating decisions under uncertain condition. The earliest application of simulation in finance was financial management, Dessislava A. Pachamanova, Frank J. Fabozzi (2010).

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The aim of this paper is to propose pyramidal decomposition of Economic Value Added of the Manufacturing industry and to predict the evolution of economic value added of the Manufacturing industry for each quarter of one year based on the results.

2. Used methods

This part of the paper is dedicated to the methods used for analysis and prediction. Firstly, economic value added as a measurement of financial performance is characterized. Then the pyramidal decomposition of economic value added is proposed and the main influencing factors are found. The last part of this chapter is dedicated to methods of prediction of financial indicators.

2.1 Economic value added

The financial performance of companies or industries can be measured in different ways. Meanwhile, some authors use traditional performance measures such as NOPAT, ROI or ROE, nowadays Economic Value Added is often used for financial performance analysis. This is because of the fact, that Economic Value Added incorporates the costs of capital and respects factor of time. Economic Value Added is based on the concept of economic profit. When the economic profit is positive, it means that the company earns more than the weighted average costs of capital. It also means that some wealth for the shareholders is created.

There are many ways how economic value added can be expressed. In this paper, the financial performance of the Manufacturing industry is analyzed according to EVA - Equity and is expressed as

$$EVA = (ROE - R_E) \cdot E, \tag{1}$$

where ROE is return on equity, E is equity and R_E are costs of equity.

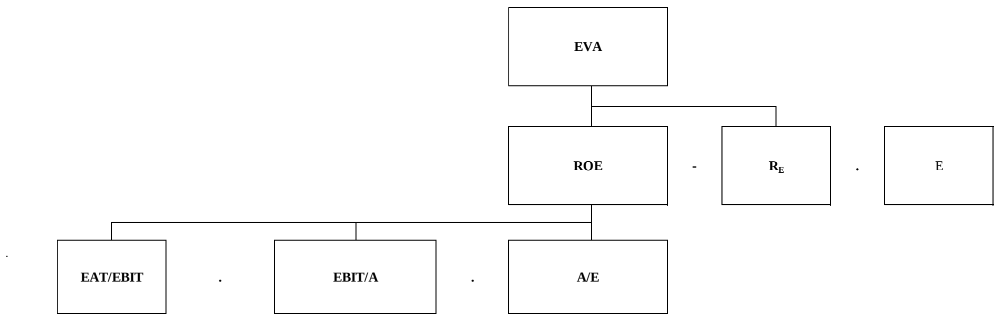
The difference between ROE and R_E is called spread. If this spread is positive, it means that industry or company earns more than the costs of equity are.

2.2 Method of pyramidal decomposition and analysis of deviation

One of the main tasks of financial analysts is to analyze the deviations of component indicators and to find and quantify the factors that contribute the most to these deviations. The method of pyramidal decomposition is usually used for quantification of the impact of component ratios on the change in the base ratio. This method also allows to determine the interactions and relationships among the component ratios.

There are many ways, how relative Economic Value Added can be decomposed. In this paper, there is proposed possible pyramidal decomposition of Economic Value Added as

Figure 1: Economic Value Added – pyramidal decomposition



Source: own calculation

where *E* is equity, *EAT* is earnings after taxes, *EBIT* is earning before interests and taxes and R_E are costs of equity.

The pyramidal decomposition together with the analysis of deviations help~~s~~ to identify the relationships between the financial ratios and to quantify the impact of selected ratios on the base ratio, Dluhošová (2010).

It is useful to apply the analysis of deviations for in-depth analysis of the impact of component ratios on the base ratio. It is possible to quantify the impact of the changes in them according to this analysis, Zmeškal (2013).

2.3 Prediction of economic value added

The essence of the prediction of the financial indicators is the estimation of the future value of the probability distribution of the partial financial indicators and the estimation of the profitability of the Economic Value Added as a base financial indicator. Financial assets are characterized by a random evolution over time, which is known as a stochastic process. Basic stochastic processes include Wiener's process, Brown's geometric process, and others, Zmeškal (2013).

Levy's models are processes whose increments are independent and stationary. For these processes, it is generally typical that the probability of a jump occurring for a given time interval is zero. The basic building elements of these models are Wiener's process and Poisson's process. Financial assets should show only positive values.

Some of Levy´s models are defined as a geometric Brown´s process, which is controlled by certain internal process. From the economic point of view, these processes can be taken as processes which are evolving in a random trading time. Due to the fact, that the time is replaced by a certain process, this is a compound process of modelling. Principles of compounding modelling using internal processes can be found in Bochner (1949) or Clark (1973). The main idea of the modeling of financial assets through these processes is the instability of the volatility of these processes as well as the occurrence of extreme scenarios which somewhat complicated the use of classical models based on the geometric Brown process, see Bertoin (1998), Tichý (2011).

In recent years, many studies have shown that the probability distribution of the yields of most of the financial variables does not correspond to the normal distribution. The yields of the financial variables are often skewed and more acute. In practice, there are many models, which allow to capture the real characteristic of profitability distribution of financial indicators on the base of third and fourth moment. Therefore, the Variance gamma process, one of the Levy´s process is used to predict the economic added value of the Manufacturing industry.

Generally, the price of the financial asset according to the Lévy´s models is determined as

$$S(t) = S\,exp[\mu t + X(t)], \tag{6}$$

where $S(t)$ is asset price, $X(t)$ is Lévy´s process, μ is deterministic increase.

The variance gamma process is one of the most frequently applied multi-parametric models. One of the advantages of this model is that it allows modeling of higher moments of profitability distribution. The gamma parameter allows to control the kurtosis of random number and omega parameter allows to control the skewness. It is necessary to determine basic moments and characteristics. These can be derived either by density function or by a characteristic function, Gurný, Richtarová (2016).

Variance gamma process is defined as follows

$$VG_t = \theta g_t + \vartheta Z(g_t) = \theta g_t + \vartheta, \tag{7}$$

where g_t is gamma process in certain time t, θ is drifting of process and υ control skewness, while $Z \in N[0,1]$.

Substituting the variance gamma process to Levy´s model in exponential form the financial asset price is determined

$$S_t = S_0 \exp(\mu t + VG_t) - \varpi t) = S_0 \exp(\mu t + \theta g_t + \vartheta\sqrt{g_t \varepsilon} - \varpi t), \tag{8}$$

where correction parameter is $\varpi = -\frac{1}{\upsilon} ln(1 - \theta\upsilon - \frac{1}{2}\vartheta^2\upsilon)$.

There are some problems when Lévy´s models are applied. One of them is the connection of marginal probability distribution, it means the probability distribution for a partial source of risk, to obtain the probability distribution of comprehensive portfolio of financial assets. The use of linear correlation was especially preferred in the modeling of credit risk and derivatives dependent on it. Currently, copula function is preferred solution, Tichý (2010).

Using copula function, it is possible to model the dependence of complex portfolios whose value depends on more diverse processes or more complicated processes, and it is not possible to use, for example Cholesky decomposition, Tichý (2010). Generally, the copula function is defined as the function between the individual probability distributions in the interval [0,1],

$$C:[0,1]^n \to [0,1] \in \Re^n, n \in \{2,3.\dots\} \tag{9}$$

According to Tichý (2010) it is basically possible to distinguish two basic groups of copula functions – copula function based on elliptical probability distributions and Archimedes copula functions. The Gaussian copula function, which is used in this paper, belongs to the elliptic copula functions. Assuming the correlation between the random variables can be defined as follows:

$$C_R^{Ga}(u,v) = \Phi_R(\Phi^{-1}(u);\Phi^{-1}(v)), \tag{10}$$

where R is correlation among the variables, Φ is inverse function to the distribution function of the normalized normal distribution, Φ_R is two-dimensional distribution function of the normalized normal distribution in given correlation.

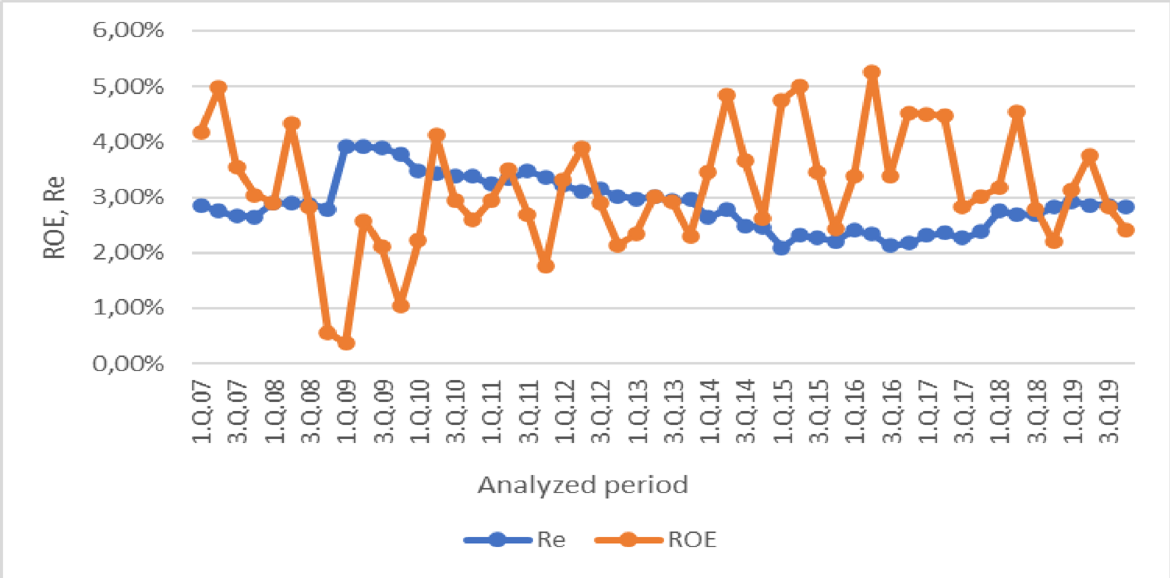
3. Application part

Application part of the paper is divided into two parts. First, an analysis of the Economic Value Added of pyramidal decomposition applications in the Manufacturing industry of the Czech Republic during the years 2007 – 2019 is performed. Then, component ratios of Economic Value added are considered in the simulation model and the Economic Value Added is predicted for one year.

3.1 Input data

The data were taken from the website of Ministry of Industry and Trade of the Czech Republic. Quarterly data are used for the analysis. In the Graph 1 there are values of ROE and costs of equity of the Czech Manufacturing industry in the period 2007 to 2019.

Graph 1: The values of ROE and costs of equity of the Manufacturing industry during the period 2007 to 2019



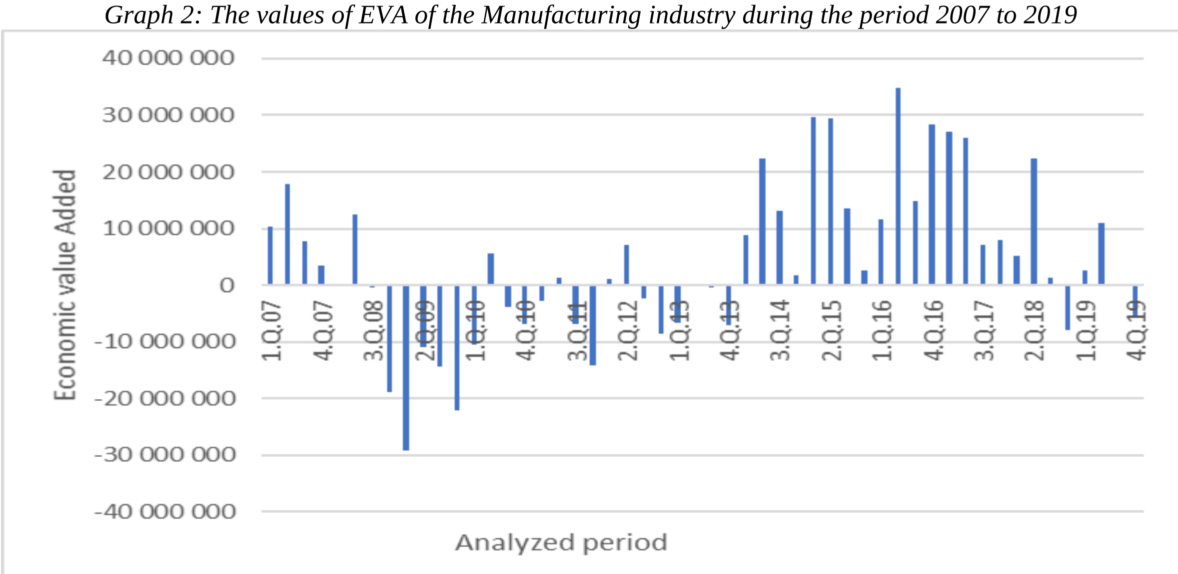
Source: own calculation

The values of ROE and cost of equity are key to determining EVA - Equity. Their difference is the spread. ROE exceeded the value of costs of equity for most in the analyzed period. The average cost of equity was 2.87%, while the average ROE was slightly higher 3.16%. Although these values are low, the return on equity in the Manufacturing industry was higher than its cost.

3.2 Analysis of EVA of the Manufacturing industry of the Czech Republic

The financial performance of the Manufacturing industry is analyzed according to Economic Value Added. This indicator belongs among modern indicators which considers the costs of capital. In this paper Eva is expressed as EVA - Equity. The values of EVA indicator in analyzed period are shown in Graph 2.

Graph 2: The values of EVA of the Manufacturing industry during the period 2007 to 2019



Source: own calculation

During the analyzed period, the Manufacturing industry went through a period of recession in between the years 2008 to 2013 to an economic recovery, which was caused by increased performance and profitability of companies operating in the Manufacturing industry. Since 2014, the Manufacturing industry has generated positive Economic Value Added. The highest performance was achieved in 2016. The analysis of quarterly data revealed that the highest EVA value was always generated in the 2nd quarter of the given year.

3.3 Prediction of EVA in the Manufacturing industry of the Czech Republic

To predict the Economic Value Added of the Manufacturing industry, the indicators of the first component are simulated according to variance gamma process. Subsequently, a thousand scenarios for the development of the Economic Value Added of the Manufacturing industry of the Czech Republic are determined and the distribution of the probabilities of Economic Value Added for the following quarters is estimated.

Quarterly data of the component indicators on which the EVA indicator was decomposed are the input empirical data for the simulation of EVA. These are quarterly data from the first quarter of 2007 to the fourth of 2019. These absolute values are converted to continuous yields and individual characteristics such as mean, standard deviation, skewness and kurtosis are determined. The values of these characteristics for each component indicators are shown in Table 1.

Table 1: The values of characteristics for each component indicators

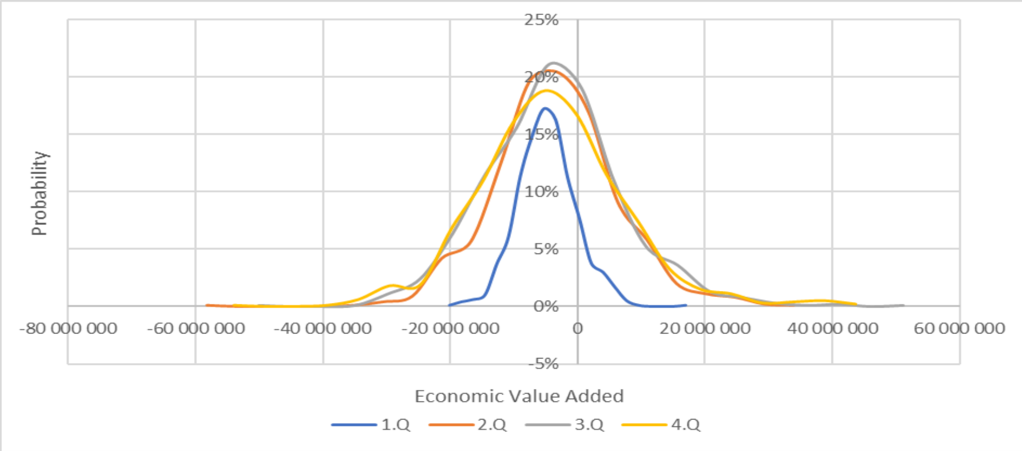
	EAT/EBT	**EBIT/A**	**A/E**	**R_E**	**E**
MEAN	0,0024	-0,0139	0,0006	-0,0002	0,0092
STDEV	0,2563	0,3156	0,0191	0,0720	0,0218
SKEW	2,6815	-0,2500	-0,0510	1,9598	1,1372
KURT	18,8551	3,3945	3,2773	11,9744	5,4434

Source: own calculation

Subsequently, the individual moments of variance gamma process and parameters of variance gamma process for each component indicators are estimated. The correction parameter is estimated, too. For the estimation generalized method of GMM moments is used.

Generally, according to Lévy′s models the asset price is determined as (6), where $X(t)$ is variance gamma process. Variance gamma process is calculated according to (7).
From the results is clear that applied variance gamma process can faithfully capture all four probability distribution moments, such as mean, variance, skewness, and kurtosis. Because of the fact, that only a thousand scenarios of EVA were simulated, minor variations may have occurred. Distributions of the probabilities of relative Economic Value Added was determined for each quarter. In the Graph 4 there is shown the probability distribution for each quarter.

Graph 4: The probability distribution of EVA for each quarter



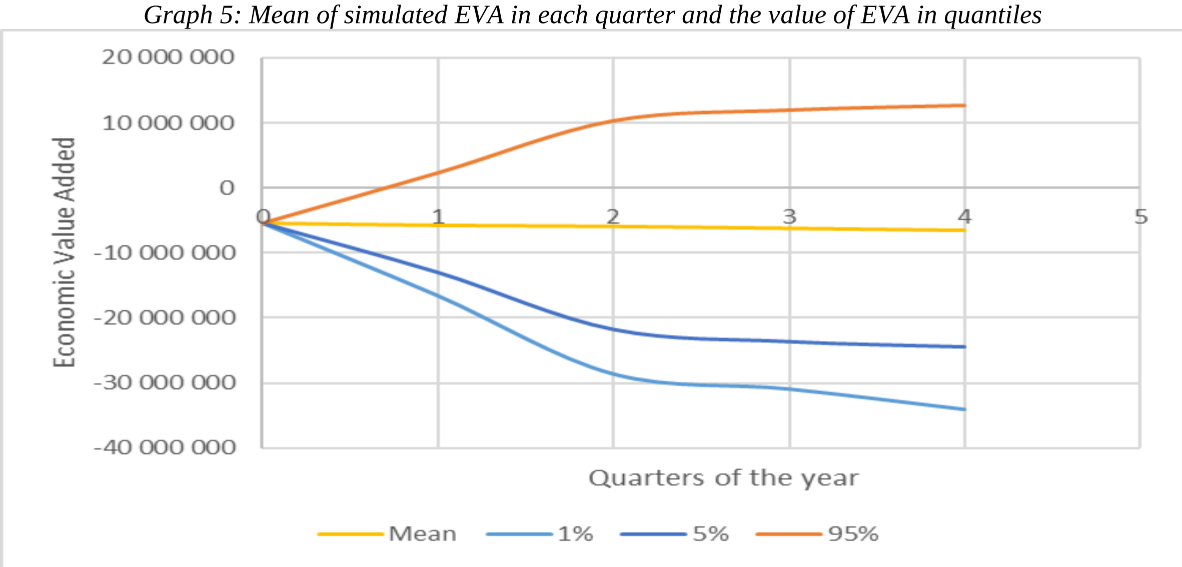
Source: own calculation

The average value of Economic Value Added for the first simulated quarter is negative (- 5 692 166 CZK). This is a slight decrease from the previous quarter. The probability distribution is positively skewed and markedly sharpened. The data of the simulated Economic Value Added from the first quarter were used for the prediction of EVA for the second quarter. Parameters are same.

The average value of the EVA for the second simulated quarter is negative again (- 5 875 901 CZK). Compared to the first predicted quarter, EVA declined. The average value of the EVA for the third simulated quarter is - 6 203 202 and - 6 548 473 for the fourth simulated quarter. According to the results of the simulation is clear, that Economic Value Added of the Manufacturing industry of the Czech Republic is expected to be negative in next year.

In the Graph 5 there is mean of simulated Economic Value Added in each quarter and the value of EVA in quantiles. These quantiles determine the limits in which the value of Economic Value Added should move. The mean of Economic Value added at the 5% probability level is -12 948 723 CZK in the first quarter and -24 451 814 CZK in the fourth quarter. The average value of EVA at the 95% probability level is 2 351 403 CZK in the first quarter and 12 642 060 CZK in the fourth quarter.

Graph 5: Mean of simulated EVA in each quarter and the value of EVA in quantiles



Source: own calculation

In the graph 5 there are shown the quantiles of probability distribution of simulated Economic Value Added for the level of probability 1 %, 5 % and 95 %. From the Graph 5 is clear that with a 95% probability the value of EVA of the Manufacturing industry of Czech Republic will be positive in all quarters. With 5% probability, the value EVA will be negative in each predicted quarter of the year and will decreased.

4. Conclusion

This paper was dedicated to financial performance of the Manufacturing industry of the Czech Republic during the period 2007 to 2018 and its prediction. For the analysis Economic Value Added was used and according to Variance gamma process EVA of the Manufacturing industry of Czech Republic was predicted for each quarter of one year.

Firstly, Economic Value Added was used for financial performance of the Manufacturing industry of the Czech Republic.

Secondly, economic value added was simulated according to variance gamma process. The variance gamma process is one of the most frequently applied multi-parametric models. One of the advantages of this model is that it allows modeling of higher moments of profitability distribution. From the results it is clear with a 95% probability the value of EVA of the Manufacturing industry of Czech Republic will be positive in all quarters. With 5% probability, the value EVA will be negative in each predicted quarter of the year and will decrease.

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The choice of the type of image for graphical processing of input data for corporate bankruptcy prediction using CNN

Monika Šebestová, Petr Dostál [1]

Abstract

This paper deals with the application of Convolutional Neural Networks (CNN) for the bankruptcy prediction of firms in the Czech Republic. It proposes several variants based on the GoogLeNet architecture that predict the bankruptcy of a company 1 to 3 years in advance. The inputs of the model are financial ratios whose values are converted into several types of images. The various types of images are searched to improve the accuracy of company bankruptcy prediction and the right type of image is found. CNN networks can effectively distinguish between active and bankrupt enterprises. The predictive accuracy of the best proposed model ranges between 85 and 93% (depending on the number of years before bankruptcy).

Key words

bankruptcy prediction, convolutional neural network, financial indicators

JEL Classification: C53, G33

1. Introduction

The issue of predicting corporate bankruptcy is one of the most widely discussed areas today and considers the creation of effective prediction models and the selection of indicators that should be entered into these models. Many studies dealing with the area of corporate bankruptcy using statistical methods have been published to the present time, but less attention has been paid to approaches based on deep learning neural networks.

Beaver (1966) is one of the pioneers of statistical bankruptcy models. The author applied univariate discriminant analysis to this problem. In the following years, bankruptcy models based on multivariate discriminant analysis (Altman, 1968; Ohlson, 1980) and regression analysis (Zmijewski, 1984) began to appear.

Qu et al. (2019) published a review of machine learning and deep learning models used in corporate bankruptcy prediction, such as multiple discriminant analysis, logistic regression, neural networks, Support Vector Machines, the Deep Belief Network, and convolutional neural networks. By analyzing the relevant articles, they summarized the characteristics, advantages and disadvantages of each technique.

Nowadays, artificial intelligence methods have become a suitable tool for solving non-linear problems and have come to the forefront as the preferred methods for predicting corporate failures. Neural networks are one of the most popular machine learning methods. López Iturriaga and Sanz (2015) used neural networks to create a model that combines a multi-layer neural network and Self-Organizing Maps to predict bank failures. Geng et al. (2015) state that neural networks are the most effective model for predicting the financial distress of companies. They outperformed other datamining techniques such as decision trees and SVM.

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Deep learning is becoming a popular tool in machine learning and artificial intelligence these days. Convolutional neural networks are one of the significant deep learning models that often achieve better results than discriminant analysis, decision trees or multilayer neural networks. They are most commonly used for image recognition and classification (Gao and Lim, 2019), voice processing (Mohamed et al., 2012) and text data processing (Mai, 2019). Convolutional neural networks can also be applied to corporate financial management and bankruptcy prediction, though only a small number of authors have worked on this issue. The reason for this is mainly due to the orientation of convolutional neural networks towards images, which results in limitations on the processing of numerical data and financial statements. Publications oriented towards the prediction of stock price fluctuations are an exception, where neural networks are used mainly for time series analysis (Aggarwal and Aggarwal, 2017).

The use of deep learning to predict corporate bankruptcy was presented in a study by Yeh et al. (2015), who predicted corporate bankruptcy using a Deep Belief Network. They used the stock returns of firms as input variables which were displayed graphically by means of line graphs. The results show that the proposed model achieves better results than traditional machine learning methods. Hosaka (2019) applied a convolutional neural network to predict corporate bankruptcy using grayscale pixel images. He used the financial statements of 102 bankrupt and 2,062 active firms over a period of 4 years. He converted the values of the financial ratios to grayscale (values of 0–255) and increased the size of the dataset using weighted averages to create additional synthetic inputs. In total, he created 7,520 images which he used to train a convolutional neural network with the GoogLeNet architecture.

This paper aims to propose a prediction model that will be able to detect bankrupt companies using a convolutional neural network based on the GoogLeNet (Szegedy et al., 2015) architecture. Financial indicators are used as model inputs. Since convolutional neural networks work efficiently with image analysis, the numerical data of financial indicators are converted into several types of images and an investigation into which graphical representation more accurately predicts the bankruptcy of enterprises is then conducted.

The remaining part of the paper is structured as follows: Section 2 presents the methods used to solve the bankruptcy prediction problem. Section 3 describes the data used to build the model. Section 4 describes the solution of the problem and the results obtained. Finally, the paper is summarized and specific results are presented.

2. Methodology

2.1 Convolutional Neural Networks (CNNs)

The Convolutional neural network was used because of its higher prediction accuracy than multilayer neural networks and other conventional methods (decision trees, support vector machines, discriminant analysis). CNNs are used for object recognition in images. They are deep learning architectures that learn from data directly and have convolutional layers at their core. The computational algorithm is based on convolution between the input data and filters that represent the characteristic features being searched for (Jirkovsky, 2018).

The convolutional neural network consists of 5 layers – an input layer, a convolution layer, a pooling layer, a fully-connected layer and an output layer. The input layer defines the size of the input images and changes this size if necessary. The convolution layer contains a set of filters that are capable of learning. The network learns to activate these filters when it sees a certain type of visual feature, such as an edge or a blob of color. The pooling layer reduces the size of the image with preservation of the contained information. The fully-connected layer is placed at the end of the network and works as a classification layer whose output is the classification decision of the input image (Phung and Rhee, 2019).

According to Gokmen et al. (2017), the training of a convolutional neural network is based on a backpropagation algorithm that uses a least squares method to find the optimal values for given parameters. The error, usually defined as the difference between the total system output and the desired output, is measured at the end of each epoch. Learning ends when an acceptable system error is reached or when a set number of epochs is reached. Jirkovsky (2018) states that the accuracy of deep learning models depends on the amount of data used to train them. The most accurate models require thousands or millions of samples for learning and this makes the learning process very long. An alternative approach is to use a pretrained network (trained for a different classification or regression task) and merely train this network for a new task. This learning method is called transfer learning. Only the layers that are closer to the end of the network and that extract features specific to a particular task need to be learned. There are a large number of pretrained networks, such as AlexNet, GoogLeNet and ResNet.

2.2 The Synthetic Minority Oversampling Technique (SMOTE)

CNN networks need large amounts of data for learning, which can be difficult to obtain in the bankruptcy prediction domain. The authors mostly encounter a disparity between the number of active and bankrupt firms. The synthetic minority sampling technique (SMOTE) introduced by Chawla et al. (2002) can be used to increase the number of cases in an unbalanced dataset. This is a statistical method that generates new instances from existing data in a balanced way and does not change the number of instances of the majority. The SMOTE method is different from classical data resampling methods. These work with data duplication and do not, therefore, provide any new information for machine learning models. In contrast, the SMOTE method produces synthetic data using the k-nearest neighbor algorithm. It first selects random data from a minority class and then determines k-nearest neighbors (usually $k = 5$) from the data. Synthetic data are then created between the random data and the randomly selected k-nearest neighbors. Authors who have used the SMOTE method to increase the number of samples in bankruptcy prediction include Veganzones and Séverin (2018).

3. Data

The research used financial statements (balance sheets and profit and loss statements) from 365 construction companies in the Czech Republic from 2011–2018 (227 active and 138 bankrupt companies). The selected firms had to meet the following requirements: The analyzed companies are located in the Czech Republic. The company is classified as a small or medium-sized enterprise. The legal form of the company is a limited liability company or joint stock company. According to CZ-NACE, the main business activity of companies must fall into the category F (Construction). The businesses included in the research must be active, inactive (bankrupt), or active in insolvency proceedings.

Due to the possibility of calculating all selected financial ratios, only complete financial statements were included in the sample. For bankrupt firms that went bankrupt from 2013–2019, these are financial statements from 1–3 years prior to bankruptcy (B-1, B-2, B-3). For some companies, data were not available 1 year before bankruptcy. The sample of active firms contains data from 2011–2018, as available. In total, the sample contains 1,768 records (1,464 for active firms and 304 for bankrupt firms). A detailed breakdown for each year is shown in Table 1.

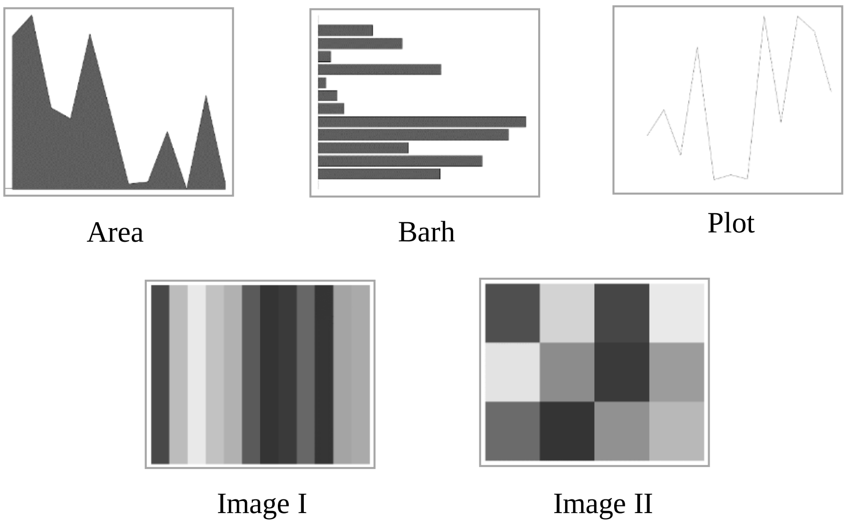
Based on a literature review, we have identified 45 financial and macroeconomic indicators that are most commonly used to predict company bankruptcy. Logistic regression was used to reduce the number of indicators to 20 ($p\text{-value} \leq 0.05$). This was followed by the application of correlation analysis, where the number of indicators was reduced to 12 (indicators whose correlation coefficient was in the interval [-0.5;0.5]). These are current liabilities/total assets, log of total assets, long-term liabilities/equity, EAT/sales, working capital/total assets, inventories/average sales, sales/fixed assets, sales/total assets, cash/total assets, sales growth, cash flow/total liabilities, and PRIBOR interest rate (%).

Table 1: Number of samples of active and bankrupt enterprises

Year	**Active firms (total)**	**Bankrupt firms (total)**	**TOTAL**	**Bankrupt firms (by years before bankruptcy)**		
				B-1	**B-2**	**B-3**
2011	144	18	162	0	7	11
2012	161	26	187	7	9	10
2013	170	39	209	10	11	18
2014	190	64	254	14	20	30
2015	212	68	280	22	28	18
2016	221	51	272	15	15	21
2017	219	26	245	7	19	0
2018	147	12	159	12	0	0
TOTAL	**1464**	**304**	**1768**	**87**	**109**	**108**

Several different images were generated from the values of the above indicators for each company in each year, (Figure 1). For Image I and Image II, the financial ratios are converted to greyscale values of 0–255 and plotted in black and white. All 5 image types were further used as inputs for training the CNN network based on the GoogLeNet architecture. In total, 15 networks (3 networks for each image type – for 1, 2 and 3 years before bankruptcy) were created and tested to see which graphical representation of the indicators predicts the bankruptcy of the companies most accurately.

Figure 1: Graphical display of input data



4. Processing and Results

4.1 Increasing the number of input data

The learning process of CNN networks achieves better results with a larger amount of training data. The number of items for training the network is small (304 companies) in the analyzed sample, especially for bankrupt enterprises, for which reason the SMOTE method was used to increase the amount of input data and the values of financial ratios were normalized within the interval [0;1] before entering the SMOTE method. The increase in the sample of bankrupt enterprises is shown in Table 2. After this change, 1,464 active firms and 1,216 bankrupt firms enter the model.

Table 2: Quantity of bankruptcy data before and after application of the SMOTE method

	B-1	B-2	B-3
Original number of bankruptcy items	87	109	108
Number of items with SMOTE method	348	436	432

4.2 Network learning

Three CNN networks were created for each image type. Each network predicts the bankruptcy of a firm in a different time frame (1, 2 or 3 years before bankruptcy). A corresponding number of images of bankrupt companies is created for each group, including synthetically generated data. The number of images of active companies is the same for all three networks, namely 1,464. From the generated images, 70% of the data was used for network learning and the remaining 30% was used for testing model accuracy. In the test sample, bankrupt firms are mainly represented by real firms and supplemented by synthetic data.

The CNN network was trained on the training data (images) presented in the previous sections. The network used for learning was based on the GoogLeNet architecture, which was also used by Hosaka (2019). Despite the fact that there are several types of CNN network architectures, Hosaka (2019) does not recommend using the powerful AlexNet architecture because the large number of parameters (about 61 million) leads to an inefficient learning process. In contrast, the GoogLeNet network was designed to save computational time with improved identification accuracy. Even though it contains a higher number of layers, the number of parameters is lower than that of AlexNet (about 7 million).

4.3 Evaluation experiments

Because the synthetically generated data for bankrupt companies does not match reality and was only used for neural network learning, the performance of the prediction model was only verified on the original data obtained from existing financial statements. The results of each model are shown in Table 3, which shows the accuracy rate of correct classification of active and bankrupt enterprises for the given type of figure. The group of active enterprises achieves higher accuracy than the bankrupt enterprises. This is mainly due to the fact that, despite using the SMOTE method to increase the number of input data, this is an unbalanced dataset and the number of cases of active enterprises exceeds the number of bankrupt enterprises (the network learns better with the larger amount of data on active enterprises).

Table 3: Accuracy rate of correct classification of active and bankrupt enterprises

Image type	Status	Years before bankruptcy		
		B-1	B-2	B-3
AREA	Active	92.70	99.73	98.09
	Bankrupt	85.06	87.16	93.52
BARH	Active	98.40	100.00	99.80
	Bankrupt	56.32	65.15	85.19
PLOT	Active	98.57	98.16	100.00
	Bankrupt	50.57	89.00	67.59
Image I	Active	98.98	92.96	98.98
	Bankrupt	21.84	81.65	69.44
Image II	Active	98.63	98.98	97.47

	Bankrupt	34.48	61.47	67.59

For the identification of bankrupt companies, it would be most appropriate to build a model using AREA images, where the prediction accuracy achieves the highest values in two periods (B-1 and B-3). The results are not so clear for the prediction of active firms, though the accuracy of the correct identification of active firms reaches values between 90 and 100% for all models. The correct prediction of firms that could fail within a few years is an important component of the functional model. Based on these results, the best input into the model would be an AREA image. However, the issue of model accuracy is much more difficult, for which reason the next section looks at metrics that measure model accuracy in a more comprehensive way.

The accuracy of the models was assessed using overall accuracy, which expresses the percentage of all correctly classified cases, and F-measures characteristics:

- Overall accuracy = $\frac{TP+TN}{TP+TN+FP+FN}$
- F-measures = $\frac{2*Precision*Sensitivity}{Precision+Sensitivity}$

where *Precision = TP/(TP+FP)* and *Sensitivity = TP/(TP+FN)*. Some models may have a higher Precision characteristic, while others may have a higher Sensitivity characteristic. The F-measures characteristic produces a harmonized average of these values. This average enables the accuracy of a model with an unbalanced dataset to be determined using a single number (Chen, 2011). For both accuracy indicators, it is possible to achieve a result within an interval of [0,1], where 1 indicates the highest accuracy.

To assess the performance of the models according to the above characteristics, bankrupt firms are labelled as positive cases and active firms as negative cases. TP, TN, FP and FN represent true positive, true negative, false positive and false negative cases, respectively. TP is the number of correctly classified bankrupt enterprises, TN is the number of correctly classified active enterprises, FP is the number of active enterprises identified by the model as bankrupt, and FN is the number of bankrupt enterprises identified by the model as active.

The overall accuracy and F-measure metrics for each model are shown in Table 4. The best results are highlighted in bold. The best results according to the given metrics are achieved by models whose inputs are composed of AREA and BARH images. The difference between these is often very small (especially for the overall accuracy metric). More significant differences occur for the F-measure metric. In general, the AREA image-based model has better predictive ability on data collected 2 years before bankruptcy, while BARH better predicts the failure of a firm 1 and 3 years before bankruptcy. In terms of the average values constructed from all 3 periods, models based on AREA images win very narrowly over models with BARH type images for the F-measure metric. In the case of overall accuracy, models with BARH image inputs perform better. It is clear from the above description that of the analyzed image types AREA and BARH type images could be good candidates for predicting the failure of a firm.

Table 4: F-measure and overall accuracy of created models

	F-measure				**OVERALL accuracy**			
	B-1	**B-2**	**B-3**	**Average**	**B-1**	**B-2**	**B-3**	**Average**
AREA	0.552	**0.913**	0.852	**0.773**	0.923	**0.989**	0.978	0.963
BARH	**0.616**	0.789	**0.906**	0.771	**0.961**	0.976	**0.988**	**0.975**
PLOT	0.579	0.833	0.807	0.739	0.959	0.975	0.978	0.971
IMAGE I	0.314	0.591	0.758	0.554	0.946	0.922	0.969	0.946
IMAGE II	0.438	0.702	0.670	0.603	0.950	0.964	0.954	0.956

A Type II error analysis was performed in order to make a final decision on which image would be the most appropriate input for the development of the bankruptcy prediction model

(Table 5). Type II error measures the proportion of cases in which bankrupt firms are incorrectly identified as active (False Negative). Its calculation is given by the relation:

$$\text{Type II error} = \frac{FN}{(TP+FN)}.$$

The results presented in Table 5 show the difference between the preferred AREA and BARH pictures above. The BARH picture results in a much higher Type II error than the model based on the AREA pictures and identifies bankrupt firms as active to a greater extent, which can impose high costs and losses on firms trusting this type of model. Therefore, the AREA-type image is considered a suitable image for predicting company bankruptcy that is better able to detect a failing firm and thus provide firms with timely information about possible business failure.

Table 5: Type II error in created models

	Type II error			
	B-1	**B-2**	**B-3**	**Average**
AREA	**0.149**	0.128	**0.065**	**0.114**
BARH	0.437	0.349	0.148	0.311
PLOT	0.494	**0.110**	0.324	0.309
IMAGE I	0.782	0.183	0.306	0.424
IMAGE II	0.655	0.385	0.324	0.455

5. Conclusions

This paper dealt with the problem of predicting corporate bankruptcy using CNNs. Five types of images were generated from the values of selected financial and macroeconomic indicators and an analysis performed as to which type of image is most suitable as an input variable for bankruptcy prediction. The transfer learning method was used to generate the models with the GoogLeNet network. Based on the results, it can be concluded that CNNs can effectively distinguish between active and bankrupt enterprises. The average overall prediction accuracy for all models ranged between 95 and 98%, but in some cases the prediction of bankrupt enterprises was low in favor of active enterprises. For this reason, Type II error, which reveals cases in which the model incorrectly identifies a bankrupt enterprise as active, was analyzed. This misprediction can have negative effects on firms due to the lack of early warning. After the analysis, the AREA image type, which can detect bankrupt companies 85–93% of the time (depending on the number of years before bankruptcy), was selected. To compare the accuracy of the model with other authors, we can mention the prediction of firm bankruptcy using SVM with an overall accuracy of 85-90% (Chen, 2011) and using CNNs with an accuracy of 88-91% (Hosaka, 2019).

For further research, it would be useful to analyze other pretrained CNN network architectures to see if any of them work more efficiently with financial ratio images and subsequent bankruptcy prediction.

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Insights for stochastic dominance extension in a multistage framework

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Abstract

Stochastic dominance has proven to be an efficient tool to compare random variables. This is particularly true in financial application when one wants to compare the outcome of an optimal strategy with the outcome of a given benchmark. A classical example is the Asset and Liability Management problem that aims at defining the optimal allocation of the assets of a Pension Fund. The recent literature proposes univariate stochastic dominance constraints to guarantee that the optimal strategy is able to stochastically dominate a benchmark portfolio. The purpose of this work is (1) to provide an extended literature of the recent findings about stochastic dominance in pension fund management and (2) to propose some hints to investigate new formulations of stochastic dominance that could be particularly important when multivariate random variables are involved and when one of their dimension is the time.

Key words

Stochastic Programming, Pension Fund, Stochastic Dominance

JEL Classification: G11, C61

1. Introduction

One of the main area of application of the Asset and Liability Management (ALM) is the Pension Fund (PF) problem, meaning the capability of optimally managing the assets of the PF to guarantee the sustainability of the liabilities. Such topic has been widely analysed in the last decades. Pflug and Świetanowski (1999) proposed an ALM model for pension funds considering specific characteristics of both assets and liabilities. Consigli and Dempster (1998a,b) and Dempster et al. (2003) presented the CALM model that considers different types of pension contracts over a long-term horizon. For a review, we suggest Ziemba and Mulvey (1998) and Zenios and Ziemba (2006, 2007). More recent and advanced formulations of ALM models can be found in Consigli et al. (2011), Consigli and di Tria (2012), Consigli and Moriggia (2014), Consigli et al. (2017), and Moriggia, Kopa and Vitali (2019). Some applications focus not only on the pension fund manager's point of view, but also on the pension fund sponsor's (the issuer of the pension fund), see e.g. Vitali, Moriggia and Kopa (2017), and the pension fund investor's, see e.g. Consigli (2007), Consigli et al. (2012), Kopa, Moriggia and Vitali (2018) and Consigli, Moriggia and Vitali (2020).

In such contest, scholars focused on an efficient tool to compare the outcome of the optimal investment strategy with the outcome that the pension fund would produce with the business-as-usual allocation strategy. Of course, on the two random variables that represent the two outcomes, several statistics can be computed and compared. Still, recently, the researches in this area prefer not to rely simply on a set of relevant statistics, and prefer to compare the whole

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distributions of the random variables. In this perspective, the tool chosen by most of the researchers is Stochastic Dominance (SD).

The concept of SD was introduced more than 50 years ago and it was firstly applied to economics and finance in Quirk and Saposnik (1962), Hadar and Russell (1969) and Hanoch and Levy (1969). Two main formulations of SD are used in financial applications: First-order Stochastic Dominance (FSD) and Second-order Stochastic Dominance (SSD). As it will discussed in next section, the FSD turns out to be a too restrictive requirement and, thus, in many applications SSD is preferred. The SSD constraints were included in stochastic programs in Dentcheva and Ruszczynski (2003) and Luedtke (2008) and in portfolio efficiency analysis in Kuosmanen (2004), Dupačová and Kopa (2012) and Kopa and Post (2015). In multistage stochastic optimization, the stochastic sources evolve in times over a stochastic tree where each realization of the uncertain variables is contained in a node of the tree and the collection of consecutive nodes from the initial one (the root of the tree) and any of the last ones (the leaves of the tree) is called scenario. The SSD constraints in multistage ALM models have been widely investigated, see e.g. Yang, Gondzio and Grothey (2010), Kopa, Moriggia and Vitali (2018), Consigli, Moriggia and Vitali (2020) and Moriggia, Kopa and Vitali (2019). In all these papers, the SD constraints have been applied either on a single stage or on multiple stages simultaneously. The idea of applying SD constraints jointly on multiple stages is called Multistage Stochastic Dominance (MSD). Such approach requires the dominance on several stages, but, in all the mentioned works, the way in which a random variable dominates another random variable in a given stage is completely independent by the dominance relation in another stage. This means that applying SD constraints on multiple stages defines a dominance relation among nodes, while, in a multistage framework, it would be much more reasonable to have a dominance among scenarios so that we could be able to say "whichever scenario will happen, the one random variable dominates the other".

2. Multivariate stochastic dominance

In the next definitions of FSD and SSD, let's assume that the stochastic tree is represented as follows. The set of stages is denoted $\mathbb{T} = \{t_h\}_{h=0,\dots,H}$, where t_0 represents the here-and-now stage, and t_H represents the final horizon. The scenario tree is represented with the nodal notation, then each node is denoted by a unique index n. For each node n we define $t(n)$ as the corresponding stage time.

The definition of FSD relation is as follows: A random variable A FSD dominates a random variable B if the cumulative distribution function of A is below that of B. Equivalently, the FSD holds if and only if no rational and insatiable decision maker prefers B to A. When random variables are discrete with equiprobable realizations (as for most of the stochastic optimization problems in a multistage environment) it is useful to formulate the FSD conditions using a permutation matrix as proposed in Kuosmanen (2004). In particular, if we define $\mathbf{A}_{t_h}$ the vector of the optimal portfolio wealth realizations occurring in all nodes at stage t_h and similarly we define $\mathbf{B}_{t_h}$ the vector of a benchmark portfolio wealth realizations occurring in all nodes at stage t_h then the optimal portfolio FSD dominates the benchmark portfolio at stage t_h if and only if

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}$$

for some matrix $\mathbf{Q}^{t_h}$ which is a permutation matrix, i.e. satisfies the following conditions:

$$\sum_i \mathbf{Q}_{i,j}^{t_h} = 1$$

$$\sum_j \mathbf{Q}_{i,j}^{t_h} = 1$$

and the elements of $\mathbf{Q}^{t_h}$ belong to the set $\{0,1\}$. This set of constraints is a mixed-integer optimization problem.

The definition of SSD relation is as follows: A random variable A SSD dominates a random variable B if the integrated cumulative distribution function of A is below that of B. Equivalently, the SSD holds if and only if no risk-averse rational decision maker prefers B to A. Again, considering random variables with discrete and equiprobable realizations, it is useful to formulate the SSD conditions using a double stochastic matrix as proposed in Kuosmanen (2004). In particular, if we define $\mathbf{A}_{t_h}$ the vector of the optimal portfolio wealth realizations occurring in all nodes at stage t_h and similarly we define $\mathbf{B}_{t_h}$ the vector of a benchmark portfolio wealth realizations occurring in all nodes at stage t_h then the optimal portfolio SSD dominates the benchmark portfolio at stage t_h if and only if

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}$$

for some matrix $\mathbf{Q}^{t_h}$ which is double stochastic, i.e. satisfies the following conditions:

$$\sum_i \mathbf{Q}^{t_h}_{i,j} = 1$$

$$\sum_j \mathbf{Q}^{t_h}_{i,j} = 1$$

and the elements of $\mathbf{Q}^{t_h}$ belong to the interval $[0, 1]$, so each row and each column represents a convex combination. This set of constraints is a linear optimization problem.

Since the vector $\mathbf{A}_{t_h}$ and $\mathbf{B}_{t_h}$ are uni-dimensional, the FSD and SSD defined above are called *univariate* SD. The simplest evolution to *multivariate* SD when the further dimension is the time, can be obtained applying jointly multiple univariate SD. Indeed, the multistage SSD (or multivariate SSD) is obtained just by selecting jointly more than one stage, i.e. defining a subset $\mathbb{T}^{SSD} \subseteq \mathbb{T}$, and then defining the constraints

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}, \qquad \forall t_h \in \mathbb{T}^{SSD}$$

The same can be done with FSD constraints. Notice that the matrices $\mathbf{Q}^{t_h}$ can differ from stage to stage. This means that the dominance relation at stage t_i is not related with the dominance relation at stage $t_j, i \neq j$. Clearly, this is in contrast with the idea that an optimal solution dominates another one *through* the stages. Indeed, let's imagine to walk through a scenario and assume that $\mathbf{Q}^{t_i}$ establishes a dominance relation between A and B such that a specific node $n^a, t(n^a) = t_i$ is *linked* with node $n^b, t(n^b) = t_i$. Now, let's move to the next stage and consider the successor $\tilde{n}^a$ of node n^a and the successor $\tilde{n}^b$ of node n^b. It is possible that the matrix $\mathbf{Q}^{t_i+1}$ does not establish a *link* between $\tilde{n}^a$ and $\tilde{n}^b$ which means that it is not possible to conclude that A dominates B *through* the stages, but only over a set of stages.

Thus, it is clear that in a multistage framework a new definition of SD is needed. Recently, some studies have been performed in Dentcheva and Wolfhagen (2015), Dentcheva and Wolfhagen (2016) and Dentcheva, Martinez and Wolfhagen (2016) where the authors propose to linearly aggregate the random variables over the stages. Then, the realizations $\mathbf{A}_{t_h}$ of the random variable A are aggregated with a convex combination to generate a unique vector $\mathbb{A}$, similarly, the realizations $\mathbf{B}_{t_h}$ of the random variable B are aggregated to generate $\mathbb{B}$ and, finally, they establish a SD between $\mathbb{A}$ and $\mathbb{B}$. Still, this formulation does not directly address the issue of imposing an SD relation among scenarios and we believe that it must be possible to define a specific multistage SD that impose a dominance among the scenarios, and not among the nodes. Probably, the best way is to work on the definitions of matrices $\mathbf{Q}^{t_h}$ that must be put into relation among themselves.

3. Conclusion

In this work, we provide an extended literature review for SD and its application in ALM models, and we remind the definition of SD for univariate random variables. The idea of SD, as it is known until now, does not provide an easy and effective tool to compare multivariate distributions, especially when one of the dimension is the time and the distributions are nested through the time. Some hints have been provided by in Dentcheva and Wolfhagen (2015) that propose to aggregate the realizations over the stages through a convex combination. Such procedure has several drawbacks among which the main two are: the difficulties in the implementation, and the too complex interpretation of the resulting dominance relation. We believe that working on the matrices $\mathbf{Q}^{t_h}$ it can be possible to define a dominance among scenarios which would be the best achievement in terms of SD when the optimal solution evolves over a multistage scenario tree.

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Out-of-Sample Performance Evaluation of Mean-Variance Model Extensions

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Abstract

In the mean-variance model proposed by Harry Markowitz in 1952, the trade-off between the return and risk of a portfolio is realized by using the mean and variance of assets' returns. Although the framework of mean-variance analysis is straightforward, several shortcomings have been demonstrated by practitioners in the applications. In this paper, we apply 6 extensions of the mean-variance model in the empirical analysis and make evaluations of the out-of-sample performances. In our study, the chosen dataset of the empirical analysis is the daily adjusted closing prices of 27 components of Dow Jones Industrial Average covering the period from January 3, 2006, to April 30, 2021. By applying the rolling window approach, we find that the minimum-CVaR strategy outperforms the minimum-variance strategy in the case of risk-minimizing, however, the maximum Sharpe Ratio strategy is proved as not robust. Last but not least, the extensions of the mean-variance model which are designed to reduce the estimation errors make almost no difference in the out-of-sample performance compared to the classical mean-variance model.

Key words

Mean-variance model, performance measures, portfolio optimization, random-weights portfolios, risk measures.

JEL Classification: G11, G17

1. Introduction

Harry Markowitz formulated Modern Portfolio Theory (henceforth MPT) in 1952 (Markowitz, 1952). It explains how to construct investment portfolios in a quantitative way. In the core idea of MPT, the trade-off between the return and the risk of a portfolio is balanced by using the mean and variance of assets' returns, so, the model proposed by Markowitz is also named as the mean-variance (henceforth MV) model. In the past, the MV model has become the foundation of the research on portfolio selection problems, however, although its framework is straightforward to understand, in practice, several shortcomings of the MV model have been demonstrated. According to the review on literature of MV analysis, we list three main shortcomings.

1.1 Limitations of using variance as the risk measure

In the optimization procedures of the MV analysis, the variance is used as the risk measure. Variance reflects both the upside and downside deviations over the mean value. In this sense, outperformance, as well as underperformance of the investments, are treated just as the same. Although the assumption of normal/symmetric distribution of assets' returns makes the application of variance more reasonable, in empirical studies the historical returns can be fat-

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tailed, for this reason, more alternative risk measures which only observe the unfavorable downside risk have been proposed, such as the semivariance that measures the dispersion of observations that fall below the mean, see Estrada (2008) and Ben Salah *et al.* (2018), or the Conditional Value at Risk (henceforth CVaR) that quantifies the amount of tail risk, see Xu *et al.* (2016) and Rachev *et al.* (2005).

1.2 Concentration of generated weights

According to the pioneering works on the MV analysis, the MV optimal portfolios have been proved as overly concentrated on few components with large expected returns, which go against the idea of diversification (Lin, 2013). To improve this shortcoming, several enhancements have been made to the classical MV model, for instance, except for the original constraints setting of MV portfolio optimization, other constraints targeting at the lower and upper bound of weights are considered.

1.3 Difficulty in estimating parameters

Another shortcoming of MV model is that there might be errors arisen in parameters' estimation. In the classical MV model, the weights of the components of a portfolio are generated by inputting the values of estimated parameters (i.e., mean and covariance matrix of the assets' returns), however, the input parameters in the MV model are estimated based on the historical trading data, which means even though the estimated parameters are descriptive, the estimation errors still might exist due to the differences between the history and the future actual status. So, to increase the robustness of the estimates, a wider range of parameter estimation methods are emerging, for instance, deep learning and machine learning models can be applied to predict future returns more effectively, see Ma *et al.* (2021). Moreover, the application of shrinkage estimators which shrink the unbiased vector of expected returns and the covariance matrix towards the biased targets can also improve the classical MV model, see Jorion (1986).

Various portfolio optimization models have been well developed in recent years, to compare the performance of the enhanced models to the classical MV model. DeMiguel *et al*. (2009) evaluate the out-of-sample performance of the MV model's extensions designed to reduce estimation error by using seven empirical datasets, while the evaluation results show that none of the extensions is consistently better than the naive portfolio in terms of the applied performance measures. Similarly, Martínez-Nieto *et al.* (2021) make a review of the diversification-based strategies and an experimental study with a complete repository of datasets with a total of 11 strategies. However, although there is already literature reviewing and evaluating the extensions of the classical MV model, there still lacks related works which systematically categorize the MV extensions and evaluate the performance by applying appropriate benchmarks, so, this paper contributes to these two knowledge gaps.

The goal of this paper is to categorize six extensions of the MV model according to the corresponding improvements, at the same time, the out-of-sample performance of the extensions is compared to that of the classical MV model as well as the generated random-weights portfolios.

This paper is structured as follows. In section 2, we describe the formulations of MV model and the studied extensions. In section 3, we introduce the performance evaluation methods which are applied in the empirical analysis. We show the empirical results in section 4. At last, the paper is concluded in section 5.

2. Portfolio Optimization Models

In this section, firstly we introduce the formulation of the minimum-variance (henceforth MVA) strategy generated from the classical MV model. Secondly, we introduce the other two strategies that are also generated from the mean-risk model in which the risk measure is replaced with CVaR and the mean absolute deviation (henceforth MAD).

2.1 MVA Strategy

In Markowitz (1952), the weights of the optimal portfolio are generated through the tradeoff between the return and risk. Under the framework of MV model, assuming that there are N assets in a portfolio, then the MV portfolio's expected return $E(R_p)$ and variance ${\sigma_p}^2$ can be calculated as follows:

$$E(R_p) = w^T \cdot E(R) = \sum_{i=1}^{N} w_i \cdot E(R_i) \tag{1}$$

$${\sigma_p}^2 = w^T \cdot \boldsymbol{Q} \cdot w = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot \sigma_{i,j} \cdot w_j \, , (i\, , j \; = \; 1, \ldots, N) \tag{2}$$

where w is the vector of assets weights, $E(R)$ is the vector of expected returns of assets, $\boldsymbol{Q}$ is a $N \times N$ covariance matrix.

In this paper, since we aim at evaluating the performance of the extensions of MV model, so, we generate the MVA strategy as a benchmark. As it literally means, in MVA strategy the objective is to minimize the portfolio variance, so, its optimization can be formulated as follows:

$$\begin{aligned} &\min \; w^T \cdot \boldsymbol{Q} \cdot w, \\ &\text{s.t.} \;\; \textstyle\sum_{i=1}^{N} w_i = 1, \\ &\qquad w_i \geq 0, i = 1, \ldots, N. \end{aligned} \tag{3}$$

2.2 Alternative Minimum-Risk Strategies

However, the variance is not always a perfect risk measure, because it makes no distinction between gains and losses, so, to overcome this shortcoming, more alternatives to variance have been proposed in the later literature. In particular, in our study, we apply two alternative risk measures in the portfolio optimization, one is the CVaR which quantifies the amount of unfavorable tail risk, the other one is the MAD which measures the absolute deviation from the mean of portfolio return.

2.2.1 Minimum-CVaR Strategy

CVaR is a risk measure that indicates the expected loss under the condition of exceeding Value at Risk (henceforth VaR). The CVaR of a portfolio can be calculated as follows:

$$CVaR_\alpha(R_p) = E\left(-R_p \middle| -R_p \geq VaR_\alpha(R_p)\right) \tag{4}$$

where α is the probability level which is usually set to 1% or 5%. Furthermore, the formulation of minimum-CVaR (henceforth MCV) strategy has the same framework as that of the MVA strategy, the only difference is that the risk measure in the objective function of MCV strategy formulation is replaced with CVaR.

2.2.2 Minimum-MAD Strategy

Comparing to the variance, MAD is more robust to the outliers of random returns. It can be calculated as follows:

$$MAD(R_p) = E(|R_p - E(R_p)|) \tag{5}$$

similarly, in the formulation of minimum-MAD (henceforth MMA) strategy, the risk measure in the objective function is changed to MAD.

2.3 Diversification Strategies

MV model is straightforward to understand, but it might generate efficient portfolios which are highly concentrated, in other words, the component assets with large expected returns might be overweighted, so, once there is an unexpected drop in the price of the overweighted asset, it could result in large losses. So, improving the diversification degree of the MV efficient portfolios is an important issue.

2.3.1 1/N Strategy

As it literally means, 1/N strategy is a strategy in which all the assets of the portfolio are invested at an equal weight $1/N$. In our study, we apply the 1/N strategy as a benchmark to compare its performance with that of the other MV extension designed to improve the diversification degree.

2.3.2 Maximum Sharpe Ratio Strategy

Under the framework of MV analysis, the optimal portfolio targeted at maximizing the expected portfolio return would invest all the wealth into the asset with the largest expected return, which means this maximum-return strategy would lose the advantage of the diversification of portfolio investment, so, in our study, instead of the maximum-return strategy, we apply a maximum-Sharpe Ratio (henceforth MSR) strategy which is also designed to maximize the performance of portfolio but expected to have a higher degree of diversification.

The Sharpe ratio (henceforth SR) is defined as the expected value of the difference between the portfolio return and the risk-free rate divided by the standard deviation of the portfolio return, it can be calculated as follows:

$$SR(R_p) = \frac{E(R_p - R_f)}{\sigma(R_p)} \tag{6}$$

where R_f is the risk-free rate. As for the formulation of the MSR strategy, it can be defined as follows:

$$\begin{aligned} &\max SR(R_p), \\ &\text{s.t.} \ \ \textstyle\sum_{i=1}^{N} w_i = 1, \\ &\qquad w_i \geq 0, i = 1, \dots, N. \end{aligned} \tag{7}$$

2.4 Strategies Reducing Estimation Errors

In the MV model, the input parameters are estimated based on the historical data, however, the future actual trading data could be different from the history. Due to this reason, errors might exist in the estimated parameters. So, to increase the robustness of the estimates, in our study, we apply the Bayes-Stein shrinkage (henceforth BSS) estimation and the Fuzzy Probability (henceforth FP) estimation which are both designed to reduce the estimation errors of the classical historical sample method.

2.4.1 MVA Strategy in BSS Estimation

BSS estimation takes the subjective (a priori) assumption of the shape of the assets returns distribution into account, and the resulting (a posteriori) assumption is then a combination of the priori assumption and the probability distribution of the observed sample. In this paper, we apply the shrinkage estimation method suggested by Jorion (1986) in Bayesian portfolio selection problem, and by applying this method, the estimated expected returns of assets and the estimated covariance matrix are reformulated on the basis of historical sample estimation, see Kresta and Wang (2020).

2.4.2 MVA Strategy in Fuzzy Probability Estimation

Tanaka *et al.* (2000) proposed a fuzzy probability model which helps to handle the uncertainty of the individual assets returns distribution by involving the fuzzy theory. In the proposed fuzzy probability model, the possibility grade is introduced to reflect the similarity degree between the future state of the asset return and the same asset's m-th historical return. For the formulations of estimated fuzzy expected returns of assets and the estimated fuzzy covariance matrix, see Kresta and Wang (2020).

For both the BSS estimation and the FP estimation, we input the reformulated parameters to the classical MV model to generate the optimal portfolios which are objective to minimize the portfolio variance, by this way, we can obtain the MVA-BSS strategy and MVA-FP strategy.

3. Approaches to Evaluate Portfolio Performance

To evaluate the performance of the strategies, we apply three performance measures and three risk measures in our empirical analysis. At the same time, we also generate 5,000,000 random-weights portfolios as a benchmark to compare the performances with that of the strategies.

3.1 Performance Measures

For the performance evaluations, we estimate the strategies' mean return (henceforth MR), SR and Calmar ratio (henceforth CR). We have already introduced the calculations of the portfolio's MR and SR in equations (1) and (6), respectively. As for the CR, it is a drawdown-based ratio which can be defined as follows:

$$CR(R_p, T) = \frac{E(R_p - R_f)}{MDD(T)} \tag{8}$$

where MDD is the maximum drawdown, to be more specific, it is the maximum of the drawdown of investment wealth evolutions during a specific period, its calculation can be shown as follows,

$$MDD(T) = \max_{\tau\in(0,T)} D(\tau) = \max_{\tau\in(0,T)} \left[\max_{t\in(0,\tau)} X(t) - X(\tau)\right] \tag{9}$$

where $D(\tau)$ is the drawdown at time τ and $X(t)$ is the investment wealth at time t.

3.2 Risk Measures

To evaluate the risk of the strategies, we apply three risk measures, which are the standard deviation (henceforth STD), CVaR and MDD. The calculations of CVaR and MDD has been introduced in equation (4) and (9), respectively. As for the STD of portfolio returns, it can be calculated as the square root of the variance.

$$\sigma_p = \sqrt{{\sigma_p}^2} \tag{10}$$

3.3 Random-Weights Portfolios

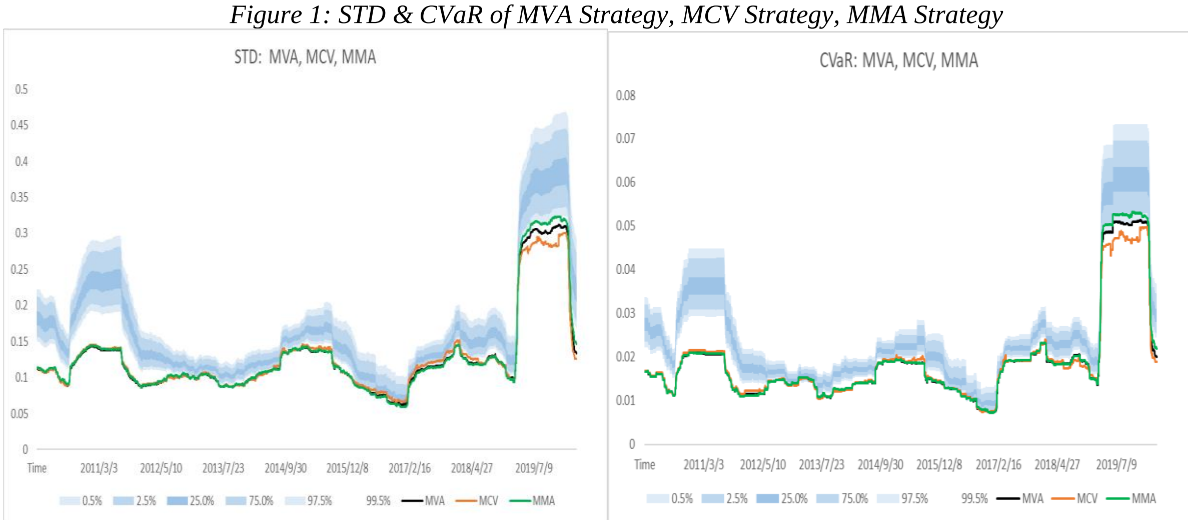
As it literally means, random-weights portfolios are the portfolios in which the assets are invested at random weights, and the method of generating the random weights can be referred to Kresta and Wang (2020). In our empirical study, we generate 5,000,000 random-weights portfolios as a benchmark, by comparing the performance of the random-weights portfolios with that of the strategies, we can verify the efficiency of the strategies.

4. Empirical Analysis

In this section, we make the empirical analysis. The chosen dataset of the analysis is the daily adjusted closing prices of 27 components of Dow Jones Industrial Average index (henceforth DJIA) which starts from January 3rd, 2006 to April 30th, 2021. Three DJIA stocks were excluded due to the incomplete data in the chosen period. In order to test the robustness of the strategies to the changes of the periods, the analysis is made by applying the rolling window approach, in particular, we always take 3 years (750 days) as the in-sample period and 1 year (250 days) as the out-of-sample period, then we move the start of the window day by day.

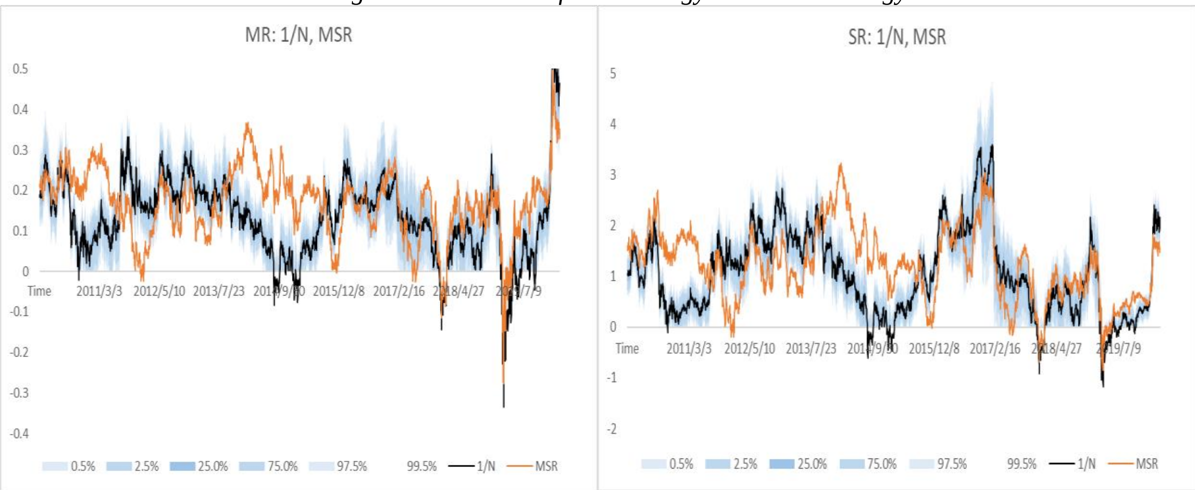
To evaluate the performance of the alternative mean-risk models, we generate the MCV strategy and the MMA strategy. In the meanwhile, to make the evaluation results more convincing, we apply the performance of MVA strategy and the performance of the random-weights portfolios as the benchmark. In Figure 1, the risks of all the strategies are evaluated by STD and CVaR, respectively. The blue shadow area is constructed by the different quantiles of the values of the corresponding risk measurement for all the 5,000,000 generated random-weights portfolios. We can see no matter for STD or CVaR, the risks of the three minimum-risk strategies are lower than that of the random investments, and by comparison, we find that the MCV strategy, intending to minimize the CVaR in the in-sample period, has the best out-of-sample performance.

Figure 1: STD & CVaR of MVA Strategy, MCV Strategy, MMA Strategy



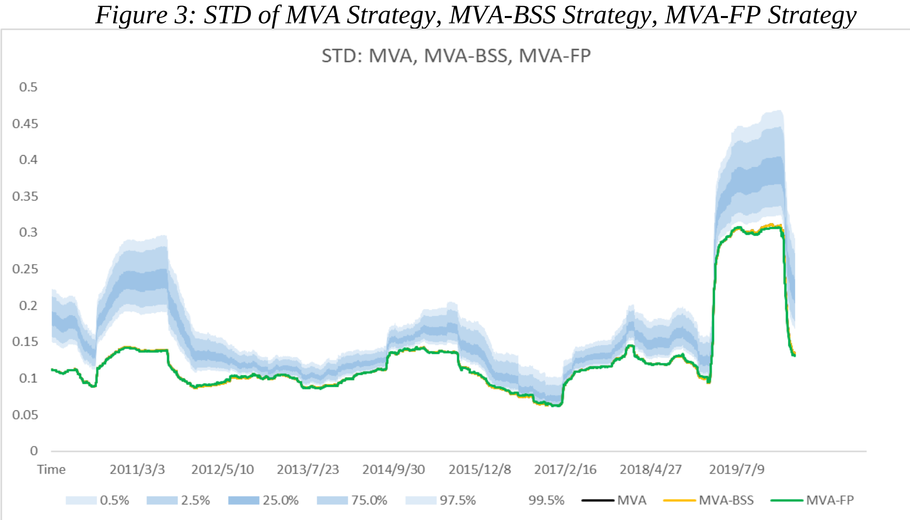
The other category of the strategies is composed of *1/N* strategy and MSR strategy. To evaluate the strategies' performance, we apply the performance measures MR and SR. From Figure 2, for both MR and SR, we can see that the performance of *1/N* strategy is on the average level of the performance of random investments, however, the performance of MSR strategy is not always better than that of the other portfolios, which means the MSR strategy aiming at maximizing the SR in the in-sample periods is not robust in the out-of-sample periods.

Figure 2: MR & SR of 1/N Strategy and MSR Strategy



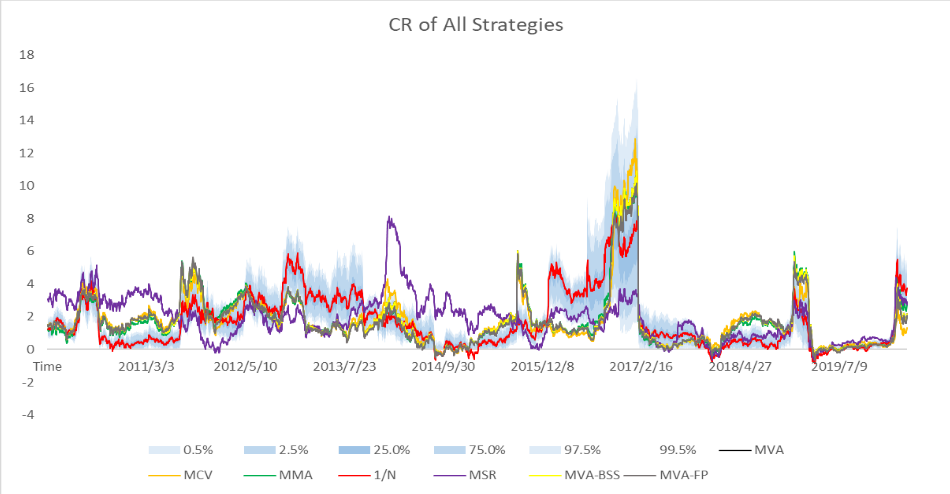
In section 2, we have introduced two extensions to the classical MV model which are designed to reduce the estimation errors. In Figure 3, we show the evolutions of STD of the applied strategies, while the results show that there is no big difference between the baseline MVA strategy and its extensions (MVA-BSS strategy and MVA-FP strategy) in variance-minimizing in the out-of-sample periods.

Figure 3: STD of MVA Strategy, MVA-BSS Strategy, MVA-FP Strategy



In Figure 4, the evolutions of CR of all the generated strategies (include the baseline MVA strategy) are presented, we can see that the values of CR of most strategies are on the average level of the random-weights portfolios, while the CR value of MSR strategy is most floating comparing to the others.

Figure 4: CR of All the Generated Strategies



5. Conclusion

In this paper, based on the review of the literature of MV portfolio optimization, we list three main shortcomings of the classical MV model. Since different extensions designed to improve the MV model have been developed in recent years, so, we choose six improved strategies and make the evaluations on the performance. In our empirical analysis, we apply the trading data of DJIA from January 3rd, 2006 to April 30th, 2021, furthermore, to verify the efficiency of the generated strategies, we compare the performance of the strategies with that of the baseline MVA strategy as well as the random-weights portfolios. From the empirical results, we make the following conclusions. Firstly, we find that when the portfolio risk is measured by STD and CVaR, the MCV strategy outperforms the MVA strategy in risk-minimizing in the out-of-sample periods. Secondly, no matter the portfolio performance is measure by MR, SR, or CR, we find the MSR strategy is not robust. Last but not least, the extensions of MV model which are designed to reduce estimation errors make almost no difference to the baseline MVA strategy in variance-minimizing in the out-of-sample periods.

Acknowledgments

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