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CONTENTS

Corporate taxation from the perspective of EU countries categorization

Alena Andrejovská[1], Veronika Konečná[2]

Abstract

Corporate taxation is an important source of revenue for the state budget of all EU countries. The article focuses on the categorization of EU countries from the perspective of corporate taxation using cluster analysis. The aim is to analyze the issue of categorization of EU member states according to their common characteristics based on selected macroeconomic determinants: unemployment, gross domestic product, public debt, foreign direct investment, tax revenues, statutory and effective tax rate. The analysis uses several statistical methods. From the hierarchical methods - Ward's method and the k-means from non-hierarchical method. The EU Member States were grouped into seven clusters according to features. The analysis has point to the despite the integration that continues, the differences between countries are visible. Differences persist both in development and in individual economic policies.

Key words

unemployment, GDP, public debt, FDI, tax revenues, statutory tax rate, effective tax rate

JEL Classification: H21, H25

1. Introduction

It is necessary to realize the taxes are the basic component of state budget revenues and ensure the functioning of the state. Taxes are used for public spending and governments can influence the country's economic development. Every country in the world has its own tax system and applies different tax principles. One thing is same for all tax systems - the negative impact on the disposable income of natural and legal person. Finding an optimal taxation system is a more and more discussed topic that is gaining in importance. The European Union's development tendencies are part of intense globalization. That leads to ever greater economic development, integration and harmonization. Company tax burden is a central concern for investors who intend to make investments in the country, as well as experts, politicians, economists and analysts. Member State governments are motivated to establish an appropriate tax system as a competitive advantage against other states. The ideal tax system promotes the economic growth of the country, reduces unemployment and eliminates barriers in the business. For potential investors, that is transparent and administratively simple. On the other hand, great emphasis is also placed on tax revenue because many Member States are suffering from long-term instability of public finance and long-term fiscal imbalances.

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2. Literature review

Corporate taxes are one of the most important factors affecting make decision of investors about investment. These factors include the statutory tax rate, which is determined by tax legislation. That is one of the criteria for obtaining country information. This method is the easiest and accessible for everyone. The important is monitoring the overall burden. That depends on business taxation, the proportion of taxes paid on total income or profits of the company in a country mentioned by Bayer (2011), Gupta (2007), Bird et al. (2008). Despite the fact the legislation determines the basics (reduced statutory tax rates in some countries), that is necessary to monitor the tax burden from a wider angle. The use of statutory rates as an objective indicator for assessing and comparing business tax rates has proven inappropriate and has led to the derivation of an effective tax rate that has much better information capacity. This issue has been addressed by Baker and McKenzie (1999) and Barrios et al. (2014). Significant information for investors, politicians and economists is the amount of the tax rate, which has a key impact on the tax burden in the form of statutory (nominal), effective and average tax rate (Bánociová et al., 2014). Other factors that assess tax deductible costs are depreciation policy, the possibility of applying tax breaks and incentives and full tax exemption. We can say the method of defining the tax base is also important in the measurement of corporate burden. In the years 1980-2004, a correlation was realised for OECD countries, reflecting the relationship between tax rates and tax revenues. Kawano and Slemrod (2016) found the raising implicit tax rates maximizes corporate earnings. Other authors, such as Clausing (2007), Devereux (2006), Devereux and Griffith (1999,2003), have also addressed this dependence. In their research, they found the higher tax rate increases tax revenue. But between the tax rate and tax revenue, they talk about a two-way negative relationship. In relation to investments, the tax rate has a negative dependence, which means the higher the corporate tax rate, the more negative for next investments. Conversely, the disadvantage of the business and economic environment may cause a reduction in corporate tax, and in particular the abolition of selective preferences and concessions. That confirmed by Johansson et al. (2008). He points out the selective tax support for small businesses is not very effective at global level and does not lead to total investment growth at all.

3. Methodology

The aim of the article is to create a categorization of EU countries from the perspective of corporate taxation for predetermined segmentation criteria using hierarchical and non-hierarchical clustering methods. The indicators were selected on the theoretical knowledge of Barro (1979), Devereux, Griffith and Klemm (2004), who examined the impact of these factors on corporate income. The reference period for analysis is 2018.

The selected determinants:
- UE = unemployment rate as a proportion of unemployed people aged 15 to 64 in the total population (in %),
- GDP = gross domestic product expressed in current prices (in mil. EUR),
- DB = public debt as a debt-to-GDP ratio (in mil. EUR),
- FDI = foreign direct investment as a ratio of FDI inflow and FDI outflow at current prices (in mil. EUR);
- TR = total tax revenues as total tax revenues from direct and indirect taxes at current prices (in mil. EUR),
- STR = nominal (statutory) income tax rate (in %),

- ETR = effective income tax rate (in %).

Statutory tax rate (STR) includes multiple information, e.g. nominal tax rate, various additional rates, tax breaks and subsidies. The composition of the statutory rate is different due to the diversity of the country's tax systems. The statutory tax rate is an inaccurate indicator for an investor who decides where to place his investment (Szarowska, 2011).
An effective tax rate (ETR) is the proportion of the tax burden to the tax base. Effective corporate tax rates take into account also aspects of tax systems that determine the total amount of taxes effectively paid. That points to how tax competition works (Blechová, 2008). An effective tax rate can be expressed by a tax wedge. The tax wedge represents the difference between profit before and after tax. That is way it is possible to measure the neutrality of the tax system of a country (Kubátová, 2011).

The data used in the analysis are from Eurostat and KPMG databases. From the econometric multidimensional methods was used cluster analysis. Specifically, from the hierarchical clustering methods - Ward method and the method of k-means from non-hierarchical methods. Cluster analysis methods were implemented in econometric and statistical software "R Studio". As a general rule, the greater the similarity within the cluster, the greater the difference between individual clusters (Tan, Steinbach and Kumar, 2006, Norušis, 2011). The analyzes were performed in the R Studio statistical language using the installed rJava, psych, GPArotation, nFactors, cluster and NbClust packages. There was performed standardization / normalization by subtracting the mean value and dividing by the standard deviation to obtain a zero mean and standard deviation unit. Subsequently, we compare the results of analyzes and the deviations, which may be caused by clustering processes.

4. Results and discussion

Hierarchical clustering by Ward method and k-means method

In 2018, the structure and abundance in clusters was varied. The first cluster consisted of Bulgaria and Ireland. The second group was the largest. There were nine countries, namely: Czech Republic, Romania, Slovenia, Lithuania, Latvia, Poland, Hungary, Cyprus and Estonia. In the third cluster: Austria, Denmark, Finland, Netherlands and Sweden. The fourth included Luxembourg, Portugal, Croatia and Slovakia. The fifth cluster includes countries such as Belgium, Spain, Greece and Malta. There are two states in the sixth cluster: Germany and the United Kingdom, and France and Italy in the last seventh cluster.

Figure 1: Dendogram – Ward method
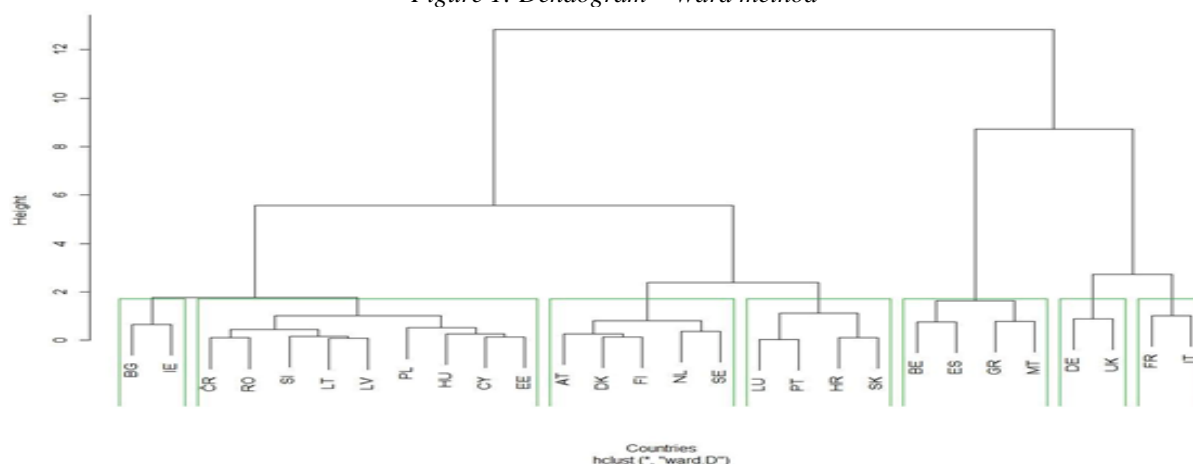
The Ward's method is most commonly used for economic purposes and its results have been confirmed by the other methods. The Figure 1 point to average values of determinants in each cluster. We compare and interpret the results that contains. Even for non-hierarchical methods we have determined the number of clusters seven as well as the hierarchical method. In determining the number of clusters, the subjective viewpoint of the solver also plays a role. None of the clusters analyzed showed signs of overlap. The clustering of countries has not changed using a non-hierarchical method (Figure 2). The integration of countries into individual clusters can be seen in Table 1.

Figure 2: Clusters– K-means method
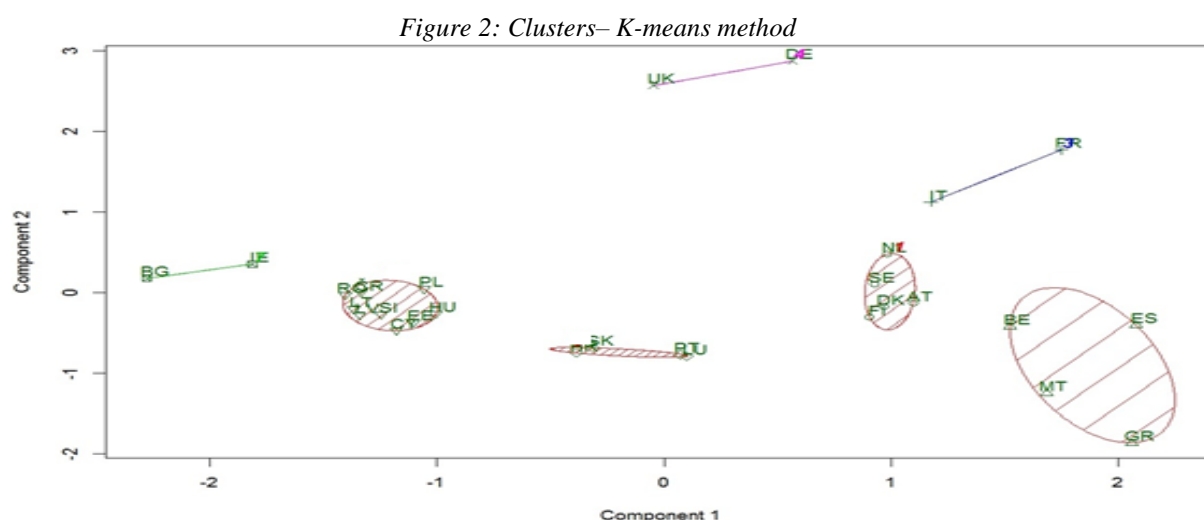
The first cluster is made up of two European countries - Bulgaria and Ireland. In 2018, unemployment in Bulgaria reached 7.7% and 7.9% in Ireland (the EU average was 9.03%). The critical indicator is the percentage of public debt that has reached 102% of GDP. That indicates the countries are suffering from macroeconomic problems and public finance instability. Bulgaria and Ireland have on average the lowest tax rates in all EU countries, which is also reflected in the amount of tax revenues. The average effective tax rate is around 10% and the statutory tax is 11.3%. FDI amounts to 141,592 mil. EUR which is not the lowest value in the clusters.

The second cluster was the largest and was made up of nine countries. These countries (Czech Republic, Romania, Slovenia, Lithuania, Latvia, Poland, Hungary, Cyprus and Estonia) have an average unemployment rate of 7.4%, which is lower than the EU-wide average. The lowest unemployment rate was reported by the Czech Republic (4%) and the highest by the Cyprus (13.3%). The positive indicator is the percentage of public debt - 55% of GDP. In 2018, this value is the lowest among all the clusters and that is in accordance with the Maastricht criteria. The average lowest value of FDI is only 14,359.9 mil. EUR. Average tax revenues were 38,579.1 mil. EUR. This is due to the low tax rates in individual countries, which are below the EU average. The average effective tax rate was 14.8% and the statutory tax rate was 16.9%.

Table 1: Countries in the clusters

Cluster	1.	2.	3.	4.	5.	6.	7.
Country	Bulgaria Ireland	Poland Czech Republic Hungary Lithuania Latvia Slovenia Cypress Estonia Romania	Netherlands Sweden Austria Finland Denmark	Slovakia Croatia Portugal Luxemburg	Belgium Spain Malta Greece	United Kingdom Germany	Italy France

The third cluster consist of five states: Austria, Denmark, Finland, the Netherlands and Sweden. An unmistakable feature that characterizes the cluster is the geographical proximity (except Austria). Average tax rates are above the EU-wide average, which is also reflected in higher tax revenues (166,363.8 mil. EUR). The average effective tax rate was 19.8% and the statutory tax rate was an average of 23%. The percentage of public debt exceeded the set Maastricht criteria (60%). The states showed 74% of GDP.

The fourth cluster grouped countries such as Slovakia, Luxembourg, Poland and Croatia. The highest unemployment was reported by Slovakia (9.7%). The fourth cluster had a relatively low average FDI value (39,191.7 mil. EUR). Average tax revenues amounted to 53,716.7 mil. EUR. The effective tax rate was 21.7% (EU average - 19.7%), is higher than the statutory tax rate, which was 17.8% but lower than the EU average. The public debt of these states is at the level of 57%. Their debt still does not exceed the specified amount.

The fifth cluster consisted of four states: Belgium, Spain, Greece and Malta. Average unemployment is the highest among all clusters. It is caused by Greece; whose unemployment was up to 28.2%. The cluster of these four states also has an enormously high public debt of 114%. Greece reported the highest debt ratio - up to 205% of GDP. Greece fought with serious macroeconomic and political problems and public finance disruption. FDI was on average 184 807.2 mil. EUR. Effective and statutory tax rates have been set high above the EU average. The average effective tax rate in these countries was 27.4% and the statutory tax rate was 31.5%. The highest tax burden is in Malta. Malta had a statutory tax rate of 35% and an effective tax rate of 32.2%.

Table 2: Average values of determinants in clusters

cluster determinant	1. cluster	2. cluster	3. cluster	4. cluster	5. cluster	6. cluster	7. cluster
UE (v %)	7.8	7.4	6.8	10.0	15.2	6.9	11.2
DB (v mil.EUR)	131 493	62 020	246 446	85 748	488 911	2 330 225	2 238 337
GDP (v mil.EUR)	128 484	113 638	330 976	150 969	430 402	2 750 133	1 948 849
FDI (v mil.EUR)	141 592	14 359	246 854	39 191	184 807	810 918	610 104
TR (v mil.EUR)	32 576	38 579	166 363	53 716	111 830	946 915	903 946
ETR (v %)	10.0	14.8	19.8	21.7	27.4	23.2	28.5
STR (v %)	11.3	16.9	23.0	17.8	31.5	19.0	32.4

The sixth cluster included only two states - Germany and the United Kingdom. Germany, as an economically strong country, had a public debt of 75% GDP. Germany along with Great Britain, had the highest tax revenue compared to the other clusters (946,915.9 mil. EUR). These countries did not have the same high tax rates as in the previous cluster. The effective tax rate was on average 23.2% and the statutory tax rate was 19%. The average public debt was 85% of GDP, which violates the Maastricht criteria. Unemployment average was about as low as for the second cluster and also below the EU-wide average.

The last cluster was made up of France and Italy. These two countries together had a relatively high unemployment rate of 11.2%. The seventh cluster showed on average the highest public debt to 115% of GDP, implying a fiscal imbalance in public finances. These two countries have similar economic performance and therefore approximately the same tax burden has been included in one cluster. The tax burden is set above the EU-wide average. The average effective rate was 28.5% and the statutory rate was 32.4%, which is also due to high tax revenues (903,946.5 mil. EUR).

We can say, the founding countries of the EU and the member states that joined the EU among the first (also called Old EU Member States) maintain an effective tax rate at a good level. They try to create the best possible conditions for the business environment. New Member States should improve the fiscal policy to achieve the most favorable conditions for companies. They would lower the effective corporate tax rate, attracting more investors, and their economy would grow at a faster pace, which would also be reflected in the living standards of the population.

5 Conclusion

The effectiveness and neutrality of corporate taxation in EU Member States is still one of the main issues in corporate taxation today. Areas that still remain a problem within the EU are often disparate national reforms and vague attitudes to tax harmonization and corporate taxation. This constant problem has a spatial and regional character and threatens the monetary union. Differences between EU Member States come from different macroeconomic situations of countries, different economic policies and disparate tax legislation. The result of the cluster analysis brought conclusions the highlight the differences in tax rates and other determinants that are part of corporate taxation in each of the countries under review. There are great and still ongoing discussions by experts about developments in the future that point to possible modifications to corporate taxation, but reality clearly shows the deal and consensus are not always achieved.

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Financing of small and medium – sized enterprises from public sources

Katarína Belanová [1]

Abstract

Small and medium-sized enterprises are a key segment of the business sector in the Slovak Republic, as well as the backbone of the economy and the basis for the country's competitiveness. A favorable business environment and the provision of appropriate forms of support is an important prerequisite for the successful development of small and medium-sized enterprises because they usually have limited financial sources and significantly higher sensitivity to the existence of different barriers in business compared to large enterprises. Under the conditions of the Slovak Republic, a number of support measures have been implemented, which direct or indirect aim is to support small and medium-sized enterprises. The aim of the article is to evaluate support programmes for SMEs in the Slovak Republic and to propose measures to improve the support system.

Key words

Small and medium – sized enterprises, public support programmes, bank loans

JEL Classification: G21, G38

1. Introduction

The small and medium-sized business sector is an integral part of any advanced market economy.

The importance of small and medium - sized enterprises (SMEs) in the Slovak economy is characterized by several indicators. Similarly to past years, even in 2017, they represented 99.9% of the total number of business entities in the Slovak economy, employed circa three quarters (73.8%) of the active labour force in the corporate economy, and contributed to the added value generation. The positive development of the national economy has also been reflected in positive impacts on the development of the said indicators, when, as compared to 2016, the number of employees in SMEs increased of 1.4%, the added value of 8.9% and the generated profit of 7.5%. The growth in value added in the SME sector has been most significant over the past seven years [1].

The significance of small and medium-sized enterprises is also recognized by individual EU countries that use a wide range of support measures to help them.

There have been many support measures implemented in the Slovak Republic in order to improve the conditions for doing business of SMEs. Creating new as well as further developing of support tools for entrepreneurs can help to sustain the growth of the economy and promote employment in the long term. However, without evaluating the impact of support measures, it is not possible to assess the degree of fulfillment of the individual priorities and objectives of these instruments.

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This was the reason why we decided to focus on evaluation of effectiveness of public funding for SMEs. The aim of the article is to evaluate support programmes for SMEs in the SR and to propose measures to improve the support system.

2. Literature Review

One of the basic prerequisites for the development of the SME sector is access to finance. Small and medium-sized enterprises, given their distinctive features, have different financial needs compared to large enterprises. Because SMEs are often perceived as relatively risky subjects by the banking sector, they try to obtain resources for the development of their business even from the public sector.

[2] dealt with issues of obtaining funds and attitudes of banks to small and medium-sized enterprises. They have shown that banks play a crucial role in financing SMEs because they have a difficult access to the capital market.

However, small and medium-sized enterprises are usually at a disadvantage compared to large firms in accessing debt financing.

It is because SMEs have different structure of assets compared to large companies – share of their fixed assets to total assets is significantly lower. On the other hand, the share of current liabilities to assets is higher, which indicates their higher financial vulnerability [3].

They tend to have less financial strength; do not have sufficient collateral, which is usually the main reason why banks refuse to provide credit to them and why such businesses obtain it so hard. Smaller businesses and enterprises with a shorter history have only short-term contacts with the banks and therefore pay higher interest rates and the banks require higher guarantees from them [4]. Later, [5] continued in studies of relations between SMEs credit availability and banks' organizational structure. They stated that companies' size and age play an important role in obtaining external finance.

[6] - [9] also reached similar conclusions in their studies.

It is obvious that support for small and medium-sized enterprises (SMEs) is very important. Their functioning and growth helps further with the multiplier effect to support the national economy, which can be tracked through indicators such as Gross Domestic Product (GDP), Gross National Product (GNP) or Unemployment Rate.

The survey, evaluating the impacts and effects of selected support measures on the SMEs themselves is also available. SBA, in [10], confirmed the positive economic impacts on supported SMEs within the framework of the measure 1.1. Innovation and technological transfers of the Operational Programme Competitiveness and Economic Growth. It confirms the importance of the structural funds of the EU for the financing of SMEs. Similarly positive results have also been shown by the recipients of its Micro-loan Programme.

3. Public Support for SMEs

Statistical data show, that from 2015 to 2016, SMEs were mainly supported by EU grants, accounting for more than 25% of the total amount of financial support for SMEs for those years. In 2017, support for SMEs through EU grants fell significantly and accounted for 1.6% of the total financial support for SMEs. The reason for such a decline is a slow start in the use of support programmes under the new 2014-2020 programming period. In 2017, financial support under loans from the budget resources represented 30% of the total amount of support for SMEs.

Table 1: Financial statement of support drawn by SMEs for the years 2015-2017

Form of financing	Relatively (%)		
	2015	**2016**	**2017**
EU grants	28,0	25,3	1,6
Support in the frame of SPP	8,0	1,4	4,9
Subsidies from the state budget	5,8	10,0	19,4
Measures of active labour policy	12,1	19,4	19,3
Loans with the support of EU	18,7	16,1	15,4
Loans from the state budget	21,1	19,8	30,0
Innovation fund	0,1	0,1	0,1
Guarantees	4,9	4,8	7,9
Venture capital	1,3	2,4	0,7
Incentives	0,0	0,7	0,7
Totally	100,0	100,0	100,0

Source: own procession

According to the results of the European Commission (EC) survey on SME access to finance [11], small and medium-sized enterprises in the Slovak Republic are characterized by a low rate of utilization of public grants and subsidised loans. In 2017, only 2.3% of small and medium-sized enterprises were using public grants and subsidised loans in Slovakia. Compared to 2016, the rate of use of public grants or subsidised loans to small and medium-sized enterprises in Slovakia increased year-on-year (from 1.3% in 2016 to 2.3% in 2017), but is still lower than in 2014 and 2015. A substantially higher rate of use of public grants and subsidised loans is achieved in the EU-28 (7.3%) and in the V4 countries (6.7%).

Small and medium-sized enterprises use public grants and subsidised loans mostly in Italy (14.1%), Hungary (12.0%) and Slovenia (11.7%). Slovakia ranks among the countries with the lowest use of public funding sources (2.3%). Only three countries, namely Estonia (1.8%), the Netherlands (1.5%) and Latvia (1.2%) were placed after Slovakia in the comparison.

Only 15.4% of small and medium-sized businesses in Slovakia considered grants and subsidised loans as the relevant source of their funding.

This is confirmed by the results of a further EC survey [12], which evaluated entrepreneurs' attitudes towards corruption. It also states that Slovak business entities are characterized by a low level of use of public support programmes designed to support entrepreneurship.

The results of the survey show that in 2017, some 6.3% of business entities in Slovakia have used some type of public support to promote business in the previous 12 months. A higher rate is achieved in the EU – 28 (8,5%) and in the V4 countries (13,3%).

In order to map the attitudes of small and medium-sized enterprises to the use of support programmes, the Slovak Business Agency conducted a representative quantitative survey of 1,000 SMEs [13].

We value positively the fact that in the period 2015-2018 the number of entrepreneurs who have enough information on the possibilities of using support programmes increased (from 39% in 2015 to 43% in 2018).

Entrepreneurs who acquired or attempted to obtain public support for their business identified the Internet (40%) as the most commonly used source of public support information. Nearly one third of entrepreneurs received information on public support from their friends, acquaintances and business partners (32%) and through a call for proposals (29%). Nearly one quarter (24%) of small and medium-sized enterprises have used the official sites of support institutions. 15% of small and medium-sized entrepreneurs used television

and print media. To a lesser extent, entrepreneurs reported social media (8%) and advertising (6%).

The most significant obstacle to the availability of public support for entrepreneurs was the high administrative burden, the demanding fulfillment of the support conditions or the long evaluation procedures of the applicants.

According to approximately half of the respondents, public sector support should be more strongly geared towards promoting entrepreneurship for start-ups (53%) and micro-enterprises (51%).

Public support was used for employment and and retention of workers (35% of SMEs that received public support). In order to support the implementation of innovation activities, support was provided for 29% of small and medium-sized enterprises. For the introduction of organic products and services, support was primarily used for 13% of respondents. Almost every tenth entrepreneur (9%) took advantage of the support received in the start-up business.

There were positive impacts prevailing at entrepreneurs who received public support for the development of their business. According to more than one third of respondents (35%), the support received helped to overcome short-term business problems. A quarter of small and medium-sized enterprises were helped by the support received to strengthen the company's market position (25%) and stimulated further expansion (24%). 16% of supported firms received the necessary information through the support provided.

Nearly two thirds (61%) of small and medium-sized enterprises in Slovakia plan to use public support for their business in the future. On the contrary, more than one third (39%) of entrepreneurs expressed no interest in future public support for entrepreneurship. More than one fifth (21%) of respondents plan to use the EU contribution/grant in the future. In the future, 18% of SME representatives plan to use the state budget contribution/subsidy. 12% of small and medium-sized enterprises plan to use a tax advantage in the future, a 10% discounted loan or loan, 9% advice and information services, and a 4% guarantee. 6% of small and medium-sized enterprises plan to use other forms of public support in the future.

4. Evaluation of Public Support for SMEs

The development described in the previous part is also in line with the assessment of government support programmes for small and medium-sized enterprises, which we elaborated according to Global Entrepreneurship Monitor data, component National Expert Survey (GEM NES). On a year-on-year basis, the evaluation of support programmes in Slovakia after previous improvement decreased by 0.6 points. The level of evaluation of support programmemes in 2017 was the lowest within the monitored period.

Table 2: Evaluation of public support programmes for SMEs

country	2013	2014	2015	2016	2017	2017/2016 (p.p.)
SR	2.2	2.3	2.3	2.6	2.0	-0.6
EU	2.8	2.8	2.7	2.8	2.8	0.0
V4	2.5	2.5	2.5	2.5	2.3	-0.2

Source: Own eleboration according to GEM NES data
Note: Assessment of government programmes for SMEs is classified on a scale from 1 (worst) to 5 (best).

Evaluation of support programmes for SMEs in V4 countries shows a relatively stable development (around 2.5 points), with a slight decrease in 2017 to 2.3 points. There is also a

relatively constant assessment in the EU countries, which oscillates 2.8 points in the period under review.

Figure 1: Evaluation of government support programmes for SMEs
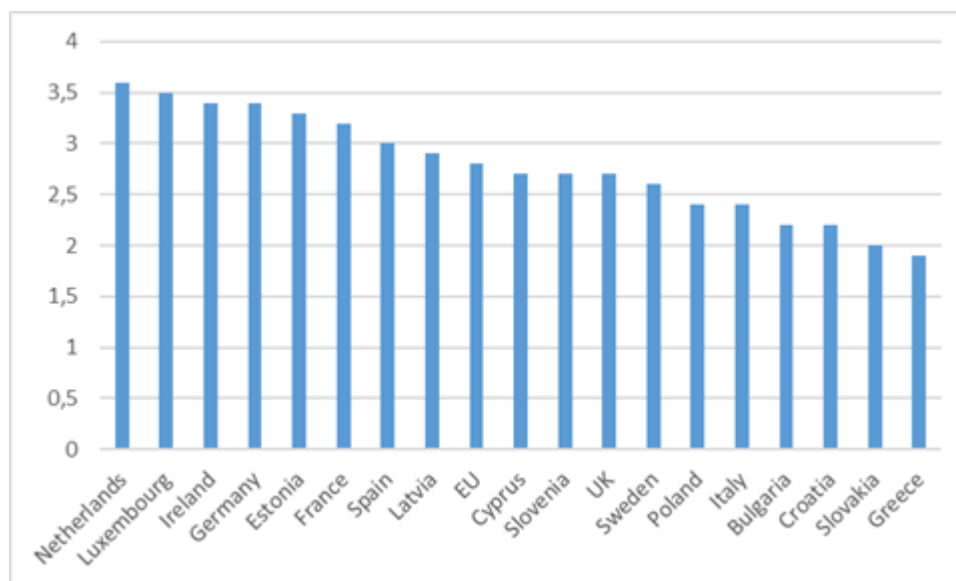
Source: Own eleboration according to GEM NES data

The significant lag of Slovakia in the status and quality of support programmes for small and medium-sized enterprises is also highlighted by the ranking of individual EU countries in the given area. In this comparison, Slovakia is ranked at the penultimate place ahead of Greece, which achieved a score of 1.9 points in 2017. Other V4 countries are also characterized by below-average levels of government support programmes. Countries such as the Netherlands (3.6 points), Luxembourg (3.5 points), Ireland and Germany (3.4 points each) rank best in government support programmes for entrepreneurs.

5. Conclusions

The position of small and medium-sized enterprises in the national economy in terms of job creation, local economy support or balancing of regional development disparities is significant in Slovakia in the long run. Not only macroeconomic development, a stable rate of economic growth, but also a range of support measures implemented within Slovakia's economic policy have had a positive influence on the business conditions of SMEs in recent years.

In order to improve the conditions for the use of support programmes, it is a prerequisite not only to increase the quantity and budget capacity of the support programmes, but also to increase their efficiency and availability for individual target groups. As the SME business support system in Slovakia is rather complex and not very transparent to business people, it is important to strengthen SME awareness at regional and local level. 97% of business entities in Slovakia are micro-enterprises, but up to two-thirds of these entrepreneurs feel a lack of information about the possibilities of using support programmes. Examples of suitable measures in this area include national projects of the National Business Centers of the SBA, which aim to provide entrepreneurs with a comprehensive range of services, including information support on support measures in the Slovak Republic.

As additional measures to improve the use of support programmes and to streamline support delivery, we propose to: reduce time and simplify administrative processes between submitting a grant application until it is approved, simplify and streamline the procurement process, reduce the difficulty of defined measurable indicators, or introduce a "pre-class" evaluation of project objectives. Furthermore, it would be appropriate to simplify the EU grant scheme so that the applicant is able to develop the project self-help, without the need for external consultancy agencies. These measures would increase the attractiveness of support programmes for entrepreneurs and the conditions for the use of support measures.

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Comparison of Insurance Operational Efficiency between the Biggest Life-insurance Company from China and Japan

Biwei Guan[1]

Abstract

Chinese insurance market built a significant contribution to the insurance industry in recent years. However, its insurance supervision system seems not complete if compared with Japan. When an insurance company faces effective supervision, it can often have higher operational efficiency. Thus, this paper selects the biggest life-insurance company from China and Japan to make a comparison of their insurance operational efficiency which measured by solvency and profitability, so simple to infer the supervision efficiency of the two markets. This paper uses SPSS software to complete the factor analysis. From the results, it can infer Nippon Life had better operational than China life before 2013, and later it was reversed. And the supervision efficiency in China is still not perfect.

Key words

Insurance industry, SPSS, Supervision efficiency, Operational efficiency, Factor analysis

JEL Classification: C38, G14, G22

1. Introduction

Allianz SE (2018) reported Chinese market attributed around 80 percent in additional premiums of life insurance in the global market in 2017. Effective supervision of insurance market can help to maintain the stability of insurance security and the development of the whole industry. Although the insurance industry develops is speediest in China, there does have some gap from the developed country.

China and Japan have many similarities in historical tradition, cultural background and so on. Japanese insurance companies have excellent management and operation. The Japanese insurance industry has a long history, and its premium income, insurance density, and insurance depth are among the best in the world. During the development of the Japanese insurance market, many problems and challenges experienced by the Chinese insurance industry are similar. Therefore, it is of great significance for the development of China's insurance industry to study the supervision of Japanese insurance industry and compare the specific situations of the two markets. Ma (2001) stated the core of insurance supervision system is the solvency supervise of an insurance company. Tao and Xu (2012) mention insurance supervision efficiency including two parts: the supervision efficiency of the supervision institutions themselves and the operational efficiency of the insurance companies.

This paper focuses on comparing the operational efficiency from 2010 to 2017 in the Chinese largest life-insurance company and the Japanese largest life-insurance company by using factor analysis, and then infer how well the supervision efficiency of the market. The paper will be processed as follows. The first part writes about the description of factor analysis and information about different markets' supervision mode. Next part will give the results from

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the factor analysis model, then infer the operational efficiency of these two companies. In the last section, will bring to the end of the paper.

2. Insurance Supervision of Different Markets

The main goal of insurance supervision is to make sure the insurance company can have proper operation. Therefore, the performance of the insurance company can prove the supervision is efficiency or not. To assess the insurance operational efficiency, need to consider two aspects: solvency and profitability of the insurance company. For the sake of policyholders, insurance companies should keep their debt at a relatively stable level and be supposed to repay their debt at any time. The business circumstance, which is described by profitability, of the insurance company is a piece of strong evidence that supervision is effective or not, the higher the difference between the insurance company's revenues and costs, the more effective supervision.

2.1 Generally Modes of Insurance Supervisory

UNCTAD (1994) defined three principles to the insurance market to make sure the ideal effectiveness and efficiency of supervision. They are atomicity, transparency, and information. The World Bank (2006) found insurance supervision relies upon the policy, well-developed financial market infrastructure, and efficient financial market. Fu, Zhang, and Liu (2016) found there are four main modes of insurance supervisory, there are decentralized free supervision, highly self-regulated supervision, supervision based on system construction and administrative supervision.

The principle of decentralized free regulation is to supervise the solvency of insurance companies, protect the interests of the insured, and maintain fair competition in the market. This mode is more suitable for federal countries where regional governments have a certain autonomy. The supervisory principles of highly self-regulatory supervision also focus on the solvency supervision of insurance companies and give full play to the role of self-regulatory organizations. The government only intervenes in "solvency standards" and "information disclosure". This mode is more suitable for mature markets with a long history of the insurance industry and relatively loose regulation. The supervision based on system construction gives consideration to safety and efficiency, guarantees the asset safety of insurance companies and protects the interests of customers on the basis of guaranteeing the safe operation of insurance companies. In this mode of supervision, there will be very strict and standard regulations. Administrative supervision is mainly based on market behavior supervision to prevent excessive competition from making the market fluctuate violently. This kind of supervision has strict management over the insurance market, adopting the "centralized and single supervision system", and setting up a special national insurance supervision institution for unified management in accordance with laws and regulations.

There are three main insurance supervision ways in the insurance industry: off-site monitoring and public information disclosure; on-site inspection; and regulatory regulation. For public information disclosure, the supervision institution needs to establish an efficient requirement. In general, the on-site inspection aims to compare the insurance company's risk structure and ability to bear the risk and find any issues that may affect the insurer's ability to take on long-term obligations to policyholders. Regulatory regulation pays attention to the legality of the form of the insurance business and punishes the illegal forms.

2.2 Insurance Supervision of the Chinese Markets

Chinese supervisory mode is a combination of administrative supervision and supervision based on system construction, which is also a "centralized and single regulatory system" and

has very strict management. The financial supervision and administration department of the State Council is responsible for the supervision and management of the insurance industry based on "Insurance Law of the People's Republic". Chinese insurance supervision has strict requirements on insurance business scope and market entry threshold. Peng (2000) mentioned the main basis for the strict supervision of the Chinese insurance market is the prominent information asymmetry, disorderly competition, and undesirable competition. SCNPC (2015) requires the smallest registered capital for the insurance company is 0.2 billion RMB; the basic insurance clauses and premium rates for the main types of commercial insurance shall be settled by the financial supervision and regulation department; if the companies operating is non-life insurance, the amount of the withdrawal and carry-over shall be equal to 50% of the premium retained in the current year and so on. The Chinese insurance market applies on-side inspection as the supervision method. Because this method has more comprehensive and effective supervision of the entire insurance industry, this way, the illegal operators can be multiplied, and the public interest can be better protected. In terms of the content of supervision, China pays more attention to the solvency of insurance companies, and also focuses on the operating behavior of insurance companies.

2.3 Insurance Supervision of the Japanese Markets

Japanese supervisory is administrative supervision due to the smaller territories and less mature market. Japanese insurance supervision pays more attention to the protection of the interests of policyholders, promotes market differentiation and moderate competition, improves the mechanism for insurance institutions to withdraw from the market, and allows insurance companies to go bankrupt. Japan is one of the countries with the most advanced insurance industry, but its supervision system is single, and the supervision department is the ministry of finance. There is a banking bureau under the ministry of finance, and an insurance department under the banking bureau, which is individually responsible for the supervision and administration of insurance. However, due to this single supervision system, although the insurance industry in Japan is very developed, it is not very open to the outside world. The insurance market has always been in the hands of a small number of insurance companies, making it difficult for foreign companies to do business. Supervision of the Japanese insurance industry is focused on the operating behavior of insurance companies. Before the financial crisis, the Japanese government controlled the premium rates very strictly; there almost no pricing competition in the market. However, in the Japanese insurance market, there is a lack of accurate judgment and treatment of solvency ability.

3. Factor Analysis

Abbas and Wasin (2019) define factor analysis a multivariate technique used to describe the relationships between different variables under study with new variables called factors. The core of factor analysis is to analyze several comprehensive indicators and extract common factor. Then build a scoring function by taking the variance contribution rate of each factor as the sum of weights and the score multiplier of the factor. In general, the mathematical representation of factor analysis is a matrix:

$$\boldsymbol{x} = \alpha \boldsymbol{f} + \boldsymbol{\beta} \tag{1}$$

where $k \leq p$; $x = (x_1,\ x_2,\ x_3,...,x_p)'$ is an observable random variable; $f = (f_1,\ f_2,\ f_3...\ f_k)$ is the common factor, the common factors in the expressions of the original observed variables are independent and non-observable theoretical variables; $A(\alpha_{ij})$ is the coefficient of a common factor, it also named factor loading matrix; α_{ij} is the covariance of x_i and f_j, the large absolute value it is, the greater the load f_j for x_i; $\beta(\beta_1,\beta_2,\beta_3...\beta_p)$ is the specific factor, this factor is also unobservable. There are two statistics in factor load matrix A which are very important for the

economic interpretation of factor analysis results; they are the common degree of variables and the variance contribution of common factors. The variable commonality is the sum of squares of elements in line i of factor load A,

$$h_i^2 = \sum_{j=1}^{k} \alpha_{ij}^2 \tag{2}$$

where $i=1, 2, 3, \ldots, p$. It measures the contribution of all common factors to the variance of x_i and reflects the influence of all common factors on the variable x_i. The larger the h_i^2, the greater the dependence of x on each component of f. Take variance on both sides of Formula (1), then the new formula is shown as follows:

$$Var(x_i) = \alpha_{i1}^2 Var(f_1) + \alpha_{i2}^2 Var(f_2) + \cdots + \alpha_{ik}^2 Var(f_k) + Var(\beta_i)$$
$$= \sum_{j=1}^{k} \alpha_{ij}^2 + \sum_{i=1}^{p} \beta_i^2 \tag{3}$$

If the result of h_i^2 is close to $Var(x_i)$, and β_i^2 is very small, the effect of factor analysis is better, and the transformation from original variable space to common factor space is better. The sum of squares of each column element in the factor load matrix is denoted as:

$$g_j^2 = \sum_{i=1}^{p} \alpha_{ij}^2 \tag{4}$$

where $i=1, 2, 3, \ldots, k$. g_j^2 refers to the contribution of the f_j to x, it is an indicator to measure the importance of the common factor. The greater the g_j^2, the stronger effect and influence of f on x. If calculate all the g_j^2, and make it sort by size, then the most influential common factor can be extracted from it.

The four main processes of factor analysis are: first, decide whether the original variables to be analyzed are suitable for factor analysis; then construct factor variables; use rotation to make factor variables more explanatory, and calculate the score of the factor variable. For Bartlett's test, when the significance level is under 0.05, it is suitable for factor analysis. For the second step, principle component analysis is applied. There is one principle to decide the number of common factors: according to the eigenvalue value, in general, several principal components corresponding to eigenvalues greater than one are taken.

4. Application of Factor Analysis

CIRC (2017) found life insurance has the most significant market shares in the insurance industry. This project will research insurance operational efficiency in different periods in the biggest life-insurance company from different markets.

4.1 Input Data

Two types of indicators are chosen as variables. Solvency factors include solvency margin ratio (SMR), recognizing asset-liability ratio (RALR), asset recognition rate (ARR), and premium receivable to provident fund (PRPF). For the profitability, there is the return on equity (ROE), net income interest rate (NIIR) and the ratio of profits to cost (RPC). The period of the data is from 2010 to 2017, Table 1 shows the data from China Life Insurance Company, Table 2 shows the data from Nippon Life Insurance Company.

Table 1: Data from China Life   (%)

	2010	2011	2012	2013	2014	2015	2016	2017
SMR	211.99	170.12	235.58	226.22	294.48	359.02	297.16	277.65
RALR	85.08	87.79	88.25	88.72	87.29	86.82	88.59	88.77
ARR	91.23	92.82	90.73	91.46	89.39	88.39	74.87	75.61
PRPF	8.31	12.04	9.06	12.58	11.12	11.1	11.84	11.9
ROE	16.06	9.56	5.05	11.24	11.49	10.96	6.37	10.07
NIIR	8.70	4.80	2.78	5.90	7.29	4.45	4.66	4.91
RPC	11.79	5.62	2.78	7.46	9.96	9.87	4.54	6.82

Table 2: Data from Nippon Life   (%)

	2010	2011	2012	2013	2014	2015	2016	2017
SMR	1006.0	966.2	567.0	696.4	779.0	930.8	903.7	896.0
RALR	94.9	95.8	95.1	92.8	91.7	88.0	90.1	90.4
ARR	86.0	87.2	86.0	81.3	79.7	73.6	73.4	75.0
PRPF	36.6	34.9	31.7	28.2	22.9	19.3	39.1	37.6
ROE	10.1	11.0	8.9	5.3	6.0	4.1	4.6	4.3
NIIR	85.9	100.3	46.	54.2	55.5	50.0	53.6	58.6
RPC	4.0	3.6	3.4	3.1	4.6	4.5	4.0	4.5

4.2 China Life

SPSS software is used to make the factor analysis of 8-year data of 7 variables of China Life. SPSS extract two common factors, and these two common factors include above 75.445% information. The result from Bartlett's test is 0.009, that means these variables are suitable for factor analysis. Figure 1 shows the total variance explained.

Figure 1: Total variance explained for China Life

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3,788	54,115	54,115	3,788	54,115	54,115	3,494	49,911	49,911
2	1,493	21,331	75,445	1,493	21,331	75,445	1,787	25,535	75,445
3	,987	14,102	89,548						
4	,546	7,801	97,349						
5	,114	1,625	98,975						
6	,070	1,005	99,980						
7	,001	,020	100,000						

Figure 1 shows the cumulative variance contribution rate of component 1 and component 2 is 75.445. The eigenvalue of component 1 is 3.494, the component 2's eigenvalue is 1.787. It is easy to find when the component number is greater than two; the eigenvalue is below one, which means that it can not become the common factors. Thus, there are two principal component factors. Table 3 shows the rotated component matrix.

Table 3: Rotated Component Matrix for China Life.

	Component 1	Component 2
SMR	0.159	0.806
RALR	-0.827	0.388
ARR	0.236	-0.790
PRPF	-0.328	0.566
ROE	0.935	-0.152
NIIR	0.882	-0.132
RPC	0.984	0.057

Table 3 describes the first common factor has a heavy load on ratio of profits to cost and return on equity. Thus, the first factor can be named "profitability factor." The second common factor has a heavy load on solvency margin ratio and asset recognition rate. Thus, the second factor can be named "solvency factor". From SPSS software calculation, the component score coefficient matrix is shown in Table 4.

Table 4: Component Score Coefficient Matrix for China Life.

	Factor 1	Factor 2
SMR	0.160	0.519
RALR	-0.209	0.127
ARR	-0.033	-0.456
PRPF	-0.027	0.305
ROE	0.275	0.033
NIIR	0.261	0.038
RPC	0.319	0.169

The calculation formula of how the get the score of each common factor are:

$$F1 = 0.160*x_1 - 0.209*x_2 - 0.033*x_3 - 0.027*x_4 + 0.275*x_5 + 0.261*x_6 + 0.319*x_7 \tag{5}$$

$$F2= 0.519*x_1+0.127*x_2-0.456*x_3+0.305*x_4+0.033*x_5+0.038*x_6+0.169*x_7 \qquad (6)$$

where x_1-x_7 represent solvency margin ratio, recognize asset-liability ratios, asset recognition rate, return on equity, net income interest rate, premium and accumulation fund ratio and reserve premium ratio.

To calculate the total score, it needs to use the score of each common factor times each common factor's rotation sums of squared loading of variance (shown in Figure 2). The results of the score of each common factor and the total score are shown in Table 5.

Table 5: Score of Each Common Factor for China Life.

	Factor 1	Factor 2	Total
2010	1.74506	-1.11690	0.58578
2011	-0.62099	-1.04394	-0.57651
2012	-1.41181	-1.13732	-0.99506
2013	-0.14242	-0.12551	-0.10313
2014	0.79268	0.33155	0.48030
2015	0.59263	0.83700	0.50952
2016	-0.72244	1.13700	-0.07024
2017	-0.23272	1.11812	0.16936

Table 5 shows how these two common factors influence the efficiency of insurance operational in China Life from 2010 to 2017. The higher the total score it has, the more effective operation in that year of China Life. It is not hard to see from the results that the operational efficiency of China's insurance market was not stable from 2010 to 2017. From 2010 to 2012, there was a significant decline, followed by a significant increase in the following two years. From 2014 to 2015, it was relatively stable, and then there was a small decline and another rise.

4.3 Nippon Life

SPSS abstract two common factors from the 7 variables. The result from Bartlett's test is 0.000, that means these variables are appropriate for factor analysis. Figure 2 shows the total variance explained for Nippon Life Insurance Company.

Figure 2: Total variance explained for Nippon Life

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3,884	55,481	55,481	3,884	55,481	55,481	3,479	49,697	49,697
2	1,854	26,492	81,973	1,854	26,492	81,973	2,259	32,276	81,973
3	,857	12,239	94,212						
4	,336	4,796	99,007						
5	,053	,753	99,761						
6	,015	,221	99,982						
7	,001	,018	100,000						

Figure 2 shows the cumulative variance contribution rate of component 1 and component 2 is 81.973. When the component number is smaller than three; the eigenvalue is above one, that is the reason why there are extract two common factors. The eigenvalue of component 1 is 3.479; component 2's eigenvalue is 2.259. And the rotation sums of squared loading of the variance of component 1 is 49.697%, of component 2 is 32.276%. Table 6 represent the rotated component matrix for Nippon Life Insurance Company.

Table 6: Rotated Component Matrix for Nippon Life.

	Component 1	Component 2
SMR	-0.439	0.869
RALR	0.947	0.287
ARR	0.933	0.247
PRPF	0.243	0.510
ROE	0.809	0.510
NIIR	0.379	0.889
RPC	-0.812	0.225

Table 6 shows the first common factor has a heavy load on recognize asset-liability ratios and asset recognition rate. Thus, the first factor can be named "solvency factor." The second

common factor has a heavy load on net income interest rate, solvency margin ratio and return on equity. Thus, the second factor can be named "profitability factor." From SPSS software calculation, the component score coefficient matrix is shown in Table 7.

Table 7: Component Score Coefficient Matrix for Nippon Life.

	Factor 1	Factor 2
SMR	-0.236	0.469
RALR	0.265	0.032
ARR	0.265	0.014
PRPF	0.019	0.219
ROE	0.196	0.155
NIIR	0.019	0.387
RPC	-0.280	0.200

The results of the score of each common factor and the total score are shown in Table 8.

Table 8: Score of Each Common Factor for Nippon Life.

	Factor 1	Factor 2	Total
2010	0.50010	1.36905	0.69041
2011	0.97707	1.40114	0.93781
2012	1.37308	-1.22845	0.28589
2013	0.63126	-1.11367	-0.04573
2014	-0.39210	-0.42941	-0.33346
2015	-1.36026	-0.37255	-0.79625
2016	-0.78598	0.08677	-0.36260
2017	-0.94318	0.28714	-0.37605

Table 8 shows the final results of operational efficiency score in Nippon Life Insurance Company. The highest score was 0.938 in 2011, the lowest was -0.796 in 2015. From 2011 to 2015, the efficiency score was decreased continuity.

4.4 Comparison between China life and Nippon Life Insurance Company

With the help of factor analysis, the insurance operational efficiency score of China Life and Nippon Life Insurance Company is clear. Figure 3 shows the comparison of these two companies.

Figure 3: Comparison of the insurance operational efficiency
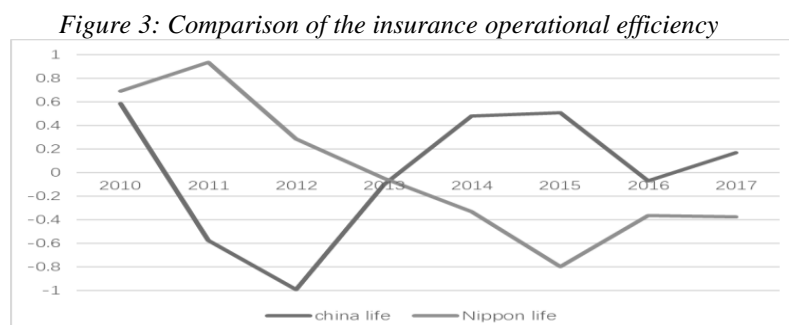
Figure 3 shows the comparison of insurance operational efficiency between China life and Nippon Life Insurance Company from 2010 to 2017. The operational efficiency of Nippon Life Insurance Company is better than China Life from 2010 to 2013, from 2013 to 2017, the situation is opposite. The main reason for this result is the change in solvency. Nippon Life's solvency before 2013 has had a positive impact on the company's operations, and has turned into a negative impact since 2013; China Life is the opposite.

From 2011 to 2013, there are two heavy earthquakes happened in China, that also affected the life-insurance market, the unusual situations let the insurance companies have to prepare more reserves than other times. This sudden and large-scale event has caused China Life's solvency to fail to meet the standards and had a negative impact on operational efficiency. After 2012, in the process of change, the role of the market mechanism is from weak to strong. and the focus of insurance supervision is to prompt change from cultivating the market to regulating the

market, to keep up the supervision and development of the industry. The solvency of insurance companies has also been better regulated, making China Life's solvency a positive impact on operational efficiency, thereby increasing operational efficiency.

5. Conclusion

From the factor analysis, it is known the operational efficiency of China Life had large fluctuation during the past 8 years. As mentioned before, the effective operation is based on the excellent supervision environment. From the comparison of operational efficiency of these two companies, it can infer the supervision efficiency in China is worse than Japan before 2013, and later surpassed it. In recent years, China has a better development of economy than Japan and the insurance market becomes more international nowadays, due to the supervision ways of the Japanese insurance market, it is not easy for foreign companies to get into the Japanese market. That is also the reason why after 2013, China Life has a better performance than Nippon Life Insurance Company.

Chinese insurance supervision is very strict, but from the results of this paper, we can see that the supervisory effect is still not ideal, because it can not keep the insurance company's operation in a stable level. One reason is that overly strict insurance regulation sacrifices market efficiency to protect the interests of policyholders and maintain market stability, but these goals cannot be achieved in practice.

Acknowledgement

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Performance Assessment of companies by Applying Decomposition Multi-Criteria Methods

Martina Borovcová, Zuzana Folvarschi [1]

Abstract

The aim of the article is to determine and verify the key assessment indicators for performance assessment of companies by applying the decomposition multi-attribute methods. Among the assessed specific indicators there are the indicators (return on assets, return on equity, debt-to-assets, reserve ratio, solvency ratio, technical coverage ratio and more). The decomposition multi-criteria AHP method (analytic hierarchy process) and ANP method (analytic network process) based on the Saaty pair comparison approach are described, including the computation procedure. The described methods are then applied to determine the preferences of the indicators for performance assessment of companies. Our findings reveal the resulting preferences of individual indicators of performance assessment of companies and key assessment indicators.

Key words

Performance assessment indicators, decomposition multi-criteria methods, analytic hierarchy proces, analytic network proces.

JEL Classification: C02, C4, G2, G11

1. Introduction

Multi-criteria decision-making is one of options how to choose the optimal variant of a certain sets of variants (Fotr, Dědina, Hrůzová, 2010), (Saaty, 1980). Only very rarely it is possible to find the optimal variant which meets all specified criteria. The solution of decision-making problem is more often a compromise variant, which meets just the most important criteria and does not meet all the specified criteria best. It is preferable to take into account more than one decision-making criterion when making the decision. However there are situations where a single evaluation criteria has been used. Conditions for the quantitative nature of the criteria would then be enough to organize a variant according to the values of the criteria and the variant with the highest or the lowest value would be the best (optimal) option. Still, there are relatively a few decision-making problems with mono-criteria character. More and more frequently it is necessary to deal with problems when the solution variants should be assessed using a larger number of evaluation criteria (Saaty, 2006). Such decision-making problems then have the character of multi-criteria decision-making.

The aim of the article is to determine and verify the key performance assessment indicators of companies on the insurance market by applying the decomposition multi-attribute methods. Among the assessed indicators there are the indicators: return on assets, return on equity, debt-to-assets, reserve ratio, solvency ratio, technical coverage ratio and more. The decomposition multi-criteria AHP method (analytic hierarchy process) and ANP method (analytic network process) based on the Saaty pair comparison approach are described, including the

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computation procedure. The described methods are then applied to determine the preferences of the indicators.

Insurance plays a key role in supporting the economic activity and a sound insurance mechanism makes a critical contribution to the sustainability and growth of the national economy. Few individuals and entities can effectively protect their property ownership and other economic rights without relying on insurance. Insurance companies are also significant institutional investors playing a significant role in providing financing to the real economy through investments in bonds, stocks and other assets (Kwon, Wolfrom, 2016).

The insurance industry is one of the most important sectors of the economy (Arena, 2008), (Pradhan, Arvin, Norman, 2015). The performance assessment of companies in insurance is done by using selected indicators. However, since the importance of the indicators is different, there is a need to identify the key, most important indicators. For this purpose, multi-criteria decision-making methods can be used. Specifically, methods for determination of criteria preference.

2. Methology

The aim of the application of the multi-criteria decision making evaluation of variants is primary finding the best (optimal) variant and layout of these variants from the best to the worst. The best option is usually a variant of the compromise. The compromise solution is the least distant one from the ideal variant, or the furthest away from the variants of basal, while the ideal option is the one that has all the criteria with the best possible value. On the contrary, variant with the worst values of the criteria is the basal variant. Ideal and basal variants are usually hypothetical. If the ideal variant really existed, it would be at the same time, a variant of the optimal solution. However, this situation usually does not occur and therefore any selected solution is the solution to the compromise. Compromise variant must be undominated in all tasks, which means that there is no dominating variant among decision-making variants (Ramík, 1999).

Alternatives are specified by using variants and the measurement of satisfaction depends on each variant. Determination of the criteria is difficult process, which requires certain knowledge of the area. The criteria used to selection of the most appropriate variants can be classified according to several aspect. Firstly it's possible to divide criteria as maximizing (income, profit) or minimizing (cost, loss) according to the level of desirable values. According to the type Secondly it is possible to divide criteria into qualitative and quantitative. These are expressed in the units of measurement.

For calculations and comparison it is usually desirable for specified criteria values yij to be normalized the unit interval, i.e. $x_{ij} \in [0;1]$. Generally, it is possible to obtain these values of the criteria from the sub-functions of the utility (value) as $x_{ij} = u(y_{ij})$. Utility of the criteria, which acquire the worst values is equal to 0 or close to 0, and the utility of the criteria with the best value is equal to 1. Saaty method AHP and ANP will be used in the application part of the study, therefore the following description will be focused on these methods.

The Saaty's method of weights determination of the criteria can be divided into two steps. The first step consists of a pairwise comparison when finding the preferential relations of criteria pairs. It is presented as so-called Saaty's matrix S. This matrix is symmetric with elements sij. It is possible to determine also the size of this preference expressed by a certain number of points from the selected point scale in addition to the direction of the preference of pair of criteria. Scale of relative importance (descriptors) was recommended by Saaty and it is shown in Table 1. Other values can be used to express sub-preferences. The strength of

preferences is expressed in the interval $s_{i,j} \in [0;9]$. The result of this step is to obtain the right upper triangular part of the matrix size preferences (Saaty's matrix). The diagonal element have to be $s_{i,j} = 1$ and for the inverse elements (in the lower left triangular part of matrix) is true the following:

$$s_{i,j} = \frac{1}{s_{j,i}}. \tag{1}$$

The elements $s_{i,j}$ Saaty matrix are estimated shares of weights of criteria v_i and v_j, so:

$$s_{i,j} \cong \frac{v_i}{v_j}. \tag{2}$$

The scales can be obtained in the following manner:

$$\min F = \sum_{i}^{n} \sum_{j}^{n} \left(s_{i,j} - \frac{v_i}{v_j} \right)^2, \tag{3}$$

with the condition $\sum_{i}^{n} v_i = 1$.

Because of difficulty it is possible to obtain the weights using an algorithm based on the geometric average.

$$\min F = \sum_{i=1}^{n} \sum_{j>i}^{n} \left[\ln s_{i,j} - \left(\ln v_i - \ln v_j \right) \right]^2, \tag{4}$$

with the condition $\sum_{i}^{n} v_i = 1$.

The final solution is based on the geometric mean of rows (Saaty, 2010):

$$w_i = \frac{v_i}{\sum_{i}^{N} v_i} = \frac{\left[\prod_{j}^{N} s_{i,j} \right]^{\frac{1}{N}}}{\sum_{i}^{N} \left[\prod_{j}^{N} s_{i,j} \right]^{\frac{1}{N}}}, \tag{5}$$

Table 1: Recommended point of scale with the descriptors by Saaty

The number of points	Descriptor
1	Element A and B are equally important
3	Element A is moderately more important than element B
5	Element A is strongly more important than element B
7	Element A is very strongly more important than element B
9	Element A is extremely more important than element B

Source: Saaty (2006), authors´own processing

The sign of relevant evaluation is the consistency of Saaty's matrix, in other words when the elements satisfy the condition of transitivity the most. It should be emphasized that in many methods this aspect is not accounted. Consistency can be measured using the coefficient of consistency CR (Consistency Ratio). The coefficient for consistent evaluation should be

$CR \leq 0,1$ (Saaty, 2012). Consistency ratio is calculated as following $CR = \dfrac{CI}{RI}$, where

$CI = \dfrac{\lambda_{\max} - N}{N - 1}$, (Saaty, 2010), (Zmeškal, 2009). The characteristic number of the matrix $\lambda_{\max}$

can be determined by various procedures. One option is $\lambda_{\max} = \dfrac{1}{N} \sum\limits_{i}^{N} (S \cdot w)_i / w_i$, while w is a

vector and $(S \cdot w)_i$ is the i-th element of the vector. Furtherly RI (Random Index) is derived from an empirical examination and reaches the following values depending on the number of criteria, see in Table 2.

Table 2: The value RI according to the number of criteria

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Source: Saaty (2009), authors´ own processing

2.1   Multi-attribute methods AHP and ANP

Weights or values of criteria are in the case of decomposition tasks set by gradual decomposition from the goal, global groups of criteria, sub-groups, to the the initial sub-criteria and variants. For AHP method these linkages may be linear and for ANP method in the shape of a pyramid or nonlinear with feedbacks. Evaluation of preferences (weights) of the criteria is carried out using the Saaty's method of pairwise comparison.

Local weights (preferences) of the subgroups or indicators with regard to the specified purpose are determined by using Saaty's method of pairwise comparison. The next step is calculation of the global weights including the initial sub-weights. The sum of all sub-weights is equal to one.

In AHP method can be used analytical procedure and also method of supermatrix. In the ANP, it is possible to calculate global weights by using only the method of supermatrix (Saaty, 2010).

For analytical method AHP the indicator subgroup weights are obtained as follows, $w_{i,j} = v_i \cdot v_{i,j}$ where $w_{i,j}$ is global weight of j-th indicator and i-th group, v_i is local weight of i-th group and $v_{i,j}$ is local weight of j-th indicator and i-th group. By this way we can gradually get all the global weights of primary indicators (Saaty, 2012).

The procedure for the calculation of sought weights in case of AHP and ANP supermatrix method can be divided into three steps (Zmeškal, Dluhošová, Tichý, 2013):

- First step is determination of default supermatrix W. The local weights $v_{i,j}$ are typed to the columns inside this supermatrix W. The weights of criteria are highlighted from $e_{2,1}$ to $e_{2,n2}$ according to the purpose (criteria) $e_{1,2}$.

- Subsequently the default supermatrix is transformed into the weighted supermatrix $\overline{W}$ so, that sums of columns are equal 1.

- The last step is the calculation of limit (final) supermatrix $\overline{W}^{\infty}$. This supermatrix can be calculated like acyclical weighted matrix as following $\overline{W}^{\infty} = \lim\limits_{k \to \infty} \overline{W}^{k}$, where $\overline{W}^{\infty}$ is limit (final) supermatrix, $\overline{W}^{k}$ is weighted supermatrix without existence cycle, and this supermatrix is k times amplified. Global weights are found in the first column considering the goal.

2.2   Data

The database for verification and performance assessment of companies in insurance consists of indicators in the period from 2006 to 2015 (ČAP, 2006-2015). The mentioned indicators are used for example by the Czech Insurance Association and OECD for evaluation of the insurance market level.

Standard indicators of performance assessment of companies (SIC), insurance companies performance evaluation indicators (ICI) and insurance market evaluation indicators (IMI) are taken into account for the purposes of the article. The following indicators are included among standard indicators of performance assessment of companies according to the subjective opinion of an expert: return on assets (ROA), return on equity (ROE), debt-to-assets (DTA), equity-to-assets (ETA). The following indicators are included among insurance companies performance evaluation indicators (Ducháčková, 2015), (Vávrová, 2014): claims ratio (CLR), solvency ratio (SOR), technical coverage ratio (TCR), reserve ratio (RER), reserve-to-equity (RTE), financial placement ratio (FPR), financial placement-to-assets (FTA) and assets leverage (ASL). The following indicators are included among insurance market evaluation indicators (Wooldridge, 2013): gross premium (GRP), net premium (NEP), number of insurance contracts (NIC).

3. Results

The goal is to determine the weights and preferences of individual indicators of the performance assessment of insurance companies by using both AHP method and ANP method and to select the key indicators.

The first step in the performance assessment of insurance companies is selection of the indicators. The selected indicators are used for the performance assessment of insurance companies and insurance market by the Czech Insurance Association and OECD. Next, the preferences of indicators are set, according to subjective approach, using Saaty´s method of multi-criteria decision making based on pairwise comparison.

Table 3: Preferences of individual indicators of the performance assessment of insurance companies

Goal	Local	Groups	Global - analytical method	Global - supermatrix method	
			AHP	AHP	ANP
SIC	56.00 %				
ICI	32.00 %	0			
IMI	12.00 %				
ROE	49.40 %		27.59 %	27.59 %	20.51 %
ROA	30.70 %	56.00 %	17.17 %	17.17 %	12.76 %
DTA	10.60 %		5.89 %	5.89 %	4.38 %
ETA	9.30 %		5.19 %	5.19 %	3.86 %
CLR	32.80 %		10.48 %	10.48 %	11.61 %
SOR	23.20 %		7.41 %	7.41 %	8.21 %
TCR	15.90 %		5.07 %	5.07 %	5.61 %
RER	10.70 %	32.00 %	3.41 %	3.41 %	3.77 %
RTE	7.10 %		2.28 %	2.28 %	2.52 %
FPR	4.80 %		1.53 %	1.53 %	1.70 %
FTA	3.30 %		1.05 %	1.05 %	1.16 %
ASL	2.30 %		0.74 %	0.74 %	0.82 %
GRP	63.70 %		7.77 %	7.77 %	14.70 %
NEP	25.80 %	12.00 %	3.15 %	3.15 %	5.96 %
NIC	10.50 %		1.28 %	1.28 %	2.42 %
$\sum$		100.00 %	100.00 %	100.00 %	100.00 %

In the case of standard indicators of performance assessment of companies view, the main influence on the insurance market level assessment was found in ROE indicator. The local weight was 49.40 %. According to the insurance companies performance evaluation indicators indicators, the highest local strength was found out in claims ratio indicator, where the influence strength was 32.80 %. In the case of insurance market evaluation indicators view, the main influence was found in gross premium indicator. The local weight was 63.70 %.

The next step was focused on calculation of the global weights according to the AHP and ANP methods. In the case of the AHP method, two ways of calculation were used the analytical procedure and the method of supermatrix. The strongest influence was found in ROE indicator, the strength was 27.59 %. According to the ANP method, the results were slightly different due to the feedback between subgroups. The order of preferences of the indicators remain almost the same, the values of the preferences are slightly different. E.g. for the above mentioned ROE, the weight was currently calculated at 20.51 %.

Table 4: Final order of indicators

Indicators	Order (AHP)	Order (ANP)
ROE	1.	1.
ROA	2.	3.
CLR	3.	4.
GRP	4.	2.
SOR	5.	5.
DTA	6.	8.
ETA	7.	9.
TCR	8.	7.
RER	9.	10.
NEP	10.	6.
RTE	11.	11.
FPR	12.	13.
NIC	13.	12.
FTA	14.	14.
ASL	15.	15.

Source: Authors´own processing

Table 4 gives an overview of the final ranking of the indicators identified by all applied methods. The order of preference of the indicators according to the AHP and ANP methods is almost identical. They differ slightly in the values of preferences, which reflects the consideration of network links between the indicators and the groups of indicators by the ANP method. The AHP method does not take the network links into account.

Table 4 also shows the range of key indicators (order 1 to 5), the band of indicators with a medium level of preference (order 6 to 10) and the band of indicators with a low level of preference (order 11 to 15). Thus, the group of key indicators for the performance assessment of companies in insurance would consist of ROE, ROA, claims ratio, gross premiums and solvency ratio.

4. Conclusions

The aim of this article was to determine the preferences of indicators of performance assessment of companies in insurance using multi-attribute AHP and ANP methods on the basis of Saaty's method of paired comparison and to select the key indicators.

The multi-attribute decomposition AHP and ANP methods on the basis of Saaty's method of paired comparison were described. Then the analytical method and supermatrix were used. It was found that both approaches to AHP and ANP methods lead to the almost same results regarding the order of preferences of indicators.

The most important indicators that can be described as the key indicators, whose preference is high, above 7%, are ROE 27.59% (20.51%), ROA 17.70% (12.76%), claims ratio 10.48% (11.61%), gross premiums 7.77% (14.70%) and solvency ratio 7.41% (8.21%). The indicated percentages were determined by the ANP method (by the ANP method). The indicators of medium preference, with a preference rate of more than 3%, include the indicator of debt-to-assets, the equity-to-assets, the technical coverage ratio, the reserve ratio and the net premium. The category of low-preferred indicators with a preference rate of less than 3% consist of the indicator of the reserve-to-equity, the financial placement ratio, the financial placement-to-assets, the assets leverage and the number of insurance contracts.

Acknowledgments

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Pension (participant) fund selection via multi-criteria evaluation method

Adam Borovička [1]

Abstract

Almost half of Czech Republic citizens have pension insurance, or supplementary pension savings. And more are coming. Then the crucial question is what pension fund (from eight in the Czech Republic) to choose. To solve this problem, the quantitative multi-criteria evaluation procedure is proposed. This flexible decision-making approach based on multi-criteria evaluation method enables to include more or less significant product's characteristics (return, risk, cost, etc.). To make a complex analysis, the various clients' strategies can be considered – savings only in conservative participant fund (early as pension insurance), or making a portfolio from a few participant funds to potentially gain a higher return. Often simplifying one-criterion analysis (according to the return or risk) aptly complements the multi-criteria perspective. Then the results can be complexly analysed. Respectively, the most appropriate fund is chosen for each strategy.

Key words

Multi-criteria evaluation, pension (participant) fund, selection, supplementary pension savings, weight

JEL Classification: C44, G11

1. Introduction

Nearly half of people in the Czech Republic have pension insurance (not already offered), or supplementary pension savings. This saving product is still very popular due to its favourable characteristics, i.e. interesting (expected) return (supported by the state contribution) with a relatively lower level of risk. In our country, there are eight companies providing this product, own and manage the pension fund, respectively. Of course, there are the strongest 'traditional' banks on the Czech market (Česká spořitelna, Komerční banka, or ČSOB). The clients of these banks usually demand the product of their 'home' bank because it is especially the easiest way. However, over the past few years in the Czech Republic, many people (more than two million) moved to the emerging banks (as Air Bank, FIO bank, mBank, etc.) which don't provide this product. In this case, deciding on a suitable pension fund becomes more open and interesting.

Thus, how to properly select pension fund, or supplementary pension savings? The clients are often focused on pension fund (expected) return, sometimes implicitly (separately) supplemented with the risk. Even sometimes, the only criterion is a state contribution, which leads to the selection of the least risky fund. However, this approach can be inadequately simplistic. It doesn't simultaneously consider (next to the return) significant characteristics as the risk related to a return variability (instability), or cost connected with the deposits and their management. It is obvious that these characteristics are not so pleasant, and the companies don't talk much about them. Nevertheless, to make a complex decision, these characteristics (criteria) should be included. Therefore, I introduce a multi-criteria evaluation method (procedure) despite its low use in this area, only in (Alptekin, 2009) or (Voronova, 2011). With the support of these analyses, I am convinced of significant usability of the multi-criteria evaluation method in this field. This non-traditional decision-making tool applied in this area can include all

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mentioned (and other) criteria simultaneously. Moreover, a various importance of these criteria can be determined (according to the client's preferences). Other advantage of such a complex procedure is user-friendliness. Its algorithm is comprehensible for a wider range of users. This method not only provides 'the best' alternative but can also rank them. From my perspective, almost all requirements are fulfilled by TOPSIS method using a minimization of the distance from the ideal alternative principle. Methods based on the evaluation principle of a preference relation are not suitable because they cannot provide a full ranking of alternatives. Moreover, they often require mostly hardly accessible additional information in the form of threshold value(s). Methods using the utility function sometimes suffer just from this function whose construction can be very hard. Many of these methods calculate the utility through some additional simplified formulas which often distort the original input data and their proportion relations. TOPSIS method also has some smaller drawback. In the spirit of the applied Euclidian metric, the normalization technique for the criteria values tends to distort the data. To eliminate this disadvantage, a simpler technique is integrated to the algorithm. Then a distance from the ideal and basal alternative is measured by a linear metric. Now the algorithm is more understandable and efficient.

Thus, the main aim of this article is to propose a complex supporting tool for a selection of the most suitable pension fund, or supplementary pension savings. This methodological approach provides a one-criterion/multi-criteria selection in various situations (various types of clients, or attitude toward the risk) on the market with pension funds. The second aim is to demonstrate its application power in a real case on the pension market in the Czech Republic. In this real-life analysis, a few typical saving strategies are specified. Still the most often strategy is a savings in only (compulsory) conservative participant fund (in terms of the pension fund). However, a possibility for potentially higher (expected) return leads to making a portfolio from other participant funds (dynamic, etc.). Multi-criteria evaluation (under client's preferences about the criteria importance) can may be suitably supplemented by a single criterion analysis in order to provide a representative view of a pension fund selection process.

The structure of the article is as follows. The introduction is followed by section of supplementary pension savings, or pension fund and their characteristics in the Czech Republic. Section 3 presents a methodology for pension fund selection based on the multi-criteria evaluation method. In Section 4, a selection of the most suitable pension fund on the Czech market is performed. In conclusion, the article is summarized and some ideas for future research are outlined.

2. Pension funds and their characteristics

In the Czech Republic, there are eight pension companies owning and managing the pension fund. Three companies are managed by the 'traditional' banks Česká spořitelna (abbreviated PSČS – Česká spořitelna pension company), ČSOB and Komerční banka (KB). Four pension companies are operated by the insurance companies Allianz, Axa, NN and Česká pojišťovna (PSČP – Česká pojišťovna pension company). The last one is owned by Conseq Investment Management (Peníze.cz, 2019). Until November 2012, a *pension insurance* was provided. This product was based on the state contribution whose amount is affected by a deposit amount. A personal (month) deposit could be changed. Even an employer could contribute to this product. Moreover, the tax base can be reduced by up to CZK 24 000. At the end of 2012, all deposits were transferred from the pension insurances to the transformation funds under the same conditions. From the beginning of the year 2013, a new product called *supplementary pension savings* have been offered under similar conditions to the pension insurance. Beyond that, a pension fund is consisted from a few so-called *participant funds* that differ in a return-risk profile determined by their investment portfolio (stocks, bonds, treasury bonds, etc.). Half

of the companies manage three participant funds. Each of them has *compulsory conservative fund*. The other two funds are mostly called *balanced* and *dynamic fund*. A second half of the companies still has a fourth fund (ČSOB – *guaranteed fund*, KB – *saving conservative fund*, PSČP – *saving fund*, PSČS – *ethical fund*). The client can choose in which shares he/she will deposit his/her money in these funds. It is possible to say that the 100% deposit into the compulsory conservative fund corresponds with then 'pension insurance' strategy.

The popularity of this product lies mainly in the generous state contribution. The amount of a state contribution is common to all. It is based on the deposit amount. However, there are other very interesting (and more or less important) characteristics. The first one is the (capital expected) *return* which is determined by a change of the funds' property. This performance rate can be identified for every participant fund, for the entire pension fund as well. Other characteristic could be the *risk* representing a return instability in time. It is usually presented on the scoring scale based on the historical variability of returns (measured via a standard deviation of them). The last one is the *cost* that usually represents the fees for asset management and appreciation of the fund property.

3. Methodology for a pension fund selection

Selection of the suitable pension fund is very important financial decision. In the case of the clients of big aforementioned banks or insurance companies, a decision-making can become easy. These clients usually use the offer of a 'home' bank, or insurance company. Maybe such an 'automatic' decision may not be the most suitable. On the other side, it is very comfortable for the client. Further, several million Czechs are the clients of the banks or insurance companies not providing this product. Then a selection of the pension fund potentially becomes more complicated.

My personal experience suggests that (expected) return is often the only criterion (sometimes implicitly supplemented with a risk) in a decision-making. And more, sometimes only the state contribution makes a decision, which leads to a selection of the least risky compulsory conservative fund. This approach is unnecessarily short-sighted. I propose a multi-criteria procedure taking into account all significant criteria simultaneously in order to make a complex decision. Besides (expected) return, the criteria risk and cost should play a significant role in a pension fund selection. Thus, these criteria actually act negatively on the clients. That is the main reason why the employees of the banks or insurance companies do not speak much about them with the clients. The importance of the criteria would be adjustable according to the various client's preferences and strategy. A mechanism of a determination of the criteria importance is integrated to the proposed multi-criteria decision-making procedure.

To include the most real-life cases, I specify the following most frequent saving situations, or types of the client. The first one can be called *conservative* that include the savings only in compulsory conservative participant fund. This strategy actually corresponds with a former product pension insurance that the most people (in the Czech Republic) still have in the transformed form. The supplementary pension savings serves for a main financial support for his/her pension age. Then the savings must be safe. Other type of the client can be specified as a *risk-averse*. His/her purpose of the supplementary pension savings is similar to the previous client. However, s/he is a little bolder. S/he wants to gain some 'additional' return. This client wants to save his/her free financial resources to a wider portfolio of the participant funds, not only in the 'low-risky' compulsory fund, respectively. His/her attitude toward the risk is 'average'. After all the risk of savings is more important that the return. Potentially higher returns are sacrificed at a lower risk level. Cost also play a significant role, but far less than the two most important criteria. The last client can be called *return-seeking*. The supplementary pension insurance is rather complementary to several other financial resources in old age. The

return is clearly the most important criterion. S/he is willing to take a higher risk to gain potentially higher level of (expected) return. Cost is mostly overlooked.

For the purpose of a comprehensive analysis, I specify three strategies. *Strategy I* is presented for a conservative client. So only the compulsory conservative participant funds are evaluated. Some new clients still make decisions only based on 'risk-free' state contribution. Then they usually choose the least risky compulsory conservative fund. If there are multiple funds with the same minimal risk, another distinguishing criterion can be used (other risk measure as standard deviation, return, etc.). This approach is valid for all strategies. On the other side, because the risk of all these funds is low, some clients prefer the compulsory fund with the highest return. Then the best fund according to the two most important characteristics (risk and return) is selected. Further, the cost can also play some smaller role in this decision-making process. Such a one-criterion perspective may ill-advisedly discriminate more interesting solution. Multi-criteria analysis provides a more complex (open) selection of the most suitable fund for this strategy. It is obvious, the risk, or cost has the highest, or lowest importance. The risk aversion rate of the conservative client can be regulated by the setting criteria weights (see the algorithm of the proposed multi-criteria method below). For the other two clients, the following two strategies are specified. *Strategy II* (as Strategy I) focuses only on one group of the participant funds. For a risk-averse, or return-seeking client, the participant funds with 'middle', or 'higher' risk are chosen. The funds with 'middle' risk are usually called *balanced*. They are situated between the compulsory funds (with the lowest level of risk) and the funds with 'higher' return (usually called *dynamic*). If there are two funds with 'middle' risk, the fund with a higher return is preferred. This distribution is proposed on the basis of a general fact that a riskier asset potentially generate a higher return. In both groups, the fund with the lowest risk and the highest return is selected. The one-criterion analysis is again supplemented by a more complex multi-criteria perspective. The risk criterion has a slightly higher weight than return for a risk-averse client. In the case of a return-seeking client, unambiguously the largest weight is assigned to the return. The strategy thus offers both the selection of a suitable fund from the point of view of the most important criterion for both clients and a more complex multi-criteria view with the possibility of a "finer" specification of preferences. Because the client can want to save his/her free financial resources into more participant funds, from which the portfolio is made, then the pension funds must be analyzed as a whole. For this purpose, the 'summary' characteristics for each fund are specified in terms of the *Strategy III*. Return, risk and cost of the pension fund are calculated as average across its all participant funds. Then the pension fund with the highest return (for a return-seeking client), or lowest risk (for a risk-averse client) can be selected. If the client is able to express multi-criteria preferences, then a multi-criteria evaluation process can again be very beneficial. As in the previous strategies, the weights of criteria can take into account the client's preferences. Investigating the effect of changing these weights (expressed the importance of criteria) for different cases can be very beneficial for a real decision-making with pension funds.

3.1 Multi-criteria evaluation method

Multi-criteria evaluation methods can be divided into three groups according to an evaluation principle. The first one is represented by methods based on the utility function, or using the weighted sum approach. The main representatives are WSA (Fishburn, 1967), AHP (Saaty, 1977) or ANP (Saaty, 1996) methods. They provide a full ranking of alternatives. The problem can be a construction of the utility function because it is potentially very difficult for a decision maker. These methods usually use some 'automatic' (transformation/normalization) procedure that can distort (more or less) the original data. The second group includes multi-criteria methods using an evaluation according to the distance from the ideal alternative. TOPSIS method (Hwang and Yoon, 1981) and their modifications (Kahraman, 2008), or

VIKOR method (Opricovic and Tzeng, 2004) provide also full ranking of alternatives. The main question is a choice of the metric for a distance measurement because this selection can significantly affect the results. The last group is represented by the methods working with the preference relation. We know many such methods – AGREPREF (Lagrèze, 1975), class of methods ELECTRE (e.g. Roy, 1968), or PROMETHEE (e.g. Brans, 1982). These methods can divide the alternatives into effective and ineffective, or into some indifference classes. Some of them do not enable determine the relation among all alternatives. Moreover, many of them require some threshold value(s) which can be difficult to obtain from a decision maker. Although these older well-known methods have recently undergone various modifications (stochastic, fuzzy form, etc.), they are still very well applicable in many real decision-making problems.

For a multi-criteria analysis in this article, the last group of methods is not appropriate mainly because of the form of the results. The client needs to know the most suitable pension fund, or their ranking in case of a selecting more than one fund. The pitfall of the first group of methods can be a construction of the utility function. In addition, from my point of view, the notion of utility is so abstract. Thus, its determination can be very complicated. Further, basic utility function method WSA can discriminate some of non-dominated alternatives. These specified drawbacks of two groups lead a choice to the second group. These methods offer a full ranking of alternatives. They don't require any additional information from a decision maker. Then TOPSIS method could be a suitable candidate. However, some partial ideas of this algorithm are also for though. Then the modified algorithm is proposed in the following form:

<u>Step 1:</u> Formulate the criterial matrix $\mathbf{Y} = (y_{ij})$, where $y_{ij}, i = 1, 2, ..., p, j = 1, 2, ..., k$, represents the evaluation of the i-th alternative according to the j-th criterion. To be comparable, the data must be normalized. The original normalization technique significantly distorts a proportion among the original input values. The relative relations among criteria values are changing. To eliminate these drawbacks, the following technique for maximizing, or minimizing criterion is integrated into this algorithm

$$y_{ij}' = \frac{y_{ij}}{\max_i(y_{ij})}, \text{or} \quad y_{ij}' = \frac{\min_i(y_{ij})}{y_{ij}} \qquad i = 1, 2, ..., n; \; j = 1, 2, ..., k \, .$$

It should be noted that this technique requires non-negative criterion values. If they are not, an adequate constant must be added. The importance of the criteria is specified by the vector of their weights $\mathbf{w} = (w_1, w_2, ..., w_k)^T$. The weights can be determined by a well-known user-friendly scoring method (Fiala, 2013).

<u>Step 2:</u> Friendly change in the normalization concept predetermines a change in an applied distance measurement. Now the virtually unfounded Euclidean metric is being replaced by a 'standard' linear metric. Thus, the ideal $I = (I_1, I_2, ..., I_k)$, or basal $B = (B_1, B_2, ..., B_k)$ alternatives are calculated by means of the following formulas

$$I_j = \max_i(y_{ij}'), \text{or} \quad B_j = \min_i(y_{ij}') \qquad j = 1, 2, ..., k \, .$$

As you can see, the normalized weighted criteria matrix is not calculated because it is unnecessary. The distance of the i-th alternative from the ideal, or basal alternative can be immediately calculated via a linear metric as follows

$$d_i^+ = \sum_{j=1}^{k} w_j(I_j - y_{ij}'), \text{or} \quad d_i^- = \sum_{j=1}^{k} w_j(y_{ij}' - B_j) \qquad i = 1, 2, ..., p \, .$$

<u>Step 3:</u> The last step is the same as in the original TOPSIS algorithm. The following relative indicator of distance from the basal alternative for each alternative i is calculated as

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \qquad i = 1, 2, \dots, p \,. \tag{1}$$

'The best' alternative reaches the highest value of this indicator. To make a ranking, the alternatives must be sorted in descending order of this indicator.

4. Pension (participant) fund selection on the Czech market

The supplementary pension insurance is offered since the year 2013 in the Czech Republic. Therefore, the data of pension funds are being tracked since this year. The return is calculated as average yearly from 2013 to 2019. The risk is determined via the integer evaluation from the interval $\langle 1, 5 \rangle$ which is derived from the return variability from the last five years (measured by a standard deviation). The cost includes the fees for asset management (percentage of fund property) and appreciation of the fund property (percentage of possible return). The data about values of the pension units, risk and fees are downloaded from the internal documents of the particular companies accessible from their web sites. Of course, the returns are calculated from values of the pension units. All data are shown in Table 1. It should be noted that the ethic fund of PSČS is excluded because of its very short history.

Table 1: Pension fund data

Company	Fund	Return [%]	Risk [point]	Fees [%]
Allianz	*Compulsory cons.*	0.43	3	10.4
	Balanced	0.81	4	15.8
	Dynamic	1.37	5	16
AXA	*Compulsory cons.*	0.26	2	10.4
	Bond	0.35	2	16
	Balanced	0.05	4	16
CONSEQ	*Compulsory cons.*	0.01	1	10.4
	Bond	1.58	2	16
	Global stock	7.21	5	16
ČSOB	*Compulsory cons.*	0.48	3	10.4
	Guaranteed	0.04	3	16
	Balanced	1.33	4	16
	Dynamic	2.12	5	16
KB	*Compulsory cons.*	-0.12	2	10.4
	Saving cons.	-1.18	3	16
	Balanced	0.37	3	16
	Dynamic	1.58	4	16
NN	*Compulsory cons.*	0.04	2	0.4
	Balanced	-0.99	3	15.8
	Dynamic	4.72	5	16
PSČP	*Compulsory cons.*	0.58	1	10.4
	Saving	0.97	2	16
	Balanced	1.54	4	16
	Dynamic	1.74	4	16
PSČS	*Compulsory cons.*	0.30	2	10.4
	Balanced	1.68	3	16
	Dynamic	2.63	5	16

4.1 Strategy I – analyses and results

In terms of this strategy, the conservative client is selecting the best compulsory participant fund from the perspective of its return. As we can see in Table 1, the best one is a fund from PSČP followed by ČSOB and Allianz. The only fund with a negative average (expected) return is from KB. For a stubborn risk averse investor, a less risky conservative fund (distinguished via a standard deviation of return) is chosen. It is CONSEQ fund. It is obvious that multi-criteria analysis cannot significantly change the results. The PSČP fund is clearly 'the best'. It is actually unique compulsory conservative participant fund thanks its best value of two most important criteria – risk (at the level 1, from the perspective of a standard deviation of returns slightly worse than CONSEQ fund) and return. If the client is very careful, weights of criteria can be set as follows (via easy applicable scoring method as mentioned above): return – 0.35, risk – 0.5, cost – 0.15. Then the ranking is the following: PSPČ (indicator (1) = 0.825), CONSEQ (0.480), NN (0.371), ČSOB (0.363), PSČS (0.355), Allianz (0.334), AXA (0.329) and KB (0.101). If the client would prefer return over risk, then the ČSOB fund will be placed in the second place despite a higher level of risk due to its second highest return. The negative score of the dominated alternative KB fund is deepening. Finally, the NN fund would have a chance to win with a very significant importance of cost. However, this cannot be expected in a real decision-making situation.

4.2 Strategy II – analyses and results

In this strategy, the most suitable fund (from the perspective of risk and return) with 'middle' risk, or 'higher' risk is selected for risk-averse, or return-seeking client. For a risk-averse strategy, the following funds have the smallest risk (at the value of 2): bond fund AXA, bond fund CONSEQ and saving fund PSČP. According to the additional distinguishing 'risk' criterion – standard deviation of returns, these participant funds can be further evaluated. Then the most suitable fund (with the lowest level of this indicator) is the bond fund PSČP. The highest return is represented by the balanced fund PSČS. Only slightly lower return is reported by the CONSEQ fund. This fact is remarkable because this fund has the smallest risk (and the highest measured by standard deviation from three funds with the risk equal 2). Only one fund even reports a negative return (NN balanced fund). For a return-seeking client, the global stock participant fund CONSEQ clearly reports the highest return (7.21%). With the great distance to others, the second one is dynamic NN fund. The worst fund is from AXA reporting virtually zero return. From the perspective of a risk level, the best funds (with level of 4) are balanced AXA, dynamic KB and PSČP. Moreover, the AXA fund has the smallest standard deviation of returns which confirms its lowest riskiness.

The results of a multi-criteria evaluation are as follows. The preferences of the risk-averse client can be expressed by the following weights: return – 0.35, risk – 0.5 and cost – 0.15. As expected, the most suitable participant fund with 'middle' risk is from CONSEQ which has the lowest level of risk and the second best return. Then its position is almost unshakable (for various level of criteria weights). On the second place, there is a saving participant fund PSČP. Besides its lowest risk, it also provides a solid return. Sovereignly worst participant fund is managed by NN company. The only one fund reports a negative return, with the middle level of risk. The second worst fund is balanced Allianz fund because of its highest level of risk which is not sufficiently offset by a higher return. The return-seeking client is motivated by a potentially higher return, so s/he determines the weights of criteria as follows: return – 0.6, risk – 0.25 and cost – 0.15. The most suitable 'higher-risky' participant fund is global stock CONSEQ fund. For a return-seeking strategy, this fund is clear because its return is significantly higher than returns of the other funds. Its position could only be jeopardized by a significant reduction in the importance of return criterion under increasing importance of the risk. Of

course, a dynamic fund Allianz is on the last place because of its practically zero return. Because the weight of return is high, the result is consistent with one-criterion (return) analysis. The cost does not play a significant role in all multi-criteria analyses because of its small importance and very similar evaluation of alternatives.

4.3 Strategy III – analyses and results

Now, here is a 'summary' selection of pension funds. The previous (partial) analyses suggested, the pension fund CONSEQ is the 'absolute' winner because of its lowest level of risk (at the level of 2.67) and the highest level of return (2.93%). The same risk is also reported by AXA pension fund. Only slightly worse risk is provided by PSČP pension fund. The Allianz fund has the worst value of risk (at the level of 4). The second best return is reported by PSČS pension fund closely followed by NN and PSČP funds. The pension funds offered by KB and AXA have the worst return. After one-criterion analyses, the multi-criteria analyses are also performed for both types of the clients. Of course, the winner of these analyses is just evident. However, a full ranking of pension funds is still interesting. The weights of criteria are determined at the same levels as in the Strategy II. For a risk-averse client, the ranking is in the following form: CONSEQ (0.99), PSČP (0.518), NN (0.443), PSČS (0.434), AXA (0.337), ČSOB (0.224), KB (0.212) and Allianz (0.163). On the second place, there is a pension fund from PSČP which reports the second lowest risk. The pension fund AXA (with the same risk as CONSEQ) is on the fifth place because of its small return. The ranking of the pension funds for a return-seeking client is as follows: CONSEQ (0.999), PSČS (0.459), NN (0.432), PSČP (0.421), ČSOB (0.261), Allianz (0.215), AXA (0.147) and KB (0.083). This result confirms the fact from the previous analyses. The first four pension funds are good thanks their two most important characteristics, the second four are worse. The first place is inviolable. The other three places, as well as the last four, may change according to the client's preferences (about the criteria importance expressed by the weights).

5. Conclusion

The article deals with a selection of the suitable pension fund, or supplementary pension savings. To make a complex (representative) decision, the multi-criteria evaluation method is designed. This procedure enables to take into account all unnecessary evaluating criteria (characteristics of the pension fund). Another advantage is that the client's preferences about the criteria importance can be considered. This multi-criteria analysis is performed to a few savings strategies on the Czech market. The results are analysed, especially from the point of view of the different settings of the weights and its effect on the most suitable product. Moreover, these results are also confronted with an often simplified one-criterion (return or risk) process. On the other side, these analyses remarkably supplement a multi-criteria perspective. The results show that the globally best pension fund is from Conseq Investment Management. This pension fund, or their participant funds slightly deny 'bigger return-bigger risk' phenomenon. On the contrary, the Allianz and Komerční banka pension funds went very bad. The proposed methodology of complex evaluation procedure is shown as a strong (quantitative) support tool for a various decision-making situations (determined by the client's preferences) on the market with pension funds, or supplementary pension savings. Moreover, the results can be the base for the second important phase – making a portfolio of the participant funds in terms of the selected pension fund, or supplementary pension savings. For this purpose, some of multiple objective programming methods can be applied.

Acknowledgements

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Relationships between monetary variables and the stock price returns: Case for Czechia, Hungary, Poland and the UK

Jiří Branžovský[1]

Abstract

This research aim examines the influence of the monetary variables to the equity markets. The first research was to analyse the relationship between interest rates, money aggregate, total banking balance sheets sizes, dummy variable and their relationship to the stock indices in three Central-European countries between Q1 2003 and Q3 2018, and to see whether those can be explained by selected regressors. For contrast, the UK was added too. The second research was run only against interest rates and dummy variable on time horizon already since Q1 2000.
As a method, vector error correction model (VECM) was selected to identify the long-term cointegration among variables. This long-run relationship was concluded only for the UK market. Short-term vector autoregressive (VAR) stochastic model was used for all the observed countries. Czech and Hungarian stock markets had neither short-term relationships, Poland was dependent on the rates.

Key words

Vector autoregressive model, vector error correction model, eViews, cointegration, monetary variables, interest rates, stock price returns.

JEL Classification: C01, C32, E44, G10

1   Introduction

This paper examines the impact of the monetary variables on the particular stock markets returns. The current approach is that monetary variables influence the real economy through several channels, e.g. liquidity channel, wealth channel or asset price channel. This research aim is to analyse the relationship between regional interest rates, money aggregate, total banking balance sheets sizes, business cycles dummies and their relationship to the stock markets in three post-Communist Central-European countries – Czechia, Poland and Hungary. Additionally for contrast, the UK as an old traditional market economy was included. Two quarterly time series analyses were run, one since Q1 2003, the second one since Q1 2000, both until Q3 2018.

The contribution of this paper is at identification of the long-term relevance and significance of particular national indicators on the selected countries' stock markets. That makes the difference visible between traditional market economy and once planned economies. Any evidence supporting economic theories at investment decision-making process would be beneficial not only for academic researchers, but investors too.

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2   Literature review

Since the 1970s, there have been written many academic papers with the focus on seek for monetary variables and analyzing their impact on the mostly developed countries.

Haitsma and Unalmis and Haan (2016) confirmed the impact of ECB monetary policy on the stock returns in the European union. Similar results were identified by Fausch and Sigonius (2017) on the German stock market. Balafas and Florackis and Kostakis (2018) also examined monetary policy and stock markets. Additionally, all three papers found the opposite relationship during the Great Recession as the lack of market and investors confidence. Kulhanek and Matuszek (2006) tested long-term influence of the MS on the stock returns in the Central Europe and concluded that it is weakening during time. Feldman (2017) ran Monte Carlo simulations on the interest rates and their impact on the stock returns in emerging markets. Chatziantoniou and Duffy and Filis (2012) also confirmed the relationship between monetary variables and stock returns in the European union, as well as in the USA.

Stock markets reflect the big picture of real economy. Even Donald Trump[2] often observes the US equity markets and claims the credit for their resilient growths as his own presidential fiscal success "to make America great again". On the other side, in the rest of the world it is mostly central banks that had received the big monetary decision-making power in the third millennium.

Macroeconomical factors are usually those representing a severe part of the stock variance, especially the monetary policy´ tools and money variables play usually the vital importance. There is however not the unique opinion about which of money variables are the most important ones – these vary based on the observed country, model used, observed period etc. Three selected regressors used in this research were also used in Branzovsky (2018).

3   Methodology

Before distinguishing VAR and VEC models there is a necessity to specify the stationarity (trends) of the variables. Time series stationarity is a characteristic typical for a low volatility around the mean value and no trend representing an autocorrelation. Stationarity is usually apparent already from the chart, followed by correlogram and unit root tests. In this paper it is processed by Augmented Dickey-Fuller (ADF) test and by Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. Stationarisation of time series not in levels is usually processed by natural logarithmisation and by first or higher-order differentiation, or by both. Non-stationary time series may be endangered by the spurious regression, and so stationary variables are a must assumption for VAR models. Besides of that, a long-run model called Vector Error Correction Model (VECM) that furthermore solves an issue of the model with cointegration, does not require stationarity of the data.

3.1 VAR model

Standard vector autoregressive (VAR) is an econometric model used for analysing, simulating and predicting linear interdependencies among multiple time series. Each of them plays is treated symmetrically once as an exogenous, and once as endogenous variable. VAR as a multi-equational model is due to possible cointegration among regressors considered as the short-run model.

A p-order VAR model, described as a VAR(p) is a set of k variables and their p time-lagged values. The specific model for endogenous variable explained by its own time lags and one another variable could be as follows:

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[2] Donald Trump (*1946) has been the 45th president of the United States of America since 2017.

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + B x_t + \varepsilon_t \tag{1}$$

where y_t are *k* observed endogenous variables, *A* are the parameters of endogenous variables parameters and their time lagged values, *B* are the exogenous variables parameters and possibly their own time lagged values and ε_t are normally distributed standard errors and *p* is the maximum time lag. The optimum maximum time of lags to be used may be found out by t-tests, F-test and mainly by information criteria.

Granger causality is a predictive concept for stochastic linear measuring whether lagged value of a stationary variable X does or not improve an explanation of another stationary variable Y – to see what variability of Y is explained by its time lags and then to see if additional lagged values of X may improve their relationship.

Cholesky´s Impulse-Response analysis is a verification method to identify the inner-behaviour among the variables in the VAR model over the time period.

Cholesky´s Variance decomposition works sort of as a dynamic coefficient of determination. That helps to propose a proportion of the endogenous variable´ variance based on own shocks and cross-variables shocks. Apparently, their own shocks are the most important of the error variances, but another ones will also be transmitted into VAR model.

Residual tests of distribution, homoscedasticity and autocorrelation are to be run. For more details about VAR models see Branzovsky (2018).

3.2 VEC model

Until cointegration has started being studied back in the 1980s, most of econometric financial models had had significant explanatory power due to the spurious regression as most of financial variables keep drifting together side by side. The reality of integrated (non-stationary) financial time series led Engle and Granger (1987) to say that the combination of such series may be still linear. If these cointegrating relationships are identified, time series are called to be cointegrated and the long-run linear relationship may be estimated in the cointegrating equation of VEC model through OLS. Cointegration has changed the way how econometrists had been thinking once for good. As Cuthbertson et al. (1993, p. 129) mention it makes economic models based on theories to be economically correct saying whether variables within the model are actually influencing / cointegrating each other.

VEC model is effective with the variables that are non-stationary in levels, but are integrated of higher orders, but all of the same order, the highest one *I(max(x1, x2, ...))*. In that case the test of cointegration is proceeded, and if time series identified to be cointegrated, there may be a long-run relationship among those meaning that any potential short-term shock will oscilate around and converge to the long-run equilibrium.

$$\Delta y_t = \prod (y_{t-1} + X_{t-1}) + \sum_{i=1}^{p-1} \Gamma_i (\Delta y_{t-i} + \Delta X_{t-i}) + \varepsilon_t \tag{2}$$

where Δy_t are the differences of non-stationary endogenous variables, x_t are non-stationary deterministic variables and ε_t are normally distributed standard errors while *p* is the maximum time lag. VECM is gathered of adjustment coefficient of cointegrating vector and its parameters based on lagged raw time series, then of short-run coefficients linked to the lagged differenced variables, error and finally by a possible intercept.

$$\prod = \sum_{i=1}^{p} A_i - I \qquad \Gamma_i = -\sum_{j=i+1}^{p} A_j \tag{3}$$

where $\prod =$ is a matrix of coefficients that is a result of multiplication of two *k x r* matrices α and β^T under the scenarios *r < k* and $\beta^T y_t$ is stationary in levels, where *r* represents the reduced

cointegration rank while β cointegrating vector. The coefficient matrix and reduced rank are tested by the Johansen method from an unrestricted VAR model.

When specifying the deterministic trend, it is natural that just as time series may have means and trends, the cointegrating equations may have intercepts and deterministic trends as well. Regarding to that, there are two types of columns – cointegrating column and outside column depending on whether the deterministic variables do or do not appear inside the cointegrating relations.

As trends are typical for financial time series, only deterministic trends cases with linear trends (case 3 and 4) of five Johansen´s were considered.

Case 3 for linear trend of the level data only:

$$H_1(r): \prod y_{t-1} + \mathrm{B}x_t = \alpha\left(\beta' y_{t-1} + \rho_0\right) + \alpha_\perp \gamma_0 \qquad (4)$$

Case 4 for linear trends of both the level data and the cointegrating equations:

$$H^*(r): \prod y_{t-1} + \mathrm{B}x_t = \alpha\left(\beta' y_{t-1} + \rho_0 + \rho_1 t\right) + \alpha_\perp \gamma_0 \qquad (5)$$

Where $\alpha_\perp$ represents the deterministic term outside of the cointegrating relation (rank).

There are two widely used tests. The Engle-Granger cointegration test is used for VEC models with one cointegrating equation model while Johansen test is suitable even for those with several equations. Regarding to the number of cointegrating relations and their statistical significance, there are two types of test statistics: *trace* statistics and *maximum eigenvalues* statistics. Estimation of cointegrating relations *r* in their first columns is proceeded in the order as follows *r = 0, 1, ... , k – 1* until the rejection of the null hypothesis. The second columns involve the estimates of the ordered eigenvalues of the matrix $\prod^=$. Trace statistics for testing the null hypothesis of *r* cointegrating ranks is calculated as below:

$$LR_{tr}(r|k) = -T \sum_{i=r+1}^{k} \ln(1 - \widehat{\lambda}_i) \qquad (6)$$

where $\widehat{\lambda}_i$ represents i-th largest eigenvalue of the matrix. The maximum eigenvalues statistic for testing alternative hypothesis of *r+1* cointegrating relations is as follows:

$$LR_{tr}(r|r+1) = -T \ln(1 - \widehat{\lambda_{r+1}}) = LR_{tr}(r|k) - LR_{tr}(r+1|k) \qquad (7)$$

Estimation of the cointegrating vector β coefficients and its adjustment coefficients follows. Identification of the cointegrating vector is based on the normalisation $\beta S_{11} \beta^T = I$

4   Data

The author´s research was processed on the quarterly data from Q1 2000 to Q3 2018. That is represented by 73 observations of the Czech, Hungarian, Polish and UK markets, gathered through Bloomberg financial terminal and Yahoo agency. Time series are collected as reported each quarter. The research since Q1 2000 was run only if banks balance sheets sums were omitted (the only variable since Q1 2003), otherwise just since Q1 2003.

The endogenous variables are the regional stock market indices. These are namely, PX index for the Czech Republic (CZ_L_INDEX), BUX index for Hungary (HU_L_INDEX), WIG 20 index for Poland (PL_L_INDEX) and FTSE 100 index for the UK stock market (UK_L_INDEX).

Among the independent explaining regressors there have been observed specific regional short-term effective interest rates CZ/HU/PL/UK_RATEQ (p.q.), regional M1 money supply indices as CZ/HU/PL/UK_L_M1, aggregate balance sheets sums of national banking system named as CZ/HU/PL/UK_L_BS (in millions EUR) and the DUMMY variable – ones except of nulls representing business contractions periods (NBER, 2019) during Q1 to Q4/2001 and

Q4/2007 until Q2/2009. Economically, monetary aggregate and banking balance sheets sum should be positively affecting the stock returns, while interest rates negatively. For VAR model, all observed variables must be stationary at *I(1)* and hence not only naturally logarithmed, but differenced at first levels as well, into these named CZ/HU/PL/UK_DL_INDEX, CZ/HU/PL/UK_D_RATEQ, CZ/HU/PL/UK_DL_M1 and CZ/HU/PL/UK_DL_BS.

5   Results

ADF and KPSS tests were proceeded in order to address this issue at 10% significance level[3]. All Czech, Hungarian, Polish and UK variables were recognised with the trend meaning being non-stationary with a unit root, *I(1)* at lower than 5% significance levels based on ADF and KPSS and depending on what was included in test equation[4] with exclusion of polish money supply that is *I(2)*.

For VEC model, the raw non-stationary time series were used, after being natural-logarithmed into L_X, except of the interest rates that were only transformed into quarterly rates. For VAR models, all variables were modified into the first differentiations of naturally logarithms (DL_X) as for VAR the time series are required to be stationarity of the first order *I(1)*. See short-run VAR models initially, and then let be verified the potential cointegration among regressors.

The **lag order** for models was selected based on several information criterions, e.g. Akaike, Hannah-Quinn or Schwarz information criterions (the most preferred one as the smallest negative coefficient), LR statistic test at 5% significance level etc. As tested by above mentioned ICs, the best models are suited for one time lag in all four observed countries.

The tests of the **Granger causality** estimate that while Polish and the UK markets monetary variables may explain some variability of their stock indices, it is the Hungarian stock index causing interest rates. See below table of just few causalities at max 5% significance level due to many equations.

CZ_DL_BS does Granger cause CZ_D_RATEQ at ** significance level.
HU_DL_INDEX does Granger cause HU_D_RATEQ at ** level.
HU_D_RATEQ does Granger cause HU_DL_M1 at *** level.
PL_DL_BS does Granger cause PL_DL_INDEX at ** level.
PL_DL_M1 does Granger cause PL_D_RATEQ at ** level.
PL_D_RATEQ does Granger cause PL_DL_M1 at ** level.
PL_D_RATEQ does Granger cause PL_DL_INDEX at *** level.
UK_DL_BS does Granger cause UK_D_RATEQ at *** level.
UK_D_RATEQ does Granger cause UK_DL_M1 at ** level.
UK_D_RATEQ does Granger cause UK_DL_INDEX at *** level.
UK_DL_INDEX does Granger cause UK_D_RATEQ at *** level.

Estimation of the **VAR** parameters was processed by OLS method. All parameters were rounded to two decimals. If not statistically significant, the particular variables were omitted and VAR model was re-tested whether not got improved. Intercepts were kept in the VAR models if significant but their relevance in models depend on the economical interpretation.

CZ_DL_INDEX$_t$ $= -0.13^{***} + 0.18$ CZ_DL_INDEX$_{t-1}$ $+ 2.18$ CZ_DL_M1$_{t-1}^* - 0.39$ CZ_D_RATEQ$_{t-1}^* + 0.10$ DUMMY$_t^{**}$

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3 This paper works with * at 10%, ** at 5% and *** at 1% statistical significance level.
[4] Model could include intercept, intercept with the trend, or none of those.

HU_DL_INDEX$_t$ $= -0.09 - 0.02$ HU_DL_INDEX $_{t-1}$ $- 0.06$ HU_DL_BS $_{t-1}$ $+ 0.02$ HU_D_RATEQ$_{t-1}$ $- 0.04$ HU_DL_M1 $_{t-1}$ $+ 0.13$ DUMMY$_t$*

PL_DL_INDEX$_t$ $= 0.43$ PL_DL_INDEX$_{t-1}$*** $- 0.25$ PL_D_RATEQ$_{t-1}$* $- 0.53$ PL_DL_BS$_{t-1}$

UK_DL_INDEX$_t$ $= -0.08$*** $+ 0.10$ UK_DL_INDEX$_{t-1}$ $- 0.02$ UK_DL_BS$_{t-1}$ $- 0.23$ UK_D_RATEQ$_{t-1}$*** $+ 0.09$ DUMMY$_t$***

All four national short-run models with one time lag as VAR(1) were **statistically verified** and found **significant**, and are updated based on the most significant variables included only. Their adjusted coefficients of determination R^2 are approximately 21 % (CZ), 2 % (HU), 21 % (PL) and 41 % (UK).

Evaluating these short-run VAR models, Polish stock index is mostly explained by its own time lagged value (1% stock index growth will lead to 0.43% growth one quarter later) and interest rates (100bp rate raise results in 0.25% stock decrease). UK stock index is negatively influenced by rates (100bp rate raise results in 0.23% stock decrease) and positively by 9 bp dummy variable. The Czech stock index is weakly dependent on dummy, money supply and interest rates.

The Hungarian model was very weak, parallely the strongest relationship was identified between its money supply as dependent variable and the others independent explaining 48 % of HU_DL_M1 variability. Compared to that in the UK, 42 % variability of the interest rates as a regressant is explained by stock index, balance sheet and dummy.

Regarding to unrestricted VARs **residual econometric verification**, there are no residual autocorrelations identified in the models based on LM and Portmanteau Autocorrelation tests at *** significance.

However p-values of White tests of residual heteroscedasticity with/without cross terms are mostly lower than 10% significance level which means rather rejecting null hypothesis of homoscedasticity – VAR models are heteroscedastic[5].

VARs residuals do not come from the normal distribution[6], they are skewed and kurtosed.

Regarding to the Cholesky´s **Impulse response** method, endogenous variable the Czech stock index reacts within two quarters positively on the growth of money supply and negatively on the rates hike. No significant relationship identified for Hungary. Polish and UK stock indices have being negatively influenced by both interest rates and surprisingly balance sheet sums as well – in the UK for up to two following quarters and in Poland for up to five.

In terms of Cholesky´s **Variance decomposition,** there is no explanatory power for stock prices obvious in Czechia and Hungary. On the other side, Hungarian interest rates are accounted for approximately 14 % of the money supply growth forecast error variance within four quarters. Polish interest rates account for 16 % of PL stock index growth variance within one year and UK rates account for 20 % within two quarters.

As the raw data are non-stationary, performing of **Johansen cointegration test** is to be performed, as well as testing the long-term model. Involving the endogenous monetary variables and the exogenous dummy one, the Johansen cointegration test and deterministic

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[5] Heteroscedasticity may have negatively influenced the accuracy of the OLS estimated parameters, but the trends ought to be still alright.
[6] Normal distribution is not a necessary requirement for suitable BLUE OLS estimated parameters. This issue gets even less important with greater observed time series sample.

trend case 3 with linear trend among data while just intercept expected in the cointegrating equation, revealed one cointegrating equation at 10% significance level. Based on ICs, the lag order for VEC models is 1 as set up from unrestricted VAR models. VECMs are tested for Czechia, Poland and the UK. Hungarian model was not the statistically significant one.

$$DL\_INDEX_t = \alpha_1 + \gamma_1(L\_INDEX_{t-1} - \lambda_1 L\_BS_{t-1} - \lambda_2 L\_M1_{t-1} - \lambda_3 RATEQ_{t-1} + c_1) + \sum_{i=1}^{k} \delta_{1,i} DL\_INDEX_{t-i}$$

$$+ \sum_{i=1}^{k} \delta_{2,i} DL\_BS_{t-i} + \sum_{i=1}^{k} \delta_{3,i} DL\_M1_{t-i} + \sum_{i=1}^{k} \delta_{4,i} D\_RATEQ_{t-i} + c_2 + DUMMY + \varepsilon_t$$

(8)

where α, c are intercepts, γ_1 is adjustment coefficient and λ_i are coefficients of the cointegration vector, and δ are coefficients of the short-run models.

VECM long-term equilibriums have not been identified in Czechia, Hungary, neither Poland as adjustment coefficients γ_1 were not statistically significant as there are some another dynamic fluctuations between series. In this case the reader can be indifferent between usage of VEC and VAR models. Czech short-run VECM revealed the significant importance of index its own lagged values*, money supply** and dummy variable**. Polish model was mostly dependent on interest rates**.

The only significant VECM was shown for UK, at 1% level enforcing the long-run cointegrating relationship among series. Adjusted coefficient of determination R^2 was 45 %, with the greatest influence by the interest rates*** (100bp rate raise results in 0.28% stock decrease) and dummy*** (economic boom is responsible for 0.12% increase of stock prices).

As generally banks balance sheets sums and most of money supplies were not statistically significant in neither of the selected countries, there was another series of tests run once again without these, only with interest rates. And because of that, time horizon was prolonged since Q1 2000. Time lags were set up at one lag, the same like in previous testing. Statistical and econometric back-testing had similar results like at the first research.

UK long-run VECM was once again confirmed as statistically significant at 1% level, with stock index lagged values*, dummy*** and rates*** relevant in the model with explanatory power R^2 of 30 %.

Short-run VAR models were run too, with mostly similar results. Czechia and Hungary were significantly influenced by dummy only, Poland by interest rates and the UK by all: lagged stock prices, dummy and even rates. Their adjusted coefficients of determination R^2 were as follow in the same order: 13 %, 5 %, 17 % and 30 %.

6   Conclusion

This research paper tested long-run and short-run relationships between monetary variables and stock prices returns in four selected countries. Test was firstly run on quarterly data since Q1 2003 until Q3 2018. After identifying statistical irrelevance of the banks balance sheets sums and mostly even money supplies, the models were once again tested against just interest rates and dummy as long as since Q1 2000 (a longer time series). But based on this research in EU, and Branzovsky (2018)´ similar research run on the US data since 2008, monetary variables are somewhat exaggerated indeed, compared to relevant macroeconomical indicators.

For all four countries, short-term VEC and VAR models were run if possible. All models worked with one time lag (a quarter delayed) and all displayed heteroscedasticity and non-normality of their residuals. Impulse responses of regressants on the shocks revealed that

stock indices are positively influenced by their own one-quarter lagged values and negatively by interest rates for mostly three to four quarters.

Based on both researches with comparably similar results, there was no long-run cointegration identified in three post-Communist Central-European countries. Czech and Hungarian stock markets were unpredictable even in short-term. Poland has been short-run influenced by interest rates. The UK stock market, as the only representative of Western countries, has had been impacted by interest rates and dummy variable. Apparently, the monetary policy has been observed there for much longer.

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Credit rating and Corruption Perceptions Index

Josef Budik, Otakar Schlossberger[1]

Abstract

The goal of this article is to analyse the dependency between the credit ratings of selected countries and their corruption perceptions indices. The credit ratings of countries compiled by credit rating agencies are intended to be a complex assessment of the operation of various influences on the economy. The three most notable credit rating agencies evaluate the risk of the individual countries according to special criteria and express those risks using their own rating scales. Credit rating agencies may use somewhat different scales for evaluating company risks. The role of objective assessor of the impact of corruption on the economies of different states is filled by the authors of the Corruption Perceptions Index. The authors of this article have reached the conclusion that linear growth on the Corruption Perceptions Index corresponds to exponential growth of credit default risk according to credit rating.

Key words:

Trends, analysis, corruption, credit rating

JEL Classification: C52, D73, E51

1. Introduction

Money laundering also includes money from corruption. Connected to corruption is the question of how corruption influences the economic performance of the individual countries. For this reason, research associated with corruption practices is important relative to the economy of the individual countries. When discussing corruption, a widely debated topic is the method of identifying and quantifying it. Publicly accessible data and statistics mostly indicate that corruption is not perceived the same in all countries, which raises the question of whether corruption can help explain the differences in the economic performance of the individual countries. And yet the selection of indicators of economic performance is not clearly predetermined. Various indicators can be used to assess impacts on the economy, one of which is the credit rating compiled and published by credit rating agencies. This article presents an analysis of the relationship between Corruption Perceptions Index, the determination of credit default risk by credit rating, and their mutual relationship.

2. Objective, theory, and methods of research

The goal of this article is to compare perceptions of corruption and the likelihood of default for a selected countries according to information available from public sources at the beginning of 2019.

One of the problems the authors of the scientific texts addressed was the mutual comparison of the credit ratings of individual agencies. Karminsky, Hainsworth and Solodkov

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(2013) proposed a method for comparing weightings and therefore creating comparison tables and graphs to evaluate the scales used by the various credit rating agencies. They determined that assessing bonds can be confusing for a range of investors, since the credit rating agencies have accepted inconsistent definitions of the individual ratings for various types of securities.

Adelson (2007) also addressed inconsistency in definitions. He states that it is less and less clear what bond ratings mean. Certain rating agencies relatively frequently accept new definitions for their ratings. Inconsistent definitions significantly hamper investors' ability to assess credit default risk for different types of securities and different maturities. In the Moody's rating scale for American municipal bonds, the scale is calibrated differently for losses expected at different levels than the rating scale for all other types of bonds. Inconsistent definitions undermine the comparability of investment instruments with fixed yields from various market sectors.

Rating scales are used by Moody's for short term municipal bonds, investment funds, or structured products. Banks publish their long-term rating scales and global short-term scales in the bank's annual reports, in part so as to show the public their elite capability to repay short-term liabilities. One example is the Spanish bank Caixa and its Integrated Corporate report (2017, p. 37). This bank combines its AAA long-term rating by S&P with its A-2 short-term rating from the same agency. According to a study conducted by Vazza and Kraemer (2014), there is a high likelihood of failure to perform obligations for companies with the lowest speculative rating. In this analytical document, the authors show from S&P that there is on average a negative correlation between the initial rating of a company and its moment of default.

Analyses of the relationship between credit rating and the country's economic situation were performed by Mareš and Kotěšovcová (2017), but they focused on only two factors in their study, gross domestic product and inflation. These were later used in their analyses.

In subsequent studies, Kotěšovcová (2018) showed that the credit ratings of the individual agencies are independent of each other, which is the assumption underlying their functioning. The rating scores assigned by the three most notable rating agencies, Moody's, Standard & Poor's, and Fitch Ratings, at first glance seem to differ from one another. And yet the differences between the rating scores assigned by these agencies are not statistically significant between companies. The proof of the fact that the assessment of credit rating agencies of a specific country are essentially the same was made on the basis of a test of statistical significance. From this test it was shown that the differences in credit ratings are not statistically significant. This result was used by the authors of this article in underlying analyses.

Other research studies was focused on the issue of corruption and corrupt activities. Corruption exists in the individual countries, but is difficult to document and effectively root out. Caruana-Galizia, P. and Caruana-Galizia, M. (2018) analysed the situation on the island of Malta. They published a unique data set from the smallest European Union member state and one of the most heavily populated countries in the world. The underlying data was gathered using automated web searching, where a regulator first rejected and then ignored requests for information that the authors needed to use in their analysis.

Other studies focused on whether the institutional structure of the political system in the country influences its economic performance (Martini and Quaranta, 2019). And yet the study did not address the relationship between corruption and economic performance.

The question of whether an association exists between the corruption index and the credit rating of a country is the subject of the current article. For this reason, its authors have sorted countries for which Transparency International (2019) has published a corruption perceptions index in ascending order according to the growing perception of corruption and for the

purposes of the analysis the authors have assigned values of probability of financial default derived from the rating agencies. To visualize the dependency of credit default risk to the extent of perceived corruption, a graph was used which listed the individual countries on the x axis (independent variable) in ascending order by growth of perceived corruption and credit default risk was used as the dependent variable.

A critical problem of the time series analyses was defining the specific type of trend functions. The decision on suitable type of function should be based substantial economic criteria, i.e. the functions of the trend should be selected after substantive analysis of examined phenomenon. Even a cursory glance at the growth of default risk by several lines indicates that exponential trend is a suitable depiction.[2] This is generally characterized by equation 1, "Exponential trend".

Equation 1 Exponential trend

$$y = be^{ax}$$

The graphic depiction of time series data enables us to roughly reveal the trend tendencies of the analysed indicator.[3] The risk of selection therefore consists in its subjectivity. Various analysts could evaluate the situation differently and select various types of trend functions.

3. Corruption and credit rating

Corruption is frequently designated as "the greatest obstacle to economic and social development. The main reason is the fact that corruption disrupts the role of a lawful state and weakens the institutional foundations on which economic growth depends." Corruption does not include merely bribery, but also various forms of abuse of functions in public administration for personal gain. The question of how to quantify corruption and its impacts in the economy is worthy of greater attention. It is addressed by the documents of international organizations such as the OECD (2011).

3.1 Perceptions of corruption

Corruption is in essence subjective, and its measurement and the measurement of corruption levels is problematic. There is no empirical data available, and for this reason it is nearly impossible to evaluate the absolute level of corruption on the basis of available statistics. If we use, for example, comparison of the amounts of bribes, number of charges filed or court proceedings heard in corruption cases, it would be shown that they most likely do not reflect the actual extent of corruption but rather emphasize the quality of the government representatives, judges, and sometimes even the media in revealing and investigating corruption.

For this reason the authors in the current article have taken advantage of the fact that corruption is examined by the non-profit organization Transparency International and their quantified Corruption Perceptions Index (CPI), the most recent version of which presented data from 2018 (Corruption Perceptions Index 2018, 2019). The CPI scale provides values between 0 - 100, where "100" designates a country with almost no corruption and "0" designates a high level of corruption. At the same time, TI has ordered countries into a single

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[2] The exponential trend is the result of a comparison of two expertly determined and published time series.

[3] A cursory look at the chart attached may act as a linear trend. However, it should be taken into account that the scale on the y-axis is logarithmic. The logarithmic scale was chosen because it shows better than linear difference of several orders in the probability of bankruptcy.

order ("rank")[4] such that the least corrupt countries are at the beginning. The lease corrupt countries traditionally include Denmark, New Zealand, and Finland (in 2018 Denmark had 88 points, New Zealand 87 points, and Finland 85 points). Each of the three countries mentioned achieved fewer points than they had in 2014, but in the summary from 2018 occupied the first three places. This indicates that they are the countries with the least corruption.

The other end of the rankings, i.e. in last place, are the countries of Somalia with 10 points, Syria with 13 points, and South Sudan with 13 points. The Transparency International analysts resolved the problem of having the same point ranking and same place in the ordering of countries by ordering the countries alphabetically. The alphabetical ordering of countries with the same ranking is used throughout the entire TI report and was adopted by this article as well.

The index is compiled on the basis of survey results in which respondents assess the capability of government institutions to limit and sanction corruption, the effectiveness of anti-corruption measures, the extent of corruption, and the openness of institutions of public administration. The commentary also refers to the extent of abuse of public finances and the forms and methods of lobbying in the public sector.

To capture perceptions of corruption, TI uses a total of thirteen surveys prepared for example by the African Development Bank, Bertelsmann Stiftung, World Bank, and others. According to TI, their studies represent the most reliable method of comparing relative levels of corruption in various countries. The Corruption Perceptions Index 2018 ranked 180 countries. Out of 31 European countries (EU member states and Norway, Switzerland, Iceland), the Czech Republic is ahead of Hungary and Slovakia at 20th place, and 28th place in the overall ranking of all 180 countries.

3.2 Rating and default probability

In today's globalized world, ratings are often used for investment decisions. As shown by Felixová (2011), a rating can be generally characterized as a method of ranking institutions or objects using metrics. In the finance sector these consist of credit ratings of financial institutions or securities. A credit rating represents the opinion of a specialized agency on the ability of the issuer to repay in full and on time the obligation arising from the issue of the given security. Credit rating agencies evaluate the various types of securities with various maturities. In comparison to this, the credit rating of an issuer applies exclusively for the immediate economic situation of the issuer. National credit ratings are used primarily when assessing the financial standing of receivables denominated in foreign currencies against the government, because as pointed out by Jílek (2009), credit risks are associated with individual countries as well. Each credit rating is presented by the rating symbol of the given scale, and this symbol represents the opinion of the rating agency of the financial standing of the rated subject.

 Ratings are created by independent rating agencies, the most notable of which are Standard and Poor's, Moody's, and Fitch. Ratings use a scale from the lowest speculative level to the highest investment level. The higher the rating, the lower the risk of default by the government or financial institutions, and for this reason the lower the yield required of the government bonds. The rating is based on numerous quantitative and qualitative indicators, such as government debt and its trends over time, the economic situation and outlook, institutional development, or political risks.

The perspective of the rating agencies are expressed verbally or using symbols. The individual rating symbols, which are brief, concise, and suitable for publication, have a verbal

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[4] Given that TI has published values and "rank" in its document, the authors of the presented analysis have chosen this order as well.

interpretation in order that less experienced readers can accurately understand the meaning of the rating symbol. It informs about how reliable the rated subject is in terms of its ability to meet its obligations.

The investor may also be interested to know the likelihood of default of the subject, or failure to repay bonds. The probability of default on bonds in the timeframe of the subsequent five years cannot be determined altogether easily, because different analysts rate different periods and on the basis of non-unified data. Four examples obtained from various sources are shown in Table No. 1 "Default risk over a five-year timeframe".

Table No. 1: Default probability in a five-year timeframe

Moody's	Standard & Poor's;Fitch	Column A [%]	Column B (21 points)	Column C	Column D [%]
Aaa	AAA	0.15	21	1 in 600	0.05
Aa1	AA+	0.27	20		0.19
Aa2	AA	0.11	19	1 in 300	0.26
Aa3	AA-	0.40	18		0.36
A1	A+	0.48	17		0.56
A2	A	0.32	16	1 in 150	0.62
A3	A-	0.82	15		0.92
Baa1	BBB+	1.15	14		1.20
Baa2	BBB	1.36	13	1 in 30	1.89
Baa3	BBB-	3.21	12		3.63
Ba1	BB+	5.79	11		5.74
Ba2	BB	6.88	10	1 in 10	8.11
Ba3	BB-	12.23	9		12.50
B1	B+	16.18	8		17.09
B2	B	24.66	7	1 in 5	21.36
B3	B-	29.16	6		27.08
Caa	CCC	41.29	4	1 in 2	37.64
Ca	CC		1		59.73
C	C		1		73.70

Sources: Nývltová and Režňáková, 2007, s. 178; Alfonso, Gomes and Rother, 2007; Meadows, 2014; CMA, 2015.

To analyse the relationship of default risk to perceptions of corruption, the rating scale was assigned numerical increments. The credit ratings of the individual agencies are recorded in the same rows (according to their verbal characteristics), as the work of Kotěšovcová (2018) indicated that differences in credit rating scores are not statistically significant. Table No. 1 "Default risk over a five-year timeframe" Summarizes the results of analyses published by various authors. The number in the table can be described briefly in the following manner:

- Column A contains data about the incidence of default in percentages from the period of 1981 to 1988 (Nývltová and Režňáková, 2007, s. 178),
- Column B shows that the designation of the quality of the subject by credit rating can be transformed into a linear scale (Alfonso, Gomes and Rother, 2007),
- Column C lists data about the probability with which it can be expected that an investor will not receive the full value of an investment placed in the rated subject on schedule due to default in the five years following their rating (Meadows, 2014),
- Column D contains data on the incidence of default, in percent (CMA, 2015).

The authors of this article have chosen data from "Column D" for the next work, as it is the newest data and better corresponds to the theory (See, for example, row for rating Aa1). Also, the data in D set is newer.

4. Discussion

To visualize the analytic results, the authors used the graph below, in which each state sorted according to the Corruption Perceptions Index was assigned a numerical value of credit default risk over a 5-year timeframe. The probability was derived from table no.1, from "Column D". Only those countries were chosen that the Czech National Bank had selected in its own summary.

Graph in Figure No. 1 "Probability values according to ratings" (annex) shows that with a growing rank in the ordering of states according to the Corruption Perceptions Index the probability increases exponentially that the country will not be capable of performing its financial obligations arising from the sale of government bonds to investors. Both analysed approaches match almost exclusively with the left side of the graph, which shows countries with minimal corruption and high investment ratings (Denmark, Finland). For most of the countries the graph shows a gap interval between a relatively high perception of corruption and favourable ratings (South Korea, Czech Republic, China), or on the contrary, a low level of perceptions of corruption and an unfavourable rating (Cyprus, Turkey, Greece). The relatively large mismatch at the beginning and at the end of the ordering of countries can to a certain extent be attributed to the use of logarithmic scales of probability in the graph.

5. Conclusion

In the current paper the authors have shown the relationship between the individual credit ratings published by credit rating agencies. They have analysed the relationship between long-term credit rating and the credit default risk of the rated subject. They consider their primary outcome to be the finding that growth in the Corruption Perceptions Index corresponds to exponential growth of credit default risk according to the credit ratings of credit rating agencies.

The research showed that in the Czech Republic efforts to make positive changes in the perception of corruption can also have a positive impact on improving the state's investment rating. Therefore, the authors concluded that it would be appropriate to establish a new research project that would address the practices and methods of combating corruption. The project would build on the already conducted research into the fight against money laundering and a forthcoming project aimed at detecting tax evasion at a decentralized level.

Acknowledgements

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Annex
Figure No. 1: Probability values according to ratings (Expon. trend $y = 0,0054e^{0,133x}$)
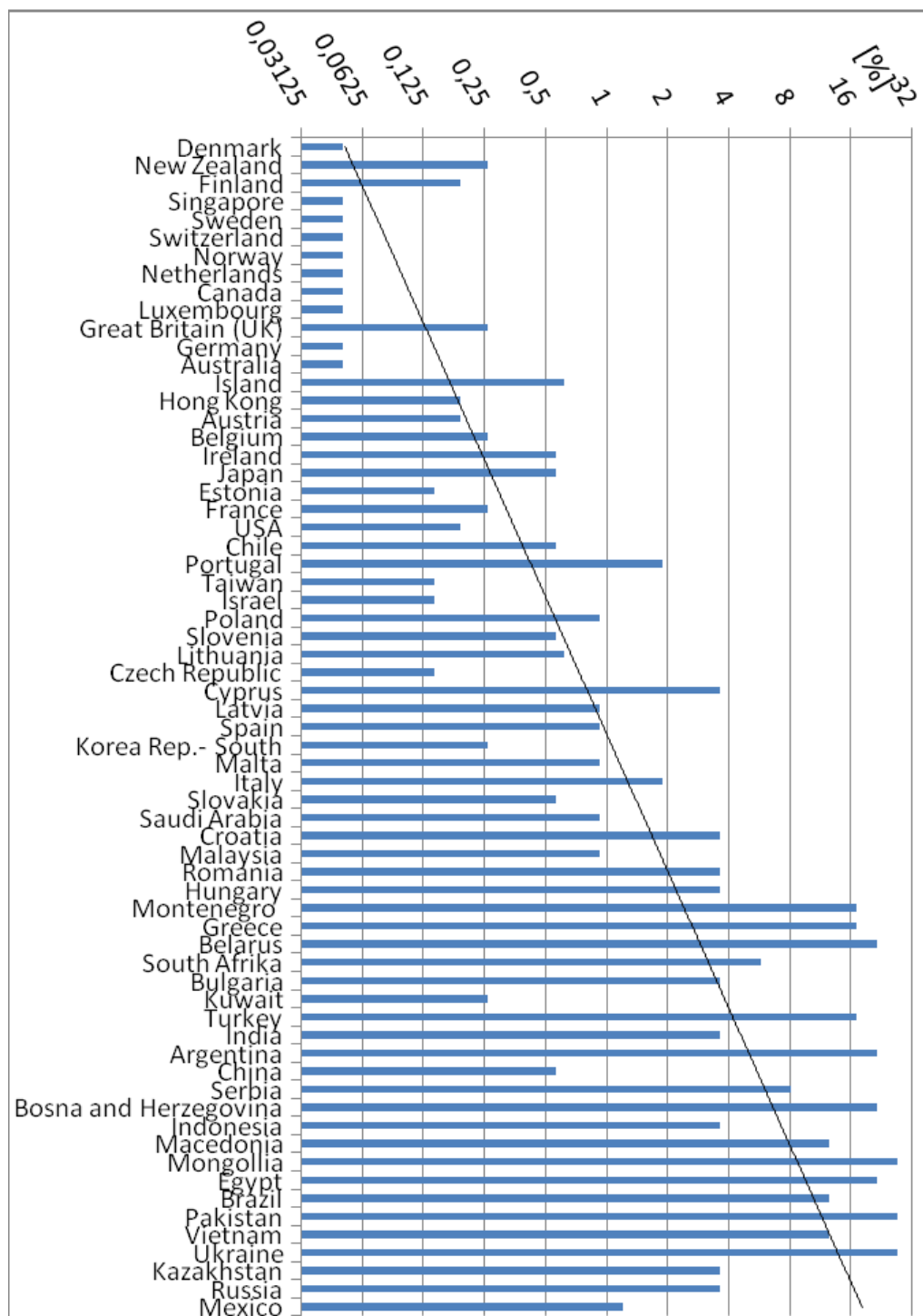
Valuation of Multi-Real Options With Multiple Sources of Uncertainty (Dual Rainbow Real Options)[1]

Miroslav Čulík, Petr Gurný, Vít Ošťádal[2]

Abstract

This paper focuses on the application of the real option approach on the investment project valuation under flexibility and risk. Particularly, comparative analysis is performed; i.e. how the presence of the portfolio of managerial actions may affect the asset (investment) value. The underlying asset (cash flow) is linked with two correlated random variables. The risk-neutral approach and discrete lattice are applied for project valuation. Managerial actions (real options) are American type and valued both in isolation (as single real options) or in combinations (multiple real options). At last, sensitivity analysis is performed.

Key words

Real option, risk-neutral, probability, exotic option, rainbow option, lattice, sensitivity analysis

JEL Classification: G31, G32

1. Introduction

One of the most commonly used mainstream and traditional methodology to determine the value of the investment projects is the income approach[3]. The income approach determines the value of the projects by looking at the future potential profit or free cash flow generating potential. These future potentials are forecasted, quantified and discounted by risk-adjusted cost of capital to a present value. The cost of implementation and acquisition is then deducted from this present value to generate the net present value.

Recently, some potential pitfalls in using traditional income approach are frequently mentioned by some authors, see for example Brach (2002), Mun (2016), Trigeorgis (2000) etc. The main shortcoming of this approach is that it assumes a single decision pathway with fixed outcomes and decisions made at the beginning of the project's lifetime without any ability of the management to alter and adjust the course of a project over time to prevailing market conditions. Such abilities represent important value-added component of projects, which is should be quantified and captured into the total project value.

Real options represent an approach that takes in the valuation process into account management's ability to make new decisions or correct the past ones. Particularly, this happen in situations when there is uncertainty involved in the future. As information becomes available and the uncertainty becomes resolved, management can choose the optimal strategies or options

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[3] The other methodologies aplicable for the value asset determination is the market approach and the cost approach.

with respect to the actual business environment. These options may have significant added-value that is necessary to quantify and include into the project's value. Real options approach enables to model these future possible decisions as formal call and put options that can be under some circumstances exercised and determines their values. For their valuation, traditional financial options valuation models are used. Compared to traditional valuation approaches, the value of an asset captures not only the present value of all directly measurable cash flows, but the value of the future options that can be by active management exercised.

This paper aims to apply the real option approach on the investment project valuation under flexibility and risk. Particularly, comparative analysis is performed; i.e. how the presence of the managerial flexibility affects the project's value. In addition, it is assumed that cash flows generated by project are affected by two random correlated variables.

The structure of the paper is following.

Chapter one provides the overview of the results of research on the application of the real options approach used for the valuation and financial decision-making issues.

Chapter two provides introduction into real options approach, real options classification and application possibilities. In the end, the generalised step-by-step description of the valuation procedure is described.

Chapter three provide illustrative study. Here, an investment project is valued in the presence of portfolio real options and two correlated random elements underlying cash flows generated by the project. For the comparative purposes, traditional approach and real option methodology is used. At last sensitivity analysis is performed, where the strength of the random elements correlation and the impact on the project value is studied. The results are compared, commented and conclusions are drawn.

2. Real options research overview

After more than 30 years of the research, the community of researches and academicians has established comprehensive theoretical framework for the application and usage of the real options methodology for the solution of wide range of the valuation and financial decision-making problems. Interesting results and findings have been achieved, both in the theoretical field, and real applications.

Following part firstly provides brief overview of some well-known pioneering works on the real options theory. Black and Scholes (1973) were the first authors to state that it is possible to take the equity in a levered company as a call option on the company value. Myers (1977) was the first one, who proposed the real options concept. He pointed out to the analogy between financial and real options and provided some theoretical backgrounds on the real options methodology. Ross (1978) studied the inherent potential of the investment opportunities and suggests considering these opportunities as real options. Then he studies and analyses the theory of using financial option pricing models on valuation of real options. The differences between traditional valuation approaches and real options theory is studied and analysed as well. Myers (1984) explains the limitations of discounted cash flow approach and the importance of the company's active strategies in the valuation process. He proposes that capital investment decisions should be evaluated as real options instead of by simply discounting the directly measurable cash flows. Hodder and Riggs (1985), Trigeorgis and Manson (1987), Ross (1995), etc. claim that traditional measurements based on the discounted cash flows (NPV, IRR etc.) may provide wrong results and incorrect investment decisions. They point out that if the uncertainty exists, these traditionally used and recommended measurements cannot capture the value of the managerial flexibility hidden in many capital investments. The result is that the true value of most investments is higher than the value provided by traditionally used valuation methods.

The real options methodology has been used for real applications as well; empirical results and findings are available in valuation and financial decision-making problems particularly in the areas with highly uncertain environment. Particularly, some strategic sectors or activities of national economies have suffered significant market, regulatory and technological changes during the last decades. In these important national sectors, the future events and business environment development affecting forecasted cash flows are difficult to forecast due to high level of uncertainty. Moreover, undertaking of investments in these sectors usually requires sequence of large capital investment outlays, that are spent over a relatively long period, have long economic life and cash flows affected by more sources of uncertainty. Typical examples of such sectors or activities are the energy sector, transport, mining of natural resources, research and development programs, etc. These sectors have moved in many countries in Europa from regulated and monopolistic sectors to deregulated, uncertain and highly competitive sectors. This change opened the way to the application of the real options theory. Examples in the following subsection illustrate the increased importance of the real options approach in some major areas.

Brennan and Schwartz (1985) were first who applied option pricing methods to the evaluation of irreversible natural resources using the Chilean copper mines. At the same time, other authors developed work in the energy sector, more specifically in the oil industry, like Siegel *et al.* (1987), Paddock *et al.* (1988) and Ekern (1988) etc. In the decade of 1990–2000, Dixit and Pindyck (1994), Trigeorgis (1999) and Brennan and Trigeorgis (2000) contributed to the development of the real options approach applications, giving an emphasis to examples and case applications in several industries and or markets, including the energy sector.

Following studies are the examples, when real option methodology is employed in the mining sector to analyse and quantify managerial flexibility in mining operations. Slade (2001), Moel and Tufano (2002), Colwell *et al.* (2003) examined managerial flexibility for cooper and gold mines in Canada, USA. Their findings are that the flexibility in mine projects is significant, leading to mining projects to open, defer or shut down under different circumstances. Blais *et al.* (2005), Dessureault (2007), Guj and Garzon (2007), etc. conducted research on projects in oil and mining industry. Their results show that shut down or temporary closure of projects has significant impact on project value when measured by employing real option methodology compared with traditional passive discounted cash flow methods. Furthermore, results of some authors shows that real options methodology is an appropriate method for mining companies because operational flexibilities are deemed an essential component of mining and oil project values; see, for example, Hall and Nicholls (2007), Dogbe *et al.* (2007) or Shafiee and Topal (2008). An overview of some recent studies on the use of real options approach in energy sector can be found in Fernandes *et al.* (2011) as well.

As the real option approach became a tool traditionally used by academicians and analysts, research interest on solving more complicated and complex problems have come into foreground. A lot of studies with interesting findings have been published. Examples are the applications on valuation of exotic real options, see Ouwehand and West (2005), Hucki and Kolokoltsov (2007), Alexander and Venkatramanan (2012), Reyes and Venegas-Martínez (2016), Wang *et al.* (2017), etc., hybrid real options, see Arasteh (2017) or Zmeškal (2013), fuzzy-real options, see Ghosh *et al.* (2013), Zmeškal (2014), etc.

3. Real options – managerial flexibility, applicability and valuation

This chapter introduces the fundamentals of the real option approach. First subsection provides brief introduction into the basics of the real options, followed by the subsection focused on the real options classification. In the last part, application of financial options

valuation models on real options is examined. Necessary mathematical background is provided as well.

3.1   Real options – the basics

Real options represent a systematic approach and integrated solution using financial, decision-making and financial option theory in valuing real assets. The main idea relies on the fact that the value of an asset is not viewed just as a present value of the single pathway of the cash flow streams, but rather as the multiple decision pathways as a consequence of high uncertainty coupled with managerial flexibility in choosing the optimal strategies. These strategies can be viewed in terms of owning the rights (options) to exercise them or allow them to expire if costs outweigh the benefits of execution. Whereas the traditional discounted cash flows approaches assume single static decision strategy "now or never",[4] the real options approach assumes dynamic series of decisions, where the management has the flexibility to adapt the real assets to changes in the business environment.

To determine the asset's value and the value of the managerial flexibility, several necessary assumptions are required.

Firstly, financial model must be created, where the model's operating, technological, market or other factors are subject to uncertainty and changes. These sources uncertainty are assumed to drive the asset's value.

Secondly, there exists managerial flexibility (strategic options), that can be either exercised or allowed to expire. Generally speaking, the options are exercised if the benefits of given decision outweigh the costs of the decision; otherwise it is better to let the options expire. Particular decision cannot be undertaken until the uncertainties driving the asset's value become resolved over time.

At last, the management must be able and willing to exercise these options when it becomes optimal to do it. It follows that the management must be rational and executes the decisions when the additional value generated by the decisions is comparable with the undertaken risks. Ignoring such opportunities result in underestimating the value of an asset. It is obvious that the real options approach determines not only the value of the future options and opportunities, but indicates when and under which conditions undertaking certain decisions is optimal.

Assets, whose value is determined only as a present value of directly measurable cash flows generated, that are "at-the-money" or "out-of-the-money", i.e. their net present values are negative or close to break-even, should be valued in terms of real options. The reason is that real options approach is able to quantify the additional value of strategic options that are overlooked by passive traditional approaches. The additional value enables to justify the acceptance of physical assets that would be otherwise rejected.

3.2 Real options categorization

There exist a lot of ways of categorizing real options. The categorization and terminology used in this paper relies on the one used by most of the authors.

The key categorization of the real options is with respect to the time they can be exercised. If the management has an option to make the strategic decision only at a given point of time during the economic life of investment projects[5], such options are called *European-style options*. In case of *American-style options* it is assumed that the strategic decision can be undertaken at any time during the economic life of investment project.

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[4] Some authors use the term "all or nothing", see Munn (2016) etc.
[5] Real assets are mostly represented by capital investment projects. From now, the term *investment project(s)* will be used, which makes is easier to explain some types of real options and flexibility they provide.

Other way how the real options may be categorized is according to the flexibility they provide to management[6]. Options related to the investment project size are *option to expand* (management can expand the scale of the production if the market conditions turn out to be more favourable than initially expected), *option to contract* (scale of production can be contracted if market conditions are worse than initially expected), *option to shut down and restart* (production can be stopped and restarted again when market temporarily drops down), etc. Options relating to investment project life and timing include *option to abandon* (project can be completely abandoned if market conditions turn overall unfavourable), *option to defer* (management has the right to postpone the investment project initiation by next year until the uncertainty is not resolved), *sequential options* (investment project can be abandoned during construction if investment outlay is not a single expenditure at the outset but sequence of outlays extending throughout the investment project's life) etc. Real options relating to investment project operations include *output mix options* (product flexibility, input mix options) *process flexibility* etc.

Previously outlined real options are so-called *plain vanilla* (non-exotic) real options; the options with no complicated payoff functions and straightforward valuation procedure. Real options relating to a given investment project can be either as a *single-real options* (management has the flexibility to make only one type of strategic decision at given predetermined time) or *multiple-real options* (more different strategic decisions may exist at given predetermined time). Valuation procedure both single real options and multiple real options is identical; the difference is that in the case of multiple real options, at each decision point management choose the best strategic decision from those available (with respect to prevailing conditions) to maximize investment project value.

Advanced (*exotic*) *options* are options with more complicated payoff functions. The advanced options could have one or more of the following features:

- payoff doesn´t depend only on the underlying asset value on maturity day, but on the average value over given time (*Asian options*), minimum or maximum value (*lookback options*), if some level is reached (*barrier options*), the price difference between two underlying assets (*spread options*), etc.
- payoff depends on the performance of the asset or assets that are in portfolio underlying: for example, as the weighted sum or average of different (*basket options*), performance of the best asset in the portfolio (*Himalaian options*), worst-performing asset in the portfolio (*Everest options*), etc.
- payoff depends on the moneyness of the option on the maturity day (*cash or share options*),
- payoff depends on the behaviour of two or more sources of uncertainty (*rainbow options*),
- payoff depends on the behaviour of other markets participants (*hybrid real option[7]*)
- etc.

3.3 Real options valuation procedure description

Real options methodology applies financial options pricing theory models in real assets valuation. Therefore, there are many similarities, but differences between financial and real options[8]. The applicability of these models relies on the assumption, that necessary parameters are defined for each type of strategic decision (real option) that needs to be valued. [9]

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[6] The same categorization can be found in Trigeorgis, Schwartz (2001), Copeland, Antikarov (2003), Damodaran (2006), Munn (2016) etc.

[7] Combination of real option methodology and game theory.

[8] see Trigeorgis, Schwartz (2001), Mun (2016) etc.

[9] Underlying variables definition for most of the real options can be found in Trigeorgis (2000), Brach (2002), Trigeorgis, Smith (2004), Damodaran (2006), Mun (2016) etc.

There exist lots of financial option valuation approaches and models to calculate an option's value. These can be classified as follows:

a) closed-form models (such as Black-Scholes model and its modifications),
b) Monte Carlo path-dependent simulation methods,
c) numerical methods (such as lattices, variance reduction, etc.)
d) partial differential equations, and so forth.

However, the widely used methods are the closed-form solutions, the path-dependent simulations and the lattices.

The closed-form solutions are models relying on the equations that can be solved under a set of assumptions. Although they are easy to implement, they have some important limitations. The most important is the limited modelling flexibility. Even though they are exact for valuation of European-style options, on the other hand, they provide just approximation for American-style options. Moreover, most of Bermudan[10] and other exotic options cannot be valued using close-form solutions.

Similar limitations apply to the part-dependent simulations. Some types of the path-dependent exotic, American-style or Bermudan-style options cannot be valued applying simulations at all.

In contrast, lattices are easy to implement, can solve all types of options, including European-style, American-style, Bermudan-style including many types of exotic options. In addition, results obtained by solving lattices converge in the limit to those obtained through the use of the closed-form solutions. Generally, the benefits of the lattices for option valuation can be summed into the following points:

▪ easy to implement and explain,
▪ applicable for valuing many types of option,
▪ suitable for modelling managerial flexibility (managerial decisions are made at discrete points rather than continuously),
▪ applicable for solution of multinomial options (more possible strategic decisions at time points they can be exercised),
▪ possible to apply if some assumptions are broken (volatility changes, the risk-free interest rate fluctuation, etc.),

Due to the outlined advantages, attention is paid just to lattices in the next subsection. Here, the underlying lattice and valuation lattice construction is described. Moreover, necessary mathematical background is provided as well.

3.3.1   Lattices as a discrete simulation of uncertainty

The lattice approach represents a discrete model of Brownian motion stochastic process. Discrete models involving two bifurcations at each node (up and down) are binomial lattices and they are usually used for valuing both financial and real options[11].

There are two sets of key equations to consider when creating a binomial lattice evolution of the underlying asset. These equations consist of upward (u) and downward (d) coefficients equation (used for creating binomial lattice of the underlying asset evolution) and the risk-neutral probability (p) equation (used in valuation through the lattice).

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[10] Bermuda options are a combination of American and European options that can be exercised on predetermined dates only.

[11] Other types of lattices are trinomial, quadranomial or pentanomial. No matter how many branches are at each node; these models provide exactly the same results at the limit. Generally, the more branches at each node, the faster the results are reached. Due to the complexity involved in solving trinomial and other multinomial lattices and easier mathematics required for binomial lattices, most real options are solved by using binomial lattices.

The upward (u) and downward coefficients (d) derived from exponential Brownian motion process[12] are calculated as,

$$u = exp(\sigma \sqrt{dt}), \tag{1}$$

$$d = exp(-\sigma \sqrt{dt}), \tag{2}$$

where σ is the volatility of underlying asset's log-returns and dt is the time-step in lattice.

If the underlying asset value at time t is S_t, the values in the nodes at time $t+dt$ (i.e. after passing dt) holds following,

$$S_{t+dt}^{u} = S_t u, \tag{3}$$

$$S_{t+dt}^{d} = S_t d. \tag{4}$$

Formula (3) and (4) can be generalized so the value of the underlying asset in any node of the binomial lattice can be obtained as,

$$S(i,j) = S_0 u^{i-j} d^{j}, \tag{5}$$

where i is the number of upward movements, j is the number of downward movements for $0 \le j \le i \le n$ and $dt = T/n$, where T is the expiration and n is the number of steps.

3.3.2   Real options valuation through discrete lattices

Regardless of what type of real option problem is solved, the solution can be obtained in one of the two ways. The first is by using the replicating portfolio and the second by using the risk-neutral probabilities. Both these procedures provide the identical results.

The usage of the replicating portfolio assumes the non-existence of arbitrage opportunities and that there exist a number of assets in the market that can be obtained to construct the portfolio that exactly replicates the payoff of the option. If the two payoffs (payoff of the replication portfolio and option) are identical, then their present values should be equal (to avoid the arbitrage opportunities).

The risk-neutral probability approach is based on the risk-adjusting of the probabilities of the possible payoffs occurring at a given time. The payoffs are subsequently discounted at the risk-free rate and the present value is obtained.

In reality, it might be difficult to find the assets for the replication portfolio construction that is why the valuation through the risk-neutral probabilities is mostly used.

If the lattice of the underlying asset evolution is constructed, the option valuation lattice construction follows in the valuation procedure.

The calculation of the valuation lattice involves following steps to get the option values:

1. Option's intrinsic values (*IV*) calculation. For European-style options, the intrinsic values must be determined just for the terminal nodes of the lattice; for the American-style options the intrinsic vales must be (due to the early possible exercise) determined for both terminal and intermediate nodes of the lattice.

2. The risk-neutral probabilities calculation. If binomial valuation lattice is applied, the risk neutral probabilities of upward movement (p^u) and downward movement (p^d) are given as,

$$p^u = \frac{(1+R_f)^{dt} - d}{u - d}, \tag{6}$$

$$p^d = \frac{u - (1+R_f)^{dt}}{u - d} = 1 - p^u. \tag{7}$$

3. Option price C_t determination. The backward-induction valuation procedure starts from the terminal nodes of the lattice (option price equals the intrinsic value) and moves

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[12] For the derivation see for example Cox, Ross, Rubinstein (1979).

through the intermediate nodes until the price in the initial node of lattice is reached. For European-style options and binomial lattice the valuation formula is defined as follow,

$$C_t = \left[C_{t+dt}^u p^u + C_{t+dt}^d \left(1 - p^u \right) \right] \left(1 + R_f \right)^{-dt}, \tag{8}$$

where C_t is the option price at time t, and C_{t+dt}^u (C_{t+dt}^d) is the option price in the subsequent period (after passing time dt). It is apparent from (8) that the option price at t is equal to the present value of the expected risk-neutral option's payoff in the subsequent period $t+dt$.

Due to the fact that the American options can be exercised at any time before the expiration, the valuation formula (8) is modified as follows,

$$C_t = max \left[\left(C_{t+dt}^u p^u + C_{t+dt}^d p^d \right) \left(1 + R_f \right)^{-dt} ; IV_t \right]. \text{[13]} \tag{9}$$

4. Illustrative example –valuation of American style dual rainbow real options

Following part examines the investment project valuation in the presence of managerial flexibility in situation when the cash flows are affected by the performance of two random variables (price and output volume).

The managerial flexibility gives the management the possibility to act and adjust the project according to the prevailing market conditions at any time during the first stage of the project life-span (American-style real option to abandon, expand or contract the project).

For the comparison purpose, the valuation procedure is performed under the following situations: existence/non-existence of managerial flexibility and presence/absence of the correlation between the underlying cash flow elements. At last, sensitivity analysis is performed. Here, the influence of the correlation of the random variables on the gross project value and flexibility value is studied.

4.1 Problem specification, inputs and assumptions

The objective is to determine the investment project value under different conditions by applying the real option methodology. The total investment (single) outlay INV (at period 0) is 30000 thousand USD; the investment project is set into the operation during the same period. The life-span of the project is 10 years. During the first stage of the life-span (period from year 0 to 3), the operating cash flows (and project gross project value) evolve randomly, during the second stage (period from year 4 to 10) the operating cash flow are non-random and are fixed at the level reached in the last year of the first stage. General formula for the operating cash flow FCF determination is given by,

$$FCF_t = \left(Q_t^i \cdot P_t^i - VC_t - FC_t - DEP_t \right) \cdot \left(1 - tr \right) + DEP_t - \Delta NWC_t \tag{10}$$
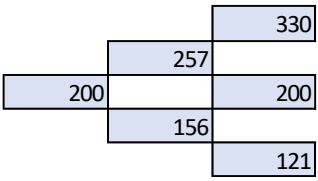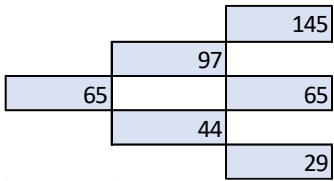
where Q is output volume, P is price of the final output and FC is fixed costs payment of 2000 thousand USD. The depreciation DEP in the first year of operation is 1650 thousand USD, in the next years 3150 thousand USD. The variable cost VC of production is 60% of revenues, the corporate tax rate tr is 19%. For simplicity we assume the investment project does not require any changes in company's net working capital components (i.e. $\Delta NWC = 0$).

It is assumed that Q and P evolve randomly according to the discrete binomial Geometric Brownian Motion with following characteristics: $Q_0 = 200$ units, $\sigma_Q = 25\%$ p.a., $P_0 = 65$ thousand USD, $\sigma_P = 40\%$ p.a. The risk-free interest rate R_f is 1.25% p.a. Given the inputs, the

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[13] The formulas for valuation lattices construction if trinomial and other multinomial lattices are applied can be found in Hoek, Elliot (2005) or Tichý (2008).

upward and downward coefficients according to (1) and (2), the risk-neutral probabilities and the underlying asset lattices are summarized in Table 1.

Table 1: Random variables (Q, P) characteristics and binomial lattice of predicted values

Output volume (Q) (units)		Price (P) (in thousands of USD)	
$Q_0 = 200$	$\sigma_Q = 25\%$	$P_0 = 65$	$\sigma_P = 40\%$
$u_Q = 1.284$	$p_Q^u = 46.3\%$	$u_P = 1.492$	$p_P^u = 41.7\%$
$d_Q = 0.779$	$p_Q^u = 53.7\%$	$d_P = 0.67$	$p_P^u = 58.3\%$

Output volume (Q) lattice:
```
                    330
            257
    200             200
            156
                    121
```

Price (P) lattice:
```
                    145
             97
    65               65
             44
                     29
```

It is obvious that if two random variables are modelled through discrete lattice and two bifurcations at each node are involved, the quadranomial (four-branch) lattice is used to capture all possible combinations.

4.2 Valuation procedure and results

The valuation procedure comprises the following steps:

1. Calculation of the *FCF* for each period and node of the underlying asset lattice. according to (10).
2. The join risk-neutral probabilities calculation. With no correlation, the join risk-neutral probability is given as a product of the individual risk-neutral probabilities of the random variables, i.e. for example $p^{uu} = p_Q^u p_P^u$ represents the join risk-neutral probability of upward movement both in output volume and price. The results for possible combinations are summarized in Table 2.

Table 2: The join risk-neutral probabilities (no correlation between random variables) (%)

Join risk-neutral probabilities (%)		Price (P)	
		up	down
Output volume (Q)	up	19.2	27.0
	down	22.4	31.4

The join risk-neutral probabilities for correlated underlying random factor are given by the following formulas (12)-(15) and are show in Table 3.

$$p^{uu} = \frac{1}{4} \frac{u^Q u^P + u^P g^Q + u^Q g^P + \sigma^Q \sigma^P \rho^{QP}}{u^Q u^P}, \tag{12}$$

$$p^{ud} = \frac{1}{4} \frac{u^Q u^P + u^P g^Q + d^Q g^P - \sigma^Q \sigma^P \rho^{QP}}{u^Q u^P}, \tag{13}$$

$$p^{du} = \frac{1}{4} \frac{u^Q u^P + d^P g^Q + u^Q g^P - \sigma^Q \sigma^P \rho^{QP}}{u^Q u^P}, \tag{14}$$

$$p^{dd} = \frac{1}{4} \frac{u^Q u^P + d^P g^Q + d^Q g^P + \sigma^Q \sigma^P \rho^{QP}}{u^Q u^P} . \text{ }^{14}$$ (15)

Table 3: The join risk-neutral probabilities (correlation between random variables, (%))

Join risk-neutral probabilities (%)		Price (P)	
		up	down
Output volume (Q)	up	27.73	23.98
	down	25.37	22.92

3. Calculation of the gross project value V. For passive project (no strategic decisions are available or possible) the backward valuation procedure starts from the end nodes of the valuation lattice, where; for the intermediate nodes the valuation formula for quadranomial lattice is as follows

$$V_t = \left[V_{t+dt}^{uu} p^{uu} + V_{t+dt}^{ud} p^{ud} + V_{t+dt}^{du} p^{du} + V_{t+dt}^{dd} p^{dd} \right] \left(1 + R_f \right)^{-dt} + FCF_t .$$ (11)

If managerial flexibility is assumed, the gross project value depends on if the available strategic decision (real option or mix of real options) exists and is undertaken. At each node of the lattice it is tested if the decision should be undertaken (real option is exercised) or keep the option open. The formula (11) is modified and has the following form,

$$V_t = max \left\{ \left[V_{t+dt}^{uu} p^{uu} + V_{t+dt}^{ud} p^{ud} + V_{t+dt}^{du} p^{du} + V_{t+dt}^{dd} p^{dd} \right] \cdot \left(1 + R_f \right)^{-dt} ; V_t \right\} .$$ (12)

Following table shows the gross project value determination for each node of the first stage of project life -span if managerial flexibility is available.

Table 4: Real options and payoff functions

Option type	Gross project value determination	Symbols
expand	$max\left[\left(1+x\right) \cdot V_t - I_{EXP.} ; V_t \right]$	x – scale of expansion, $I_{EXP.}$ – costs on expansion
contract	$max\left[\left(1-y\right) \cdot V_t + I_{CON.} ; V_t \right]$	y – scale of contraction, $I_{CON.}$ – costs saved
abandon	$max\left[V_t ; A_t \right]$	A – salvage (selling) price
mix of options	$max \begin{bmatrix} \left(1+x\right) \cdot V_t - I_{EXP.} ; \\ \left(1-y\right) \cdot V_t + I_{CON.} \\ A_t \\ V_t \end{bmatrix}$	

4. Project *NPV* calculation and final decision-making (accept/reject a project). Results are summarized in the Table 5 and 6.

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[14] The return of the output volume is given as $g^Q = R_f + \left(\sigma^Q \right)^2 / 2;$ for the price return holds $g^P = R_f + \left(\sigma^P \right)^2 / 2;$

Tab. 5: Valuations results: no correlation between random elements

		Gross project value	NPV	Flexibility value
Passive project		34620	4620	0
Project with option to	abandon	37157	7157	2536
	expand	37007	7007	2386
	contract	39234	9234	4614
	mix of options	42009	12009	7389

Tab 6: Valuations results: correlation between random elements

		Gross project value	NPV	Flexibility value
Passive project		48288	18288	0
Project with option to	abandon	49762	19762	1474
	expand	54774	24774	6486
	contract	50191	20191	1903
	mix of options	57324	27324	9036

4.3   Sensitivity analysis

In this section, sensitivity analysis is performed. The objective is to analyse how the degree of the correlation of random variables affect the value of given managerial flexibility. Results obtained are shown in Table 7. It follows that positive correlations generate higher volatility of operating cash flows resulting in increase in project value (and value of managerial flexibility). In addition, the number of the nodes where the option is exercised increases as well. The opposite is true for negative correlation degrees. Furthermore, positive correlation degrees increase the join risk-neutral probabilities of occurrence of extreme values.

Table 7: Influence of correlation of random elements on the flexibility value

		Correlation degree							
		-1	-0.75	-0.5	-0.25	0.25	0.5	0.75	1
Passive project		0	0	0	0	0	0	0	0
Project with option to	abandon	1304	1339	1407	1440	1474	1507	1539	1572
	expand	6306	6342	6414	6450	6486	6523	6559	6595
	contract	1900	1900	1901	1902	1903	1904	1910	1917
	mix of options	8624	8707	8872	8954	9036	9118	9199	9281

5. Conclusion

This paper aims at the project valuation generating cash flows that are determined by the behaviour of two underlying elements and the presence of managerial flexibility (real options). Particularly, the option to expand, contact abandon and the portfolio of these options are considered. The real options are exercisable in situations when the market conditions evolve in different manner (more favourably or unfavourably) than expected. Due to the presence of two underlying random elements and managerial flexibility, the quadranomial lattice and risk-neutral probability approach is applied.

The valuation is performed under different conditions: with/without the abandonment option and with/without the correlation of the underlying variables. Moreover, the combination of managerial flexibility and the correlation of the underlying variables is subject to sensitivity analysis.

As it is evident from the results, that managerial flexibility captures significant part of the total asset value (here the project value) regardless of the degree of the correlation of underlying variables. It confirms the fact that projects with negative *NPVs* or close to break-even should be valued in terms of real options. It enables to accepts more projects that would be in terms of passive traditional approaches otherwise rejected.

In reality, most of the project´s cash flows are affected by two (or more) underlying variables that are correlated. The results in Table 6 and 7 confirm the fact that the positive correlation generates higher volatility and overall underlying assets value (underlying elements moves in the same direction) that provides higher *NPVs* and higher option values. In contrast, negative correlations reduce the *NPVs* and option values. The results coincide with the generally known influence of the underlying asset volatility on the option value.

These findings are considered as important especially in the long-term projects valuation whose cash flows are associated with high degree of risk (more possible scenarios in the future can appear) and are affected by more underlying factors (possible changes in demand prices, operating costs etc.).

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Searching for employees through social networks in companies and financial institutions in Slovakia

Hana Gažová Adamková[1]

Abstract

The subject of the article is the use of social networks in the personnel area of the company - companies and financial institutions. As social networks are considered to be a future in the HR environment, staff members try to understand the principle of its functioning. The aim of the article is to point out the degree of use of modern methods and social networks in the selection of employees for specific jobs. Primarily, the article focuses on analysing data obtained through e-communication in the process of searching and recruiting staff. The article also presents new trends in the use of social networking, describes the social networks Facebook and LinkedIn, which are mostly used. The results also specify the benefits and limitations of the use of social networking as a tool for finding employees. These facts are predicting the use of social networks as a primary step in finding a candidate that is combined with traditional methods of search and selection.

Key words

social networking, employee search, employee search methods, social networking benefits, personal activity

JEL Classification: M12

1. The need for social networks in managing personnel processes

Current trends in the management of companies and financial institutions is becoming a focus on active use of digital technologies and social channels. In the personnel area, they are used to acquire and retain the end user. The end user may be an employee, client, customer, business partner. Social networks are used in the company in two directions - in and out. Inward means finding and recruiting new employees or increasing corporate loyalty. Outward is the focus on finding and retaining clients and consumers, consulting, exchanging information, and maintaining an active interaction with the outside environment.

Surveys show that almost every company or financial institution uses social networks for passive or active communication with an external environment. It is also a good and useful source of information. Based on experience, it can be assumed that the strategic focus on the interconnection of social networks with the personnel and information area can bring companies a long-term financial effect in both primary and secondary levels. Many companies perceive the use of virtual and digital space as part of a competitive and financial strategy.

1.1. Social networks as a personal resource

Social networks are increasingly becoming a place where companies are looking for new employees. The number of employee searches in the virtual environment is increasing every year. Companies can communicate with 95% of active and non-active labor market through social networks.[2] This is a direct or mediated provision of information. The social networking

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environment allows you to bid faster and get feedback. The whole process of communication is very time-efficient. In terms of the number of information, we can get more information, but it is necessary to select and verify it.

The use of social networks gives companies in the field of human resources a large area with great potential. Total employment now has a slow upward character. Slovakia's unemployment rate fell to 5,03% in March 2019, the lowest level since 1993.[3] Job vacancies are increasing and employers are creating new jobs. Candidates have a greater opportunity to choose a job as required. Companies need not only to attract and attract the attention of the candidate, but to be faster than the competition. There is an urgent need to work actively and innovatively with potential candidates to attract and acquire quality human resources.

A new Y generation is emerging into the labor market, working continuously with new information media and information resources. It is a generation that grew up with digital technologies at the time of social networking. This generation is online most of the productive time. That's why finding employees and communicating through these media is seen as an obvious and undeniably important step.

Search and recruitment of candidates gradually takes on a new direction, and companies need to actively reflect on this fact. People, potential employees, communicate more and more over the Internet and use social media and social networks of various types. They share a wealth of information that needs to be properly selected from a professional perspective.

Companies and financial institutions are increasingly trying to adapt to new trends and are increasingly using modern technologies and social networks in the personnel area. They apply them not only to recruit, select and train employees, but also to promote and inform job vacancies and to create relationships between individuals and companies. The use of new technologies allows applicants and employees to interact with HR and managers. At the same time, they provide new opportunities for human resources management. Namely to reduce the administrative burden, improving efficiency and contributing to the strategic management of the company.[4]

On the basis of several statistics, it can be stated that the social-web revolution is still underway. It takes on a high dimension, with an ever-increasing tendency. In 2014, the number of social network users worldwide exceeded 1.9 billion, in 2016 the number increased to 2.28 billion users, in 2018 it is around 2.62 billion, and by 2021 the number of users is estimated at 3,02 billion.[5] It is a great space for a personal database for professional and business purposes. Personal source social networking allows you to work with quantity from which selects quality. An elaborate personnel system and HR specialists play an important role, able to process and analyze large amounts of data. An important role is played by a sophisticated personnel system in the company. HR specialists know to process and analyze large amounts of data. A new professional symbiosis is created between HR and IT specialists. It is a professional elevator, in which HR specialists need to accurately and accurately specify the evaluation and

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recruitment criteria for each position. IT specialists create a technical field for primary database modeling and candidate selection. Given the increasing number of users of social networks, the classification of candidate groups is necessary for at least two reasons. One reason is the need to separate candidates who meet and do not meet the required criteria. At the same time create the order of suitability of candidates. The second reason is to select candidates and to create an internal career database of candidates that would be suitable for other jobs.

Although the social networks and the virtual world in the human resources area make a lot of space, it turns out that the companies do not use it to the full. When comparing Slovak and foreign companies, it can be stated that Slovak business entities use the space of social networks to do business less than European countries. In 2017, two-thirds of large European businesses had an account on at least one social network. On his promotion it used the social networking 43% of small firms. Slovak companies use social networks less. Only 35% of Slovak companies use active communication through social networks. Although their number is growing every year, it is relatively low compared to other European countries.[6] At the same time, the number of people active on social networks is growing every year. It is a great opportunity and reason to actively map and use virtual space.

2. Comparing the use of employee search methods to the aspect of the company's financial effect

The aim of the survey was to highlight the use of digital communication tools, through social networks in companies and financial institutions. It points to new opportunities and limitations of social networks in practice, focusing on perception of financial effect. The results can serve as information support for the selection of alternative and modern methods in the process of finding new employees.

The use of social networks in the area of search and selection of employees of Slovak companies was compared on a sample of 121 respondents. Respondents were from companies and financial institutions. In size, large (16%), medium (53%) and small (37%). We focused on job positions from the perspective of formal structure and expertise (managers, specialists and employees). Data was collected through a structured questionnaire in both written and electronic form. The questions in the questionnaire focused on identifying the methods companies use to find employees. Questions were also focused on the experience with the use of social networks in the area of human resources and evaluation of their benefits and limitations.

From the results it can be seen that companies in Slovakia use different types of methods in the field of employee search. They can be divided into two groups - traditional and modern methods. In the group of traditional methods, we included e.g. advertising, employment offices, recruitment agencies, educational institutions, personal references and job fairs. In the group of modern methods, we have included technologies such as computers, internet, websites and portals, social networks. Modern methods have grown considerably, especially in recent years. The modern methods are mostly used so-called. social media like LinkedIn and Facebook. Their advantage is high popularity of users and topicality of profile information. As a result, the quantity of potential candidates increases and the opportunity to verify their quality (education, experience, stimuli, etc.). On the network LinkedIn is most often look for people in the field of information   technology,   financial   services,   advertising   and   marketing,   and

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telecommunications. Likewise, Facebook is a suitable media for finding candidates and is equaling traditional job vacancies.

Figure 1: Traditional and modern methods of recruitment (%)

own sourcing
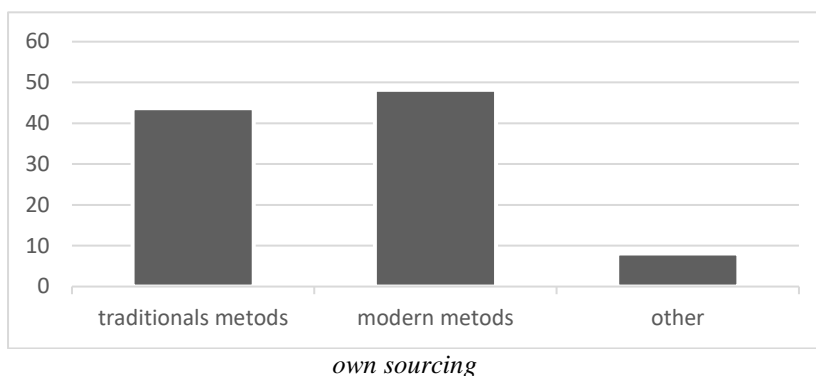
In Figure 1, we can see that companies and financial institutions are trying to follow current trends. They also use modern technologies to find and recruit employees. Traditional methods are used by 43.7% of respondents and 48.2% of respondents use modern methods. When responding with "other methods" (8.1%), respondents did not specify specific methods. The results showed that these methods are used by companies in combination, the data are complementary and verified. Data from social networks and Internet portals are verified by traditional methods (references, interview). Conversely, information from candidates (CVs, references) is verified through social networks.

Given the job position, there are differences in the use of modern and traditional methods (Figure 2). In managerial positions, the use of traditional and modern methods is relatively balanced - traditional (13.3%) and modern (13.2%). Traditional methods mainly use direct methods - reference sources. They are personal references from a specific source or information from recruitment agencies.

Figure 2: Traditional and modern methods of recruitment - working positions (%)

own sourcing
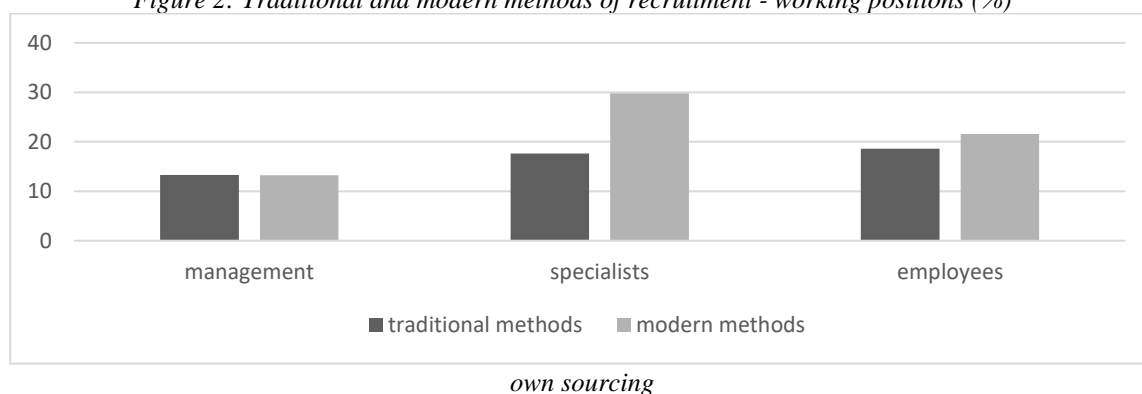
A more detailed use of specific methods can be seen in Figure 3. Management positions are most often sought based on personal references (41.4%). Enterprises also cooperate with recruitment agencies (18.2%), which have their database of candidates and other sources of management positions. Modern methods (e.g. social networks and the Internet) are used to a lesser extent. If they use them, they are mostly career sections on the web (14.9%). In the search for managers, the least popular use is regular advertising in the press (3.9%) and cooperation with labor offices (5.5%). This is obvious because management positions are seen as major.

Partially different is the situation when choosing methods of getting work positions - specialists. It is a specific category of employees. Due to their expertise and uniqueness, it is

81

necessary to use different sources and methods. E-recruitment is the most widely used, such as Internet work portals (35.9%), career sections on the web (28.2%), social networks (25.4%) and internship program (24.3%). Specialist references (27.6%), educational institutions (17.7%) and recruitment agencies (17.1%) play an important role in the search for specialists.

In normal positions, more candidates are searched. These job positions require consistent specifications, such as the amount of entitlement to their education, experience and specific expertise. These positions are largely occupied by the labor office database (29.8%). An important source is also electronic communication with a potential employee. It is often also used for advertising in the press (37.6%), social networks (23.8%), career section on the website (20.4%) and Internet job portals (21.5%).

Figure 3: Methods of recruiting employees (%)

own sourcing
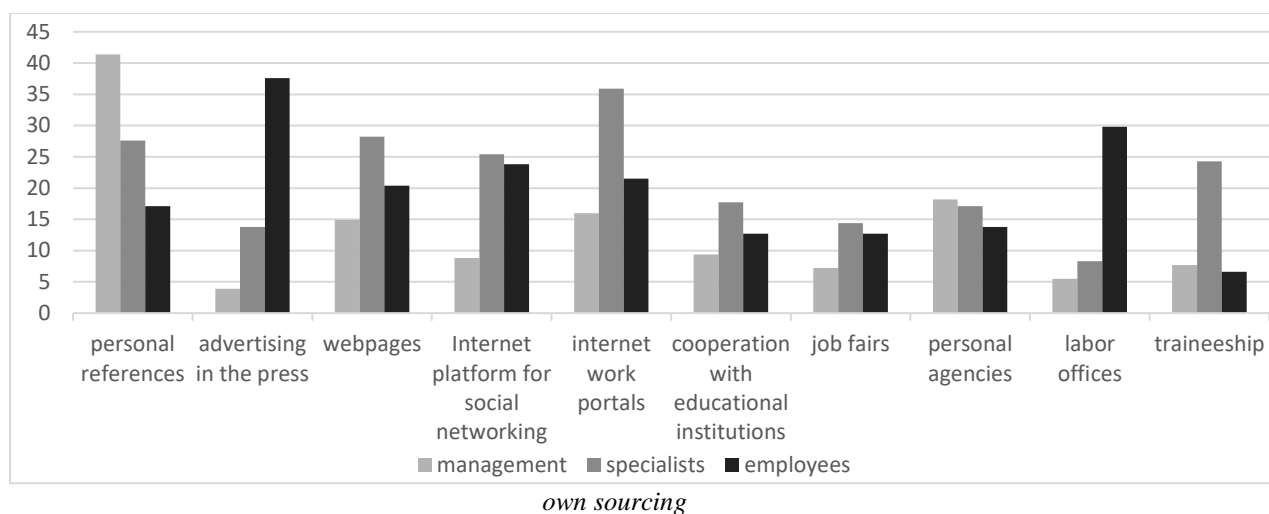
By comparing data, it can be stated that companies and financial institutions are trying to exploit the opportunities of social networks and digital technology in the search for employees. They search for information from social networks and work with Internet portals. They perceive these data as necessary but at the same time as complementary to traditional methods. They use traditional and modern methods as a necessary and effective combination. At the same time, the results show that many Slovak companies and financial institutions are still quite cautious and more oriented towards traditional methods. Thus, there is sufficient room to complement the necessary information and skills with the use of modern methods. Increasing courage to actively use modern methods will also gradually increase.

The frequency of using modern methods is associated with the perception of benefits, limitations and risks. Companies compare these factors: advantages - disadvantages. The results of the comparison then influence their attitude towards the use of these techniques in practice.

The results of the survey show (graph 4, 5) that the greatest contribution of modern methods is the possibility of verifying information about a potential candidate (30.6%). They are mostly used for this purpose by medium-sized enterprises (42.1%) and large enterprises (35.1%). Respondents also see a reduction in costs (28.8%) and easy access to candidate data (23.8%). Cost reduction is most appreciated by medium-sized enterprises (32.5%). Small businesses (28.2%) appreciate the availability of information about applicants.

Figure 4: Benefits of using social networks to find employees (%)

own sourcing
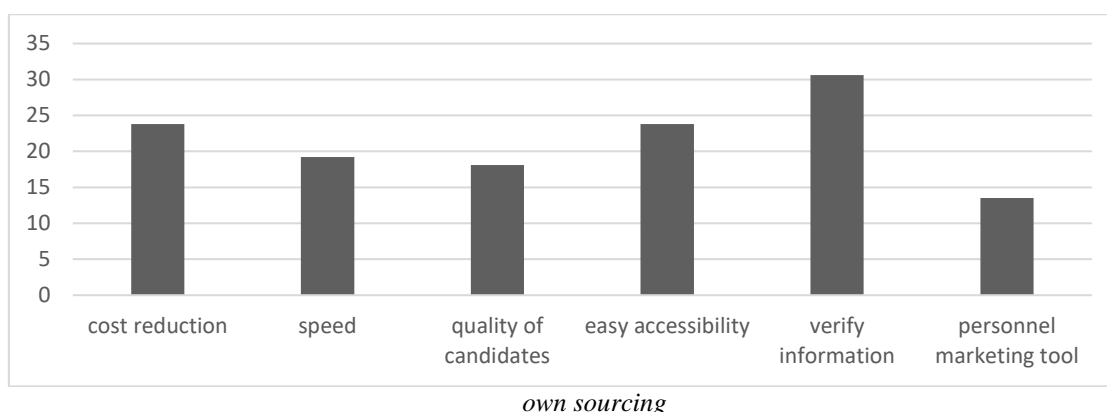
Figure 5: Benefits of using social networks to find employees - company size

own sourcing
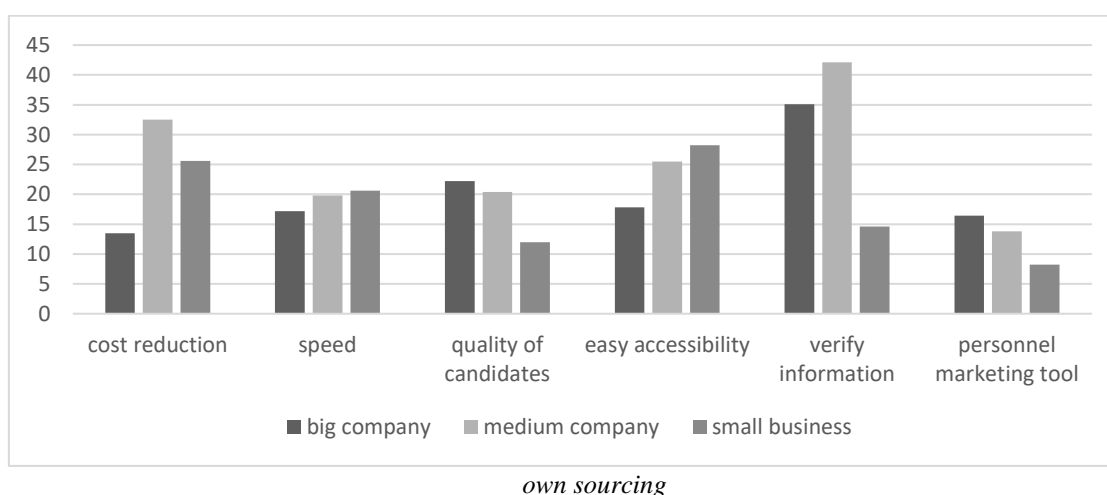
The benefits of social networking as a source of information in the search for staff from the personnel and work point of view, are presented in Table 1.

Table 1: The benefits of social networks from a personal and working perspective

1. Contacts with people
• similar professional interests
• online meetings of different groups of people, discussions,
• Provide a system of recommendations through acquaintances
• sharing different information
• use for HR managers to find and select a suitable job candidate
• use for job seekers to find jobs
• information about work and organization
2. Create a profile and promote the candidate
• personal data and user profile, necessary information
• a job seeker is expected to be sufficiently broad and to provide factual, concrete, and truthful information
• the possibility of adequate photographs
• a user profile can gradually replace a professional resume
• the HR manager uses the candidate profile data, but needs to be verified

3. Create an organization profile
• organizational profiles and publication of specific job offers
• information about the organization, its products and services, the organizational culture, the activities of the organization
• opening discussions on a topic, or setting up a specific problem, task - Tracking how to deal with them and people who volunteer
• the HR manager will review and then select from participants who have had an interesting approach to solving tasks
4. Social networks as a working portal
• publish job vacancies, job fairs
• providing tips and advice for job seekers
• allowing participants to discuss
• allows you to reach a candidate from different locations - distance is not an obstacle
• people who are interested in professional contacts and information about work choose professional social networks, especially LinkedIn
5. User references
• possibility of publishing user references or evaluating their capabilitie.
• the possibility of obtaining more information on candidates
6. Profile analysis and user confidence
• a supportive tool for recruiting and recruiting staff
• verifying candidate information or getting more information
• information found on the internet may be inaccurate or incorrect
7. Financial modesty
• search and work on social networks is mostly free of charge
• minimal costs, e.g. registration, targeted promotion and reaching out to candidates
• lower financial risks

own sourcing

From the point of view of the financial efficiency of using modern methods of searching for employees (social networks and available internet portals), companies see the benefits in two ways. Primary and secondary cost reduction. The primary cost reduction involves investing in setting up and managing a company portal and registering on social networks. Companies say that these costs are about 13% lower than when looking for candidates through advertisements and recruitment agencies. Secondary cost reductions are seen in a significant reduction in candidate search time and a greater selection of candidates. They estimate higher efficiency by approximately 18%. Respondents say that financial efficiency needs to be compared with the expertise. To what extent is the reduction of financial efficiency adequate to the quality of the candidates. It is necessary to monitor the comparison of the quantity and quality of candidates in searches through social networks and internet portals. Respondents perceive it necessary to gradually accept the financial costs of using digital communications (internet, social networks, etc.) as part of the company's financial and personnel strategy. They agree that the human factor in finding new people is still essential for analyzing information gathered from social networking resources.

The primary advantage of social networks for HR managers is the high popularity of users and the topicality of profile information. This increases the number of potential candidates. At the same time, it is possible to verify the applicant's quality - education, experience, complaints and so on. The advantage is that candidates are immediately available on social networks. This is the starting point that speaks for the use of these media. On the other hand, false or poorly set data may distort the information obtained from candidates. This increases the risk of a wrong decision.

3. Conclusion

It appears that in Slovakia the use of data from social networks for personal purposes, for example when searching for and reaching people, is still in the beginning phase. These media are being used, but not yet sufficiently. Research shows that companies and financial institutions rely more on classical than traditional methods. From modern search methods (social networks, internet portals, information communication), they are most used to search for specialist positions or at regular employee positions. Personal references and recruitment agencies are more used for management positions.

It can be stated that the use of information from social networks still has considerable potential. At the same time, it is necessary to take into account not only its advantages but also the possible risks that can be eliminated to some extent.

Based on this information, it can be stated that there are several possibilities of using social networks in the field of human resources. It is a great source of information about potential but also passive job candidates. Although many recruiters, as well as job candidates, consider social networks to be a progressive way of providing employment, this method is still used only to a limited extent. Its use is currently rather supportive. The future of social networks as a tool of HR managers will be influenced by the quality of experience of the company as well as people seeking work. Also, an attitude towards the publication of personal data on the Internet and the possibilities of their protection. At the same time, the position and level of job portal for job placement.

Acknowledgement

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Review and analysis of buyback in US equity market

Haochen Guo[1]

Abstract

The largest source of US equity demand has consistently been repurchasing. Goldman forecasts companies will spend a record $ 940 billion, with $ 1.1 trillion in buyback announcements on buybacks in 2019, up 16 percent from the previous record hit in 2018. However, during a rising legislative chorus calling for a halt to corporate buybacks, with the US Congress realizing that most of the funds released by tax cut and offshore tax repatriation were not used for capex or hiring but merely for levitating stock prices. Therefore, if the US Congress banned buybacks, the market would probably crash. Potential buyback restrictions would likely have five implications for the US equity market: slow growth in EPS; boosting cash spending on dividends, M&A, and debt payouts; widening trading ranges; decreasing demand for shares; and lower corporate valuations. This paper presents US equity market buyback theoretical and practical analysis.

Key words

Buyback, Share Repurchase, Volatility, US Equity Market.

JEL Classification: G10, G14, C5

1 Introduction

Buyback, also called share repurchase, it is the largest source of US equity demand. Recently, increasing stock repurchase cases have caused widespread concern on the market. We believe that repurchasing stocks is beneficial in order to increase EPS but may also result in tight cash flow in listed companies. Goldman forecasts firms will spend a record $940 billion (with $1.1 trillion in buyback announcements) on buybacks in 2019, up 16% from the previous record hit in 2018. From the impact of stock repurchases on stock prices, the potential buyback restrictions would likely have five implications for the US equity market: slow growth in EPS; boosting cash spending on dividends, M&A, and debt payouts; widening trading ranges; decreasing demand for shares; and lower corporate valuations.

During a rising legislative chorus calling for a halt to corporate buybacks (an activity that was illegal until 1982), with the US Congress realizing that most of the funds released by tax cut and offshore tax repatriation were not used for capex or hiring but merely for levitating stock prices. Therefore, the market would probably crash if the US Congress banned buybacks. One of the goals of research work is in unifying knowledge and terminology in the area of US equity market buyback within theoretical and practical analysis.

2 Literature Review

Buyback is like a dividend on cash allocation to investors, they were the legal owners of the business they retain. Shareholders spent their money in the company's share in the ex-anticipated return on their assets earned. The dividend, however, is pervasive in that context; businesses have been paying periodic money dividends since the founding of joint stock firms about three hundred years ago. Initially, it is the only way to pay the shareholders the excess. (Miller & Franco, 1961) first time suggests that the company value remains the same before and after

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payment of the dividend. So, in commercial finance, dividend payment is regarded a mystery. Before 1980, businesses chose dividend as a preferred technique of payment despite the comparative tax benefit of buying back (Barclay & Smith, 1988). But the repurchase development in the US was enormous after 1980. Buyback as spending rose from 4.8% in 1980 to 41.8% in 2000. The median buyback yield is nearly four times higher than the median dividend development. Buyback rose from 13.1% in 1980 to 113.1% in 2000 as a proportion of the complete dividend. In 1999-2000, businesses spent more on repurchase than dividends for the first moment in US history (Michaely & Grullon, 2002). Following this development, as an option technique of payout, many nations embraced buyback.

Buyback's primary aim was to allow firms to use their gain in two respects. In the form of dividends or buyback, one portion of the earnings can be transferred to investors. Retained profits are laid aside for additional investments in the company's potential development. (Baker, Powell, & Veit, 2003) used study technique to examine why US companies have recently purchased shares. They evaluated five hypotheses, i.e. signaling, free money stream agency costs, capital market allocations, tax-motivated dividend substitutions and changes to capital framework and discovered that undervaluation was the main motivation behind repurchase followed by absence of investment possibilities.

(Andriosopoulos & Hoque, 2013) reported that firm size, money dividend and concentration of property have an important effect on businesses ' buyback decisions in all of these nations (UK, Germany and France). The findings indicate that big and commonly owned businesses are more probable to announce share buyback and discovered a supplementary connection in the UK and Germany between dividend payment and share buyback. (Yarram, 2014) several variables affecting Australia's open business repurchases were examined. The research promotes the agency's hypotheses of signaling and leveraging and does not promote surplus money flow and hypotheses of replacement. This study's distinctive input is to mix corporate management factors with other company-specific parameters to examine the effect of corporate governance on Australia's repurchase choices. The findings indicate that the autonomy of the board has a beneficial impact on the choices of repurchase.

3   Theoretical Analysis

Buying away its stocks from an investor by a business that puts up venture assets for the firm's data. The stocks are purchased back at a cost that satisfies the shareholder, the amount that the firm is prepared to pay for its autonomy. If the firm is openly listed or passed over, the buy-back may happen. A corporation's buying back of its shares, particularly in the US, to reduce the number on the market, either to increase the return on those still available shares or to remove threatening shareholders. Action by the govt of a developing country to decrease some or all its debt to foreign banks by purchasing back that debt at the exchange cost or at a significant discount. The bank's appeal is the suppression of a harmful and adverse loan that may have already been made available in its balance sheet. A return to creditworthiness and the chance of obtaining fresh credits is the benefit for the nation in debt.

The Motivation of buyback

Based on the cases of buyback, there are four main reasons for buyback motivation: Prevent others company's M&A; Revitalize the stock market; Maintain or increase EPS and stock price; Recapitalization.

Buyback mode

There are five mode for buyback: According to the location of stock repurchase, it can be divided into two types: open public acquisition and over-the-counter agreement; According to the financing method, it can be divided into debt repurchase, cash repurchase and mixed repo; According to the scope of asset replacement, it is divided into the sale of assets to repurchase

shares, the use of handheld bonds and preferred stock exchange (repurchase) of the company's common stock, debt equity swap; According to the method of determining the repurchase price, it can be divided into fixed price offer repurchase and Dutch auction repo; Transferable sale right repurchase method.

Buyback blackout period

Since the big outflows of over $100bn in October 2018, US investment funds this week saw inflows rise over $31bn in the first three days, helping push the S&P 500 back to the top of its latest spectrum. March 2019's second-largest inflow after $38.30bn compares with a combination of tiny inflows and outflows including a $5.83bn outflow in the previous week. What to make of this sudden fund flow reversal? These episodes were of two types: In many episodes surging inflows were a contrarian indicator; Exceptions were when inflows were just catching up after lagging the market.

So as firms are about to go cold turkey on what was another year of record buyback, who is going to buy-or sell? The reply is blended: Vol Control's risk is downside down. Vol Control resources are close to complete allocations of capital, so if volatility increases, danger is downside; CTAs are ready to purchase within 7-10 days if a sell-off does not materialize. In other words, if they do not sell, they will purchase; Risk Parity remains to purchase tailwind equities.

The potential restriction on buyback in 2019

In the midst of a growing parliamentary chant calling for a stop to corporate buybacks, with Congress knowing that most of the resources issued through tax cuts and overseas tax repatriation were not used for capex or recruiting but simply to levitate inventory rates, the prospective buyback limitation would probably have five consequences for the US equity market: slow EPS growth; boost cash spending on dividends, M&A, and debt paydown; widen trading ranges; reduce demand for shares; lower company valuations.

Clearly concerned that the anti-buyback movement is gaining momentum in Congress – another study document on the "Buyback Realities" was released, in which it paradoxically attempted to mitigate the part buybacks play in stock creation and asset misallocation just one week after explaining how prohibiting buybacks would have catastrophic effects on shares. (Goldman Sachs, 2019) In short, buybacks were illegal until 1982 for a reason - currency manipulation - and then gradually became commonplace, with inventory buybacks and dividends increasing to 90% of the S&P 500 profit cumulative payout percentage in the 2002-2018 period. The cherry on top: Goldman forecasts firms will spend a record $ 940 billion (with $ 1.1 trillion in buyback announcements) on buybacks in 2019, up 16% from the previous record run in 2018.

4   Buyback Model Analysis

Basic buyback model

(Vermaelen, 1981) presented the basic model of share repurchase and assumptions. There are seven assumptions (the basic conditions of equilibrium pricing of securities) of this basic share repurchase model: the market is efficient in the sense that at any time market prices reflect all publicly available information relevant for the pricing of securities; after the announcement date, shareholders have homogeneous expectations with respect to the change in value, the fraction of shares tendered, and the fraction of shares purchased by the company; individual investors are price-takers and cannot influence the outcome of an offer; offers are 'maximum limit' offers: if the offer is undersubscribed the firm will buy all shares tendered (if any). If the offer is oversubscribed, the company will buy all shares tendered or will allocate the shares pro rata; undersubscribed offers are not expected to be extended; in making their decisions, shareholders maximize the value of their wealth, after personal taxes and transaction costs; the expected or realized price change caused by market wide events during the tender offer period can be ignored.

Dividend payout ratio

What are the consequences for the system of dividend payment? Focusing solely on dividends paid as the only money transferred to stockholders exposes us to the danger of losing important money transferred in the form of inventory buybacks to stockholders. The easiest route to include inventory buybacks in a dividend discount system is by adding them to the dividends and calculating an altered payout ratio:

$$\text{Dividend Payout Ratio} = \frac{Dividends\ Paid}{Net\ Income} \tag{1}$$

$$\text{Modified Dividend Payout Ratio} = \frac{Dividends + Stock\ Buybacks}{Net\ Income} \tag{2}$$

While this adjustment is simple, the subsequent one-year proportion may be skewed by the reality that inventory buybacks are not rounded out, unlike dividends. Consequently, considering the median value over a four-or five-year span, a much stronger assessment of the altered payout percentage can be achieved. Furthermore, sometimes firms can buy back inventory as a manner to increase economic leverage. This could be adjusted by netting fresh bonds released from the above calculation:

$$\text{Modified Diviend Payout} = \frac{Dividends + Stock\ Buybacks - Long\ Term\ Debt\ Issues}{Net\ Income} \tag{3}$$

Adjusting the payout ratio to include stock buybacks will have ripple effects on the estimated growth and the terminal value. In particular, the modified growth rate in earnings per share can be written as:

$$\text{Modified Growth Rate} = (1 - \text{Modified Payout Ratio}) \times ROE \tag{4}$$

Even the return on equity can be affected by stock buybacks. Since the book value of equity is reduced by the market value of equity bought back, a firm that buys backs stock can reduce its book equity (and increase its ROE) dramatically. If we use this ROE as a measure of the marginal ROE (on new investments), we will overstate the value of a firm. Adding back stock buybacks in recent year to the book equity and re-estimating the ROE can sometimes yield a more reasonable estimate of the ROE on investments.

Value of growth

Investors pay a price premium when they acquire companies with high growth potential. This premium takes the form of higher P/E or P/BV ratios. While the proposal that development is important will not be contested by anyone, it is feasible to pay too much for development. In fact, empirical studies that show low P/E ratio stocks earning return premiums over high P/E ratio stocks in the long-term supports the notion that investors overpay for growth. The value of the equity in any firm can be written in terms of three components: extraordinary growth, stable growth and assets in place.

Extraordinary Growth

$$
= \frac{DPS_0 \times (1 + g) \times (1 - \frac{(1 + g)^n}{(1 + K_{e,hg})^n})}{K_{e,hg} - g} + \frac{DPS_{n+1}}{(K_{e,st} - g_n)(1 + K_{e,hg})^n}
$$
$$
- \frac{DPS_1}{(K_{e,st} - g_n)} \tag{5}
$$

$$\text{Stable Growth} = \frac{DPS_1}{(K_{e,st} - g_n)} - \frac{DPS_0}{K_{e,st}} \tag{6}$$

$$\text{Assets in Place} = \frac{DPS_0}{K_{e,st}} \tag{7}$$

Estimating the Value of Growth (P_0) = Extraordinary Growth + Stable Growth + Assets in Place $\tag{8}$

where DPS_t = Expected dividends per share in year t,
K_e = Required rate of return,
$P_n = Price\ at\ the\ end\ of\ year\ n,$
g = Growth rate during high growth stage,
$g_n = Growth\ rate\ forever\ after\ year\ n.$

5   Case Study

In 2018, stock buybacks collapsed across the roof. Companies in the Standard & Poor's 500-stock survey alone announced plans to buy back nearly $1 trillion in stocks–a strategy that not only adds value to the residual inventory, but also increases economic per-share metrics in their annual accounts. For 2019 to be another powerful year for stock repurchases, the climate is correct. Indeed, several organizations have produced their announcements already. We are reviewing the chosen business that just since the start of 2019 has imitated or enhanced inventory buybacks.

Banking and wealth management company Comerica (CMA) declared that its board has initiated a stock buyback plan on 22nd January 2019, which authorizes the company to repurchase 15 million shares. This repurchase authorization authorizes the company to buy shares of its stock through open market purchases. Shares repurchase plans are often a sign that the company's management believes its stock is undervalued. Following Tables and Figures are present the analyze stock buyback effect.

Table 1 Buyback Impact	Pre-buyback	Post-buyback	Change
Revenue USD Mil	3,328	3,379	2%
Net Income USD Mil	1,235	1,293	5%
Earnings Per Share (EPS) - Basic	1.91	7.83	310%
Earnings Per Share (EPS) - Diluted	1.88	7.72	311%
Dividend Per Share	1.84	2.68	46%
Buyback Yield %	8.39	13.49	61%
Total Yield %	11.07	16.37	48%
Payout Ratio %	25.8	28.6	11%
Book Value Per Share	48.64	48.06	-1%
EBT Margin	47.72	49.16	3%
Net Margin %	36.87	38.03	3%
Return on Assets %	1.72	1.8	5%
Financial Leverage (Average)	9.43	9.54	1%
Return on Equity %	15.86	16.68	5%
Financial Leverage	9.43	9.54	1%
Debt/Equity	0.86	0.92	7%

Figure 1 Valuation Impact   （*Price/Earnings, Price/Book, Price/Sales, Price/Cash Flow*）

Price/Earnings	pre-buyback	post-buyback
CMA	11,5	9,9
S&P 500	17,2	20,6

Price/Book

	pre-buyback	post-buyback
■ CMA	1,4	1,6
■ S&P 500	2,9	3,3

■ CMA ■ S&P 500

Price/Sales

	pre-buyback	post-buyback
■ CMA	3,6	3,8
■ S&P 500	1,9	2,2

■ CMA ■ S&P 500

Price/Cash Flow

	pre-buyback	post-buyback
■ CMA	9,8	9,9
■ S&P 500	11,7	13,3

■ CMA ■ S&P 500
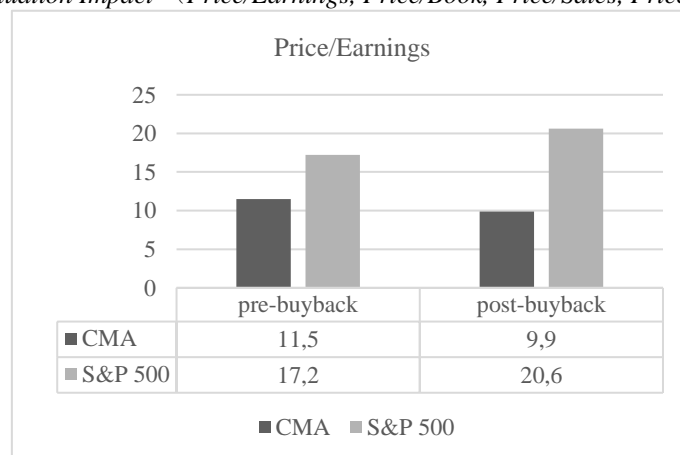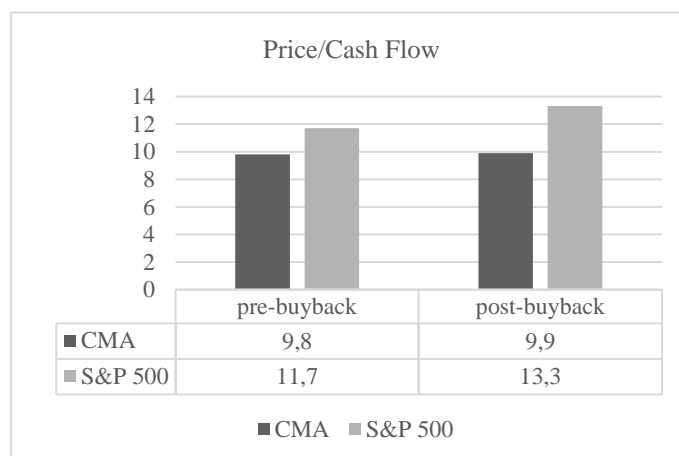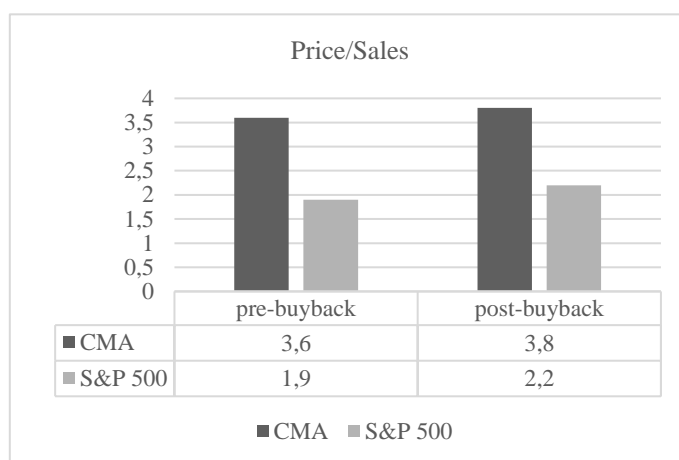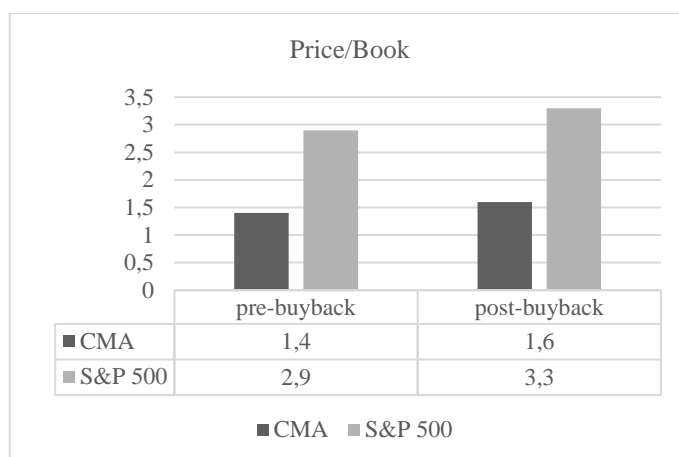
Comerica is now a successful inventory to boost profitability through GEAR up projects owing to its diligent impacts. The organic and inorganic development policies of the company also render it well-prepared for future efforts. In addition, the opportunities for income development and favorable assessment render it an appealing choice. Comerica not only hit projections in the last quarter recorded, but also witnessed changes of upward projections showing the optimism of analysts about their opportunities.

Revenue Strength: Comerica remains to advance steadily towards enhancing its top line. Revenues witnessed a 9.1% compounded annual growth rate over the last five years (2014-

2018). Also, the company's projected sales growth of 4.59% (the industry average was 2.79%) indicates constant upward momentum in revenues.

Earnings Growth: Over the past three-five years, Comerica has seen earnings growth of 19.87% relative to the 10.87% growth of the industry. Additionally, the long-term (three-five years) company's approximately 19.6% EPS development level offers long-term benefits for shareholders. In addition, in the previous four quarters, it produced an average favorable earnings shock of 7.23%.

Involvement in Strategic Growth Initiatives: Comerica's focus on enhancing operational effectiveness led to the launch of GEAR Up projects in mid-2016. The implementation of these projects, together with a rise in short-term prices, resulted in an effectiveness proportion of 53.6% in 2018, up from 58.6% in 2017. Also, in 2019 and beyond, these projects are expected to offer extra advantages in $35 million pre-tax income.

Steady Capital Deployment Activities: The board of managers increased the annual dividend by 12% in January 2019 and endorsed a scheme to repurchase a further 15 million popular stocks. In addition, the debt / equity ratio and dividend payout ratio of the firm compare positively with that of the wider sector showing the sustainability of such dividend hikes.

Strong Leverage: The debt/equity ratio of Comerica is estimated at 0.86 relative to the sector median of 0.92, suggesting a comparatively reduced debt burden. It also emphasizes the company's financial sustainability despite an uncertain economic environment.

Superior Return on Equity (ROE): Comerica's ROE of 15.84% relative to the sector median of 12.83% highlights the com-mendable status of the company over its colleagues.

Stock is Undervalued: Comerica feels undervalued in terms of price-to-earnings (P/E) and PEG ratios. The 10.17 P/E ratio of the company is below the 10.75 sector median. The company's PEG ratio is also 0.52 versus the sector median of 1.26.

6   Conclusion

This paper showed that the buyback of share activities in US equity market. The introduction of buyback of shares in US showed clearly that the main purpose was the profits that can be distributed to shareholders in the form of dividends or share repurchases. This paper covered the buyback of shares played an important role in the companies i.e., researchers have explained the share repurchases in all over the world indicated the signaling to the investors or market and it has the substitute or alternative for dividends. Based on analysis presents there are many factors influence the company to go buyback of shares.

If the most malignant use of corporate resources is to pay excessive CEO wages, inventory buybacks may well take second position. Conventional knowledge is that to manipulate the short-term inventory cost, CEOs purchase back inventory. By reducing capital, they finance the buyback, and so strong valuation declines in the long run. The state that buying back inventory forces companies to cut investment places the cart in front of the horse. A more plausible perspective is as follows: First, firms allocate resources to investment based on accessible possibilities. If after getting all value-creating business possibilities, they have spare money left over, then they can use it for buybacks. For other purposes, repurchase flexibility is appealing. Credit card consumers and businesses with rotating loan lines appreciate the choice to repay their debt at any moment. They overpay especially when the interest level–the rate of exchange that the bank requires–is large, just as companies especially repurchase when the inventory cost is small and therefore the rate of exchange needed by investors is large. If a loan card permitted customers to make the minimum monthly deposit only, few would bring the card out. Similarly, if companies are limited from purchasing back stocks, they may not initially issue capital. Instead of funding themselves by carrying on more debt, fewer businesses would go public. For a second reason, debt is a helpful metaphor. A borrower who pays back debt, by decreasing her

potential interest commitments, makes an investment that pays off in the future. Likewise, in the future, a business that purchases back inventory will have to pay less dividends.

Another benefit of dividends repurchases is that they result in more focused property. If a business purchases back inventory, the CEO now has a larger proportion of the residual shares and therefore now has stronger incentives to enhance firm quality. Typically, higher CEO ownership shares enhance long-term yields on stocks. And buybacks focus not only on the CEO's property, but also on the ongoing investors. A prevalent government company problem is that it is held by millions of distributed investors whose shares are too tiny to motivate them to look beyond short-term income. They generate block owners – big shareholders – by focusing the ownership of ongoing investors. Because these investors are encouraged to look beyond income and look instead at the long-term development possibilities and intangible resources of a company. But, while the proof indicates buybacks add importance, the concept that some buybacks might be short-term. A thorough latest research demonstrates that buybacks that enable a business to match the income forecasts of analysts when they have lost, otherwise they are combined with job and capital cuts. While we don't know if cutting is bad or good investment, this evidence certainly doesn't rule out short-termism. But the issue here is not so much buyback, but payment systems for CEOs that encourage them to reach objectives for earnings. (Edmans, 2017). These agreements result in capital reductions because such reductions assist the CEO fulfill the aim, irrespective of whether the saved money is used for repurchases.

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Tax havens and company liquidity: Evidence from Slovakia

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Abstract

The purpose of this contribution is to analyse and compare the liquidity ratios between Slovak companies with direct ownership links to tax havens to Slovak companies with no ownership links to tax havens. As a direct ownership link to a tax haven we consider the first ownership level (direct equity linkage) with selected jurisdictions, which are further divided into three categories (onshore, midshore and offshore) in our analysis. We investigated the selected liquidity ratios (current, quick and operating cash flow ratios) for the years 2014 and 2015. During the analysis we matched two databases. The first one is provided by the company Bisnode, ltd. and contains the list of Slovak companies with foreign ownership links (selected jurisdictions). The second database is provided by Finstat, ltd. and contains the financial statements of all Slovak companies. Our results suggest that Slovak companies with ownership links to tax havens report statistically significant lower liquidity ratios compared to their counterparts.

Key words

Liquidity ratios. Foreign ownership link. Tax haven. Tax planning.

JEL Classification: F23, H25, H26

1. Introduction

A tax haven according to the "classic" definition is a country that imposes low or no taxes. Typically, tax havens are used by corporations willing to avoid paying taxes in countries with a high or higher level of corporate income tax burden. Common characteristics are a lack of transparency in legislative operations, lack of information exchange and a high anonymity of the ultimate beneficial owner (UBO) (European Parliament, 2018). Murphy (2018) recognizes 7 types of tax havens based on a variety of specific features (incorporation locations, secrecy locations, specific geographic market suppliers, specialist service providers, market entry conduits, high net worth providers and the tax raider). Tax havens are probably most often linked with multinational profit shifting activities. These activities are perceived to be a major threat to the tax base of high-tax economies around the world. According to Dharmapala and Riedel (2013), the existing studies employ indirect identification approaches that analyse shifting behaviour by estimating firm responses (adjustments in reported pre-tax profitability, intra-firm transfer prices or debt-equity structure) to changes in the corporate tax rate or the difference in tax rates between home and host countries. Saleem and Rehman (2011) state that the liquidity ratios affect the profitability ratios. Liquidity and profitability are closely related

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because when one increases the other decreases. Madushanka and Jathurika (2018) based on the correlation analysis identified the positive relationship (impact) of liquidity ratios on a firm's profitability. Fuest and Riedel (2010) focused on developing countries and showed how firms belonging to a multinational group obtained a lower ratio of return on assets than domestic firms, while multinationals with related parties in tax havens did not obtain a significantly lower ratio than national firms. Potin, Silva, Reina and Neto (2016) found a relationship between tax planning and return on assets (ROA). Companies practising aggressive tax planning are related to a low ROA, companies practicing moderate tax planning report high values of ROA. Nwaobia and Jayeoba (2016) emphasise that not all tax planning strategies necessarily lead to incremental cash flow (liquidity) for the firm. Under certain conditions, the profit-shifting techniques based primarily e.g. on debt (interest expenses) may lead to descending cash flow (liquidity) for the firm. Ištok and Kanderová (2019) provide empirical evidence on using debt (interest expenses) as an often-used profit-shifting technique among Slovak companies with direct links to tax havens.

2. Results and discussion

The results of the analysis are structured into two complementary parts. The first one deals with the analysis of liquidity ratios between companies in tax havens in comparison to companies in Slovakia with no ownership links to tax havens. The second one is focused on a more detailed analysis of achieved results among companies in three different types of low tax jurisdictions and those with no links. Both analyses were carried out for the years 2014 and 2015.

Based on the selected empirical studies mentioned above and based on their mutual comparison and synthesis we assume that companies in tax havens will show worsened liquidity ratios. There is data available for 164,923 Slovak companies for both periods, 2014 and 2015, since we have excluded companies that reported incorrect data (missing values or zero value of denominator of liquidity ratios - current liabilities). In addition, we have also excluded companies where at least one of the indicators was showing outlier (2%), which could distort the results of statistical tests. We have data available for 2,105 Slovak companies with a direct ownership link to a tax haven (based on matched databases of Bisnode and Finstat). We reduced the large difference between number of companies with links (2,105) and without links to tax havens (159,520) by conducting a random selection of 5% of the ordinary companies (7,976).

The companies´ ability to pay off debt obligations without raising external capital is usually determined via liquidity ratios. The coverage of short-term debt is evaluated by current ratio which analyses current liabilities in relation to most liquid assets – financial accounts. Quick ratio analyses current liabilities in relation to financial accounts and short-term receivables while operating cash flow ratio considers also short-term assets such as inventories and prepaid expenses. In general, the higher the ratio is the better the company´s liquidity position.

To test statistically the significant difference between selected liquidity ratios, the nonparametric tests were used because the data did not meet the assumptions of parametric t-tests. We used Mann Whitney U test and Kruskal-Wallis H test, which is considered to be an extension of the Mann-Whitney test, to allow the comparison of more than two independent groups. Because we performed several statistical tests simultaneously what can increase the like-hood of incorrectly rejecting a null hypothesis we also used the Bonferroni correction.

The results of the Mann-Whitney test (statistically significant level of 5%) between the selected groups of companies with and without ownership links to tax havens are shown in Table 1.

Table 1: Mann-Whitney Test (liquidity ratios)

Test Statistics		
Liquidity ratios 2014		no links vs. with links
Current ratio	Z	-17,274
	Asymp. Sig. (2-tailed)	0,000
Quick ratio	Z	-9,851
	Asymp. Sig. (2-tailed)	0,000
Operating cash flow ratio	Z	-9,583
	Asymp. Sig. (2-tailed)	0,000
a. Grouping variable: Links to tax havens		

Test Statistics		
Liquidity ratios 2015		no links vs. with links
Current ratio	Z	-14,813
	Asymp. Sig. (2-tailed)	0,000
Quick ratio	Z	-6,312
	Asymp. Sig. (2-tailed)	0,000
Operating cash flow ratio	Z	-6,118
	Asymp. Sig. (2-tailed)	0,000
a. Grouping variable: Links to tax havens		

Source: own processing by the software SPSS

Nonparametric Mann-Whitney tests showed a statistically significant difference in the values of all three liquidity ratios for both investigated periods (p-value 0.000). The basic descriptive statistics of current ratio, quick ratio and operating cash flow ratio are shown in Table 2.

Table 2: Descriptive characteristics of liquidity ratios

Current ratio_2014	With links	With no links	**Current ratio_2015**	With links	With no links
N Valid	2105	7976	N Valid	2105	7976
Missing	0	0	Missing	0	0
Mean	6,0662	6,8196	Mean	6,5717	5,5855
Median	0,1319	0,4585	Median	0,1564	0,4498
Std. Deviation	69,35574	93,2221	Std. Deviation	71,3517	64,7818
Quick ratio_2014	With links	With no links	**Quick ratio_2015**	With links	With no links
N Valid	2105	7976	N Valid	2105	7976
Missing	0	0	Missing	0	0
Mean	27,6976	9,9454	Mean	38,1972	23,7836
Median	0,8818	1,142	Median	0,9521	1,1229
Std. Deviation	587,0113	131,8423	Std. Deviation	646,7675	1343,801
Operating cash flow ratio_2014	With links	With no links	**Operating cash flow ratio_2015**	With links	With no links
N Valid	2105	7976	N Valid	2105	7976
Missing	0	0	Missing	0	0
Mean	29,367	10,3728	Mean	40,6667	24,1038
Median	1,0442	1,3051	Median	1,126	1,2915
Std. Deviation	587,6196	132,4537	Std. Deviation	648,2745	1343,806

Source: own processing by the software SPSS

The arithmetic average has a low reporting ability due to the extremely high variability of the liquidity ratios, so we have focused on median values in the evaluation with respect to the diameter. In general, the recommended value of the current ratio is <0.2-0.6>. Companies with links to tax havens have a current ratio lower than the specified interval in both periods under review, so the threat of being unable to repay their liabilities is actual. Enterprises with no links to tax havens have a given liquidity ratio at the optimum interval. Similar results are obtained from the quick ratio analysis. The median of a given business indicator with links to tax havens is slightly lower than the recommended interval <1.0-1.5> in both investigated

periods. At the same time, businesses with no links to tax havens reach a value of quick ratio in a given interval during both periods. As the recommended interval for the operating cash flow ratio is <2.0-2.5>, it is clear that no group of companies has sufficient liquidity for that level. Overall, businesses with links to tax havens get worse results from all three liquidity ratios than businesses with no links to tax havens, although in 2015 the indicators are closer to optimal than 2014 levels. Companies that have moved or established their headquarters in jurisdictions marked as tax havens generally have worse liquidity and thus a greater risk of being unable to repay their liabilities in the near future. This fact can be explained by the fact that companies often use debt as a profit-shifting technique, thus increasing their current liabilities in addition to interest expenses, which is also the denominator of all three liquidity ratios. As a result, the liquidity ratios are lower than those of conventional firms.

Tax havens are most often divided into three categories: onshore, midshore and offshore[2]. Each category has different characteristics and on the first ownership level are used to reach different goals. Offshore jurisdictions are used despite a relatively poor image to hide the UBO. Tax optimization on the first ownership level when using an offshore company is very limited as there are relatively high withholding taxes applied (19%, resp. 35%). For tax optimization, jurisdictions from onshore and midshore categories are mainly used due to the possibility to use selected EU directives or signed double taxation treaties (DTTs). Midshore jurisdictions are used mainly in aggressive tax planning. The main advantage of midshore jurisdictions is also the relatively low costs of forming and managing the companies. Other than potential tax planning, onshore jurisdictions also offer a good image and benefits in asset management and asset protection. For a more detailed picture of the effect of tax optimization on liquidity ratios we performed testing of statistically significant differences between companies with ownership links in different categories of tax havens and companies with no ownership links. The results of the Kruskal-Wallis test are shown in Table 3.

Table 3: Kruskal-Wallis Test (liquidity ratios)

Test Statistics		
Liquidity ratios 2014		JURISDICTION
Current ratio	Chi-Square	311,566
	Asymp. Sig.	0,000
Quick ratio	Chi-Square	103,352
	Asymp. Sig.	0,000
Operating cash flow ratio	Chi-Square	101,324
	Asymp. Sig.	0,000
a. Kruskal-Wallis Test		
b. Grouping variable: Company Jurisdiction		

Test Statistics		
Liquidity ratios 2015		JURISDICTION
Current ratio	Chi-Square	241,475
	Asymp. Sig.	0,000
Quick ratio	Chi-Square	55,271
	Asymp. Sig.	0,000
Operating cash flow ratio	Chi-Square	54,753
	Asymp. Sig.	0,000
a. Kruskal-Wallis Test		
b. Grouping variable: Company Jurisdiction		

Source: own processing by the software SPSS,

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[2] Jurisdictions have been divided into three categories in our analysis:
  a) OFFSHORE JURISDICTIONS (OFF): Bahamas, Belize, Bermuda, British Virgin Islands, Gibraltar, Guernsey (United Kingdom), Jersey (United Kingdom), Cayman Islands, Marshall Islands, the Netherlands Antilles, Panama, Man Island, and Seychelles;
  b) MIDSHORE JURISDICTIONS (MID): Hong Kong, Cyprus, Malta, United Arab Emirates, United States of America;
  c) ONSHORE JURISDICTIONS (ON): Liechtenstein, Latvia, Luxembourg, Monaco and the Netherlands.

The Kruskal-Wallis Test confirmed statistically significant differences between individual groups of companies, with all three liquidity ratios for both periods examined. The Mann-Whitney Test was used to determine the specific differences between the categories of tax havens compared to those with no links to tax havens. Often, the first type of error occurs when performing the test, so Bonferroni's correction is recommended (Field, 2005). The first type of error is where we reject the null hypothesis of equality of mean values in a set even if it is true. Since we divide businesses into four independent files, using Bonferroni's correction, the original 5% significance level is reduced to 0.83%. We assume that the most significant differences will be in the onshore and midshore categories, as the offshore category is primarily used to acquire the real owner's anonymity. If the offshore category is used on the first ownership level, then the use of profit-shifting techniques is limited due to the withholding taxes. The results of the Mann-Whitney test are shown in Table 4.

Table 4: Mann-Whitney Test (selected liquidity ratios among each category of jurisdictions)

Test Statistics[a]		2014			2015		
Jurisdiction		Current ratio	Quick ratio	Op. Cash flow ratio	Current ratio	Quick ratio	Op. Cash flow ratio
with no links-OFF	Z	-4,627	-2,994	-3,149	-2,457	-0,112	-0,427
	Asymp. Sig. (2-tailed)	0,000	0,003	0,002	0,014	0,910	0,669
with no links-MID	Z	-12,388	-8,435	-8,778	-11,591	-6,822	-7,111
	Asymp. Sig. (2-tailed)	0,000	0,000	0,000	0,000	0,000	0,000
with no links-ON	Z	-13,374	-6,044	-5,163	-11,451	-3,592	-2,787
	Asymp. Sig. (2-tailed)	0,000	0,000	0,000	0,000	0,000	0,005
a. Grouping variable: company links to tax heaven							

Source: own processing by the software SPSS

Based on the test results, it is obvious that our assumption was confirmed. Enterprises in offshore jurisdictions reached statistically significant differences in liquidity ratios only in 2014, and in 2015 the differences are no longer statistically significant based on our significance level (0.0083). Thus, enterprises in offshore jurisdictions have increased their level of all three ratios liquidity. If we compare liquidity ratios between businesses with no links to tax havens and businesses in midshore or onshore jurisdictions, it is clear from the test that they are achieving statistically significant differences in both periods under review.

For a more detailed picture of the impact of tax havens at the first stage of ownership structure on liquidity ratios, we have further carried out descriptive statistics of liquidity ratios in each category of jurisdiction, the results being shown in Table 5.

Table 5: Descriptive characteristics of liquidity ratios in given categories of jurisdictions

Current ratio_2014		OFF	MID	ON	Current ratio_2015		OFF	MID	ON
N	Valid	363	866	876	N	Valid	363	866	876
	Missing	0	0	0		Missing	0	0	0
Mean		13,221	7,0345	2,1441	Mean		11,4042	7,7012	3,4526
Median		0,1859	0,1236	0,1229	Median		0,3360	0,1207	0,156
Std. Deviation		107,7601	80,4207	19,9649	Std. Deviation		82,1062	87,31025	43,5444
Quick ratio_2014		OFF	MID	ON	Quick ratio_2015		OFF	MID	ON
N	Valid	363	866	876	N	Valid	363	866	876
	Missing	0	0	0		Missing	0	0	0
Mean		19,375	55,4208	3,7397	Mean		27,8233	75,594	5,5262
Median		0,931	0,7784	0,9656	Median		1,1031	0,8305	1,0058
Std. Deviation		117,6642	911,3353	20,9907	Std. Deviation		177,429	999,8193	46,0065
Operating cash flow ratio_2014		OFF	MID	ON	Operating cash flow ratio_2015		OFF	MID	ON
N	Valid	363	866	876	N	Valid	363	866	876
	Missing	0	0	0		Missing	0	0	0
Mean		25,6318	55,9377	4,6474	Mean		38,3956	75,988	6,6897
Median		1,0515	0,9656	1,1478	Median		1,2121	1,0163	1,2349
Std. Deviation		133,3891	911,951	23,3259	Std. Deviation		206,531	911,3514	48,1773

Source: own processing by the software SPSS

Significantly more businesses are located in onshore and midshore jurisdictions. Of the 2,105 businesses we surveyed with links to tax havens, only 17.3% (363) are in offshore jurisdictions. At the same time, companies in offshore jurisdictions achieve the slightest differences in liquidity ratios compared to businesses with no links to tax havens.

The median current ratio is closest to the recommended interval in both periods, and in 2015 it is within the given interval. Although the median quick ratio reaches almost the same value as enterprises in onshore jurisdictions, for both types of jurisdictions the medians are very close to the recommended interval <1.0-1.5>, but it is important to look at the average value of that indicator. In offshore jurisdictions, it is significantly higher (similar to those with no links to tax havens) than in onshore jurisdictions, again confirming that the smallest differences are between offshore businesses and businesses with no links to tax havens. The results of the operating cash flow indicator are similar. Offshore jurisdictions are even slightly lower than onshore, but average values are several times higher, causing businesses in offshore jurisdictions to achieve similar liquidity ratios as businesses with no links to tax havens. The results are in line with the assumption in offshore jurisdictions where businesses base their headquarters mainly because of the actual owner's anonymity, so their liquidity results are similar to businesses with no links to tax havens (no direct use of aggressive tax planning techniques is envisaged using tax havens) .

Companies in midshore jurisdictions achieve the lowest values for all three liquidity ratios, with values significantly lower than the recommended intervals for those indicators. The result is consistent with the assumption that Slovak companies with direct ownerships to the midshore category show the worst liquidity ratios results.

As mentioned above, midshore jurisdictions at the first level of ownership are used for aggressive tax planning. According to a study by Ištok and Kanderová (2019), Slovak companies use debt (interest expenses) and significant profit-shifting methods out of the Slovak Republic. Due to the increased level of capabilities, worse liquidity ratios could be expected. Thus, our results are consistent with the outcomes of Nwaobia and Jayeobia (2016) and Silva, Reina and Neto (2016), who argue that the relationship between liquidity and profitability is not always valid in tax optimization. This relationship is also influenced by the

profit-shifting technique used. In the case of intensive use of debt (high debt/asset ratio) then lower values of both liquidity and profitability ratios could be expected.

Our analysis faced several limitations. The first limitation is the period under review, which was only two years (2014 and 2015). Another limitation was, in our opinion, an incomplete list of tax havens that was available for our research. We consider the absence of data of Slovak companies with the owner at the first level from Great Britain as a significant limitation. According to available data (Bisnode, 2019a), in March 2019 there were 1,466 registered companies in the Slovak Republic which had direct foreign ownership structured through the United Kingdom (owned by share capital in the amount of EUR 1,061 billion). In the authors' opinion, research into a similar focus should be relatively cautious in formulating generalizations and rules, since foreign direct ownership through tax havens is not always linked solely to "classical economic reasons", e.g. tax optimization. Tax havens are often used for reasons that only the UBO itself knows or a very limited range of people. We don't know the reason for the ownership link to a tax haven, but we can at least identify the trends. Furthermore, according to the authors, it is necessary to pay more attention to tax havens, not only on the level of providing empirical evidence on profit-shifting techniques, but also on the use of individual techniques with regard to the overal tax system settings and their consequences not only on a macroeconomic but also a microeconomic level.

3. Summary

This work aimed to compare liquidity ratios between Slovak companies with direct ownership links to tax havens to their counterparts. Our results confirmed statistically significant differences in the values of all three liquidity ratios for the investigated years 2014 and 2015. The median values of all three liquidity ratios are lower in the case of Slovak companies with ownership links to tax havens compared to those without these links. Among all liquidity ratios, the worst median values were recognized for midshore jurisdictions, which are on the first ownership level used mainly for tax optimization purposes. As for the comparison between companies with ownership links in different categories of tax havens and companies without ownership links, the statistically significant difference in median values was confirmed for offshore category only in 2014 (for onshore and midshore categories confirmed for both periods). Overall, the median values of liquidity ratios in onshore category achieve the lowest variability. For both investigated periods, Slovak companies with ownership links to tax havens further exhibited lower liquidity ratios compared to the recommended interval, which relates to the threat of a potential inability to repay their liabilities in the future. The reported values of liquidity ratios improved in 2015 by Slovak companies with links to tax havens. Observed improvement could be explained mainly by imposed thin-capitalization rules in the Slovak Income Tax Act (in force since 2015). The output of our analysis suggests that the applied profit-shifting technique has an impact on the relation of reported liquidity ratios and profitability indicators. In case that the debt/asset ratio is high (debt is intensive used as profit-shifting technique), then both impaired values of liquidity ratios and profitability indicators could be expected.

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The role of BankTechs in developing mobile payments in Poland – the case of BLIK

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Abstract

The retail payment market is a crucial determinant of the efficient payment system and influences the whole domestic economy. The increase of the noncash payments brings advantages to all market players. This market development is undoubtedly connected with new technology application. The paper describes the role of banks' technological initiatives focused on procedures and payments scheme standardisation. The aim of the paper is to presents the contribution of such initiatives established by Polish banks as BankTech called BLIK to mobile payment market development in Poland. The results develop the knowledge concerning the role of technological standardisation for payment market development and presents the relationship between the standardisation and the number and volume of mobile payments as well as give some insights for policymakers and society concerning mobile payments' development.

Keywords

BankTech, banking technology, mobile payments, payment standardisation,

JEL Classification: G2, L8, O3

1. Introduction

During the last few decades, the retail payment market has been developing dynamically. The experience of many countries shows that the increase in the noncash transaction is very beneficial for the whole economy as the cost of a cashless payment transaction is definitely lower in comparison with the cost of cash transaction. In the European Union, such a cost per year is estimated at the level between 0,4 and 0,6 per cent of GDP. Despite that, the number of cashless transaction is still lower than cash transactions (EPC, 2009).

Increasing the number and volume of cashless transaction requires changes on both sides of the market – the demand and the supply. The demand is generated by customers, enterprises, public institutions and merchants who accept cashless payment instrument as a payment wage for goods and services. Whereas the supply is shaped by banks, payment cards issuers and nonbank institutions (Harasim, 2013: 27-18). They influence not only the accessibility and diversity of payment instruments but also their form and functionality. But finally, the customers decide which payment instruments have satisfactory usability for them (Begg, et al., 2007: 139). On the one hand, the high level of particular payment instrument's acceptance and usage is one of the conditions for its dissemination. On the other hand, the adequate number of transactions made by a given payment instrument enables payment services providers to generate profits. The standardisation of payment processes helps both the maximisation of usability for customers and reaching the critical mass of the transactions' number and volume.

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The standardisation is a necessary prerequisite to cashless payment transactions. As far as in the case of traditional payment instruments as direct transfers, direct debit and payment cards such standards have been already established, implemented and harmonised, the mobile payments are at the start of their dissemination and harmonisation.

The aim of the paper is to present the contribution of BLIK, a banks' initiative in the field of mobile payment standardisation, and its impact on the development of the mobile payment market in Poland. The paper begins with the literature review on payment market standardisation and the role of technology in this process. The second section present Polish banks initiative called Polish Payment Standard BLIK that is one of the most developing BankTech in Poland. The third part of the paper includes the analysis of the BLIK – its development and impact on the mobile payment market in Poland.

2. The retail payment market standardisation and the role of technology in this process

The retail payment market is a two-sided market which means affecting the total volume of transactions by the price structure or the share that each end-user pays the platform (Rocher and Tirole, 2004). Concurrently the profits of the platform depend on the number of those end-users (Harasim, 2013: 35). Standardisation helps in achieving the critical mass what is especially important in the process of innovation implementation. It also brings some benefits for customers as it causes product compatibility with complementary products which enables increasing the product usability. In the case of the payment market, the standardisation leads to developing the payments acceptant network and payment infrastructure, facilitating the processing of payments and may influence the cost of transactions. On the other hand, it may impact the payment market negatively by reducing the competition, delaying the implementation of innovation and discriminating more efficient solutions (Kemppainen, 2003). Moreover, standardisation reduces the problem of market fragmentation and enhances the recognition of payment services among banks' clients (NBP, 2015: 94).

According to Bank for International Settlement, the standardisation may be executed on three dimensions: technical such as payment messages at ATM machines, business as a single course of transactions, and operational including internet payment intermediaries enabling payments between the payment instruments issuers and merchants (BIS, 2000).

The standards may be implemented by bottom-up market initiatives or the industry as well as laid down by legislation. Their implementation and development are strictly connected with technology advancement. Undoubtedly, the range of product and services available by contemporary mobile banking services is the result of technology application (Table 1).

Table 1: The range of banking products and services available due to technology (Klimontowicz, 2018: 86)

Technology	Banking services
SMS (Short Messaging Service)	– information on the balance on the account – information on the recent, completed operations – funds transfers
WAP (Wireless Access Protocol)	– information on the balance on the account – funds transfers – opening bank deposits
Lite Website	– information on the balance on the account – searching and consultation of data concerning the history of operations – funds transfers – pre-paying the phone bill – submitting the request for payment cards' issuing – payment card's activation

	– stopping payment cards
Mobile applications for JAVA and Android	– information on the balance on the account – searching and consultation of data concerning the history of operations – funds transfers – opening bank deposits – repayment of credit facilities – payment and credit cards' support – searching the branch and ATMs location
NFC (Near Field Communication)	– POS payments – P2P payments
QR Codes	– online payments – POS payments – P2P payments – sending checks
RWD (Responsive Web Design)	– information on the balance on the account – searching and consultation of data concerning the history of operations – funds transfers – opening bank deposits and saving accounts – pre-paying the phone bill – repayment of credit facilities including credit cards – payment and credit cards' support – searching the branch and ATMs location – loans and credit applications

On the Polish payment market, there are three main technologies which have the potential to be used in the process of market standardisation. The first is Near Field Communication Technology (NFC). The technology has been applied by both banks and telecommunication operators. Using this technology the payment cards are installed in the telephone SIM card. The payment process uses the cards schemes and is similar to contactless payments. Payments are accepted by the same POS terminals which give the opportunity to be used by a broad network of merchants. The only barrier is the necessity to own the smartphone which is equipped with an NFC antenna. Another solution is the Host Card Emulation (HCE) implementation. HCE enables the bank to process payments without the necessity to cooperate with telecommunication operators as the payment information are not stored on the SIM card but in the cloud. The third technology applies digital and QR codes. Initially, the number of merchants was thought to be a barrier to further development of payments made using this technology. At the beginning it was implemented by two largest banks operation on the Polish banking market – PKO BP and PeKaO SA. The PKO PB application called IKO application has been the foundation for Polish Payment Standard establishment and implementation under the BLIK trademark.

3. The Polish Payment Standard BLIK as an example of BankTech

BankTech is a contemporary, highly emergent phenomenon and there is still a lack of a commonly accepted definition in both theory and practice. BankTech is a special kind of FinTech which refer to technology and innovations implemented by banks. The term refers to the application of technology to the banking sector as an industry in which services are changed with technology (Dimler et al. 2018; Das, 2018). In this sense, the BankTech means banks that use technology and operate inside and outside of the traditional, regulated banking system. They can compete, cooperate and have a coopetitive relationship with other banks and entities on the banking market.

In Poland, technology-oriented banks, are engaged in the following segments: banks and their FinTech accelerators, interbank entities, and nonbank entities. It is assumed that in the nearest future the next segment of neobanks will appear (Cashless.pl et al., 2018).

The BLIK is the trademark of the Polish Payment Standards (PPS) which is an example of coopetition between banks. The joint venture was established in 2013 by the six largest banks operating in the Polish banking market. The shareholders of PPS include Alior Bank (PZU Group), Bank Millennium (Banco Comercial Portugues Group), Santander (previous BZ WBK), ING Bank Śląski (ING Group), mBank (Commerzbank Group) and PKO Bank Polski (the biggest retail bank in Poland and Middle and Eastern Europe). PPS is a Polish mobile payment system and scheme. It is open for other banks and other payment providers. Today it cooperates with 11 bank and 13 acquirers (Table 2). Affiliated banks serve about twenty million customers which are approximately 80% of the Polish market (Finextra, 2018).

Table 2: BLIK partners

Banks		Other payment providers	
– Alior Bank	– PBN Paribas	– Blue Media	– PayLane
– Credit Agricole	– PeKaO SA	– Bill	– PayTell
– Getin Bank	– PKO Bank Polski	– DotPay	– PayU
– ING Bank Śląski	– Santander Bank	– eCard	– Planet Cash
– mBank	– Tmobile Usługi	– eServise	– Przelewy 24
– Millennium Bank	Bankowe	– Euronet Worldwide	– tpay
		– FirstData	

BLIK is thought to be very convenient, secure and flexible payment method that enables its user to pay in-store and make a deposit or withdraw cash at ATMs using their smartphones. Among other payment services allowed by BLIK are: generating checks, instant money transfers, online payments in e-commerce and m-commerce as well as P2P transactions using the phone number. Each transaction has a unique, six-digit code that can only be used for two minutes after it is generated. Additionally, users must confirm the value and the recipient of the transaction in their bank's mobile application.

4. The impact of BLIK on mobile payments development in Poland Poland

Mobile banking is becoming increasingly popular and appreciated by consumers in Poland. This is the result not only of changes in consumer behaviour but also of high-quality mobile applications with multifunctional capabilities offered by Polish banks. For customers, mobile banking application in their smartphone is increasingly often just "a bank in a hand". At the end of the Q1/2019, the banks had 11.8 million users of mobile banking already. This is over 400 thous. more than in the previous quarter and 2.4 million more than in the corresponding period of last year. Traditionally, PKO Bank Polski is ranked first in the list of mobile banking customers with 2.9 million. The second place is taken by mBank, which exceeded the barrier of 2 million mobile banking customers. Third place belongs to ING Bank Śląski, which has 1.8 million smartphone banking customers (Boczoń, 2019b) – see figure 1. All these three banks were among the institutions which launched the mobile payments system BLIK four years ago. Currently, observed development of BLIK is so dynamic that we can talk about the phenomenon of this system success. The number of BLIK users increased from 1.4 million in Q1/2015 to 9.5 million in Q1/2019 and BLIK system is widening its scope in different dimensions (Table 3).

Figure 1: Number of mobile banking users who log into the bank from the mobile device at least once a month in years 2014-2018 (in thous.) (Boczoń, 2017 and 2019a)
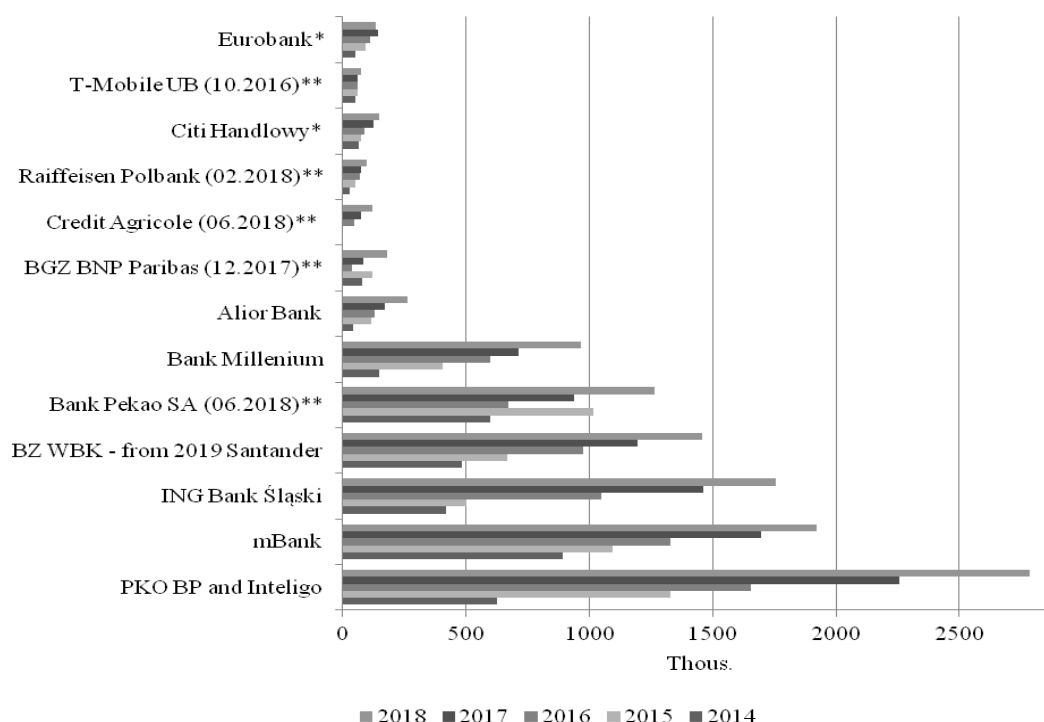
*Banks not in BLIK
**Date of BLIK accessibility

Table 3: The scope of BLIK (NBP, 2015a, 2016a, 2017a, 2018a, 2019a).

	Q4/2015	Q4/2016	Q4/2017	Q4/2018	Q1/2019
Number of banks	6	7	9	11	11
Number of users (million)	1.4	3.1	6.1	8.8	9.5
Number of retail and service outlets (thous.)	136.9	185.3	255.4	384	413.6
Number of payment terminals (thous.)	132.9	177	232.2	413.8	452.8
Number of online stores (thous.)	28.1	42.9	79.9	91.8	98.4
Number of ATMs (thous.)	14.3	15.9	17.2	19.8	18.2

Since the beginning of BLIK functioning we can observe (period Q4/2015-Q1/2019) the immense increase in: the total value of transactions (from PLN 159.1 million to PLN 4.96 billion), average daily turnover (from PLN 1.73 million to PLN 55.1 million), the total number of transactions (from 615 thousand to 39.8 million), the average daily number of transactions (from 6,688 to 442,472) and gradual decline in the average value of a single transaction (from PLN 259 to PLN 124) (NBP, 2016a: 30 and NBP, 2019a: 31).

The key market segment for BLIK are payments in the e-commerce environment. This has changed significantly in recent years: in the last quarter of 2015 the non-cash payments on the Internet were accounted for 16.9% of all mobile transactions compared to almost 75% in Q1/2019. During the year (period Q1/2018-Q1/2019), the number of BLIK transactions on the Internet tripled (from 10.5 million to almost 30 million). It seems that it is not only the result, of the popularization of BLIK itself but also to the growing importance of e-commerce in the Polish economy (Uryniuk, 2019a). According to the latest data from the Statista Digital Market Outlook report, Poland is ranked 13th among the fastest developing e-commerce markets in the world. In 2018-2022, its value is expected to increase in Poland by USD 6 billion and this year it will probably increase to PLN 50 billion (Interaktywnie.com, 2019: 3).

Figure 3 shows the value of the executed transaction in BLIK against the background of e-commerce market value in Poland.

Figure 3: Value of the e-commerce market in Poland and value of executed transactions in BLIK payment system in years 2015-2018 (Szafranek, 2018 and NBP, 2015b, 2016b, 2017b, 2018b, 2019b)
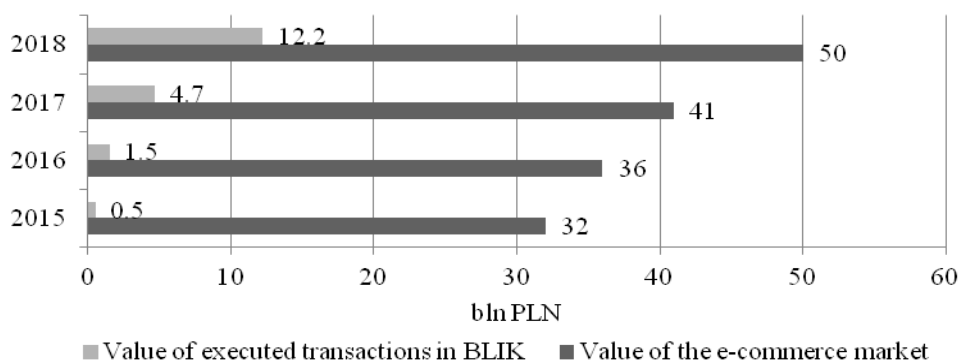
The Q4/2018 was the first quarter when customers paid more often on the Internet with BLIK (25.3 million transactions) than with payment cards (24.1 million transactions). This trend continued also in Q1/2019 (payments in the BLIK system increased by 18% and decreased by 1.5% in case of payment cards transactions (NBP, 2019a, 2019c). Significant increase in the use of BLIK could be noticed in transactions carried out by m-consumers (mobile consumers): comparing Mays 2018-2019, increase by +18 p.p. (BLIK used by 33% of mobile buyers) (eizba, 2019: 77). In 2019 BLIK has introduced a completely new way of shopping on the Internet - buying directly from an advertising banner, which can be displayed on any website. It is not only a great convenience for the buyer but also a significant acceleration of the whole purchasing process (Rudzik, 2019). The share of each channel in the number of BLIK transactions has changed since the system launched in 2015 (Figure 4).

Figure 4: The number of BLIK transactions in all channels in Q4/2015 and Q1/2019 (NBP, 2016a, 2019a)
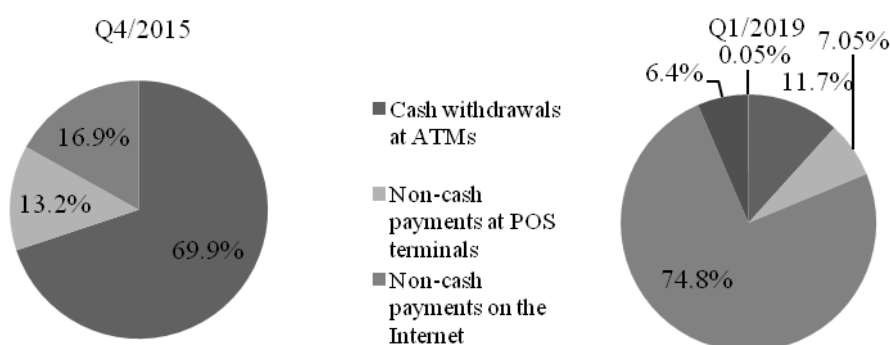
The aforementioned increase in a number of transactions on the Internet, on the one hand, is accompanied by a significant decrease of a number of transactions at ATMs and POS terminals (however the value of transactions is increasing). Furthermore, the new channels have emerged such as above all: P2P payments (increase in the number of transactions from 658 thous. in Q1/2018 to 2.5 million in Q1/2019) and others (terminal cash operations: cash back and cash advance). BLIK development is based on innovation and cooperation among the most important market players in the Polish banking sector. Recently however BLIK "crosses borders" and at the end of 2018 PPS has established a strategic alliance with Mastercard that will enable BLIK users to use contactless technology at all points of sale

worldwide, accepting contactless payments from Mastercard (Polish Payment Standard, 2018). Additionally BLIK is going to introduce International Phone Number Money Transfers as the first European operator to join the new system (Uryniuk, 2019b). And in cooperation with DXC Technology, PPS will implement the mobile payment system BLIK in the United Arab Emirates (BiznesPolska, 2019).

5. Conclusions

The important feature of banks in Poland is that they develop and quickly adapt innovative technologies and aspire to perceive themselves as BankTechs. Setting the interbank standard for mobile payments - the Polish Payment Standard – was a revolutionary step in the creation of modern mobile payment system based on state-of-the-art technology. BLIK payment system is an expression of the possibility of cooperation between competing institutions. It shows that everyone can benefit from it, both banks and consumers. BLIK has been evolving at tremendous speed during last months, being one of the fastest payment methods in online and mobile shops. It is crossing the borders, entering foreign markets and introducing new services.

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Stochastic dominance analysis of Slovak 2nd pillar pension funds

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Abstract

The paper analyses the returns of Slovak 2nd pillar pension funds using several approaches. The main goal of the paper is to compare these funds among each other and to find the most attractive ones. Since the fund returns are random, we are working with the probability distributions of the returns estimated from the data parametrically (assuming a stable distribution) and non-parametrically (assuming an empirical distribution). Moreover, we consider several ways of comparisons (descriptive statistics, performance ratios and stochastic dominance relations) focusing mainly on the stochastic dominance analysis. We employ stochastic dominance rules of the first order, second order, third order as usually. Moreover, we consider the most attractive stochastic dominance relation generated by utility functions with decreasing absolute risk aversion – DARA stochastic dominance, Finally, we compare the results of all these approaches.

Keywords

Pension funds, Slovakia, Stochastic dominance

JEL Classification: D81, G11

1.   Introduction

1.1 Slovak 2nd pillar pension funds

2nd pillar in Slovakia was established in 2005 as a defined contribution pension saving scheme (Mešťan et al. 2017). Enrolment into pillar is currently full voluntary and eligible for persons up to 35 years of age. Current level of contribution in 2019 is 4.75 % from gross salary and contribution rate will increase by 0.25 b. p. per year until it will reach 6 % in 2024. Balco et al. (2018) and Andersen et al. (2018) mention that individuals' savings in 2nd pillar are collected in pension funds which are administrated by pension fund management companies licensed by National Bank of Slovakia (Andersen et al., 2018). Pension funds invest savings into different investment vehicles available on financial markets (for example stocks, bonds, ETFs, mutual funds, precious metals, etc.) taking into account the quantitative limits set by law.

According to the applicable law in Slovakia (the Act on Old-Age Saving n. 43/2004), each pension fund management companies is obligated to operate at least two pension funds which can be divided into two main groups:
•   Bond guaranteed mandatory pension fund;
•   Mixed mandatory pension fund (since March 2005);
•   Equity mandatory pension fund (since March 2005);
•   Index mandatory pension fund (since April 2012).

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As Balco et. al. (2018) mention, after the legislative changes became effective in May 2013, Mixed and Index pension funds became optional, and some of these pension funds have been merged with obligatory Equity (non-guaranteed) mandatory pension funds.

In Slovak 2nd pillar, 6 pension fund management companies operated at the end of 2018. Together they managed 19 pension funds (6 guaranteed and 13 non-guaranteed). In Table 1, there is a list of analyzed pension companies with offered pension funds, broken down by law.

Table 1: List of pension fund management companies and type of funds offered by them in Slovak 2nd pillar at the end of 2018

Pension companies	Guaranteed mandatory pension fund	Non-guaranteed mandatory pension fund		
	Bond fund	Mixed fund	Equity fund	Index fund
Aegon	x	-	x	x
Allianz	x	-	x	-
AXA	x	-	x	x
Postova banka	x	-	x	x
NN	x	x	x	x
VUB Generali	x	x	x	x

Source: the authors according to National Bank of Slovakia, Pension fund management companies list above and www.manazeruspor.sk, 2019

1.2 Stochastic dominance

The basics of stochastic dominance go back to 1960th, see Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970). In these papers, the most popular stochastic dominance relations, so called the first-order (FSD), the second-order (SSD) and the third-order (TSD) stochastic dominance were introduced and analyzed. Moreover, Post et al. (2015) introduced a tractable way how to employ stochastic dominance based on the decreasing absolute risk aversion (DARA SD), which seems to be the most attractive order of stochastic dominance. As Kopa (2016) summarizes, in last years, a substantial development of the stochastic dominance applications in finance was realized, mainly in the following directions:

• portfolio efficiency with respect to stochastic dominance criteria was analyzed, see e.g. Post and Kopa (2013), Grechuk (2014), Kopa and Post (2015) or Post et al. (2015).

• portfolio enhancement using stochastic dominance rules, mainly the second-order stochastic dominance criterion, see e.g. Roman et al. (2013), Hodder et al. (2015), Post and Kopa (2017), Kopa et al. (2018), Moriggia et al. (2019).

• more robust version of stochastic dominance relations and stochastic dominance efficiency tests, see e.g. Dentcheva and Ruszczynski (2010), Kopa (2010), Dupačová and Kopa (2012, 2014).

• more general stochastic dominance (ordering) rules, see e.g. Ortobelli et al. (2013, 2016) and references therein, or Post (2016).

• Data Envelopment Analysis equivalent models in the sense that a portfolio is classified efficient with respect to a stochastic dominance relation if and only if it is efficient with respect to a particular Data Envelopment Analysis model, see Branda and Kopa (2012, 2014, 2016) for more details.

Following this evidence, in our paper, we apply the stochastic dominance theory to the Slovak 2nd pillar pension funds in order to identify the most attractive funds. In particular, we consider FSD, SSD, TSD and DARA SD for a pairwise comparison of any pair of funds. Moreover, despite of the usual assumption of empirical distribution of returns in the references above, we consider a parametric approach (assuming a stable distribution), too. Finally, we compare the results of all approaches and identify the winners.

The remainder of this paper is structured as follows. Section 2 presents the basics of FSD, SSD, TSD and DARA SD with focus on empirical and stable distribution of returns. Section 3 presents the results of the numerical study, including descriptive statistics and performance ratios of the returns. The paper is summarized and concluded in Section 4.

2.   Stochastic dominance relations

We say that a random variable X dominates a random variable Y with respect to the N-th order stochastic dominance, $N = 1, 2, 3$ if

$$Eu(X) \geq Eu(Y) \quad for \quad all \quad u \in U_N$$

where the set of admissible utility functions for the N-th order stochastic dominance is defined as follows:

$$U_N = \{u(x) : (-1)^k u^{(k)} \leq 0, \quad \forall x, \quad k = 1, \dots, N\}.$$

Moreover, a random variable X dominates a random variable Y with respect to DARA SD if

$$Eu(X) \geq Eu(Y) \quad for \quad all \quad u \in U_{DARA}$$

where now the set of admissible utility functions is restricted by the assumption of decreasing (non-increasing) Arrow-Pratt coefficient of absolute risk aversion:

$$U_{DARA} = \left\{ u(x) \in U_3 : \frac{d}{dx}\left(-\frac{u''(x)}{u'(x)} \right) \leq 0, \quad \forall x \right\}.$$

See Levy (2016) and references therein for more details. In the rest of the paper, the random variables will represent daily returns of the pension funds. And we say that one pension fund dominates the other one if stochastic dominance between their weekly returns holds true.

2.1   Stochastic dominance for empirical distributions

Let r_i be the random return of the i-th pension fund with distribution function $F_i(x)$. Then i-th fund dominates j-th fund with respect to:

- FSD if and only if $F_i(x) \leq F_j(x) \ \forall x \in \mathbb{R}$
- SSD if and only if $\int_{-\infty}^{z} F_i(x) dx \leq \int_{-\infty}^{z} F_j(x) dx \ \forall z \in \mathbb{R}$
- TSD if and only if $\int_{-\infty}^{z} \int_{-\infty}^{y} F_i(x) dx dy \leq \int_{-\infty}^{z} \int_{-\infty}^{y} F_j(x) dx dy \ \forall z \in \mathbb{R}$.

If r_i takes value $r_{i,t}$, $t = 1, \dots T$ with the same probabilities, such that

$$r_{i,1} \leq r_{i,2} \leq \cdots \leq r_{i,T}$$

then i-th fund dominates j-th fund with respect to FSD if and only if

$$r_{i,t} \geq r_{j,t} \quad t = 1, \dots, T$$

and with respect to SSD if and only if

$$\sum_{s=1}^{t} r_{i,s} \geq \sum_{s=1}^{t} r_{j,s} \quad t = 1, \dots, T.$$

Since similarly easy conditions hold neither for TSD nor for DARA SD we apply algorithms in Levy (2016) and Post et al. (2015) for testing TSD and DARA SD among the pension funds, respectively.

2.2   Stochastic dominance for stable distributions

When providing the pairwise stochastic comparisons, the distributions of returns need to be estimated first. We showed Kopa et al. (2019) that based on the distribution assumptions, the results may differ. In this paper, we fit our data to alpha-stable distribution.

More general fat-tailed and asymmetric distributions are widely used in financial mathematics, see Hoechstoetter et al. (2005), Rachev & Mittnik (1993), Samorodnitsky & Taqqu (2000) and Fama & Roll (1968). A subclass of stable distributions is known since 1919 and has many versatile properties comparing to other distributions (see e.g. Bertocchi et al. (2005),

Hoechstoetter et al. (2005)). However, application of alpha-stable distribution in practice is complicated because it hasn't got a closed form of probability distribution function in general (see Kabasinsas, 2009 for more details) and one needs to use numerical integration or apply simulation techniques.

Recently, few papers have appeared, where stochastic dominance rules for symmetric case of alpha-stable distribution were used (Ortobelli et al. (2016) and Kouaissah & Ortobelli (2017)). However, none of them formulated a sufficient and necessary condition for SD in the general case (asymmetric, different alpha's and beta's). In this paper we use numerical SD computation technics for any alpha-stable distribution.

3.   Empirical study

3.1   Data description

Data about pension funds' performance in 2nd pillar in Slovakia were collected from database of National Bank of Slovakia and individual pension fund management companies' websites. Information about portfolio structure was obtained from database on web portal Manazeruspor.sk which aggregate information about each pension funds managed by pension fund management company in 2nd and 3rd pillar in Slovakia from official monthly fund reports from inception of both pillars. The analysis period covers six years starting from May 1, 2012. There existed nineteen 2nd pillar pension funds, which may be arranged to four risk categories based on the investment share in stocks: R0 – conservative funds (0% share), R1 – low risk funds (less than 30% in stocks), R2 – moderate risk funds (less than 70% in stocks), R3 – high risk funds (up to 100% in stocks) (see Table 2).

Table 2. Descriptive Statistics and Performance Ratios

PF*	Mean	St.Dev	Skewness	Kurtosis	VaR.99	CVaR.99	Sortino	Sharpe	Rachev
Dlhopisový_R0	0.00020	0.0013	-1.5630	6.3531	0.0042	0.0058	0.1999	0.1486	0.5655
Garant_R0	0.00025	0.0012	-1.1128	3.2862	0.0040	0.0044	0.3093	0.2166	0.6594
Klasik_R0	0.00041	0.0011	-0.6523	2.2853	0.0029	0.0037	0.5950	0.3600	0.9595
Solid_R0	0.00025	0.0009	-0.7736	2.6072	0.0027	0.0030	0.4334	0.2779	0.7919
Tradícia_R0	0.00014	0.0008	-0.8076	3.4264	0.0026	0.0029	0.2593	0.1807	0.8490
Harmónia_R1	0.00068	0.0056	-0.5034	1.8058	0.0164	0.0179	0.1774	0.1227	0.9112
Mix_R1	0.00083	0.0071	-0.4434	2.1215	0.0220	0.0240	0.1674	0.1165	0.9003
Stabilita_R1	0.00047	0.0024	-0.9765	3.3749	0.0077	0.0090	0.2837	0.1980	0.7204
Akciový_R2	0.00078	0.0061	-0.5029	1.8258	0.0181	0.0197	0.1850	0.1279	0.8606
Dynamika_R2	0.00091	0.0080	-0.6035	1.9851	0.0251	0.0268	0.1632	0.1145	0.8498
Profit_R2	0.00100	0.0095	-0.5874	2.3548	0.0300	0.0346	0.1481	0.1049	0.7997
Progres_R2	0.00117	0.0148	-0.4633	2.2268	0.0416	0.0495	0.1110	0.0790	0.8873
Prosperita_R2	0.00081	0.0127	-0.7266	3.6952	0.0468	0.0502	0.0875	0.0642	0.7726
Vital_R2	0.00078	0.0083	-0.6666	2.8912	0.0248	0.0305	0.1328	0.0948	0.7628
IndexAegon_R3	0.00188	0.0181	-0.5539	1.4773	0.0474	0.0605	0.1476	0.1039	0.7330
Indexový_R3	0.00194	0.0180	-0.5316	1.3843	0.0475	0.0598	0.1532	0.1073	0.7428
Perspektíva_R3	0.00165	0.0186	-0.6628	1.5785	0.0462	0.0649	0.1232	0.0886	0.6780
IndexNN_R3	0.00156	0.0235	-0.4956	0.7958	0.0596	0.0766	0.0926	0.0663	0.7069
IndexVSB_R3	0.00181	0.0181	-0.4855	1.3024	0.0486	0.0600	0.1428	0.0999	0.7536

* Symbols after underscore denote the risk category

Considering the mean return and risk measured by standard deviation (St.Dev), Value-at-Risk (VaR.99) and Conditional Value-at-Risk (CVaR.99), one could notice that the low mean is

accompanied by low risk, and vice versa, which is the expected result known from the theory. Taken into consideration the values of skewness and kurtosis, the funds such as Dlhopisový_R0, Garant_R0, Prosperita_R2, and Vital_R2 would become unattractive for the investor, while Harmónia_R1, Mix_R1, and Akciový_R2 are the examples of funds demonstrating a good performance, comparatively. The performance ratios estimated with Sortino and Sharpe risk-adjusted measures show that one should consider the funds belonging to the group of conservative ones (risk category R0), while the Rachev values approaching to one suggest to choose the fund from other risk categories as well. In either case, the choice depends from the risk-return preferences of the investor, which becomes enough complicated by a long list of risk and performance measures to be estimated.

3.2   Empirical distribution

In this section, the SD rules are applied for PFs empirical distributions of weekly returns. The procedure includes all possible pairwise comparisons to be checked between any two funds X_i and X_j for all $i \neq j$. Table 3 lists all pairs of PFs such that at least one SD rule holds.

Table 3. Stochastic Dominance Results for Empirical Analysis

Pair of PFs			FSD	SSD	TSD	DARA SD
Klasik_R0	≽	Garant_R0	Yes	Yes	Yes	Yes
Garant_R0	≽	Dlhopisový_R0	–	Yes	Yes	Yes
Klasik_R0	≽	Dlhopisový_R0	–	Yes	Yes	Yes
Solid_R0	≽	Dlhopisový_R0	–	Yes	Yes	Yes
Mix_R1	≽	Prosperita_R2	–	Yes	Yes	Yes
Mix_R1	≽	Vital_R2	–	Yes	Yes	Yes
Dynamika_R2	≽	Prosperita_R2	–	Yes	Yes	Yes
Dynamika_R2	≽	Vital_R2	–	Yes	Yes	Yes
Profit_R2	≽	Prosperita_R2	–	Yes	Yes	Yes
Indexový_R3	≽	IndexAegon_R3	–	Yes	Yes	Yes
Index_VSB_R3	≽	Perspektíva_R3	–	Yes	Yes	Yes
Index_VSB_R3	≽	IndexNN_R3	–	Yes	Yes	Yes
IndexAegon_R3	≽	IndexNN_R3	–	Yes	Yes	Yes
Indexový_R3	≽	Perspekíķva_R3	–	Yes	Yes	Yes
Indexový_R3	≽	IndexNN_R3	–	Yes	Yes	Yes
Perspektíva_R3	≽	IndexNN_R3	–	Yes	Yes	Yes
IndexAegon_R3	≽	Perspektíva_R3	–	–	Yes	Yes

With a few exceptions, almost all pairs obtained consist of funds belonging to the same risk category. As can be seen from Table 3, a less risky fund Mix_R1 dominates two more risky pension funds, namely Prosperita_R2 and Vital_R2. The interesting result to emerge from the data is that the dominance of Klasik_R0 over Garant_R0 by FSD criterion is determined. Furthermore, Garant_R0 dominates Dlhopisový_R0, which results in the chain dominance. Within risk category R3, in total three chains of SSD dominance including three funds can be determined, which implies that the fund IndexNN_R3 is the most inefficient one. Given that, no risk averse participant would consider dominated funds as the optimal choice. Comparing to SSD, only one additional pair of PFs holding TSD was found. Relaxing to DARA SD brings no more dominance among the PFs.

3.3   Stable distribution

We start with the estimation of alpha-stable distribution parameters, the results are collected in Table 4.

Table 4. Parameters of alpa-stable distributions for Slovak pension funds, grouped by risk class

	alpha	beta	mu	sigma
Dlhopisový_R0	1.6297	-0.49	0.0004	0.0007
Garant_R0	1.4416	-0.32	0.0004	0.0006
Klasik_R0	1.5683	-0.35	0.0005	0.0006
Solid_R0	1.5052	-0.09	0.0003	0.0005
Tradícia_R0	1.4225	-0.23	0.0002	0.0004
Harmónia_R1	1.3996	-0.08	0.0010	0.0028
Mix_R1	1.3564	-0.20	0.0015	0.0035
Stabilita_R1	1.4552	-0.42	0.0009	0.0012
Akciový_R2	1.3743	-0.15	0.0013	0.0029
Dynamika_R2	1.3874	-0.04	0.0013	0.0040
Profit_R2	1.3657	-0.23	0.0020	0.0046
Progres_R2	1.2756	-0.13	0.0023	0.0066
Prosperita_R2	1.2294	-0.18	0.0021	0.0052
Vital_R2	1.2331	-0.04	0.0010	0.0033
IndexAegon_R3	1.5332	-0.41	0.0044	0.0101
Index_NN_R3	1.7448	-0.66	0.0047	0.0150
Index_VSB_R3	1.5582	-0.40	0.0041	0.0103
Indexový_R3	1.5436	-0.40	0.0043	0.0102
Perspektíva_R3	1.4888	-0.43	0.0046	0.0103

In the next table, we present the results for pairs from Table 3 and for other pairs for which at least one SD relation holds.

Table 5. Stochastic dominance results for parametric analysis within risk group

Pair of PFs			FSD	SSD	TSD
Dlhopisový_R0	≽	Tradícia_R0	-	-	Yes
Klasik_R0	≽	Garant_R0	-	Yes	Yes
Garant_R0	≽	Dlhopisový_R0	–	-	-
Klasik_R0	≽	Dlhopisový_R0	–	-	-
Klasik_R0	≽	Tradícia_R0	–	-	Yes
Solid_R0	≽	Dlhopisový_R0	–	Yes	Yes
Solid_R0	≽	Garant_R0	-	Yes	Yes
Solid_R0	≽	Tradícia_R0	-	Yes	Yes
Harmónia_R1	≽	Mix_R1	-	Yes	Yes
Stabilita_R1	≽	Mix_R1	-	-	Yes
Mix_R1	≽	Prosperita_R2	–	-	-
Mix_R1	≽	Vital_R2	–	-	-
Akciový_R2	≽	Profit_R2	-	Yes	Yes
Akciový_R2	≽	Progres_R2	-	Yes	Yes
Akciový_R2	≽	Prosperita_R2	-	Yes	Yes
Akciový_R2	≽	Vital_R2	-	Yes	Yes
Dynamika_R2	≽	Profit_R2	–	Yes	Yes
Dynamika_R2	≽	Progres_R2	–	Yes	Yes
Dynamika_R2	≽	Prosperita_R2	–	Yes	Yes
Dynamika_R2	≽	Vital_R2	–	Yes	Yes
Profit_R2	≽	Prosperita_R2	–	Yes	Yes
Profit_R2	≽	Progres_R2	–	Yes	Yes
Profit_R2	≽	Vital_R2	–	-	Yes
Progres_R2	≽	Prosperita_R2	–	-	Yes
Progres_R2	≽	Vital_R2	–	-	-

Vital_R2	≽	Progres_R2	–	Yes	Yes
Vital_R2	≽	Prosperita_R2	–	Yes	Yes
IndexAegon_R3	≽	IndexNN_R3	–	-	-
IndexAegon_R3	≽	Perspektíva_R3	–	Yes	Yes
Index_NN_R3	≽	IndexAegon_R3	-	-	Yes
Index_NN_R3	≽	Indexový_R3	-	-	Yes
Index_NN_R3	≽	Index_VSB_R3	-	-	Yes
Index_NN_R3	≽	Perspektíva_R3	-	-	Yes
Index_VSB_R3	≽	IndexAegon_R3	–	-	Yes
Index_VSB_R3	≽	Perspektíva_R3	–	Yes	Yes
Index_VSB_R3	≽	IndexNN_R3	–	-	-
Indexový_R3	≽	IndexAegon_R3	–	Yes	Yes
Indexový_R3	≽	Perspektíva_R3	–	Yes	Yes
Indexový_R3	≽	IndexNN_R3	–	-	-
Perspektíva_R3	≽	IndexNN_R3	–	-	-

From risk group R0 fund Solid_R0 SSD dominates nearly all other funds with exception of Klasik_R0. Moreover, Klasik_R0 additionally dominates Garant_R0. When moving to TSD, two other pairs could be found. In the second risky group, only Harmonia_R1 SSD (and TSD) dominates Mix_R1. While fund Stabilita_R1 dominates MIX_R1 only by TSD. In the third group (R2), there are two funds (Akciový_R2 and Dynamika_R2) that SSD (and TSD) dominate all other funds with exception of each other. Moreover, Profit_R2 and Vital_R2 SSD dominates Progres_R2 and Prosperita_R2. Relaxing to TSD, two more pairs can be found.
The largest difference between SSD and TSD results was observed for the last risky group (R3). Fund Index_NN R3 dominates all other R3 funds by TSD but not by SSD. On the contrary, funds Index_VSB_R3, Indexovy_R3 and IndexAegon_R3 dominates the same fund Perspectiva_R3 no matter whether SSD or TSD is used.

4.   Conclusion

In this paper we have analysed the performance of 2nd pillar Slovak pension funds using return-risk characteristics, performance ratios and mainly stochastic dominance relations. Four different types of stochastic dominance (FSD, SSD, TSD, DARA SD) and two distributional assumptions were considered. First, we have assumed that the weekly returns follow empirical distribution coming directly from the data, then we assumed alpha-stable distributions of returns. The parameters of the alpha-stable distributions were estimated from data. The goal of the paper was to compare the identified SD pairs for various types of stochastic dominance and distributional assumptions.

Summarizing the results of Table 3 and Table 5, we can see that the identified pairs for SSD and TSD differ between each other more substantially under assumption of alpha-stable distributed returns. Moreover, more SSD and TSD pairs are identified under this assumption than under assumption of empirical returns. This is not a surprise because the parametric approach is always less strong, so more pairs could be found, no matter which distribution or type of dominance is considered. Finally, we can conclude that for half of the pairs from Table 3, the stochastic dominance relations are not fulfilled when changing to the parametric approach. All these findings show, that one should be very careful in formulating the distributional or preference assumptions for comparisons of the 2nd pillar Slovak pension funds, because the results heavily depends on these assumptions.

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Evaluation of finished products and its impact on corporate's profit and value of assets

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Abstract

Knowledge of the value of finished products can be important part of financial decisions. It influences total sum of costs and consequently total corporate's profit. The evaluation process depends on correct way of assignment of all production costs and volume of real or planned capacity. The value of fixed assets used for production is the next fact which may influence final production costs. Specific situation occurs if fixed assets are financed due to accepted grants. Submitted article concentrates on comparing impact of this fact on setting the value of finished products according to the Czech accounting rules and according to the rules of International Financial Reporting Standards. Additionally, there is pointed out impact of this fact on details of balance sheet and profit and loss statements.

Key words

Evaluation of finished products, grants, fixed assets

JEL Classification: M 21, G31

1. Introduction

Business can be supported by government in different ways. One of them is usage of possibility to provide grants for assets acquisition or to finance consumed sources i.e. to finance individual costs and expenses. Regarding the accepted grants, certain problem arises, specifically what is an impact of accepted grants on evaluation of unfinished or finished products. The way of products assessing has been changed since the year 2016 with regard to amended text of the Act no. 563/1991 Coll., on Accounting and with regard to amended text of Implementing Decree no. 500/2002 Coll., by which some provisions of Act no. 563/1991 Coll., on Accounting, as amended, for accounting units which are entrepreneurs that keep accounts by a double entry accounting system as amended, are carried out. According to § 25 article no. 1 of Accounting Act created products must be assessed respecting the value of total production costs. In the case of § 49 article no. 5 of Implementing Decree production costs includes direct cost and may include part of variable and fixed indirect costs. Moreover, there must exist undeniable reason of assigning these costs to the certain product and as well as these costs must be costs of this period of activity (Mruzková, 2013). Selling costs must not be used for assessing of finished or unfinished products and the way of valuation of them depends on the decision of individual entrepreneur respecting the principle of materiality and faithful and fair representation of property.

As to fact that production activities influence the value of costs and final value of products stocks in certain moment, the correct evaluation of them is an important part of economic

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decisions. Finally, this fact has been supported by issuing interpretation no. I-35 of National Accounting Council of the Czech Republic that more precisely specified details of direct and indirect costs used for assessing of individual products and details of costs which must not be used in assessing process. This interpretation moreover specifies solution of situations when level of used capacity differs among individual periods and differs from planned capacity value. When it comes to indirect costs of production, they must be allocated to certain product through appropriate allocation base. It means that there must be an understandable relationship between indirect costs and product. Depreciation of tangible or intangible assets can be one of the indirect costs which are necessary to allocate to products. It means that value of this kind of assets plays a significant role in an appraisement of finished or unfinished products.

Completely specific situation occurs if long term assets financed by grants are used in production activities. In this connection, there are certain accounting rules, which must be applied. The basic approach respects the fact that the value of used long term assets must be reduced with the value of accepted grants. Therefore, calculation of depreciation is affected according to the value of assets lower than acquisition price. In this moment it is necessary to realize the main role of depreciation in financial management. Depreciation has to create sources to finance completely worn assets. The evaluation process of products should respect this main idea and the value of depreciation as allocated indirect cost should be derived from not reduced long term assets value. Unfortunately, Czech accounting standards don´t support this manner. On the contrary, according to § 25 article no. 1 and § 25 article no. 5 of the Act no. 563/1991 Coll., on Accounting it is possible to use only acquisition price or reproductive acquisition price as an input value of depreciation process. This fact - the possibility of usage only acquisition price reduced by value of accepted grants - impacts on assessing of final production. On the contrary, International Financial Reporting Standards offer to use fair value of long-term assets and in case of accepted grants do not give up possibility to work with no reduced value of them. In response to above mentioned facts the main aim of this article is the comparison and emphasis of differences between Czech accounting standards and International financial reporting standards in setting value of products if used fixed assets has been financed via provided grants.

2. Setting value of finished and unfinished products

As it has been already mentioned, the important part of financial decisions is the ability to know value setting process of products. Hence the process of determination of stock value has an impact on total sum of costs and consequently on value of realized profit and on final tax liability, because allocated costs of finished products are judged as tax costs. When it comes to costs which can be accepted as production costs, according to Accounting Act, its Implementing Decree and Interpretation I-35 of National Accounting Council of the Czech Republic, products are evaluated respecting the real value of costs or planned value of costs. Aside from that, it is determined that indirect costs may include fixed and variable indirect costs which can be assigned to product and which has occurred in the period when products have been produced (Fibírová, 2005). When it comes to selling costs, administrative costs and finance costs, they can not be included to the cost allocation (Kresta, 2017). In response to economy requirements, unusual waste, avoidable damage, unnecessary work and unused capacity costs must not be included in cost allocation. Referring to the degree of unused capacity, it is necessary to work with capacity, taking into account the level of planned maintenance.

As it has been mentioned above, the value of final products is influenced by the value of costs which rised because of production (Fibírová, 2015). Depreciation of fixed assets which are used for production, it is one kind of these costs. Depreciation costs are fixed indirect costs which can be allocated to final products. It means that the value of depreciation influences the value of stock of products. If fixed assets are financed due to external sources like grants, there is an important effect on setting value of final products and on evaluation of fixed and current assets. According to the main principles of Czech Accounting Standards grants decrease the value of fixed assets which are financed by this way. But as far as the main rules of International Financial Reporting Standards solution of it is different and it makes possible to work with acquisition price (Ficbauer, 2016).

2.1 Setting price of products according to the Czech rules

For emphasizing main disparities between Czech standards and international standards details of individual accounting steps will be demonstrated. Let us assume that fixed asset has been used for production activity. Acquisition price was 100,000 CZK and this asset can be used for ten years. Grant has been used for financing of this asset and it has been provided at an amount corresponding to the acquisition price of the fixed asset. In order to stress impact of the Czech accounting standards we take into account only the fact that product has been produced with usage of the fixed asset and we eliminate the remaining costs that may affect production costs. There is not any depreciation because of book value of fixed asset is equal to zero respecting the Czech rules. The main reason of this is the reduction of book value by accepted grant. To point out total impact of this situation we moreover take into account selling of product to final consumer. The value of this sale is 10,000 CZK and it has been derived from value of consumption of fixed asset in reality and it has been determined as one tenth of the acquisition price. The accounting procedure respecting the Czech accounting standard can be observed in following table 1.

Table 1: Accounting procedure according to the Czech accounting standards

Item	Description	Amount	Drs	Crs
1.	Acquisition of the fixed asset	100	042	321
2.	Payment of invoice	100	321	221
3.	Entitlement to grant	100	378	346
4.	Use of grant as finance source of an asset	100	346	042
5.	Payment of grant to a bank account	100	221	378
6.	Putting the fixed asset into use	0	022	042
7.	Depreciation of the fixed asset	0	551	082
8.	Transfer of finished products to stock	0	123	583
9.	Dispensing of products from warehouse	0	583	123
10.	Invoice for customer	10	311	601

Source: Authors` processing

Items 1. – 5. of table 1 describe standard situation and 6th item corresponds with the fact that all grant covered acquisition price of the asset. It is deed which caused that there is not any value of the asset in balance sheet and consequently there is not any value of depreciation, because acquisition price of the asset has been completely reduced by financing source. Moreover, there is a problem with a lack of depreciation costs which should be allocated to the finished products.

What are the consequences of above-mentioned procedure? It is clear that there is a problem with the valuation of finished or unfinished products. The current real value of production costs should be derived from the value of depreciation. Because depreciation is zero, the real value of finished product is zero as well. In addition, considering value of the

asset we have to notice that the value of asset is distorted by this approach. Both of these facts influence informative ability of balance sheet and profit and loss statement. The consequence of it is obvious from the following pictures. To stress the impact of this approach on evaluation and evaluation process, all statements will be presented two times. Firstly, before selling of products and later after selling of products.

Picture 1: Profit and loss statement according to the Czech rules before selling

Profit and Loss Statement			
Depreciation	0		
Change in stock volume	0		
		Profit	0

Source: Authors` processing

According to details of picture 1 and table 1 operations 1 - 8, we can observe, that there is distortion of information provided by profit and loss statement. Details of this statement declare that depreciation costs do not exist. Moreover, assessment of profitability is unclear and there is a lack of information important for determining of changes of finished products.

Picture 2: Balance sheet statement according to the Czech rules before selling

Balance sheet Statement			
Fixed asset	0	Grant	0
Products	0		
Bank Account	0		
Receivables (grant)	0		
Total Assets	0	Equity and Liabilities	0

Source: Authors` processing

According to the Czech regulations, balance sheet (Picture 2) shows the reduced book value of fixed assess by the received grant. Above that, respecting appraisal approach the real costs determining book value of finished product is distorted as well and does not reflect reality. Following picture 3 and 4 shows final balance sheet statement and income statement after selling of products.

Picture 3: Profit and loss statement according to the Czech rules after selling

Profit and Loss Statement			
Depreciation	0	Sales of product	10
Change in stock volume	0		
		Profit	10

Source: Authors` processing

Eliminating the book value of fixed assets by grant influenced income statement as well. There is no possibility to compare turnover with certain costs because they are not completely mentioned. Profitability is not reported correctly by this way as well.

Picture 4: Balance sheet statement according to the Czech rules after selling

Balance sheet Statement			
Fixed asset	0	Profit	10
Products	0	Accepted grants	0
Bank account	0		
Receivables (grant)	0		
Receivables (sales)	10		
Total Assets	10	Equity and Liabilities	10

Source: Authors` processing

The inaccuracy in capturing the real financial situation is also evident from the picture 4. Balance sheet statement on contrary to reality declares any fixed asset. This information should be disclosed to detailed description of the financial statements. On the other hand, the zero value of product can be accepted. Also, the zero value of grant refers about the fact that total value of grant has been used by recipient of subsidiary.

2.2 Setting price of products according to the IFRS rules

According to the International Financial Reporting Standards, the company has the possibility to report the amount of the accepted grant in two ways. The first is identical to the Czech regulations and the second one uses a time resolution to display it (Dvořáková, 2014). However, the choice may not be very user friendly in practice, where the financial statements of individual companies may be incompatible.

Details of accounting proceedings are mentioned in the following table 2. Input data are the same. Value of grant which completely cover the acquisition price of fixed asset is 100,000 CZK, the value of depreciation is 10,000 CZK. Finished products will be assessed only according to the real value of depreciation (eliminating the remaining costs that may affect production costs) and will be sold for selling price derived from allocated costs 10,000 CZK.

Table 2: Accounting procedure according to the International Finance Report Standards

Item	Description	Amount	Drs	Crs
1.	Acquisition of the fixed asset	100	042	321
2.	Payment of invoice	100	321	221
3.	Entitlement to grant	100	378	346
4.	Use of grant as finance source of an asset	100	346	384
5.	Payment of grant to a bank account	100	221	378
6.	Putting an asset into use	100	022	042
7.	Depreciation of the asset	10	551	082
8.	Aliquot part of grant as deferred revenues	10	384	648
9.	Transfer of finished products to stock	10	123	583
10.	Dispensing of products from warehouse	10	583	123
11.	Invoice for customer	10	311	601

Source: Authors` processing

According to this procedure it is clear that this approach does not distort reality and does not make it difficult to assess finished products. The book value of the fixed asset corresponds with its acquisition price and stock of products is derived from costs (depreciation) consumed. Items 10 and 11 show selling of products. Before selling of these products we can identify income statement and balance statement to be more reliable as can be seen in picture 5 and picture 6.

Picture 5: Profit and loss statement according to the international rules before selling

Profit and Loss Statement			
Depreciation	10	Other operating income	10
Change in stock volume	-10		
		Profit	10

Source: Authors` processing

Data of picture 5 respect first nine mentioned accounting steps of table 2. There is observed the value of depreciation in profit and loss statement and moreover, the value of other operation income which expresses aliquot part of the grant as deferred revenues. It is the value of the grant which has been used in this period. Therefore, there is no problem with evaluation of the finished products respecting the real value of depreciation and consequently total sum of costs more precisely describes the costs situation.

Picture 6: Balance sheet statement according to the international rules before selling

Balance sheet statement			
Fixed asset	90	Profit	10
Products	10	Grant	0
Receivables (grant)	0	Deferred revenues	90
Bank Account	0		
Total Assets	100	Equity and Liability	100

Source: Authors` processing

Following pictures 7 and 8 are devoted to situation after selling products to final consumer. It can be observed that data of profit and loss statement provides better information compared to pictures 3 and 4. With regard on details mentioned here, it is understandable that the fixed asset has been used and it is clear the value of this consumption. On the other hand, other operating income respects general principle of accounting processing of accepted grants which declares, as long as grant is used throughout consumption of assets and services, it must be covered by expression of accounting revenues.

Picture 7: Profit and loss statement according to the international rules after selling

Profit and Loss Statement			
Depreciation	10	Other operating income	10
Change in stock volume	0	Sales of product	10
		Profit	10

Source: Authors` processing

Picture 8 provides detailed information about balance sheet statement. Moreover, it is better way how to describe the reality. This statement shows depreciated value of the fixed assets. Because of customers have already consumed all products, it is obvious that there is not any finished product. On the contrary, we can identify receivables from customers. Item deferred revenues determines as in previous picture 6 value of grant which will be consumed in future.

Picture 8: Balance sheet statement according to the international rules after selling

Balance sheet statement

Fixed asset	90	Profit	10
Products	0	Grant	0
Receivables (grant)	0	Deferred revenues	90
Bank Account	0		
Receivables (sales)	10		
Total Assets	100	Equity and Liability	100

Source: Authors` processing

3. Conclusion

As it has been mentioned in the introduction, setting the value of products may consequently influence costs, revenues, profit, assets, equity and even liabilities. This article compared the different approaches to determining the value of finished products if products have been made with usage of fixed assets financed by provided grants. Firstly, the methodology of the Czech accounting standards has been highlighted. According to the lacked depreciation costs total value of profitability was influenced. Moreover, there was a problem with determining the value of finished products and with the value of fixed assets. Total sum of assets is underdetermined and the value of assets does not provide relevant and reliable information. In this situation it is important to point out that this distortion should be mentioned and explained in the notes of financial statements of corporates.

On the other hand, respecting the methodology of international finance report standards more reliable information provided by both statements could be observed. Additionally, there has not been problem with setting the value of finished products because of knowing the value of depreciation. Moreover, the way which works with deferred liabilities and other operation income improved information about value of fixed and current assets and deferred revenues as accrued liabilities declares rest value of grant which should be consumed in a future. According to these facts it is clear that usage of international finance standard provides more reliable information which more and better describes and respects reality.

Acknowledgments

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Comparison Fat Tails for PX and FTSE Indices

Jiří Málek[1], Tran van Quang[2]

Abstract

Three distributions (stable, normal inverse Gaussian, Student) with heavy tails are selected as candidates for modeling the returns of two stock market indices. First of them is from developing markets (Czech index PX), and the second (London FTSE) represent developed stock markets. Using daily data of the selected indices from year 2000 to 2018, the parameters of these distributions are estimated. Normal distributions is used for comparison. Then the suitability of each chosen distribution for index returns is examined with χ^2 goodness of fit test. The obtained results show that if correctly specified, the tail power of the distribution of stock market index returns depends on the development of the stock market.

Key words

Stock market indices, Heavy tailed distributions, Parameter estimation, Goodness of fit test.

JEL Classification: C58, G14

1. Introduction.

It has been known for quite a long time that the distribution of financial asset returns in general as well as stock price returns has heavier tails and sharper peak than the corresponding normal distribution. Not only can the correct choice for their distribution help to find the answer to our problem, but it is also of great importance for risk management and asset pricing. So far many efforts of researchers as well as practitioners have been devoted to this task. There are two ways how to deal with it.

The first one, which is less inconvenient but may not yield the needed accuracy, is to replace the normal distribution by an alternative distribution with the same number of parameters as the normal one which exhibits the leptokurtic property. The second way how to solve this problem is to use a candidate distribution with more than two parameters. In this case, the additional parameter(s) will capture the tail and peak behavior of the distribution of financial asset returns. However, additional parameters also make estimation procedure more complicated. A candidate distribution for the second approach often comes from either of these three families of distributions: alpha stable distribution, generalized hyperbolic distribution and skewed generalized t-distribution.

As the estimation of parameters of multiparameter distributions used in the second approach mentioned above tends to be more complicated and computationally extensive as well as intensive. A typical research often looks like as follows: picking a family of distributions, estimating its parameters for several financial assets and using one (or a few of them at once) of available goodness of fit tests to validate the use of the examined distribution family. For example, for the alpha stable distribution family, see works of Nolan (1997), Borak, Hardle and

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Weron (2005), Borak, Misiorek, and Weron (2010), for the generalized hyperbolic distribution family, see Prause (1999), and Eberlein and Keller (1995), for the skewed generalized t-distribution family, see Theodossiou (1998), Zhu and Galbraith (2012) and Guo (2017). By doing the research this way, one might obtain a set of suitable candidates for the distribution of financial asset returns. It is evident that there is a lack of inter-group comparison whose results can lead to a choice of the most suitable candidate which can be used for various purpose including financial engineering application.

To find the answer to our research question raised above, we choose s three candidate distributions potentially suitable for both approaches of capturing heavy tails. We also use two stock market indices: Prague Stock Exchange index PX (Czech Republic) representing stock markets of developing economies, and index FTSE (Great Britain) representing stock markets of developed economies. The data used for this research are daily series of these indices from 2000 to 2018. These series are converted into return series and then they are used to estimate the parameters of each chosen distribution will be estimated by maximum likelihood technique. Then the suitability of each distribution for each series will be tested by χ^2 goodness of fit test. They will be tested both locally for left loss tails. Based on this result, the appropriate candidate distribution for returns will be recommended. As such, a procedure of how to select a most suitable distribution for a returns series as well as the pattern of distributional behavior of each type of stock market will be presented.

2. Overview of used probability distributions

In this section a brief description of heavy tailed distributions of our interest is provided. We will consider alpha stable distribution, Student t-distribution and normal inverse Gaussian (NIG).

2.1 Stable distributions

This probabilistic distribution is formulated by its characteristics function because density function doesn't exists in explicit form:

$$\Phi(t) = \exp\left\{-\sigma^\alpha |t| \left(1 - i\beta \, sign\,(t) \tan\frac{\pi\alpha}{2}\right) + i\mu t\right\} \quad \text{for } \alpha \neq 1,$$

$$\Phi(t) = \exp\left\{-\sigma |t| \left(1 - i\beta \frac{2}{\pi} sign(t) \log|t|\right) + i\mu t\right\} \quad \text{for } \alpha = 1,$$

where parameters $\alpha \in (0, 2], \beta \in [-1, 1], \mu \in \mathbb{R}$ and $\sigma \in [0, \infty)$.
So the α-stable distribution has four parameters. These parameters may be interpreted as

α …tail power (tail index), as α decreses tail thicknes increases,
β …skewness parameter, determines asymmetry, a positive β indicates that right tail is fatter then left   tail and vice versa, $\beta = 0$ corresponding to a symmetric distributions,
μ …location parameter (corresponding to a mean for $\alpha > 1$),
σ …scale parameter, generalized   standard deviation, for $\alpha = 2$ corresponding to a standard deviation of normal distribution.

2.1.1 Power of the tails

The power of the tail is the index α which approximately means that[3] $P(X < x) \approx c_\alpha |x|^{-\alpha}$ as $x \to -\infty$.

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[3] The exact formula for c_α can be found in Nolan (1997).

2.2 Normal inverse Gaussian and Student-t distributions as special cases of generalized hyperbolic distributions

This generalized hyperbolical distributions was introduced by Barndorff-Nielsen (1977) and at first applied them to model grain size distributions of wind-blown sands. Eberlein and Keller (1995) were the first to apply these distributions to finance. The probability density function is as follows:

$$f(x) = \frac{(\alpha^2 - \beta^2)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\left(\lambda-\frac{1}{2}\right)}\delta^\lambda K_\lambda\left(\delta\sqrt{(\alpha^2 - \beta^2)}\right)} (\delta^2 + (x-\mu)^2)^{(\lambda-1/2)/2} K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right) \exp\left(\beta(x-\mu)\right),$$

where $K_\lambda(x)$ is modified Bessel function of the third (sekond) kind with index $\lambda \in \mathbb{R}$. It can be defined as

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty s^{\lambda-1}\exp\frac{x(s+s^{-1})}{2}\, ds.$$

The following distributions are the special cases of the generalized hyperbolic distribution (GH). For $\lambda = 1$ we get the hyperbolic distribution, for $\lambda = 1/2$ we get the normal inverse Gaussian distribution (NIG). So the probability density function of NIG is (using some properties of Bessel functions):

$$f(x) = \frac{\alpha\delta\, K_1\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\pi\sqrt{\delta^2 + (x-\mu)^2}}\exp\left(\delta + \beta(x-\mu)\right).$$

For $\lambda = -\frac{\nu}{2}, \nu > 0, \alpha = \beta = 0,\ \sigma = \sqrt{\nu}, \mu$ we get Student t-distribution where ν is the number of degrees of freedom :

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}.$$

2.2.1 Fat tails of GH

The asymptotic probability of GH is the following

$$P(X \le x) \approx |x|^{\lambda-1}\exp[(\alpha+\beta)x]\ \text{ as } x \to -\infty.$$

We see that t-distribution has thicker left tail than NIG.

3. Results

For our empirical analysis two stock market indices are chose. They are indices PX for stock markets of developing economy and FTSE 100 representing the stock markets of developed economies. Data are series of a daily close index values prom 2000 to 2018. Indices are obtained from Bloomberg database. Time series of index PX is retrieved from Quandl database which receives it directly from Prague Stock Exchange. As time series acquired from Bloomberg

database contain artificially injected data for nontrading days at stock exchanges with values for the last trading days.

Table 1: Descriptive statistic of original time series

	PX	FTSE 100
mean	985.12	5737.8
median	979.42	5861.9
maximum	1936.1	7778.6
minimum	320.10	3287
st. deviation	361.46	949.87
skewness	0.328	-0.284
kurtosis	2.782	2.347
number of observations	4554	4735

Table 2: Descriptive statistics of daily log-returns

	PX	FTSE 100
mean	1.31e-4	1.81e-4
median	2.13e-4	5.73e-4
maximum	0.124	0.094
minimum	-0.162	-0.093
st. deviation	0.0137	0.0117
skewness	-0.222	-0.478
kurtosis	16.106	9.573
number of observations	4553	4458

4. Estimation of parameters of distributions

The return series is first used to estimate parameters of hypothesized distributions described in the previous section by maximum likelihood estimation technique. The normal distribution is also included for comparison as well as it is a special case of all multi-parameter distribution classes. All computation is implemented in Matlab. The estimation results are displayed in Tables 3, 4, 5. Besides the values of the parameters of each distribution, the asymptotic standard errors (S.E) of the estimates are also computed and they are displayed under the estimated values of the parameters in the tables.

Table 3: Parameters alpha stable estimation

		PX	FTSE
α	Coeff	1.591	1.461
S.E.		3.02e-2	2.38e-3
β	Coeff	-0.125	-0.132
S.E.		3.30e-2	1.01e-2
σ	Coeff	6.17e-3	5.45e-3
S.E.		9.46e-5	9.04e-5
μ	Coeff	-1.27e-4	-1.22e-4
S.E.		6.25e-5	1.40e-4

Table 4: Parameters Student t-distribution estimation

		PX	FTSE
μ	Coeff	2.89e-4	5.21e-4
S.E.		3.64e-4	1.21e-4
σ	Coeff	7.62e-3	6.50e-3
S.E.		1.26e-4	9.12e-5
ν	Coeff	3.09	2.36
S.E.		0.176	1.71e-2

Table 5: Parameters NIG distribution estimations

		PX	FTSE
α	Coeff	63.290	46.431
S.E.		2.975	3.565
β	Coeff	-5.444	-4.798
S.E.		0.380	1.781
δ	Coeff	8.8e-3	6.66e-3
S.E.		3.13e-4	1.33e-3
μ	Coeff	7.69e-4	8.23e-4
S.E		1.37e-4	9.04e-4

5. Pearson's chi squared goodness of fit test

In this section the Pearson chi squared goodness of fit test is used to verify the suitability of each distribution. The advantage of this choice is that it can be applied globally as well as on individual segments of a distribution. It also takes into account the number of parameters of a distribution. The essence of the test is as follows. This test tests the null hypothesis whether data comes from a certain distribution. The measure of goodness of fit which is also test statistic compares the observed frequencies with the expected ones by summing up their differences as follows

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i},$$

where
O_i is observed frequency for the *i*-th bin,
E_i is expected frequency for this bin
and E_i is computed as follow

$$E_i = N\big(F(ub)_i - F(lb_i)\big),$$

where N is total number of observations, *F* is the CDF of the hypothesized distribution, *lbi* and *ubi* are the lower bound and upper bound of the *i*-th bin respectively. The test statistic under the null hypothesis has χ^2 with *n –k -1* degrees of freedom, where *n* is the number of bins and *k* is the number of parameters of the tested distribution (see Snedecor and Cochrane 1989).

The testing using chi squared goodness of fit test is proceeded as follows. First we test for the whole distribution. The whole interval [0,1] is divided into forty subintervals of length 0.025. If the inverse CDF of a distribution exists, then the boundary points for each bin are computed with the inverse CDF, otherwise, it is determined numerically. Then the boundary points for each bin are computed by interpolation. After that the number of observed frequencies is counted and so is the test statistic. The results are displayed into Table 6.

We also use the chi squared goodness of fit test to investigate the validity of the null hypothesis in several segments, namely the left tail. We test for the validity in segment (0, 0.025). This interval is divided into ten bins of equidistant length 0.0025 and the testing is proceeded as described above. The results are summarized in Table 6.

Table 6: *χ2 statistics*

Distribution	PX	FTSE
Normal		
Test stat	155.18	113.63
p-value	0	0
Studen t		
Test stat	11.07	17.92
p-value	0.135	1.24e-2
Alpha-stable		
Test stat	22.11	46.72
p-value	1.16e-3	2.13e-8
NIG		
Test stat	15.93	8.220
p-value	1.41e-2	0.222

The left end of the empirical CDF of returns of index and PX is above both t-distributions and NIG distribution while the ones of returns of index FTSE lie between the CDF of two t distributions and NIG distribution and for index FTSE they are closer to the NIG distribution than to the two t distributions. This indicates that the distribution of returns of index PX has heavier tail than the distribution of returns of index FTSE.

Integrating the test results together, it is clear that the returns of, PX can be modeled by t distribution while for the returns of index FTSE the NIG distribution is quite suitable.

6. Conclusion

In this paper we have examined whether there is a difference in the distribution of returns of stock market index from two groups of economies: developed ones and developing ones. For the former, two index FTSE is chosen and for the latter the index PX of stock markets from central European countries is chosen. Daily data series of close values of these indices are used. First, the returns of these indices are used to estimate parameters of their distribution. Four distributions are selected for this purpose taking into account the leptokurtic tail. They are normal distribution, alpha stable distribution, t-distribution and normal inverse Gaussian distribution. The maximum likelihood estimation technique is applied to estimate parameters of these distributions from the data. After that we use chi squared goodness of fit test to examine the suitability of these distributions for our data. This test has been applied for their left tail of distribution. The test results show that the distribution of returns of indices of developed stock markets tend to have lighter tails. For index FTSE its returns can be appropriately modeled by normal inverse Gaussian distribution while for index PX of developing stock markets, their returns can be modeled by t-distribution which has strong tail than the previous one. In our opinion, the type of distribution with its tail power, when correctly identified, can be used as a relative measure to evaluate the efficiency of a stock market and for risk management (computing VaR and CVaR).

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Determination of Fuzzy Relations for Economic Fuzzy Time Series Models by Neural Networks

Dušan Marček [1]

Abstract

Most models for the financial time series have centered on autoregressive (AR) processes. Traditionally, fundamental Box-Jenkins analysis have been the mainstream methodology used to develop time series models. We describe developing a fuzzy regression model, i.e. determining and developing of fuzzy time series models, calculating of fuzzy relations, calculating and interpreting the outputs. The fuzzy rules are generated using the neural network with SCL-based product-space clustering. Forecasts of a fuzzy time series model is compared with statistical AR(1) model.

Key words

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1. Introduction

The fuzzy time series models have been in use in analyses for many years. Lots of issues of journal Fuzzy Sets and Systems as well as many others have been articles whose analyses are based on the fuzzy regression and fuzzy time series models. From reviewing of these papers, it became clear that in economic applications the use of method is not on the same level as analyses using classical linear regression. Computers play an important role in fuzzy regression analyses and dynamic processes. Economic time series analysis is concerned with estimation of complicated relationships among groups of variables. Very frequently, in such cases more sophisticated approaches are considered, which are based on the human experience knowledge and consist of series linguistic express in the form of an if … then" fuzzy rule. Most of these approaches are based on the use fuzzy/non fuzzy neural networks (Horikawa, et.al, 1992, Zadeh, L.A., 1992).

The primary objective of this paper is a focused introduction to the fuzzy time series model and its application to the analyses and forecasting from classical regression model of view. In Section 2, we introduce the conventional and fuzzy time series modelling methods. Section 3 shows how to combine neural and fuzzy systems to produce fuzzy rules. Concluding remarks are given in Section 4.

2. Quantitative Fuzzy Time series Modeling Methods

Quantitative modelling approaches of both conventional and fuzzy time series can be grouped into two types: time series method and causal methods. In practice, there are many time series in which successive observations are dependent, i.e. there exists an observational relation

$$R = \{(y_t, = f(y_t, y_{t-1}), (y_{t-1}, y_{t-2}), \ \dots \ \} \subseteq Y_t \times Y_{t-1} \tag{1}$$

where Y_t, Y_{t-1} denote the variables and y_t, y_{t-1} denote the observed values of Y_t, Y_{t-1} respectively.

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The most often used model is, however, an explicit function

$$f: \ Y_{t-1} \rightarrow Y_t, \tag{2}$$

This means that we look for some relation instead of function, or for function f such that the condition $f(y_{t-1}) = y_t$ for $t = 1, 2, \ldots, N$ are violated. Very often the linear function (Markov process)

$$y_t = f(y_{t-1}, \phi_1, \varepsilon_t) = \phi_1 y_{t-1} + \varepsilon_t \tag{3}$$

is used, where ε_t is white noise with mean zero and variance σ^2 and is normally distributed. Equatin (3) is called an autoregressive process of the order p = 1 abbreviated AR(1).

The AR(1) is special case of a stochastic process which is known as ARMA(*p, q*) (Auto-Regressive Moving Average) model defined as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{4}$$

where $\{\phi_i\}$ and $\{\theta_i\}$ are the parameters of the autoregressive and moving average parts respectively, and ε_t is white noise with mean zero and variance σ^2 and is normally distributed. ARMA(*p, q*) process can be considered as an approximate AR(p) model of the form

$$y_t = \phi_1 y_{t-1} +, \phi_2 y_{t-2} + \ldots \phi_p y_{t-p} + \varepsilon_t \tag{5}$$

In the case of fuzzy time series the fuzzy relational equations can be employed as the models. Analogously to conventional time series models, it is assumed that the observation at the time t accumulates the information of the observation at the previous time, i.e. there exists a fuzzy relation such that

$$y_t^j = y_{t-1}^i \circ R_{ij}(t, t-1), \tag{6}$$

where $y_t^j \in Y_t$, $y_{t-1}^j \in Y_{t-1}$, $i \in I$, $j \in J$ and J are indices for set Y_t and Y_{t-1} respectively "$\circ$" is the sign for the *max-min* composition, $R_{ij}(t, t-1)$ is the fuzzy relation among the observations at t and $t - 1$ times. Then we can written

$$Y_t^j = Y_{t-1}^i \circ R_{ij}(t, t-1) \tag{7}$$

Equation (6) is equivalent to the linguistic condition

$$\text{"if } y_{t-1}^i \text{ then } y_t^j\text{"} \tag{8}$$

we have $R_{ij}(t, t-1) = y_{t-1}^i \times y_t^j$, where "$\times$" is the Cartesian product and therefore

$$R(t, t-1) = \max_{i,j} \{ \min(y_{t-1}^i, y_t^j) \} \tag{9}$$

Equation (9) is called a first-order model of the fuzzy time series Y_t with lag $p = 1$.

The first-order fuzzy time series model can be extended to p-order model in the form

$$Y_t = (Y_{t-1} \times Y_{t-2} \times \ldots \times Y_{t-p}) \circ R_a(t, t-p), \tag{10}$$

or

$$y_t^j = (y_{t-1}^{i1} \times y_{t-2}^{i2} \times \ldots \times y_{t-p}^{ip}) \circ R_p(t, t-p) \tag{11}$$

where $R_a(t, t-p)$ is fuzzy relation among Y_{t-1}, Y_{t-2}, ..., Y_{t-p}.

Equation (10) is equivalent to statement

$$\text{if "} y_{t-1}^{i_{i1}} \text{ and } y_{t-2}^{i_{i2}} \text{ and } \ldots \text{ and } y_{t-p}^{i_{ip}} \text{ then } y_t^j\text{"} \tag{12}$$
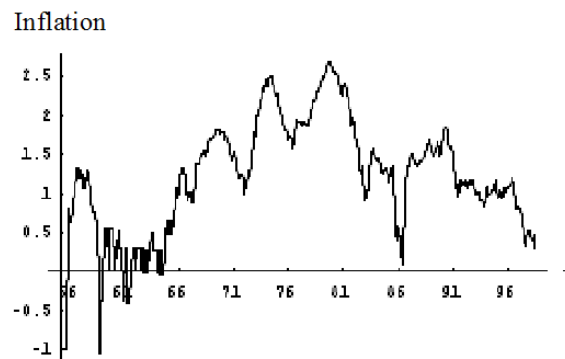
All of the univariate fuzzy time series models can be extended to the econometric fuzzy time series model. See Marcek, 2015 for details.

3. Extracting of Fuzzy Relation Using Competitive Neural Networks

To find exact of fuzzy relations, we will use methods based on a model of a process and illustrate how sophisticated obtain the fuzzy rules of the type of (8) or (11). Neural networks can adaptively generate the fuzzy rules purely from data using the fuzzy sets theory and neural networks. a simple example.

Let as consider the 514 monthly inflation observations for the forty-three years 1956-1998[2] (see Fig. 1). This time series a no apparent trend or periodic structure. To build a forecast model the sample period for analysis $y_1,…,y_{344}$ was defined, i.e. as the period over which the forecasting model can be developed and estimated, and the ex post period $y_{345},…,y_{514}$ as validation data set.

Figure 1: Natural logarithm of monthly inflation from February to November 1998

Using time series data and traditional statistical tools as the autocorrelation function (ACF), the partial autocorrelation function (PACF) and the Akaike Information Criterion the model is estimated as (Therien, 1992, Marcek, 2003)

$$\hat{y}_t = -0,1248\, y_{t-1},\ t = 1, 2, ..., 344,$$

where y_t are the calculated values of inflation. We will further suppose that the potential inputs, which were chosen based on statistical analysis, are crisp data. Sometime it may be advantageous to convert them into fizzy sets (linguistic values). Then the fuzzy time series modelling procedure consist on an implementation several steps. In the literature this modelling approach is known as the fuzzy rule based system (see Figure 2).

The fuzzy rule based system in Figure 2 has three blocks: block for fuzzification of input variables, knowledge base block, and defuzzification block.
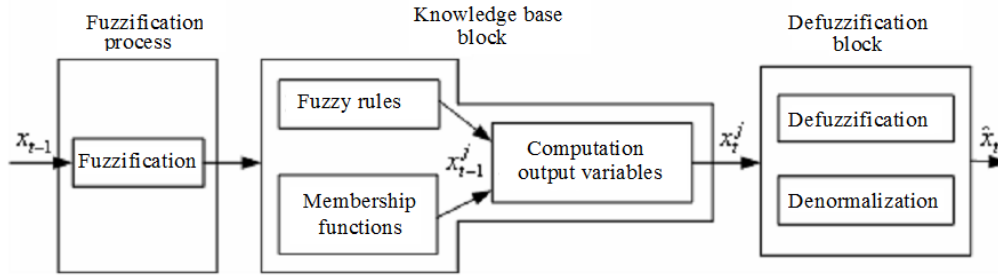
Firstly in the fuzzification block, we specified input and output variables. The input variables x_{t-1} is the lagged first difference of inflation values and is calculated as $x_{t-1} = y_{t-1} - y_{t-2.}$, $t = 3, 4,…$. The output variable x_t is the first difference of inflation values and it is calculated as $x_t = y_t - y_{t-1.}$, $t = 2, 3,…$. The variable ranges are as follows:

$$-0.75 \le x_t, x_{t-1} \le -0.75$$

Figure 2: Structure of fuzzy system for inflation forecasts

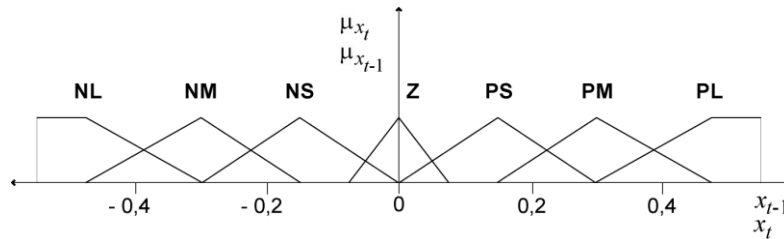[2] http://neatideas.com/data/inflatdata.htm

These ranges define the universe of discourse within which the data of x_{t-1} and x_t are, end on which these fuzzy sets to be, specified. The universe of discourse we divided into the seven intervals.

Next, we specified the fuzzy-set values of the input and output fuzzy variables. The fuzzy sets numerically represent linguistic terms. Each fuzzy variable assumed seven fuzzy-set values as follows: NL: Negative Large, NM: Negative Medium, NS: Negative Small, Z: Zero, PS: Positive Small, PM: Positive Medium, PL: Positive Large.

Fuzzy sets contains elements with degrees of membership. Fuzzy membership functions can have different shapes. The triangular membership functions were chosen. Figure 3 shows membership function graph of the above fuzzy sets.

Figure 3: Fuzzy membership functions of fuzzy variables x_{t-1} and x_t

The input and output spaces we divided into the seven disjoint fuzzy sets. From membership function graphs μ_{t-1}, μ_t Figure 3 shows that the seven intervals [-0,75; 0,75], [-0,375; -0,225], [-0,225; -0,075], [-0,075; 0,075], [0,075; 0,225], [0,225; 0,375], [0,375; 0,75] correspond to NL, NM, NS, Z, PS, PM, PL, respectively.

Next, we specified the fuzzy rule base or the fuzzy relation bank. The above specified interval $-0.75 \le x_t, x_{t-1} \le -0.75$ portioned into seven non-uniform subintervals that represented the seven fuzzy sets values NL, NM, NS, Z, PS, PM, PL assumed by fuzzy variables x_{t-1} and x_t. The Cartesian product of these subsets defines $7 \times 7 = 49$ fuzzy cells in the input-output product space R^2. As mentioned in (Kosko, 1992) these fuzzy cells equal fuzzy rules. Thus, there are total 49 possible rules and thus 49 possible fuzzy relations.

We can represent all possible fuzzy rules as 7-by-7 linguistic matrix (see Figure 5). The idea is to categorize a given set of distribution of input vector $\mathbf{x}_t = (x_{t-1}, x_t)$, $t = 1, 2, …, 344$ into $7 \times 7 = 49$ classes, and then represent any vector just by the class into which it falls.

The neural network shown in Figure 5 was used to generate structure knowledge of the form "if A, then B" from a set of numerical input-output data.

We used SCL (Supervised Competitive Learning) (Marcek, 2002) to train the neural network in Figure 5. We used 49 synoptic quantization vectors. For each random input sample

$\mathbf{x}_t = (x_{t-1}, x_t)$, the winning vector $w_i = (w_{1i'}, w_{2i'})$ was updated by the SCL algorithm according to

Figure 4: Distribution of input-output data (x_{t-1}, x_t) in the input-output product space $X_{t-1} \times X_t$ (a). Bank of fuzzy rules of the time series modelling system (b)
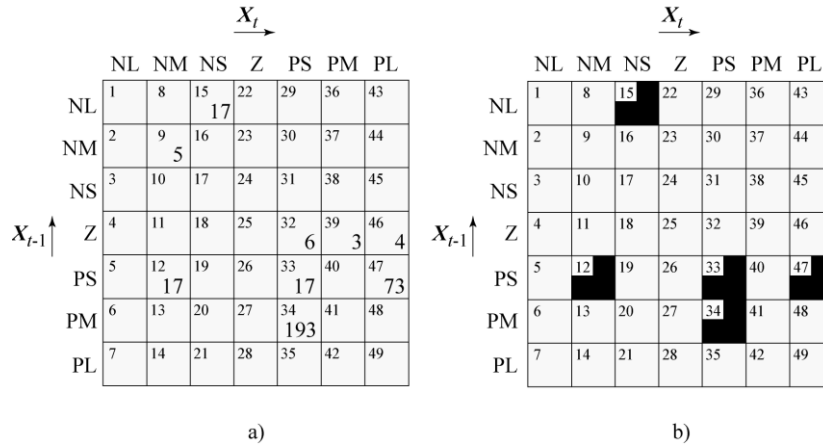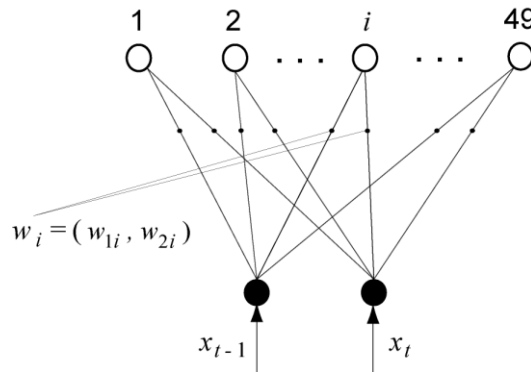

a)   b)

Figure 5: The topology of the network for fuzzy rules generating by SCL-based product-space clustering

$$\left.\begin{array}{l}\tilde{w}_{1i'} \leftarrow \tilde{w}_{1i} + \eta\,(\tilde{x}_{1t} - \tilde{w}_{1i}) \\ \tilde{w}_{2i'} \leftarrow \tilde{w}_{2i} + \eta\,(\tilde{x}_{2t} - \tilde{w}_{2i})\end{array}\right\} \quad \text{if } i = i' \qquad \left.\begin{array}{l}\tilde{w}_{1i'} \leftarrow \tilde{w}_{1i} - \eta\,(\tilde{x}_{1t} - \tilde{w}_{1i}) \\ \tilde{w}_{2i'} \leftarrow \tilde{w}_{2i} - \eta\,(\tilde{x}_{2t} - \tilde{w}_{2i})\end{array}\right\} \quad \text{if } i \neq i' \ ,$$

where i' is the winning unit defined as

$$\left\|\widetilde{\mathbf{w}}_{i'} - \widetilde{\mathbf{x}}_t\right\| \leq \left\|\widetilde{\mathbf{w}}_i - \widetilde{\mathbf{x}}_t\right\|$$

for all i, and where $\tilde{w}_i$ and $\tilde{x}_i$ are normalized versions of w_i and x_i, respectively, and η is the learning coefficient.

Supervised Competitive Learning (SCL)-based product-space clustering classified each of the 344 input-output data vectors into 9 of the 49 cells as shown in Figure 4 (a). Figure 4 (b) shows the fuzzy rule bank. For example, the fuzzy rule of 34th block corresponds to the following fuzzy relation

$$\text{IF } x_{t-1}^i = \text{PM} \ \text{THEN} \ x_t^j = \text{PS}. \tag{13}$$

141

We added a rule to the rule bank if the count of input-output vectors in particular cells was larger than the value 0.05 N, where $N = 344$ is number of data pairs (x_{t-1}, x_t), in the input and output series. For example, the most frequent rule represents the cells 34. From most to last important (frequent) the fuzzy rules are (PM; PS), ((PS; PL), (NL; NS), (PS; PL) a (PS; PS).

Finally, we can determine the output action given the input conditions. The Mamdani´s implication (Mamdani, 1977) was used. Each rule produces the output fuzzy set clipped at the degree of membership determined by the input condition and the fuzzy rule. When the input value, say $x_{t-1}^i = x_{344}^i$, is applied to the model (6), the output fuzzy value $x_t^j = x_{345}^j$ can be calculated. It is possible to compute the output fuzzy value x_t^j by the following simple procedure consisting of three steps:

- Compute the membership function values $\mu_{NL}(x_{t-1}), \dots \mu_{PL}(x_{t-1})$ for the input x_{t-1} using the membership functions in Figure 3.
- Substitute the computed membership function values in fuzzy relations (8), (12).
- Apply the max-min composition to obtain the resulting value of fuzzy relations. x_t^j

Following the above principles, we have obtained the predicted fuzzy value for the inflation $x_t = x_{345}^j = 0.74933$.

The inflation values in the output x_t^j, $t = 345, 346, \dots$ are not very appropriate for a decision support because they are fuzzy sets To obtain a simple numerical value in the output universe of discourse , a conversion of the fuzzy output is needed. This step is called defuzzification. The simplest defuzzification scheme seeks for the value $\hat{x}_t$ that is of middle

Membership in the output fuzzy set. Hence, this defuzzification method is called the Middle of Maxima, abbreviated MOM. Following this method, we have obtained the predicted value for the $\hat{x}_{345} = -0.15$. The remaining forecast for ex post forecast period $t = 346, 347, \dots$ may be generated in a similar way.

As a final point, let us examine what has been gained by use of fuzzy time series model over an ordinary AR(1) model for the output $\hat{x}_{345}$. For this purpose, we have computed prediction limits on the one-step-ahead forecast from the AR(1) model, and fuzzy time series model. The 95 percent interval around the actual inflation value based on the statistical theory is

$$\hat{x}_{345} = \pm u_{1-\alpha/2}\hat{\sigma}_\varepsilon (1+\phi_1^2)^{1/2} = 0.00312 \pm 1.96 \ 0.15476(1+(-0.1248)^2)^{1/2} = (-0.0442; 0.05043),$$

where $\hat{x}_{345}$ represents the forecast for period $t = 345$ made at origin $t = 344$, $u_{1-\alpha/2}$ is a 100(1-$\alpha/2$) percentage of the standard normal distribution, and $\hat{\sigma}_\varepsilon$ an estimate of the standard deviation of the noise. An intuitive method for constructing confidence intervals for fuzzy time series model is simply the defuzzification method First of Maxima and First of Minima to obtain the prediction limits on the one-step-ahead forecast. In our example, the "confidence" interval for fuzzy time series value $\hat{x}_{345} = 0.00312$ is (-0-30256 to 0.3088). The actual value for the AR(1) model does not fall within the forecast interval, and moreover, its sign is opposite to the forecast value sign.

4. Conclusion

In this paper, we have presented an application of the fuzzy tine series model to forecast an autoregressive process. This formal framework is based on the statistical approach, the AR(1) model and the neural network with the SCL clustering technique.

The comparison of proposed techniques with statistical approaches, the AR(1) model generates worse one-step-ahead forecasts. Furthermore, pure statistical models will involve more greater computational effort, and will more difficult to modify.

The method may be of real usefulness in practical applications, where the expert usually cannot explain linguistically what control actions the process takes or there is no knowledge of the process. In principle a neural network can derive this knowledge from data.

Acknowledgement

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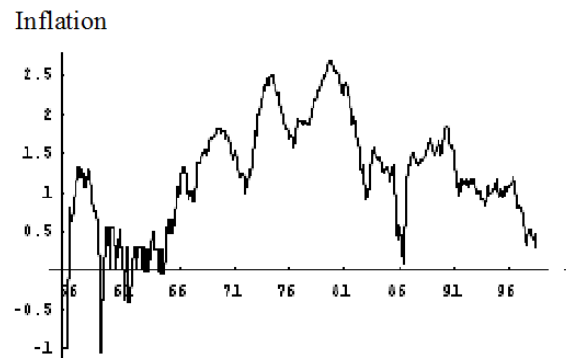
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[2] http://neatideas.com/data/inflatdata.htm

Figure 2: Structure of fuzzy system for inflation forecasts

These ranges define the universe of discourse within which the data of x_{t-1} and x_t are, end on which these fuzzy sets to be, specified. The universe of discourse we divided into the seven intervals.

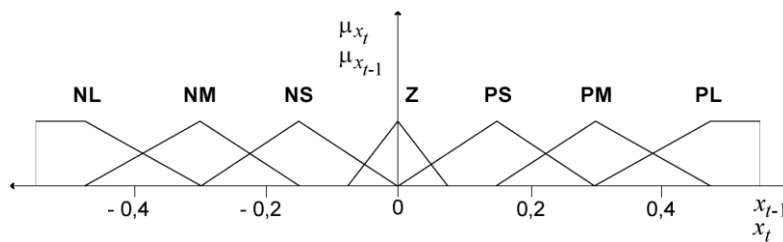Next, we specified the fuzzy-set values of the input and output fuzzy variables. The fuzzy sets numerically represent linguistic terms. Each fuzzy variable assumed seven fuzzy-set values as follows: NL: Negative Large, NM: Negative Medium, NS: Negative Small, Z: Zero, PS: Positive Small, PM: Positive Medium, PL: Positive Large.

Fuzzy sets contains elements with degrees of membership. Fuzzy membership functions can have different shapes. The triangular membership functions were chosen. Figure 3 shows membership function graph of the above fuzzy sets.

Figure 3: Fuzzy membership functions of fuzzy variables x_{t-1} and x_t

The input and output spaces we divided into the seven disjoint fuzzy sets. From membership function graphs μ_{t-1}, μ_t Figure 3 shows that the seven intervals [-0,75; 0,75], [-0,375; -0,225], [-0,225; -0,075], [-0,075; 0,075], [0,075; 0,225], [0,225; 0,375], [0,375; 0,75] correspond to NL, NM, NS, Z, PS, PM, PL, respectively.

Next, we specified the fuzzy rule base or the fuzzy relation bank. The above specified interval $-0.75 \le x_t, x_{t-1} \le -0.75$ portioned into seven non-uniform subintervals that represented the seven fuzzy sets values NL, NM, NS, Z, PS, PM, PL assumed by fuzzy variables x_{t-1} and x_t. The Cartesian product of these subsets defines $7 \times 7 = 49$ fuzzy cells in the input-output product space R^2. As mentioned in (Kosko, 1992) these fuzzy cells equal fuzzy rules. Thus, there are total 49 possible rules and thus 49 possible fuzzy relations.

We can represent all possible fuzzy rules as 7-by-7 linguistic matrix (see Figure 5). The idea is to categorize a given set of distribution of input vector $\mathbf{x}_t = (x_{t-1}, x_t)$, $t = 1, 2, \ldots, 344$ into $7 \times 7 = 49$ classes, and then represent any vector just by the class into which it falls.

The neural network shown in Figure 5 was used to generate structure knowledge of the form "if A, then B" from a set of numerical input-output data.

We used SCL (Supervised Competitive Learning) (Marcek, 2002) to train the neural network in Figure 5. We used 49 synoptic quantization vectors. For each random input sample $\mathbf{x}_t = (x_{t-1}, x_t)$, the winning vector $w_i = (w_{1i'}, w_{2i'})$ was updated by the SCL algorithm according to

Figure 4: Distribution of input-output data (x_{t-1}, x_t) in the input-output product space $X_{t-1} \times X_t$ (a). Bank of fuzzy rules of the time series modelling system (b)

a)   b)

Figure 5: The topology of the network for fuzzy rules generating by SCL-based product-space clustering

$$\left.\begin{array}{l} \tilde{w}_{1i'} \leftarrow \tilde{w}_{1i} + \eta\,(\tilde{x}_{1t} - \tilde{w}_{1i}) \\ \tilde{w}_{2i'} \leftarrow \tilde{w}_{2i} + \eta\,(\tilde{x}_{2t} - \tilde{w}_{2i}) \end{array}\right\} \quad \text{if } i = i' \qquad \left.\begin{array}{l} \tilde{w}_{1i'} \leftarrow \tilde{w}_{1i} - \eta\,(\tilde{x}_{1t} - \tilde{w}_{1i}) \\ \tilde{w}_{2i'} \leftarrow \tilde{w}_{2i} - \eta\,(\tilde{x}_{2t} - \tilde{w}_{2i}) \end{array}\right\} \quad \text{if } i \neq i' ,$$

where i' is the winning unit defined as

$$\left\| \tilde{\mathbf{w}}_{i'} - \tilde{\mathbf{x}}_t \right\| \leq \left\| \tilde{\mathbf{w}}_i - \tilde{\mathbf{x}}_t \right\|$$

for all i, and where $\tilde{w}_i$ and $\tilde{x}_i$ are normalized versions of w_i and x_i, respectively, and η is the learning coefficient.

Supervised Competitive Learning (SCL)-based product-space clustering classified each of the 344 input-output data vectors into 9 of the 49 cells as shown in Figure 4 (a). Figure 4 (b) shows the fuzzy rule bank. For example, the fuzzy rule of 34th block corresponds to the following fuzzy relation

$$\text{IF } x_{t-1}^i = \text{PM} \text{ THEN } x_t^j = \text{PS}. \tag{13}$$

We added a rule to the rule bank if the count of input-output vectors in particular cells was larger than the value 0.05 N, where $N = 344$ is number of data pairs (x_{t-1}, x_t), in the input and output series. For example, the most frequent rule represents the cells 34. From most to last important (frequent) the fuzzy rules are (PM; PS), ((PS; PL), (NL; NS), (PS; PL) a (PS; PS).

Finally, we can determine the output action given the input conditions. The Mamdani´s implication (Mamdani, 1977) was used. Each rule produces the output fuzzy set clipped at the degree of membership determined by the input condition and the fuzzy rule. When the input value, say $x_{t-1}^i = x_{344}^i$, is applied to the model (6), the output fuzzy value $x_t^j = x_{345}^j$ can be calculated. It is possible to compute the output fuzzy value x_t^j by the following simple procedure consisting of three steps:

- Compute the membership function values $\mu_{NL}(x_{t-1}),....\mu_{PL}(x_{t-1})$ for the input x_{t-1} using the membership functions in Figure 3.
- Substitute the computed membership function values in fuzzy relations (8), (12).
- Apply the max-min composition to obtain the resulting value of fuzzy relations. x_t^j

Following the above principles, we have obtained the predicted fuzzy value for the inflation $x_t = x_{345}^j = 0.74933$.

The inflation values in the output x_t^j, $t = 345, 346, \ldots$ are not very appropriate for a decision support because they are fuzzy sets To obtain a simple numerical value in the output universe of discourse , a conversion of the fuzzy output is needed. This step is called defuzzification. The simplest defuzzification scheme seeks for the value $\hat{x}_t$ that is of middle

 Membership in the output fuzzy set. Hence, this defuzzification method is called the Middle of Maxima, abbreviated MOM. Following this method, we have obtained the predicted value for the $\hat{x}_{345} = - 0.15$. The remaining forecast for ex post forecast period $t =346, 347, \ldots$ may be generated in a similar way.

As a final point, let us examine what has been gained by use of fuzzy time series model over an ordinary AR(1) model for the output $\hat{x}_{345}$. For this purpose, we have computed prediction limits on the one-step-ahead forecast from the AR(1) model, and fuzzy time series model. The 95 percent interval around the actual inflation value based on the statistical theory is

$$\hat{x}_{345} = \pm u_{1-\alpha/2} \hat{\sigma}_\varepsilon (1+\phi_1^2)^{1/2} = 0.00312 \pm 1.96 \ 0.15476(1+(-0.1248)^2)^{1/2} = (-0.0442; 0.05043),$$

where $\hat{x}_{345}$ represents the forecast for period $t = 345$ made at origin $t = 344$, $u_{1-\alpha/2}$ is a 100(1-$\alpha/2$) percentage of the standard normal distribution, and $\hat{\sigma}_\varepsilon$ an estimate of the standard deviation of the noise. An intuitive method for constructing confidence intervals for fuzzy time series model is simply the defuzzification method First of Maxima and First of Minima to obtain the prediction limits on the one-step-ahead forecast. In our example, the "confidence" interval for fuzzy time series value $\hat{x}_{345} = 0.00312$ is (-0-30256 to 0.3088). The actual value for the AR(1) model does not fall within the forecast interval, and moreover, its sign is opposite to the forecast value sign.

4. Conclusion

In this paper, we have presented an application of the fuzzy tine series model to forecast an autoregressive process. This formal framework is based on the statistical approach, the AR(1) model and the neural network with the SCL clustering technique.

The comparison of proposed techniques with statistical approaches, the AR(1) model generates worse one-step-ahead forecasts. Furthermore, pure statistical models will involve more greater computational effort, and will more difficult to modify.

The method may be of real usefulness in practical applications, where the expert usually cannot explain linguistically what control actions the process takes or there is no knowledge of the process. In principle a neural network can derive this knowledge from data.

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Predictive HR analytics: case of employee turnover

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Abstract

Human resource management is one of the most important elements within organization's decision processing and represents strategic approach to the effective management of people in an organization. Human resource management decisions are becoming more frequently based on data which is the area of human resources analytics. This paper presents a quantitative approach of human resources analytics to solve organization's problem with employee turnover using multivariate logistic regression analysis. In the current conditions of the Czech labour market, both private and public organizations are experiencing high undesirable employee turnover with increasing trend. Estimated predictive model of the risk of undesirable employee fluctuation is verified on the actual data from Q1 2019. The possibilities and advantages of practical application of the HR analytics approach to employee turnover are discussed.

Key words

Human resource management, predictive analytics, logistic regression, employee turnover

JEL Classification: C53, M1, M54

1. Introduction

The concept of human resource management (HRM) is defined as a strategic and complex approach to managing people working in the organization and who individually and collectively contribute to achieving the goals of the organization (Horváthová et al., 2014). Human resources analytics (HRA) also called people analytics is currently a very dynamically developing part of HRM and represents a data-based, goal-setting approach based on measurable inputs and outputs and implementation of quantitative methods for informed decision making. The need for HRA is increasing with the gradual transformation of the HRM function from supportive functions of providing staff administration, recruitment and remuneration to the strategic functions. Advanced responsibilities include strategic workforce planning and its variations, managing employee turnover, management and measurement of human capital, ensuring company´s development through talent management, education, engagement and employee motivation and influencing the success or failure of the company in a current, globally competitive environment (Edwards and Edwards, 2016). Fast development of HRM is based also on increasing availability of technological capabilities in the area of processing multivariate data and large data sets in the field of HRM. HR analysts combine this data from various sources (BI records, questionnaires, etc.) to create a comprehensive overview from which the management can derive following action steps and response in advance to possible future scenarios. HR analytics development moved from focus on quantitative applications in human resource management and basic metrics, ways of measuring employee training and development, return on investment in measurement of human capital (Fitz-enz, 2009) to predictive analytics and the application of statistical and econometrical models to human resources management (Fitz-enz and Mattox, 2014) and

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(Edwards and Edwards, 2016) and became already regular part of human resource management (Bauer, Erdogan, Caughlin and Truxillo, 2019). In general, HR analytics can be divided into three categories (Fitz-enz and Mattox, 2014):

Descriptive HR analytics reveals and describes relationships and contemporary and historical patterns and trends. It includes traditional metrics such as turnover, time to occupy positions, recruitment costs, recruitment and training rates, etc., whose primary purpose is to expose cost-saving areas. Descriptive analytics outputs are presented in the form of structured reports, dashboards, scorecards, etc.

Predictive HR analytics combines number of areas such as statistics, modelling, data mining, etc., which use historical and current data to predict future developments. The purpose is to identify probable scenarios and their potential impact. Examples of practical applications are models that increase the likelihood of choosing the right employee to recruit, educate or promote; forecasting employee with the right managerial abilities or employee who is more likely to leave the organization.

The prescriptive HR analyst goes beyond predictions and outlines the possibilities of decision making and optimization in human resource management and human capital. It is used to analyse complex data to predict the output and to present alternative impacts on the organization based on possible decision-making processes. For example, models used to understand how alternative investment in education can affect an organization.

The aim of this paper is to estimate predictive model of employee turnover using multivariate logistic regression model. Undesirable and high employee turnover leads not only to high direct and indirect costs but also presents a challenge for organizations to address this issue. Both private companies and public organizations, including hospitals (Waldman, Kelly, Aurora, Smith, 2004), financial, manufacturing and hi-tech businesses spend a large amount of money each year to manage and deal with employee turnover and associated productivity losses (O'Connell, Kung, 2007). Employee turnover and retention are addressed by both private sector specialists and professionals, HR and finance departments, as well as many academics. Healy, Lehman and McDaniel (1995) research on the topic identified several factors affecting employee turnover, and in their study of a large sample of individuals demonstrated that age is not significant for fluctuation, although it has been found to be a significant factor in several previous studies. Griffeth, Hom and Gaertner (2000), in addition to the impact of age on employee turnover, also looked at wages and wage-related factors such as occupation and level of education, combining these factors with employee performance assessments and demonstrating functional interdependencies between variables. Rosemary and Valcour (2003) assumed family background and gender can influence intention of employee to leave or stay in an organisation. Glumbíková, Vávrová, Nedělníková (2018) accentuate the role of reflection of setting the rules of the organization as an important factor for the intention to leave or stay. Huang (2011) demonstrated the interdependence of working characteristics following blue collars and white collars, job satisfaction and the employee's intention to leave the company. Dostie (2014) examined the link between wage, fluctuation and length of employment or seniority in the Canadian labor market and pointed to individual correlations but overall heterogeneity in relation to jobs. Zhang, Xu, Cheng, Chao and Zhao (2018) applied the machine learning model to predict fluctuation, and one of the important factors of employee turnover was the amount of overtime worked in the analysed period.

2. Logistic regression

Logistic regression can be applied to any combination of discrete or continuous variables, assuming the dependent variable and the independent variables of the analysed selection. Logistic regression is an alternative classification method if the assumptions of a multidimensional normal model are not met. Logistic regression predicts whether a given event has or has not happened, i.e. that the probability is equal to 1 or 0. To create such a binding condition, LR uses a so-called logit transform that leads to a sigmoidal relationship between the explained variable *y* and the explanatory vector of the independent variables *x* (Meloun, Militký, Hill, 2017).

For LR model parameters, it is classically used the maximum likelihood iterative method assuming a multinomial distribution for the general model and binomial distribution for the standard two-class model. Dependent variable in binary or categorial expression may acquire a limited amount of values. If the feature which is followed occurs than the expected value comes with probability π. The opposite value equals to $1 - \pi$. The value of chance can be expressed as $\pi/(1 - \pi)$, where π represents the probability that given event or feature occurs. Logarithm of previous equation is equal to the value from interval (-∞; +∞). Logistic model can be expressed as

$$\pi(x) = \frac{e^{\beta_0 + \beta_1 \cdot x}}{1 + e^{\beta_0 + \beta_1 \cdot x}}. \tag{1}$$

where β_i is a parameter of each independent variable. $\pi(x)$ can be transformed by logit transformation with the domain (-∞; +∞) as following

$$g(x) = \ln\left[\frac{\pi(x)}{1 - \pi(x)}\right] = \beta_0 + \beta_1 \cdot x \tag{2}$$

So called logit is created by this transformation as a result of weighted sum of independent variables which can have value from interval (-∞; +∞).

Hosmer, Lemeshow (2001) propose optimal maximum likelihood function which can be under assumption of independent observations in case of binomial distribution assuming independent observations can be described as following

$$l(\beta) = \prod_{i=1}^{n} \pi(x_i)^{y_i} [1 - \pi(x_i)]^{1-y_i} \tag{3}$$

The objective of maximum likelihood method while estimating each parameter is to maximize the likelihood function. This function expresses a probability of obtaining a specific random selection. Logarithm of previous function can be expressed as following

$$L(\beta) = \ln[l(\beta)] = \sum_{i=1}^{n} \{y_i \ln[\pi(x_i)] + (1 - y_i) \cdot \ln[1 - \pi(x_i)]\}. \tag{4}$$

To find a value β maximizing $l(\beta)$, it requires to differ between β_0 and β_1 and sum up the equations to zero. *B* value is given by solving previous equations. It is called maximum likelihood estimation and final logistic model is defined as following

$$\ln\left(\frac{L_{(1)}}{L_{(0)}}\right) = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \cdots + b_p \cdot x_p \tag{5}$$

Where $\ln\left(\frac{L_{(1)}}{L_{(0)}}\right)$ represents the logarithm likelihood ratio (logit) and b_i are individual regression coefficients.

3. Predictive model of employee turnover

HR analytics approach is applied on the real data from production company to estimate a predictive model of employee turnover using multivariate logistic regression model. In order to address the goal, a predictive model of risk of employee turnover will be estimated in the second part of this chapter using the multiple logistic regression analysis method with a combination of Edwards and Edwards (2016) and Sharma (2018):

1. Testing variables in the model, overview of descriptive statistics, correlation analysis.
2. Preparing and modifying data in a matrix where relevant factors are attached to each individual employee.
3. Estimating the model using multivariate logistic regression analysis.
4. Optimizing the model.
5. Calculating the risk of employee turnover at an individual level.
6. Evaluating the quality of the predictive capabilities of the model on the out-of-sample analysis of employee turnover during first quarter 2019.

Variables were chosen based on studies described in chapter 1. Risk of loss of employee is predicted using other variables as independent in the model. Dependent variable is binary representing with 0 an employee and 1 leaver with undesirable turnover. Independent variables are divided into two groups: work related and sociodemographic. Work related variables are Organization, Category, Work contract, Seniority, Performance, Tariff and Wage. Sociodemographic variables are Age, Gender, Education and Distance to work.

Organization is categorical variable defining at which part of an organization an employee works: 1= primary production, 2 = secondary/finishing production, 3 = maintenance, 4 = engineering and 5 = headquarter.

Category is binary variable defining whether an employee is a 0 = blue collar professions or 1 = white collar professions.

Work contract is binary variable defining whether an employee has an unlimited employment contract = 0 or limited employment contract = 1.

Seniority is continuous variable which represents the duration of employment within the organization, with a minimum value of 0 and a maximum value of 47, which are empirical values that correspond to actual values in the conditions of the analysed company.

Tariff corresponds to the placement of an employee within a certain tariff level. The organization uses twelve-scale scales to rank employees with a minimum of 1 and a maximum of 12. the tariff level is used for the purpose of work rather as an indicator of the inclusion of an employee in a certain group of professions with a similar tariff rating.

The wage represents the total basic wage of the employee. The total basic wage may differ from the final wage of the employee at a certain month depending on the amount of overtime worked, the amount of bonuses and individual remuneration.

Age is continuous variable which represents the chronological age of an employee based on empirical values from the analysed company.

Gender is binary variable with values 0 = man or 1 = woman.

Education represents the highest level of education obtained by an employee. There are three groups in this variable, 1 = elementary education, 2 = high school education and 3 = university education.

Distance to work is binary variable representing whether the employee is resident in the same municipality where the organization is located or does not have a longer commute to work. For the purposes of the model 0 = residence in the same municipality as the headquarters of the organization and 1 = residence outside the municipality of the seat of the organization.

3.23.1 Model estimation

A total of 11 independent variables divided into two groups are used in the model to predict employee turnover as independent variable representing employees with undesired turnover from the analysed organization during 2015 – 2018. Following Table 1 shows summary indicators of model significance.

Table 1: Summary indicators of model significance

	Chi-square	df	Sig.
Step	480,695	15,000	0,000

	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
Step	2308,902[a]	0,091	0,214

Table 1 shows that the model is significant on 0,000 level and Nagelkerke R-squared with value 0,214 can be interpreted as amount of variability explained by independent variables. It is a substantial amount of clarified variability considering several factors that cannot be influenced, identified or quantified. The statistical significance of the individual factors in the model and their influence or significance in the form of coefficients in relation to the undesirable turnover of employees is shown in Table 2.

Table 2: Variables in the equation

	B	S.E.	Wald	df	Sig.	Exp(B)
HQ			60,913	4	0,000	
Primary	-1,019	0,222	20,983	1	0,000	0,361
Finishing	-0,96	0,25	14,727	1	0,000	0,383
Service	-0,907	0,185	24,177	1	0,000	0,404
Engineering	0,033	0,202	0,026	1	0,871	1,033
Category	0,32	0,244	1,711	1	0,191	1,377
Work contract	0,72	0,193	13,898	1	0,000	2,054
Seniority	-0,005	0,009	0,303	1	0,582	0,995
Performance	-0,658	0,17	14,976	1	0,000	0,518
Tariff	-0,179	0,063	7,998	1	0,005	0,836
Wage	0	0	0,022	1	0,881	1
Age	-0,036	0,008	22,094	1	0,000	0,964
Gender	-0,327	0,19	2,957	1	0,086	0,721
Elementary			31,319	2	0,000	
High school	-2,274	0,565	16,21	1	0,000	0,103
University	-1,135	0,221	26,492	1	0,000	0,321
Dist. to work	0,342	0,118	8,359	1	0,004	1,407
Constant	3,48	0,703	24,503	1	0,000	32,466

Table 2 shows the results of multiple logistic regression analysis for each variable and their achieved coefficients (*B*), standard deviation (*S.E.*), Wald's test of significance (*Wald*), number of degrees of freedom (*df*), significance (*Sig.*) and odds ratio (*Exp (B)*).

Variable Organization as a categorical variable compares whether employees of any part of the organization have a higher probability of undesired fluctuation and is significant for the model. Employees of headquarter with negative coefficient *B* -1,019 with odds ratio *Exp (B)* 0,361 for primary production means that employees from primary production are 2,77 times (1 / 0,361) less likely to show undesired fluctuation than employees from headquarter. Similar

results are for employees of finishing production with 2,6 times lower probability and service workers with a 2,5 times lower probability of undesirable departure employees in headquarter. Engineering employees are not significant for the model and there is no difference in the likelihood of unwanted fluctuation towards headquarter employees. Similarly can be explained odds ratios of each variable. It is concluded that variables Category, Seniority and Wage are not significant for the model on 0,1 level of importance and needs to be extracted from the model or adjusted for better fit and variable Gender is not significant on 0,05 level of importance.

3.33.2 Prediction evaluation

Estimated coefficients from the predictive logistic regression model are used to estimate a number between 0 and 1 to each employee based in his / her individual variable settings. This number represents the risk of loss of employee or in other words the probability of an employee to undesired turnover. Employees are divided according to their risk of loss into deciles to better assess a predictive potential of the model. The results based on the real data from analysed production company from Q1 2019 are available in Table 3.

Table 3: Prediction evaluation Q1 2019

Decile	Leavers Q1 2019	% Leavers Q1 2019	Total in decile	Avg. Risk of loss
1	14	50,00%	464	0,271
2	6	21,43%	464	0,116
3	2	7,14%	464	0,076
4	1	3,57%	464	0,055
5	1	3,57%	465	0,042
6	2	7,14%	464	0,034
7	1	3,57%	464	0,028
8	0	0,00%	464	0,024
9	1	3,57%	465	0,019
10	0	0,00%	464	0,012
Celkem	28	100,00%	4642	0,068

There were 28 employees with undesirable turnover during the first quarter of 2019 and it is visible in Table 3 that more than two-thirds of employees (71.4%) who left the companyare in the first two deciles and 50% in the first decile of the total amount of employees included in the estimation of risk of loss of employee based on the logistic regression model. Therefore, it is concluded that the predictive model is well estimated and correctly assess the risk of an employee leaving the organization. The predictive characteristics of the model with a certain degree of accuracy correctly classify employees who are at greater risk of leaving the company and correctly classify employees with a higher risk of unwanted fluctuation.

For real application of the predictive model in the practice of private organizations, it is necessary to refine the predictive capabilities of the model by adding a number of different linearly independent variables to more accurately specify and classify employees with a higher risk of undesired fluctuation and estimate the model separately for predefined groups of employees with less heterogeneity.

4. Conclusion

The paper presents a very actual topic based on HR analytics approach combining statistical modelling, econometric applications and analytical approaches with human

resources management, and currently have a lot of attention from private organizations and academics. HR analytics have the potential to change the overall approach to human resource management. Factors with expected impact on undesirable employee turnover are determined based on current academic contributions in this area and available literature, factors with expected impact on undesirable employee fluctuations and discussed. Predictive model of undesirable employee turnover is estimated, and it is concluded that it represents potentially very important tool in human resource management policy and proactive approach to data-driven employee turnover and retention outputs. Model described in the paper presents a straightforward and systematic approach to address employee turnover and to allow managers to actively manage undesirable employee turnover. Procedure of model estimation is described and can be applied in any company or public organization only resulting in different coefficients for different variables based on the focus company or organization. Also, it is recommended to add more variables from different background to furtherly improve the predictive accuracy of the logistic regression model to be better applicable in practice.

Acknowledgements

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Swedish scenario for monetary policy normalization process

Aleksandra Nocoń[1]

Abstract

In February 2015, when the Bank of Sweden (*Sveriges Riksbank*) decided to lower the main interest rate - *the repo rate* to a negative level, also implemented the large-scale purchases of government bonds under the *Quantitative Easing* policy, as well as announced contingency plans to intervene on the foreign exchange market if necessary. From the beginning, the primary objective of Sveriges Riksbank's non-standard interventions was to stabilize inflation around the inflation target and support the general economic policy of the state, aimed at maintaining sustainable growth and maximizing employment. Currently, more than a decade after the historical date of the outbreak of financial crisis, economic condition in Sweden seems to be improving. Therefore, the central bank's thoughts increasingly revolve around the normalization process of its monetary policy. The main aim of the paper is identification of the possibile scenario of monetary policy normalization process of the Swedish central bank – *Sveriges Riksbank*. Moreover, the article presents evolution of the expansionary monetary policy in Sweden after the outbreak of the global financial crisis.

Key words

Sveriges Riksbank, monetary policy, normalization process, exit strategies

JEL Classification: E50, E52, E58

1. Introduction

It has been almost a decade since central banks, in the face of the global financial crisis, implemented a set of unconventional initiatives. The end of the first decade of the 21st century has started a new period in central banking - a period of non-standard and unconventional monetary policy instruments, used in so far unprecedented scale and scope. Monetary authorities' actions, undertaken in response to the spreading out of the global economy instability, have included unprecedented interventions that have led to a reduction of main interest rates to historically low (zero or, in some cases, even negative) levels, huge expansion of central banks' balance sheet and changes in their communication system with stakeholders. At the same time, extraordinary monetary policy tools have become the foundation of lively discussion, both of their supporters and opponents, for further fate and framework of modern central banking.

The problem with a return to traditional (standard) monetary policy is all the more complicated and ambiguous, because till 2019 some of the central banks sill have not withdraw non-standard instruments and have not returned to the procedures and strategies, used before the global financial crisis[2]. Not only monetary policy, but also the whole economic policy of many developed countries, has not completed unconventional activities, implemented after 2008. Financial markets are still not functioning properly and interest rates in many countries

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[2] However, some of the major central banks (like the Federal Reserve System or Bank of England) have discontinued unconventional Quantiatative Easing so far. And they try to implement normalization assumptions in practice.

are at almost zero level. There is also a certain degree of uncertainty, among the participants of economic life, reducing their propensity to invest. Therefore, it can be presumed that if more than a decade after the outbreak of the crisis, central banking has not returned to the *old normal*, then over time, these changes will be extremely difficult to reverse. It seems more likely that we will observe creation of a new reality in monetary policy, defined in the literature as the *new normal* - the new framework of central banking. However, it should be noted that the economy framework in the *new normal* will be different from those after previous periods of destabilization. Scale and intensity of the recent crisis, which has affected almost every economy in the world (with varying intensity), place the global financial crisis of the first decade of the 21st century as one of the most powerful in the history of the world economy.

Normalization - in general terms - is a process of returning to traditional monetary policy. Most often, it is identified with the restoration of monetary policy to pre-global financial crisis framework. On the other hand, it can mean a transformation of monetary policy towards a new quality, so far unknown to central banks - *new normal*. In practice, it means implementation of the *exit strategies* assumptions. The lack of previous experiences of central banks in realization of normalizing activities is undoubtedly a factor hindering their implementation. Furthermore, literature studies indicate a negligible number of studies and publications in this field (Nocoń, 2018; Pyka, Nocoń and Cichorska, 2016; Societe Generale, 2018; BIS, 2018). Appropriate definition and implementation of *exit* assumptions is extremely difficult in the absence of a theoretical framework (Ceciono, Ferrero and Secchi, 2011). In turn, normalization without clearly defined assumptions, *ad hoc* implemented, may destabilize banking sector and finally the whole financial system. That is why the main aim of the paper is identification of the possibile scenario of monetary policy normalization process of the Swedish central bank – *Sveriges Riksbank*. Moreover, the article presents evolution of the expansionary monetary policy in Sweden after the outbreak of the global financial crisis.

2. Expansionary monetary policy of the Sveriges Riksbank after the global financial crisis

In 2008-2009 the oldest central bank in the world, with the longest tradition of conducting monetary policy (www 1) - Swedish central bank (*Sveriges Riksbank*) entered to the path of low interest rate policy. It decided to reduce main interest rates to almost zero level (see figure 1)[3]. As a response to the outbreak of the global financial crisis, the main interest rate - *repo rate*, which is the rate of interest at which banks can borrow or deposit funds at the *Riksbank* for a period of seven days, was reduced by 400 basis points to only 0.25 % (in July 2009). It also provided fixed-rate loans for commercial banks at a value of 300 bln SEK with maturity of approximately 12 months. Their aim was to reduce interest rates on loans for households and companies. However, in 2010 along with the improvement of Swedish economy and – as it was believed at that time - too low level of interest rates and risk of rising inflation, Swedish central bank decided to start increases of the main interest rate. It was made systematically by 25 basis points in 2010 and 2011. In November 2011, the *repo rate* was at a level of 2.00% (www 2). However, significant dependence of Swedish economy on the trade exchange with European Union, caused that Sweden was strongly affected by the effects of public debt crisis in the euro area. *Riksbank* has returned to interest rate cuts in December 2011. Reductions were made systematically in subsequent years. Finally, the *repo rate* was decreased to 0.00% in October 2014. Such a policy was called as *Zero Interest Rate Policy* (ZIRP). It was argued that the aim of expansionary monetary policy was to stimulate inflation towards 2% (in September 2014 the CPI index in Sweden was -0.4%) – see figure 1.

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[3] In February 2008 the main interest rate in Sweden was at a level of 4.25%.

Even earlier (in July 2014), it was implemented a negative deposit rate of -0.5%, decreased also later this year (in October 2014) to -0.75%. This meant that commercial banks, which make an overnight deposit on an account in the Swedish central bank, would have to pay for their storage. This decision was aimed at motivating banking institutions to expand their lending, which in turn would cause an increase of inflation. However, the negative interest rates did not affect households' deposits.

Figure 1: Main interest rate, inflation rate and GDP growth in Sweden between 2007-2019
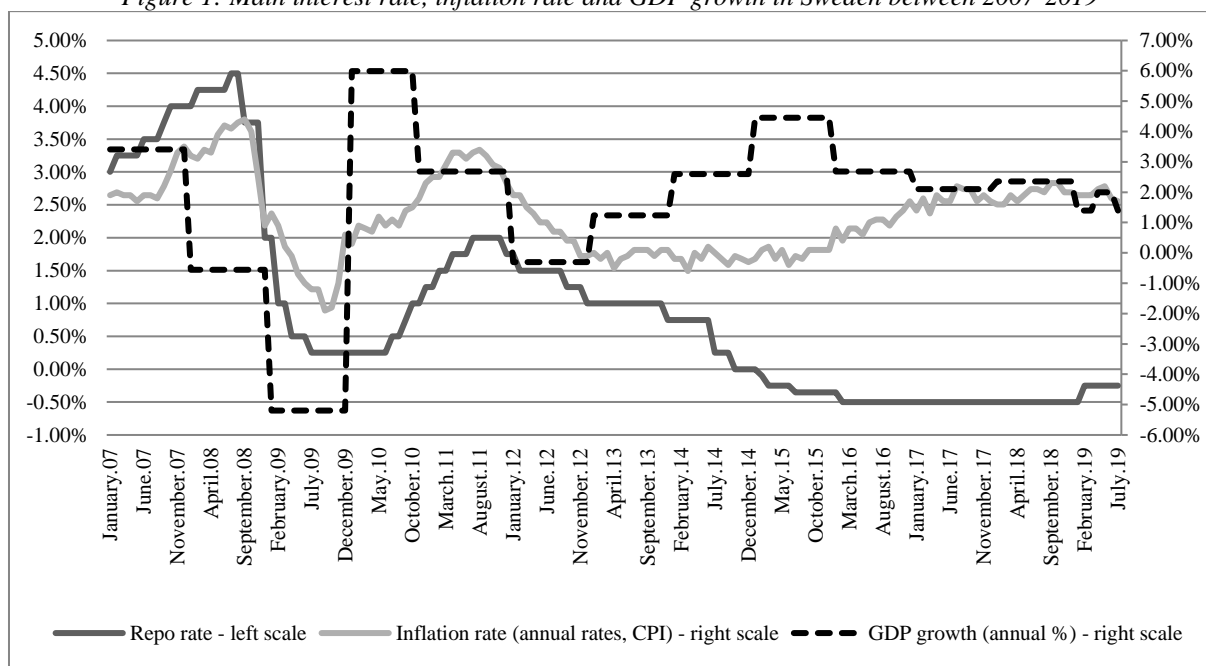
*Source: own work based on: Sveriges Riksbank, Statistics Sweden and World Bank data*

In February 2015, *Zero Interest Rate Policy* in Sweden was turned into *Negative Interest Rate Policy* (NIRP). The *repo rate* has been reduced to -0.1%. Further reductions in the following months decreased the interest rate in February 2016 to -0.5%, the lowest level in a central bank's history. At that time a deposit rate was -1.25%, while a lending rate was 0.25%. These decisions were still targeted to stimulate inflation. Therefore, the case of Sweden shows that non-standard zero interest rate policy was also implemented even in those economies, which banking sector did not suffer such negative consequences of the recent financial crisis.

In February 2015, along with implementation of NIRP policy, the *Riksbank* also decided to implement *Quantitative Easing* policy (QE policy). The program of monetary policy easing included the purchase of 10 bln SEK of government bonds with a maturity between one and five years. In the assumption, these actions were aimed at – in addition to stimulating growth of consumer prices after many months of deflation, also lowering yields of long-term debt securities, while they caused even their increase[4]. After subsequent meetings of the central bank's representatives, they increased the scale of bonds purchase to 30 and then to 45 bln SEK (from July 2019 to December 2020). It was argued that although the policy contributed to increase of inflation, the pace of price growth was still unsatisfactory, and additionally strengthening of the crown, threatening inflation growth, determined further monetary policy easing.

As a result of QE policy, the central bank's balance sheet total as a percentage of GDP increased from less than 7% in 2006 till about 19% in 2017 (see figure 2). On the asset side the

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[4] In April 2015, the yield of 10Y Swedish government bonds was 0.2%, while in June 2015 it was 1.1%. In turn, the yield of 5Y bonds in April 2015 was at fluctuated below zero, and in June increased to 0.4%.

most important change was a significant increase of the share of government bonds, purchased through QE programs. While on the liabilities side, which reflects sources of their funding, bank reserves increased, and constituted more than 8% of the GDP in 2017. They were created to finance the special asset purchase programs, supporting Swedish economic growth and stimulating inflation towards the target.

Figure 2: Central bank's balance sheet in 2006 and 2017 as a percentage of GDP
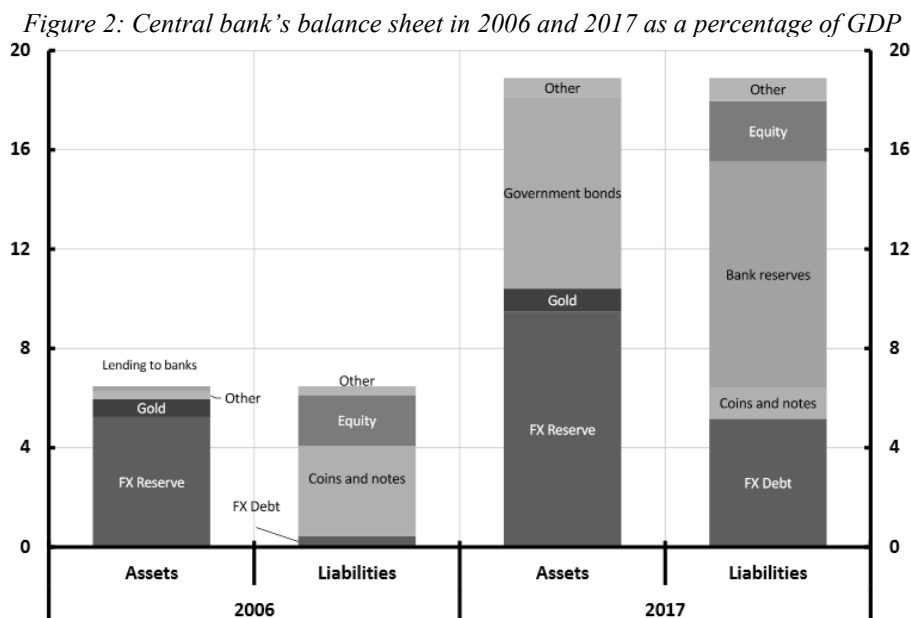
Source: (Flodén, 2018, p. 12).

3. Exit strategies of modern central banks

On June 18-19, 2009, the European Council pointed out that "*there is a clear need for a reliable and credible exit strategy, inter alia by improving the medium-term fiscal framework and through coordinated medium-term economic policies*". At the G20 summit in Pittsburgh on September 24-25, 2009, they called for "*cooperative and coordinated exit strategies*" (von Hagen, Pisani-Ferry and von Weizsäcker, 2009, p. 15). Therefore, a year after the outbreak of the global financial crisis and implementation of extraordinary monetary policy tools, various transnational bodies and institutions pointed to the need to define *exit strategies* from non-standard monetary policy. At that time, it was not assumed that unconventional activities will be used for many consecutive years, in the face of lack of improvement in macroeconomic conditions, risk of deflation and further spreading of banking sector instability. Moreover, it was feared that too long period of implementation of non-standard monetary policy instruments could significantly affect the general paradigm of modern central banking. That is why, special attention was paid to the need to define assumptions of *exit strategies* from non-standard monetary policy.

"*Exit strategy*" is the concept of *exit* from the global financial crisis, developed by the political will of the G-20 states, expressed in the documents of *Financial Stability Board*. It is defined as a return to "*normal*" monetary policy, identified with the pre-global financial crisis policy. In the documents, Financial Stability Board also indicated that *exit strategies* should not be implemented prematurely and that monetary policy tightening should be technically feasible (Nocoń, 2018, pp. 94-99). It was emphasized that *exit strategies* are the greatest challenge facing economic policy after 2007 (Eichengreen, 2009). At the same time, although coordination of international policy on withdrawal of non-standard monetary policy is desirable, it might be difficult to achieve.

In practice, uncertainty about future path of economic growth, inflation rate, or functioning of monetary transmission mechanism may complicate *exit strategies* implementation. In particular, this refers to the timing and pace of their implementation, as well as sequencing of selected actions. Moreover, to raise a level of a main interest rate, central banks first are forced to eliminate excess bank reserves from their liabilities. Their creation was aimed at financing special asset purchase programs implemented through QE policy, or at least neutralizing potentially unwanted effects, that could impact on credit growth or an inflation rate. When it comes to assets, purchased and held by monetary authorities, a part will expire maturity. However, others will have to be resold in order to restore central bank's balance sheet to the size from before unconventional initiatives. "*Exit strategy*" also requires a change from expansionary to restrictive monetary policy. As a consequence, it causes inhibition of domestic money supply and an increase of their market price (interest rate), which raises fears of negative economic consequences.

Limiting negative effects, associated with implementation of *exit strategies* in a banking sector and the economy, requires precise identification of (Przybylska-Kapuścińska, 2010, pp. 198-201):

- moment and scale of completion of expansionary monetary policy - *Quantitative Easing* policy,
- its impact on economic growth,
- synchronization of restrictive policy of money supply with interest rate hikes,
- nature and timing of transitional support,
- operational changes – re-definition of monetary policy instruments,
- methods how to solve balance sheet problems, given the high share of high risk assets.

Therefore, the appropriate implementation of *exit strategies* plays a key role. A kind of recommendation for modern central banks in terms of formulation and realization of *exit strategy* assumptions can be their even greater orientation on objectives and strive for them in a consistent and independent manner.

Exit strategies should be formulated in advance and detailed specified by monetary authorities, even if they are not immediately implemented. Taking into account the wide range of interventions, used by central banks, *exit strategies* should also depend on the nature of a given monetary authorities and, on the other hand, specificity of the selected country. Nevertheless, the following principles should help to formulate *exit strategies* from unconventional monetary policy (Kozicki, Santor and Suchanek, 2011, p. 21):

1. Monetary policy should be guided by objectives for inflation or price stability.
2. Monetary policy should be conditioned on information regarding the economic outlook, including fiscal paths. In this context, fiscal authorities need to plan and communicate their intentions to the public. This would allow central banks to condition monetary policy on the fiscal outlook and help reinforce central bank credibility.
3. Policy authorities need to understand how the monetary transmission mechanism may have changed.
4. Policy credibility and central bank independence must be maintained to ensure the effectiveness of future policy.
5. Communication regarding *exit strategies* should be clear and should include timely reporting of balance sheet developments.

Moreover, A. Belke emphasizes the following prerequisites for the *exit strategy* guidelines (Belke, 2016, p. 287):

1. The exit should be step-by-step rather than a one-off.
2. Communicating about the *exit strategy* must be an integral part of the strategy.
3. Price stability should take precedence in all decisions.

4. Due to vagabonding global liquidity, there is a strong case for globally coordinating monetary *exit strategies*.

The fact that central banks formulate general outlines of the *exit strategies* does not mean that they intend to begin their practical implementation. Appropriate preparation is crucial. The more precisely prepared for the implementation of *exit strategies*, potentially the higher effectiveness. There is no doubt that a return to normality of national economies will have to involve normalization of conditions for conducting monetary policy. However, the key question is what is meant by the term of "*normality*" in post-crisis financial reality. There is empirical evidence suggesting that financial crises typically generate significant and long-term costs in terms of production and employment, and lead to significant declines in asset prices (Reinhart and Rogoff, 2009, pp. 466-472). In the case of the recent financial crisis, there is a risk that the cost of losing financial, physical and human capital around the world can be very significant. As a result, the crisis may have a long-term impact on potential growth of many economies (González-Páramo, 2009). At the same time, this means that a return to "*normality*" can take very long time.
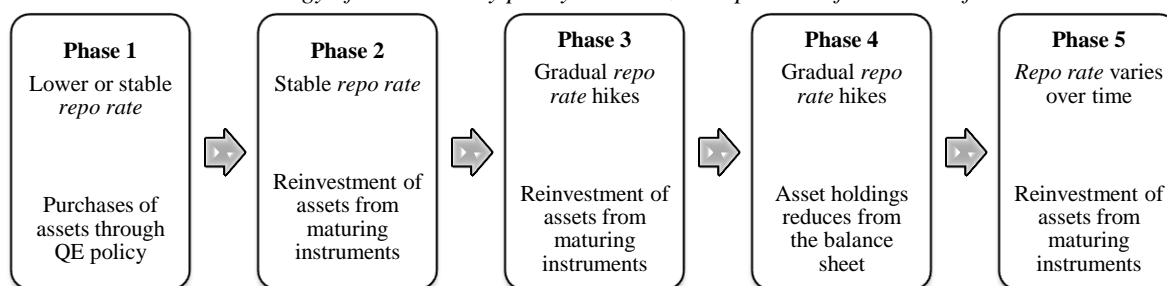
4. Strategy of the normalization process of the Swedish central bank

After the outbreak of the global financial crisis, monetary policy of the Bank of Sweden, like many other major central banks in the world, became strongly expansionary. These activities were aimed at increasing economic activity and bringing inflation towards the inflation target. More than a decade after the escalation of negative events that started on the *subprime* mortgage market in the United States, monetary policy in Sweden is expected to become less expansionary. The change in monetary policy towards the normalization process raises questions how these normalizing activities will affect central bank's monetary policy and how increases of the *repo rate* may impact on market participants' behavior, and thus the real economy.

Any decision of the Bank of Sweden regarding the direction of its monetary policy and implemented instruments is determined by actions aimed at stabilizing inflation around the target, maximizing employment and the long-term path of sustainable development. However, the Swedish central bank, as compared to other large central banks, as a monetary authority of a small, open economy, must take greater account of the actions undertaken by other central banks of the global economy. If *Riskbank's* monetary policy was significantly different from the monetary policy pursued in other countries (mainly European Union ones), this could have an impact on the Swedish crown, which in turn could have a negative impact on both inflation and the growth of the Swedish economy. Thus, the strategy of normalization process of the Bank of Sweden monetary policy is largely determined by the strategies of other central banks.

In 2017, the central bank of Sweden presented the general, possible assumptions of the strategy of monetary policy normalization. It was indicated that the normalization process may proceed according to the following scenario (see scheme 1).

Scheme 1: Strategy of the monetray policy normalization process of the Bank of Sweden

| **Phase 1**		**Phase 2**		**Phase 3**		**Phase 4**		**Phase 5**
Lower or stable *repo rate*		Stable *repo rate*		Gradual *repo rate* hikes		Gradual *repo rate* hikes		*Repo rate* varies over time
Purchases of assets through QE policy		Reinvestment of assets from maturing instruments		Reinvestment of assets from maturing instruments		Asset holdings reduces from the balance sheet		Reinvestment of assets from maturing instruments

Source: own work based on: Sveriges Riksbank, 2017, p. 16.

The strategy of monetary policy normalization of the Bank of Sweden assumes that the central bank will initially continue its expansionary monetary policy, maintaining a very low (negative) level of key interest rates and *Quantitative Easing* policy. In the second phase, a level of interest rates will stabilize, so the main *repo rate* will not be further lowered. In turn, asset purchase programs will be terminated, but the central bank will continue reinvestment of funds due to maturing instruments. In the third phase, monetary policy is supposed to become less and less expansionary, through gradual increases of the main interest rate, while continuing reinvestment of funds from the maturing instruments. These activities will also be continued in the next phase - phase four, which may be conducive to a gradual but slow shrink of the balance sheet. The fifth phase is a situation in which monetary policy returns to "normality", and therefore it is assumed that a level of interest rates will be close to pre-crisis level (above 2%) and reinvestments of funds will be completed. Timing of individual phases has not been determined. Like the boundaries between different phases. In practice, decision to move to the next phase will be determined by development of the Swedish economy. However, monetary policy can gradually shift from one phase to another. In addition, it can not be ruled out that the macroeconomic situation may require re-entering to the path of loosening monetary policy (so-called *enter strategy*), which means coming back to the use of non-standard and unconventional monetary policy instruments (Sveriges Riksbank, 2017, p. 16).

Figure 3 presents the forecast of the size of government bonds, maintained in the Bank of Sweden assets and a pace of their decrease due to maturity dates, in accordance to the above scenario of the monetary policy normalization process.

Figure 3: The Riskabnk's government bonds holdings (in bln SEK)
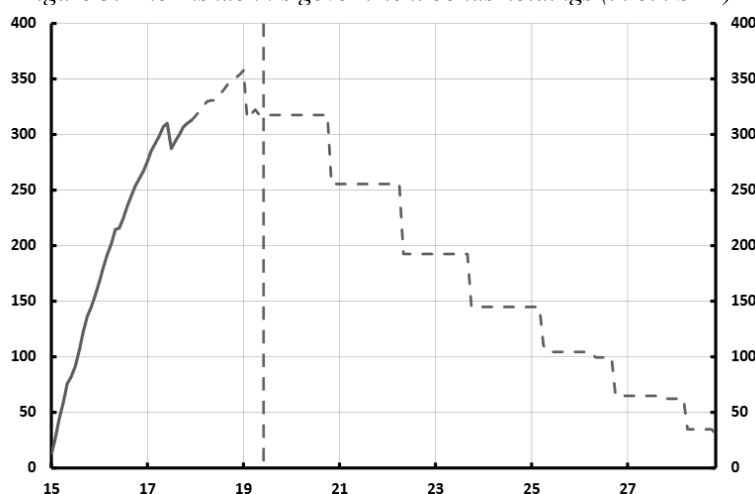
Source: (*Flodén, 2018, p. 14).*

The central bank of Sweden also assumes that the *repo rate* due to the normalization process will be increased before the next unfavorable times come. Monetary authorities' representatives predict that the *repo rate* will reach a level of 0.5-0.75% at the end of 2020. Although it does not seem to be particularly high, if we look at it from a historical point of view. The last decade shows that the overall level of main interest rates in the world have decreased. Thus, it is becoming increasingly difficult to indicate what is currently the neutral or "normal" level of central bank's key interest rate. Certainly, this level is much lower than 10-15 years ago.

In the event of another economic slowdown, the Swedish central bank allows the possibility of re-entering to the path of negative interest rates. In the *Sveriges Riksbank's* history, the lowest *repo rate* was -0.5% (2016-2019). If the economic situation requires, the monetary authority is ready to even deepen this level and decrease the main interest rate, e.g. to -0.75%, as in the case of the Swiss National Bank. However, in this situation, commercial banks could implement a negative interest rate on household deposits, which has not occurred so far. This could cause a number of problems, especially as the society in Sweden is becoming increasingly accustomed to living without cash (the so-called *cashless society*).

5. Summary

Situation of the Bank of Sweden is significantly different from the leading central banks in the world, which in response to the global financial crisis implemented non-standard and unconventional monetary policy instruments. It seems that in Sweden, financial markets and financial sector as a whole now operate relatively well, loans to households and enterprises are growing, short- and long-term interest rates are low, and the crown is not excessively strong. In addition, economic growth is strong and stable, while GDP and employment rate are growing at a satisfactory pace. Therefore, macroeconomic conditions justify the possibility of taking actions to normalize monetary policy. The normalization scenario of *Sveriges Riksbank* seems to take into account domestic macroeconomic conditions and links between the Swedish economy with European Union countries. In the first place, the normalizing activities will focus on ending of the QE policy. Afterwards, the central bank will start hikes of the main interest rate and gradual, very slow reduction of its balance sheet.

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Non-Financial Determinants of Corporate Survival in the Czech Republic

Martina Novotná [1]

Abstract

The knowledge of corporate survival helps lenders assess the credit quality of borrowers and predict potential problems of an undesirable event such as bankruptcy, default or the change of credit rating. Survival analysis is a statistical method suitable for the measurement of firm-based credit risk and it is related to the analysis of time to event data. The aim of this paper is to analyse survivor data of Czech companies and assess the impact of selected corporate characteristics on the event of bankruptcy. The empirical study is based on the use of nonparametric and semiparametric methods of survival analysis to assess the association between economic sector, legal form, company size and the probability of corporate survival.

Key words

Bankruptcy, Cox proportional hazards model, hazard function, survival analysis

JEL Classification: G21, G32, G33

1. Introduction

The bankruptcy of companies is typically analysed on the basis of credit score models, which are statistically derived models based on methods such as discriminant analysis, logistic regression, classification trees or neural networks. Survival analysis provides an alternative perspective on the measurement of firm-based credit risk and it can be used for modelling corporate survival.

The topic of this paper is related to the analysis of time to event data. While there is a vast academic research on the prediction of corporate bankruptcy through the credit score models, the use of survival analysis has not received much attention compared to the previous ones. However, there are research studies that significantly contribute to the application of survival analysis for the prediction of corporate failure in different countries. For example, Lane et al. (1986) employed the Cox model to predict bank failure. As the author suggests, the overall accuracy of their model is similar to the results of the discriminant analysis. Laitinen and Kankaanpää (1999) discuss the six most popular alternative methods to financial failure prediction, including survival analysis. The results of their study propose that there is no superior method even though the failure prediction accuracy varied depending on the prediction method applied. Agarwal and Audretsch (2001) focused on the effect of companies' size on their survival. In their research, they find that smaller companies face a lower likelihood of survival when compared to larger companies. Similarly, Glennon and Nigro (2005) examined the effect of time on the probability of default of medium-maturity loans under a loan guarantee program of small firms. The authors find that the default behaviour of these loans is time-sensitive and that the probability of default is conditional on the borrower, lender, loan characteristics and changes in economic conditions. De Leonardis and Rocci (2008) used a discrete-time survival analysis approach for assessing the default risk

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of small and medium-sized Italian companies and suggest that the prediction accuracy of the duration model is better than that provided by a single-period logistic model.

Overall, there is a vast literature on the prediction of corporate bankruptcy using a variety of techniques, however, there is still not much attention to the estimation of corporate bankruptcy using some methods. The aim of this paper is to analyse survivor data of Czech companies and assess the impact of corporate characteristics on their survival using selected methods of survival analysis. The main contribution to the current research is the focus on real data of Czech companies and the comparison of the used models. The structure of this paper is as follows. Firstly, the used methodology is described and then the application on real data is conducted. The focus is paid to the use of different models and their comparison. Finally, the main findings are summarised in the conclusion.

2. Description of Survival Analysis

Survival analysis should be used to analyse the data in which the time until the event is of interest. The response variable, the time until that event, is typically called failure time, survival or event time (Harrell, 2010). If the time until the event is not important for our study purposes, the event can be analysed as a binary outcome. The survival analysis allows the response to be incompletely determined for some subjects, for example, we are not able to follow all observations in the dataset. This is based on the mechanism of censoring when censored and uncensored observations are defined. For example, Hosmer et al. (2008, p. 18) describe a censored observation as one whose value is incomplete due to random factors for each subject.

Based on the assumptions about the distribution of failure times, we can use nonparametric, semiparametric and parametric methods of survival analysis, which are in short described in this chapter. We focus on fundamental and basic theoretical background that can be supplemented by a variety of relevant literature, for example Gourieroux and Jasiak (2007), Tabachnik and Fidell (2007), Hosmer et al. (2008), Cleves et al. (2010), Harrell (2010), Royston and Lambert (2011) or Klein et al. (2014).

Klein et al. (2014) define the distribution function of the survival time, commonly called the failure function, as the probability of failure up to time t,

$$F(t) = \Pr(T \leq t), \tag{1}$$

where T is a non-negative random variable denoting the time to a failure event and $F(t)$ refers to cumulative distribution. For practical reasons, it is often more suitable to use a complementary function in survival analysis, the survival function $S(t)$, which is the probability of surviving beyond time t,

$$S(t) = 1 - F(t) = \Pr(T > t). \tag{2}$$

Based on the survival function, we can estimate the probability that there is no failure event prior to t. The density function $f(t)$ can be obtained both from $S(t)$ or $F(t)$:

$$f(t) = \frac{dF(t)}{dt} = \frac{d}{dt}\{1 - S(t)\} = -S'(t). \tag{3}$$

The hazard function or rate $h(t)$ at time t can be explained as the probability that the subject will die very shortly after reaching time t, provided that it reaches time t (Gourieroux and Jasiak, 2007). Cleves et al. (2010) explain the hazard rate as the conditional failure rate or the intensity function. As they emphasize, the hazard rate represents the instantaneous rate of failure with $1/t$ units:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t + \Delta t > T > t | T > t)}{\Delta t} = \frac{f(t)}{S(t)} \,. \tag{4}$$

The hazard function can range from zero (no risk) to infinity (the certainty of failure at that instant) and can be decreasing, increasing, or constant, or it can even take on other different shapes.

Based on the assumptions about the distribution of failure times, there are various methods of survival analysis. Cleves et al. (2010) specify three approaches and relevant models as follows: (i) Nonparametric models (Kaplan-Meier, Nelson-Aalen), (ii) semiparametric models (Cox-proportional hazards model) and (iii) parametric models (e.g. exponential, Weibull, lognormal, log-logistic, gamma, Gompertz). While parametric models require assumptions about the distribution of failure times, semiparametric models are parametric in the sense that the effect of the covariates is assumed to take a certain form. In this case, no parametric form of the survival function is specified, yet the effects of covariates are parametrized to modify the baseline survivor function. Compared to the previous approaches, nonparametric models do not require any assumptions about the distribution of failure times.

Nonparametric methods can be used to estimate the probability of survival past a certain time or to compare survival experiences for different groups (Cleves et al, 2010). The common characteristic of nonparametric models is that they do not make any assumptions neither about the distribution of failure times nor how covariates change the survival experience. The Kaplan-Meier estimator of the survivorship function at time t can the from the equation

$$\hat{S}(t) = \prod_{t_{(i)} \le t} \frac{n_i - d_i}{n_i} \,, \tag{5}$$

where n_i is the number at risk of dying (company failure) at $t_{(i)}$, d_i refers to the observed number of failures, and $\hat{S}(t) = 1$ if $t < t_{(i)}$. If we assume that the time variable is absolutely continuous, then the survival function may be expressed as

$$S(t) = e^{-H(t)} \,, \tag{6}$$

where $H(t)$, the cumulative hazard function, can be written as

$$H(t) = -\ln(S(t)) \,. \tag{7}$$

Aalen, Nelson and Altshuler proposed the indicator $H(t)$ that is referred to as Nelson-Aalen estimator (Hosmer et al., 2008). The Nelson-Aalen estimator of $H(t)$ is given by

$$\hat{H}(t) = \sum_{t_{(i)} \le t} \frac{d_i}{n_i} \,. \tag{8}$$

Semiparametric models do not assume any parametric form of the survival functions, however, the effects of covariates are parametrised to modify the baseline survival function. According to Hosmer et al. (2008), the regression model for the hazard function can be expressed as

$$h(t, x, \beta) = h_0(t) r(x, \beta), \tag{9}$$

where $h_0(t)$ characterizes how the hazard function changes as a function of survival time and $r(x, \beta)$ describes how the hazard function changes as a function of subject covariates. The model makes no assumptions about the shape of the hazard over time, however, the general

shape of the hazard is the same for everyone. One subject's hazard is a multiplicative replica of another's, which is constant. The quantities estimated from the model are hazard ratios, which measure how much a covariate increases or decreases the rate of a particular event.

Parametric models are used when the distribution of survival time has a known parametric form. They generally provide smooth estimates of the hazard and survival functions for any combination of covariate values. According to Hosmer et al. (2008), using these models may have the following advantages. Full maximum likelihood may be used to estimate the parameters, the estimated coefficients or their transformations can provide clinically meaningful estimates of effect, fitted values from the model can provide estimates of survival time, residuals can be computed as differences between observed and predicted values of time.

3. Estimation of Survival Models

The empirical study is based on the data of Czech companies extracted from the Bisnode Magnusweb database and from the Czech government portal Justice.cz. The sample comprises data of 16,733 subjects, including 1,163 bankruptcies considered as failure events, the end of the study is March 15, 2015. If the company did not bankrupt until the end of the study or if the company was not listed in the dataset any more, it is assumed to be a censored observation. Otherwise, the observation is uncensored. Each record in our data sample documents the time span of a particular company, the original duration time is measured in days. The companies are grouped according to the following characteristics: sector, legal form, and business size, which is based on the number of employees (Table 1).

Table 1: Description of groups

Group	Sector	Legal form	Size
1	Services	Joint-stock comp.	Micro (0-9)
2	Industrials	Cooperative	Small (10-49)
3	Agriculture	Limited-liability comp.	Medium (50-249)
4	Utilities	Other	Large (>250)

Firstly, the Kaplan-Meier estimates are used to calculate the cumulative bankruptcy rates (CBR) of all companies. For illustration, they are compared with the cumulative default rates of rating groups (CDR). Although the bankruptcy and default events are not directly comparable, the results in the table suggest that the overall credit quality of analysed companies measured by the means of the probability of bankruptcy at a specified time deteriorates over time (Table 2). During the first six years, the bankruptcy rates are close to default rates of the best quality rating groups. Beyond that time, they decrease faster and they are eventually comparable with BBB rating.

Table 2: Overall cumulative bankruptcy rates and average rating cumulative default rates

Day	Year	CBR	CDR_AAA	CDR_AA	CDR_A	CDR_BBB	CDR_BB	CDR_B
365	1	0.00%	0.00%	0.02%	0.06%	0.18%	0.72%	3.76%
730	2	0.02%	0.03%	0.06%	0.15%	0.51%	2.24%	8.56%
1095	3	0.04%	0.13%	0.13%	0.25%	0.88%	4.02%	12.66%
1460	4	0.14%	0.24%	0.23%	0.38%	1.33%	5.80%	15.87%
1825	5	0.33%	0.35%	0.33%	0.53%	1.78%	7.45%	18.32%
2190	6	0.59%	0.46%	0.44%	0.69%	2.24%	8.97%	20.32%
2555	7	0.93%	0.52%	0.54%	0.88%	2.63%	10.26%	21.96%

2920	8	**1.43%**	0.60%	0.62%	1.05%	3.01%	11.41%	23.23%
3285	9	**1.92%**	0.66%	0.69%	1.23%	3.39%	12.42%	24.37%
3650	10	**2.45%**	0.72%	0.77%	1.41%	3.76%	13.33%	25.43%
4015	11	**2.95%**	0.75%	0.85%	1.57%	4.16%	14.06%	26.34%
4380	12	**3.46%**	0.78%	0.91%	1.73%	4.48%	14.71%	27.03%
4745	13	**4.08%**	0.81%	0.98%	1.89%	4.79%	15.29%	27.64%
5110	14	**4.92%**	0.88%	1.05%	2.03%	5.10%	15.80%	28.21%
5475	15	**5.60%**	0.94%	1.11%	2.20%	5.43%	16.34%	28.80%

Source: S&P Financial Services (2017). *Default, Transaction and Recovery: 2016 Annual Global Corporate Default Study and Rating Transitions (p. 61-62)*, author's calculations

The estimated mean survival times of all considered categories suggest that there are differences among groups (Table 3). Since the results indicate that sector, legal form and size are associated with the probability of corporate survival, we use the Cox proportional hazards model to further explore our data.

Table 3: Estimated mean survival time (95% CI)

Group	Sector	Legal form	Size
1	14,656 [14,504;14,809]	20,419 [19,668;21,170]	17,685 [16,854;18,516]
2	8,887[8,805;8,969]	22,117 [21,502;22,731]	20,603 [20,211;20,994]
3	22,045[21,523;22,567]	11,785 [11,634;11,935]	21,676 [21,233;22,120]
4	11,427[11,276;11,577]	9,005 [8,886;9,124]	21,953 [20,710;23,195]

All three corporate characteristics are considered as categorical variables in the following analysis. Firstly, we estimate three individual models, each with just one variable to check the potential effect of these variables on the probability of survival. The results confirm the main findings from our previous model and suggest that all used variables are associated with corporate survival. The estimated cumulative hazard functions of all three models are presented in Figure 1. In each graph, the hazard for each group within a category (HI – sector, HL – legal form, HS – size) is compared with a relevant baseline hazard function. The baseline hazard function is the function for group coded as 1 in our case. For example, in the first graph, we compare the hazard of industrials (HI2), agriculture (HI3) and utilities (HI4) with services, where HI1 stands for services (baseline hazard function). Similarly, we compare the hazard of different legal forms with joint-stock companies (HL1) and the hazard of larger-sized with micro-companies (HS1).

Based on the estimated coefficients for each model, we quantify and compare the hazard of companies with different characteristics (Table 4). For example, the hazard of sector 2 (industrials) increases by 125% compared to sector 1, the baseline category (services). Similarly, the hazard of sector 3 (agriculture) decreases by 62% compared to services. Thus, the models can be used to compare the hazard for companies with different characteristics.

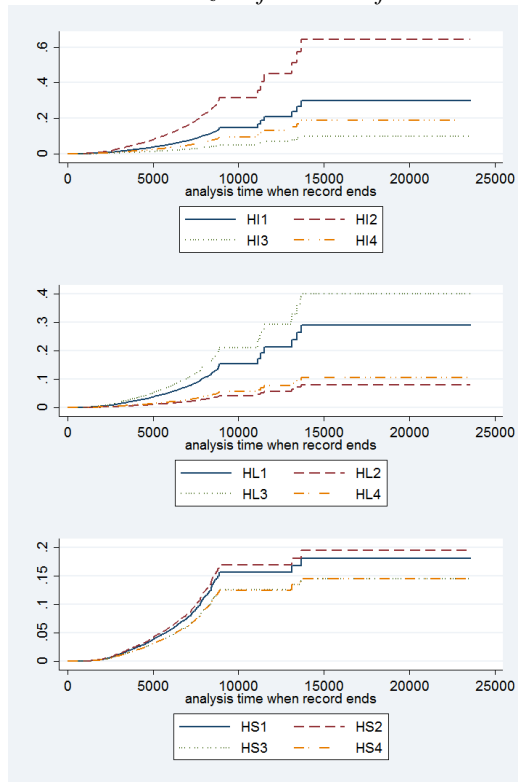*Figure 1: Cumulative hazard functions of estimated models*

Table 4: Coefficients of estimated models

Model	Coefficient	Standard error	Confidence interval (95%)	
Sector				
2	0.76*	0.073	0.621	0.908
3	-1.10*	0.165	-1.420	-0.774
4	-0.46*	0.231	-0.912	-0.006
Legal				
2	-1.32*	0.300	-1.904	-0.727
3	0.32*	0.103	0.121	0.524
4	-1.03**	0.585	-2.178	0.116
Size				
2	0.08	0.086	-0.092	0.247
3	-0.22**	0.126	-0.468	0.025
4	-0.22	0.274	-0.761	0.312

* significant at 0.05, ** significant at 0.1

Next, we estimate the Cox proportional hazards model based on all categorical variables (Table 5). Compared to the individual models, the estimated parameters are not statistically significant for some groups. Still, we can see a strong association between sector, legal form and the probability of corporate survival. There is also a clear difference between the hazard of micro and medium-sized companies.

Table 5: Coefficients of multivariable model

Categorical variable	Coefficient	Standard error	Confidence interval (95%)	
Sector				
2	0.81*	0.090	0.639	0.988
3	-0.98*	0.208	-1.391	-0.575
4	-0.41	0.261	-0.919	-0.104
Legal				
2	-2.33*	0.747	-3.798	-0.870
3	-0.13	0.119	-0.364	0.101
4	-1.98*	1.006	-3.949	-0.005
Size				
2	-0.02	0.088	-0.190	0.156
3	-0.24**	0.132	-0.501	0.016
4	-0.39	0.281	-0.944	0.160

* significant at 0.05, ** significant at 0.1

4. Conclusion

The aim of this paper was to examine the impact of the economic sector, legal form and business size on the probability of corporate survival. The association between the used non-financial determinants and the probability of corporate bankruptcy was examined by selected methods of survival analysis in this study. The main findings of the empirical study based on the data of Czech companies provide evidence that there is an association between the used corporate characteristics and the probability of corporate survival.

Both nonparametric and semiparametric models confirm that the selected non-financial determinants, considered as categorical variables, are related to the probability of corporate survival. Generally, we conclude that industrial companies tend to be riskier than services, utility, and agriculture based on our data. Regarding the legal form, cooperatives and joint-stock companies have a higher estimated survival time compared to limited-liability companies and other types and thus they can be considered as less risky. The company size was classified according to the number of employees in our study. The main findings suggest that micro and small-sized companies are associated with a higher hazard of corporate bankruptcy compared to medium and large-sized companies.

To summarise, survival analysis provides a useful tool to examine survivor data and the relationship between a time variable and the probability of failure. The knowledge of corporate survival over some period helps investors and lenders access the credit quality of borrowers and predict potential problems of default, insolvency or bankruptcy and prevent financial losses in the event of corporate failure.

Acknowledgment

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The Effective Rate of Income Tax in the Banking Sector of the Czech Republic over the period 2016 - 2017

Josef Novotný[1], David Ohanka [2]

Abstract

In this paper is analyzed and evaluated the effective tax rate (ETR) of the Corporate Income Tax in the banking sector of the Czech Republic for the years 2016 - 2017. The analysis is performed on a data set of 50 banks operating in the Czech banking sector. The effective tax rate analysis includes an assessment of the impact of seven indicators commonly used within banks financial position assessment (bank size expressed as total asset value, return on assets, fixed assets ratio, leverage ratio, credit claims ratio to total bank assets, total productivity indicator and bad debts ratio indicator).

Key words

Effective Tax Rate, Czech Banking Sector, Tax Burden

JEL Classification: H20, G20, K34, H25, G21, G29

1. Introduction

The process of globalization that the international economy has experienced throughout of the last decades has greatly conditioned the evolution of the national tax systems. This is particularly true for taxes that fall on capital and commercial profits.

In this new context, the tax on profits is a fundamental tool within the tax system, not only because of its tax collection capacity, but also because of the influence it may have on the economic decisions of the entities subject to its assessment.

Throughout the world, this tax has been characterized by having experienced a gradual decrease in the legal Statutory Tax Rate (STR) in recent years.

In the last annual report of the audit company KPMG (2018), it is shown that for the 116 countries analyzed, the STR has fallen 4,95 percentage points in the last ten years, going from 28,95 % in 2004 to 24,00 % in 2018, which represents a reduction of almost 17 %. Czech Republic have not been spared out of this process as well as STR of 28 % in 2004 have been reduced to 19 % in 2018.

The goal of the paper is to analyze Effective Tax Rate on Income tax in the Czech banking sector during the period 2015 – 2017 and to investigate what variables are the most important drivers in the determination of the Income Tax

2. Approaches to effective tax rate calculations

There are three basic methods for calculating the effective tax rate:
- micro-backward looking;

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- the macro-backward looking;
- micro-forward looking.

The difference between them lays in the use of different data on which each method is based. Macro approach essentially uses macro-economic data; i.e. national income or product accounts. On the other hand, micro approach is based on company level data extracted from financial statements either from individual companies or aggregate industrial sectors (Nicodeme, 2001). While backward looking approach is using ex-post data, forward looking approach can be derived based on utilization of ex-ante data as this approach deals with future gains of current investment.

As we find micro-backward approach the most suitable one to apply within our paper, further description of approach is in place. For this method, corporate financial statements are the main source of data. The effective tax rate is defined as the ratio between corporate income tax paid and the tax base. Three approaches to determining the tax base are used:

- profit before tax (taxable profit from ordinary activities before tax);
- net turnover (i.e. total operating income);
- gross operating profit (i.e. total operating revenue less operating costs less other operating charges and taxes less personnel costs).

The first of the above mentioned approaches is appropriate when comparing the effective corporate tax rate with statutory rates. However, comparisons at international level may be problematic, mainly due to different national accounting legislation. In the second option, it is necessary to take into account that the low value of the calculated effective tax rate may be due to the high costs of the company that must be covered by operating revenues. The result is therefore low, as well as the tax paid, even though the statutory tax rate may be high. There is a distortion because the cost information is missing in this calculation. In the last of the variants, the profit denominator is before deducting write-offs. This is advantageous for international comparisons since it eliminates the problem of different methodology and rules for asset depreciation

3. Methodology of the effective rate of income tax

In this paper the effective tax rate (ETR) is calculated using a micro backward-looking approach. ETR can be calculated as:

$$ETR = \frac{tax\,payable}{EBT}, \tag{1}$$

where *EBT* is earning before taxes.

Following variables are analyzed as a possible determinants of ETR on the basis of a review of various related studies. Diaz, Rodriguez and Arios (2011) in their study examined possible relationship between ETR and bank size, profitability measured as return on assets, degree of own funds and proportion of bad loans to total assets of a bank. Salaudeen (2017) added to this matter variables such as proportion of loans to clients or total productivity measured as EBIT per one employee.

Based on those studies, following variables are selected following variables: : banks size (SIZE); return on assets (ROA); capital intensiveness (CAPIN); financial leverage (LEV); loans to clients on total assets (LOA); total productivity (PROD); and bad debts ratio (BAD).

The primary hypothesis tests whether there is a statistically significant relationship between the effective rate of income tax of banks and selected parameters.

The null and alternative hypothesis is determined as follows:

H0: there is a statistically significant relationship between ETR and selected parameters,

H1: there is no statistically significant relationship between ETR and selected parameters.

$$ETR_{it} = \beta_0 + \beta_1 SIZE_{it} + \beta_2 ROA_{it} + \beta_3 CAPIN_{it} + \beta_4 LEV_{it} + \beta_5 LOA_{it} + \beta_6 PROD_{it} + \beta_7 BAD_{it},,$$

$$(2)$$

where β_0 is the constant, $\beta_1, \beta_2 \dots \beta_7$ are coefficients, *SIZE* represents bank size, *ROA* is return on assets, *CAPIN* is capital intensiveness, *LEV* is firm leverage, *LOA* is loans to clients on total assets, *PROD* is total productivity, *BAD* is bad debts ratio.

4. Analysis of the effective rate of income tax

In this part the effective tax rate is calculated using a micro backward-looking approach

4.1 Input data

Due to chosen approach of micro-backward looking study, unconsolidated financial statements data from banks operating in the Czech banking sector were used. The financial statements were obtained from www.justice.cz. At 30. 9. 2018 (according to CNB) 50 banks were operating in the Czech banking sector. For the analysis, profitable banks with complete financial statements during the period have been used.

4.2 Effective tax rate and deskriptive statistics

Effective Tax Rate (ETR) of banks in the Czech banking sector for period 2015 – 2017 is presented in Table 1. Statutory Tax Rate (STR) for period of study is 19 %.

Table 1: Deskriptive statistics 2015 - 2017, original data and adjusted data

Deskriptive statistics	Original data			Adjusted data		
	2015	2016	2017	2015	2016	2017
N	33	33	30	26	28	26
Minimum	-0,2982	0,01	0,0425	0,0947	0,1277	0,104
Maximum	1,1832	0,3028	0,5793	0,3784	0,2221	0,2808
Mean	**0,1939**	**0,1829**	**0,1947**	**0,1921**	**0,1885**	**0,1867**
Std. Deviation	0,2141	0,0532	0,0922	0,0539	0,0217	0,04
Variance	0,0458	0,0028	0,008	0,0029	0,0005	0,0016
Skewness	2,8637	-1,0913	2,444	1,2691	-0,7136	0,4162
Kurtosis	15,0862	3,9214	10,328	5,315	1,2037	1,3809
Median	0,1937	0,1905	0,1885	0,1958	0,1906	0,1885
1. kvartil	0,1277	0,1704	0,1646	0,1744	0,1805	0,1695
3. kvartil	0,2073	0,2041	0,2034	0,2062	0,2017	0,2018

Adjusted for extreme values and values where the numerator or denominator contained zero or negative values, the number of observations decreased to 26 in 2015 and 2017, respectively to 28 observations in 2016.

In the period of 2015 – 2017, calculated Effective Tax Rate of Czech banks is stable with values similar to Statutory Tax Rate.While the mean Effective Tax Rate in 2015 is above 19%, in 2016 and 2017 Effective Tax Rate is already below Statutory Tax Rate.

So it is obvious that analyzed banks adjusted the resulting tax base through tax deductible and non-deductible costs.

4.3 Nornormality testing

First approach to testing normality is graphical analysis, where the histogram of our sample data is compared with a normal probability curve. The results of the graphical analysis are in the following figures (1-3 original data and 4 – 6 adjusted data).

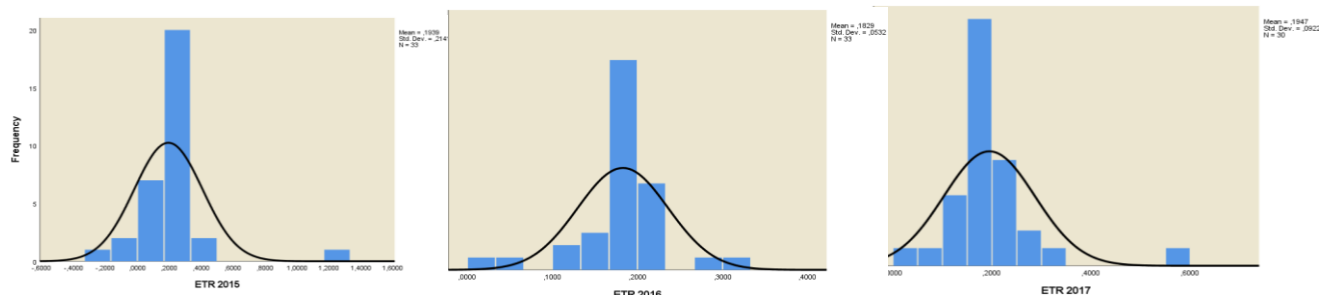*Figure 1- 3: Histogram of ETR 2015 – 2017, original data*
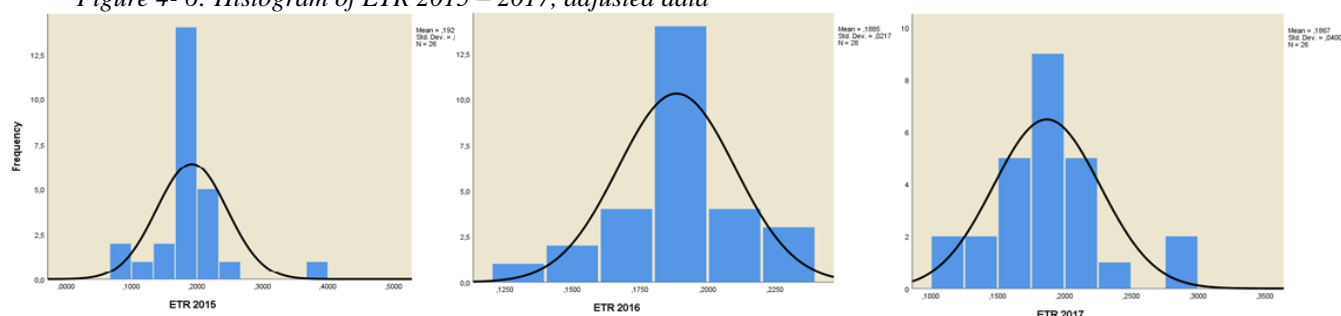

Figure 4- 6: Histogram of ETR 2015 – 2017, adjusted data

The empirical distribution of the data (the histogram) should be resemble the normal distribution. Based on the visual assessment of the graphs, it is unclear whether the sample has a normal distribution. For further analysis is used the Jarque-Bera test. The results of the Jarque-Bera test are in following Table 2.

Table 2: Results of the Jarque-Bera test

Jarque-Bera test	Jarque-Bera test	
	Original data	Adjusted data
2015	245,9613	12,7851
2016	7,7182	6,0413
2017	96,9902	3,5905

If the results are higher than the critical value and far from zero, it is signal, the data do not have a normal distribution. The calculated critical value is 5,9915. Based on the calculated results, a normal distribution is confirmed for the adjusted data 2017. In the case of adjusted data 2016 based on the Jarque-Bera test, the normality of the data is almost confirmed.

4.4 Correlation analysis

Because the data does not have a normal distribution, Spearman correlation coefficients is used for correlation analysis. The results of the Spearman correlation coefficients are in following Table 3 – 4.

Table 3: Spearman correlation coefficients – original data

	SIZE	ROA	CAPIN	LEV	LOA	PROD	BAD
ETR 2015	-0,067	-0,125	-,415*	-0,036	-0,042	0,001	0,267
ETR 2016	-0,240	-0,227	-0,118	0,066	-0,068	-0,223	0,157
ETR 2017	-0,349	-,548**	-0,170	-0,244	-0,064	-0,340	-0,234

According to the results in the Table 3, statistically significant correlations are only confirmed between ETR and the CAPIN 2015 (capital intensiveness) expressed as capital intensiveness is calculated as ratio of fixed assets to total assets and between ETR and ROA 2017 (return on assets) expressed as the ratio of earnings before tax to total assets. Otherwise, we cannot confirm that the remaining selected variables have an impact on the effective tax rate of banks in the Czech banking sector during the period 2015 – 2017.

Table 4: Spearman correlation coefficients – adjusted data

	SIZE	ROA	CAPIN	LEV	LOA	PROD	BAD
ETR 2015	-0,439*	-0,260	-0,164	0,030	0,011	-0,177	0,019
ETR 2016	-0,424*	-0,385*	0,124	0,165	0,005	-0,357	0,011
ETR 2017	-0,416*	-0,603**	-0,186	-0,275	0,169	-0,223	-0,295

* correlation is significant at the 5%
** correlation is significant at the 1%

According to the results in the Table 4, statistically significant correlations are only confirmed between ETR and ROA 2016, 2017 and between ETR and the SIZE 2015, 2016, 2017 (size of bank) expressed as total value of its assets. Otherwise, we cannot confirm that the remaining selected variables have an impact on the effective tax rate of banks in the Czech banking sector during the period 2015 – 2017. There is a negative relationship between ETR and SIZE. This result is consistent with (Siegfied, 1972), which predicts a negative relationship for the largest companies, that have more influence on regulators to lower their tax rates.

4.5 Regression analysis

Regression analysis is a statistical method by which we estimate the value of the dependent variable (ETR) based on the knowledge of the independent variables (SIZE, ROA, CAPIN, LEV, LOA, PROD, BAD. A prerequisite for regression analysis is normal distribution. Based on the Jarque-Bera test, this assumption is met only by the adjusted data 2016 and the adjusted data 2017. The results of regression analysis are in following Table 5.

Table 5 : Regression analysis

				Sig. (t-test)							
M	Year	R^2	Sig. (F-test)	Konst.	SIZE	ROA	CAPIN	LEV	LOA	PROD	BAD
Model with untransformed variables	2016	0,3300	0,2571	0,0000	0,0132	0,4006	0,4051	0,2133	0,6772	0,4992	0,4388
	2017	0,4821	0,0443	0,0000	0,4073	0,0151	0,8545	0,5811	0,2575	0,5522	0,8649
Logarithmic model	2016	0,3644	0,1740	0,0000	0,0064	0,0271	0,5469	0,2649	0,6524	0,3563	0,5241
	2017	0,0391	0,1862	0,0000	0,0680	0,0130	0,8449	0,5003	0,5533	0,7126	0,4201

Based on the results shown, the outcomes of the regression analysis are not satisfactory. An acceptable value for the coefficient of determination was only achieved for the model with untransformed variables in 2017, namely 48.21%. The same model is also statistically significant at the 5% significance level. This means that 48.21% of the variability of the dependent variable is explained by the regression model and 51.79% is a random component.

Other models, ie the model with untransformed variables in 2016 and logarithmic models in 2016 and 2017, do not show very satisfying results in terms of the coefficient of determination and according to the F-test results they are not statistically significant at the defined significance level. From the results of the t-test of the individual explanatory variables, it is apparent that only two variables show a Sig. value below five percent of significance level - SIZE variable for the 2016 model with untransformed variables and 2017 logarithmic model; and ROA for the 2017 model with untransformed variables and logarithmic model in 2016 and 2017

Based on the results of the regression analysis, it can be argued that the only model with a sufficiently high determination coefficient, statistically significant at a significance level of 5% and a statistically significant variable - ROA, is the model with untransformed variables in 2017.

5. Conclusion

The aim of paper was to analyze and evaluate the effective tax rate (ETR) of the Corporate Income Tax in the banking sector of the Czech Republic for the years 2016 - 2017. The analysis is performed on a data set of 50 banks operating in the Czech banking sector. The effective tax rate was calculated using a micro backward-looking approach, followed by the definition of statistical model and primary hypothesis used. Further, the effective tax rate was characterized using descriptive statistics, normality of analysed data was tested and a correlation analysis was performed using the Spearman correlation coefficient.

Based on the results of performed analyses and statistical tests, it was evident that the analysed relationship between the effective rate of income tax and selected bank indicators was not confirmed as the results of the tests did not confirm the primary hypothesis of the paper. Despite that, there were following findings included within this paper. Firstly, the analysed data of 50 banks and foreign bank branches operating in the Czech banking sector were not normally distributed. Secondly, both analysed data sets (original and adjusted) did not differ significantly from the statutory tax rate (19 %). It was also found that the values of effective tax for all data sets did not differ statistically from one year to another in the period 2015–2017. Moreover, a weak relationship was found between ETR and bank size and moderate relationship was found between ETR and indicator of return on assets for both original and adjusted data sets. As for the other parameters analysed, i.e. ratio of fixed assets to total assets, leverage ratio, ratio of credit claims to total bank assets, total productivity ration and ratio of provisions (to client loans) to the total value of credit claims of the bank, relationship between them and ETR was not proved.

Acknowledgments

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High-tech start-up and valuation methods

Anna Polednáková [1]

Abstract

High-tech start-up companies are among the riskiest investments, while offering potentially high returns. The key to such investments is to make the selection of the appropriate valuation methodology. Correct evaluation of companies begins with a strategic analysis of the internal and external environment of the company in order to identify strengths and weaknesses. Thorough business planning with constant updates is essential to monitor progress and reevaluate the ever-changing likelihood of future success. Evaluating high technology start-ups is a very complex process with a high degree of subjectivity. It is imperative that there is a precondition for these companies to assess themselves correctly and ultimately to decide on their further development. For valuing start-up companies, we can use both traditional and alternative approaches that use a number of factors that influence the value of the company. Using the methods presented in this paper, we tried to reveal some of the secrets surrounding the valuation of start-up technology companies, point out the strengths and weaknesses of each method, and finally recommend methods / scenario methods and real options analysis / start-up company.

Key words:

start -up, traditional valuation methods, alternative methods, market approach, asset approach, venture capitals, methods equity allocation

JEL Classification: G 32, D 46

1   The need for valuation in technology start-up companies

Measuring and managing value in high-tech start-up companies to maximize shareholder value is a unique challenge for valuation. When evaluating a high-tech start-up company, it is necessary to first understand the differences in comparison with established companies, to evaluate the strategic position of the company, to identify its stage of development as defined in the conditions of operational development, to find out its stage of development such as. defined in the terms of funding and required returns and consider its path to sales or initial public offering (IPO).

Evaluations are needed in many areas and situations involving high-tech start-ups. The provision of share options to employees as a form of incentive compensation is a common trigger of valuation. When start-up companies lack sufficient capital to pay competing wages, stock options are often an attraction used to attract skilled workers; the effectiveness of the options depends on the state of the economy and the markets. The assessment is the basis for a correct estimate of the market value of the options at the issue date and preparation for a possible future revision.

Start-up is a term that refers to an emerging project or a start-up company that has the potential to grow rapidly and a high level of innovation. For the first time this term became popular at the time of internet fever (1996-2001), when many companies of this type were

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established overseas, mainly on the Internet in Silicon Valley. He is experiencing his renaissance again now, when any starting project can be labelled with this term. A start-up project can appear in any field, but most often it is technology or Internet. Their features include:

1. Low initial costs,
2. Higher (than "standard") business risk,
3. Potentially higher returns when a company establishes itself.

Technology firms also face significantly stricter consumer protection rules, which are not yet fully unified across the EU. These factors mean higher complications and costs associated with doing business within the common European market, especially when compared to major US and Chinese innovative rivals. This category also includes the lower cultural and linguistic homogeneity of the common European market. It is easier for innovative companies to start and then scale their business quickly in markets where there are no language barriers.

1.1 Key differences in technology start-up companies

Developed companies usually have limited results, little or no revenue, and no operating profit. Top-technology start-up companies share these features, but often with a higher degree of uncertainty because they operate in new or emerging markets or in sectors where there are no traditional business models and performance criteria. Another increase in uncertainty is the fact that a company can develop new products that are experimental or totally unknown to their potential customer base or market.

Because the success of a start-up is so closely linked to the time and cost of developing, manufacturing and marketing products, forecasts need to be accurate and detailed. The unwillingness to identify and address key forecast parameters, including prices, volume, cost and capital investment at each stage of development, is often the first step towards miscalculating the company's actual performance and ultimately its value.

Continuous technological changes and short product life cycles challenge accurate forecasting and contribute to value volatility. These problems require a lot of attention to be paid to competing factors in order to identify the strategic advantages and disadvantages of the company, which ultimately determine the rate of return or multiples that are chosen to calculate the value. The possession of tangible assets and the size of the company's assets are less important in a technological society. When technology is the primary asset, most or all of the company's value comes from intangible assets, including intellectual property and human capital. Such fragile and dynamic assets can quickly shrink, more than assets, machinery and equipment. Therefore, it is important to identify newly created intellectual property and obtain appropriate patents and legal protection.

Lack of adequate capital is a common constraint, since initial funding, whether from entrepreneurs, angel investors, or venture capitalists, is generally undertaken to move the company to the next stage of development at a specified time and cost. Companies that have high technology generally have little or no credit capacity. The lack of proven products, services, or customer base and several tangible assets means that a start-up business can only use limited debt financing at best, which usually leads to high interest rates and limits management.

1.2 Value management and competitiveness analysis

Traditionally, investors look at significant financial indicators such as sales, sales growth, operating margins, asset efficiency and cash flow - as the basis for their decisions. However, these indicators reflect results rather than causes. For more thorough valuation, it must be identified what constitutes cash flow and value in it. This is done through external and internal

evaluation of the business strategic position - a competitive analysis. These basic principles can also be applied to technology companies.

External analysis. For start-ups, this analysis begins with an analysis of extremes or a sector that is often complex because competitors are small-scale or large corporations, or relatively unknown. For example, numerous software companies go from one application of their technology to another because competing or collateral applications are emerging and their target market changes are part of the process. Customers appear and quickly disappear when the distribution channel redefines the end user of the technology.

Sales growth prospects can change rapidly with technology developments and resulting products and customers. The value fluctuates accordingly. This uncertainty raises high requirements for investors' venture capital during the initial financing of new technology. Without more information about potential company products, customers and competitors, there is a higher risk associated with higher returns.

Companies that acquire the necessary skills can dramatically change their value as technology moves from concept to product, to customers, and turns them into cash flow. Management must therefore constantly identify missing features that hinder success. It must constantly review the cost of launching a product, which can cause significant fluctuations in value due to the company's technology failure. Failures and delays can lead to society being particularly vulnerable.

Internal analysis. The most important in the internal analysis is the continual examination of how technology will lead to products or services and ultimately cash flows. This analysis begins with a review of the occupancy and forecast plans, in particular the assessment or financial investment needed to complete and complete the product. Competitive advantages that support predicted volume, prices and margins need to be carefully examined. Uncertainties may require adjustments to the forecast in time lag or a thorough application of probability analysis to quantify possible outcomes.

As key people are usually necessary in business in the development phase, special attention must be paid to both management and technical staff. Leaders are often "scientists" with little expertise or management expertise, and there may be gaps in sales and marketing, manufacturing, or finance. While competences in these functional areas may be less critical at an earlier stage, the management team must have or develop the required expertise and experience in order for the company to progress in growth and typically in liquidity. Competency gaps do not justify society. However, they create limits and often signal the need to end a strategy that puts the business in the next phase of growth.

Most start-ups offer action options to attract and keep key people, and these options can have a major impact on value per action. Management should also recognize how the share option value is calculated for non-publicly traded shares. Known Black Scholes option pricing model generally overestimating the value of shocks to private companies because of their lack of liquidity, appropriate adjustments to the model or other valuation procedures should be considered.

Maximizing value in start - up high - tech companies often means building a project and convincing buyers or markets that the project has unique potential and significant value. In start-up companies, it is important to place emphasis on planning for valuation. Planning can lead to more efficient use of cash, which will reduce the need for finance and lead to increased returns for investors.

In order to understand the main differences in start - up high - tech companies in the analysis of the competitive position, it is necessary to determine its relevant development

phase. This is an essential part of the valuation of a start-up company. Determining the degree of development of a company will help to determine which method or combination of methods is most appropriate for valuing these companies, while helping to assess risk, achieve growth plans or potential sales. The discount rate plays a practical role in assessing return on each stage of development. The discount rates are higher for start-ups due to the higher risk of generating future cash flows and longer periods.

By understanding the background of the company, its development, including the analysis of its external and internal factors and risk, it is possible to move to the evaluation of the company.

1.3 Start-up and traditional valuation methods

As is known, in order to focus on value, investors must be able to measure this value; valuation should be an integral part of strategic planning. The valuation analysis quantifies the consequences of risk and return - changing the value of each external and internal competing factor. This process creates a management plan to increase cash flow while minimizing risk in order to maximize shareholder value. The valuation of start-ups is based on traditional methods based on revenue, market and assets. Alternative valuation techniques include, in particular, Monte Carlo simulation and real options analysis. However, there are various considerations when applying these methods and valuation techniques to start-ups. In our contribution we will focus on the analysis of the use of traditional methods for evaluating start up companies.

Income method. The revenue approach measures the value of an enterprise based on the expected flow of cash benefits attributable to the entity. Within the income approach, there are two methods of capitalization technique in one period and the method of multiple discounting - the difference is the one-year forecast compared to several years. There are two methods in the income approach - one-time capitalization method and multi-period discounting - the difference is one year in comparison to several years.

The one-off capitalization method (SPCM) is rarely suitable for evaluating start-ups, especially high-tech businesses. Start-up incomes or cash flow, if any, hardly represent long-term potential, and successful start-ups are experiencing very rapid growth, after which increasing competition or new technology slows growth to a more normal level. The capitalization process cannot precisely limit these changes.

The multi-period discounting (MPDM) method, typically discounting cash flows (DCF), is much more effective because it reflects expected future growth in terms of revenue, revenue and ultimately cash flows. The main advantage of DCF - carefully thought out, forecasted future returns - is also its biggest drawback. This disadvantage - the reliability of these forecasts is worsening for start-ups. Often, start-ups are characterized by having never created a positive cash flow, have limited or no financial history, and / or are in such a unique space that there are no similar companies from which to make adequate comparisons.

The application of the DCF method can be performed using a traditional two-stage model or a three-stage model. In a two-stage model, the value is calculated based on the sum of the current value of earnings over several (usually five) years and the current value of the terminal value. Often, the final value is determined on the basis of a multiple of a certain level of revenue or cash flow and usually reflects most of the initial value. In the three-stage model, the continuous growth rate over the next period (possibly the next five years) is estimated, after which the final value is calculated. In any case, real financial forecasts are necessary for the process in terms of management capabilities, market potential, technology and industry. Ultimately, a DCF-based revenue approach is an effective way to evaluate a start-up company. When applying the forecast of expected future cash flows, DCF may reflect changes in the

company's performance as it progresses through developmental stages. It also suitably matches sensitivity and probability analysis.

Market approach. The market approach an indication of value by comparing a company under review with guidance from publicly traded companies or guidance firms that were recently acquired in normal business transactions. Market multiples are often used in valuing start-ups because they are relatively easy to understand, market-based and easy to apply. The basic problem in using multiples for start-ups is that when applied in static applications to a very volatile situation. Profit multiples are usually based on unrealistic low returns compared to potential profits, with different short and long-term business growth. The resulting value makes no sense.

The traditional price to earning indicator (P / E) is rarely applicable to "valuing start-ups, because start-ups often do not have the revenue that this technique can apply. To be effective, there must be a relative homogeneity of the market sample operations with the company being evaluated.

In general, when analysing multi-company public shares, it is important to note that there may be significant differences between public companies and closed private companies that attract public investment, usually enjoying above-average revenue growth. Multiple profit can also not accurately capture expected changes in growth from very early to high growth to slow growth rates with competition. Multiples derived from strategic transactions may suffer from just two factors just described. In addition, strategic transactions often reflect synergies that only a particular buyer could achieve. Similar distortions occur when multiples are derived from industry leaders.

Access to assets is a general method of determining the value of an enterprise's assets and / or equity interest using one or more methods based directly on the value of the assets of the enterprise, net of liabilities. In particular, the least relevant approach to asset start-up valuation is the fact that virtually all early-stage assets are intangible that are difficult to identify and not recognized in the balance sheet. If appropriate for start-ups, access to assets could be used if the company has significant cash balances but has not reached any significant milestones or liquidation scenarios.

Although traditional valuation methods are applicable in the valuation of start-up companies in the field, alternative techniques are often used. These include, in particular, Monte Carlo simulation methods and analysis of real options, venture capital, and equity allocation. While traditional valuation methods are applicable to the valuation of start-ups in high technology, venture capital investors use traditional valuation methods such as: the conventional valuation method, the first Chicago method, the basic method, the multiplication method.

The conventional valuation method is a residual valuation method that takes into account only one scenario at two time points: the current date and the future payout date. It also requires assumptions regarding a compound annual rate of return growth.

The first Chicago method initially employs venture capital techniques and then probability - weighs three output scenarios: success, lateral survival and failure. This method requires the evaluator to consider potential alternative outcomes and not to rely on multiple simplified incomes or earnings. This method is intuitive.

The basic method is similar to discounted CFs, which considers several years in the future, before and after the liquidity event. Like FCM, this method is intuitive.

Revenue multipliers are often used for companies in the early stages of development for which little or no profit was made. They also suffer from the same weaknesses in terms of disregarding differences in profit margins, capital expenditure and working capital requirements, R&D time and intensity, and marketing and production time. However, they are

an extremely simple and fast method that can be used as a value control derived from other methods.

For valuations that use traditional valuation approaches, it suggests that more valuation methods should be used in the valuation of start-ups and weighed according to their likelihood and relevance for the company to be valued, based on the degree of development of the particular company and other characteristics.

Traditional probability analysis requires identifying probable outcomes (eg, Optimistic, Most Probable, and Pessimistic) and then considering the probability of each occurrence. Of course, these results depend on the company's ability to achieve key indicators - most commonly targeted revenues, operating margins, capital reinvestments and ultimately net cash flow.

The valuation of high-tech start-ups should take into account all approaches that should be considered as indicators of value in different possible scenarios and the probability should be weighted as expected at the date of value. In particular, the probability-weighted scenario method should be used to value start-ups in high technology. This method, which considers several outcome scenarios, is an effective way of rewarding a high-tech start-up company. This method makes it possible to assess different exits at different times in terms of sales or IPOs, the remaining private scenario where the long-term forecast is used to determine the value indication, and the liquidation scenario if the business model did not materialize.

The valuation methods described in the paper relate to the valuation of the start-up company as a whole. The valuation of a company's common stock is often required. Although in such situations there are many ways to measure both common and preferred shares, two methods of distributing equity are the most commonly used to allocate total assets - the expected weighted expected return method and the option pricing method (PDT). The value of the equity of the company to the components of its capital structure and thus to assign value to its ordinary shares.

2   Conclusion

High-tech start-up companies are among the riskiest investments, while offering potentially high returns. The key to such investments is to make the selection of the appropriate valuation methodology. Correct evaluation of companies begins with a strategic analysis of the internal and external environment of the company in order to identify strengths and weaknesses. Thorough business planning with constant updates is essential to monitor progress and reevaluate the ever-changing likelihood of future success. Evaluating high technology start-ups is a very complex process with a high degree of subjectivity. It is imperative that there is a precondition for these companies to assess themselves correctly and ultimately to decide on their further development. For valuing start-up companies, we can use both traditional and alternative approaches that use a number of factors that influence the value of the company. Using the methods presented in this paper, we tried to reveal some of the secrets surrounding the valuation of start-up technology companies, point out the strengths and weaknesses of each method, and finally recommend methods / scenario methods and real options analysis / start-up company.

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Reporting of payables in bankruptcy

Monika Randáková, Jan Hospodka [1]

Abstract

This paper analyzes the information needs of different stakeholders of the insolvency proceedings, such as creditors, bankruptcy administrators or insolvency court, towards financial statements. It discusses the points and information that are required by the different parties in order to determine, whether financial statements of entities undergoing bankruptcy by liquidation or reorganization should have a different structure. This paper is part of a broader research which aims to establish a new structure of financial statements for entities which are insolvent.

Key words

Reorganization; Bankruptcy; Accounting; Reporting.

JEL Classification: M41, G33

1. Introduction

Insolvency and the way the debtors are treated is currently a very popular topic in the Czech Republic. Most of the attention is focused on the bankruptcy of non-entrepreneurial individuals through a so called debt relief. This is mostly driven by the fact that there is currently hundreds of thousands of individuals in the Czech Republic who are insolvent and for whom it is very difficult to return to normal life. This phenomena, which is quite unique in the EU, is the main reason why the primary focus of the debate is on debt relief. It has been often debated even in the world media, that this is a large social issue that needs to be tackled (e.g. Tait, 2018)

This does not mean, however, that academic literature would not also analyze the corporate insolvency and related issues. Recently, this has been thoroughly discussed for example by Haskin & Haskin (2012). Considerable importance of financial reporting is discussed thoroughly by Scorte et all (2009) or Peek at all (2010). At a local level, accounting experts openly discuss the need for changing the accounting treatment in the Czech accounting framework (which includes Czech Accounting Act, Accounting Decree and the Czech Accounting Standards). The fact that the accounting and reporting rules for companies which are undergoing insolvency proceedings (in our case, either bankruptcy by liquidation or reorganization) have not been significantly updated for over a decade, speaks for itself. The Czech Accounting standards (CAS) include Standard 21, which aims at giving accounting

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guidelines for companies in insolvency. The problem is, however, that it has undergone a last major update before 2008, i.e. before the reorganization was introduced for the first time into the Czech law. Simply put, this means that there is no mention about reorganization throughout the whole CAS as well as the Czech accounting Act. Even though bankruptcy by liquidation is covered to a limited extent, Czech accounting rules do not provide any specific treatment of the individual line items in the financial statements. This leads to the fact that the financial statements for companies in bankruptcy by liquidation or reorganization have precisely the same format as the statements of healthy entities that do not have any financial difficulties. This stands in a very stark contrasts to e.g. US GAAP, which provides entities undergoing either bankruptcy by liquidation or reorganization with a different set of statements (in case of bankruptcy by liquidation) or new items on the balance sheet, P&L and cash flow statements which refer specifically to the insolvency proceeding (in case of reorganization). This should increase the reader's ability to assess the financial situation of the entity. Authors believe that the main statement affected by the bankruptcy is the balance sheet, the payables in particular. Based on this assumption, authors try to identify and present proposed changes to the balance sheet in order the meet the information needs and requirements of the readers of the financial statements, especially the creditors.

We have analyzed and searched for any specific balance sheet structure of companies in bankruptcy in the domestic as well as foreign legal and accounting requirements. We have identified that only the US GAAP formulates requirements on specific balance sheet items as well as structure in such cases. In light of this analysis, as well as analysis of Czech insolvency rules, we have suggested a new balance sheet structure for liabilities for bankruptcy by liquidation and reorganization.

2. Bankruptcy by liquidation

In case of bankruptcy by liquidation, it is necessary to provide the readers of the financial statements with information on the true classification of the payables as these are usually affected the most during the insolvency process. At the same time, we consider the creditor to be the main reader of the financial statements in the bankruptcy by liquidation and because of that, the balance sheet should reflect their needs.

Czech Accounting Act, as well as most of the reporting frameworks around the world, structure the payables in the following light:

- Based on the maturity of the payables (long term vs short term)

- Based on the type of the creditor (e.g. suppliers, bank, state employees, shareholders)

Firstly, this structure partly loses its purpose in the bankruptcy by liquidation as based on the Insolvency Act, all liabilities of the debtor which are not yet due at the day of the declaration of bankruptcy, become due. (Insolvency Act, section 250). That means that all payables at this moment become short term. Secondly, the structuring by creditors is still meaningful as there are different creditors present in the proceedings.

We believe that in the bankruptcy proceeding, the structure of payables reported in the balance sheet should reflect the needs and requirements of the creditors, as various types of payables are treated differently in the process. In our opinion, the following four primary distinguishing features need to be reflected:

- Whether the payable has emerged before the bankruptcy declaration or afterwards

- Whether the payable has been submitted to the court by the creditor

- Whether the court has allowed the payable

- Whether the payable is secured or unsecured

In order for creditors to have their pre-petition claims satisfied during the insolvency proceeding, they need to submit their claim to the court. If the court allows the claim, then it is going to be satisfied by the monetization of the secured assets in case of secured claims at any point during the proceedings, and from the monetization of the remaining estate in case of unsecured creditors in the final settlement. In case the secured assets have not sufficed to cover the secured claim, the rest of the claim is reclassified as unsecured claim and settle in the final settlement.

Also, two new types of creditors' claims emerge in the insolvency proceedings:

- Claims against the debtor's estate (statutory payables from the debtor's perspective)

- Claims with the same treatment as claims against the debtor's estate (which have emerged before the bankruptcy declaration)

Payables that have come to existence only after the proceeding had started, are classified as statutory payables and have preferential treatment. That means that they can be settled at any point during the proceeding in full with the means from monetization of the estate. At the same time, insolvency act classifies certain payables (or claims, from the creditors' perspective) as payables with the same preferential treatment as statutory payables. These are for example payables related to salaries, alimony or health damages. Both these types of payables should have a similar treatment in the balance sheet as they have a specific treatment in the insolvency process.

Therefore, we suggest that the balance sheet of a company that is entering bankruptcy by liquidation, should take the following form (which is primarily driven by legal aspects of payables in the proceedings of bankruptcy by liquidation):

Table 1 – Proposed structure of payables in the balance of entities where bankruptcy by liquidation was declared

Structure of payables[2]

Submitted and allowed payables

- Statutory payables
- Payables with the same treatment as statutory payables
- Secured payables
- Unsecured payables

Submitted and not yet allowed payables

- Secured payables
- Unsecured payables

Other

- Provisions according to specific regulations
- Liabilities related to subsidies

Not submitted or rejected payables

- Secured payables
- Unsecured payables

Source: Authors' own analysis

All the presented categories should still include the breakdown of payables based on the type of creditor. The balance sheet should also include information about the submitted but not yet allowed payables. These payables have either not yet been allowed by the court, or had been rejected by the court and creditors have appealed against that decision. At the end, this category should be empty and the payables should either move into the category "submitted and allowed payables" or into the category "Not submitted or rejected payables". Category Other would primarily include payables related to provisions according to specific regulations (company is obliged to keep financial means on its bank accounts in the same amount) and liabilities according to specific regulations (e.g. payables related to subsidies).

Last category includes payables that either not been submitted into the insolvency proceedings or have been rejected by the court. These will not be satisfied at any point during the proceeding.

We believe that such structure of payables reflects the actual economic situation in which the entity finds itself much better. It also provides the creditors (who should be the main focus of the statements) with much clearer and more transparent view on the entity.

3. Reorganization

In comparison to bankruptcy by liquidation, we believe that the main stakeholders in the reorganization process are not only the creditors, but because the assumption of going concern is still valid, also other parties, such se the insolvency administrator, employees or business partners of the entity. However, as a very significant change and an extraordinary event for the

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[2] For simplification purposes, we have not included conditional and subordinated obligations in the proposed structure

entity, the balance sheet should also take into account specifics of the reorganization process in order to provide the readers with more transparent information about the financial position of the entity. In our opinion, the following two primary distinguishing features need to be reflected:

- Whether the payable is to be classified as long term or short term

- Whether the payable will be subject to compromise

The major driver of reorganization in the Czech Republic is the reorganization plan (similar case to e.g. the United States). According to the reorganization plan, some of the payables might have their maturity postponed or their amount lowered (so called Payables subject to compromise). We believe that payables subject to compromise should be reported separately as they are directly linked to the reorganization plan. Also, entity should report all unsecured payables at their full amount among payables subject to compromise until the moment the reorganization plan comes into effect. In that moment, based on the reorganization plan, some payables might decrease in value or move into the "not subject to compromise" category. That should also be reflected on the balance sheet. The proposed balance sheet is inspired by the US GAAP (mainly in the sense of split of payables into payables subject to compromise and not subject to compromise) as well as by the legal aspects and classification of payables in the reorganization proceedings, as defined by the reorganization plan and the Insolvency Act.

Table 2 – Proposed structure of payables in the balance of entities where reorganization was declared

Structure of payables[3]
Long term payables
Submitted and allowed payables
•   Statutory payables
•   Payables with the same treatment as statutory payables

Payables not subject to compromise

Payables subject to compromise

Short term payables
Submitted and allowed payables
•   Statutory payables
•   Payables with the same treatment as statutory payables

Payables not subject to compromise

Payables subject to compromise

Source: Authors' own analysis

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[3] For simplification purposes, we have not included conditional and subordinated obligations in the proposed structure, as well as payables towards shareholders arising from their ownership interest (which equal to zero as per section 335 of the Insolvency Act)

All the presented categories should still include the breakdown of payables based on the type of creditor. It is a matter of discussion whether an entity undergoing reorganization should report only a new balance sheet structure as suggest in the Table 2, or whether the entity should at the same time report regular set of financial statements based on the Accounting Decree. Generally, we believe that an additional balance sheet on top of the one that we suggest would be an unnecessary administrative burden for entities which anyway struggle with their cost and other capacities. However, it is very important to bear in mind that these entities, when they emerge from reorganization, should again start following regular reporting requirements as any other entity. In order to be able to understand the reconciliation between the regular balance sheet statement and the special one which we propose in this paper, we suggest including a reconciliation table in the disclosure to the financial statements. This reconciliation sheet would explain, how the specific structure and content of the balance sheet ties to the regular one.

It is also important to understand that the maturity of payables can change based on the reorganization plan. The fact that a payable was past due on the date of bankruptcy declaration, does not mean that it will remain to be short term also after the acceptance of the reorganization plan. The reorganization plan can set the maturity of payables regardless of their original status. Because of that, even short term payables can become long term and as such be reclassified within the balance sheet.

The proposed balance sheet puts primary focus on the reorganization plan and its classification of receivables. However, as per the Insolvency Act (section 337), each secured creditor should be classified in a separate category. It would be, therefore, understandable to include a separate line item for each such creditor in the balance sheet. However, we believe that this would again only bring unnecessary administrative burden on the entities, while not increasing the added value to a significant extent.

4. Conclusion

We believe that suggested structures of balance sheet would reflect much better what the readers of the financial statements require to understand. In case of bankruptcy by liquidation, it is primarily the creditors who need to see the status of their claim in the context of other claims. Especially to be able to distinguish between secured, unsecured claims and statutory claims, the debtor needs to present these types of payables separately in the balance sheet as they are all treated differently. Also, the split between long term and short term payables becomes completely irrelevant in the bankruptcy by liquidation, as all the payables become due regardless of their maturity. Similar logic applies to the balance sheet for companies in reorganization, when the information requirements of the readers also change. It is important to distinguish payables which will be subject to compromise and those that will be not. This allows the reader to see whether the reorganization plan is being met successfully.

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Analysis of Economic Value Added development of Manufacturing Industry: the case in the Czech Republic

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Abstract

This paper is focused on the analysis of Economic Value Added development of manufacturing industry in the Czech Republic and statistic approach is applied to quantify the influence of component ratios to economic value added. The aim of this paper is to analyze the financial performance of the individual branches of manufacturing industry in the Czech Republic. First, Economic Value Added and its pyramidal decomposition as static method for quantify the influences of manufacturing industry is reviewed over the period 2008 to 2017. Years data are used for analysis. Next, factors affecting the Economic Value Added are analysed by using the pyramidal decomposition approach. In the end, comments on the results of the influence quantification are provided.

Key words

Financial performance, economic value added, pyramidal decomposition

JEL Classification: C1, C5, C58, G3, G30

1. Introduction

The financial performance of the sector is very important for assessing the economy. Evaluation of financial performance is one of the core activities of each company. The financial performance of a sector or a company is a random process that can be divided into individual indicators. It can also be evaluated by accounting, economic or market indicators and is addressed by many authors, see Copeland (1994), Maříková, Mařík (2005), Neumaierová (2005), Vernimmen (2005) or Brealey (2014).

Economic value added is one of the most prefered modern economic indicators which is used because of its ability to take into account costs of capital, factor of time and factor of risk. The preference of this approach can be found in many publications, see Ehrbar (1998), Young (2001), Vernimmen (2005) or Ross (2013). Most of the authors solve problems of company performance valuation but only in few publications the valuation of performance of industry can be found, see Dluhošová (2004) or Dluhošová, Ptáčková, Richtarová (2018). When assessing financial performance, it is also necessary to identify the factors and influences that affect it. The effects can be determined using static or dynamic methods.

The aim of this paper is to analyze the financial performance of manufacturing industry and find out which individual divisions of the manufacturing industry have the greatest influence on the economic value added of this industry. The proposed pyramidal breakdown of the EVA

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indicator is used to quantify the effects of sub-indicators of selected industries on the EVA indicator during the analyzed period 2008-2017.

2. Methodology

This part of the paper is divided in two subchapters. Firstly, economic value added as a measurement of financial performance of industries is characterized. Then pyramidal decomposition of economic value added is proposed as one of the static method for quantifying the effects of selected ratios.

2.1   Economic value added

Economic value added is based on the concept of the economic profit. When the economic profit is positive, it means that company earns more than the weighted average costs of capital are, which also means that some wealth for the shareholders is created.

There are many ways how economic value added can be expressed, see Ehrbar (1998), Young (2001), Dluhošová (2010) and Mařík (2005). It can be distinguished EVA – Entity, EVA – Equity or relative economic value added. In this paper financial performance of branches of manufacturing industry are analyzed according to EVA – Equity. Economic value added is expressed as

$$EVA = \left(ROE - R_E \right) \cdot E, \tag{1}$$

where ROE is return on equity, E is equity and R_E are costs of equity.

The difference between ROE and Re is called spread. Spread is very important parameter influencing EVA. If this spread is positive, it means that industry or company earns more than the costs of equity are.

2.2 Static approach – method of pyramidal decomposition

Static approach is based on a longer time series of the performance indicator. For in-depth analysis of the impact of component ratios on the base ratio, it is useful to apply the analysis of deviations. According to this analysis it is possible to quantify the impact of the changes of the component indicators to the base indicator, Zmeškal (2013). Among the ratios it is possible distinguish two operations - additive relationship and multiplicative relationship, Zmeškal (2013).

Additive relationship can be expressed as

$$x = \sum_i a_i = a_1 + a_2 + ... + a_n. \tag{2}$$

Quantification of the impact under the additive relationship is generally applicable and the total impact is divided in proportion to the changes in the component ratio as

$$\Delta x_{a_i} = \frac{\Delta a_i}{\sum_i \Delta a_i} \cdot \Delta y_x, \tag{3}$$

where $a_{i,0}$ and $a_{i,1}$ are the values of the i-th indicator for the initial period and the subsequent period $\Delta a_i = a_{i,1} - a_{i,0}$.

Multiplicative relationship can be expressed as

$$x = \prod_i a_1 = a_1 \cdot a_2 \cdot ... \cdot a_n. \tag{4}$$

According to the way in which the multiplicative relationship is handled, it can be distinguish five basic methods: a method of gradual changes, a decomposition method with a residue, a logarithmic method, functional method and integral method, Zmeškal (2013).

In this paper, for quantification of the impact of components indicators on base indicator integral method is used.

Integral method (Zmeškal, 2013) is derived as follows

$$\Delta x'(a_{1,0}, a_{2,0}, a_{3,0}) = a_{2,0} \cdot a_{3,0} \cdot \Delta a_1 + a_{1,0} \cdot a_{3,0} \cdot \Delta a_2 + a_{1,0} \cdot a_{2,0} \cdot \Delta a_3,$$

$$\frac{\Delta x'}{x_0}(a_{1,0}, a_{2,0}, a_{3,0}) = \frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \frac{\Delta a_3}{a_{3,0}}. \tag{5}$$

By substituting to

$$\Delta y_x = \frac{\Delta a_1}{a_{1,0}} \cdot \frac{\Delta a_2}{a_{2,0}} \cdot \frac{\Delta a_3}{a_{3,0}} \cdot \frac{x_0}{\Delta x'} \cdot \Delta y_x, \tag{6}$$

for any three factors,

$$R_{a_j} = \frac{\Lambda a_j}{a_{j,0}}, R_{x'} = \frac{\Delta x'}{x_0}, \Delta y_x = (R_{a_1} + R_{a_2} + R_{a_3}) \cdot \frac{1}{R_{x'}} \cdot \Delta y_x. \tag{7}$$

Then the impact of component indicator to base indicator is quantify as follows

$$\Delta x_{a_1} = \frac{R_{a_1}}{R_{x'}} \cdot \Delta y_x \text{ and } \Delta x_{a_2} = \frac{R_{a_2}}{R_{x'}} \cdot \Delta y_x, \text{then } \Delta x_{a_3} = \frac{R_{a_3}}{R_{x'}} \cdot \Delta y_x,$$

$$\text{where } R_{a_i} = \frac{\Delta a_i}{a_{i,0}} \text{ and } R_{x'} = \sum_{i=1}^{N} R_{a_i}, R_x. \tag{8}$$

3. Application part

For the analysis of financial performance of manufacturing industry individual divisions of the manufacturing industry of the Czech Republic were chosen. Financial performance of these divisions is analyzed according to economic value added and this indicator is decomposed to component financial indicators using pyramidal decomposition method.

There are many ways, how economic value added can be decomposed. The pyramidal decomposition of the EVA indicator is shown in Figure 1. The primary pyramidal decomposition factors are highlighted. The integral method is used to quantify the effects of partial indicators.

In this paper, there is proposed possible pyramidal decomposition of economic value added to four levels as

Figure 1: Economic value added – pyramidal decomposition
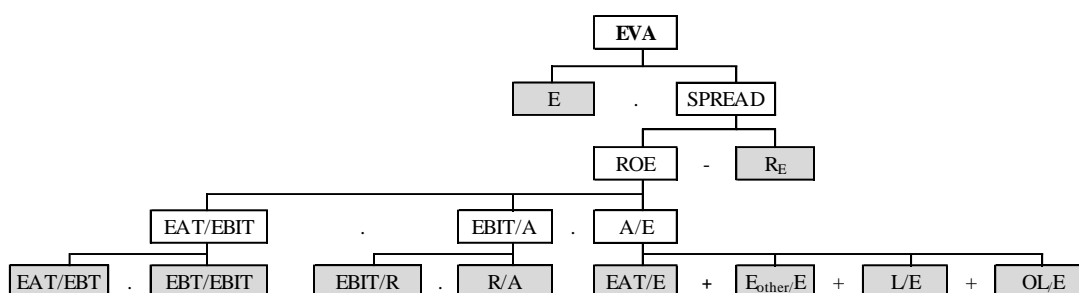
Source: own calculation

where E is equity, EAT is earnings after taxes, $EBIT$ is earnings before interests and taxes, EBT is earnings before taxes, A are assets, R are revenues, E_{other} is other equity (E - EAT), L are liabilities, OL is other liabilities and R_E are costs of equity which are determined according to a build-up model of Ministry of industry and trade of the Czech Republic.

3.1   Input data

Financial performance of individual divisions of the manufacturing industry of the Czech Republic are analyzed according to economic value added in the period 2008 to 2017. Method of pyramidal decomposition is applied and component indicators are calculated on the base of the input data of the manufacturing industry (MI) in the Czech Republic, which includes 24 divisions. These input data were taken from the website of Ministry of industry and trade (MIT) of the Czech Republic. Annual data are used for the analysis.

Table 1: Economic value added (thousand CZK) – individual branches of Manufacturing industry

CZ-NACE	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
10	-5 971 497,64	-4 653 468,34	-3 353 206,09	-4 440 988,91	-5 216 819,26	-4 228 877,18	-3 333 909,82	-2 975 386,53	-252 732,53	-1 985 799,97
11	-2 463 591,38	-1 744 750,25	-2 160 824,69	-723 659,28	417 466,67	-2 701 070,65	-3 999 045,44	-450 449,86	-1 006 999,74	-855 929,42
13	-5 819 109,80	-4 458 391,35	-855 795,79	-1 580 760,82	-579 920,26	-1 305 590,13	-98 106,94	-492 716,89	-303 270,76	-892 998,00
14	-1 260 852,62	7 583,30	305 562,14	479 892,84	1 016 272,11	693 493,41	1 023 689,02	1 145 708,87	1 091 659,68	1 015 087,34
15	-159 039,61	-67 949,36	-82 546,85	-25 358,72	-12 177,71	-81 526,87	-62 091,95	134 656,07	3 601,33	-23 707,25
16	-692 450,49	-2 399 957,59	-1 365 704,33	-1 948 637,62	-1 045 838,58	-1 006 075,50	473 723,59	1 126 220,72	2 300 427,86	2 068 836,56
17	-66 983,74	-195 685,16	804 238,25	-468 680,59	630 669,16	-321 365,47	1 249 613,69	1 877 543,32	2 920 304,53	2 963 832,30
18	238 002,29	-13 983,63	-308 275,09	-475 895,25	109 196,30	187 238,41	1 017 164,56	1 612 715,59	1 169 206,19	875 627,78
20	-171 942,35	-122 207,88	-49 298,38	-186 304,55	-22 896,68	-106 058,08	104 365,75	637,61	-60 998,40	-88 130,40
21	3 274 777,12	-2 262 701,62	2 639 785,84	-152 856,81	561 933,96	319 317,71	1 484 035,11	2 698 114,27	2 184 601,62	840 529,52
22	-3 452 160,68	-978 294,83	3 714 326,13	4 915 168,54	8 482 069,67	10 663 137,10	15 285 524,68	19 632 580,67	19 964 189,27	15 173 639,79
23	-1 288 791,01	-3 464 374,14	-2 366 014,62	-2 652 891,28	-2 712 703,23	-2 688 584,22	355 725,30	2 732 883,65	4 312 744,84	1 211 365,05
24	-4 988 897,07	-26 284 075,23	-17 266 387,35	-14 674 737,82	-17 467 262,26	-7 594 953,18	1 412 516,94	-3 860 022,50	-3 213 121,28	-4 815 429,94
25	-2 419 398,66	-2 639 623,26	-917 996,34	-1 291 040,55	-303 057,33	305 569,74	1 525 275,11	1 502 168,97	2 169 585,05	902 587,96
26	-11 313 083,58	-9 952 809,70	-4 276 227,33	-4 629 057,92	-2 897 084,43	-2 083 048,06	4 195 124,61	778 449,06	-698 321,40	-1 721 155,81
27	-3 296 757,86	-1 879 448,19	4 397 856,96	3 584 365,42	3 537 430,97	4 711 550,70	8 124 411,76	10 255 170,91	8 073 675,66	7 337 115,28
28	-1 248 128,48	-2 301 239,35	-1 518 953,16	-689 582,15	597 484,67	-414 799,08	-42 222,78	-50 495,69	-2 075 261,26	-2 432 338,22
29	-11 312 728,27	-17 720 875,42	3 067 227,32	11 671 451,13	9 720 536,75	4 686 414,51	26 214 400,07	42 528 320,69	38 436 255,50	39 402 921,85
30	-1 765 056,67	2 811 847,25	1 872 755,77	2 239 305,75	2 098 858,09	1 651 853,80	1 144 120,38	985 961,16	2 177 646,76	-1 862 150,70
31	-429 926,15	-852 728,67	-1 602 736,97	-1 424 142,66	-552 476,28	-564 418,27	-326 919,31	-171 011,81	176 463,71	-121 357,20
32	929 078,47	1 901 936,12	1 873 222,35	1 476 405,62	1 516 908,05	1 354 802,06	766 179,24	1 079 298,33	2 308 995,27	1 194 614,42
MI	-68 308 336,13	-106 346 396,38	-22 912 660,46	-16 412 842,18	1 911 450,18	4 629 187,27	76 510 323,89	105 835 138,66	90 648 793,52	71 942 976,05

Source: self-elaboration based on MIT data, own calculation

One of the most important segments of the Czech economy is manufacturing industry. Manufacturing industry contains 24 divisions which has a code number CZ_NACE. There are 10 Manufacture of food products, 11 Manufacture of beverages, 13 Manufacture of textiles, 14 Manufacture of wearing apparel, 15 Manufacture of leather and related products, 16 Manufacture of wood, products of wood and cork, articles of straw and wicker materials, except furniture, 17 Manufacture of paper and paper products, 18 Printing and reproduction of recorded media, 20 Manufacture of chemicals and chemical products, 21 Manufacture of basic

pharmaceutical products and pharmaceutical preparations, 22 Manufacture of rubber and plastic products, 23 Manufacture of other non-metallic mineral products, 24 Manufacture of basic metals, metallurgy; casting of metals, 25 Manufacture of fabricated metal products, except machinery and equipment, 26 Manufacture of computer, electronic and optical products, 27 Manufacture of electrical equipment, 28 Manufacture of machinery and equipment n.e.c., 29 Manufacture of motor vehicles (except motorcycles), trailers and semi-trailers, 30 Building of ships and boats, 31 Manufacture of furniture and 32 Other manufacturing. Data for divisions 12 Manufacture of tobacco products, 19 Manufacture of coke and refined petroleum and 33 Repair and installation of machinery and equipment are not reported in the statistics.

Table 1 presents values of EVA of individual divisions during the analysed period 2008-2017. Positive values of EVA are highlighted. Most sectors generated negative values of |EVA. EVA of manufacturing industry was negative in period 2008 – 2011. The manufacturing industry has been improving its performance and since year 2012 EVA has been positive. The highest positive value of EVA was reported in 2015. Only branch number 32 generated positive value of EVA during whole analyzed period but its share on total value of EVA of manufacturing industry is minimal.

The economic value added was calculated for each sector. Based on the determination of the mean value of the EVA indicator, the individual sectors were ranked. For the deeper analysis, 3 sectors with the highest positive mean value of EVA and 3 sectors with the highest negative mean value of EVA were selected.

The sectors with the largest negative mean value of EVA include sectors 24, 10 and 26. Both of these values were caused because of the low value of equity and negative value of spread. The return on equity was much lower than the costs of capital. By contrast, the sectors with the highest positive mean value of EVA are 29, 22 and 27. The automotive industry (29) is the industry that has the greatest influence on the development of the EVA indicator.

In the Figure 2 there are shown values of EVA of selected divisions of Manufacturing Industry, which have the highest positive (29, 22 and 27) or negative (24, 10 and 26) mean value of economic value added during the period 2008 – 2017.

Figure 2: Economic value added (thousand CZK) – selected divisions of Manufacturing industry (MI)
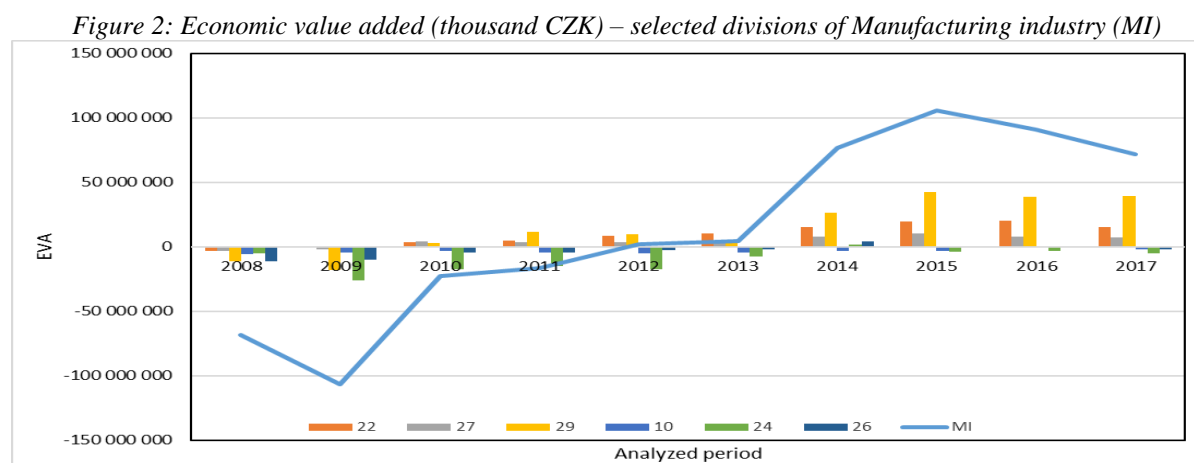
Source: own calculation

3.2 Pyramidal decomposition of EVA

Method of pyramidal decomposition was applied for deeper analysis of the factors affecting economic value added of manufacturing industry and of three divisions, which had the highest positive and negative influence to EVA of manufacturing industry. The pyramidal

decomposition was made according to the Figure 1. Integral method was used for quantification influences according to the formula (8). In the Table 2 change of EVA indicator, equity and spread during the analyzed period 2008 - 2017 to the selected divisions is presented. The first level of this decomposition represents the impact of equity and spread of economic value added. The economic value added increased in all divisions during the analysed period 2008 - 2017. The automotive industry (29) is the industry that has the greatest influence on the development of the EVA indicator. The growth in EVA was positively influenced by an increase in the equity and the spread. Spread increased due to rising return on equity and decreasing value costs of equity. Only for sector 26 the effect of own capital and spread was opposite.

Table 2: First level of decomposition of EVA – selected divisions of manufacturing industry

CZ-NACE divisions/ indicator	27	22	29	14	10	26
EVA	10 633 873	18 625 800	83 255 704	2 275 940	3 985 698	9 591 928
E	15 517 140	30 827 402	110 883 236	3 155 438	11 695 021	-17 396 133
Spread	-4 883 266	-12 201 602	-27 627 532	-879 498	-7 709 324	26 988 061

Source: own calculation

After applying method of pyramidal decomposition it was found that economic value added of selected divisions is distributed to 10 proposed component financial ratios (according to the Figure 1).

In the Figure 3 there are presented values of deviation of selected component indicators of EVA for divisions that increase the manufacturing industry's EVA during the analysed period. Only 3 ratios had negative influence and 7 ratios had positive influence. Positive influence means that if influence of component indicators is growing, then the value of change of EVA is increasing. On the other hand, negative influence works in reverse.

*Figure 3: Values of deviation of component ratios of EVA – divisions with positive EVA*
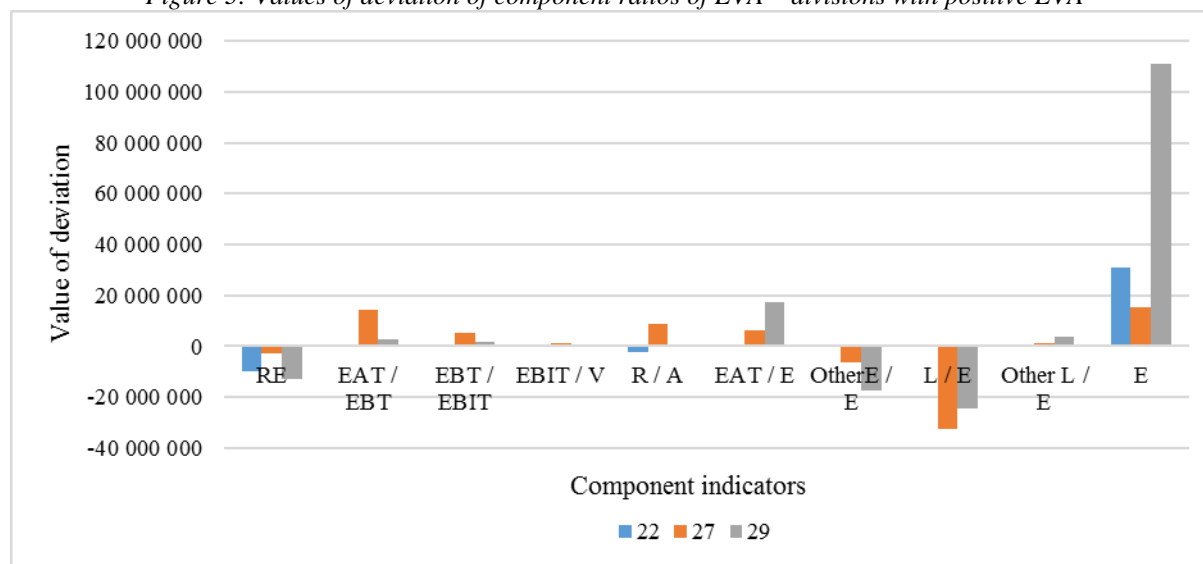
Source: own calculation

In sector 27 and 29, the share of liabilities to equity has the largest negative influence to EVA of manufacturing industry, in sector 22 it is the indicator of costs of equity. The equity is the indicator with the greatest positive impact on EVA. The share EAT to EBT and the share of revenues to assets are other indicators with positive influence. In these sectors, assets were

used more efficiently and equity was increased. The increase in return on equity was given by increasing profit. Divisions 22, 27 and 29 generated positive value of EVA throughout the analysed period.

The Figure 4 shows values of deviation of component ratios of EVA of selected divisions with negative influences to EVA of manufacturing industry (MI). In the Figure 4 values of deviation of selected component indicators of EVA for divisions that reduce the manufacturing industry's EVA during the analysis period are presented.

Figure 4: Values of deviation of component ratios of EVA – divisions with negative EVA
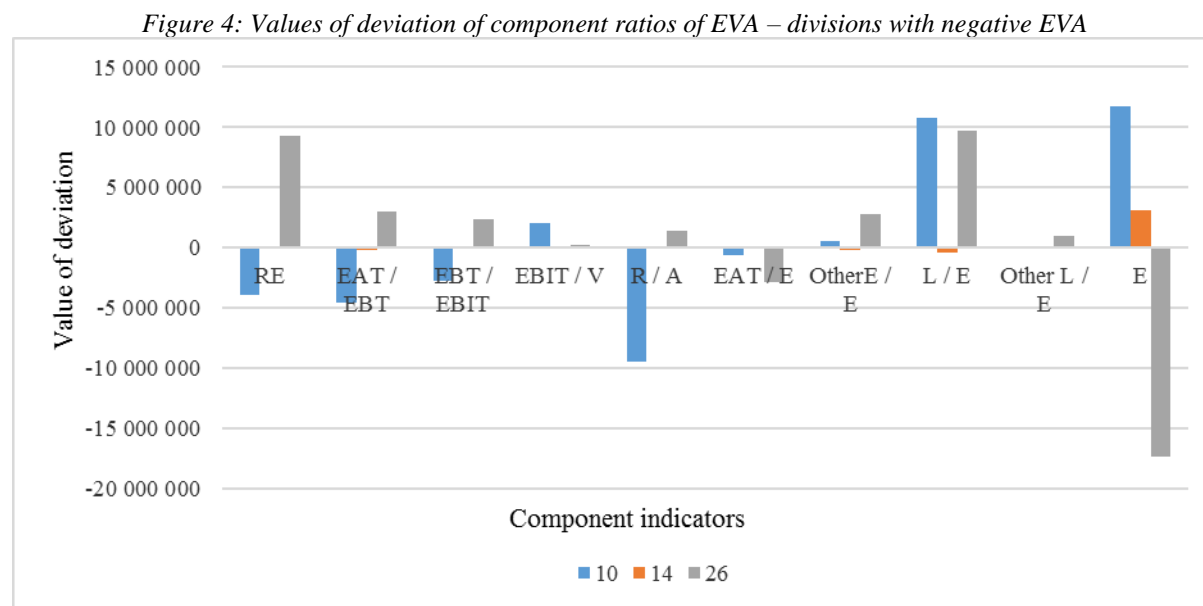
The division 10 generated negative economic value added throughout the analyzed period. Divisions 14 and 26 generated positive value of economic value added only in years 2014 and 2015. The share of EBIT to assets had positive influence for divisions 10, 14 and 26. Influences of other indicators were different. The division 10 was most negatively affected by the share of revenues to assets, the share of EAT to EBIT and costs of equity. The equity and the share of liabilities to equity has the highest positive influence.

4. Conclusion

The contribution was focused on performance evaluation of selected divisions of manufacturing industriy in the Czech Republic. The aim of this paper was to identify factors, which has the most important influence on the economic value added of selected industries for the period 2008 – 2017 by using pyramidal decomposition of EVA.

Firstly, economic value added of manufacturing industry and individual divisions was clarified. The mean value of the EVA was established for each sector. Based on the determination of the mean value of the EVA indicator, the individual sectors were ranked. For the deeper analysis, 3 sectors with the highest positive EVA and 3 sectors with the highest negative EVA were selected. According to the pyramidal decomposition method, the economic added value of selected divisions was divided into 10 proposed financial ratios of components. Subsequently, the effects of partial indicators on the economic value added of selected sectors were quantified and clarified.

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Impact of Google Trends on stock prices

Kamonchai Rujirarangsan[1] and Sergio Ortobelli Lozza[2]

Abstract

This study seeks to investigate the searching for information based on Google Trends for a choice of investment strategy. We gather this information by using Google query volume that reacts to the public investors' behavior. Then we examine the information about some components of S&P500. In particular, we use the cointegration test to investigate the potential of the behavioral information over the price and return of assets. The preliminary results evaluate the dynamic relationships between the stocks price and Google Trends volume on daily data basis.

Key words

Google Trends, Cointegration, behavioral finance.

JEL Classification: C32, G12, G41

1. Introduction

Over the past decade, the big data became an important key to justify the various analysis of human behavior, for example, health care, information services, economic and finance. The big data has been widely implemented in searching for information based on Google Trends (Jun, Yoo, & Choi, 2018). Recently, the data used: in medical science (III, Goldstein, & Lester, 2019; Lazer, Kennedy, King, & Vespignani, 2014) helps to monitor the suicidal behavior and influenza; in economic context helps to predict unemployment positions (Askitas & Zimmermann, 2009; DaAmuri & Marcucci, 2017; Francesco, 2009); in financial context helps to forecast of the equity markets behavior, (Bank, Larch, & Peter, 2011; Bijl, Kringhaug, Molnar, & Sandvik, 2016; Kim & Villa, 2017; Kristoufek, 2013; Preis, Moat, & Stanley, 2013; Takeda & Wakao, 2014). In recent studies, Google Trends data are considered as an additional decision-making tool to quantify the financial market analysis.

This study aims to investigate the relationship between Google Trends and stock prices. In our methodology, we combine the selected Google Trends and stock from some S&P500 components into each pairwise. The assets are carefully chosen from the case sensitive of searching query to perceive the unique meaning of words. Then, we to examine the dynamic relationship between Google Trends data and stock market data by using the cointegration analysis.

2. Literature review

According to the efficient market hypothesis (EMH), all relevant information should reflect the asset price on a risk-adjusted basis (Fama, 1970). However, several information

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factors may affect the market inefficiency. Barberis, Shleifer, & Vishny (1997) found that the investors' reaction generates new information which reflects EMH. For instance, trading volumes, extreme returns, news and company's announcement (Barber & Odean, 2008; Chemmanur & Yan, 2019) indicated the information effects of the investor's decision and asset price on the market.

In the age of information, the data plays a significant role in the analysis of human behavior. For example, in the medical context, (Eysenbach, 2006) found that the number of queries search on Google Trends are highly correlated to the cases of influenza in Canadian hospitals. Recently, the use of data from Twitter mood found the evidence on the prediction of the stock market (Bollen, Mao, & Zeng, 2011). In this study, they first examine the plain text used to construct the analysis, to understand the human interaction on financial markets. Another compelling case is to apply the data to predict consumer behavior. By using the searching volume of online customers, the search counts show significant predictive outcomes of the firm (Goel, Hofman, Lahaie, Pennock, & Watts, 2010).

Moreover, several studies have been proposed to analyze the investors' behavior that the dataset of the Google query index is opened access from Google Trends interface or API. Furthermore, Google published a report that used the query index to predict retail sales, automotive sales, home sales, and travel (Choi & Varian, 2012). The report showed that Google queries data and business data based on seasonal autoregressive models and fixed-effect models can be performed as a prediction model.

In the financial context, the Google query index data has been used in several studies, for example, liquidity and return of German stock (Bank et al., 2011), early warning signs of financial markets (Preis et al., 2013), stock returns (Bijl et al., 2016; Kim & Villa, 2017), strong correlation on trading volume (Takeda & Wakao, 2014) and risk diversification (Kristoufek, 2013).

3. Data and a preliminary analysis

The Google Trends data is retrieved from Google query search on a specific geographic location and category. The data demonstrates in term of query index, which derives from the total query volume of search term divided by the total number of queries at a time point. Then the result is normalized by the query number from the starting point of this service from January 01, 2004. That means Google Trends data provides the query index instead of the raw data of queries searches in each specific search term. So, the entire data will be dynamically adjusted by every new coming of query search.

In the S&P500 stock selection, we are carefully selected from the case sensitive of searching query. For instance, the International Business Machines (IBM) which trades on the New York Stock Exchange (NYSE) market as the same name as IBM. The investors may search meaning for recruitments, products, shops nearby, chipsets, ect., which are not relevant to the searching term for trading. Thus, this study filters some case sensitive words to ascertain the focus of the investors.

After that, we retrieve the daily data from the selected stocks and its Google queries. In the case of Google queries, we can download staring from January 01, 2004, with the limited download length of daily data. To overcome this problem, we split the download length of daily data into monthly and then aggregate them back into a single file. This methodology allows us not only to download the limited data but also to normalize the daily data with the monthly data which opens for download in entire length.

The data collections with excluded weekends and holidays contain 3,774 days. In the descriptive statistics of Google Trends returns, from table 1, the means explain almost at zero. The standard deviations are close to its means. In the measures of shapes, the skewnesses

relatively explained the symmetric of the datasets, and the kurtosises revealed the heavy-tailed distribution.

Moreover, the normality tests using the Jarque-Bera test show all data normally distributed. The serial correlation tests using the Ljung-Box test shows that all the data and squared data are independence. Figure 1 shows the correlation diagrams of stock returns and Google Trends. The stock returns indicate quite high correlation; meanwhile, Google Trends indicates a relatively low correlation with negative correlation in CMCSA, ISRG, and VLO.

Table 1: Descriptive statistics of Google Trends returns

Name	Mean	Stdev	Skewness	Kurtosis
AAPL	5.74E-04	0.7611	0.5979	0.7472
ADBE	-6.88E-04	0.5329	0.0541	1.8398
AMGN	7.10E-04	0.5241	0.0316	1.7955
AMZN	2.68E-03	0.6808	0.3046	1.2572
AXP	-7.11E-04	0.5271	0.0289	1.3614
BDX	1.08E-03	0.3943	0.0586	3.3555
BMY	-3.50E-03	0.5599	0.1511	1.9780
BRKB	1.76E-03	0.5254	0.1665	1.5279
CMCSA	1.02E-03	0.4866	0.0527	3.0464
CSCO	5.00E-06	0.7219	0.3034	0.9141
CVX	-5.32E-04	0.5909	0.2365	1.3881
INTC	-2.68E-03	0.7598	0.3781	0.8638
ISRG	-2.80E-03	0.5114	0.0816	2.4146
JNJ	-1.41E-03	0.5530	0.1640	0.7581
JPM	-7.56E-04	0.6702	0.1874	1.1584
KSS	7.40E-04	0.5087	0.0306	1.0257
MCD	-2.49E-04	0.4517	0.1320	1.0602
MRK	1.66E-03	0.5640	0.0691	0.8227
MSFT	2.53E-03	0.7218	0.3429	0.8508
NVDA	1.56E-03	0.6539	0.2977	1.4831
ORCL	-1.49E-03	0.6528	0.1938	1.3506
QCOM	4.10E-04	0.6488	0.2813	1.5513
SBUX	-2.43E-04	0.6424	0.1196	1.1172
SLB	5.14E-03	0.5505	0.0222	0.8474
TXN	-5.30E-03	0.5506	0.0929	1.4336
UTX	6.50E-04	0.5538	0.1821	1.8433
VLO	4.36E-03	0.5230	0.0224	0.9907
WFC	1.04E-03	0.5728	0.1245	1.1294
WMT	-2.42E-03	0.5920	0.2854	0.8894
XOM	-1.02E-03	0.5505	0.2916	1.1204

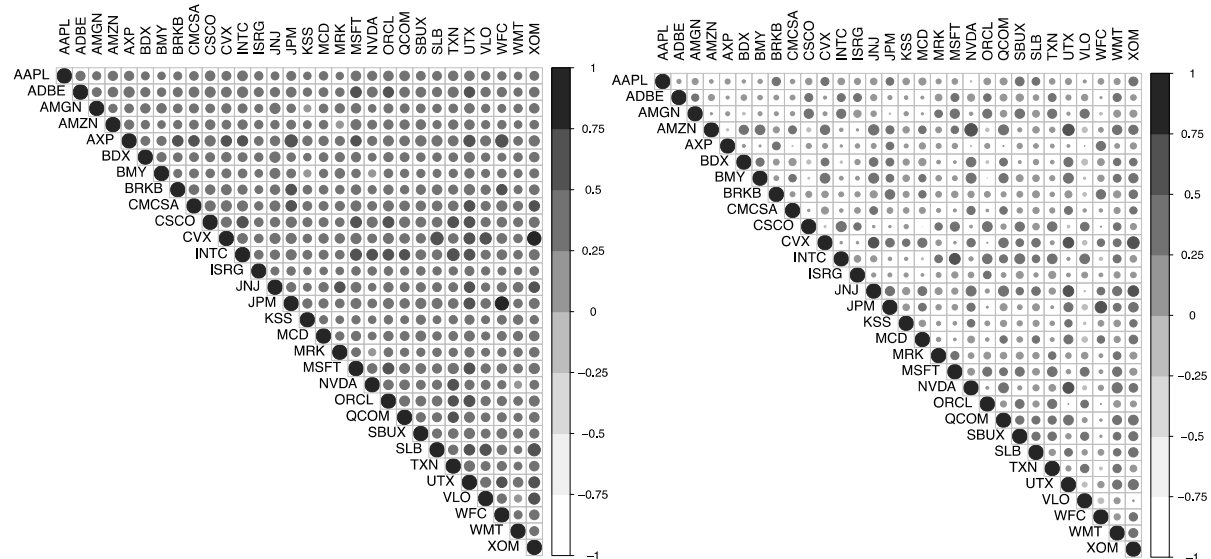*, **, *** indicates significance at the 10%, 5%, and 1% levels, respectively.

Figure 1: Correlation of stock returns (left) and Google Trends returns (right)

From table 2, we analysis the pairwise between the stock and Google Trends by using the price and the return without the lagged of variables. The correlation column shows that the stock return and Google Trends indicates the highest correlation among other pairs. However, the correlation results cannot provide a dynamic relationship between the samples.

The cointegration test that the linear combination between two or more series is individually integrated need to be tested for the stationary by using the unit root test.

In particular, this test serves the long-run relationship that the difference between the means approach to constant rather than how well the direction between two variables explains. The cointegration test proposed by Johansen, (1991) analyses the error correction of vector autoregressive model as shown follow;

$$\Delta x_t = \Pi x_{t-1} + \Sigma_{i-1}^{p-1}\Gamma_i^*\Delta \mathbf{x}_{t-i} + e_t,$$

where Δx_t is the matrix (K × 1), Π defines as the function of $\Pi = -(I - \Gamma_1 - .. \Gamma_p)$, Γ_i^* defines as the function of $\Gamma_j^* = -\Sigma_{i=j+1}^{p}\Gamma_i, j = 1, ..., p - 1$, p is the lag order terms of VEC model, e_t is vector of error term (K × 1) as follows $e_t \sim N(0, Q)$, where Q defines as the innovations covariance matrix. The null hypothesis of no cointegration can be identified by the trace statistic test (r). By comparison with the correlation, the cointegration analysis is similarly considered without the lagged of variables.

The results show that the set of stock return, Google Trends, and Google Trends return are stationary except the stock price. We then apply the Johansen test procedure to evaluate the cointegration relationships. As a result, from table 3, the pairs of stock price vs. Google Trends and stock price vs. Google Trends return are found the cointegration relation at least a pair from the trace test statistic (r<=1) except for the ORCL that is excluded.

Table 2: Correlation and unit root test

Name	Correlation				Unit root test			
	Stock Price vs GT	Stock Price vs GT return	Stock return vs GT	Stock return vs GT return	Stock Price	Stock return	GT	GT return
AAPL	0.1432	0.0025	-0.0301	0.0289	-0.1426	-44.5555***	-58.3339***	-14.4832***
ADBE	0.2164	-0.0015	0.0031	0.0045	1.4161	-46.3918***	-64.3071***	-28.2674***
AMGN	-0.0399	-0.0029	0.0233	0.0383	0.1348	-46.0196***	-67.7270***	-18.9216***

AMZN	0.8682	0.0015	0.0100	0.0741	1.2366	-45.3127***	-62.0501***	-8.5417***
AXP	-0.2011	-0.0005	0.0048	-0.0125	-0.6677	-47.6455***	-67.6354***	-25.2212***
BDX	0.5037	0.0017	-0.0007	-0.0201	0.5488	-46.8068***	-64.2039***	-22.2431***
BMY	0.5298	0.0101	-0.0463	0.0176	-0.8857	-45.4479***	-66.9153***	-18.0836***
BRKB	0.1900	0.0017	0.0167	0.0338	0.2056	-43.3476***	-69.4367***	-15.5143***
CMCSA	0.3801	-0.0091	0.0123	0.0058	-0.1477	-47.5996***	-65.6740***	-33.2573***
CSCO	-0.0414	-0.0002	0.0025	0.0186	-0.0889	-46.2416***	-60.2839***	-17.8584***
CVX	0.5765	0.0000	0.0186	0.0168	-1.3903	-48.0183***	-67.6073***	-14.1893***
INTC	-0.0925	0.0028	0.0002	0.0630	0.0700	-45.7128***	-64.2086***	-15.4449***
ISRG	0.0492	-0.0117	0.0142	0.0146	0.7845	-43.1778***	-60.7036***	-27.9067***
JNJ	0.6088	-0.0022	-0.0127	-0.0274	0.0486	-48.3161***	-71.4510***	-17.4358***
JPM	0.3102	0.0045	0.0242	0.0121	-0.1156	-46.7981***	-63.0421***	-16.6278***
KSS	0.1190	-0.0024	-0.0188	-0.0335	-2.2862	-45.1087***	-67.8075***	-28.7981***
MCD	0.4014	0.0002	0.0100	0.0088	1.2242	-48.1664***	-67.4497***	-14.7334***
MRK	0.0451	0.0000	0.0012	0.0301	0.3929	-45.8879***	-70.5533***	-26.6865***
MSFT	0.1252	0.0027	0.0093	0.0277	1.7146	-47.3118***	-63.6505***	-15.8563***
NVDA	0.8736	0.0004	-0.0062	0.0076	-0.4689	-44.1709***	-67.4131***	-10.0564***
ORCL	-0.1618	-0.0007	0.0287	0.0390	-0.9636	-46.1651***	-65.0637***	-21.3190***
QCOM	0.3949	0.0003	0.0124	0.0591	-2.1124	-46.3671***	-65.0244***	-17.6808***
SBUX	0.0966	-0.0008	-0.0131	0.0410	0.2446	-44.2796***	-67.1187***	-17.5703***
SLB	0.3886	-0.0031	0.0048	0.0329	-2.3901	-45.7011***	-72.2712***	-23.7630***
TXN	0.1136	-0.0010	-0.0044	0.0107	0.5758	-46.5154***	-67.5821***	-21.2402***
UTX	0.6868	-0.0012	-0.0041	0.0131	-1.1003	-46.7210***	-67.2053***	-16.1430***
VLO	0.0586	-0.0123	-0.0083	-0.0072	-0.9788	-45.5535***	-65.0934***	-24.3427***
WFC	0.0189	-0.0008	0.0542	0.0147	-1.3204	-45.5461***	-67.3485***	-11.8078***
WMT	0.2860	0.0037	-0.0019	0.0218	-0.2946	-46.8951***	-70.4394***	-18.1965***
XOM	0.5634	-0.0109	-0.0278	-0.0278	-2.3420	-50.5155***	-69.1673***	-14.5878***

*, **, *** indicates significance at the 10%, 5%, and 1% levels, respectively.

Table 3: Cointegration test

Name	Stock Price vs GT		Stock Price vs GT return		Stock Return vs GT		Stock Return vs GT return	
	r=0	r<=1	r=0	r<=1	r=0	r<=1	r=0	r<=1
AAPL	236.6413	5.9999***	2427.68	5.9662***	217.3332	1611.617	1600.542	2436.338
ADBE	851.4995	4.7375***	2794.883	4.7289***	724.8056	1708.557	1705.147	2795.246
AMGN	364.3714	5.5391***	3004.693	5.5286***	351.1362	1684.865	1680.394	3004.898
AMZN	416.1537	5.0926***	2654.702	5.2638***	131.2168	1642.913	1643.652	2667.242
AXP	681.4129	4.2841***	2998.887	4.2914***	589.5133	1781.822	1778.668	2996.981
BDX	726.7832	3.5374***	2788.696	3.5429***	677.782	1728.793	1728.218	2787.564
BMY	513.4479	6.5197***	2955.637	6.5542***	500.8359	1649.659	1648.511	2955.686
BRKB	244.2243	5.4365***	3113.524	5.4389***	242.1586	1527.125	1525.946	3108.562
CMCSA	1286.2665	5.8041***	2881.211	5.8086***	1256.1022	1778.259	1776.211	2879.594
CSCO	352.0433	5.0978***	2553.031	5.1052***	333.863	1700.337	1695.720	2549.326
CVX	424.3273	11.7025**	2989.935	11.7468**	400.0176	1800.97	1795.565	2990.937
INTC	244.5540	8.7110***	2796.154	8.6445***	240.91	1667.093	1665.191	2790.430
ISRG	728.5936	3.4477***	2574.073	3.4532***	708.7037	1517.151	1517.678	2574.274

JNJ	560.3668	4.4936***	3231.592	4.5300***	538.3594	1820.978	1818.439	3231.689
JPM	341.4311	4.1042***	2716.413	4.1002***	338.8742	1730.02	1723.118	2723.139
KSS	872.2807	7.5866***	3006.166	7.5721***	866.9261	1634.539	1626.620	3007.813
MCD	279.3980	4.6686***	2985.31	4.6871***	275.758	1809.804	1809.688	2986.727
MRK	655.6255	7.3251***	3176.559	7.3441***	655.3979	1678.819	1676.303	3177.003
MSFT	310.1348	6.0222***	2756.266	6.0115***	248.7945	1761.773	1759.045	2758.922
NVDA	516.3571	2.4340***	2984.945	2.4361***	149.5923	1573.667	1574.208	2985.327
ORCL	449.3360	24.6229	2845.862	24.6740	449.5746	1698.469	1688.807	2851.633
QCOM	394.1223	12.6636*	2837.956	12.6160*	392.5926	1704.018	1691.582	2844.242
SBUX	304.1721	4.3453***	2967.842	4.3096***	294.448	1587.9	1575.321	2972.100
SLB	680.0365	5.7198***	3281.658	5.7250***	605.4905	1668.676	1667.603	3282.537
TXN	495.8603	4.9088***	2994.688	4.8984***	430.4414	1714.503	1712.209	2996.788
UTX	603.6214	8.5850***	2974.079	8.5742***	559.5352	1724.563	1718.152	2978.239
VLO	815.5878	2.2397***	2844.852	2.2406***	637.7146	1655.039	1654.660	2850.022
WFC	199.3752	7.7947***	2982.601	7.7824***	146.5894	1671.269	1655.068	2987.320
WMT	356.5624	8.2447***	3168.986	8.2333***	352.5697	1735.96	1734.837	3170.185
XOM	334.2786	10.6563**	3093.968	10.6639**	312.5112	1953.116	1951.782	3092.113

*, **, *** indicates significance at the 10%, 5%, and 1% levels, respectively.

3.1 Preliminary experiments on portfolio selection

According to the observation of the previous analysis, we try to study the effects of Google Trends on optimal portfolio choices. We use daily data of the stock prices and Google Trends from January 01, 2004 till December 31, 2018. In particular, we maximize the Sharpe ratio by following Google Trends and stock returns. Then we recalibrate the portfolio monthly for a total of 55 recalibrations. As the main result of this preliminary analysis, we obtain that the final wealth is accumulated around three times the initial wealth maximizing the ordinary Sharpe ratio on Google Trends. Meanwhile, we get about 2.5 times the initial wealth when we maximize the Sharpe ratio on original stock returns. Therefore, this first comparison appears promising for future portfolio selection analysis that should include several different strategies.

4. Conclusion

This study investigates the dynamic relationship between the searching information based on Google Trends and the selected stock from S&P500. We found that the price of the chosen assets is cointegrating with Google Trends and its returns. As a result, we found that there is the potential of the behavioral information over the price of selected stocks from S&P500. In summary, the data from Google Trends can be a potential choice to construct alternative promising portfolio strategies.

Acknowledgements

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Accounting policy and its impact on the classification of an accounting entity into a size group

Petra Srnišová [1]

Abstract

The classification of an accounting entity into a size group affects the entity's bookkeeping and the scope of information required to be disclosed in the financial statements. An entity can control the decisive criteria for the classification into a size group by choosing an appropriate accounting policy, thereby influence the scope of obligations and the level of administrative burden associated with a specific size group of accounting entities. The objective of this paper is to examine if an entity has the possibilities to influence the decisive criteria for the classification into a size group by applying a chosen accounting policy.

Key words

Accounting policy, accounting choices, size groups

JEL Classification: M40, M41

1. Introduction

In some economic transaction, which are subject to accounting, the accounting legislation allows the entity to choose one of several alternatives of measuring and accounting. The entity is obliged to select one alternative and follow the chosen option throughout the accounting period and in subsequent accounting periods.

The legislation is not able to create an ideal accounting model for all entities which carry out diversified activities in various economic conditions. Therefore, managers have the right to choose their own legally accepted accounting principles, in order to present a true and fair view of their financial results in financial statements. However, the freedom to choose accounting principles may also lead to use such tools of accounting policy that have a direct impact on the manner of recognizing revenues and costs and allow the creation of financial performance and other elements of financial statements. (Kludacz-Alessandri, 2016) This puts the importance of selecting and adopting an accounting policy to the spotlight. Accounting policy is the application of specific and deliberated regulatory options. The International accounting standard *IAS 8 – Accounting policies, changes in accounting estimates and errors* defines accounting policy as "the specific principles, bases, conventions, rules and practises applied by an entity in preparing and presenting financial statements". Accounting policy means the choice of legal accounting practises and measurement methods in order to influence the opinion of the financial statements' users "in the desired direction". By deliberate application of the chosen accounting policy, the internal interests and goals of the entity can be controlled "in the desired direction" as well.

At the same time, it is evident that the process of measuring, accounting and recognizing cannot be left to the full responsibility of the entity. Legitimate requirements on accounting

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information are not only their truth and fairness, but also transparency and comparability. The purpose of the accounting regulation is to set constraints within which an entity has the possibility to choose an alternative in measuring or accounting in order to achieve the true and fair view.

Accounting regulation takes place in different ways and to a different extent in each country. Its level depends on the habits and traditions of the country and on engaging in a broader politico-economic group such as the European Union. (Šlosárová et al., 2016) Accounting regulation in the Slovak Republic is governed primarily by Act no. 431/2002 Coll. on Accounting, as amended (hereinafter referred to as the „Act on Accounting"), which is followed by Ordinances issued by the Ministry of Finance of the Slovak Republic. The legislation regulates both bookkeeping and financial statements in detail. When applying an accounting policy, an entity is required to choose only procedures and methods that comply with the law.

Accounting policy is partly based on the information asymmetry between the entity and the external environment, as the management's information and knowledge are incomparably more detailed than the data and information presented to the public through the financial statements. Significant disparities between the management and the wide range of financial statements' users are not only in access to information but also in their interests. The existence of diverse users and stakeholders with often conflicting interests and requirements leads to different approaches to accounting and financial statements. For example, unlike the creditors who are willing to assess the paying ability of their debtors, the investors would rather know the actual and prospective returns of their investments. (Tumpach et al., 2015).

Accounting policy uses legal options to influence the opinion of users of accounting information. The intentions of the reporting entity may vary: to give the impression of solvency or a high level of liquidity, for example to facilitate access to credits, to negotiate better terms with supplier and customers, or to achieve a higher price when selling the business or a part of it. We also meet the opposite intention: an entity wishes to give the impression of a less favourable financial situation in order to avoid employees' wage increases; wants to avoid distributing profit between the owners or eliminate the competitor's attention. The objectives of accounting policy are often contradictory and mutually exclusionary, which means that there is the need to sort the goals and find compromises.

We may consider the classification of accounting entity into a size group as one of the objectives of the accounting policy. The Act on Accounting distinguishes three size groups of accounting entities: a micro accounting entity, a small accounting entity and a large accounting entity. The purpose of this categorization is to avoid an unreasonable administrative burden on entities that are classified as micro accounting entities and small accounting entities by setting lower requirements in accounting. From this point of view, the most profitable is the micro accounting entity, which has reduced obligations on measuring, accounting and recognising.

2. Impact of accounting policy on the classification of an accounting entity into a size group

The role of accounting is no longer only to provide information about a particular transaction or its impact on the entity's financial position and its economic results, but also to deliver information that enables the management to take decisions in the future. (Mateášová, 2016) Choosing an appropriate accounting policy can affect not only the opinion of the accounting information's users, but also the criteria that give rise to legal claims or vice versa, create obligation, limitations or prohibitions.

When choosing the accounting policy, it is advisable to model future performance outcomes proceeding from past events and allowed alternative accounting methods. Modelling the process of choosing an accounting policy incorporates accounting data, statistical data, and marketing information, which proves the indissoluble connection between accounting and managerial decision-making. (Sysoieva et al., 2019)

The classification of an accounting entity into a size group affects measuring, accounting and recognising and is directly related to the administrative burden in terms of accounting. The consequences of chosen accounting policy thus interfere with the daily activities of the entity. With increasing obligations, the costs of ensuring all statutory requirements on accounting increase proportionally.

The objective of this paper is identifying the accounting choices in measuring and accounting, which may affect some of the critical criteria for classifying an accounting entity into a size group. The decisive criteria are (1) the total net assets, (2) the net turnover for the accounting period, and (3) the average number of employees over the accounting period.

2.1 Incentives for influencing the classification of an accounting entity into a size group

The accounting legislation in the Slovak Republic, as stated above, distinguishes three size groups of accounting entities: a micro accounting entity, a small accounting entity and a large accounting entity. This categorization is based on Directive 2013/34/EU of the European Parliament and of the Council of 26 June 2013 on the annual financial statements, consolidated statements and related reports of certain types of undertakings. The directive requires each member state to distinguish small businesses from large ones and, in connection to this, enable a regulatory burden in proportion to the size of the company (Mamić Sačer et al. 2015). The purpose of this categorization is, by setting lower requirements in accounting, to avoid an unreasonable administrative burden on entities that are classified as micro accounting entities and small accounting entities. From this point of view, the most profitable is the micro accounting entity, which has reduced obligations on measuring, accounting, and preparing and presenting financial statements.

Accounting procedures and measuring methods given by the accounting legislation in the Slovak Republic apply to all size groups. In certain cases, there are remissions for micro accounting entities and, on the contrary, additional obligations for large accounting entities.

Lowered requirements on micro accounting entities in the area of **measuring** relate to securities, derivatives and equity interests. They are not measured at fair value, neither at the initial measurement nor at the balance sheet date. Micro accounting entities do not use the equity method.

We can find differences between the size groups in **accounting**, too. Micro accounting entities are relieved of some obligations. For example, they do not account for deferred tax and have not to differentiate an insignificant, recurring accounting transaction that relates to costs or revenues between two accounting periods. Derivatives are accounted in a simplified manner. In the contrary, large accounting entities have additional obligation compared to small accounting entities: to account for receivables and payables to related parties in special analytical accounts.

In this subchapter, we will orient towards the most significant part of the accounting: the **financial statements**. For the area of financial statements, there is a separate ordinance of the Ministry of Finance for each size group, regulating the structure, content and scope of the financial statements. It is evident from the existence of separate ordinances that accounting entities classified in different size groups have different obligations. The simplification in reporting has led to the introduction of a simplified form of the micro accounting entity's financial statement, as well as a significantly shortened range of information in the notes.

(Parajka and Ondrušová, 2014) The set of financial statements of a micro accounting entity, a small accounting entity and a large accounting entity consists of three components: a balance sheet, an income statement and the notes. The *balance sheet* of a micro accounting entity contains only 45 lines, the balance sheet of a small and large accounting entity contains 145 lines. A micro accounting entity presents in the balance sheet figures in net terms for the current and immediately preceding accounting periods. From the balance sheet of a micro accounting entity, we can only determine the carrying amount of assets. We cannot detect the original measurement of the asset nor the amount of the adjustment (cumulative depreciation, value adjustments). (Blahušiaková, 2018) Asset's values for the current accounting period are shown in the balance sheet of a small and large accounting entity in three columns: gross values, correction and net values. The *income statement* of a micro accounting entity consists of 38 lines, the income statement of a small and large accounting entity contains 61 lines. Listed numbers of lines in the balance sheet and in the income statement indicate that the financial statements of a micro accounting entity contain less detailed or, in other words, more aggregated data. It has a lower information value than the financial statements of a small or a large accounting entity. An independent user of the financial statements is able to collect less information from the cumulative financial statements of a micro accounting entity. The most significant difference between the financial statements of each size group is in *the notes*. The lowest disclosure requirements for the notes follow micro accounting entity. At the other end of the scale, there is a large accounting entity that, in the notes, presents an incomparably larger amount of data in a more detailed specification. The classification of an accounting entity into a size group also affects further obligation: for example, a large accounting entity has the obligation to prepare and make public a Report on payments to governments.

From the above, it is evident that the classification of an accounting entity into a size group affects not only the accounting and the financial statements. It is linked to other administrative obligations, too. We consider the variety of obligations arising from the classification of an accounting entity into a size group as a justified objective of the accounting policy.

2.2 Accounting policy options to influence the accounting entity's classification into a size group

The decisive criteria for classifying an accounting entity into a size group are (1) the total net assets, (2) the net turnover for the accounting period, and (3) the average number of employees over the accounting period. Criteria limits for each size group are given by the Act on Accounting. Compliance with at least two criteria in two consecutive accounting periods is considered.

We will examine the extent to which these criteria can be affected by the accounting policy.

The first examined criterion is the **total net assets** value. It represents the carrying amount of assets recognized in the balance sheet. It is the gross amount of assets less the correctios. Corrections are cumulative depreciation and value adjustments and they are considered as accounting estimates. They are ideal accounting policy tools to manipulate the entity's financial position and profitability.

Depreciations are entity's cost and they reflect a reduction in the economic benefits of fixed assets caused by its physical or moral wear. Based on the anticipated use of fixed assets, the entity compiles a depreciation plan. The depreciation method is determined by whether the asset's wear is associated with the length of its useful life or with the performance, that can be achieved using the asset. The existence of alternative methods of depreciations create a legal scope for the subjective determination of the costs' amount which affects the carrying amount

of the asset and, through profit or loss, the amount of the equity during several subsequent accounting periods.

Value adjustments are recognized at the balance sheet date whenever an entity determines that an asset's measurement does not conform to the reality as a result of an impairment loss. Value adjustments are created for all types of assets. The amount of the value adjustment is determined by an estimate that represents the amount of justified assumption of asset's impairment against its carrying value. The estimated amount of the value adjustment is based on the management's subjective judgement. Value adjustments are a simple and highly effective accounting policy tool that allows the entity to increase or decrease the carrying amount of the assets, the profit or loss and the equity over several accounting periods.

By means of value adjustments, the management can easily get the impression that they are being offered a number of permissible solutions. It is therefore essential to place great emphasis on ethics. Even the best legislation cannot replace ethical behaviour when measuring and accounting cannot be avoided accounting estimates. (Podmanická, 2008)

An alternative way to influence the total net assets value is, in accordance with the Slovak accounting legislation, the option to book tangible fixed assets whose valuation is equal or lower than 1 700,- € and intangible fixed assets whose valuation is equal or lower than 2 400,- € directly to consumption, which means their *full cost recognition*. Another legal way of not exceeding the limit for the total net assets value is to *prefer to lease an asset* instead of owning it. Nowadays, operative leasing is widely available and less valuable assets, such as office technology, are commonly leased, too.

Securities and shares held for sale may be measured at initial measurement by acquisition cost or by fair value. The selected measurement base will be used subsequently at the balance sheet date, too. Subsequent measurement in fair value allows revaluation which is higher than the initial measurement, while revaluation of securities and shares held for sale, which were initially measured by acquisition cost, cannot exceed their initial value. The chosen method affects the value of financial investments and thereby the total net assets value.

Slovak accounting legislation allows to measure *shares and interests in a subsidiary undertaking or participating interests* by the cost used at the initial measurement or by the equity method to the balance sheet date. When using the equity method, the value of shares and interests is adjusted to the value that corresponds to the equity participation rate. Using this method, the measurement may be lower, but also higher than the initial measurement. When measuring the shares and interests by the cost used at the initial measurement, it is not allowed to exceed this value. The chosen method has to be applied to measure all such shares and interests. Again, the decision affects the value of financial investments and thereby the total net assets value.

Inventories created by own activities are measured by production cost. Act on Accounting defines the production cost as "direct costs of production (or other activities) and, optionally, a part of indirect costs proportionally related to the production (or other activities)". It is the responsibility of the entity to determine whether the inventories created by own activities will be measured only in the amount of direct costs, or the measurement will include a part of the indirect costs related to the production of the inventory. The inclusion or non-inclusion of indirect costs affects the total net asset value.

The second examined criterion is the **net turnover for the accounting period**. It includes revenues from sales of products, goods and services net of discounts. In order not to exceed the legal limit influencing the classification of the accounting entity into a size group, we assume the entity seeks to reduce the revenues by booking them and subsequently recognizing them at a lower amount or as revenues from other economic activities, that are not included in the net turnover. However, these approaches are not in line with the true and fair view

principle and are contrary to the law, as the legal nature of the transaction is modified and the legal procedures are not followed. The net turnover for the accounting period is influenced by the entity's pricing policy. It is based on accounting data, but the determination of the selling price – and thus the revenues from the sale of products, goods and services – is not related to the accounting policy.

An entity does not have the choice of accounting policy that would affect the **average number of employees** over the accounting period. However, it does not mean that this criterion is not easily to control. The management has the power to decide on the number of employees at any time. However, we consider it inappropriate and unethical to reduce, respectively not to increase the number of employees, just for the reason to affect the classification of the entity into a size group.

3. Conclusion

Accounting policy can affect not only the opinion of the accounting information's users, but also the criteria that give rise to legal claims or vice versa, create obligation, limitations or prohibitions. The classification of an accounting entity into a size group affects measuring, accounting and recognising. It is directly related to the administrative burden in terms of accounting. The consequences of chosen accounting policy thus interfere with the daily activities of the entity. With increasing obligation, the costs on ensuring all statutory requirements on accounting increase proportionally.

The aim of this paper was to examine the accounting choices in measuring and accounting that may affect some of the critical criteria for classifying an accounting entity into a size group: the total net assets, the net turnover for the accounting period, and the average number of employees over the accounting period.

The **total net assets** value is influenced by several options in measuring and accounting: the choice of the measurement base used for securities and shares held for sale; the measurement of shares and interests in a subsidiary undertaking or participating interests to the balance sheet date; the method of recognising tangible assets valued equal or less than 1 700,- € and intangible assets valued equal or less than 2 400,- € as fixed assets or as costs; the calculation of depreciation: the depreciation period, depreciation rate and depreciation methods; the method of determining the production cost of inventories; determination of the estimates of an impairment loss and value adjustment. Another legal way to influence the total net assets value is to prefer to lease an asset instead of owning it. Based on the above findings, we may conclude that the criterion of the total net assets value is easily to be affected by the chosen accounting policy.

We have not identified any accounting choice in measuring and accounting with an impact on the **net turnover** for the accounting period without violating the true and fair view principle. The net turnover is influenced more by the pricing policy than by the accounting policy.

The entity's management is free to influence the third criterion – the **average number of employees** over the accounting period – at any time. However, this criterion cannot be affected by the choice of an appropriate accounting policy.

Based on the findings, we conclude that a targeted accounting policy provides limited opportunities to influence the entity's classification into a size group. The limitation is related to the fact that in accordance with the Act on Accounting, at least two criteria out of three are considered to be met. We have identified only one criterion that can be influenced by the accounting policy. However, there are other factors outside the accounting which affect the classification of an entity into a size group. Therefore, business decision on pricing or human

resources thus have as serious consequences on accounting as the choice of an accounting policy. We recommend to all entities to consider all existing option and compare their effect with the entity's long-term strategy and objectives.

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Empirical Macaulay's Duration Concept

Bohumil Stádník [1]

Abstract

The value of Macaulay's duration (one of the most popular quantification method for sensitivity issues) could be roughly financially interpreted as the percentage change of a bond price if the shift of interest rate equals 1% along the whole zero-coupon curve (along the whole spectrum of maturities), when initial price of the bond is equaled to 100% and the rate shift is 1 p.p. Based on this idea we may conclude, by the way of example, that the ratio of average price changes of two assets is approximately the same as the ratio of their Macaulay's durations. But in financial practice this duration is problematic as the parallel curve shift is very rare. The main contribution of this research is to find a certain measure which can be handled in the same way as conventional Macaulay's duration, for example: in the equation for the change of ΔP, ratio of volatility of two assets, or in the equation for the duration of portfolio. We also calculate its values for EUR interest rate markets. We name this measure as "Empirical Macaulay's duration".

Keywords

Empirical Macaulay's duration, conventional duration, Macaulay's duration, Empirical Macaulay's duration of portfolio, zero-coupon yield curve

JEL classification code(s): G1, G12

1 Contribution of the Research and Brief Literature Review

The main contribution of this financial engineering study is to define a certain measure for interest rates sensitive assets, lets denote it as "Empirical Macaulay's duration", which can be handled in the same way as Macaulay's duration, by the way of example: in the equation for the change of $\mathit{\Delta P}$, ratio of volatilities of two assets (bonds), or for calculation of portfolio sensitivity with respect to interest rates changes. The new measure respects empirical zero-coupon rate curves shifts, so the result is much closer to reality than in the case of Macaulay's duration which could be roughly financially interpreted as the percentage change of a bond price after interest rate shift of 1% along the whole zero-coupon curve (along the whole spectrum of maturities) when initial price of the bond is equaled to100 and the shift is 1 p.p. Based on this idea we may conclude that in this situation the ratio of average price changes is approximately the same as the ratio of durations. In practical life this value has no significant reason as the parallel curve shift is very rare.

In this financial engineering study we use zero-coupon interest rate changes based on empirical data.

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Study is important for determination of bond interest rate sensitivity and, of course, for resulting volatility of bond. Volatility issues have important impact on risk management of a bond portfolio.

This study belongs to a strand in the financial literature focused on basic bonds behaviour such as Fabozzi 1993, 1995, 2010; Smit, Swart 2006; Málek, Radová, Štěrba 2007, bonds volatility (Litterman, Scheinkman, Weiss 1991), Litterman, Scheinkman 1991, volatility of bonds' determinants (Fuller, Settle 1984), bond market price development over medium-run and long-run periods (Chance, Jordan 1966; Kang, Chen 2002; Tvaronavičienė, Michailova 2006; Křepelová, Jablonský 2013; Visokavičienė 2008) or behaviour of a bond portfolio (Dzikevičius, Vetrov 2013).

There are many potential areas for application of our approach. One example being one certain bond life during which its term to maturity is decreasing and the price volatility is changing. One can ask a following question to be answered: How does the volatility development style depend on the time to maturity, coupon rate and on the level of interest rate (yield to maturity)? Because of similarities, another application is a straightforward extension, that being a portfolio of bonds with different maturities at a certain point of time.

Serious research in the area of volatility is provided in Stádník 2012, 2014; Stádník, Žďárek 2017 for a practical portfolio management. This text introduced a definition of three different regimes of common bond clean price volatility and examines theoretical and practical repercussions of such phenomena as an extension to the existing literature. A way of determining values of switching points between these regimes with respect to the level of interest rates using numerical calculations are presented and explained. The text includes numerical solving for switching points for maturities from 1 up to 1200 years that show that the switching point 1 (between regime of the "typical" development and other regimes) is of lower value for higher maturities, which is also in accordance with Fuller, Settle (1984). We can also state that for higher maturities the "switching" point has its practical value within the meaning of today's very low levels or even negative interest rates. The switching point 2 (starting point of "inverse" volatility development) is not of value less than 50%. This value is given by switching point 1 between the lowest maturities for each coupon. If the clean price of a bond is developing inside the volatility envelopes, its sensitivity (volatility) increases/decreases according to the shown shape of the envelope.

For connected research see also Steeley 2006; Janda, Rausser, Svárovská 2014; Janda, Svárovská 2010, Ortobelli, Tichý, 2015; Giacometti, Ortobelli, Tichý 2015; Blahová 2015; Brůna, Blahová 2016; Webb 2015.

2   Methodology

Methodology could be divided into following steps:
1. We take USD empirical zero-coupon curve rates on daily basis, in this research from USD market (figure1).
2. Based on the empirical data we calculate the daily change of price of fixed coupon bonds with maturities 1 to 30 years (equation 2b). The coupon is, for the demonstration, set to be 3% p.a.
3. Based on the daily price changes we calculate adequate daily Empirical Macaulay's duration in the same way how the Macaulay's duration is calculated (equation 9).
4. We define and calculate Empirical Macaulay's duration as mean of daily Empirical Macaulay's durations.
5. We calculate the values of Empirical Macaulay's duration for different coupons.

2.1 Basic Theory

$$P(YTM) = \left(1 + YTM\frac{l}{T}\right)^{-1}\left[c + \frac{c}{(1+YTM)} + \frac{c}{(1+YTM)^2} + \cdots + \frac{c+100}{(1+YTM)^{n-1}}\right] \qquad (1)$$

where *P(YTM)* is the dirty price of a bond determined in the percentage of its face value on purchasing day, c is the coupon rate per the coupon period, *YTM is* the yield to maturity per the coupon period, l is the number of days to the next coupon payment, n is the number of coupon payments till the maturity and T is the number of days inside the coupon period.

In the special case when we purchase a bond on the day with zero accrued interest (could be for example an ex-coupon day) and the clean price equals to the dirty price we can use the formula (2a) for the approximation of the required clean price development.

$$P(YTM) = \frac{c}{(1+YTM)} + \frac{c}{(1+YTM)^2} + \cdots + \frac{c+100}{(1+YTM)^n} \qquad (2a)$$

We may consider the total price also to be the sum of total prices of n zero-coupon bonds

$$P(i_1, i_2, \ldots i_{mat}) = \frac{c}{(1+i_1)} + \frac{c}{(1+i_2)^2} + \cdots + \frac{c+100}{(1+i_{mat})^n} \qquad (2b)$$

where $i_1, i_2, \ldots i_{mat}$ are zero-coupon rates for maturities 1,2,….years and all other variables have the same interpretation as in equation (1).

Based on the Taylor's theorem for a real-valued function f differentiable at the point a, there is a polynomial approximation of a higher degree (quadratic, cubic, quartic…) at the fixed point a. Taylor´s theorem provides this approximation in a sufficiently small neighbourhood (h) of the fixed point *a*:

$$f(a + h) = f(a) + f'(a)h + \frac{f''(a)h^2}{2!} + \frac{f'''(a)h^3}{3!} + \cdots + R \qquad (3)$$

where formula (3) is applied to formula (2a) and with the substitution: *h for* Δi and *f(a)* for *P*. Consequently, it can be shown that, the expression (4) holds:

$$\Delta P(YTM) = P'(YTM)\Delta YTM + \frac{P''(YTM)\Delta YTM^2}{2!} + \frac{P'''(YTM)\Delta YTM^3}{3!} + \cdots + R \qquad (4)$$

where R is the remainder of the series and Δi is the change in the market interest rate. Using the formula (4) for ΔP (as percentage of its face value) as the general measure of volatility and only up to the second order approximation:

$$\Delta P(YTM) \cong -DUR_{MAC}\frac{P(YTM)}{(1+YTM)}\Delta YTM + \frac{1}{2}P(YTM)CONV\Delta YTM^2 \qquad (5)$$

where DUR_{MAC} is the Macaulay's duration and the term *CONV* stands for the convexity of the bond. Macaulay's duration can be specified using the following shorthand notation:

$$DUR_{MAC} = \frac{\sum_{k=1}^{n}\left[\frac{kc}{(1+YTM)^k}\right] + \frac{100n}{(1+YTM)^n}}{P} \qquad (6)$$

2.2 Empirical Macaulay's duration on Daily Basis

Using empirical data (figure1) we may calculate Empirical Macaulay's duration on daily basis using the first term of equation (1)

Figure. 1 USD zero coupon yield curve development
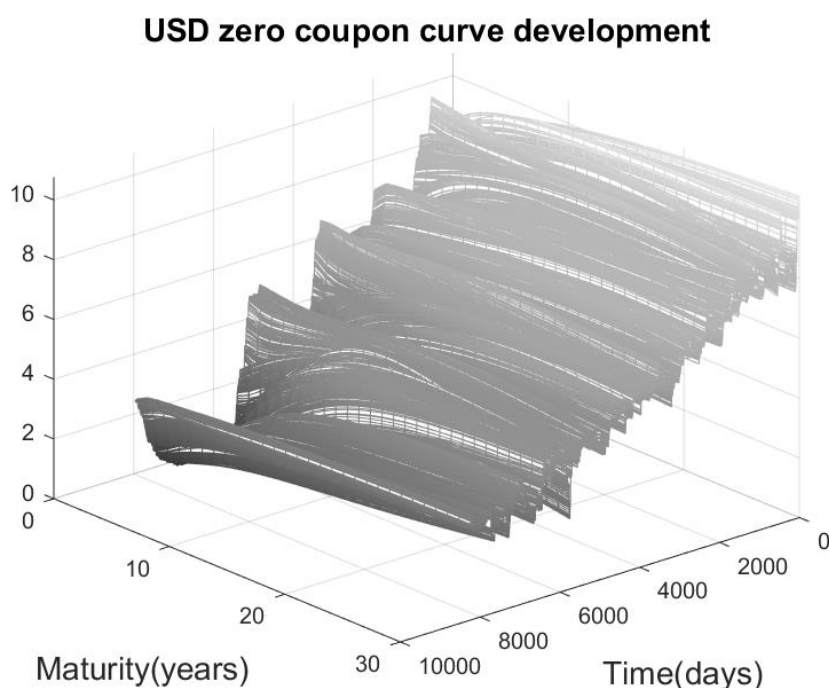
$$DUR_{EMP_d}(mat, d, YTM) \cong -\frac{YTM}{P_{mat,d}}\frac{\Delta P_{mat,d}}{\Delta YTM} \qquad (7)$$

where

$DUR_{EMP\_d}(mat,d,YTM)$	Empirical Macaulay's duration on daily basis as function of maturity, day, yield to maturity (figure1)
YTM	yield to maturity
ΔYTM	change of YTM
$P_{(mat,d,)}$	price of bond with respect to its time to maturity and the structure of zero coupon curve on day d
$\Delta P_{(mat,d,)}$	change of price between d and $d-1$ (according to figure 1)
d	number of days (from the beginning of time series, figure1)

2.3 Empirical Macaulay's duration Definition

Lets define Empirical Macaulay's duration like mean value with respect of d

$$DUR_{EMP} = E[DUR_{EMP\_d}(mat, d, YTM)] \qquad (8)$$

2.4 Empirical Macaulay's duration of Portfolio

With Empirical Macaulay's duration we may deal in the same way as with Macaulay's duration and express Empirical Macaulay's duration of portfolio:

$$DUR_{EMP\_PORTF} \cong \sum_{j=1}^{n} w_j DUR_{EMP\,j} \qquad (9)$$

Where w_j is the weight of jth asset in the portfolio

3   Results

3.1 USD zero-coupon rate case

217

Figure. 2 Inverse interest rate shift example
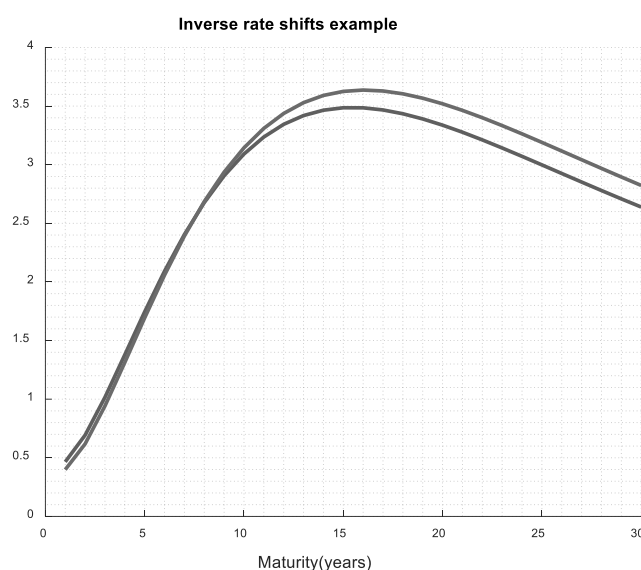
We may observe many cases, by the way of example in the figure2, of inverse shifts along zero-coupon yield curve which could decrease the change of price in case of typical coupon bond in comparison to our estimation using Macaulay's duration, which use the same shift along the whole curve.

Ratio of inverse price shifts is in the figure 3. From the figure it is clear that approximately in 30% cases the change of price of long fixed coupon bond (longer than 15 years) is in the opposite direction than the price movement of on year bond.

Figure. 3 Ratio of inverse price shifts
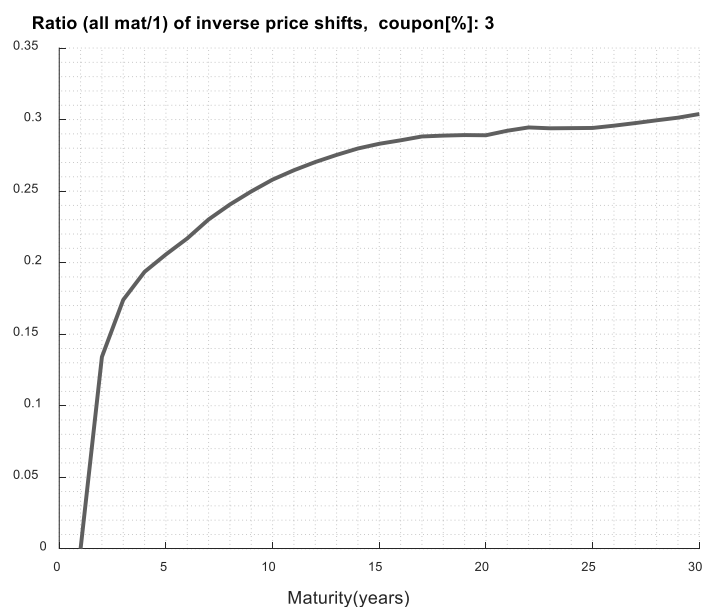
In the figure 4 there is ratio of change of price for different maturities.

Figure. 4 Ratio of inverse price shifts

Average value of delta P(A)/delta P(B), coupon[%]: 3
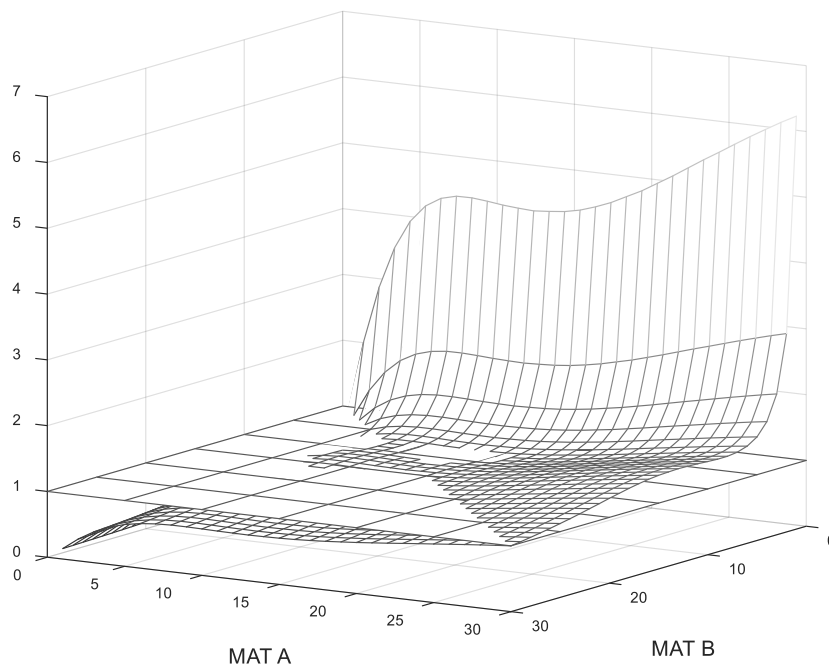
Values of Empirical Macaulay's duration in comparison to Macaulay's duration is in the figure 5.

Figure. 5 Empirical Macaulay's and Macaulay's duration (coupon=3)

Emp. Macaulay´s(thick line) and Macaulay´s duration, coupon[%]: 7

Maturity(years)
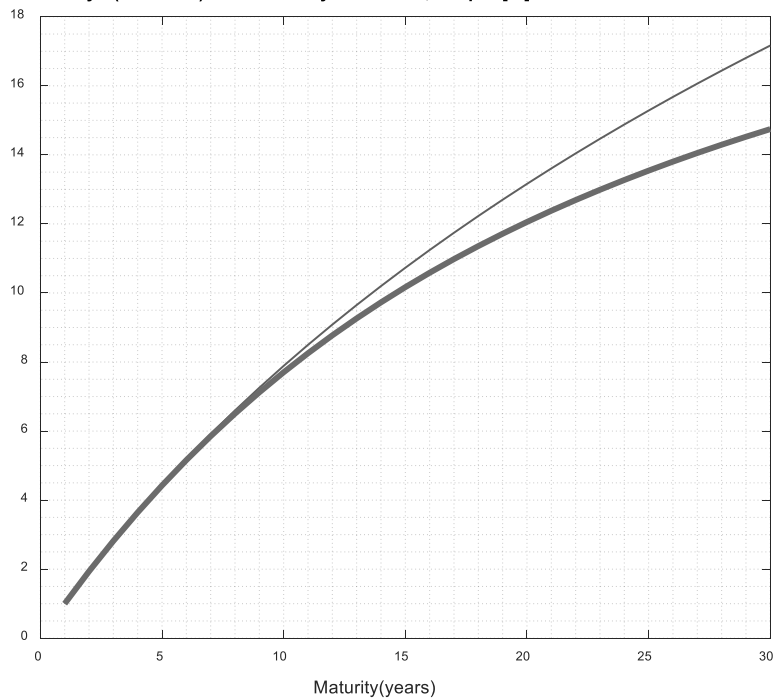
4   Conclusion

We may conclude that the question formulated in the title has negative answer. Typical one, among conventional durations, is Macaulay's duration. Its main problem is, that it assumes the parallel zero-coupon curve shifts along the whole curve which is not realistic presumption.

In this research we have defined and also quantified (for USD zero-coupen rates) a new duration – Empirical Macaulay's duration which is based on the empirical development of zero-coupon curve. Empirical Macaulay's duration is a certain measure which can be handled in the same way as conventional Macaulay's duration, by the way of example: in the equation for the change of ΔP, ratio of volatilities of two bonds, or in the equation for duration of a portfolio.

As the curve movements are in approximately in 30% of all the cases inverse we expect the price volatility of fixed coupon bonds with long maturities to be lower than it should be according Macaulay's duration. Values of Empirical Macaulay's duration, which is basically quantification of such effect, have supported these expectation. The value of Empirical Macaulay's duration also decreases with higher coupon and increases with longer term to maturity as in the case of Macaulay's duration.

Based on the reasons above we may conclude that long bonds are not as risky as it is considered to be based on Macaulay's duration.

In the case of zero-coupon bond the value of Empirical Macaulay's duration is the same as Macaulay's duration.

The values of Empirical Macaulay's duration are based on empirical data of zero-coupon rates for certain currency.

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Reporting of Equity Financial Assets by Czech TOP100 Companies: True and Fair (or Far) View of Reality?

Jiří Strouhal[1] Josef Horák [2] Jiřina Bokšová[3]

Abstract

The submitted paper focuses on the valuation of purchased stakes by TOP100 Czech companies. Accounting treatment in this particular area is similar under Czech accounting legislature as well as under International Financial Reporting Standards (IFRS) and therefore we aimed to analyse which valuation models are preferred by practitioners (companies) compared to theory represented here by legislature. As a data sample was chosen companies in TOP100 ranking. This data sample covered companies reporting both under local Czech legislature as well as under IFRS framework. Results show that models preferred by legislature are not models preferred and applied by companies nevertheless they are reporting under Czech or IFRS referential. Vast majority of companies prefer to valuate stakes under cost model, despite this information could not be considered as timely and perfectly relevant one.

Key words

Shares, measurement, fair value accounting, cost model, equity method

JEL Classification: M41, G30

1. Introduction

The quality of accounting data has a significant impact on the decision of financial statements users. From this perspective there is visible the never-ending battle between the supporters of cost model compared to mark-to-market approach mostly represented by fair value model.

Supporters of cost model claim its verifiability and easiness, supporters of mark-to-market approach claim its reliability and timeliness. A question occurs if we are dealing with non-transparent capital markets – are data from these markets reliable? How fair values are computed? Can we trust to professional valuers and information provided by them?

The perfect area where is possible to test the usage of these models in current practice is reporting of financial assets. For illiquid markets there is logical to use the cost model, for other alternatives there is required by world-wide legislation (local standards, IFRS, US GAAP) to apply fair value model or other alternatives (e.g. equity model for valuation of major interests).

For this paper we focused on the valuation of non-current stakes among Czech companies applying International Financial Reporting Standards or local accounting legislature. The reason for choosing Czech Republic is the less capitalisation of local capital market where only shares of few companies are listed. From this perspective there is interesting to find out

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how wide-spreaded is mark-to-market approach supported by legislation compared to cost model.

2. Theoretical Background

Accounting conservatism has been a salient characteristic of accounting information for centuries, and recent research into this area has documented a significant increase in conservatism in the last few decades (Givoly & Hayn, 2000). Several explanations are presented to justify the existence of conservatism, and all of them coincide in highlighting that conservatism benefits the users of financial information. These explanations may be grouped into four categories: (i) contracting motivations; (ii) litigation risk; (iii) tax optimisation; and (iv) asymmetries in the loss functions of regulators. The existing empirical evidence provides support for the contractual and litigation explanations, although taxation and regulation also contribute to the existence of conservatism (Watts, 2003b).

Watts (2003a) explains that, from the point of view of contracting and litigation risk, conservatism originates from the existence of (i) information asymmetries; (ii) asymmetries in compensation contracts; (iii) limited liability; and (iv) the fact that the parties to the firm have different time-horizons.

Accounting conservatism provides significant benefits for the users of financial information. The use of conservative accounting numbers in contracts among the different parties to the firm reduces information asymmetries and moral hazard problems derived from agency conflicts (Watts, 2003b). Contracts written using prudent numbers reduce the probability of managerial expropriation of firm resources or excess distribution of these resources. Therefore, it is expected that timely incorporation of economic losses (bad news) into earnings will reduce the opportunities for managerial opportunistic behaviour, facilitating managerial monitoring, as well as the monitoring of firm contracts, such as debt contracts. Due to these beneficial effects, accounting conservatism is commonly considered as an indicator of earnings quality or a desirable property of accounting earnings (Watts, 2003b; Francis et al., 2004; Ball & Shivakumar, 2005).

Shareholders are expected to demand conditionally conservative accounting numbers. Conservative accounting aligns the interests of managers and shareholders, reducing the tendency of managers with short-term horizons to invest in negative-NPV projects, because managers are aware that they will not be able to defer the recognition of losses to the future. Watts (2003a) points out that conservatism prevents overcompensation of managers. All of these favourable outcomes of conservatism contribute to increase firm value and create barriers to the expropriation of shareholders by managers.

2.1 Accounting Data Quality

Accounting theory argues that financial reporting reduces information asymmetry by disclosing relevant and timely information (Frankel & Li, 2004).

International accounting literature provides evidence that accounting quality has economic consequences, such as costs of capital (Leuz & Verrecchia, 2000), efficiency of capital allocation (Bushman et al., 2006; Sun, 2006) and international capital mobility (Young & Guenther, 2002).

Legal systems vary significantly within the EU, and consequently we would expect accounting quality to vary across borders after the IFRS adoption. In addition, IFRS are principles-based, which means that auditors and accountants need to follow general principles rather than detailed standards and adapt these principles to specific situations (Ball, 2006). In countries with strong creditor protection, we expect interpretation to satisfy contracting

demands of banks, such as conservative approaches to record assets but aggressive approaches to record liabilities.

Barth et al. (2006) suggest that accounting quality could be improved with elimination of alternative accounting methods that are less reflective of firms' performance and are used by managers to manage earnings. They also investigate the value relevance of earnings by comparing the R-squared from two regressions: (i) price regressed on book value and earnings; and (ii) earnings regressed on positive and negative returns. They find that R-squared increased after IFRS adoption, providing evidence of greater value relevance for IFRS earnings.

2.2 Legislative Treatement

Considering the previous literature review we can state that within the approach research area – valuation of financial securities, concretely purchased stakes, Czech accounting legislature has basically similar treatment of the accounting for interests like the international referential in the form of the International Financial Reporting Standards (IFRS).

In case that company is willing to own the stake for the period longer than one year (i.e. non-current financial asset), we can divide this stake onto two groups: (i) minority interests (up to 20 % stake) and realisable shares (corresponding with standard IFRS 9), (ii) major interests (more than 20 % stake) (corresponding with standards IFRS 10 and IFRS 11). Minority interests shall be initially measured at cost, however upon balance sheet date they have to be revalued at fair value through other comprehensive income using the revaluation surplus (capital reserve in equity). In case of major interests there is applied an equity method. For this model there is expected that the interest would be revalued on the proportion of the equity of an issuer (Strouhal, 2018). This revaluation is realised also through other comprehensive income using the revaluation surplus (Strouhal & Bokšová, 2015).

Practical problem might be the availability of data and willingness of issuers to provide the information about the value of equity (Bokšová, 2011). As per fair value model there shall be quoted its structure according to IFRS 13 Fair Value Measurement (IFRS Foundation, 2019). There are three types of the information about fair value: (i) level 1 – market data, (ii) level 2 – other observable inputs (i.e. professional valuation based on respected valuation models), (iii) level 3 – unobservable inputs (i.e. best company's management assumption about the possible fair value of an asset). From the perspective of the valuation of stakes it makes sense to apply levels 1 and 2. However for non-transparent markets cannot be underestimated the use of level 3. Level 3 is prohibited by Czech accounting legislation. According the Czech Accounting Act there is required in such case rather to follow the cost model

3. Research Objective and Methodology

There was applied qualitative analysis of the information provided by reputed Czech TOP100 companies (ranking based on total sales in 2016). CZECH TOP100 rating is annually published since 1994 and brings the perspective of a reliable data source for local and foreign investors. Data about companies were retrieved from the Business Register (www.justice.cz).

To gain the relevant research sample we followed next five steps:

[1] Do all Czech TOP100 companies present their financial statements in Czech business register?

[2] Does company present individual or consolidated financial statements?

[3] Does company prepare its financial statements according to International Financial Reporting Standards (IFRS) or according to Czech accounting legislature?

[4] Does company report non-current financial assets, concretely purchased stakes?

[5] How company measure these stakes?

Based on the financial statements we can say that 14 companies are reporting under IFRS framework and 83 companies according to Czech accounting legislature. 3 companies were directly eliminated from our research sample (Bosch Group ČR, Veolia Voda ČR, and Ředitelství silnic a dálnic ČR) as there was impossible to retrieve relevant data about these companies. These companies did not meet their legislative obligation and did not publish their financial statements in the Collection of Documents in the Business register. From this perspective our initial sample consisted from 97 companies.

Out of 14 companies reporting under IFRS framework 11 companies present their individual financial statements and 3 companies present consolidated financial statements only. All companies do present non-current financial assets in the form of purchased stakes in their financial statements.

Out of 83 companies reporting under Czech accounting legislature 32 companies do not present any non-current purchased stakes in their financial statements. As for other companies, 50 companies present their individual financial statements and 1 company present consolidated financial statements only.

Our final data sample was eliminated for companies presenting consolidated financial statements only and of course for companies do not having any purchased stakes in their accounting books. Our final sample therefore consists from 61 companies (see Fig. 1).

Figure 1: TOP100 Czech Companies Sampling
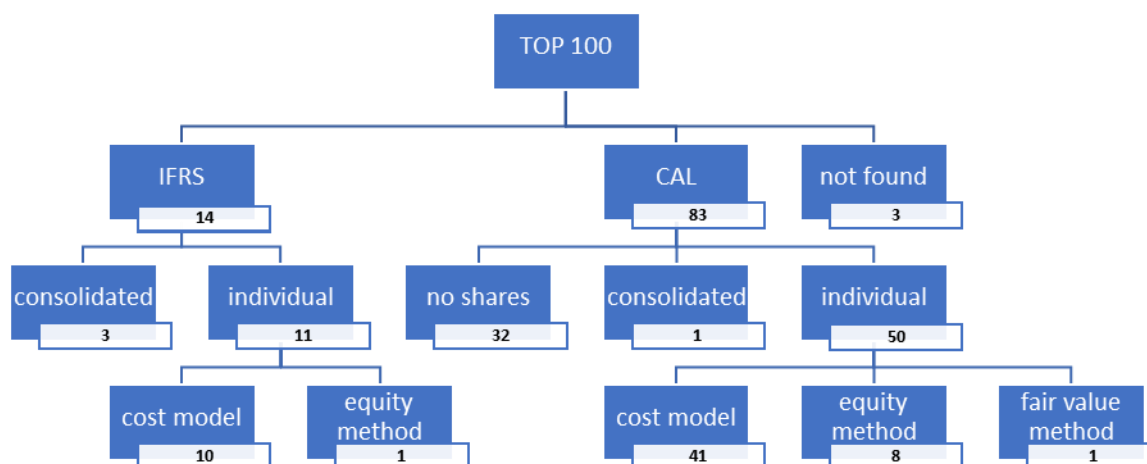
4. Results and Discussion

In our initial sample we did have 14 companies reporting under IFRS framework, out of which 3 companies present consolidated financial statements only. 91 % of analysed companies present their purchased shares at cost with possible recognition of the impairment.

The only company applying different model is Pražská plynárenská. This company strictly follows international standards (IFRS 10 and IFRS 11) and measures its non-current purchased stakes using equity method.

The results could be surprising considering the requirements of respected international standards where cost model is not very supported. The reasoning might be given by the cost over benefit criterion where companies see much more complications to find out the value of purchased stakes under equity method or even fair value of purchased stakes compared to easily retrievable cost.

4.1 Companies Reporting under Czech Accounting Legislature

Majority of companies reporting under local practices, i.e. according to Czech accounting legislature prefer cost model as a measurement base. Some of those companies did accounted for impairment for certain shares due to their significantly lower current value, however this impairment could not be recognised as a tax-deductible item according to local tax legislature. Only 8 companies opted for the equity method and one for fair value model (Lesy ČR). From this perspective there is an interesting story of the company Sokolovská uhelná which used to apply equity method until 2015, however since 2016 changed its accounting policy for the cost model providing better true and fair view.

4.1.1 Companies Applying Equity Method

According to local legislature companies holding stake of 20 % or higher shall apply equity method for the valuation of the interest in subsidiary. From our analysis there is visible that this legal requirement is not applied by majority of companies – from our sample only 8 companies measures purchased stakes by equity method. All these companies revaluate their purchased stakes through revaluation surplus (capital reserve being part of an equity).

This measurement method is probably applied by a small number of companies because of the higher demands of information needed and the associated significant costs. Obviously, this cannot be problem of parent dealing with its subsidiary, however if we aim to valuate a stake in company where we do have just around 21 % interest, then of course the information need would be much higher and much more expensive. Moreover, there could be quite non-transparent ownership structure where our company being significant investor could be on opposite site of the barrier and the information would not be provided for our accounting needs.

In case that company uses for one stake the valuation under equity method, there is a legislative requirement to valuate all such stakes under the same method. This may cause a certain disadvantage for the company in case it's holding more interests in various companies. From this perspective may be accuse to apply the cost model rather than equity method.

4.1.2 Companies Applying Cost Model

Despite not promoted much by legislature, majority of companies are applying cost model. It's mostly applied by companies holding significant proportion of various stakes. The reasoning might be provided by the last paragraph about equity method – if company applies equity method for one stake, it shall be applied for all stakes. As a typical example of the user of cost model might be given company Agrofert holding more than 10 stakes in Czech and also abroad.

Other reasons for the adoption of cost model instead of equity method might be strong simplicity, or also sometimes an inability to determine the equity value of an issuer of shares upon balance sheet date. Finally, this method is significantly less costly compared to other valuation methods. On the other hand, the companies reporting under Act on Accounting, Decree on Accounting and the Czech Accounting Standards should comply with valid legislation and prefer the equity method or fair value method rather than historical cost concept.

4.1.3 Non-Transparent Reporting at Fair Value

The only company quoting the application of fair value model from our research sample is Lesy České republiky. Why we do mention "quoting the application" instead of "applying"? The reason is the interesting valuation of its subsidiaries – Hradecká a dřevařská společnost and H.F.C. In 2014 was prepared professional valuation of the stake at H.F.C. (from the

perspective of IFRS 13 Fair Value Measurement we are dealing with level 2 disclosures). The value of H.F.C. was computed as significantly lower compared to book value. For the difference company Lesy ČR reported the impairment of 14 930 400 CZK (approx. 750 000 USD). However, when company applies fair value model, it cannot account for the impairment, so this model was misused. For the quoted amount shall be decreased the revaluation surplus and it cannot be accounted as a loss of the period.

In 2017 there was prepared professional valuation of the stake at Hradecká a dřevařská společnost. The result for this stake was similar to the book value, therefore company didn't account for any difference, despite its treatment in such case might surprise the users as well.

5. Conclusion

Based on the previous analysis there could be stated that vast majority of companies are applying cost model for the valuation of their purchased stakes. This finding is similar for companies reporting under Czech accounting legislature as well as for companies reporting under International Financial Reporting Standards framework. Cost model is focused on the reporting of the past cost of purchase. From this perspective we can say that companies rather prefer verifiable rather than reliable information about the current price of their purchased stakes. This prudence treatments supports findings of Ball (2006). Results negatively correlate with findings of Frankel & Li (2004), respectively Barth et al. (2006) who do rather support the mark-to-market approach.

For realisable shares or interests under than 20 % shall be applied according to legislature (Czech or IFRS) the fair value model. The main reason for its extremely low spread is the necessity of expert opinions because of the lack of market data. This cause significant costs for the companies and also based on previous studies there is an average of 30 % fault of experts from the real mark-to-market value. Some experts are too optimistic, some of them much more pessimistic.

Low proportion of the analysed companies apply equity method for the valuation of purchased stakes. Despite this valuation method is much more precise and up-to-date compared to cost model, there might be a problem of the availability of data for issuers where company has around 20 % stake. This cause additional costs for finding out the value under equity method is the obligation to apply equity methods for all purchased stakes.

From the capital structure perspective cannot be directly derived if the application of cost model, fair value model or equity model has a more significant impact than others. Companies mostly using cost model are obviously not applying impairment tests and from this perspective cannot be visible any impact on corporate profit and loss and equity remains unchanged.

Potential limitation of this research is size of our data sample; however, we do believe that testing TOP100 companies for their reporting of shares is more representative compared to analysis of SMEs only. Within our research sample were all listed companies reporting under IFRS as well as other significant players in Czech business environment. Second limitation might be the pure focus on individual financial statements instead additional analysis of consolidated financial statements. This may be a potential topic for the future research.

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Appendices

Appendix 1. Measurement of Financial Assets by TOP100 Czech Companies Reporting under IFRS

Valuation Method	Companies
Equity method (1)	Pražská plynárenská
Fair value method (0)	---
At cost (10)	ČD Cargo; České dráhy; ČEZ; Philip Morris; SAZKA; ŠKODA AUTO; ŠKODA TRANSPORTATION; T-Mobile Czech Republic; UNIPETROL; Veolia Energie ČR

Appendix 2. Measurement of Financial Assets by TOP100 Czech Companies Reporting under Czech Accounting Legislature

Valuation Method	Companies
Equity method (8)	BILLA; Continental Barum; DEK; FAST ČR; Ferona; HP TRONIC Zlín; Mountfield; PHARMOS
Fair value method (1)	Lesy České republiky
At cost (41)	AAA Auto International; Adient Czech Republic; AGROFERT; Alliance Healthcare; ALPIQ ENERGY; Alza.cz; AVAST Software; Bidfood Czech Republic; CARBOUNION HOLDING; ČEPRO; ČEPS; ČESKÁ LÉKÁRNA HOLDING; Česká pošta; Česká telekomunikační infrastruktura; DEZA; DHL Express (Czech Republic); Energetický a průmyslový holding; EP ENERGY TRADING; EUROVIA CS; FOXCONN CZ; GECO; HP Invest, innogy Česká republika; Iveco Czech Republic; Johnson Controls Autobaterie; JUTA; METALIMEX; Metrostav; MOL Česká republika; PHOENIX lékárenský velkoobchod; Pražská teplárenská; Pražské vodovody a kanalizace; PREOL; Severočeské doly; Siemens; Sokolovská uhelná; SWS; Tesco Stores ČR; VEOLIA Česká republika; ViaPharma; WITTE Nejdek

Forecast quality of interest rate risk measures in cases of increasing yield curves – A comparison between the Historical Simulation and the EBA IRRBB scenarios

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Abstract

Driven by the ECB interest rate policy, there is a historically low level of yields in Europe. For future development, three scenarios are in focus: a Japanization with an ongoing low level of yields, a further decline in the yield curve and a scenario of increasing yields. Against the background of an inverse relationship between market rates and bond prices, especially a future rising scenario of interest rate risk affects present value risk measurements. Strengthened by the EBA's regulation on interest rate risk in the banking book (IRRBB) is most relevant for banks. IRRBB regulation provides six interest rate risk scenarios. In addition to internal models, like the most common Historical Simulation, there are six EBA scenarios which should be considered in risk management. Hence, we analyze the future forecast quality of the Historical Simulation and the ECB IRRBB interest rate risk scenarios in a mirrored scenario of increasing yields.

Keywords

Value at Risk, EBA, IRRBB, forecast quality

JEL Classification: G21, G24, G28, G32

1. Introduction

Especially for the German banking sector, interest rate risk is one of the most relevant risk categories for bank management and also for regulators [(Reuse (, 2018),, pp. 11 ff.].). In distinction, the current maturity transformation of long term credits and the high level of sight deposits characterize the interest rate risk in the banking book. Driven by these maturity transformations of banks the current regulation of interest rate risk arising from non-trading book activities addresses banks themselves and also competent regulators was published in 2018 [(EBA (, 2018),, p. 44].). In contrast, interest rate risk from the trading book is comparatively subordinated in the risk structure of German banks [(Deutsche Bundesbank (, 2018),, pp. 81 ff.]) and regulated by the so-called Basel III regulation since 2013 [(CRR (, 2013),, Art. 102 ff.].).

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Focussed on the interest rate risk from the banking book, the EBA defined six scenarios (NII scenarios), which state an adequate risk measure [(EBA (, 2018),, p. 34].). Referring to several characteristics and research of interest rate risk management like

- the distinction of net present value and scenario techniques in risk management [Reuse (, 2017),, pp. 138],
- the maturity transformation [(Reuse and Svoboda (, 2014a),, pp. 37 ff.;; Reuse and Svoboda (, 2014b),, pp. 376 ff.],),
- state-of-the-art risk measures like a Copula [(Reuse, Boka and Rüder (, 2019),, pp. 272 ff.;; Reuse, Rüder and Boka (, 2017),, pp. 80 ff.],),
- the adequacy of the six EBA interest rate risk scenarios [(Svoboda, Reuse, Rüder and Boka (, 2018a),, pp. 712 ff.;; Svoboda, Reuse, Rüder and Boka (, 2018b),, pp. 511 ff.])
- and the forecast quality of the Historical Simulation in case of increasing yields [(Boka and Rüder (, 2018),, pp. 28 ff.],),

this article extends the mentioned research towards an analysis of the future forecast quality of the six EBA scenarios. The following academic aspects should be answered:

a) *Is the Value at Risk, based on the six EBA interest rate risk scenarios, an adequate measurement approach?*
After the previous research contributions answer the questions if the six scenarios are an adequate measurement of interest rate risks, this research derives a present value changes from the scenarios. Hereupon, we used the Historical Simulation to derive the VaR and analyze the forecast quality.

b) *Which data history of the EBA-EVE-VaR-approaches fits the best forecast quality?*
We defined a 5-year- and a 10-year-gliding-period and, additional, an increasing data history used for the VaR-measurement.

c) *Which VaR-approach has an adequate forecast quality in times of increasing yields?*
Driven by a scenario of increasing yields, we furthermore analyze which of the used VaR-approaches of the Historical Simulation has an adequate forecast quality. Therefore, we create this scenario by data mirroring and apply our VaR-approaches on this new forward-looking period.

2. IRRBB Management

2.1 Interest-rate-risk scenariosRate-Risk Scenarios of the EBA Guideline

Because the EBA Guideline provides a lot of different requirements for the management of interest rate risk in the Banking Book, let us confine ourselves in this article to describe just the requirements for the supervisory outlier test and the six scenarios.

To a supervisory comparison and classification of interest rate risk in the Banking book, the EBA defined a supervisory outlier test. This test was earlier known as a sudden parallel +/- 200 basis points shift of the yield curve [(Reuse (, 2018),, pp. 148 ff.;; EBA (, 2018), p, pp. 4 f].).

These two scenarios got added with four more scenarios to provide a supervisory early warning indicator, which is not only sensitive to a parallel shock. Therefore, four more movements in the yield structure should be calculated [(EBA (, 2018),, pp. 31 ff.].). The changes in the economic value of equity (EVE) is set in relation to the institution's Tier 1 capital. The quotient of these six scenarios should be reported the competent authority through the ICAAP.

Institution's which IRRBB-quotient is higher than 15% typically should hold additional capital buffers for their high IRRBB-risks.

Focussing on the six scenarios, the standardized scenarios are described in Annex III of the EBA Guideline. Following scenarios are required [(EBA (, 2018),, pp. 40 ff.]:):
- parallel shock up
- parallel shock down
- steepener shock (short rates down and long rates up)
- flattener shock (short rates up and long rates down
- short rates shock up
- short rates shock down.

For the Euro yield curves, there are three shift parameters set [(EBA (, 2018),, pp. 49 ff.]:):
- parallel: 200 bp
- short: 250 bp
- long: 100 bp.

Other currencies should account other shift parameters. The way of estimating these shock parameters are set in die Annex III, too [(EBA (, 2018),, pp. 49 ff.].). Additional, the parameters are based on data from 2000 to 2015.

2.2 Measuring IRRBB with the Historical Simulation

Generally, the Historical Simulation refers to a process that takes historical changes in risk factors as representative of the future. For this purpose, the historical, maturity-specific changes in interest rates are deducted and applied to the current interest rate structure. New valuation curves are created, and the interest rate book economic values are determined. The difference between these newly created economic values and the expected economic value is calculated. Based on the differences between the created economic values of historical risk factor changes and the actual present value, a historical distribution of present value changes is derived. The quantile of the historical present value distribution presents the risk value in the corresponding confidence interval and holding period [(Schierenbeck, Lister, and Kirmße (, 2008),, pp. 96 ff. and; Jendruschewitz (, 1999),, pp. 64 ff.].).

The changes in the risk factors could be calculated in an absolute, a relative or a logarithmic way. A simple subtraction of two key date observations separated by the holding period h is set as an absolute change.

It should first be noted that the relative and the logarithmized risk factor changes are independent of the market interest rate level. The absolute risk factor changes refer to the prevailing interest rate level at the time the differences are formed [(Boka (, 2018),, pp. 35 ff.].).

It has to be specified that no uniform procedure for the methodical calculation of the risk factor change has prevailed and that the methods in the various interest rate phases offer their advantages. Although the absolute difference method is dependent on the interest rate level, the history includes a significantly higher interest rate level, which significantly determines the currently good forecasting quality. In contrast, the relative and logarithmized risk factor changes prove to be level-independent. They, therefore, always adapt themselves to the current interest rate level. In particular, in a phase of rising interest rates, the benefits of level-independent risk factor changes could become apparent. In contrast, absolute changes show an adequate forecast quality and a better prediction than relative changes [(Boka and Rüder (, 2018),, pp. 28 ff.].).

About the introduced holding period h, it should be further noted that this should be consistent with the chosen planning horizon of the entire risk measurement. Typically 1, 20, 63, or 250 days are used. Based on this, a conflict of objectives consists of a short holding

period with a large number of scenarios or a lower number of scenarios, possibly even nested, risk factor changes with the consequence of methodologically induced autocorrelation effects. In particular, using shorter holding periods, the partial confirmation of the normal distribution assumption, along with the resulting possibility of root-time scaling, should already be mentioned here. [(Boka (, 2018),, pp. 35 ff. and sources specified there].).

In addition to this short introduction into the Historical Simulation, we would like to mention that some enhancement like the so-called ‚Modern Historical Simulation' with an exponentially weighted moving average to an overweight risk factor or present value changes of recent dates and corresponding underweight older changes exist.

Great advantages of the Historical Simulation are the simplicity of handling and the freedom of distribution and correlation assumptions. Only the empirical distribution and also the empirical dependency structure is used as the basis for risk prediction. Nevertheless, it also leads to the most weighty assumption and also the sticking point of the Historical Simulation: The ex-post representativeness of the data for the ex-ante movements of the yield curve.

This can be considered as a trend-free or stationary time series [(Choudhry, Moskovic, and Wong (, 2014),, p. 485 ff.].). Of course, the current yield curve movements within the continuously falling yields curve since 1990 could lead to the assumption that there is no stationarity of the risk factors. Various researches show that the forecast quality of the Historical Simulation, especially with absolute risk factor changes has a good forecast quality [(Svoboda, Reuse, Rüder and Boka (, 2018a),, pp. 712 ff.;; Svoboda, Reuse, Rüder and Boka (, 2018b),, pp. 511 ff. And; Boka and Rüder (, 2018),, pp. 28 ff.].). Also, in times of falling interest rates, the inverse relationship between interest rates and economic value prevents increasing present values.

Of course, there are more advantages and also disadvantages like the tail-dependence, the data quality and the ability to predict asymmetric risks like options and other derivatives. The Historical Simulation may not reflect hypothetically possible extreme scenarios [(Wiedemann and Lüders (, 2006),, pp. 140 f.;; Hager and Romeike (, 2010),, pp. 10 f. and; Klausnitzer (, 2015),, pp. 160 f.].).

3. Data and Methodology

3.1 Used Data

We use Euro swap rates from 1 up to 10 years that are used to be a risk-free yield curve [(Bloomberg (, 2019)].). We use a data history from 31.08.2004 to 29.03.2019. To predict the future forecast quality and especially the forecast quality in times of increasing yields within expected economic value losses, we mirrored the ex-post yield curve changes into ex-ante changes. With the history of yield curve changes of 15 years, we extrapolate a forward-looking yield curve from 31.03.2019 to 30.04.2029. This leads to 120 forward-looking data points. This is a very simple form of predicting future development, but it still meets the assumption of representativeness. The yield curve changes are computed as absolute changes. The scenario shocks are simulated on 31.03.2019. Afterward, the yield curve is simulated via the forward rates. Figure 1 shows the historical yield curve, the computed EBA scenarios as well as the mirrored scenarios, which we use for our analyze.

Figure 1: Historical yield curve, EBA interest rate risk scenarios and mirrored scenario
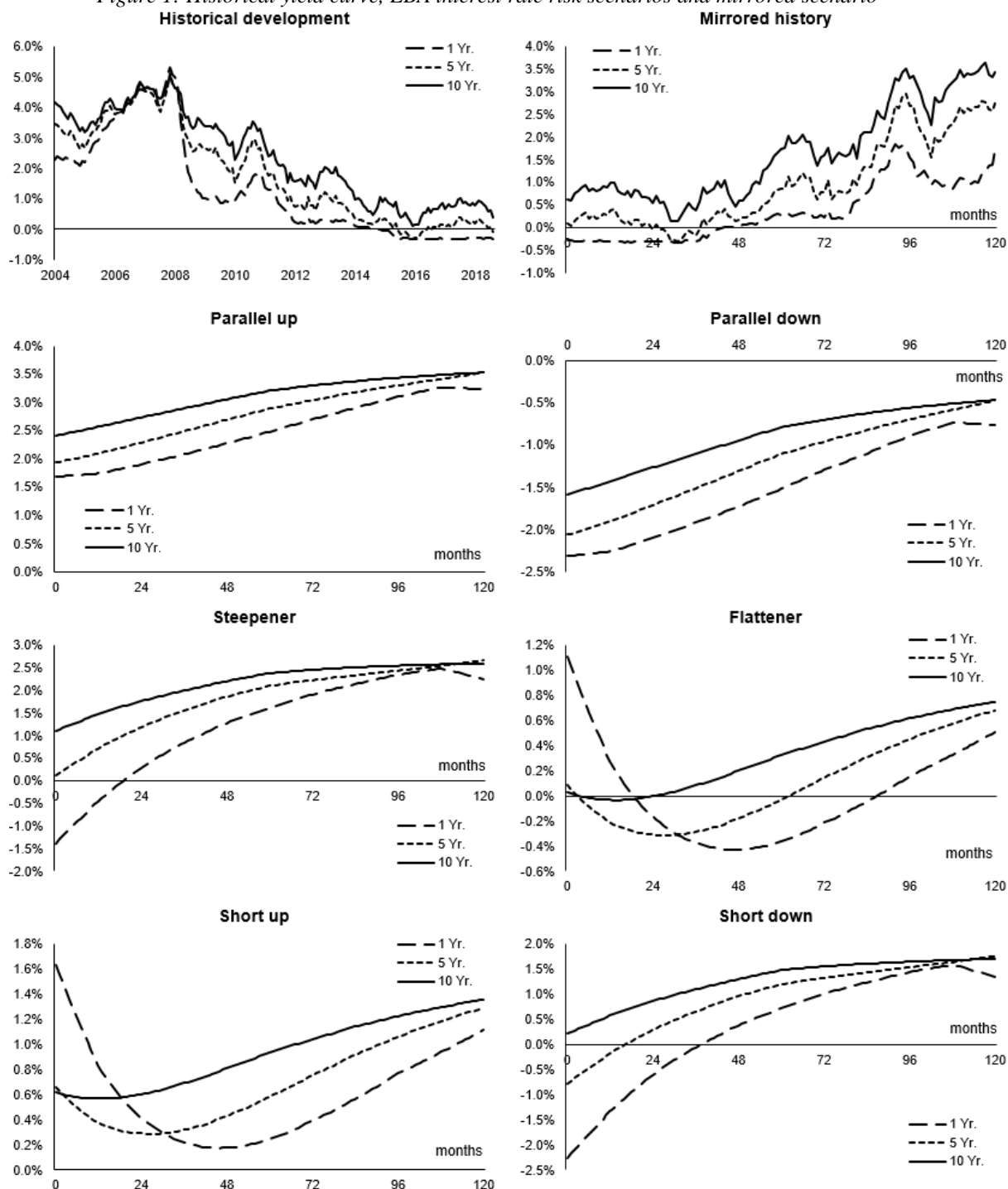
Source: Own calculations

Comparing the different IRRBB-strategies, we defined a 10-years-gliding-cashflow portfolio with a maximum cash flow period of 10 years. (see table 1). The 10-years-gliding-cashflow is a typical IRRBB-strategy [(Reuse and Svoboda (, 2014b),, pp. 376 ff.].).

234

Table 1: IRRBB-strategies

	1 Yr.	2 Yr.	3 Yr.	4 Yr.	5 Yr.	6 Yr.	7 Yr.	8 Yr.	9 Yr.	10 Yr.
gliding 10 Yr.	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000

Source: own table

3.2 Description of Methodology

Within the idealized portfolios, we eliminated the effects of reinvestments or the reduction of the residual maturity. We use a holding period of one month and a confidence level of 95.0%. We use different approaches to calculate the Historical Simulation:

- 5-years-gliding history
- 10-years-gliding history
- Increasing history.

The forecast quality for each of the 120 forward-looking data points is analyzed by comparing the predicted risk with the ex-post performance. The long data history is used to ensure a sufficient input data for the risk prediction. We do not evaluate the forecast quality between 31.08.2004 to 29.03.2019.

4. Discussing the resultsResults

If we look at the different EBA scenarios, we can assume that the forecast quality tends to be worse, especially for the parallel up shift of the yield curve. (see table 2). Only the short-down scenario is generally conservative enough to avoid VaR-outliers throughout the scenario length. It also shows that the assumed Steepener scenario has a good forecast quality. Only the calculation of the Historical Simulation with a gliding 5-year history shows one VaR-outlier. In particular, the scenarios Short up, Flattener as well as the two parallel shock scenarios (parallel up and parallel down) show a worse prognosis quality.

In particular, the introduction of the results does not take the different calculation methods of the Historical Simulation into account. Over almost all scenarios, there is no sufficient predictive quality of the gliding 5-year perspective. Nonetheless, this leads to the realization that there is no need to choose short gliding scenarios, which may well absorb short-term volatility. The inclusion of short-term volatility, however, leads to worse results. Sometimes a good forecasting quality is already evident for all scenarios with a gliding 10-year perspective. Only the parallel-up scenario is not sufficiently conservative. In essence, the increasing and the gliding 10-year history are sufficiently conservative in their forecasting quality in the scenario of rising interest rates. Nevertheless, the question which of the two histories might be more appropriate to optimize the allocation of the risk capital can be examined in further research. Considering a retrograde perspective, one of the two methods significantly overstates the risk.

Interestingly, the mirrored VaR scenarios shows a poorer forecast quality compared to the steepener and the short-down scenario. In contrast, the forecast quality of the steepener and the short-down scenario is too conservative. Similarly, by using the mirrored history to calculate the historical simulation VaR-outliers can be observed in all three data horizons. This is in contrast to various quoted research, which has assumed a good prognosis quality, especially with an increasing history. It should be noted at this point that the two types of research differ in particular by the length of the history or the data used.

It can be linked to further research to analyze the question, which length of the history is necessary to ensure a sufficient prognosis quality.

In particular, the parallel up scenario−a very extreme scenario with +200 basis points−shows a very conservative forecasting quality and no outliers can be observed. This result has

already been determined in existing research. It is surprising that other scenarios also have a sufficiently conservative forecasting quality in the future scenario.

Overall, it can be shown that the used data history provides a decisive contribution to the forecast quality. In this regard, a gliding 5-year history is too short. In the case of the gliding 10-year and the increasing history, further research is necessary to ensure an adequate allocation of risk capital.

Figure 2: VaR Excesses through the EBA scenarios and mirrored Historical Simulation
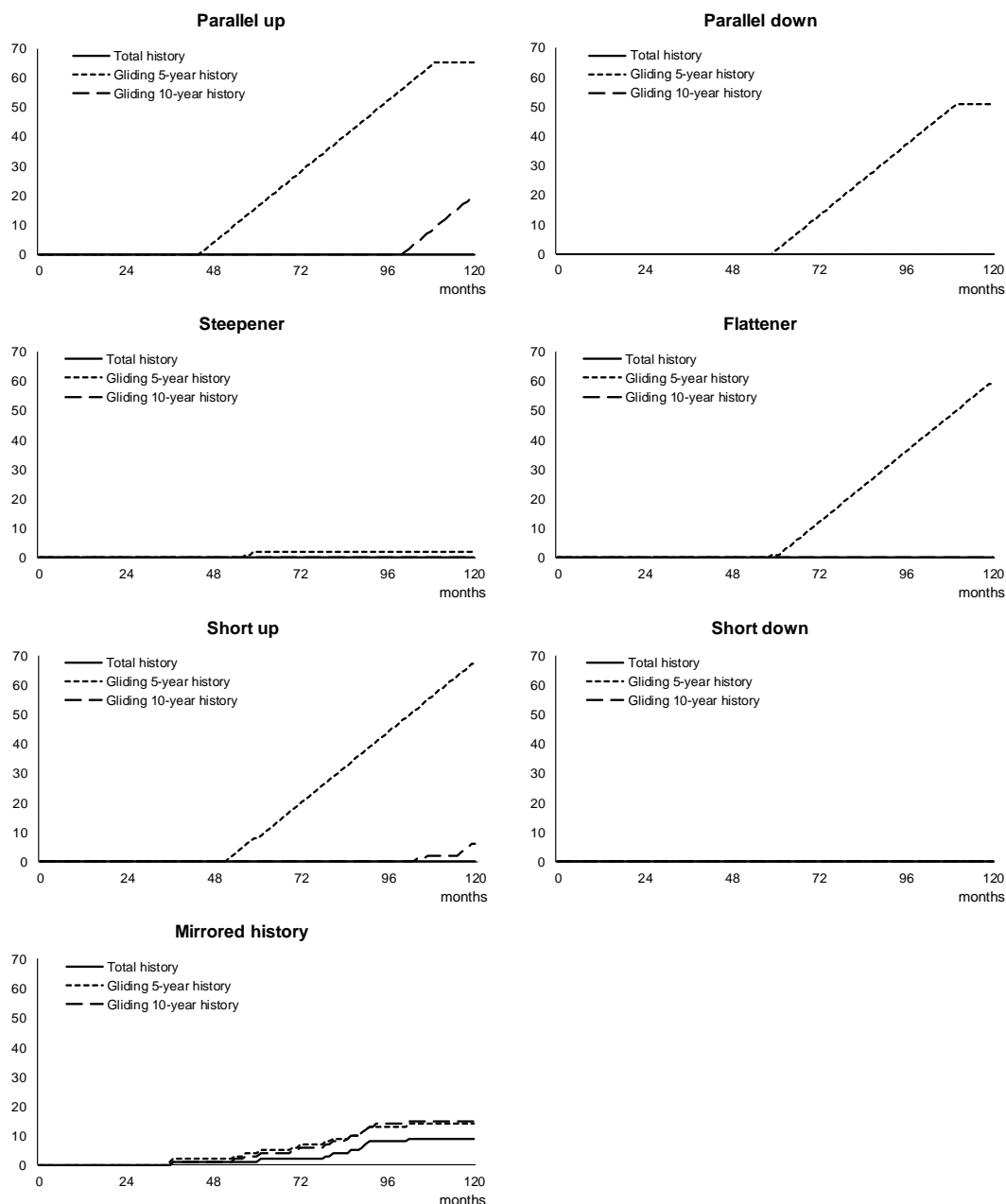
Source: Own calculations

5. Conclusions

Summing up the main results lead to the following aspects:

- Although the gliding 5-year history picks up the short-term volatility faster, it does not show sufficient forecast quality.
- The scenarios short down and steepener show a good forecast quality in the future scenario of rising interest rates in all calculation methods of the Historical Simulation.
- The scenarios short down and steepener show a better forecasting quality than the mirrored Historical Simulation. The increasing and the gliding 10-year history show a good prognosis quality.
- The scenario parallel up shows the worst forecast quality, and even with a longer data history, the danger of an insufficient forecast quality is shown.

Last, the academic questions should be answered:

a) *Is the Value at Risk, based on the six EBA interest rate risk scenarios, an adequate measurement approach?*
Yes, taking into account the different calculation methods of the Historical Simulation, the Historical Simulation based on the EBA scenarios shows a good forecasting quality.

b) *Which data history of the EBA-EVE-VaR-approaches fits the best forecast quality?*
The best forecast quality is reached with an increasing history. But also the gliding 10-years history leads to a sufficient risk forecast.

c) *Which VaR-approach has an adequate forecast quality in times of increasing yields?*
The best predictive quality is achieved with the short down scenario. But also the steepener scenario has a sufficient forecast quality, too.

For further research, the optimal length of history as a parameter of good forecasting quality is particularly interesting. This raises the question of an optimal allocation of risk capital. The question also arises in which market situations the 5-year data history has advantages.

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Social networks in the system of communication tools in internal communication used in the Slovak banking sector

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Abstract

Communication tools, including social media, are currently generally seen as key factors that enter internal systems and management processes and guide, dynamize and improve them in varying levels. In banking houses, the use of social media has several advantages, disadvantages and risks. The paper analyses the place and role of communication tools including social media in the management system in banking houses operating in the Slovak Republic. It focuses on the benefits of using communication tools in management, but also the risks that communication tools can bring to management processes. The paper presents the results of monitoring of the place, tasks and changes that have occurred in the selection and use of communication tools in the last ten years in the banking sector in Slovakia. It compares the research results obtained in the project APVV SK-CZ -0108-09 with the results of the research conducted in 2018-2019 under the project VEGA 1/0309/18.

Key words

Communication tools, internal communication, social media, social networks, intranet, email

JEL Classification: G21, M12, M2

1   Introduction

Current management practice confirms that if a company wants to compete in a market competitive environment, communication, internal or external, cannot be done at a lay level, but requires a professional approach, whether in the use of communication tools, definition and compliance with communication competences, the implementation of managerial communication skills at every level of business management. We can find many examples of how communication, respectively not/applied communication tools, communication competences and communication skills of managers whether within the internal or external environment influenced the processes of financial management in the company as in scientific literature, whether foreign (Hudson, 2017) or domestic literature (Beláňová, 2016; Sedláková, 2015; Szarková et.al., 2004) as well as in enterpreneur and business practice.

The theory and practice have confirmed that the business entity, which laxes to the selection of communication tools, the definition of communication competences and the

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consistent professional application of communication skills of managers in the internal processes, does not pay attention to their choice or the way of their use, development and improvement; it negatively affects not only internal environment (Szarková et.al., 2004), but also the external environment in which the company implements its goods, whether products or services (Sedláková, 2015). Conversely, a company that pays proper attention to communications, including the selection and use of communication tools, specifies communication skills, and improves managers' communication skills, achieves significantly better results in both internal and external environments, which are reflected not only in the quality of the internal environment that motivates higher performance of employees, including managers, but also in the perception of the bank house by the public, its clients and customers.

In the internal environment of multinational and multicultural corporations, which has long been characterized by a high degree of interculturalism and multilingualism, the choice and way of using communication tools, the application of communication skills and communication skills is the alpha and omega of their functioning. The term of social media, several authors (Hobbs, Fowler et al., 2016; Hudson, 2017; Long, 2016) characterize as a new and unlimited communication space for quick, efficient and real-time exchange of informations, knowledges and experiences. In particular, the benefits of social media are that they enable the sharing of this informations and knowledges, support internal formal and informal communication, especially in multinational companies and their subsidiaries (Szarková, 2018), with which bank houses operating in Slovakia are.

The paper analyses the place and role of communication tools including social media in the management system in banking houses operating in the Slovak Republic. It focuses on the benefits of using communication tools in management, but also the risks that communication tools can bring to management processes. The paper presents the results of monitoring of the place, tasks and changes that have occurred in the selection and use of communication tools in the last ten years in the banking sector in Slovakia. It compares the research results obtained in the project APVV SK-CZ -0108-09 with the results of the research conducted in 2018-2019 under the project VEGA 1/0309/18. The aim of the authors is not to provide a comprehensive view of the issue, but to highlight and outline the main developments in the area. They also subordinated to this purpose the selection and demonstration of acquired knowledge and information.

2   Research Results

Commercial banks operating in Slovakia were selected to explore the use of communication tools, especially social media in financial management, for two reasons. First, the banking sector in the Slovak Republic consists mainly of subsidiaries of multinational banking institutions, which are equipped with communication tools in view of the Slovak situation above standard and the managers in them are forced to use these communication tools. The second reason was the fact that bank houses operating in the Slovak Republic are moving in a relatively strong competitive environment, which creates pressure on managers to select such communication tools that ensure that they obtain, process and respond to the information obtained objectively, both internally and externally the communication environment of these entities.

The less important reason for the selection of commercial banks operating in the Slovak Republic as an object of investigation was the fact that commercial banks are an important component of the double-stage banking system, which currently operates in Slovakia and carries about 20,000 jobs in the labor market, which are mostly filling of university-educated employees, while up to 67% of employees are university-educated employees with economic

specialization. The selection of commercial banks as an object of investigation was also conditioned by the structure of the Slovak banking sector, consisting of the central bank (NBS - National Bank of Slovakia), 14 commercial banks based in the Slovak Republic, 17 branches of foreign credit institutions, 3 representations of foreign credit institutions 4 banks in a separate and 287 entities freely providing cross-border services, institutions supporting business banks (ŠÚ SR, 2011; 2019).

The main aim of the research was to find out which communication tools are used in the Slovak banking sector, their advantages, disadvantages and risks. The sample of respondents consisted of top managers of 14 commercial banks based in the Slovak Republic, of which 3 were building savings banks and 1 specialized bank. With the exception of the two mentioned, all commercial banks approached were subsidiaries of multinational financial institutions. At the same time, the managers of all the addressed commercial banks had a comparable structure of communication tools that were accepted and recommended by the parent company and were not limited by the parent company in the choice of communication tools, on the contrary, they had the opportunity to choose the communication tool that most closely matched the cultural habits and patterns used in the communication process, both in internal and external communication environments.

The research was conducted in two stages. The aim of the first stage of the research was to get answers on two basic research questions about which communication tools are used by managers in internal communication in management processes and why. The aim of the second stage was to find out which communication tools do not use, respectively which communication tools they use only sporadically and why.

In the primary and secondary research, basic scientific methods were used, mainly the descriptive method and the method of content analysis, methods of synthesis, comparison and classification. A questionnaire method with a built-in range of possible answers, a structured interview method and an observation method was used to collect data. The questionnaire and data collection were conducted electronically. In addition to basic statistical methods, T-test and non-parametric Mann-Whitney test, Microsoft Excel spreadsheet and SPSS Statistics software were used to evaluate the data. Furthermore, the method of boxplots, factor analysis, based on which causal relationships between individual variables were identified, were used.

3   Social networks in the system of communication tools in internal communication used in the Slovak banking sector

The analysis of the structure of communication tools used in internal communication, together with theoretical knowledges formed the basis for monitoring communication tools and information technologies used in banking houses operating in Slovakia. The research focused on the detection of forms of communication, application of communication tools and information technologies in internal and external communication of examined subjects. The sample of respondents consisted of randomly selected top managers of banking houses - subsidiaries operating in the Slovak Republic. Top managers (140) in 14 banking houses operating in the SR were approached using the questionnaire method. Almost all managers - 138 respondents responded to the questionnaire, representing 97, 2%, and 2 respondents did not respond to the questionnaire. All 138 respondents filled in the questionnaire completely (97,2%). All respondents were men from 56 to 62 years and worked in the bank for at least 10 years of which at least 3 years in top management. We re-conducted the research in 2019 to see the shift in the use of communication tools. We tried to reach the same target group of respondents. After sorting out, we evaluated 135 questionnaires in 2019.

Based on the analysis of the research results, the 9 most widely used communication tools in the internal communication systems of banking houses operating in the Slovak Republic were generated and based on the frequency of use and importance in the communication system. Compared to 2009, a mobile phone has been added to the list, the frequency of which, despite its internal communication, has increased significantly. It can be said that it almost replaced the in-house telephone.

In the second stage of the research, it was observed information charge and communication noise, respectively the degree of distortion in the information generated by their use in the in-house communication system for selected communication tools. We have also tried to find correlations between the communication tools used and the applied management style in the company. The results obtained by the questionnaire method and the structured interview method on a sample of respondents of banking houses operating in the SR enabled to formulate the following conclusions and compile the following table.

In 2009, respondents included the most frequently used and most important traditional communication tools in the communication system: work meeting, regulation guidelines, directions and personal interview. There were no statistically significant differences in the other characteristics of the communication tools that were the subject of the research. Once again, in 2019, traditional communication tools retained their position, but e-communication tools such as intranet/internet, e-mail, internal databases have been at the forefront. In this group can also include a mobile phone, which has become the most widely used communication tool.

Furthermore, the information charge of the most frequently used communication tools and the communication noise that can be used in the in-house communication system were investigated. The analysis showed that, according to respondents, traditional communication tools again had the greatest information charge in 2009: work meetings, guidelines, regulations, directions and personal work interviews, while e-communication tools were not considered very significant by the respondents in terms of information hub. According to respondents, the greatest communication noise was caused by communication tools with minimized feedback: noticeboards, education trainings, but also work meetings. These findings were highly correlated with other findings in the use of communication tools. Also in 2019, there has been a significant shift in relation to e-communication tools. The results of the analysis are summarized in Table 1, which shows the use of communication tools according to frequency of use, perception of their importance in the internal communication system, as well as the level of information charge and communication noise perceived by respondents.

Table 1: Percentage of the use of each communication tool in relation to the transfer of information content and importance in the internal company communication system

Communication tool	Frequency (%)			Importance (%)			Information charge (%)			Communication noise (%)		
	2009	2019	Trend	2009	2019	Trend	2009	2019	Trend	2009	2019	Trend
Working meetings	90,5	91,3	→	73,44	74,32	→	65	64,5	→	31,1	28,5	→
Guidelines, Regulations, Directives	58,2	63,9	→	79,08	82,40	→	57,9	58,7	→	8,5	6,3	→
Personal conversation, Personal contact	49,8	47,9	→	84,36	75,32	→	36,5	32,8	→	13,1	12,6	→
Noticeboards, Bulletinboards	20,2	10,9	↓	6,72	3,25	↓	9,5	2,5	↓	49,2	35,8	↓
In-house telephone	15,8	5,3	↓	26,16	3,5	↓	3,1	2,8	→	25,5	34,5	→
Mobile phone		97,4	↑		94,45	↑		75,5	↑		17,52	→
Intranet/internet (e-mail)	12,1	89,5	↑	9,24	78,54	↑	4	56,5	↑	6,7	7,3	→
In-house computer databases	8,16	17,7	↑	6,72	16,25	↑	5	13,5	↑	3,2	2,3	→
Company newspapers and Magazines	3,96	1,26	↓	4,08	2,35	↓	5,5	1,5	↓	15,6	17,54	↓
Education trainings, Education programs	2,52	15,3	↑	23,76	35,23	↑	16,5	23,5	↑	33,1	15,8	↓
Other (Social media, Social networks)	0,6	2,5	↑	0,60	1,35	↑	0,5	0,75	→	23,9	34,2	↑
Trend: ↑ - rising, ↓ - decreasing, → static												

Source: own processing

Figure 1: Rate of use of communication tools in relation to importance, information charge and communication noise (year 2019)
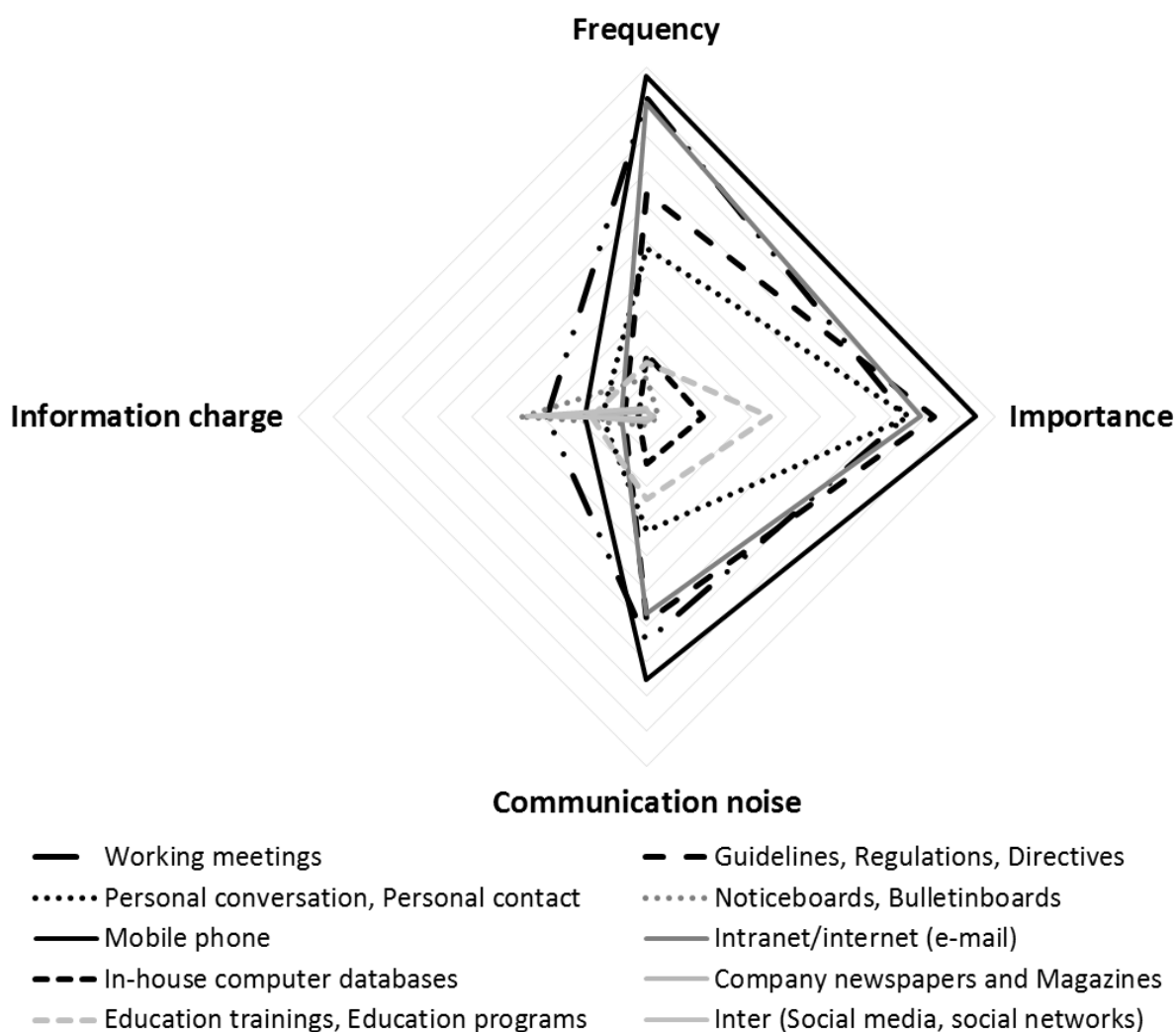
Source: own processing

Figure 2: Ranking of the most used communication tools in the internal communication systems of banking houses operating in the SR down and up (year 2009)
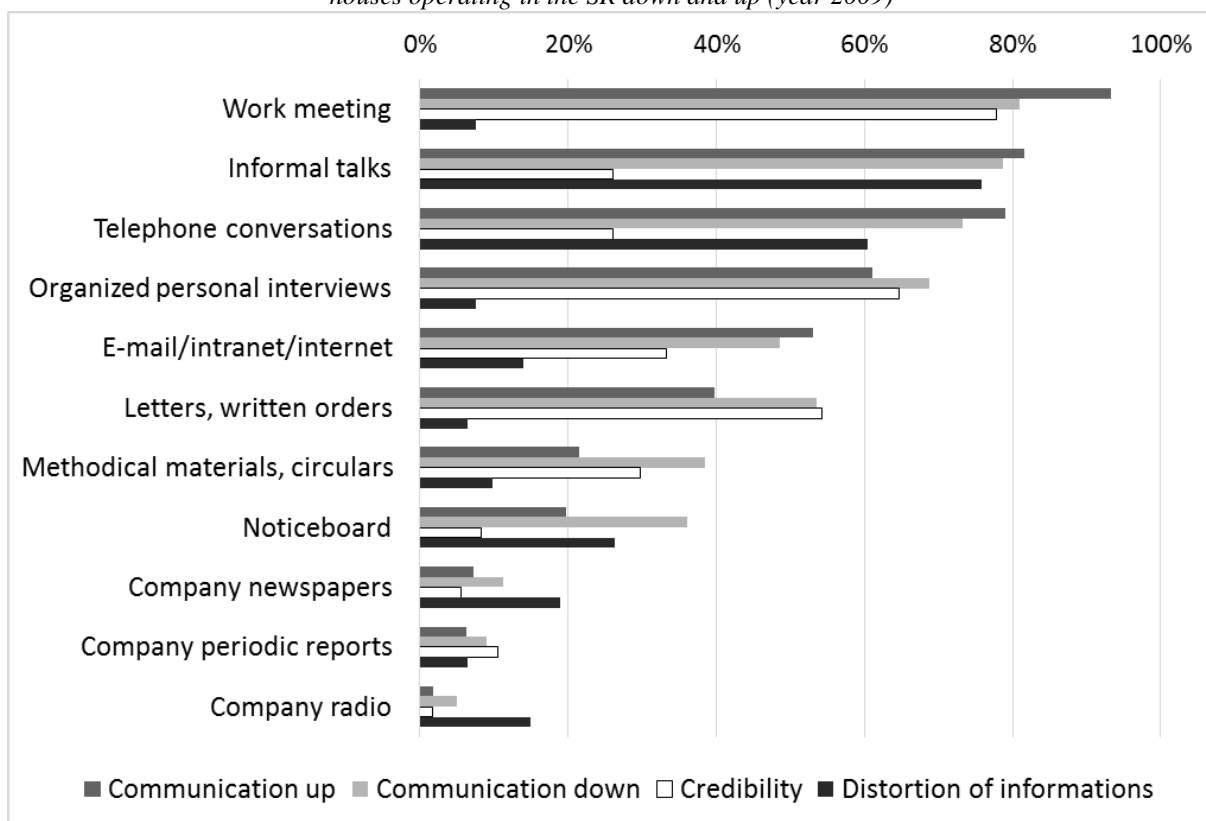
Source: own processing

Figure 3: Ranking of the most used communication tools in the internal communication systems of banking houses operating in the SR down and up (year 2019)

Source: own processing

245
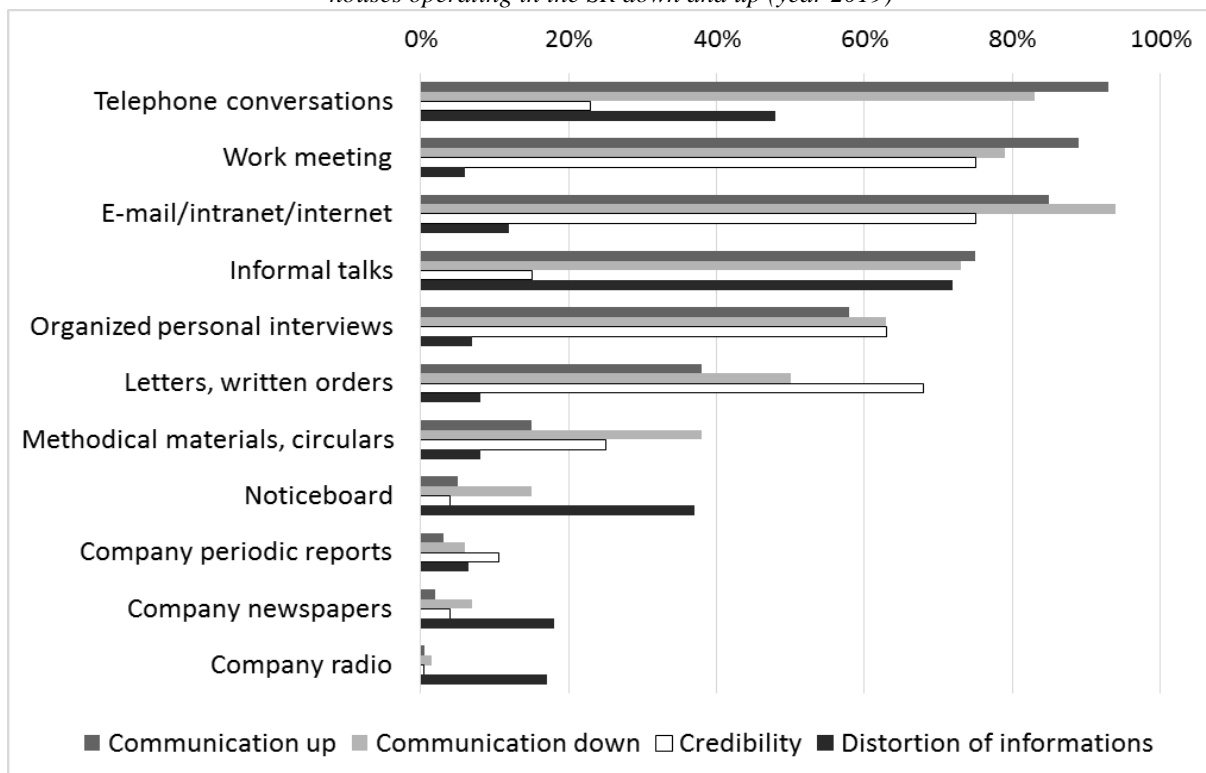
Furthermore, the frequency of use of communication tools in downstream and upstream communication systems was investigated, while the credibility and distortion of the information conveyed in relation to the management system applied in the bank house was monitored. The results of the analysis are shown in Figure 2 and 3.

From the results obtained, we found that respondents in management very little, rather marginally use social media. Over the last ten years, this increase has been minimal. The results obtained show that in regard to the subject of the activity as well as the internal communication environment and the applied management style in the banking houses under investigation, respondents characterized the social media "rather" as an unreliable communication tool at five basic points: they provide anonymous feedback that limits addressing practices in managing, providing space for false, subjective, misleading information (hoaxes), in social networking individuals lose their social and personal identities, and, last but not least, increase dependence on social networks, which can be a negative obstacle to management (Figure 4).

Figure 4: Reasons why respondents do not use social networks in their management activities
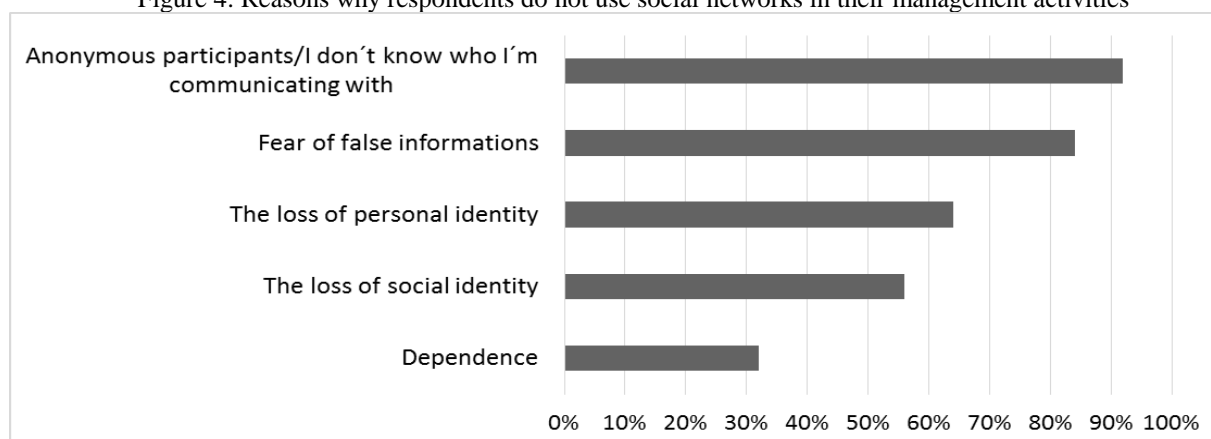
Source: own processing

4 Conclusion

The issue of investigating the place and role of communication, its individual components, forms and types in the system of managing banking houses is currently highly actual. This is due to the above-mentioned causes and also to the origin and development of the new, development of the internal processes of the determined needs, whose complex or partial saturation is enabled only by communication and correctly selected communication tools. This moment was confirmed by the results of the secondary (Heller, 2002; Hobbs – Fowler et al., 2016) and the primary research (Beláňová, 2016; Sedláková, 2015; Szarková et.al., 2004), which show that while the top managers in the banking houses operating in the Slovak Republic used traditional communication tools ten years ago (Beláňová, 2016; Sedláková, 2015; Szarková et.al., 2004), despite the fact that bank facilities are equipped with modern communication tools and communication information systems at a relatively high level (all the examined banks were 100% equipped with complete and functioning ICT), over the last ten years they have started to use e-communication tools more intensively. We assume that this phenomenon is primarily related to the persistence of communication stereotypes in internal company communication systems and their slow degradation in internal formal communication. It may also be due to some "preservation" of the respondents' rigid communication skills, as suggested by other findings.

Furthermore, the results of the primary research showed that e-communication tools in a particular management activity are less used by respondents because of the fear of leakage of sensitive information, also from the fear of "distortion" or misunderstanding of shared and provided informations and some have said that e-communication tools create stress on subordinates, because they are forced to remain alert, flexibly react, distracted at work, distract them from the task being solved. This issue corresponds to the data reported by Park and Haun (2018) and is also highlighted by Matkovčíková (2015). According to these authors, classical communication tools are much more beneficial to the management process, because they allow them to get different information and inspiration to solve their work tasks from their subordinates and also contribute to the positive formation of interpersonal relationships in the workplace. Of course, they also stated that the use of classical communication tools requires having good communication competences and communication skills. The research indicated that prioritizing traditional communication tools by top managers in internal company communication over the use of e-communication tools is also more conditional on the nature of communication that takes place in an interdependent social context of interpersonal relationships and shows elements of informality rather than formality. More than half of respondents (66, 5% of respondents) also stated that e-communication tools, especially social media, are more suitable for external communication than internal communication. To conclude, these knowledges and informations that the research brought is a valuable contribution to the management theory and practice and communication theory. The authors did not aim to provide a comprehensive view of the issue, the aim of the paper was to point out and outline the main developments in the area and subordinate the selection of acquired knowledge and information. These results are original, they were included in the comprehensive research from which those that have not yet been published have been selected.

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The efficiency of a deposit guarantee scheme- a psychological component

Łukasz Szewczyk[1]

Abstract

The aim of this study is to present the theoretical approach to the efficiency of a deposit guarantee scheme through its psychological context. This context has been widely discussed by the policymakers and it is believed that it plays a prominent role in affecting the efficiency of deposit guarantee scheme. Public awareness may be very important tool that builds trust among depositors. This trust may directly impact the efficiency of DGS.

Key words

deposit guarantee schemes, efficiency, financial component, psychological component

JEL Classification: G01, G21, G28,

1. Introduction

The efficiency of deposit guarantee scheme (DGS) is a complex problem. Nowadays it has been widely discussed, both on national and international level. It's mainly connected with the fact that the wide range of functions conducted by deposit guarantee schemes raises the problem of their efficiency, which is undoubtedly related to the design of the system and the impact of individual elements on its proper functioning. It is emphasized that despite the fact that the guarantee systems constitute the core of the financial safety net, the global financial crisis has shown their weaknesses in many areas.

The aim of this study is to present the theoretical approach to the efficiency of deposit guarantee scheme through its psychological context, which is connected with public awareness of depositors and other groups of stakeholders (e.g. policymakers, financial institutions and investors).

2. Key elements of deposit guarantee schemes

The basic asppect of establishing a deposit guarantee scheme (DGS) is to ensure the safety of depositors, reducing risk during financial crisis and supporting the soundness of the banking sector. The presence of deposit insurance eases concerns by depositors about the safety of their money and in this context it may eliminate the problem of so-called bank runs (Kusairi, Sanusi and Ismail, 2018, p. 34). Technically, the depositor protection may be organised in two ways, as an implicit and explicit one.

S. Talley (1994) states that the choice between implicit and explicit depositor protection is not obvious. Among the most important advantages of implicit solutions he mentions:
- small likelyhood to erode market discipline; it's connected with the fact, that depositors are never sure if their deposits are protected and they have a reason to

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evaluate the situtation of financial institution (however it is worth noting that such evaluation is not a common practice),

- greater flexibility in resolving crisis situation in financial institutions because the government has total discretion in choosing a proper solution (whether to protect, the form of protection and how much to protect)

On the contrary, the explicit deposit insurance system (Talley, 1994, p. 2):

- provides greater protection against bank runs due to legal obligation (the depositors are sure that there is a deposit guarantee scheme which is responsible for reimbursement),
- helps smaller depositors to cover their entire outstanding balance (up to guarantee sum),
- shifts costs onto the banking system through the premium system,
- operates more smoothly and more consintently due to procedures and rules, which are set up by the policymakers,
- provides protection to depositors of all financial institutions which accepts deposits (both private and state-owned).

Very important work which undertakes the problem of the construction of deposit guarantee scheme was the one prepared by G. Garcia (1999). In this paper the most important issues that should be taken into account when constructing and mantaining a DGS were analyzed. It is stated that a well- designed DGS has many advantages in normal circumstances but if it's designed unproperly, many negative effects may arise (e.g. moral hazard or adverse selection). The main best practices that should be taken into account along with issues to be resolved are presented in table 1.

Table 1: Best practices when designing a DGS and issues to be resolved

Best practce	Practical issues to be resolved
Avoiding incentive problems	How to incorporate the incentives in regulations?
Defining the system explicitly in regulation	How to modify the laws and regulations to ensure transparency?
Ensuring the supervisor resolved the failed institutions instantly	The types and importance of closure policies
Providing the level of coverage	What is the appropriate level of coverage and should there be a co- insurance?
Makeing membership obligatory for depository institutions	How to avoid adverse selection?
Paying deposits as fast as possible	How to provide instant payment?
Ensuring adequate sources of funds	What are the appropriate levels of premiums and fund size?
Adapting risk- adjusted premiums	How to set premiums according to risk?
Organizing good information	Whata data is necessary to collect by DGS?
Establishing an explicit, limited DGS when the banking system is sound	How to resolve banking problems so that the DGS can be established?

Source: (Garcia, 1999, p. 9).

G. Garcia also points out main issues that should be considered to make DGS efficient. Among them are (Garcia, 1999, p. 10):

- „confining eligibility for banks and other depository institutions,
- setting and adjusting coverage levels,

- choosing between ex-ante (funded) and ex-post type of funding,
- introducing risk adjusting premiums,
- selecting a financial target for the fund,
- providing supplementary funding,
- deciding whether to assign back-up powers for the DGS,
- achieving cooperation between DGS, central bank and supervisory authority".

3. The effieciency of DGS- the public awareness

The concept of efficiency can be variously defined depending on the adopted approach, i.e. (Kruk, 2018, p. 218):
- „in economic terms, it is understood as a relation between results and inputs expressed by such measures as profitability or productivity,
- in the general approach, this is an assessment of the degree of implementation of the adopted organizational objectives taking into account the economic aspect,
- in the systemic approach, it involves assessing the degree of use of organizational resources and creating specific relations with the environment,
- in the comprehensive approach is the entity's ability to achieve operational goals".

A. Rutkowska postulates that the concept of efficiency should refer to two dimensions: operational and strategic. Efficiency in the operational sense means implementing specific activities better than other entities that implement the same business concept. The strategic dimension of effectiveness is connected with the recommendation to act in a different way, thus realizing the unique concepts of business operation. It is worth noting that to achieve the specific enterprise goals, which include development and in some conditions- survival, requires efficiency both in the operational and strategic dimensions (Rutkowska, 2013, p. 442).

The effectiveness of deposit guarantee schemes is an important criterion for assessing the suitability of systems in the financial safety net. Such an assessment is carried out by various international organizations, including International Association of Deposit Insurers (IADI), European Forum of Deposit Insurers or the European Commission. The latter commissioned one of the first formal studies in this field already in 2006. The effect was "Report on the effectiveness of deposit guarantee schemes in EU countries". The report emphasizes that the concept of system efficiency has many dimensions. When comparing deposit guarantee schemes in different countries, different measures and indicators are used to assess their effectiveness. The system effectiveness assessment, taking into account the differences in financing mechanisms, the level of guarantee and the size of the guarantee system, enables the system's ability to properly perform the functions entrusted to it (protection of the depositor and ensuring financial stability).

The potential of the DGS comprises of two aspects: its effectiveness expressed by the ability to prevent bank runs or the ability to ensure an effective intervention. Two basic components, a psychological and financial component, determine this potential. The first of these is to shape the rational behaviour of the depositors and convince them about the security of their funds deposited in the financial institutions (Kerlin, 2016, p. 133). The financial efficiency of DGS is connected with the financial capabilities of DGS (the ability to reimburse depositors and to handle a crisis situtaion in a financial institution- if the DGS has a broad mandate and can act as a lender for its members).

It is worth noting that problems with the functioning of the psychological component of the guarantee systems occurred during the global financial crisis. It led to numerous bank runs (table 2).

Table 2: Selected bank runs during the global financial crisis

Year	Country	Institution	Characteristics
2007	USA	*Countrywide Financial*	Information on the worsening financial situation of the institution due to the subprime crisis and the fact that this institution played a crucial role in the market as the largest single mortgage lender in USA.
2008	USA	Washington Mutual	Disclosure of information about the bank's poor situation resulting from the provision of low quality mortgage loans. The depositors withdrew more than 10 bln USD over two weeks.
2008	USA	Wachovia	Disclosure of information about the poor financial and economic condition of the bank caused by granting of low quality mortgage loans. The bank runs were of a periodic nature.
2008	USA	IndyMac	A number of factors led to bank run, e.g. its aggressive strategy, insufficient underwriting and territorial credit concentration in residential real estate (California and Florida).
2008	Iceland, Great Britain, the Netherlands	Landesbanki, Icesave, Kaupting Edge	Disclosure of information about the poor condition of Icelandic banks in the British press. Banking panic occurred in Iceland and in the branches of Icelandic banks conducting cross-border activities, including in the Netherlands and Great Britain.
2009	The Netherlands	DSB Bank	The massive withdrawal of funds from the bank was a consequence of the public appeal of one of the bank's clients regarding the withdrawal of funds.
2011	Latvia	Swedbank	Publication of information on Twitter about the bank's poor condition, including liquidity problems. More than 10 000 Latvians withdrew their deposits.
2013	Cyprus	Cyprus Popular Bank, Bank of Cyprus, Hellenic Bank	Announcement of the authorities' proposals on the participation of all depositors in the losses of banks in Cyprus
2014	Bulgaria	CorpBank First investment Bank	Disclosure of irregularities in the bank, consisting in performing by the bank of suspicious financial transactions

Source: (Kerlin, 2016, pp. 135-136; McKinley, 2014, p. 13-17).

When analyzing the cases described in the table above, it can be noticed that the origins of the bunk runs in recent years were different. In the case of American banks, the decisions made by depositors most often related to the disclosure of information about the bad financial standing of banks, which was most often the result of the deterioration in the quality of the mortgage loan portfolio. In European banks, the reason for decisions made by depositors was was not always related to the financial crisis (Kerlin, 2016, p. 137). H. J. Kiss notes that social

networks play an increasingly important role in fostering bank runs. The social network effect can be seen nowdays among users of different social media (the example is case of Swedbank) (Kiss, 2018).

A particularly interesting example described in the table is the case of Icelandic banks that provided their services within the so-called cross-border provision of services, in Great Britain and the Netherlands. In this case, the decision to withdraw deposits was the result of information provided by the Icelandic deposit guarantee scheme that it would not pay the guaranteed sums accumulated by depositors serviced by branches located outside Iceland. Ultimately, these commitments were covered by the guarantee systems of the host countries, which nevertheless claimed reimbursement of the costs incurred (Bafia, 2011, p. 49).

Interesting is also the situation that took place in the Netherlands, in the case of DSB Bank. Here the reason for the banking panic was a statement in the media of one of the bank's clients, who stated that the bank's clients who had been treated badly by it would have a greater opportunity to satisfy their claims if the bank collapsed. The effect of this appeal was the withdrawal of over 600 million euros in deposits in a very short time (between October 1 and October 10, 2009). This situation shows that even a single statement can have serious consequences for the liquidity of a large bank if it reaches a sufficient number of recipients (in this case, in the public television program)

Contemporary banking panic is hardly observable. E. Kane calls it a silent run. He understands it as a massive withdrawal by sophisticated depositors which weakens bank balance sheet (Kane, 2008, p. 12). To sum up this aspect which affects the effectiveness of guarantee systems, it should be emphasized that there are several elements in construction of the DGS which support the psychological component. These include (Kerlin, 2016, p. 138):
• „good communication of the rules of protection provided by the deposit guarantee scheme,
• ensuring an appropriate amount of protection provided,
• maintaining the credibility and reliability of the guarantee system,
• liquidation of co-insurance,
• ensuring quick payment of funds".

A. Jurkowska- Zeidler notices that the meaning of the psychological component has increased during the global financial crisis. The result was, among others the increase of the guarantee sum up to 100 000 euro and shortening the payout time (to 7 days) in EU (Jurkowska- Zeidler, 2016, p. 174). S. Schich points out that public awareness is one of the four issues that result from the global financial crisis. The others are coverage, financial safety net interrelations and bank failure resolution. He points that explicit DGS may be effective when „depositors understand the extent of and limits to existing protection schemes" (Schich, 2008, p.77).

The problem of public awareness has been also indicated in IADI's Core Principle for Effective Deposit Insurance Systems (2014). According too this document, the public should be informed constantly about the benefits and limitations connected with deposit insurance systems. The most important information that should be provided is (IADI, 2014, p. 32):
  • where, how and when depositors will be provided with their funds,
  • the information that depositor has to provide in order to get payment,
  • if advance or interim payments are being made,
  • how to make a claim for the uninsured funds.
The most important issues that should be taken into account are (IADI, 2009, p.3):
  • to build credibility with depositors and stakeholders by the DGS,
  • to define the principle target groups by DGS,
  • information that may affect depositors' savings should be integrated into public awareness programme,

- a wide variety of tools and channels of communication should be employed by the DGS,
- budget allocations have to be made to build and maintain the desired level of public awareness,
- awareness level should be evaluated regularly,
- members institutions hould promote information about deposit insurance,
- public awareness program should be built with other sfatey net institutions.

To implement a proper awareness programme, many issues should be considered (table 3).

Table 3: Determinants of an effective public awareness campaign

Determinant	Characteristic
Target audience	The principal target audience groups must be identified. Among them there are depositors, general public, institutional investors, member institutions and policymakers.
Responsible parties	It is important to anser the question; who will be responsible for promoting public awareness? The primary party should be DGS but other safety net players should also be involved.
Content	Public awareness programme should cover all deposit insurance information that affects the interest of depositors.
Communication tools	The widest reach through different channels should be achieved.
Frequency and timing	The frequency should be determined by budghet, objectives and types of promotional tools.
Budget and resources	Budget for public awareness programme should be determined on the basis of the desired level of visibility.
Evaluation	Evaluationg the effect of the programme is the crucial aspect; it is not easy to do it and the preffered method is to conduct a survey before and after the programme.

Source: (IADI, 2009, pp. 8-16)

4. Conclusions

It is obvious that the effectiveness of the guarantee system is an important criterion for assessing the suitability of the system as part of the financial safety net. Two basic components: a psychological and financial component, determine the potential of these systems. The first of these is to shape the rational attitudes of the depositor and convince them about the security of funds deposited in the banking system. However, it is difficult to measure it. Deposit guarantee schemes have to take into account many issues which can enable the proper level of efficiency in this area. Among them are: adequate budget for the public awareness programme, conducting qualitative and quantitative research to get information about awareness level, identyfying specific target groups and implementing a proper public awareness programme to achieve its goals.

It is worth mentioning that these theoretical research should be treated as an introduction to broader research, which hopefully will show different tools that are used by deposit guarantee schemes all over the world to increase the level of public awaraness in the area of depositor protection.

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Tariff Analysis in Claim Frequencies Models

Adéla Špačková [1]

Abstract

The non-life insurance pricing consists of establishing a premium or a tariff paid by the insured to the insurance company in exchange for the risk transfer. The subject of the contribution is tariff analysis, where each empirical model is compared and the categorical model is going to be compare with continous model. The results of this contribution can be interesting and it can contribute to a deeper understanding of this problem and importance of the tariff analysis. All empirical models are estimated on the real-world sample data of czech insurance company collected during the years 2005-2010. Estimation is performed by using generalized linear models. Regression analysis allows the identification of the risk factors and the prediction of the expected frequency of claims given the characteristics of policyholders. It depends on many individual rating factors (e.g. based on individual characteristics of vehicle and driver). The aim of this paper is to find out ideally suited model for estimation claim frequencies model and point out the importance of tariff analysis.

Key words

tariff analysis, generalized linear models, claim frequency, individual rating factors

JEL Classification: C13, G22

1. Introduction

The fundamental role of insurance is to provide financial protection, offering a method of transferring the risk in exchange of an insurance premium. Considering that not all the risks are equal, it is natural that every insured will pay a premium or tariff corresponding with the gravity of the risk. Tariff groups are homogenous groups of insurance contracts, where the insured risk is approximately identical, so in each tariff group is possible to require united insurance rate. Each tariff group corresponds to a certain risk level. At first it is necessary to find out the factors influence the claim frequency and calculate it according to the values of these factors. On the basic of these facts insurance company calculate the tariff which has to fulfill the following general principles. The tariff has to be correct as possible and the structure of tariff groups has to be simple (linear function or multiplicative). The aim of this paper is to find out ideally suited model for estimation claim frequency and point out the importance of tariff analysis.

Historically, actuarial science has been limited by using the standard Gaussian linear regression in order to quantify the exogenous variables impact over the phenomenon of interest. The linear model, proposed by Legendre and Gauss in 19th century, has taken the lead in econometrics, but the applicability of this model in insurance has been found to be difficult. In this context, the linear modeling implies a series of hypothesis that are not compatible with the reality imposed by the frequency and cost of the damages generated by the risks occurrence. Considering this, the most important assumptions are the Gaussian probability density, the linearity of the predictor and homoscedasticity. An important

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milestone of the non-life insurance pricing development is considered to be the minimum bias procedure implemented by Bailey and Simon (1960). The principle of this method consists of defining randomly the link between the explanatory variables, the risks levels and the distance between the predicted values and the observed ones. Once these elements are established, an iterative algorithm calculates the coefficient associated with each risk level using the minimizing distance criterion. Although it was created outside a recognized statistical framework, this algorithm has been found subsequently to be a particular case of the GLM models.

2. Generalized linear models

In statistics, the generalized linear model (GLM) is a flexible generalization of ordinary linear regression that allows for response variables that have error distribution models other than a normal distribution. The GLM generalizes linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value.

Generalized linear models were formulated by John Nelder and Robert Wedderburn as a way of unifying various other statistical models, including linear regression, logistic regression and Poisson regression. They proposed an iteratively reweighted least squares method for maximum likelihood estimation of the model parameters. Maximum-likelihood estimation remains popular and is the default method on many statistical computing packages. Other approaches, including Bayesian approaches and least squares fits to variance stabilized responses, have been developed.

Generalized linear models (GLM) is a standard estimation method using in insurance practice. Generally, GLM includes three main assumptions:

- A probability distribution have to be form an exponential family;
- A linear predictor is a transform by a link function;
- Link function can be diverse (identity, logarithm, power..).

For the purposes of this paper the logarithm link function is choosen.

$$\ln\left(\frac{\mu}{n}\right) = x'\beta = ln\mu = \ln n + x'\beta \tag{1}$$

where $\ln n$ is called an offset, μ is mean, β is estimated parameter by maximum likelihood method(Cipra 2015).

All probability distribution can be described by the general form as following:

$$f(y) = c(y,\phi)exp\left\{\frac{(y\theta - a(\theta))}{\phi}\right\} \tag{2}$$

where θ is the canonical parameter and ϕ is called the dispersion parameter, $a(\theta)$ and $c(y,\phi)$ are determining the actual probability function Gray,Pitts (2012).

Claim frequency is estimated by Negative–binomial distribution, as it is shown at Table 1.

Table 1: Distributions and parameters

Distribution	θ	$a(\theta)$	ϕ	$E(y)$	$V(\mu) = \dfrac{Var(y)}{\phi}$
Negative-binomial (μ, κ)	$ln\dfrac{\kappa\mu}{1+\kappa\mu}$	$-\dfrac{1}{\kappa}\ln(1-\kappa e^{\theta})$	1	μ	$\mu(1+\kappa\mu)$

2.1 Claim Frequency

The random dependent variable is discrete and conditioned by a vector of explanatory variables. (characteristics of risk based on individual characteristics of shareholders) is negative-binomial distributed. The probability of random variable y has to be fit into the exponential family framework as following (Long, Freese 2014):

$$\ln\{f(y)\} = y\,\ln\frac{\mu}{1+\kappa\mu} - \frac{1}{\kappa}\ln(1+\kappa\mu) = \frac{y\theta - a(\theta)}{\phi} \tag{3}$$

According to Table 1, the dispersion parameter is equal to one and canonical parameter is $\ln\frac{\kappa\mu}{1+\kappa\mu}$.

Mean and variance function is denoted:

$$E(y) = \dot{a}(\theta) = \frac{e\theta}{1 - \frac{\theta}{e^{\theta}}\kappa e^{\theta}} = \mu \tag{4}$$

$$Var(y) = \phi\ddot{a}(\theta) = \frac{e^{\theta}}{(1 - \kappa e^{\theta})^2} = \mu(1 + \kappa\mu) \tag{5}$$

Where $\dot{a}(\theta)$ and $\phi\ddot{a}(\theta)$ are first and second derivates of $\phi a(\theta)$ with respect to θ.

2.1.1 Standard method of parameters estimation

All models are going to be estimated with standard method called maximum likelihood method. If the maximum likelihood estimation is exponential family distribution, then the probability function is following (Long, Freese 2014):

$$\ell(\beta, \phi) = \sum_{i=1}^{n} \ln f(y_i ; \beta, \phi) = \sum_{i=1}^{n}\left\{\ln c\,(y_i, \phi) + \frac{y_i\theta_i - a(\theta_i)}{\phi}\right\} \tag{6}$$

Maximization of likelihood called log-likelihood is a logarithm of the likelihood with respect to β_i:

$$\frac{\partial\ell}{\partial\beta_j} = \sum_{i=1}^{n} \frac{\partial\ell}{\partial\theta_i}\frac{\partial\theta_i}{\partial\beta_j} \tag{7}$$

2.2 Quality testing of selected models

The next step is selection of a suitable model. An important measure of the quality of the model is its predictive ability, therefore the ability to best estimate the value of new or unknown observations. The task of statistics is to choose what might best model, which will use the information contained in the data, but not to overstate the importance of random fluctuations. A simple tool for the selection of a quality model is a statistic known as the deviance.

2.2.1 Deviance

Deviance allows comparison of selected regression model with a saturated model through the logarithm of the likelihood function. In the case of saturated model the estimated values are equal to observed and the value of the logarithm is the highest as possible. Deviance is defined as(Jong, Heller 2008):

$$\Delta = 2(\iota^{max} - \iota) \tag{8}$$

where ι^{max} is logarithm of saturated model and ι is logarithm of selected model. If the difference between the values of both credibility sufficiently small, then the model can be considered of good quality. Another measure of the quality of the model can be Akaike information criterion.

2.2.2 Akaike information criterion

Information criteria are the measurements that compare the differences between models and serves for the assessment of their relatively quality. Akaike information criterion is defined:

$$AIC = -2lnL + 2k \qquad (9)$$

where L is logarithm of the likelihood function k is number of estimated parameters. It should be selected the model, where is the value of AIC is the lowest (Hardin, Hilbe 2002).

3. Tariff analysis

According to the framework of the tariff analysis, its going to be estimated a total 3 models. At first it will be estimated the model with continuous data, then continuous variables to be converted to categorical variables. We have to calculate the premium as a function of tariff variable choosen according to Cipra (2006) and Valecký (2015), in accordance with the above mentioned factors, i.e:

$$P = P(x_i ..., x_k) \qquad (10)$$

Our aim is is to find out ideally suited model for estimation claim frequency and point out the importance of tariff analysis. We first have to search for the tariff factors and then construct the tariff. In searching for the factors influencing the tariff we make no assumptions concerning the structure of the tariff.

3.1 Data

For the purposes of this paper it was used a random selection of a motor hull insurance portfolio collected during the years 2005-2010 in Czech republic territory. The file contains 18 111 contracts. All regression factors are shown in Table 2.

Table 2: Parameters description

Variable	Description	Value
Frequency	Claim frequency	0,1,2,3
Fuel	Type of fuel	1,2,3,4,5
Gender	Gender of driver	0,1
Agecar	Age of car	0-28
Ageman	Age of driver	19-88
Price	Vehicle price	15 000 – 6 704 000
Volumkw	Engine power	2-265

In following picture it is shown empirical and predicted frequency by above mentioned variables.

Figure 1: Empirical vs predicted frequency
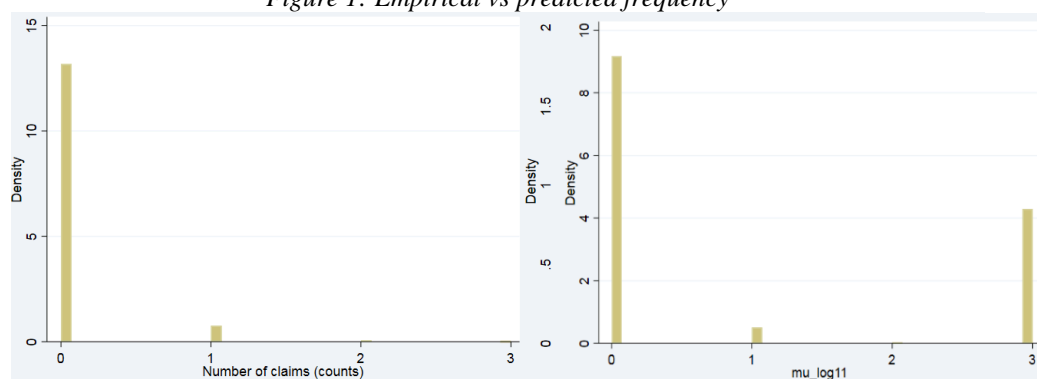
According to the histogram, it can be argued that the estimated frequency is located between zero and number two, which is comparable with the empirical histogram. In the following picture it is shown deviance of above mentioned model 1.

Figure 2: Empirical vs predicted frequency
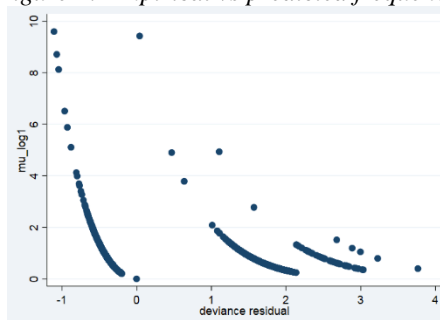
The value of AIC for model 1 is 5445,135. In the following Table 3 is shown empirical vs. predicted frequency by countinous frequency model1.

Table 3: Observed vs. predicted frequency

Number of claims	Observed	Predicted by Model 1
0	17,045	11,864
1	984	656
2	76	35
3	6	5,556

As we can see, predicted 3 number of claims can be overestimated. In the following Table 4 is shown category of 2 following frequency model. The categories of Model 2 are calculated according subjective opinion, the tariff of model 3 is calculated by quartiles.

Table 4: Tariff analysis – frequency

Variable	Tariff model 2	Tariff model 3
Frequency	0,1,2,3	0,1,2,3
Fuel	1,2,3,4,5	1,2,3,4,5
Gender	0,1	0,1
Catgecar	(0-1;2-4;85-9;10-28)	(0-7;8-14;15-21;22-28)
Catgeman	(19-25;26-35;36-50;51-88)	(19-36;37-53;54-70;71-88)
Catrice	(15 000-100 000;-100 001-300 000;300 001-1000 000;1 000 001-6 704 000)	(15 000 –1672 250; 1 672 251-3 344 500; 3 344) (501-5 016 750; 5 016 750 - 6 04 000)
Catvolumkw	(2-10;11-100;101-200;201-265)	29-99; 100- 198; 199-298; 299-426

In the following picture we can see the histograms of two predicted frequencies. Model 2 is on the left and the deviances of both models

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Figure 3: Predicted frequency
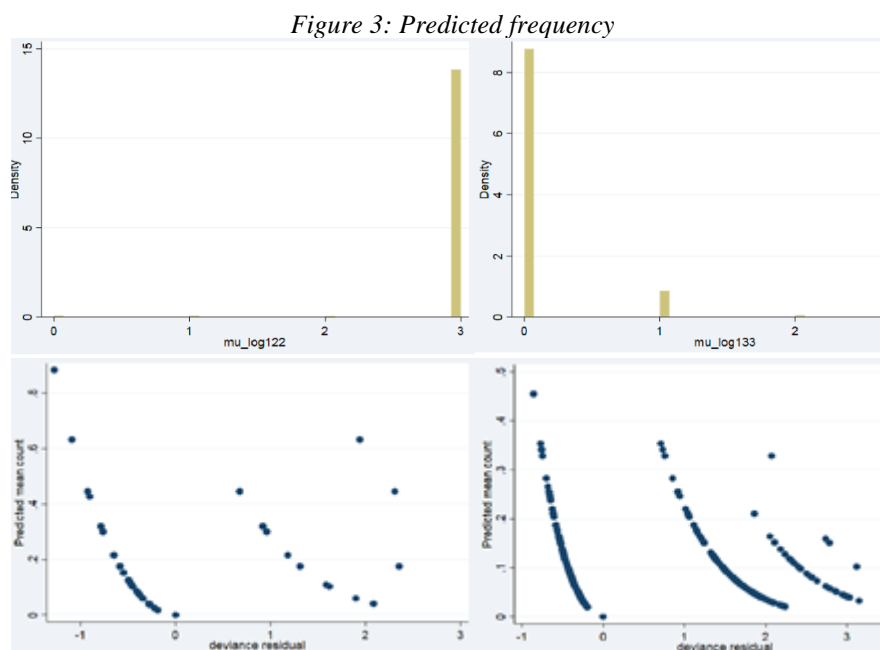
As we can see, the deviance of model 3 is lower and also predicted frequency looks like more realistic. The tariff according to quantiles is more appropriate, as we can see the histograms of two predicted frequencies. Model 2 is on the left. and the deviances of both models. In the following table is shown predicted frequency of both models.

Table 5: Predicted frequency

Number of claims	Predicted by model 2	Predicted by model 3
0	95	11,350
1	84	1,132
2	12	72
3	17,920	5,557

The value of Akaike criterion is in the case of model 2 = 5 398,434 and in the case of model 3 it is equal to 216,0583. It can be argued, that model 3 is more appropriate.

4. Conclusion

Based on the results, it was found that the creation of tariff groups is in the insurance practice very important. The formation of tariff groups can help to calculate cost of risk in individual groups. It has been shown that the tariffing according to the quartiles is more appropriate.

This article can be used as part of calculating insurance premiums. It can be applied in practice, as part of the internal risk-management model in the insurance company. The question still remains how in detail the sort of risk, since the formation of tariff groups the final result of the very affects.

Acknowledgements

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Penalized Expectiles Optimal Portfolios

Gabriele Torri[1], Rosella Giacometti[2]

Abstract

Expectiles are risk measures increasingly popular in recent years among academics and practictioners, thanks to their good theoretical properties: they are the only risk measure that is both coherent and elicitable. Moreover, they have an intutive economic explanation and interesting algebraic connections can be established with Value at Risk (V aR) and Expected Shortfall (ES). Recent works explored their usage in portfolio optimization, showing how to build optimal risk-return portfolios using expectiles as risk measure. However, real-world application to portfolios with a large number of assets are limited by estimation error, that typically leads to bad out of sample performances. In this work, we propose a novel derivation of the linear programming formulation of the minimum expectile portfolio, similar to the ones available in the literature, but computationally faster and characterized by a straightforward economic interpretation. We also introduce a ridge penalization to the portfolio weights in order to improve the finite sample performances, and we test the model on a variety of datasets.

Key words

Expectiles, portfolio optimization, regularization, linear programming

JEL Classification: G11, C44, C55, C61

1. Introduction

Expectiles can be computed as the quantity that solves the following optimization problem:

$$\xi_\tau^* = arg \min_{x \in \mathbb{R}} \left(\mathbb{E}\left[(X - \xi)_+^2 + \frac{1 - \tau}{\tau} (X - \xi)_-^2 \right] \right), \qquad (1)$$

where X is a random variable representing a financial time series, $(\cdot)_+^2 = \max(\cdot, 0)$ and $(\cdot)_-^2 = \max(-\cdot, 0)$. Following the notation proposed by [4], we can then define $EVaR_{1-\tau}(X) = -\xi_\tau^*(X)$. It can be shown that for $\tau < 0.5$, $-EVaR_{1-\tau}$ is a coherent risk measure in the sense described by [1]. Moreover, expectiles are the only coherent risk measures that are also elicitable.

2. Portfolio optimization

Here we propose a strategy to estimate the weights for the optimal minimum expectile portfolio. Our strategy consists in finding the portfolio weights that minimize a quantity $-\xi$, constraining ξ to be the expectile of the portfolio. From the definition of expectile, and since the expression in (1) is a convex function of ξ, we have that the following first order condition is necessary and sufficient to characterize the τ-expectile of X. exploiting the fact that $\mathbb{E}\left[X\left(\mathbb{I}_{X>\xi} + \mathbb{I}_{X \leq \xi} \right) \right] = \mathbb{E}[X]$, we have:

$$\frac{\partial}{\partial \xi} \mathbb{E}[(X - \xi)_+^2] + \frac{1 - \tau}{\tau \mathbb{E}[(X - \xi)_-^2]} = 0$$

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$$\frac{1 - 2\tau}{\tau T} \mathbb{E}\big[X\mathbb{I}_{X<\xi}\big] + \mathbb{E}[X] - \xi = 0, \qquad (2)$$

where $\mathbb{I}_{\{.\}}$ is an indicator function. Considering $X = Rw$, where R is a vector of asset returns, we see that by imposing (2) as a constraint, we have that the quantity ξ is the expectile of the portfolio, and we can directly minimize $-\xi$ in the objective function. Including also a budget constraint, the portfolio optimization is structured as follows:

$$\min_{w\in\mathbb{R}^n, \xi\in\mathbb{R}} -\xi$$

$$s.t. \quad \frac{1 - 2\tau}{\tau} \mathbb{E}\big[Rw\mathbb{I}_{Rw<\xi}\big] = \xi - \mathbb{E}[R]w$$

$$w'\mathbf{1} = 1.$$

Considering discrete data, it can be rewritten as a linear program by introducing T new variables $z_i, i = 1, \dots, T$:

$$\min_{w\in\mathbb{R}^n, \xi\in\mathbb{R}, z\in\mathbb{R}^T} -\xi$$

$$s.t. \quad \frac{1 - 2\tau}{\tau T} \sum_{i=1}^{T} z_i = \xi - \mathbb{E}[R]w$$

$$z_i \geq \xi - R_i w \quad \forall i$$

$$w'\mathbf{1} = 1.$$

$$z_i \geq 0 \quad \forall i$$

Note that the problem formulation is equivalent to the one proposed in a recent working paper by [3], that suggest a formulation with two sets of variables, $d_i^+, d_i^+, i = 1, \dots, T$, instead of z_i $1, \dots, T$, and only equality constraints. In practice our formulation is computationally faster using the *linprog* function in matlab 2017a.

This base formulation can then be extended by considering additional constraints such as a constraint on expected returns. In practical application however it has been shown in the literature that the estimation of expected returns is highly sensitive to estimation error, and therefore we focus on the global minimum risk portfolio.

3. Penalized portfolio optimization

Due to estimation error, the out of sample performances of optimal portfolios are typically inferior to the in-sample ones [see e.g. 7]. This well known issue has been addressed in the minimum variance portfolio optimization framework in several ways, such as covariance matrix shrinkage [12], random matrix theory filtering [11], portfolio weight penalization [8], or regularization of the precision matrix [17]. The problem is even more pronounced in portfolio optimization that focus on tail metrics, such as CVaR [see e.g. 14].

We propose here to regularize the solution by imposing a penalization on the portfolio weights. Penalization methods have been developed first in regression frameworks (see e.g.

[16] in OLS regression, [5] for quantile regression, and [13] for expectile regression). Such methods allow to reduce the effect of estimation error and, depending on the shape of the penalty, to perform model selection in the same step as model estimation (i.e. selecting only the relevant regressors, setting the coefficient of the others exactly to zero). In a portfolio setting, the penalization is applied on the portfolio weights in the objective function. Such strategies under the mean-variance optimal portfolio framework have been discussed among the others by [8, 10], and under quantile based measures by [6].

In this work, we consider a ridge penalty (a quadratic penalty), in the objective function. Considering a finite number of discrete observation, the problem becomes then:

$$\min_{w\in\mathbb{R}^n, \xi\in\mathbb{R}, z\in\mathbb{R}^T} -\xi + \lambda w'w$$

$$s.t. \quad \frac{1-2\tau}{\tau T}\sum_{i=1}^{T} z_i = \xi - \mathbb{E}[R]w$$

$$z_i \geq \xi - R_i w \quad \forall i$$

$$w'\mathbf{1} = 1.$$

$$z_i \geq 0 \quad \forall i$$

The problem is quadratic with linear constraints, and it can be solved using standard off-the-shelf solver. In particular, we use the function *quadprog* in Matlab 2017a. Finally, we see that the solution of the problem is always bounded given that the problem is quadratic, while this was not guaranteed for the unpenalized problem.

3.1 Tuning of parameter λ

A key aspect of the penalized optimization problem is related to the tuning of the parameter λ, that controls the amount of shrinkage applied to the optimization problem. In classical penalized regression framework, the λ is typically tuned using cross validation or information criteria [see e.g. 9]. For our portfolio optimization setting we propose a cross validation procedure suitable for time series, and focused on the out of sample performances of the portfolio. Adopting a rolling window approach for the optimization, we perform the cross validation over the previous 10 in sample windows as training, using the previous out of sample periods (that are then in the past) as test. We then choose the parameter λ that minimizes the average expectile computed out of sample.

Our procedure is similar in spirit to the performance based tuning proposed by [2], although it differs both in the target variable (they use sharpe ratio instead of expectile), and they select the cross validation periods differently.

4. Alternative Models

Together with the global minimum EVaR (MEVAR) and penalized global minimum EVaR (PMEVAR) portfolios, we can consider other asset allocation schemes, that focus on different risk measures. In particular, we consider the following three portfolios:

- **Equally weighted portfolio (EW):** A simple approach that assigns equal weight to all assets. It is known for its robust out-of-sample performances and it can be considered an extreme form of regularization.
- **Minimum CVaR portfolio (MCVAR):** We compute the weights that minimize the CVaR of the portfolio using the algorithm from [15].

$$\min_{w\in\mathbb{R}^n,\xi\in\mathbb{R},z\in\mathbb{R}^T} \quad \xi + \frac{1}{(1-\tau)T}\sum_{i=1}^{T} z_i$$

$$s.t. \quad z_i \geq -\xi - R_i w \quad \forall i$$

$$w'\mathbf{1} = 1.$$

$$z_i \geq 0 \quad \forall i$$

- **Penalized minimum CVaR portfolio (PMCVAR):** We consider a ridge penalized version of the previous algorithm. Thr problem is quadratic with linear constraints:

$$\min_{w\in\mathbb{R}^n,\xi\in\mathbb{R},z\in\mathbb{R}^T} \quad \xi + \frac{1}{(1-\tau)T}\sum_{i=1}^{T} z_i + \lambda w'w$$

$$s.t. \quad z_i \geq -\xi - R_i w \quad \forall i$$

$$w'\mathbf{1} = 1.$$

$$z_i \geq 0 \quad \forall i$$

By comparing the different asset allocations, we can test the performances of expectile portfolios against alternative models, and assess the effect of ridge penalization.

5. Simulation analysis

We propose a simple rolling windows analysis based on simulated data. The estimation windows have a length of 500 observations, and the asset allocation is held for 40 observation, for a total of 30 windows. In case of the penalized asset allocations, the parameter λ is tuned using the previous 10 windows with the procedure outlined above. The data are generated using a multivariate Gaussian distribution whose covariance matrix is equal to the empirical covariance of the largest 30 constituents of the S&P100 equity index. We report in Table 1 some statistics related to the out-of-sample portfolios.

We see that the performances of the penalized portfolios (PMEVAR and PMCVAR) are the best in terms of all the risk measures considered, and that they are very similar to each other. The unpenalized portfolios MEVAR and MCVAR have worse financial performances and are characterized by higher turnover (leading to higher transaction expenses), and higher gross exposures. All the portfolios in any case perform better that the equally weighted portfolio.

Table 2 reports the same analysis, performed this time with 91 assets (the data are always simulated using a multivariate Gaussian with correlation matrix equal to the one of the constituents of S&P100). We see that overall the distance between penalized and non-penalized models increase, with penalization offering greater advantages for both EVaR and CVaR portfolios. Comparing the penalized portfolios, we see that in this example characterized by

worse dimensionality, the PMEVAR portfolio obtains slightly better performances than the PMCVAR one according to all the risk measures considered. It appears therefore more robust to estimation error.

Table 1: Out of sample performances of the portfolios on simulated data (30 assets, rolling windows of 500 observations, multivariate Gaussian distribution with covariance matrix of the largest 30 assets of S&P 100 index). Tau is equal to 0.05 for all the measures.

	VaR	EVaR	CVaR	std. dev.	avg. turnover	avg. gross exp.
MCVAR	0.0177	0.0126	0.0228	0.1716	1.0546	2.6740
PMCVAR	0.0164	0.0110	0.0198	0.1644	0.4899	2.1285
MEVAR	0.0180	0.0125	0.0230	0.1688	0.8555	2.5562
PMEVAR	0.0163	0.0110	0.0197	0.1650	0.2605	1.8012
EW	0.0240	0.0170	0.0305	0.2301	0.0000	1.0000

Table 2: Out of sample performances of the portfolios on simulated data (91 assets, rolling windows of 500 observations, multivariate Gaussian distribution with covariance matrix of constituents of S&P 100 index). Tau is equal to 0.05 for all the measures.

|	VaR	EVaR	CVaR	std. dev.	avg. turnover	avg. gross exp.
MCVAR	0.0174	0.0124	0.0227	0.1603	3.5425	5.2244
PMCVAR	0.0144	0.0099	0.0179	0.1299	1.2873	2.6498
MEVAR	0.0162	0.0113	0.0201	0.1481	2.6166	4.7213
PMEVAR	0.0136	0.0095	0.0172	0.1234	0.7857	2.5224
EW	0.0227	0.0162	0.0288	0.2207	0.0000	1.0000

6. Conclusions

We introduce a novel LP formulation of the minimum EVaR portfolio optimization problem, coherent with a recent contribution to the literature. Our specification is more computationally efficient and has an intuitive interpretation. We also propose to introduce a ridge penalization to the optimization problem to improve the out of sample performances of the asset allocation, and we developed a technique for the tuning of the penalty coefficient λ. We finally consider other penalized and non penalized asset allocation techniques for comparison. A simple simulation scheme shows the performances of the model, and the advantages of the penalized model, especially in presence of bad dimensionality. The penalized minimum EVaR portfolio is characterized by better performance compared to the penalized minimum CVaR one, especially in setting with more assets.

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The Role of Financial Information in Measuring Firm-level Competitiveness for SMEs[1]

Katalin Varga Kiss, [2]

Abstract

The paper aims to investigate whether SMEs' company performance and competitiveness may exclusively be measured by financial information. The study synthetises the findings of recent research into competitiveness, financial indicators and performance measurement. The concept of competitiveness is interpreted at firm level and national level underlining their interdependence. Firm-level competitiveness indicators of different models are compared and evaluated highlighting the importance of the adequately allocated and managed financial resources resulting in competitive performance. In case of performance measurement, the paper does not only present financial indicators and underlines their importance but also depicts authors' different approaches to them. Taking into account the components of evaluated firm-level competitiveness models as well as the perspectives of the widely used balanced scorecard model, the analysis of financial metrics is also augmented by the assessment of non-financial indicators. The paper also attempts to reveal whether and to what extent SMEs' financial management practices may enhance the application of performance measurement systems.

Key words

competitiveness, financial information, SMEs, balanced scorecard, management accounting

JEL Classification: G30, L25, M20

1. Introduction

Competitiveness has gained widespread attention in economic literature over the past decades. Studies particularly focus on competitiveness at macro-level comparing national economies with each other based on various models set up by the World Economic Forum (WEF)[3] and the IMD World Competitiveness Centre[4]. However, national competitiveness and firm-level competitiveness are interrelated with each other because firm-level competitiveness serves as a basis for macro-level competitiveness, while competitiveness enhancers at macro-level may also support firm-level competitiveness.

SMEs as the backbone of the EU's economy with a key role in providing employment, increasing value added and enhancing innovation constitute a heterogeneous group in terms of size and sector diversity (Hillary, 2000 cited in Klewitz and Hansen, 2013, p.3).

According to the *Annual Report on European SMEs 2017/2018*, in 2017 SMEs in the EU-28 non-financial business sector accounted for 99.8% of enterprises, 66.4% of total employment and 56.8% of the value added generated by the business sector (p.13). The Report also reveals that "many SMEs are currently operating in sectors which are characterised by either low

knowledge or technology intensities or low export intensities: about two-thirds of SMEs (in terms of the number of SME enterprises in the EU-28 non-financial business sector) were active in either low knowledge intensive service industries or low-tech manufacturing industries" (ibid, p.19).

Similar to the European economy, SMEs in Hungary are also regarded as the main pillar of the economy providing a potential source for jobs and economic growth. However, in terms of productivity these enterprises fall behind large corporations and their development is still substantially hindered by the low level of innovation and corporate R&D, the lack of properly qualified labour force, and the absence of production processes or enterprise organisation experience (cf. Eurostat, 2011, p.20).

According literature modern financial management considers wealth maximisation of the owners as a primary objective where managers are supposed to be involved in the interrelated tasks of strategic management, operations management and risk management (Atrill, 2009, p.1) being responsible for financial planning, investment project appraisal, financing decision, capital market operations and financial control. Although this may well apply to large companies, SMEs may lag behind in a lot of aspects in this field.

Competition in a continuously changing business environment requires SMEs to monitor firm performance. Performance measurement is a complex field, which requires a lot of efforts both in terms of human capital and financial resources. While it is a common practice in case of large companies, SMEs usually face difficulties implementing it.

By highlighting the weaknesses of SMEs, especially those of micro companies in the field of strategic management accounting and performance measurement, evaluating and synthesising the recent findings of literature on firm-level competitiveness and the role of financial indicators in performance measurement the paper indicates some of the obstacles and challenges the sector is exposed to and puts forward further directions how they may develop their financial management practices.
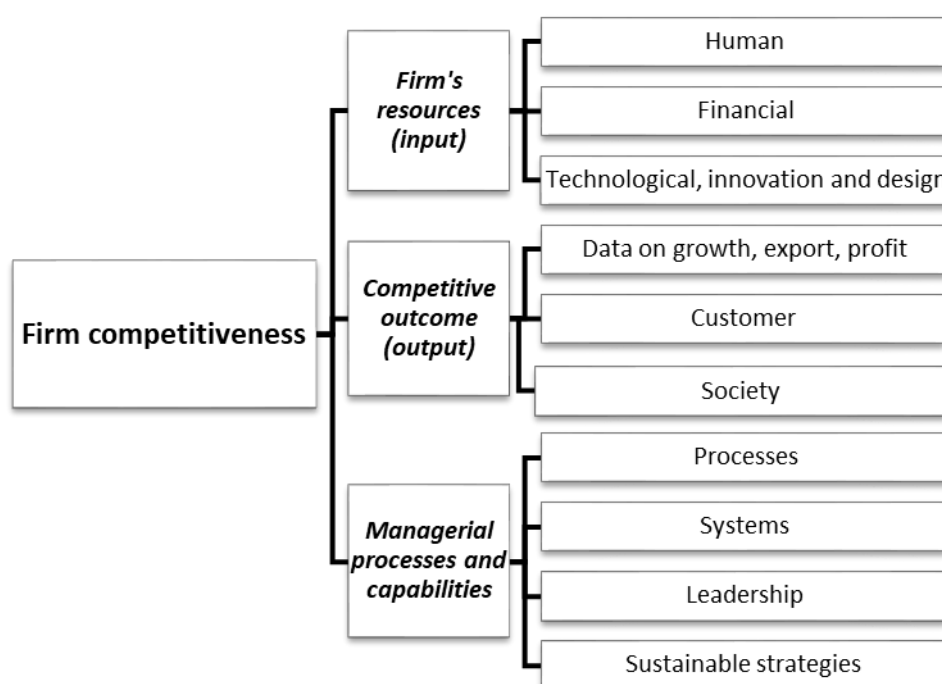
2. Firm-level competitiveness

The concept of competitiveness may be investigated both at company and national level. As competitiveness is usually measured at national level with the Global Competitiveness Index (GCI) of the WEF as a benchmarking tool, several authors (Chikán, 2008; Cetindamar and Kilitcioglu, 2013) indicate the importance of measuring firm-level competitiveness pointing to a research gap in this field. Chikán (2008, p.21) argues that "there is no competitive national economy without competitive companies and developments in national economies have a very strong influence on companies' competitiveness", while Centindamar and Kilitcioglu (2013, p.20) suggest that "it is the firms, not nations, which compete".

Firm-level competitiveness is regarded as "a capability of a firm to sustainably fulfil its double purpose: meeting customer requirements at profit" (Chikán, 2008, p.24) assuming that the goods and services offered by the firm to customers are given higher preference than those offered by the competitors. All this requires the firm to continuously adapt to changes in the economy and society. In contrast, *national competitiveness* is *"a capability of a national economy to maintain an environment for its companies and other institutions to create, utilize and sell goods and services meeting the requirements of global competition and changing social norms"* (ibid., p.24).

According to findings by Centindamar and Kilitcioglu (2013, p.21) a firm's competitiveness is based on three pillars "competitive performance (output)", "firm resources (input)" and "managerial processes and capabilities", which can be further divided into smaller units *(Figure 1)* serving as a measurement model for data collection.

*Figure 1: Measurement model for firm competitiveness*

Source: Compiled by the author based on Cetindamar and Kilitcioglu (2013, p.21)

A background document on measuring competitiveness by WIFO and ZEW[5] (2017, p.12) mentions different indicators of firm-level competitiveness such as *market share* expressed "in either physical terms (shared in the total quantity sold) or in monetary terms (share in total sales)" that may be augmented by *export share* indicating the company's ability to sell its product in a more competitive environment than its home market. Further measures include *profit margin* (the profit made per unit sold) and *return on capital employed (ROCE),* the latter of which seems to be less sector-sensitive, *market structure and growth.* While market share, profits, survival and growth are indicators of current or past competitiveness, *innovation* is regarded as a competitive potential comprising both *product* and *process innovation* measured by a firm's R&D expenditure. As regards competitiveness firm capabilities mostly involve human and organisational capital and other intangible assets that can be measured by qualitative methods. External factors are considered to be only the drivers of competitiveness but they can also hamper it, some of them may affect all the businesses in a country, e.g. legal system, inflation, exchange rate; while others may vary by regions, e.g. skilled labour and infrastructure *(Figure 2)*. The authors of the background documents also suggest taking market structure and external factors into consideration for sector-wise and international comparison, respectively.

[5] The report was completed by the WIFO: Austrian Institute of Economic Research; ZEW: Centre for European Economic Research for the European Commission.

271

Figure 2: A hierarchy of firm-level competitiveness indicators
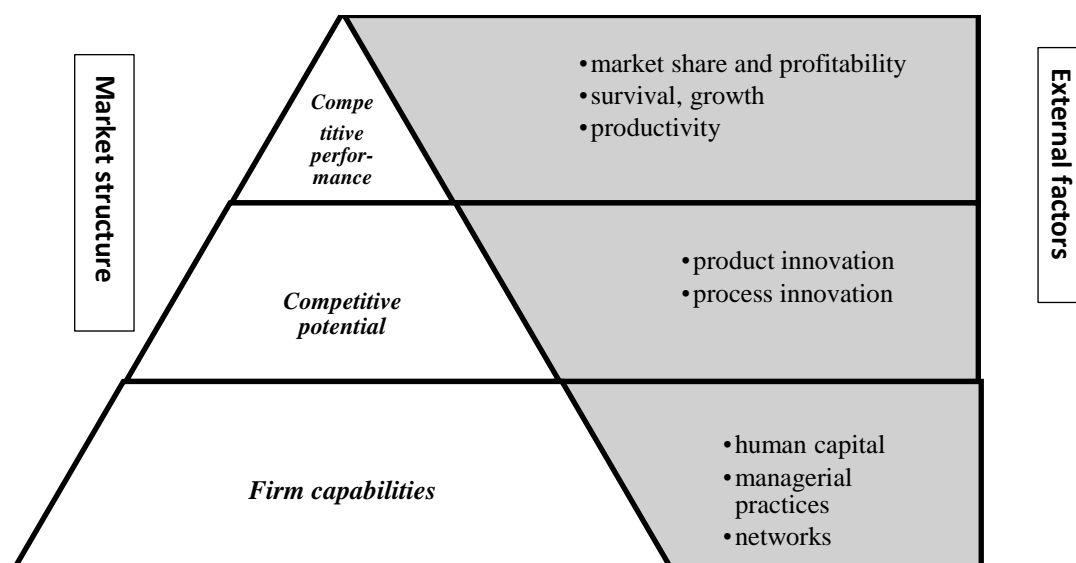
Source: WIFO and ZEF (2017, p.18)

The comparison of the two models presented by *Figure 1* and *Figure 2* reveals similarities and differences, alike. The major difference is that Centindamar and Kilitcioglu (2013) consider the firm's resources (input), the competitive outcome (output) as well as the managerial processes and capabilities as equal contributors to competitiveness. In case of WIFO and ZEF (2017) firm capabilities are regarded as drivers of competitiveness through innovation, as a result of which a firm's competitive performance can be measured by financial indicators. While the model by Centidamar and Kilitcioglu (2013) reflects a rather static approach to competitiveness, that by the WIFO and ZEF (2017) shows a dynamic approach underlining the importance of intangible assets – also viewed as non-financial indicators – as a basis for competitiveness. Although the categories applied in the two models vary from each other, the key performance indicators at sublevel tend to be similar.

In the author's view both models could be further refined. In case of the model by Cetindamar and Kilitcioglu (2013) the firm's resources may lead to competitive outcome if they are coupled with managerial processes and capabilities and financial performance is continuously monitored and measured with the help of financial information. The WIFO and ZEF model (2017) could also be augmented by the financial resources which should serve as a basis for the firm capabilities including human capital, managerial practices and networks. When adequately allocated and managed these financial resources may support product and process innovation resulting in competitive performance that can be measured both by quantitative and qualitative indicators.

3. Performance measurement

To date research into performance measurement for competitiveness has led to the creation of various models. In spite of the fact that these models are suited to the needs of large companies, they might also be applicable for smaller entities such as medium-sized companies and smaller firms where the number of employees, the amount of turnover requires the owner-manager to set up a finance department and employ a financial manager, internal accountants and bookkeepers. This is, however, not always manageable if we take the category differences per employee of the SME sector into consideration.

Several authors (Buhovac and Groff, 2012 cited in Waśniewski, 2017, p.216) point out that SMEs do not seem to pay much attention to measure their performance or even realise its potential benefits. In order for a company to become or remain competitive on the market the evaluation of performance and the effective utilisation of resources are highly important. This is, first of all, based on analysing financial performance as an indicator of a firm's financial soundness and profitability. To measure and improve company performance financial management should not merely rely on information based on financial accounting but also take the results of management accounting into consideration, which – over the past decades – has developed a strong strategic perspective "supporting strategic plans and decisions within the business" (Atrill and McLaney, 2009, p.318).

While information by financial accounting provides a broad view of performance and aggregated results, information by management accounting is both of financial and non-financial nature with considerable details supporting management decisions (ibid., p. 30). The analysis and interpretation of financial statements with the help of horizontal and vertical analysis, as well as, financial performance metrics may contribute to the prediction of the future based on past performance. Preparing projected financial statements establishing and analysing key external and internal variables also help reveal the probable financial effect of implementing strategic objectives (Atrill, 2009; Weaver and Weston, 2001).

3.1. Financial indicators of company performance

In spite of the fact that traditional performance models are often criticised because they focus on financial measures (Garengo, Biazzo and Bititci, 2005; Brem, Kreusel and Neusser, 2008), Garengo, Biazzo and Bititci (2005) draw attention to the importance of financial ratios for SMEs as they reveal limited financial resources that may be regarded as a hidden risk factor.

Financial performance can be estimated or calculated with various tools but each measure draws or portrays a different aspect of financial performance (Vohra and Dhillon, 2014, p.7). SMEs financial profitability is the conceived result of financial management practices (Dess, Lumpkin and Eisner, 2012 cited in Vohra and Dhillon, 2014, p.6).

Financial measures are direct reflections of current profitability and operating efficiency functioning as a "dashboard" to monitor and continually enhance the firm's financial performance (Simons, 1990 cited in Teeratansirikool, et al., 2013, p.180). Maverick (2016) suggests considering a number of financial metrics to accurately evaluate the financial health and long-term sustainability of a company including liquidity, solvency, profitability and operating efficiency, out of which he regards profitability as the best measurement of a company's health. However, he warns that standalone numbers such as total debt or net profit are less meaningful than financial ratios that connect and compare the various numbers on a company's balance sheet or income statement. As a result, it is highly important to examine whether financial ratios are improving or deteriorating over time.

Analysts and investors tend to focus on return on equity (ROE) as their primary measure of company performance. However, Hagel III, Brown and Davison (2010) point out that this may cause a lot of problems as companies may apply "financial strategies to maintain a healthy ROE for a while", as a result they support return on assets (ROA) as another bottom-line metric for corporate financial performance.

In their study *Indicators of Successful Companies* Johnson and Soenen (2003 cited in Petkovič and Rac, 2010, p. 1291) define different indicators which have bigger or smaller influence on the performance including (1) book-to-market ratio, (2) total assets, (3) sustainable growth rate, (4) capital structure, (5) liquidity, (6) cash conversion cycle, (7) earnings volatility, (8) profitability, (9) research and development expenditure and (10) advertising expenditure.

Akben-Selcuk (2016) uses the firm's financial performance as a proxy for its competitiveness with the following financial ratios: ROA, gross profit margin and Tobin's Q (TQ) and comes to the conclusion that several firm specific variables like leverage, liquidity,

size, exports, R&D and growth have a significant impact on the financial performance of the companies.

As indicated above, financial indicators based on accounting information provide a wide scope of financial metrics that should be suited to the strategic needs of the company. However, recent research indicates that in addition to the application of the traditional financial ratio analysis to reveal company performance, financial analysis should take the strategic and economic developments for the firm's long-run success into consideration (Weaver and Weston, 2001, p. 139)

3.2. Non-financial indicators of company performance

Firm-level competitiveness models analysed in *Section 2* of the paper have already indicated that competitive output (referring to measurable quantity) or outcome (denoting performance) may also be affected and measured by non-financial indicators.

The Balanced Scorecard (BSC) model by Kaplan and Norton (1996) – the most popular in the literature and practice – shifts focus from the sole financial perspective integrating non-financial indicators with a focus on customer, internal process, innovation and learning perspectives. Analysing the BSC model Mackay (2005, p.13) indicates that the concept of using a balanced portfolio of both financial and non-financial measures does not detract from the importance of financial outcomes since financial results have their own message, which is also underpinned by the fact that "even Kaplan and Norton (1996) see the financial quadrant as acting as the focal point or culmination of all the objectives and measures in the other three scorecard quadrants".

Non-financial measures can help managers to cope with an uncertain environment: the greater the uncertainty of the environment, the greater the extent to which non-financial measures are likely to be of value (Atrill and McLaney, 2009, p.396). R&D, human resources, product and service quality, market share are some of the areas covered by non-financial measures (ibid, p.397) Arnold (2005, p.146) suggests that companies' competitive edge is the result of some of their extraordinary resources with the help of which when combined with ordinary resources competitors may be outperformed. His TRACK system of extraordinary resources (ibid., p. 152) which – except for companies' tangibles (T) – also denote non-financial indicators such as relationships (R), reputations (R), attitude (A), capabilities (C) and knowledge (K) and cannot be measured by monetary terms, are still "important drivers of value creation".

As opposed to financial indicators which may also be referred to as 'lag' indicators, as they show the results of management decision activities (Atrill and McLaney, 2009, p.334), literature describes non-financial measures as leading indicators and drivers of future performance supporting the understanding of business results and set the path to improve financial metrics. Customer satisfaction, market share, company reputation and innovation are regarded as the major non-financial indicators, although in the view of some experts even competitiveness may belong to this group. Providing a systemic analysis of previous research into the nature of non-financial indicators Kotane and Kuzmina-Merlino (2011, p.218) conclude that non-financial indicators reflect the individual elements of company's intellectual capital. Compared with financial information expressed by quantitative terms, non-financial information is in most cases qualitative.

4. Obstacles and opportunities for SMEs to use performance measurement systems

Even if most companies realise the importance of financial and non-financial performance measures, they fail to include them in a balanced framework (Gunasekaran, Patel and

McGaughey, 2004). Maskell (1991 cited in Gunasekaran, Patel and McGaughey, 2004, p.335) suggests that companies should understand that, while financial performance measurements are important for strategic decisions and external reporting, day-to-day control of manufacturing and distribution operations is often handled better with non-financial measures.

The author of the paper shares the view of Madsen (2015, p.3) that as opposed to larger firms, small companies primarily focus on financial measurement with very little or almost no attention to the measurement of non-financial issues, thus fail to apply integrated performance measurement system (PMS). The reasons may range from the role of the owner, the management practices, through mentality and orientation to organisational structure and resource availability *(Table 1)*.

Table 1: A comparison of SMEs and large firms

	LARGE FIRMS	SMEs
Role of owner	Limited role Little contact with employees	Owner-manager (boundary spanning role) Close contact with employees
Management process	Formal and impersonal	Informal and personal
Strategic process	Formal and structured	Informal and fluid
Mentality	Preventive (proactive)	Fire-fighting (reactive)
Orientation	Long-term	Short-term (survival and making ends meet)
Organizational structure	Tall and bureaucratic	Flat and flexible
Resource availability	Organizational slack	Resource limitations and time constraints (e.g. finances, human resources)

Source: Madsen (2015, p.3)

SMEs can clearly obtain value from PMS but there are significant barriers to implementation lying in resource limitations and the fact that it can be too strategically orientated (Hudson, Smart and Bourne, 2001). In comparison with large organisations, SMEs are fundamentally different in three aspects: uncertainty, innovation, and evolution (Garengo, Biazzo and Bititci, 2005). Furthermore, research into performance measurement systems for SMEs indicates that SMEs do not use PM models or if they do, they use it incorrectly.

Brem, Kreusel and Neusser (2008) state that there has been no performance measurement system to date that is widely accepted as being especially developed for small-to-medium enterprises. Even the BSC is criticised for not being suited to implementation in SMEs (Garengo, Biazzo and Bititci, 2005). Still, the greatest benefit of performance measurement systems like the BSC is that they draw attention to measuring non-financial indicators[6] such as consumers' loyalty, quality of the products/services, motivated employees, loyal employees, development of new products/services, training of employees, and company's reputation.

In case of SMEs the issue of performance measurement may form a less coherent picture as a result of the huge difference in their size categories of SMEs. Due to their small size (less than 10 persons) micro companies[7] have difficulty implementing strategic financial management. This can be explained by the lack of management level and the fact that decisions are usually based on intuitions and are of a subjective nature. The owner who also takes the role of the manager is fully engaged in the daily operational management and lacks the knowledge of how to use and interpret financial information. The occupation with operational tasks, as well

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[6] Non-financial indicators listed are based on research by Kotane, 2015, p.232

[7] See: Micro-, small- and medium-sized enterprises: definition and scope. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=LEGISSUM%3An26026

as, the lack of management skills and finance knowledge are supposed to be some of the obstacles why micro companies may not develop and change size class.

Micro companies and small enterprises[8] employing only slightly more staff than micro companies usually have an external accountant who records the accounts, performs double-entry bookkeeping, prepares abbreviated versions of financial statements according to the legal provisions and provides firms with basic financial information. The role of the manager is usually taken by the owner, who meets the accountant on a monthly basis to submit the accounts for computing VAT payable. Most accountants only have expertise in financial and tax accounting. As a result, micro and small companies are without the expertise of management accounting, which may identify, measure, analyse, interpret, and communicate information to managers to achieve the company's goals. Also, owners are not aware of the importance of more sophisticated use of financial information, such as capital budgeting and investment appraisal methods, the application of Enterprise Resource Planning (ERP) systems.

Small enterprises employing a higher number of staff close to the size category of medium-sized companies usually employ an internal accountant with some finance staff. In case of medium-sized enterprises to measure company performance management accounting seems highly recommended, as it is able to "perform managerial functions of planning, controlling and decision-making in an effective and efficient manner" by collecting, processing and analysing data from external and internal sources and communicating those to the management (Bhattacharyya, 2011, p.1).

Expertise in financial management is a pre-requisite for firm-level competitiveness. In case of SMEs its presence, however, seems to be of various extent and focus as a result of the sector's specificity.

5. Conclusion

The paper analysed the role of financial information in measuring company performance in the case of SMEs. Interpreting the concept of competitiveness at firm and national level, the author accepts the results of previous research stating that firm-level and national-level competitiveness are interdependent. Neither of them can exist on their own. Firm-level competitiveness may be illustrated by different approaches comprising both financial and non-financial indicators. These approaches should regard financial resources as a basis for firms' capabilities including human capital, managerial practices and networks may bring about a competitive output or outcome that can be measured by quantitative and qualitative metrics.

As SMEs play a significant role in the EU's economy, enhancing their competitiveness is a priority according to the EU's SBA[9] policy. Sound financial operation is a prerequisite for competitiveness, and it should be supported by appropriate performance measurement.

Based on findings in literature the study comes to the conclusion that the existing performance measurement systems seem not to comply with the needs of SMEs. It has also been revealed that company performance, first of all, may be judged on financial information. This, however, shall be interpreted in a broader context taking non-financial indicators into consideration. In addition, business owners should be encouraged to improve their financial skills not to be solely dependent on accountants' information. SMEs should be made aware of the importance of management accounting practices and ERP systems that may assist them to sustain and become competitive.

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[8] Small companies employing 10-49 persons. The size categories of SMEs are based on the SME definition of the European Commission. See: http://ec.europa.eu/growth/smes/business-friendly-environment/sme-definition/

[9] See: The Small Business Act for Europe. https://ec.europa.eu/growth/smes/business-friendly-environment/small-business-act_en

Regarding larger scale small enterprises and medium-sized companies measures should involve a change in organisational structure and operational activities employing finance staff with an expertise in management accounting and the application of a software package that suits to the sectoral specificity and size of the company. In case of micro enterprises owners should acquire and broaden their knowledge of business finance and management, which would enable them to get the relevant pieces of financial information from their external accountants for setting objectives and financial planning. In addition, digitising their companies to store and retrieve data may also encourage them to manage their business processes more efficiently.

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Minimum Risk Portfolio Optimization

Anlan Wang [1]

Abstract

The classical Mean-Variance model has been widely used to solve the portfolio optimization problem with the considerations of the inter-relationships between return and risk. In this paper we apply the Mean-Variance optimization method as well as the Mean-CVaR optimization method to obtain minimum risk portfolios based on the chosen data. To verify the efficiency of the obtained strategy portfolios, we also generate random-weights portfolios and make the hypothesis tests. The risk performance measure in the hypothesis tests is the Maximum Drawdown. According to the empirical analysis results, we conclude that for both of minimum variance portfolio and minimum CVaR portfolio, they work efficiently comparing to the random-weights portfolios under the 2007-2008 financial crisis.

Key words

Portfolio optimization; Mean-Variance model; Mean-CVaR model; Minimum risk portfolios; Financail crisis; Hypothesis tests; Random-weights portfolios; Maximum drawdown

JEL Classification: C12, G11

1. Introduction

The trading market is always full of opportunities and risks, on the premise of liquidity and security of investment funds, the most common investment strategy is to invest in a portfolio consists of different securities to spread the risk, and minimizing the risk of portfolio is always one of the most important objectives for the risk-averse investors.

The goal of this paper is to verify the efficiency of the obtained minimum risk portfolios based on the chosen dataset. There are two portfolio optimization methods are applied in this paper. The first one is the Mean-Variance optimization method, the second one is the Mean-CVaR optimization method. To make our research practical significant, we choose the dataset which covers the 2007-2008 financial crisis in the empirical analysis.

The paper is divided into 5 sections. In following section, we make a literature review of the applications of benchmarks in pioneers' researches. The theoretical basis of this paper is introduced in section 3. To verify the efficiency of the applied optimization methods in this paper, the empirical analysis is made in section 4. In section 5, we conclude the paper.

2. Literature review

Portfolio optimization involves the efficient investment strategy considering optimal allocation of limited funds. Markowitz (1952) proposed the Mean-Variance model. In the Modern Portfolio Theory, it states that a portfolio's expected return and risk should not be viewed alone, moreover, a higher degree of risk means a higher potential return. Along with the considerations of real-life conditions and enhancements of algorithms, additional new constraints have been developed to the early classical Markowitz model in the later studies.

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What's more, rather than only the single-objective Mean-Variance model is applied, the multi-objective optimization models have also attracted attentions since financial crisis.

According to the scientific publications of portfolio optimization problem analysis, we find that different models have been proposed to address the investors' various needs, and in these researches, the applied performance measures indicate that the models fit their experimental data well based on specific dataset. However, at the point of scientific precision, measuring the performance of the proposed optimization model is just one side of the work, for another side, it's significant to involve a benchmark in the analysis to verify the efficiency of models. In this section, a review on the applications of benchmarks in the portfolio optimization problems are studied. A total number of 30 published research articles are selected from the website of *ScienceDirect* according to the proposed keyword " Portfolio Optimization" in this review. We make the statistics of the benchmarks applied in these papers, the details are shown in Fig. 1.

Figure 1: Statistics of the benchmarks applied in the reviewed articles
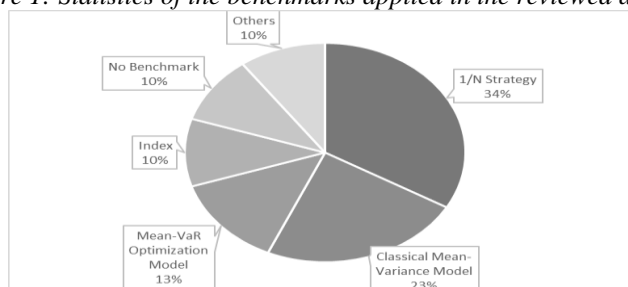
From the chart we can see the most used benchmark in the portfolio optimization researches is the 1/N strategy, which is also named as naive diversification strategy. The 1/N strategy is easy to implement because it does not rely on estimations of the asset returns, the components assets of the naive portfolio are invested at equal weights. DeMiguel, et al. (2009) evaluated and compared the performance of several optimization methods with respect to the performance of the 1/N strategy, they found that the effect of estimation error on return probability distribution is large in those optimization models, but this type of error can be avoided by using the 1/N weights. Owing to these good points of 1/N strategy, Xidonas, et al. (2017) applied the uniform 1/N strategy as a benchmark to gauge the robust allocation strategy performance in their research. Levy, H. and Levy, M. (2014) suggests two new portfolio optimization methods: The Variance-Based Constraint (VBC) method and the Global Variance-Based Constraint (GVBC) method, in their research they optimize the portfolio by allowing the weights of assets to deviate more from the "zero information" benchmark-1/N weight.

The second most used benchmark is the classical Mean-Variance model. By considering real-life conditions, the classical model has been improved with additional constraints. However, the classical model is still widely used as a benchmark in the researches. Fulga (2016) presented an approach which incorporates loss aversion preferences in the Mean-Risk framework, and the efficiency of this new approach is tested against the classical model.

Another common used benchmark is the Mean-VaR optimization model. Ranković, et al. (2016) proposed a novel approach to mean-VaR portfolio optimization method when VaR is estimated by the analytical univariate GARCH VaR, to illustrate the effectiveness of the proposed Univariate-GARCH-VaR model, they compared the results with two benchmarks, the first one is the Mean-Historical-VaR model, the second one is the Mean-Multivariate-GARCH-VaR model. Lwin, Qu and MacCarthy (2017) also replaced the risk measure variance of the Markowitz model into VaR in order to better assess market risk exposure, in their work an efficient learning-guided hybrid multi-objective evolutionary algorithm (MODE-GL) is proposed to solve mean-VaR portfolio optimization problems with real-world constraints, the optimization method with proposed algorithm is compared with the benchmark, which is the Mean-VaR model combined with the Non-dominated Sorting Genetic Algorithm (NSGA-II).

Similar to this research, the Mean-VaR portfolio optimization model incorporated NSGA-II is also used as a benchmark in the research of Babazadeh and Esfahanipour (2019).

In our review, we find there are also some researchers have applied the indices as benchmark to compare the performance of proposed methods. For instance, Solares, et al. (2019) used the dataset of 13 years' historical monthly prices of stocks in the Dow Jones Industrial Average index (DJIA), and they also made an extensive evaluation comparing the performance of the proposed approach with respect to the DJIA index, interestingly, in their research they pointed out that the main contraindication of using market indices as benchmark is that the profitability of portfolios is often compared to that of popular indices, so most investors expect to reach or exceed the yields of these indices over time, but the problem with this expectation is that they are at a disadvantage because no guarantee that the characteristics of the stocks in their portfolio coincide with that of the stocks contained in the index, so, to avoid this trap, it is recommended to incorporate into the portfolio only the stocks of the index being considered as the benchmark. Chandrinos and Lagaros (2018) also compared the performance of their proposed currency portfolio optimization strategy with the well-known indices-S&P 500 and Barclay CTA.

Rather than the types of benchmark mentioned above, Zhang, Jin and An (2017) used the non-robust strategy as a benchmark in their paper investigating the optimal portfolio choice in the presence of transaction costs and ambiguity aversion. Kumar and Mishra (2017) presented a novel co-variance guided Artificial Bee Colony algorithm for portfolio optimization problem, and its efficiency is tested on the portfolio optimization benchmark from the OR-library.

However, from Fig. 1, we can see in some papers the application of benchmark is missing, which is not rigorous from the basis of scientific evaluation, because the verification of a new finding is a significant step in the research work, which tests the efficiency of the proposed approaches when applied in the real-life.

To evaluate the performances of the proposed approaches in our research, in this paper, firstly we make the performance measurements of the obtained portfolios; Secondly, to verify the efficiency of the methods applied, we also generate random-weights portfolios in our case to make hypothesis tests by comparing their performances with those of the strategy portfolios.

3. Minimum risk portfolio selection

Portfolio optimization aims at a portfolio with maximum return or with minimum risk exposure, for risk-averse investors, minimizing the risk of a portfolio is a significant objective. In this paper, we apply the Mean-Variance optimization method and the Mean Conditional Value at Risk (henceforth Mean-CVaR) optimization method to the minimum risk portfolio selection problem.

3.1 Mean-Variance optimization method

Mean-Variance model is based on the framework of analyzing the inter-relationships between return and risk in a portfolio. We denote x_i as the weight of asset i in a portfolio investment, and in our case, we exclude short sales, so the values of x_i satisfy $x_i \geq 0$ for all assets. We suppose that the expected stock return is identical to the average of the historical stock returns within the chosen period according to Zmeškal, et al. (2004). If we denote $E(R_i)$ as the expected return of asset i in the sample period, then the expected return of a portfolio $E(R_p)$ is the weighted average of $E(R_i)$.

$$E(R_p) = \sum_{i=1}^{N} x_i \cdot E(R_i) = x^T \cdot E(R) \tag{1}$$

where N is the totoal number of assests in a portfolio, $x = [x_1, x_2, ..., x_N]^T$ and $E(R) = [E(R_1), E(R_2), ..., E(R_N)]^T$. The sum of x_i in a portfolio equals to 1.

The Mean-Variance model regards portfolio's variance or standard deviation as the risk measure, and they are calculated by the covariances $\sigma_{i,j}$ of the component assets for all asset pairs (i, j), we denote a $N \times N$ matrix as Q, and $Q = [\sigma_{i,j}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, N]$, we show the calculations of variance $\sigma_p{}^2$ and standard deviation σ_p of a portfolio separately in equation (2) and equation (3), where the standard deviation is the square root of variance.

$$\sigma_p{}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot \sigma_{i,j} \cdot x_j = x^T \cdot Q \cdot x \tag{2}$$

$$\sigma_p = \sqrt{\sigma_p{}^2} \tag{3}$$

From the Mean-Variance model, we know that if an investor requires higher expected return, it means he or she must take on higher risk. Based on the assumption of risk aversion, if the investors meet the situations when the risk levels are different while the value of expected return is fixed, investors prefer to choose the portfolio with the minimum risk. In order to achieve this goal, the efficient frontier is applied.

The efficient frontier is the set of efficient portfolios that shows the highest expected return for a given level of risk and the lowest risk for a given level of expected return, to satisfy these conditions, we can construct an efficient frontier of Mean-Variance model based on the mathematical formulation (4),

$$v = \begin{cases} \arg\min_{x} k \cdot x^T \times Q \times x - (1 - k) \cdot r^T \times x \\ \\ \sum_{i=1}^{N} x_i = 1 \\ \\ x_i \geq 0, i = 1, \ldots, N \end{cases} \tag{4}$$

in this formultion, v is the generated weights, r is the expected returns matrix of stocks, and the value of k is minimum in the portfolio of the lowest point on the efficient frontier and it increases as the expected return of portfolio increases.

As the lowest point on the Mean-Variance model's efficient frontier, a Minimum Variance portfolio which does not require the forecast of mean returns has become a popular investment alternative, it consists of volatile assets with low correlations to each other. According to Haugen and Baker (1991), it is demonstrated that the Minimum Variance portfolio outperforms the market portfolio with greater average return and less risk.

3.2 Mean-CVaR optimization method

Under the Mean-Variance model's framework, the risk measure is the variance or standard deviation of the portfolio's expected returns, however with the development of the measurements of financial risk, vale at risk (henceforth VaR) has also been widely used. VaR is defined as a worst-case loss associated with a probability and a time horizon. In this paper, rather than the use of VaR, the mean CVaR which indicates the average of VaR (or the expected loss under the condition of exceeding VaR) is applied as the risk measure, because minimizing the value of VaR in the portfolio optimization problem may stretch the tail exceeding VaR according to Larsen, et al. (2002).

The CVaR for a portfolio which is also known as the expected shortfall, it's defined as follow,

$$CVaR_\alpha(x) = \frac{1}{1 - \alpha} \int_{f(x,y) \geq VaR_\alpha(x)} f(x, y) p(y) dy \tag{5}$$

where x is a portfolio satisfies $x \in X$ (X is the set of available portfolios), α is the probability level such as that $0 < \alpha < 1$, $f(x, y)$ is a loss function for a portfolio x and asset return y, $p(y)$

is the probability density function for asset return y, VaR_α is the VaR of portfolio x at probability level α and it can be defined as follow,

$$VaR_\alpha(x) = \min\{\gamma : \Pr[f(x, Y) \leq \gamma] \geq \alpha\} \tag{6}$$

an alternative formulation for CVaR has the form as below,

$$CVaR_\alpha(x) = VaR_\alpha(x) + \frac{1}{1-\alpha} \int_{R^n} \max\{0, (f(x, y) - VaR_\alpha(x))\} p(y) dy \tag{7}$$

the probability level α is set to 0.95 in the empirical analysis of this paper, the choice of α implies that the $VaR_\alpha(x)$ for portfolio x is the portfolio return such that the probability of portfolio returns falling below this level is $1 - \alpha$. Given $VaR_\alpha(x)$ for a portfolio x, the CVaR of the portfolio is the expected loss of portfolio returns above the VaR return.

We also construct the efficient frontier of Mean-CVaR model based on the mathematical formulation, we show this as below,

$$v = \begin{cases} \arg\min_x k \cdot CVaR_\alpha(\mathbf{R} \times x) - (1-k) \cdot E(\mathbf{R} \times x) \\\\ \sum_{i=1}^N x_i = 1 \\\\ x_i \geq 0, i = 1, \ldots, N \end{cases} \tag{8}$$

similar to the statement of the construction of Mean-Variance efficient frontier, the lowest point on the Mean-CVaR efficient frontier indicates the portfolio with minimum CVaR, and the value of k is minimum at this lowest point portfolio, k increases as the expected return increases.

3.3 Hypothesis tests

To evaluate the performances of the strategy portfolios under the applied portfolio optimization methods, we calculate the strategy portfolio's maximum drawdown (henceforth MDD) of wealth evolutions, for the profitability measures, we caluculate strategy portfolio's Sharpe ratio (henceforth SR) and average annual return (henceforth AAR).

However, although the applied methods are committed to optimize the portfolios, as we mentioned in section 2, we still need to verify the efficiency of strategies under the optimization methods. So, to make the verification conclusive, not only the performances of the strategy portfolios are evaluated independently, the random-weights portfolios are also generated in our case to make the hypothesis tests by comparing their performances with those of the strategy portfolios. The performance measure applied in the hypothesis tests is MDD. As we know, MDD indicates the maximum loss from a peak to a trough of an investment's wealth evolutions up to time T, so, the smaller the MDD, the better performance of the portfolio.

As it literally means, the weights of assets in each random-weights portfolio are generated randomly, in our case we set up 50,000 random-weights portfolios, and in each portfolio the sum of weights equals to 1. We know that a hypothesis test relies on the method of indirect proof, Arson (2007). That is, to prove the hypothesis that we would like to demonstrate as correct, we show that an opposing hypothesis is incorrect. In our case, the strategy portfolios under the optimization methods are more likely to be demonstrated as efficient, so according to the rule of hypothesis tests, we can make the null hypothesis and alternative hypothesis as belows:

null hypothesis—H_0: $MDD_s = MDD_r$,
alternative hypothesis—H_A: $MDD_s < MDD_r$.

MDD_s is the MDD of wealth evolutions of strategy portfolio, and MDD_r is the MDD of wealth evolutions of random-weights portfolio. In our hypothesis test, the p-value is the proportion of the random-weights portfolios which meet $MDD_s = MDD_r$. We set the significance level at 10%, if p-value $< 10\%$, we reject H_0, which means the performance of strategy portfolio is better than

that of the random-weights portfolio, so the strategy is efficient; if p-value $\geqslant 10\%$, we fail to reject H_0, which means the performance of strategy portfolio makes no difference from that of the random-weights portfolio, so the strategy is inefficient in this case.

4. Empirical Analysis

In this section the empirical analysis is made by applying the methods introduced in section 3. We obtain the effcient potfolios and measure their performances, we also generate random-weights portfollios to make the hypothesis tests to verify the efficincy of the obtained portfolios.

4.1 Input data

The dataset is the daily closing prices of the components stocks of Dow Jones Industrial Average (DJI). There are 29 stocks included in our analysis, and the missing one is the stock of Visa Inc. due to the incomplete data in the chosen period. The time duration is 3 years. We divide the whole sample evenly into two periods, the in-sample period is from March 7, 2006 to August 31, 2007 and the out-of-sample period is from September 4, 2007 to March 2, 2009. In the applications of Mean-Variance optimization method and Mean-CVaR optimization method, we both obtain optimal portfolios based on the in-sample data, from Fig. 2, we can see in the in-sample period the DJI price shows an increasing trend. In the out-of-sample period, we make the back-tests of the obtained portfolios, we find the price keeps decreasing in this period because of the cover of the 2007-2008 financial crisis, in this sense we can test the introduced minimum risk portfolio selection methods effectively. As a comparison to the applied optimization methods, a naive strategy portfolio is also constructed. The naive strategy portfolio and the random-weights portfolios are both analyzed only based on the out-of-sample data. We assume the initial wealth to be 1 dollar in all portfolio investments.

Figure 2: Historical price evolutions of DJI
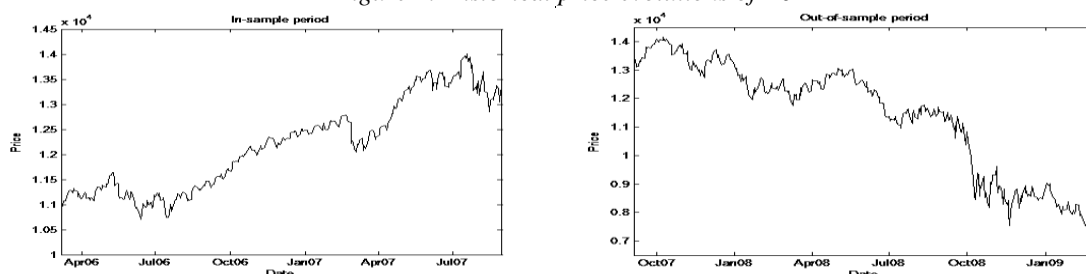
4.2 Mean-Variance method & Mean-CVaR method

The Mean-Variance efficient frontier and Mean-CVaR efficient frontier are both constructed based on the in-sample data, and for each efficient frontier there are 10 efficient portfolios are constructedon it. We show them separately in Fig. 3 and Fig. 4.

Figure 3: Mean-Variance efficient frontier   *Figure 4: Mean-CVaR efficient frontier*
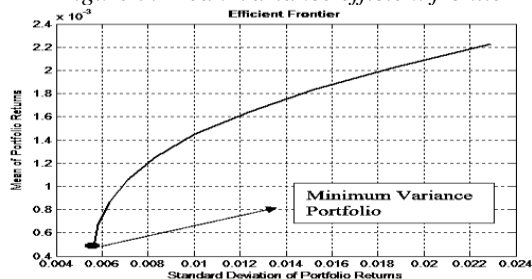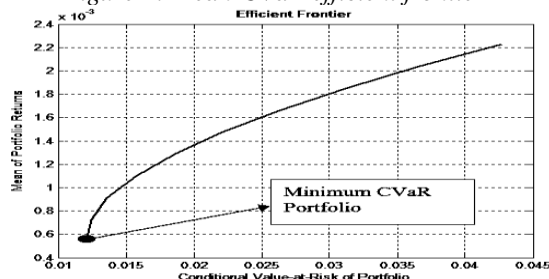

From Fig. 3, we can see the under the Mean-Variance optimization method, the mean values of daily portfolio returns are between 0.048% and 0.222%, the standard deviation of daily portfolio returns are between 0.565% and 2.292%. In Fig. 4, by applying the Mean-CVaR optimization method, on the efficient frontier, we can see the mean values of daily portfolio

returns are between 0.054% and 0.222%, the CVaR of daily portfolio returns are between 1.206% and 4.273%. For both of Mean-Variance effcient frontier and Mean-CVaR efficient frontier, their lowest point has the minimum risk. We make the back-tests of the minimum-variance portfolio and minimum-CVaR portfolio in the out-of-sample period, the performances of these two efficient portfolios are shown in Table 1. We can see the values of all profitability measures in strategy portfolios are negative due to the financial crisis during the out-of-sample period, and we also find that the value of MDD of minimum variance portfolio is lower comparing to the minimum CVaR portfolio and naive portfolio. The absolute value of AAR of the naive strategy portfolio is the highest, it might cause a high loss because it's negative.

Table 1: Efficient portfolios' performances of out-of-sample period

	Minimum Variance portfolio	Minimum CVaR portfolio	Naive portfolio
E(Rp)	-0.079%	-0.077%	-0.130%
σ_p	1.661%	1.708%	2.234%
AAR	-24.948%	-24.559%	-37.764%
SR	-156.44%	-152.080%	-118.62%
MDD	35.418%	36.058%	47.159%

4.3 Hypothesis tests by using MDD

In the hypothesis tests, we use the MDD as the performance measure. We show the tests results of p-values in Table 2, and according to the calculations, we also know the critical value of the tests is 42.777%. Based on the hypothesis assumed in section 3, we find that for both of minimum variance portfolio and minimum CVaR portfolio, we reject H_0, which means these two strategy portfolios perform better than the random-weights portfolios in the out-of-sample period, while for the naive strategy portfolio, the conclusion is contrary.

Table 2: Hypothesis tests by using MDD

	Minimum Variance portfolio	Minimum CVaR portfolio	Naive portfolio
p-value	0.050%	0.084%	47.812%

5. Conclusion

The objective of this paper is to verify the efficiency of chosen strategies obtained by applied portfolio optimization methods. The chosen sample period covers the 2007-2008 financial crisis, so minimizing the risk of portfolios based on this dataset has a practical significance. In the empirical analysis, we obtain the minimum variance portfolio and the minimum CVaR portfolio in the in-sample period, and then we make the back-tests of these two portfolios in the out-of-sample period. The performance of naive strategy portfolio is also measured as a comparison. On one hand, we find that all the obtained portfolios result in losses in the out-of-sample period due to the decreasing price evolutions, on the other hand, from the hypothesis tests, we find that the minimum variance portfolio and the minimum CVaR portfolio are both efficient, which means if people invest in the 29 stocks of DJI followed by these two portfolios, there are high probabilities to take less downside risk than investing in the random-weights portfolios.

Acknowledgements

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Application of the grey relation systems in multi-attribute decision making and forecasting in finance

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Abstract

Objective of the paper is to verify Grey relation analysis methods and compare with known models. The first part is devoted to MADM (multi-attribute decision methods) and the second one to grey forecasting methods. Localization GRA and globalization GRA multi-criteria models are described. The forecasting GRA(1,1) model based on cumulative data is described and derived. The models are verified on the data set. It was found out that localization GRA model and TOPSIS, both constructed on similarity measures, present similar and comparable results. Globalisation model based on pair-wise comparison of alternatives is different and ranking of alternatives is fully different. The forecasting model GRA(1,1) is computed and compared with the growth model. Results show that the growth model is the trendy model and non-trendy series overestimate or underestimate, so GRA model looks more robust.

Keywords

Multi-attribute decision making, grey relation analysis, grey relation grade, TOPSIS, grey forecasting, growth prediction model

JEL Classification: C4, C02, G 3, G11,

1. Introduction

Financial decision-making and management is a crucial problem in economic systems managing. There is a scale of methods applicable. Multiple attribute decision making (MADM) and forecasting methods are frequently applied. There are many references concerning of MADM. As examples of survey monographs we can introduce, Hwang a Yoon (1981), Chen and Hwang (1992), Fiala (1997), Triantaphyllou and Sánchez (1997), Černý and Glükaufová (1997), Ramík (1999), Saaty (2000), Tzeng and Huang (2011), Pomerol and Sergio (2012). Well-known methods are e. g. utility function methods WAM, AHP, ANP, similarity methods TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution), VIKOR (Compromise Ranking Method), GRA (Grey Relation Analysis), preferential relation methods Oreste, Electre, Promethe. The multi-attribute problems are solved as well in Zmeškal (2003,2001, 2014), Dluhošová et al (2010, 2015).

Forecasting is a significant topic and many researchers are dealing with, see e. g. Makridakis et al (1998), Hyndman and Athanasopoulos (2018). Well-developed methods and procedures stem from the assumption of having good data, so long series and precise. In financial practice this assumption is not often fulfilled, usually, we have short data series and non-precise data introduced in intervals. This problem is solved and modelled by various approaches and conceptions. One of the conception is Grey Relation Systems Theory founded by Deng (1989). The theory includes many systems methods, grey relational analysis (GRA),

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grey MADM, grey forecasting, grey decision-making, grey control. The basic idea is that there is a short data series, less than 30, which is minimum for application and reliability of statistical methods, furthermore impreciseness of data is given by intervals. Precise data are termed white one, an imprecise shadow data.

The goal of the paper is to describe and apply grey relation systems of MADM in multi-attribute evaluation of company financial health and performance and grey relation forecasting applied for financial measures forecasting. The paper is organised as follows: grey relation theory is described, chosen models of MADM and forecasting are introduced, verification examples are presented.

2. Grey relation systems theory and application

The grey MADM methods were presented e. g. in Deng (1989), Lin Chin-Tsai et al (2004), Kung Chaang-Yung et al (2007), Kuo Yiyo et al (2008), Tzeng Gwo-Hshiung et al (2011). The crucial term of grey relation space is grey relation grade depicted $\Gamma(x_i, x_j)$ expresses distance, similarity of two data series, x_i and x_j. The elements of the series are $x_{ik} \in x_i$, or $x_{jk} \in x_j$. The grade $\Gamma(x_i, x_j) \in [0;1]$ fulfils four axioms: normality, duality symmetric, wholeness and closeness. We distinguish due to series comparison two grey relation grades: localization grey relation grade $\Gamma(x_0, x_i)$ if one sequence is a reference set depicted x_0, and globalization grey relation grade $\Gamma(x_i, x_j)$ if each series should be reference series. The $\Gamma(x_i, x_j)$ is computed indirectly and directly.

Indirect approach is based on calculation the weighted average of grey relation coefficients, and input data are of an ordinal type, $\Gamma(x_i, x_j) = \sum_k w_k \cdot \gamma(x_{ik}, x_{jk})$. Here $\gamma(x_{ik}, x_{jk})$ is grey relation coefficients and w_k is weight.

Coefficient $\gamma(x_{ik}, x_{jk})$ is calculated on deviation basis and two basic conceptions are proposed.

 Deng´s conception is formulated as follows,

$$\gamma(x_{ik}, x_{jk}) = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{ij,k} + \zeta \cdot \Delta_{\max}}$$, where $\Delta_{ij,k} = |x_{jk} - x_{ik}|$ is deviance of ith series with reference series j and k^{th} element, $\Delta_{\min} = \min_{ijk} \Delta_{ij,k}, \forall i, \forall j, \forall k$, $\Delta_{\max} = \max_{ijk} \Delta_{ij,k}, \forall i, \forall j, \forall k$, it means minimum and maximum deviances of all deviances, $\zeta \in [0;1]$ is distinguishing coefficient, usually, value equals 0,5.

Wong´s conception is following,

$$\gamma(x_{ik}, x_{jk}) = \frac{\Delta_{\max} - \Delta_{ij,k}}{\Delta_{\max} - \Delta_{\min}} \cdot \zeta$$, the meaning of symbols is the same as for Deng´s conception.

The direct approach consists in the direct calculation of a grey relation grade, pair-wise comparison alternatives and cardinal input data are supposed. A grade on Deng´s conception,

$$\Gamma(x_i, x_j) \equiv \Gamma_{ij} = \frac{\Delta_{\min} + \Delta_{\max}}{\overline{\Delta}_{ij} + \Delta_{\max}}$$, here $\overline{\Delta}_{ij} = \left[\sum_k w_k \cdot (\Delta_{ij,k})^p \right]^{\frac{1}{p}}$, $p \in [1;\infty)$ is Minkowski norm

level, for $p = 1$, it is Manhattan norm and we get Wen´s grade, for $p = 2$ it is Euclidean norm and we obtain Wu´s grade.

The direct approach based on Wong's conception,

$$\Gamma(x_i, x_j) \equiv \Gamma_{ij} = \frac{\Delta_{max} - \overline{\Delta}_{ij}}{\Delta_{max} - \Delta_{min}}.$$ For $p=1$ we obtain Hsia's grade and for $p=2$ then Nagai-

Yamaguchi's grade.

The ranking of alternatives is given in case of indirect ordinal approach by ranking due to grey relation grade to reference series $\Gamma(x_0, x_i)$ from the highest to the lowest one.

The ranking of alternatives in case of direct cardinal approach is given by mutual comparison of series. The symmetric grey relation grade matrix R is constructed,

$$R = \begin{bmatrix} \Gamma_{11} & & \Gamma_{1M} \\ & \Gamma_{ij} & \\ \Gamma_{M1} & & \Gamma_{MM} \end{bmatrix}, \qquad v = \begin{bmatrix} v_1 \\ \\ v_M \end{bmatrix}.$$

Ranking is derived from eigenvector v for the maximum eigenvalue λ_{max} of the matrix R, here $R \cdot v = \lambda_{max} \cdot v$.

The grey forecasting methods are presented and verified e. g. in Deng (1989), Chiou Huan-Kai et al. (2004), Tseng Fang-Mei et al (2001). The grey forecasting method is given under assumption of the short data series (less than 30) and non-precise data given by intervals. The accumulated series is generated from the initial series and applied for forecasting. The forecasting model is derived from the linear differential equation of cumulative data. For instance in the case of the linear first-order differential equation

$$\frac{dx^1}{dt} = -a \cdot x^1 + b,$$ here x^1 is the cumulative element of data series.

3. Description of models

There is in the section introduced the description of the chosen models applied in the next section. Two GRA MADM models are presented, GRA localization method and GRA globalization method and for comparison, TOPSIS MADM method is described. One forecasting method GRA(1,1) model based on linear differential equation is presented.

3.1 GRA the localization method (similarity with ideal (required) solution) description and procedure

In the method, one sequence (ideal one) is the reference sequence, Deng's conception is applied. Conception and procedure of MADM method GRA is to be described in the following steps.

Step1 Construct of input decision matrix $y_{ij} \in Y$ and weights vector $w_j \in w$.

Step2 Calculation of normalised decision matrix $x_{ij} \in X$ given

$$x_{ij} = \frac{y_{ij} - \min_i y_{ij}}{\max_i y_{ij} - \min_i y_{ij}}$$ for beneficial criteria (the greater the better)

$$x_{ij} = \frac{\max_i y_{ij} - y_{ij}}{\max_i y_{ij} - \min_i y_{ij}}$$ for cost criteria (the lesser the better)

$$x_{ij} = \frac{\left| y_{ij} - y_{oj} \right|}{\max\left[\left(y_{oj} - \min_i y_{ij} \right), \left(\max_i y_{ij} - y_{oj} \right) \right]}$$ for objective (moderate) criteria y_{oj} (the precise

the better)

Step3 Calculation of grey relation coefficient

Deng´s conception of grey relation coefficient

$$\gamma(x_{ik}, x_{ok}) = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{io,k} + \zeta \cdot \Delta_{\max}}, \text{ where } \Delta_{io,k} = |x_{ik} - x_{ok}| \text{ is deviance of } i^{th} \text{ series with reference}$$

series o and k^{th} element, $\Delta_{\min} = \min_{ik} \Delta_{io,k}, \forall i, \forall k$, $\Delta_{\max} = \max_{ik} \Delta_{io,k}, \forall i, \forall k$, it means minimum and maximum deviances of all deviances, $\zeta \in [0;1]$ is distinguishing coefficient, usually, value equals 0,5.

Step4 Calculation of the grey relation grade

$$\Gamma(x_i, x_0) = \sum_k w_k \cdot \gamma(x_{ik}, x_{0k})$$

Step 5 Ranking of alternatives due to $\Gamma(x_i, x_0)$ in descending order, the best alternative

$$i^{best} = \arg\max_i \Gamma(x_i, x_0)$$

3.2 GRA the globalization method (pairwise similarity alternatives comparison) description and procedure

In the method, all alternatives sequences are the reference sequences, Wu´s conception is applied. Conception and procedure of MCDM method GRA is to be described in the following steps.

Step1 Construct of input decision matrix $y_{ij} \in Y$ and weights vector $w_j \in w$.

Step2 Calculation of normalised decision matrix $x_{ij} \in X$ given

$$x_{ij} = \frac{y_{ij} - \min_i y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \text{ for beneficial criteria (the greater the better)}$$

$$x_{ij} = \frac{\max_i y_{ij} - y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \text{ for cost criteria (the lesser the better)}$$

$$x_{ij} = \frac{|y_{ij} - y_{oj}|}{\max\left[\left(y_{oj} - \min_i y_{ij}\right), \left(\max_i y_{ij} - y_{oj}\right)\right]} \text{ for objective (moderate) criteria } y_{oj} \text{ (the precise}$$

the better)

Step3 Calculation of grey relation grade for all reference alternatives

$$\Gamma(x_i, x_j) \equiv \Gamma_{ij} = \frac{\Delta_{\min} + \Delta_{\max}}{\overline{\Delta}_{ij} + \Delta_{\max}}, \text{ here } \overline{\Delta}_{ij} = \left[\sum_k w_k \cdot \left(\Delta_{ij,k}\right)^2\right]^{\frac{1}{2}}, \text{ we obtain Wu´s conception of}$$

grade.

Step4 Construction the grey relation matrix R and eigenvector v for the maximum eigenvalue $\lambda_{\max}$ of the matrix R,

$$R = \begin{bmatrix} \Gamma_{11} & & \Gamma_{1M} \\ & \Gamma_{ij} & \\ \Gamma_{M1} & & \Gamma_{MM} \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ \\ v_M \end{bmatrix}.$$

Step 6 Ranking of alternatives due to eigenvector v in descending order, the best alternative is

$$i^{best} = \arg\max_i v_i$$

3.3 TOPSIS method (Technique for Order Preference by Similarity to an Ideal Solution) description and procedure

Conception and procedure of MCDM method TOPSIS is to be described in the following steps.

Step1 Construct of input decision matrix $y_{ij} \in Y$ and weights vector $w_j \in w$.

Step2 Calculation of normalised decision matrix $x_{ij} \in X$ given by Euclidean metric

$$x_{ij} = \frac{y_{ij}}{\sqrt{\sum_i y_{ij}^2}}$$

Step3 Calculation of weighted normalised decision matrix $r_{ij} \in R$

$$r_{ij} = x_{ij} \cdot w_j$$

Step4 Calculation of positive ideal solution (PIS) vector and negative ideal solution (NIS) vector

$$PIS = A^+ = \left[r_1^+, r_2^+ .... \right], \text{ where } r_j^+ = \left\{ \left(\max_i r_{ij}^+ \mid j \in J_1 \right), \left(\min_i r_{ij}^+ \mid j \in J_2 \right) \right\}$$

$$NIS = A^- = \left[r_1^-, r_2^- .... \right], \text{ where } r_j^- = \left\{ \left(\min_i r_{ij}^- \mid j \in J_1 \right), \left(\max_i r_{ij}^- \mid j \in J_2 \right) \right\}$$

here J_1 is a set of profit criteria J_2 is a set of cost criteria.

Step5 Calculation of the separation measures form PIS and NIS on Euclidean metric

$$D_i^+ = \sqrt{\sum_j \left(r_{ij} - r_j^+ \right)^2} \ , \ D_i^- = \sqrt{\sum_j \left(r_{ij} - r_j^- \right)^2}$$

Step 6 Calculation of similarity to positive ideal solution PIS

$$C_i^+ = \frac{D_i^-}{D_i^+ +_i^-}, \ C_i^+ \in [0;1]$$

Step6 Ranking of alternatives due to C_i^+ in descending order, the best alternative

$$i^{best} = \arg\max_i C_i^+$$

3.4 GRA forecasting method description, GRA(1,1) model

The forecasting method is based on accumulated (cumulated) input initial series. Single variable (one-factor) first-ordered grey model and procedure is described.

Step1 Initial input sequence of data is given, vector $x^0 = \left[x_1^0, x_2^0 .... \right]$

Step2 The accumulated series is generated from the initial series, $x^1 = \left[x_1^1, x_2^1 .... \right]$,

in the way $x_i^1 = \sum_{j=1}^{i} x_j^0$.

Step3 The first ordered differential equation is formulated with cumulative variables x^1

$$x^0 \equiv \frac{dx^1}{dt} = -a \cdot x^1 + b \text{, here } x_{t+dt}^0 = x_{t+dt}^1 - x_t^1$$

We can remark, that sometimes the dependent variable is given as average of values $x_{t+dt}^{*1} = 0,5 \cdot \left(x_{t+dt}^1 + x_t^1 \right)$.

Step 4 Solution of the differential equation for x_1^0 initial value

$$x_{t+1}^1 = \left(x_1^0 - \frac{b}{a} \right) \cdot e^{-a \cdot t} + \frac{b}{a} \, .$$

The common solution of the linear differential equation of type $\dfrac{dy}{dt} = \alpha(t) \cdot y + \beta(t)$ is

the one, $y(T) = F(T) \cdot e^{A(T)} + C \cdot e^{A(T)}$, where $A(T) = \displaystyle\int_t^T \alpha(t) \cdot dt$, $F(T) = \displaystyle\int_t^T \beta(t) \cdot e^{-A(t)}$,

parameter C is calculated from the initial condition for the given value $y(t)$.

Applying the common procedure for the equation $\dfrac{dx^1}{dt} = -a \cdot x^1 + b$,

the general solution is $x_{T+1}^1 = F(T+1) \cdot e^{A(T+1)} + C \cdot e^{A(T+1)}$, $A(T+1) = \displaystyle\int_t^{T+1} a \cdot dt = -a \cdot (T+1)$,

$F(T+1) = \displaystyle\int_t^{T+1} b \cdot e^{-A(t)} = \dfrac{b}{a} \cdot e^{a \cdot (T+1)}$,   so   after   the   substitution   we   obtain

$x_{T+1}^1 = \dfrac{b}{a} \cdot e^{-a \cdot (T+1)} \cdot e^{a \cdot (T+1)} + C \cdot e^{-a \cdot (T+1)} = \dfrac{b}{a} + C \cdot e^{-a \cdot (T+1)}$.   Now   for   the   initial   value

$x_1^1 = \dfrac{b}{a} + C \cdot e^{-a \cdot 1}$ the   unknown   parameter   is $C = \left(x_1^1 - \dfrac{b}{a} \right) \cdot e^{+a \cdot}$.   After   final   substitution

$x_{T+1}^1 = \dfrac{b}{a} + \left(x_1^1 - \dfrac{b}{a} \right) \cdot e^{+a \cdot} \cdot e^{-a \cdot (T+1)} = \left(x_1^1 - \dfrac{b}{a} \right) \cdot e^{-a \cdot T} + \dfrac{b}{a}$,   it   is   valid   that $x_1^0 = x_1^1$ so   we   get
introduced forecasting equation.

Step5 Statistical estimation of the model parameters is made by LSE (least square error) method from differential equation, parameters a and b are estimated.

Step6 Measures of forecasting error are calculated (e. g. MSE -mean square error).

4. Application examples of the GRA models

There is in the chapter applied MADM methods for financial performance evaluation and GRA forecasting method for forecasting sales.

4.1 Financial performance evaluation by MADM methods

The objective of the problem is to rank companies according to financial performance. Four measures (ROE, Indebtedness, Liquidity and Credibility) characterise performance. Evaluation is given by three methods: localization GRA Deng´s conception (chapter 3.1), globalization GRA Wu´s conception (chapter 3.2), TOPSIS similarity method (chapter 3.3). Input data are presented in Tab. 1.

Tab. 1 Input data - financial ratios and characteristics of companies

measure	ROE	Indebtedness	Liquidity	Credibility
criteria type	max	min	max	max
weight	0,25	0,25	0,25	0,25
a1	0,15	0,5	2	5
a2	0,07	0,6	1,2	8
a3	0,11	0,3	2,5	3
a4	0,04	0,7	1,5	4
a5	0,09	0,2	2,3	6

Solution procedure illustrate Tab. 2, Tab. 3 and Tab. 4 for GRA methods, furthermore Tab. 5 for the TOPSIS method.

Tab. 2 Normalised matrix-GRA

	ROE	Indebtedness	Liquidity	Credibility
a1	1	0,4	0,615385	0,4
a2	0,272727	0,2	0	1
a3	0,636364	0,8	1	0
a4	0	0	0,230769	0,2
a5	0,454545	1	0,846154	0,6

Tab. 3 Grey relation coefficients matrix

	ROE	Indebtedness	Liquidity	Credibility
a1	1	0,454545455	0,565217	0,454545
a2	0,407407	0,384615385	0,333333	1
a3	0,578947	0,714285714	1	0,333333
a4	0,333333	0,333333333	0,393939	0,384615
a5	0,478261	1	0,764706	0,555556

Tab. 4 Grey relation grades matrix

	a1	a2	a3	a4	a5
a1	1	0,571756907	0,38735	0,580502	0,433236
a2	0,571757	1	0,78934	0,449342	0,622301
a3	0,38735	0,789340137	1	0,647432	0,337908
a4	0,580502	0,449342465	0,647432	1	0,660551
a5	0,433236	0,622301009	0,337908	0,660551	1

Tab. 5 Normalised matrix-TOPSIS

	ROE	Indebtedness	Liquidity	Credibility
a1	0,676252226	0,450835	0,456079	0,4082483
a2	0,315584372	0,541002	0,273648	0,6531973
a3	0,495918299	0,270501	0,570099	0,244949
a4	0,180333927	0,631169	0,342059	0,3265986
a5	0,405751336	0,180334	0,524491	0,4898979

The results of compared methods are presented in Tab. 6. We can see that ranking of method based on similarity to an ideal alternative (localization Deng´s GRA, TOPSIS) is similar, the best and worse alternatives are the same. On the other side, Wu´s GRA globalization method ranks alternative quite differently because of the pairwise comparison approach and quite a different conception.

Tab. 6 Comparison of results due to methods

| Method	Localization GRA		Globalization GRA		TOPSIS	
	Deng´s GRA		Wu´s GRA			
Alternative	Measure	Ranking	Measure	Ranking	Measure	Ranking
a1	0,618577	3	0,410622	5	0,603419	2
a2	0,531339	4	0,482464	1	0,42688	4
a3	0,656642	2	0,447541	3	0,553101	3
a4	0,361305	5	0,466004	2	0,120204	5
a5	0,699631	1	0,425636	4	0,657917	1

4.2 GRA financial forecasting

There is in the chapter verified GRA forecasting GRA(1,1) with one explained and one explaining variable. The sales short series is investigated. The GRA(1,1) model, see chapter 3.4, is compared with the traditional growth model of type $\frac{dx}{dt} = g \cdot x$, where x is sale, g is the average growth rate. Forecasting model given by equation solution is the following, $x_T = x_0 \cdot e^{g \cdot T}$. The growth rate is estimated as an average of yearly growth. Input data and models results are in the tab. 7. We can see that we know sales for 9 years and forecasting period is 3 years. GRA model parameter b= 15,48465687 and parameter a=-0,013422009 both and are estimated by the Least Square Method. Parameter g=9,48% of growth model is estimated as the average yearly grows.

Tab. 7 Input data and forecast

Year	Known sales	Forecast models	
		GRA	Growth
1	10	10	10
2	12	15,72	12,09
3	18	15,94	13,29
4	14	16,15	14,61
5	20	16,37	16,07
6	15	16,59	17,66
7	10	16,82	19,42
8	11	17,04	21,35
9	15	17,27	23,47
10		17,51	25,81
11		17,74	28,38
12		17,98	31,20

Graph 1 Input data and models forecast
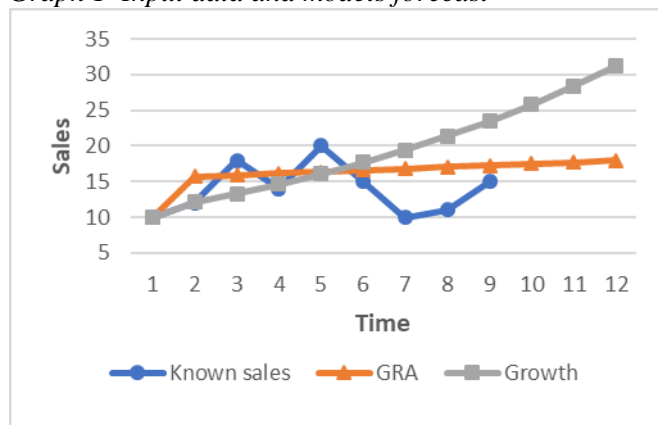
From Tab. 7 and Graph 1 it is apparent how the models spline historical data and forecast. Standard deviation on historical data is for GRA model 2,927305577 and for growth model 3,98587527, so the GRA(1,1) model was better and probably in more examples because growth model more shows a trend and overestimate or underestimate the data.

5. Conclusion

The objective of the paper was to verify the possibilities of GRA models application and comparison with other known models. MADM GRA models are one possibility of how to solve multi-attribute problems in finance. The models are based on similarity measures. The result of the localization GRA models was comparable with the TOPSIS approach. Interesting was the globalization GRA model with fully different conception. It needs patient interpretation. Forecasting model was based on cumulative variables which are more stable and forecasting results were interesting. Probably the model better predicts non-trended data and is more robust.

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