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DG method for the Hull-White option pricing model

Jiří Hozman[1], Tomáš Tichý[2]

Abstract

The main disadvantage of the Black-Scholes framework is that this approach can capture neither real nor risk-neutral market behaviour. Therefore many various extensions, such as incorporating of stochastic volatility into Black-Scholes model or replacing diffusion processes by pure jump or jump-diffusion processes, have been suggested. Among them, the Hull-White model used to be quite popular due to its ability to extend the Black-Scholes framework easily by letting the volatility itself be modeleled as a lognormal process as opposed to its constant value. A common mathematical approach to this model leads to the pricing equation represented by a degenerate parabolic partial differential equation of two spatial and one temporal variables accompanied by the system of boundary and initial conditions. The presented numerical valuation process is based on the utilization of the discontinuous Galerkin method for space discretization and the Crank-Nicolson scheme in the time coordinate. Finally, reference numerical experiments on real market data illustrate comprehensive empirical findings on options with stochastic volatility.

Keywords

Option pricing, stochastic volatility, Hull-White model, discontinuous Galerkin method, Crank-Nicolson scheme.

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1. Introduction

In this paper we continue in our research [9-13] focused on numerical pricing of options via Discrete Galerkin (DG)approach. Contrary to previous research we focus on stochastic volatility models, i.e. an additional source of risk (and dimension) is added.

After introducing the Black-Scholes (BS) model in [2, 15], the approach to option pricing based on normal (Gaussian) distribution with constant parameters became a standard market

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practice. Notwithstanding, the empirical results show that log-returns of securities at the market are normally distributed only rarely. As a result, many researchers have suggested various alternative models that relax this key drawback of BS model.

In this contribution we consider one such extension coming from [14] – the parameter of volatility, instead of being constant, is replaced by additional stochastic process so that the overall distribution is not Gaussian anymore. We also reflect the fact that such assumption generally prevents one from obtaining analytical solution of the valuation problem, ie., some kind of numerical approximation is essential (finite difference methods [5], finite element approaches [1, 19], etc.) – keeping the line of our previous research, via constitute a numerical framework based on discontinuous Galerkin (DG) approximation technique.

We proceed as follows. Section 2 is devoted to the generalization of BS formula for the case of stochastic volatility. In Section 3 we develop discontinuous Galerkin approach for this special case. Finally, in Section 4 numerical examples based on reference market data are provided.

2. Generalizations of the Black-Scholes model

Nowadays it is accepted that the BS model is not sufficiently accurate in capturing the real world features of the markets, because its idealized assumptions do rarely hold in practice. The deterministic behavior of its model parameters and/or absence of jumps processes in this model are usually essential weaknesses of BS framework. Therefore, from this point of view, one can generalize this approach in two ways – stochastic interest rate/volatility models and/or models with jump processes.

The limitation of a constant interest rate and a volatility can be relaxed by models, which presume that these parameters are defined by stochastic processes. There is a number of stochastic models, e.g. models by Hull and White [14], Stein and Stein [18] or Heston [8]. The corresponding pricing PDE can be derived in the similar manner as in the BS model. However, the aforementioned stochastic approach makes the model very robust and hence addresses the shortcomings of the BS model, i.e., it provides a framework to price a variety of options that is closer to reality.

The second possible extension of these models is based on the fact that the usual assumption of a geometric Brownian motion in BS framework should be improved by including discontinuous jump processes. Such models were originally introduced by Merton in the pioneering paper [16]. The addition of jumps into the model allows large sudden changes in the underlying asset. A suitable stochastic process that allows jumps and diffusion is the Lévy process, which has similar properties as the Brownian motion, but also allows discontinuities; for survey see [4]. The valuation of options under models with jump processes requires solving a degenerate partial integro-differential equation (PIDE), i.e., a functional equation which involves both integrals and derivatives of an unknown pricing function. The integral term arising from jumps leads to new theoretical and numerical issues regarding solving of such pricing PIDE in comparison with the standard approach for pricing PDEs.

2.1 Hull-White stochastic volatility model

In the rest of the paper, we concentrate on valuing options under one of the firstly introduced stochastic volatility models. This model was constructed by J. Hull and A. White in 1987 as a generalization of a deterministic volatility approach in the BS model. Therefore, we assume here that the asset price process S and the variance process Y obey (at each time instant $\tilde{t}$) the following two stochastic differential equations:

$$dS_{\tilde{t}} = \mu S_{\tilde{t}}\, d\tilde{t} + \sqrt{Y_{\tilde{t}}}\, S_{\tilde{t}}\, dW_{\tilde{t}}, \quad S_0 \geq 0, \tag{1}$$

$$dY_{\tilde{t}} = \alpha Y_{\tilde{t}}\, d\tilde{t} + \beta Y_{\tilde{t}}\, dZ_{\tilde{t}}, \quad Y_0 \geq 0, \tag{2}$$

where it is assumed that standard Brownian motions $W(\tilde{t})$ and $Z(\tilde{t})$ are uncorrelated. The parameter $\mu \in I\!R$ is the drift coefficient of the asset price, $\alpha \in I\!R$ is the drift coefficient of the variance, and $\beta > 0$ is the volatility-of-variance parameter, for a detailed explanation see the book [6]. It is also important to mention that the stochastic variance governed by (2) cannot take on negative values. Moreover, since S and Y are the only state variables affecting the price of the option V, the risk-free interest rate $r > 0$ must be constant.

Next, we consider reversal time running $t = T - \tilde{t}$, where T is the maturity date. Then we denote by $V(S, Y, t)$ the price of a European option on the underlying asset S at time t, which will also depend on the driving process Y. By using a common approach introduced in [14], we recall that the pricing function $V(S, Y, t) : I\!R_+^2 \times (0, T) \to [0, +\infty)$ must satisfy the partial differential equation

$$\frac{\partial V}{\partial t} - \frac{1}{2} Y S^2 \frac{\partial^2 V}{\partial S^2} - rS\frac{\partial V}{\partial S} + rV - \frac{1}{2}\beta^2 Y^2 \frac{\partial^2 V}{\partial Y^2} - \alpha Y \frac{\partial V}{\partial Y} = 0. \tag{3}$$

Since the governing PDE (3) is of the first order in time, the associated Cauchy problem is equipped with the initial condition given by the payoff function of the corresponding European vanilla option at maturity date T, i.e.,

$$V(S(0), Y(0), 0) = V_0(S, Y) := \begin{cases} V_0^C(S, Y) = \max(S - \mathcal{K}, 0), & \text{if } V \text{ is call}, \\ V_0^P(S, Y) = \max(\mathcal{K} - S, 0), & \text{if } V \text{ is put}, \end{cases} \tag{4}$$

where $\mathcal{K}$ denotes the strike price.

Furthermore, to numerically solve the problem (3) with (4), an unbounded spatial domain has to be truncated on $(0, S_{max}) \times (0, Y_{max})$, where S_{max} and Y_{max} stand for the maximal asset price and its variance, respectively. In practice we usually encounter the situation $S_{max} \gg Y_{max}$, thus the degeneracy of a spatial domain in a straight line is overcome by introducing new variable $x = [x_1, x_2] = [S/\mathcal{K}, Y]$. Then we obtain new transformed spatial domain $\Omega = (0, S_{max}/\mathcal{K}) \times (0, Y_{max})$ and new price function $u(x, t) : \Omega \times (0, T) \to [0, +\infty)$ defined as $u(x, t) = V(S, Y, t)/\mathcal{K}$. In a similar way the initial states $u_0(x) : \Omega \to [0, +\infty)$ transform into

$$u_0^C(x) = V_0^C(S, Y)/\mathcal{K} = \max(x_1 - 1, 0), \quad u_0^P(x) = V_0^P(S, Y)/\mathcal{K} = \max(1 - x_1, 0). \tag{5}$$

Consequently, the governing equation (3) can be rewritten in terms of the new price function using the divergence form as

$$\frac{\partial u}{\partial t} - \nabla \cdot \big(I\!D(x)\nabla u\big) + \nabla \cdot \big(\vec{b}(x)u\big) + c(x)u = 0 \quad \text{in } \Omega \times (0, T). \tag{6}$$

The equation (6) represents a degenerate second order parabolic equation and exhibits a convection-diffusion-reaction character. The diffusion term is proportional to the diagonal matrix

$$\mathbb{D}(x) = \frac{1}{2}\begin{pmatrix} x_1^2 x_2 & 0 \\ 0 & \beta^2 x_2^2 \end{pmatrix}, \tag{7}$$

which is singular on axes $x_1 = 0$ and $x_2 = 0$, respectively. The convection term is defined by vector $\vec{b}(x)$ representing the vector field of physical fluxes, i.e.

$$\vec{b}(x) = (b_1(x), b_2(x))^T = \big((x_2 - r)x_1, (\beta^2 - \alpha)x_2\big)^T \tag{8}$$

and the scalar function $c(x) = 2r + \alpha - x_2 - \beta^2$ stands for the variable reaction coefficient.

Concurrently, the Cauchy problem (6)–(8) with (5) has to be supplied with suitable setting of boundary conditions on appropriate parts of the boundary $\partial\Omega$. This setting plays an important role to achieve highly accurate results. We follow the common approach, where the boundary conditions are chosen in accordance with the direction of the convective flow on $\partial\Omega$ given by (8). Then we prescribe boundary conditions of a mixed type as follows. On far-field boundary $x_1 = S_{max}/\mathcal{K}$, we prescribe Neumann boundary condition corresponding to slopes of European options as $S \to +\infty$. It means that the call option has a unit slope with respect to the asset price S and the put option has a zero slope, respectively. Since the relation $b_2(x) > 0$ is not violated on $x_2 = Y_{max}$ in practice, the convection has a velocity pointing outward the domain Ω. Therefore, it is possible to set an artificial boundary condition of a Dirichlet type given by the extrapolated value of the solution u from Ω. On the other hand, no boundary conditions have to be imposed on $x_1 = 0$ and $x_2 = 0$, because $b_i(x) = 0$ for $x_i = 0$, $i = 1, 2$.

Finally, note that PDE (6) is convection-dominated when x_1 and x_2 are close to zero. This fact is the source of many numerical difficulties, thus one has to take it into account when numerical solution is constructed, see DG method.

3. DG method for option pricing

We recall application and utilization of the DG method (see [17] for a complete overview) to numerical solving of the studied option pricing problem with stochastic volatility from Subsection 2.1. Our aim is to construct solution $u_h = u_h(t)$ from the finite dimensional space S_h^p consisting from piecewise polynomial, generally discontinuous, functions of the p-th order defined on the domain Ω.

This DG discretization in spatial coordinates leads to a system of the ordinary differential equations for unknown price function u_h, i.e.,

$$\frac{d}{dt}(u_h, v_h) + \mathcal{A}_h(u_h, v_h) = l_h(v_h)(t) \quad \forall\, v_h \in S_h^p,\ \forall\, t \in (0, T), \tag{9}$$

where the initial condition $u_h(0)$ is given by (5), $(\cdot,\cdot)$ denotes the inner product in $L^2(\Omega)$, the bilinear form $\mathcal{A}_h(\cdot,\cdot)$ stands for the discrete variant of the spatial partial differential operator from (6) and the form $l_h(\cdot)(t)$ arises from boundary conditions, the detailed derivation regarding to option pricing is presented in [13].

Consequently, we realize the discretization in time by Crank-Nicolson scheme an obtain at each time level $0 = t_0 < t_1 < \cdots < t_s = T$ from (9) the sparse matrix equation, see [12], i.e.,

$$\left(\mathbf{M} + \frac{\tau}{2}\,\mathbf{A}\right) U_{m+1} = \left(\mathbf{M} - \frac{\tau}{2}\,\mathbf{A}\right) U_m + \frac{\tau}{2}\,\left(F_{m+1} + F_m\right), \tag{10}$$

where τ is a constant time step and $U_m = \{\xi_k^m\}_{k=1}^{DOF} \in I\!R^{DOF}$ is an unknown vector of coefficients of basis functions $\{\varphi_k(x)\}_{k=1}^{DOF}$ of S_h^p, i.e., $u_h(t_k)(x) = \sum_{k=1}^{DOF} \xi_k^m \varphi_k(x)$. The system matrix is a composition of the mass matrix $\mathbf{M}$ and the matrix $\mathbf{A}$ arising from the spatial differential operator defined as

$$\mathbf{M} = \left\{m_{ij}\right\}_{i,j=1}^{DOF}, \quad m_{ij} = (\varphi_j, \varphi_i), \qquad \mathbf{A} = \left\{a_{ij}\right\}_{i,j=1}^{DOF}, \quad a_{ij} = \mathcal{A}_h(\varphi_j, \varphi_i). \tag{11}$$

Since the supports of basis functions φ_k are small, the system matrix is sparse and has a block structure. The right-hand side of (10) contains an average of two following vectors

$$F_m = \{l_h(\varphi_k)(t_m)\}_{k=1}^{DOF}, \qquad F_{m+1} = \{l_h(\varphi_k)(t_{m+1})\}_{k=1}^{DOF}, \tag{12}$$

which enforce the fulfillment of boundary conditions at time levels t_m and t_{m+1}, respectively.

Finally, note that the system matrix is nonsingular, which implies the existence and uniqueness of the solution of the algebraic problem (10) related to the price function u. Since the system is sparse and non-symmetric, it is suitable to use some iterative solvers, e.g., the restarted GMRES solver.

4. Reference numerical benchmark

In this section, the properties of the option price obtained by the numerical approach from Section 3. are examined on the reference benchmark from [14]. The numerical simulations are done by software Freefem++ [7], in which the proposed DG approach is implemented. We use piecewise linear approximations on structured (consecutive refined) triangular grids. To specify the implementation settings the time step is set proportional to one calendar day as $\tau = 1/360$ and GMRES is used as a sparse solver for (10).

To be consistent with the reference experiment we consider European call with a unit strike, 180 days to maturity and the following model parameters:

$$r = 0.0, \qquad \alpha = 0.0, \qquad \beta = 1.0, \qquad S_{max} = 6\mathcal{K}, \qquad Y_{max} = 1.0, \tag{13}$$

where S_{max} and Y_{max} are chosen sufficiently large to be far enough from the zone of practical interest, i.e., it is justified to use artificial boundary conditions on the far-field boundary specified in Subsection 2.1 without a great influence on the numerical solution in the rest of the computational domain.

The first task is to investigate the behaviour of call values with respect to different grid spacing. Therefore, we compute discrete solutions on a sequence of consecutive refined meshes and comparative results are evaluated at given reference nodes $[S_{ref}, Y_{ref}]$, where $S_{ref} \in \{0.8, 0.85, 0.9, 0.95, 1.0, 1.05, 1.1, 1.15, 1.2\}$ and $Y_{ref} = 0.01$, see Table 1. These results are associated with reference ones from [14]. One can easily observe that the numerical results

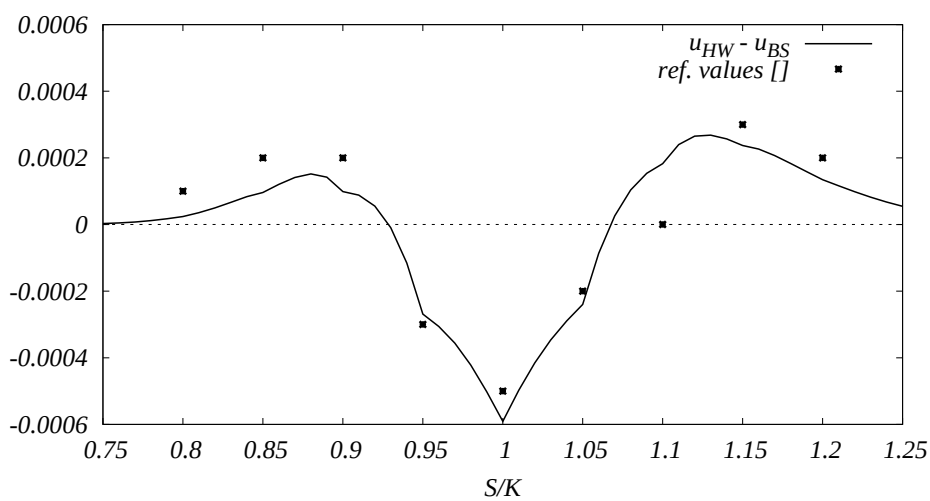
are of higher accuracy as the grid spacing decreases and give fairly the same values as in [14]. More precisely, since the reference results are rounded to only four decimal places, the absolute errors may be seem as distorted. However, the relative errors are under 1% barrier except for ITM options.

Table 1: Comparison of call option values evaluated at reference node $[S_{ref}, Y_{ref}]$ for different Y-spacing with BS prices and reference results

$\frac{S_{ref}}{\mathcal{K}}$	BS value	ref. value [14]	Y-spacing			
			$h_y = 0.02$	$h_y = 0.01$	$h_y = 0.005$	$h_y = 0.0025$
0.80	0.0000	0.0001	0.000000	0.000000	0.000000	0.000038
0.85	0.0002	0.0004	0.000000	0.000000	0.000251	0.000336
0.90	0.0019	0.0021	0.001595	0.001922	0.002038	0.002109
0.95	0.0094	0.0091	0.009031	0.009405	0.009204	0.009158
1.00	0.0281	0.0276	0.028141	0.028134	0.027725	0.027618
1.05	0.0603	0.0601	0.060876	0.060714	0.060390	0.060285
1.10	0.1030	0.1030	0.103537	0.103346	0.103262	0.103228
1.15	0.1506	0.1509	0.151050	0.150800	0.150901	0.150922
1.20	0.2001	0.2003	0.200298	0.200079	0.200225	0.200256

For a complete overview, BS prices are also included in Table 1 as nonstochastic volatility setting where $\alpha = 0$ and $\beta = 0$. Moreover, Figure 1 shows general relationship between the option price under Hull-White stochastic volatility model and the BS price. It is found that BS prices frequently overprice the correct option value. These price differences are low deep for ITM and OTM options and high for ATM options. The largest absolute differences occur at or near at the strike price. This behaviour is in quite good agreement with results from [14].

Figure 1: Option price differences between Hull-White model and BS model with volatility 10%

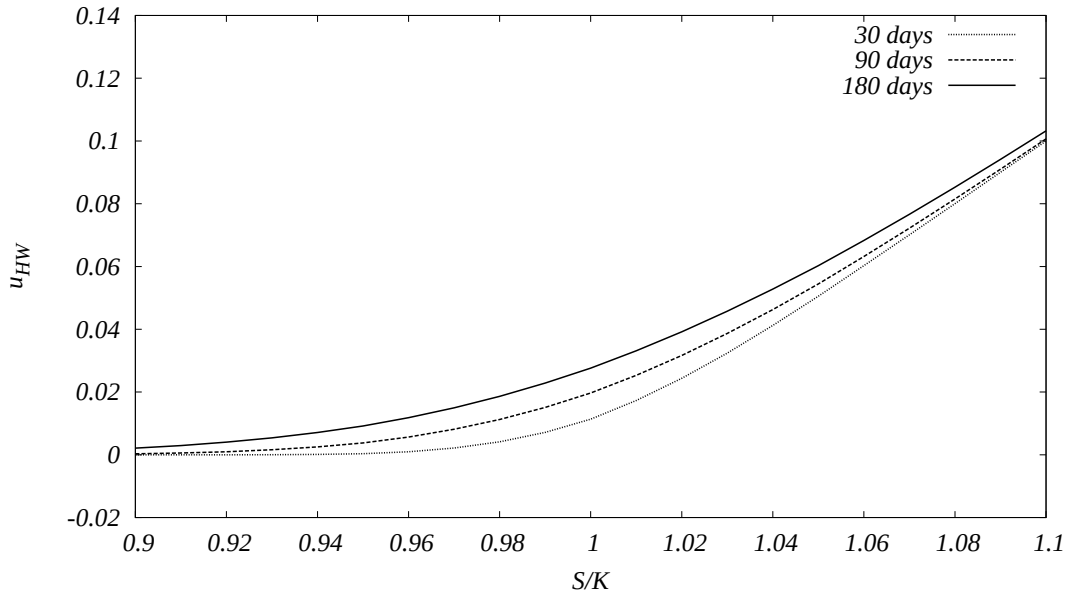


The second experiments provide the deeper insight to the properties of Hull-White option pricing problem with respect to maturities. To capture several scenarios we consider the cases with different times to maturity, in which the mesh is fixed (e.g., the finest grid from previous

experiments) and the remaining model parameters are the same as in (13). In Figure 2 it is apparent that discrete option prices increase as the time to maturity raises. This effect is the most evident for ATM or near ATM options.

Finally, note that our observations illustrate typical findings common for the European options priced under the Hull-White stochastic volatility model.

Figure 2: Effect of different maturities on discrete option prices



5. Conclusions

Although the BS model of option pricing is a part of standard market practice, it has several drawbacks. One of the most serious one is that the log returns of market securities are generally not normally distributed. In this paper we considered one suitable extension – geometric Brownian motion of the BS model is accompanied by additional stochastic process in line with Hull-White model and complex numerical treatment of the problem via DG approach has been provided. Complex analysis of proposed approach via numerical examples proof its efficiency as well as competitive advantage against standard approach.

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Managing foreign exchange exposures in the context of ending the currency commitment of the CNB

Adam Chlíbek [1]

Abstract

This paper deals with problematics of managing foreign exchange risk during the period of the CNB's exchange rate commitment. The paper includes recommendations and suggestions of solutions for Czech export enterprises. Specifically, the paper recommends methods of alternative scenarios for quantifying foreign exchange risk during the period of foreign exchange intervention of the CNB. The elimination of transaction exposure is recommended by hedging via short-term forward contracts, *i.e.* one months, two months, three months forward. The elimination of pre-transaction exposure is advised by hedging with options contracts.

Key words

foreign exchange risk, foreign exchange exposure, financial derivatives, method of alternative scenarios, CNB's exchange rate commitment

JEL Classification: F31, G15

1. Introduction

Czech firms must cope with a complete range of external influences on the economic environment. One of the notable effects is influencing the exchange rate, which significantly affects the competitiveness of companies, even those companies that not trade at all with foreign countries.

The change in the rate of a few pennies can mean a loss for one and profit for another. For export firms, the depreciation of the Czech crown is favourable, unlike with import firms, for which the depreciation of the Czech crown is adverse. The awareness of this essence is the basis for a successful management of foreign exchange risks. It is quite logical that management is interested in this issue especially in those cases where the impact of foreign exchange risk largely affects their bottom line.

In view of long-lasting adverse price development in the Czech Republic[2] the Czech national bank (CNB) was forced to use the exchange rate as a monetary policy instrument, and therefore the CNB introduced a one-sided exchange rate commitment[3] at 27 CZK/EUR on 7 November 2013.The main reason why the CNB weakened the Czech crown was achieve two percent inflation. On the other hand, it had also number of other effects. One of them was to

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[2] The Czech economy underwent an economic recession in 2012-2013. The Czech National bank has responded to this by reducing interest rates up to technical zero, *i.e.* 0,05 %. It did not help and according to CNB there was danger of deflation.

[3] The CNB was willing to intervene against appreciation of the Czech crown. It means that the second side of the Exchange rate commitment, *i.e.* the weak one, is managed according to the law of supply and demand.

increase competitiveness of exporters, but for importers it was negative, because they were less competitive.

Exporters trading with Eurozone did not have to manage foreign exchange risk during the period of foreign exchange intervention of the CNB, because they could calculate on the worst possible exchange rate of 27 CZK/EUR. However, this was not valid with the approaching end of the CNB intervention, so there was a period of uncertainty with the further development of the Czech crown against the euro. There was a question of how to prepare for the end of the interventions. To prepare for this, we needed to know the date of exit and the subsequent development of the Czech crown against the euro. Unfortunately, there wasn't unambiguous agreement among experts in answering these fundamental questions.

According to the CEO of food giant - Bidfood Bohumil Volf, the opinions of experts were so contradictory that they decided not to do anything at all. They decided to wait for what would come. Other traders shared this approach. This was confirmed by Ivor Poprach, the CEO of the metallurgical Boltjes Group. He said that he decided not to hedge too (E15, 23. 2. 2017).

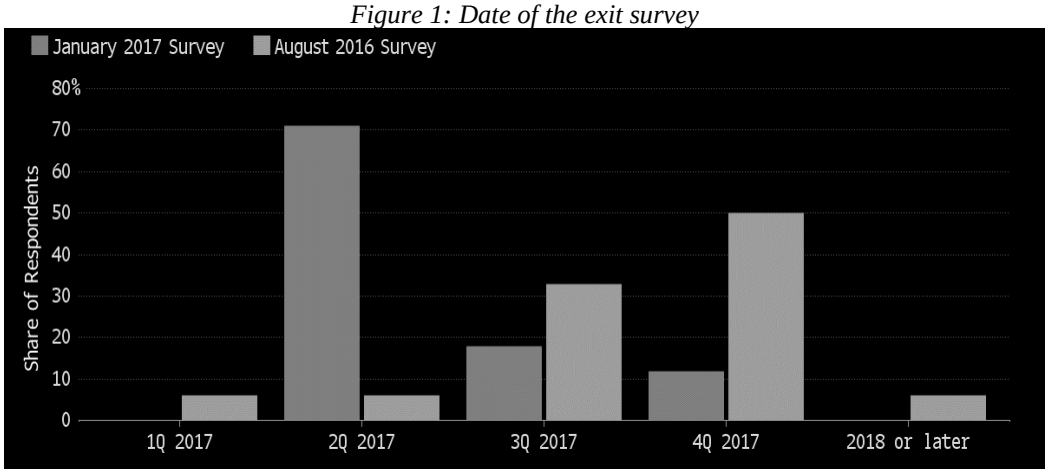
On the other hand, the CEO of Linet, Zbyněk Frohlík said that he avoided the Czech crown and he uses natural hedging to reduce foreign exchange exposure[4]. The rest of the foreign exchange exposure is hedged by banks (HN, 31. 3. 2017).

2. Date of the CNB's exit

In October 2016, the CNB could be satisfied with price development in the Czech Republic, because its two percent inflation target was fulfilled. In February 2017, inflation was even more than half above the inflation target. The CNB therefore lost its reason for continuing the foreign exchange intervention. At this time, the CNB announced a "hard" commitment, *i.e.* to not discontinue foreign exchange intervention before the second quarter of the year 2017.

Economists' opinions on the timing of the exit were different, but most expected exit in the second quarter of this year. The following figure shows that twelve of the seventeen foreign economists questioned in the Bloomberg poll believed that the exit would probably be in the second quarter of this year. Five of them even thought it was going to happen in April 2017. A similar poll was also conducted in August 2016, when economists assumed that the exit would not take place before October 2017.

Figure 1: Date of the exit survey



Source: Bloomberg (2017)

[4] It means that revenue that company obtains in euro closely matched to the expenses in euro.

For the exact date of the exit, the sustainability of two percent inflation was key. The sustainability was very important, because if the inflation rate dropped below two percent, then intervention would be pointless. Nevertheless, the exit could be anytime.

The specific dates of the expected end of the CNB's exchange rate commitment could be ascertained from the survey of the Exporters' Association, which includes the following dates: 6. 4. 2017, 13. 4. 2017 and 30. 6. 2017.

3. Czech crown to euro exchange rate after exit

Uncertainty was not just about the date of exit but also about subsequent development of the czech crown to the euro after exit. Forecasting exchange rates is one of the most difficult things for managers. In practise, there are two approaches to predicting exchange rates: technical and fundamental analysis. It should be noted that none of them achieves 100% accuracy.

The future exchange rate of the CZK/EUR pair must be known for effective hedging. Unfortunately, the predictions of the future development of the Czech crown after the exit were varied. The problem was that the Czech crown has not traded under market principles for the fourth year. It shows that companies were forced to rely only on fundamental analysis because technical analysis predicts future exchange movements based on past and current values.

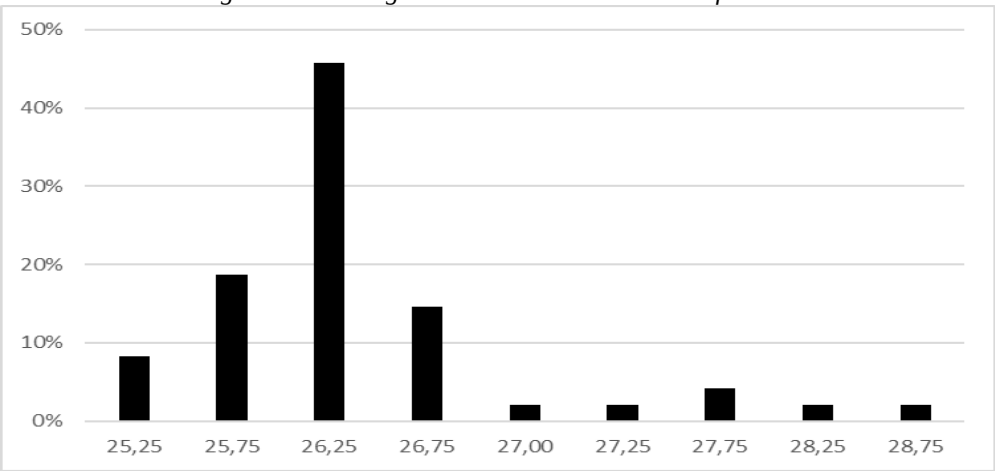
According to economist Jan Bureš (13. 9. 2016) from Patria Finance, it was better to monitor the probability of future CZK exchange rates after the CNB's interventions, rather than the specific numerical estimates.

3.1. Exchange rate CZK/EUR one month after exit

During March, the Association of Exporters compiled a short survey, which was a major input for the export index and Raiffeisenbank. In this survey, the issue of the future development of the Czech crowns against the euro was also resolved for one month after the end of CNB's interventions.

The chart below shows that roughly two percent of respondents estimated that the CZK / EUR exchange rate will remain unchanged, *i.e.* 27 CZK/EUR. Ten per cent of respondents even claim that the Czech crown will temporarily weaken. However, most of the respondents, *i.e.* 87%, expected that the Czech crown to strengthen.

Figure 2: Exchange rate CZK/EUR one month after exit



Source: Association of Exporters

4. Hedging a fictitious enterprise

In the following text, a Czech fictitious enterprise will be introduced with exposure to one foreign exchange. A Czech company exports goods to Germany priced in euros. The company intends to convert its euro income into Czech crowns. During the period of the CNB's exchange rate commitment the company didn't hedge its exposure, because no one expected an exit sooner than the second quarter of 2017 (as mentioned above) and most companies had calculated with an exchange of 27 or more CZK/EUR so far. The question of hedging became relevant on January 10, 2017. On this date, an inflation report was released for December 2016. The two percent inflation goal was fulfilled and therefore the CNB ceased to have a reason to weaken the crown.

A fictitious company recorded a receivable in its accounting books on 6. 3. 2016. The due date of this receivable was on 6. 5. 2017 and the company expected 400 000 euros, *i.e.* 10 800 000 Czech crowns (at the time of the transaction the exchange rate was 27 CZK/EUR). The company assumed that the due date of the receivable would be one month after the end of the CNB interventions and it was true. In its calculation, the Czech company calculated with 3 % profit, which means that the cost of contract was 388 000 euros, *i.e.* 10 476 000 Czech crowns.

4.1. Quantification currency risk

For effective risk management, the risk must be also quantified and not just identified. For quantification, the currency risk is used the method of alternative scenarios. The size of the currency risk varies on the basis of the size of foreign exchange exposure and the probability of adverse development of the foreign exchange rate. This method calculates the total assumed change in the value of the foreign exchange position as the weighted average of the individual changes (*i.e.* exchange rate loss or profit) in the given scenarios, the weights being the likelihoods of the arise of the individual scenarios (Černohlávková et al., 2007).

The scenarios were determined on the basic of survey of the Association of Exporters. The likelihood of entering the individual scenarios was determined by the proportion of respondents that predicted the given scenario and the total number of respondents.

Table 1: Calculation of expected exchange rate loss/profit in each scenario

Scenario	Expected change of the exchange rate	Exchange rate loss/profit [in thousands of crowns]	Likelihood	Outcome [in thousands of crowns]
1	-1,75	-700	0,0833	-58,31
2	-1,25	-500	0,1875	-93,75
3	-0,75	-300	0,4583	-137,49
4	-0,25	-100	0,1458	-14,58
5	0	0	0,0208	0
6	0,25	100	0,0208	2,08
7	0,75	300	0,0417	12,51
8	1,25	500	0,0208	10,4
9	1,75	700	0,0208	14,56
Overall				**-264,58**

If the company did not close its foreign exchange position, it would expect an average exchange rate loss -264,58 in thousands of crowns. This loss would also decrease part of the expected profit. If a company didn't hedge its exposure, it would receive just 10 535 420 Czech crowns (400 000*26,339) unlike the prior calculated 10 800 000 Czech crowns (400 000*27). A difference of 264 580 Czech crowns would reduce expected profit. The profit wouldn't be 324 000 Czech crowns but just 59 420 Czech crowns. It means that the profit was reduced from 3 % to 0,5 %.

4.2. Hedging foreign exchange exposure

After the quantification and identification of currency risk followed management's decision to hedge or not. In view of the fact that the enterprise calculated using low profits, the management wanted to conduct hedging. Nevertheless, the final decision was based on a comparison costs and assets of hedging. In this chapter are used middle rates from portal Patria and option premium was obtained via Bloomberg application based on Black-Scholes model.

4.2.1. Receivable

The company decided to hedge a receivable amounting to 400 000 euros on February 6, 2017, to select an appropriate three-month currency derivative. The company had two options to hedge this receivable, either by currency forward or by currency option.

First, the management's company evaluated hedging via three months forward. By using the forward contract, the company would be committed to sell 400 000 euros on May 6, 2017 for forward rate of 26,9014 CZK/EUR. After hedging the company would calculate with revenue of 10 760 560 Czech crowns. If we compare it with expected revenue without hedging, we conclude that after hedging we increased expected revenue of receivable by 225 140 Czech crowns, i.e. 2,137 % more.

Table 2: comparison between unhedged and hedged exposure via three months forward

Unhedged exposure		**Hedged exposure**		**Profit/loss**
Expected spot rate	Transaction volume (czk)	Forward rate	Transaction volume (czk)	Difference (czk)
26,339	10 535 420	26,9014	10 760 560	225 140

No fees are usually charged for arranging forward contract. The only one cost for arranging forward contract are forwards points, in our case it is -98,6. The hidden costs can be opportunity costs. It would arise if the exchange rate was more than 26,9014 CZK/EUR or the bank wanted to put up a collateral[5].

The second option how to hedge this exposure is via currency option. Given to necessary individual conditions, the company used the over-the-counter (OTC) option. In this variant of hedging, the company did hedging via tree month option to sell 400 000 euros. On the following table in last column, we can see that the option at the money (ATM) is the most favourable, specifically the option with the strike rate 27 CZK/EUR. This variant of hedging could enable to calculate with revenue amounting to 10 650 950 Czech crowns, i.e. 115 560 Czech crowns more than in the case without hedging. This variant of hedging costed 115 560 Czech crowns.

[5]In this case we supposed that the company can arrange a forward contract without collateral.

Table 3: comparison between unhedged and hedged exposure via three months European put options

Unhedged exposure		**Hedged exposure**			**Profit/loss**
Expected spot rate	Transaction volume (czk)	Strike rate	Transaction volume (czk)	Option premium (czk)	Difference (czk)
26,339	10 535 420	**27**	**10 800 000**	**115 560**	**149,020**
		26,75	10 700 000	712 800	93,300
		26,5	10 600 000	507 600	13,820
		26,25	10 500 000	421 200	-77,540

In both cases, we can see that both variants of hedging are favourable for the company. However, with regard to costs, the hedging via forward is more favourable than hedging via option. The hedging via forward enabled calculate by 225 140 Czech crowns more compared to unhedged exposure. It is also by 72 120 Czech crowns more compared to hedging via option. Therefore, the management's company chose variation of hedging via forward.

5. Assessment of hedging

As we know today the estimates of date of the CNB's exit were relatively precise, because the CNB ended its exchange rate commitment on April 6, 2017. In the first few minutes after the end of the CNB interventions, the Czech crown lost almost 0,5 % to the euro, but at the end of the day the exchange rate was 26,75 CZK/EUR. A temporary weakening of the Czech crown could be due to the effect of the absence of the euro counterparty[6].

The company expected that the date of CNBs exit would be on April 6, 2017. The assumption of the company was right and now we can assess whether the hedging operation was profitable or losing.

The company hedged exposure via three month forward contract with forward rate 26,9014 CZK/EUR. In the case that the company didn't hedge its exposure, according to method of alternative scenarios we expected exchange loss of 264 580 Czech crowns. At the time of maturity forward contract, i. e. May 6, 2017, the spot rate was 26,67 CZK/EUR[7]. This means that the crown was strengthened by 0,33 crown. The forward transaction ended with a profit of 92 560 crowns (0.2314*400 000).

Table 4: real comparison between unhedged and hedged exposure via three months forward

Unhedged exposure		**Hedged exposure**		**Profit/loss**
Spot rate	Transaction volume (czk)	Forward rate	Transaction volume (czk)	Difference (czk)
26,67	10 668 000	26,9014	10 760 560	92 560

[6] In other words, speculators were probably unable to close their short position on the EUR/CZK currency pair because of the absence of the euro counterparty. The short positions of the speculators could have been too big and at one point too many speculators might want to close their positions. The question was who wanted to offer them the euro for a new rate. Enough euros had the CNB, but it did not sell the euro to anyone. It could be expected that CNB will not sell any euro for 27 crowns or less. This means that speculators could not make their profit and had to wait for the moment when the exchange rate got to a stronger value, *i.e.* temporary weakening of the Czech crown (Seznam, 17. 1. 2017).

[7] It is the spot rate of May 9, 2017 – the first working day after May 6, 2017

6. Discussion and recommendation

The method of alternative scenarios was used for quantifying foreign exchange risk. I did not use Value at risk (VaR) - one of the most popular and traditional methods for quantifying foreign exchange risk. The mentioned method usually uses historical data for the development of the exchange rate and it assumes that the exchange rate will behave similarly as in the past. However, the problem was that the Czech koruna against the euro was not trading according to market principles for the fourth consecutive year, and so the method would be based on distorted values. This is the main reason why the VaR method was not used in this paper.

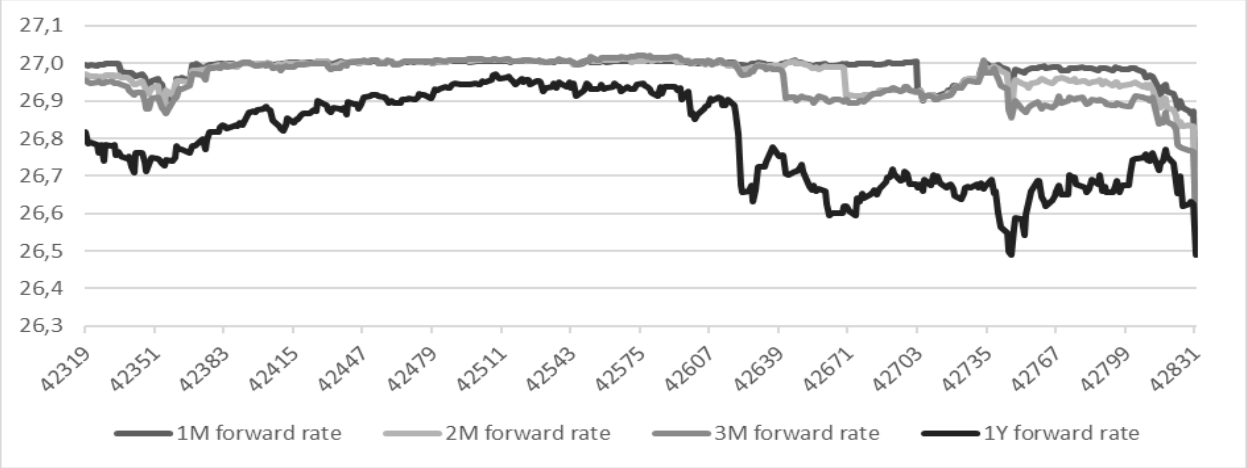
In this paper, I recommend hedging via forward contracts after natural hedging. However, there is an exception and it is hedging income from a tender. Imagine a company which tenders for a foreign contract and a price must be quoted in a foreign currency. In doing so, it creates a potential exposure. Whether or not the foreign exchange exposure will arise depends on whether or not the enterprise wins the contract. If the enterprise wins, the potential exposure transforms into real exposure. For effective hedging, it is necessary to hedge exposure just in time, *i.e.* potential exposure. In this case I recommend hedging via option contracts, because the option gives the holder the right, but not the obligation to sell a particular amount of foreign currency via tender. We can hedge income from tender via forward contracts but if we do not win the tender, it creates additional exposure. This is because forward is a binding contract and we would be forced to deliver the euros even if we did not the tender. The transaction costs could be high because we would be forced to buy euros on the spot market. If we use option contracts and we don’t win a tender, we can let the option lapse and it will only cost the option premium. We can also hedge this exposure via zero cost option strategy. This strategy does not allow participation in the positive development of the exchange rate, but it still leaves the chance of lapsing the option without paying the option premium.

Due to the expected date of the exit, the second half and part of the third quarter 2016 were the best times for arranging a one-year forward. At this time, the exporters could be hedged at 26.90 - 26.95 CZK/EUR. Journalists and bankers have often said that hedging is more expensive every day. It might seem that whoever hesitated with hedging made it much more expensive. However, it was not entirely true.

Let us assume that on March 6, 2016 an enterprise concluded a contract. The contract was payable on April 6, 2017 and the enterprise expected income in euros. At the due date, most companies expected to know the date of the exit. If the company hedged income on April 6, 2016, it would fix the exchange rate of 26,9035 CZK/EUR. However, if the company hedged exposure via a month forward on March 6, 2017, it would fix the exchange rate of 26.9846 CZK/EUR. The months forward would hedge not only for the higher value of the forward rate, but also to mitigate the risk of a later term of date of the exit than our assumption (April 6, 2017).

In the following chart, we can see that during the period of the exchange rate commitment, short-term forward rates were less volatile and offered hedging at higher values.

Figure 3: Forward rates CZK/EUR



Source: own graph, data from Patria

7. Conclusion

In this paper, I recommend a method of alternative scenarios for quantifying foreign exchange risk during the period of the CNB's exchange rate commitment. I advised hedging a payable or a receivable in foreign currency via short-term forward contracts, *i.e.* one months, two months, and three months forward. For potential exposure, for example potential income from a tender, I recommend hedging via option contract.

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Is the Financial Literacy Level of Finance and Law Students the Same?

Barbora Chmelíková, Martin Svoboda[1]

Abstract

The aim of this paper is to examine whether there is a difference in the level of financial literacy between finance students and law students. The financial literacy questionnaire was presented to the students of the Faculty of Economics and Administration and to the students of the Faculty of Law at Masaryk University in the Czech Republic. The study was conducted in the academic year 2015/16. The paper shows the results of the comparative analysis examining the difference in financial literacy among sophomore finance students and law students. The measurement of financial literacy consisted of score measuring basic financial knowledge, financial behaviour score, and financial attitudes score following the OECD methodology. The results indicated the need of financial education among both finance students and law students as the comparative analysis showed no statistically significant difference in the financial literacy level between the sophomore students majoring in finance and sophomore students majoring in law.

Key words

Financial literacy, financial knowledge, financial behaviour, financial attitudes, university students

JEL Classification: A10, A22, A23, I23

1. Introduction

The body of the literature on financial literacy is exponentially growing and its focus shifted over the time from pure financial knowledge to the complex definition of financial literacy including not only financial knowledge but also financial behaviour and attitudes (OECD, 2005). According to the OECD the financial literacy is "*a combination of awareness, knowledge, skill attitudes and behaviour necessary to make sound financial decisions and ultimately achieve individual financial wellbeing*." (Atkinson and Messy, 2012)

Various tools which measure the level of financial literacy were developed, including the OECD toolkit measuring financial literacy of adult population (OECD/INFE, 2015), high-school students (OECD, 2014), or the level of financial literacy in low- and middle-income countries (Scott *et al*., 2013). Apart from these target groups, the financial literacy studies targeted also university students, for instance, in USA (Mandell, 2008), Australia (Cull and Whitton, 2015), South Africa (Shambare and Rugimbana, 2012), China (Moon *et al*., 2014), Turkey (Ozdemir *et al*., 2015), Brazil (Mendes da Silva *et al.*, 2012), and Hungary (Luksander *et al*., 2014).

Among the factors which affect the financial literacy belong gender (Hung et al., 2012), age (Tannahill, 2012), region (Clark, 2014), income (Lusardi and Tufano, 2009), and

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educational attainment (Xu and Zia, 2012). There is a positive correlation between financial literacy and the level of education which implies that the higher is the level of education, the higher is the level of financial literacy (Bumcrot *et al*., 2013). When it comes to the university students, the major is another factor when students with business major are likely to have a higher level of financial literacy (Marcolin and Abraham, 2006). Therefore, the tested hypothesis is whether there is a difference in financial literacy between university students of different majors, particularly between finance and law students.

2. Methodology and Data

The OECD methodology (OECD/INFE, 2015) was used to measure financial literacy. This complex measurement tool was developed by the leading experts and working groups on financial literacy and was used internationally in the measuring of the level of financial literacy among the adult population. This measurement was conducted at least in three waves in different countries worldwide. The measurement itself is complex and contains questions to capture the level of financial literacy from different angles. In particular, it consists of three parts – financial knowledge, behaviour, and attitudes – combined into overall financial literacy score. Each part has a different number of items (questions) which imply that they are implicitly weighted. The maximum of financial knowledge score was 7 points, the highest possible financial behaviour score was 9 points, and the maximum of financial attitudes score was 5 points. All together result in overall financial literacy score with the maximum of 21 points.

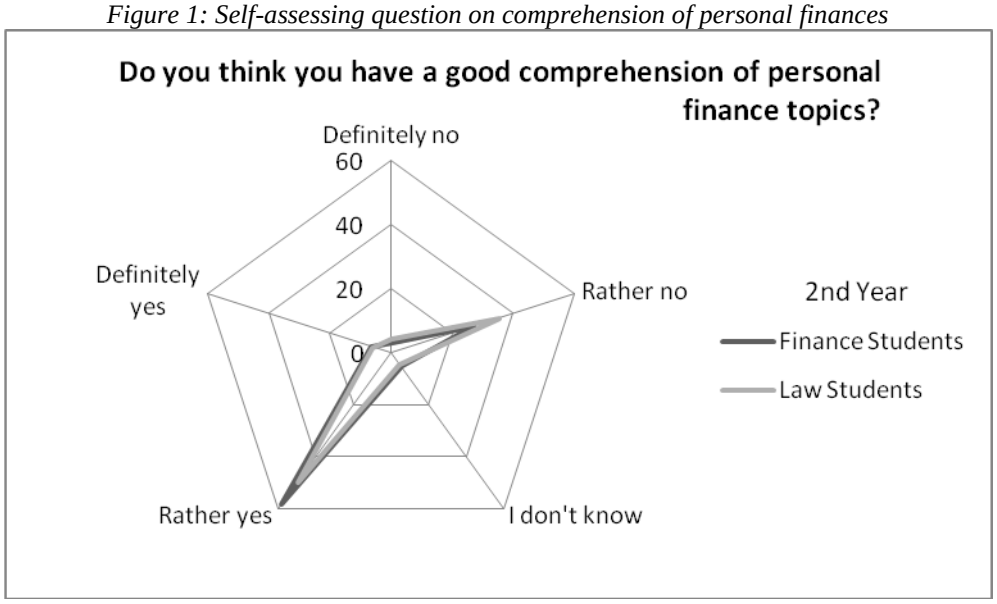
The financial knowledge questions tested the basic financial knowledge and principles, e.g. inflation, time value of money, simple and compound interest, risk and return relation, diversification, interest paid on a loan. The financial behaviour part comprised of attitudinal questions asking respondents, e.g. about their financial habits and behaviour regarding paying bills on time, watching carefully their finances, setting financial goals and following them. The financial attitude questions tested whether the respondents were more forward looking in the terms of their finances and think in the long-term.

The questionnaire was presented via an online questionnaire to the university students at Masaryk University in the Czech Republic, namely at the Faculty of Administration and Economics and at the Faculty of Law in the academic year 2015 and 2016. For the purpose of the comparative analysis was chosen students in the same year of study. The selection was limited to the sophomore students (2nd year). The sample size of finance students was n=153. The sample size of law students was n=359. The age of the majority of students ranged from 19 to 27 years, however, some of the students were older than that as there is no age limit at the university. The gender ratio was relatively balanced when 59% of finance students sample were females, and 50% of law students sample were females. Regarding the work status of sophomore finance students, the 56% of them stated that they study only, while 44% stated that they study and work at the same time. Of the sophomore law students 37% study only, while 63% of them study and work at the same time.

3. Results and Discussion

Prior to the financial literacy questions, the first question in the questionnaire was designed as self-assessment question. In Figure 1 below the students were asked to assess their financial knowledge regarding personal finances. Figure 1 shows that both finance and law students in their 2nd year of study were rather confident and answering positively that they have a good knowledge.

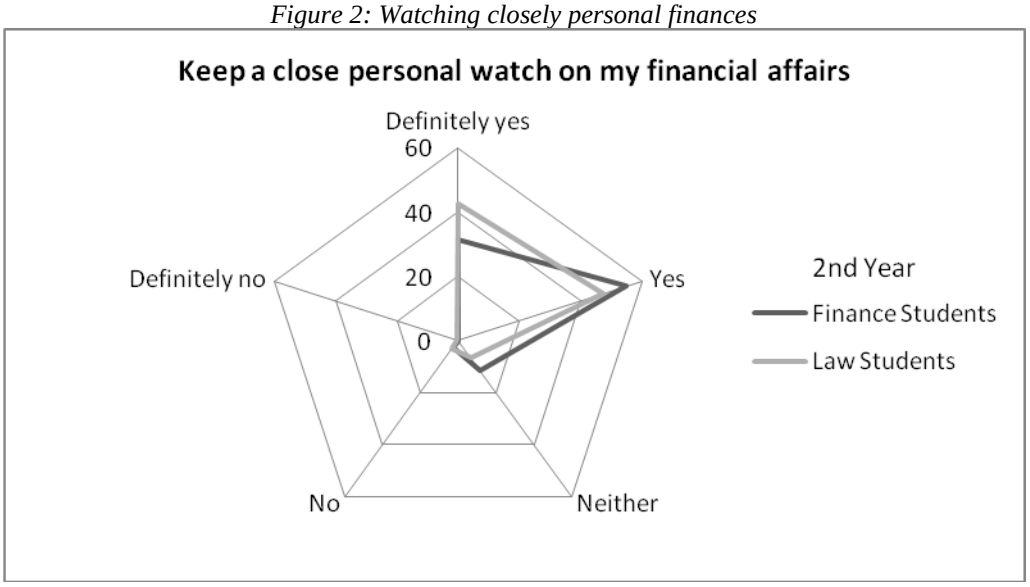
Figure 1: Self-assessing question on comprehension of personal finances



Source: Author's data and computations in SPSS 24.0

In terms of financial behaviour, the students were asked whether they closely watch their personal finances 89% of the law students answered positively (42% definitely yes and 47% yes). Of the finance students, 86% stated that they care about their financial affairs (31% of them answered definitely yes and 55% answered yes).

Figure 2: Watching closely personal finances



Source: Author's data and computations in SPSS 24.0

To examine whether there is a difference in financial literacy between sophomore finance students and sophomore law students, a statistical comparative analysis was conducted. In particular, an independent sample t-test was computed to detect whether there are statistically significant differences between the given groups in financial literacy, financial knowledge, financial behaviour, and financial attitudes. The results of the analysis are presented in the below. Table 1 provides the means, standard deviations, t-statistics with corresponding significance level. The results of independent sample t-test showed that there was not a statistically significant difference (p= .135) between sophomore finance students and sophomore law students in financial literacy.

Table 1: Comparison of Finance and Law Students in 2nd year of their financial literacy

Variable	M	SD	*t*	*df*	*p*
Financial Literacy			1.497	509	.135
Finance Students	15.58	2.386			
Law Students	15.23	2.389			
Financial Knowledge			5.468[a]	347.9[a]	.000
Finance Students	6.13	0.994			
Law Students	5.59	1.166			
Financial Behaviour			-0.586	509	.558
Finance Students	6.38	1.561			
Law Students	6.47	1.435			
Financial Attitudes			-1.339	509	.181
Finance Students	3.07	0.840			
Law Students	3.18	0.836			

[a] The *t* and *df* were adjusted because variances were not equal.

Source: Author's data and computations in SPSS 24.0

Table 1 shows that finance students at the time of measuring in their 2nd year of study scored on average 15.58 points on financial literacy out of 21 points when law students scored on average 15.23 points. However, there was a statistically significant difference between the given groups in their financial knowledge. The finance students scored higher in financial knowledge than law students but the difference, on average, was only around 0.5 points. The average financial knowledge score of sophomore finance students was 6.13 points out of 7 points and the average score of law students was 5.59 points. Furthermore, the difference in financial behaviour was tested revealing no statistically significant difference between finance and law student in their 2nd year of study regarding the financial behaviour score. Finance students scored, on average, 6.38 points out of 9 points and law students scored 6.47 points. Also, the given groups showed as indifferent in their financial attitudes when the analysis did not show any statistically significant difference between finance and law students. The average financial attitudes score of finance students was 3.07 points out of 5 points and the average score of law students was 3.18 points.

The results of the comparative analysis indicated the difference in financial knowledge between sophomore finance students and law students. Therefore, the financial knowledge questions were examined further and in Table 2 the percentage of correct answers of the groups to each financial knowledge question are shown.

Table 2: Financial Knowledge Questions

Question testing:	Finance Students 2nd year[a]	Law Students 2nd year[b]
Time value of money	92.8	88.9
Interest on a loan	99.3	99.4
Calculation of simple interest	96.1	95.0
Compound interest	75.7	71.6
Risk and return principle	96.1	84.7
Definition of inflation	64.5	62.1
Diversification	88.2	57.1

[a,b] percentage of students who answered correctly to the question

Source: Author's data and computations in SPSS 24.0

Sophomore finance students were the most successful in answering correctly the question on interest on a loan, as well as the sophomore law students. The most challenging question was for finance students to define the inflation when more than one third did not know the correct answer. The second challenging question for finance students was to answer the question regarding the principle of compounding interest when the whole one quarter did not answer correctly. As can be noted from Table 2, law students scored lower than finance students on financial knowledge question, hence their higher education curricula did not require to understand the finance basics as is required for finance students since their first year at university. Sophomore law students had difficulties with diversification; more than two fifths did not answer correctly. The definition of inflation was challenging for sophomore law students as well. Almost one third of law students did not know the correct answer regarding the compound interest. Consult Table 2 for more detail.

4. Conclusions

The comparative analysis indicated that there was no statistically significant difference in financial literacy between the sophomore finance students and sophomore law students. Even though, curricula for finance students require to pass courses focused on finance, finance students did not outperform law students who did not have such courses. This implies the need of financial education targeted especially on the financial literacy, not only involving financial knowledge but financial behaviour and attitudes, and sound decision-making.

Acknowledgments

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Efficiency of financial institutions in the Visegrad Group according to Data Envelopment Analysis with dual-role variable

Lucie Chytilová [1]

Abstract

The situation in the financial industry is still problematic all around the world. The region of the Visegrad Group is trying to keep up with another bigger and more stable economies. There are many studies which evaluate efficiency of financial industry in the Visegrad Group. Generally, these studies evaluate efficiency under certainty and for small sample. The problem of missing data or the lack of data which may cause imprecise results, are not taken in mind. The aim of this paper is based on previous research of the risk component. In this case, the aim is to determine the efficiency of the financial industry in the Visegrad Group with inclusion of the risk component as a dual-role variable and see if the new role of the risk component improves the results. The technical efficiencies with/without dual-role variable is estimated and compared for the year 2015. The special DEA models for the risk component and dual-role variable are used. The results of calculated technical efficiencies are analysed in the cross section among the units, country and based on size. Mainly, the role of the risk variable is closely analysed.

Key words

Data Envelopment Analysis, dual-role variable, efficiency of financial institutions, risk component

JEL Classification: C6, G21

1. Introduction

All financial institutions are having difficult time nowadays. The trading activities of the financial industry are rising and structure of the industry is continuously innovating. Therefore, the risk of investment for financial institutions increases. The role of financial institutions is getting more difficult. The main goal of them is to make profit. On the same hand, they have to develop a range of investments products which are attractive to clients. This leads to higher potential of profits which involve more risk and higher probability of a bankruptcy. For this reason, financial institutions are now paying more attention to the high risks attached to the high potential profits. This is also seen in general higher activity of the risk management.

Data Envelopment Analysis (DEA) is mathematical method for efficiency measurement. For the first time, this method was introduced by Charnes et al. (1978). The method has been widely applied in many types of industries – banking, health care and so on. The first applications in the financial industry were mainly upon the economies of scale and scope (McAllister and McManus, 1993), total productivity (Fukuyama et al., 1999), and the efficiency effect (Casu and Molyneux, 2003). Later authors become to model the models

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which included the risk into the equations. There are two views how to treat the risk: exogenously or endogenously. The results with exogenous treatment (Ataullah et al., 2004; Chang and Chiu, 2006) show that the efficiency level is significantly correlated with the risk indicators. The second treatment is in order to analyse banks' efficiency (Chiu and Chen, 2008; Girardone et al., 2004). However, the majority of studies adopt the overdue loan ratio as the substitute variable for risks. Certainly, this does not reflect the characteristic of uncertainty that risks display.

In previous research by Chytilová (2016), there had been used a demographic risk component. The average salary (of each country for the year 2015) has been detected as the best demographic risk component, there had been use the average salary of each country for the year 2015. This component had been used as output variable. It had been seen that this variable is useful for the analysis and it gives more information about the industry. On the other hand, there had been discussions if the strict use of the risk component had been an appropriate. According to these discussions, there had been use new DEA model. DEA model with dual-role variable, model by Cook et al. (2006). In this model, the risk component may be use as output or input. This special model is compared with two basic DEA models. One model where the risk component is define as fixed output and another model, where the risk component is not used at all. This gives better information about the situation how important is the use of the risk component and also if the position of it influence the efficiency as well.

The rest of the paper has the following structure: Section 2 provides the formulations of DEA models and dual-role DEA model. In Section 3, input and output variables are defined. As well as the applications of these variables in financial institutions in the Visegrad Group for the year 2015. Section 4 provides overall conclusions and remarks.

2. Research methodology

Data envelopment analysis (DEA) is non-parametric linear programming based technique for measuring the relative efficiency of a set of similar decision making units (DMUs). Since the work of Charnes et al. (1978), DEA has demonstrated an effective technique for measuring the relative efficiency of set homogenous DMUs. In application, DMUs may include banks, hospitals, different types of industries and other. Each DMU allocates its resources into a number of input variables to produce various output variables. The relative technical efficiency of the unit is define as the ratio of its total weighted output variable to its total weighted input variable or vice versa. DEA allows each production unit to choose its own weights of input variables and output variables in order to maximize its efficiency score. The technically efficient production unit is able to find such weights that it lies on the production frontier. The production frontier represents the maximum amounts of output that can be produced by given amounts of input in the output maximization model or, alternatively, the minimum amounts of inputs required to produce the given amount of output in the input minimization model. DEA calculates the efficiency score for each production unit and identifies peers for each production unit that is not technically efficient.

2.1 Classic DEA models

The first two known DEA models are called CCR and BCC models. CCR model is formulated for the assumption of constant return to scale (CRS). The origin model was extended by Banker et al. (1984) for the assumption of variable return to scale (VRS). There are also other types of models – additive, super efficiency, two stage DEA or DEA models which deal with dual-role variable. All of them are looking for an efficiency frontier that

envelops data and classify DMUs as efficient and inefficient. In this paper, the basic type of CCR model is used and this model is extended by the dual-role variable.

The mathematical formulation of CCR DEA model was done by Charmes et al. (1978). Suppose that there are T DMUs (DMU_k for $k = 1, ..., T$), let input variable and output variable data be $X = \{ x_{ik}, i = 1, ...R; k = 1, ...T \}$ and $Y = \{y_{jk}, \ , j = 1, ...S; k = 1, ...T \}$, u_i, for $i = 1, ...R$ and v_j, *for* $j = 1, ..., S$ be the weights of i-th input and j-th output, respectively. The mathematical model to measure the efficiency score of the under evaluation unit, DMU_Q where $Q \in \{1, ..., T \}$ is following:

$$\begin{aligned} \max\ & e_Q = \sum_{j=1}^{S} v_j y_{jQ} \\ s.t.\ & \sum_{i=1}^{R} u_i x_{iQ} = 1 \\ & \sum_{j=1}^{S} v_j y_{jk} \le \sum_{i=1}^{R} u_i x_{ik} \quad k = 1, \ldots, T, \\ & u_i \ge 0, v_j \ge 0 \qquad i = 1, \ldots, R, \ j = 1, \ldots, S. \end{aligned} \tag{1}$$

As it was mentioned, this model must be solved for each DMU. Notice, DMU_Q is CCR-efficient if and only if $e^* = 1$ and if there exists at least one optimal solution (u_i^*, v_j^*) with $u_i^*>0$ and $v_j^*>0$. Inefficient units have a degree of relative efficiency in interval (1,0]. The model (1) is called a multiplier input-oriented model.

However, for computing and data interpretation is preferable to work with a model that is dual associated to model (1). The model is referred as envelopment input oriented model is following:

$$\begin{aligned} \min\ & \theta_Q \\ s.t.\ & \sum_{k=1}^{T} \lambda_k x_{ik} \le \theta_Q x_{iQ} \qquad i = 1, \ldots R, \\ & \sum_{k=1}^{T} \lambda_k y_{jk} \ge y_{jQ} \qquad j = 1, \ldots S, \\ & \lambda_k \ge 0 \qquad \forall k, \\ & \theta_Q \in (-\infty, \infty), \end{aligned} \tag{2}$$

where λ_k is the weight for DMU_k for $k = 1, ..., T$. It is dual-variable unit. The θ_Q represents the efficiency score of DMU_Q. It can also be interpreted as a reduction rate of inputs to reach the efficient frontier. There are also multiplier or envelopment output-oriented models. These models are not used in this paper and that is why they are not more discussed.

2.2 DEA model with dual-role variable

Original DEA models (described above) assume that the function of input and output variables are strictly defined. Nevertheless, there are cases, where the use of the variables are discussed and not clear. For example, in this paper the used risk component (average sale) may be used as input variable (to be decreased) or as output variable (to be increased).

To see what is the better role, there may be used many types of models. There are even more DEA models which deal with the dual-role variable. In this paper, there had been chosen the model by Cook et al. (2016).

Consider a situation discussed above, where for existing input and output variables, there is a particular factor (for example, average salary) held by each DMU in the amount w_k. This factor serves as both - input and output factor (variable). Adopting CRS model (1) of Charnes et al. (1978) with the knowledge of previous research, the measure of efficiency of DMU "Q" by Cook at al. (2016) model is the solution to the following problem:

$$\max\ e_Q = \alpha \sum_{j=1}^{S} v_j y_{jQ} + \gamma w_Q - \beta w_Q$$

$$
\begin{aligned}
& s.t. \quad \sum_{i=1}^{R} u_i x_{iQ} = 1 \\
& \sum_{j=1}^{S} v_j y_{jk} + \gamma w_k - \beta w_k - \sum_{i=1}^{R} u_i x_{ik} \leq 0, \quad k = 1, \ldots, T, \\
& u_i \geq 0, v_j \geq 0, \beta, \gamma \geq 0 \qquad i = 1, \ldots, R, j = 1, \ldots, S,
\end{aligned} \tag{3}
$$

where γ is the weight for the output variable and β is the weight for input variable. If just the γ is equal to 0, the dual-role variable represents the input variable and if just the β is equal to 0, the dual-role variable represents the output variable. If both weights (γ and β) are equal to 0, then the dual-role variable is in equilibrium. For more details, see work by Cook et al. (2016).

3. Empirical results

This chapter is looking at required data set and the proper use of the risk component. The required data set of all variables have been collected from the Bankscope[2] and Eurostat[3]. This research uses the sample of 25 financial institutions of the Visegrad Group countries in the year 2015.

3.1 Data sources

The definitions of the production process of financial institutions or banking can use different approaches. There are two basic approaches. The first, production approached, was introduced by Sherman and Gold (1985). This approach perceived banks as producers of deposit and loan accounts. The second approach was introduced by Sealey and Lindley (1997) and is called an intermediation approach. This approach sees banks as institutions that transform money from deposits into funds provided to borrowers in the form of loans. Analysis of efficiency in this research is primarily based on the intermediation approach. The bank is seen as an institution that accepts deposits from diverse types of depositors and with the help of fixed capital these funds are converted to loans to various economic subjects.

Another used variable in this research is the demography risk component - the average salary for each country. This variable has been used in previous research by Chytilová (2016) as the output variable. The problem is that after another discussion, there had been found that the use of the risk variable may be different. To see the best use of the risk component, this research uses the risk component as the dual-role variable. Let's say, the model itself decides the best use of the variable.

Table 1 presents the description of all used variables.

Table 1: Description of variables.

Variables	Description in the balance sheet	Unit of measurement
Inputs		
Labour (x_1 – EMP)	Number of employees	Number
Physical capital (x_2 – FA)	Fixed assets = Tangible + Intangible assets	Thousands of Euro
Loanable funds (x_3 – DEP)	Deposits + short term funding's	Thousands of Euro
Outputs		
Advances (y_1 – ADV)	Loans and advances to banks	Thousands of Euro
Investments (y_2 – SEC)	Other securities	Thousands of Euro
Non-interest income (y_3 –	Non-earning assets	Thousands of Euro

2 https://bankscope.bvdinfo.com/

3 http://ec.europa.eu/eurostat

NEA)		
Dual-role		
Salary (z_1 – SAL)	Average salary	Euro

The financial institutions are classified by size and country where they are located (origin). The sample contains 8 financial institutions from the Czech Republic (CZ), 6 financial institutions from Hungary (HU), 6 financial institutions from Poland (PL) and 5 financial institutions from Slovakia (SK)[4]. The classification by size is based on quartile value of fixed assets:

- micro financial institutions - fixed assets is less than quartile Q1;
- small financial institutions - fixed assets is between quartile Q1 and median Q2;
- medium financial institutions - fixed assets is between median Q2 and quartile Q3;
- large financial institutions - fixed assets is greater than quartile Q3.

3.2 Empirical results

Table 2 shows the results for all models. There are 3 types of models:

- Model 1 – model (3) with risk component which is the dual-role variable;
- Model 2 – model (1) with risk component which is used just as output;
- Model 3 – model (1) where risk component is not used at all.

All models are applied for the Visegrad Group financial institutions in year 2015.

From Table 2, it is seen that there are 17 DMUs efficient by Model 1. Model 1 gives the highest efficiency score. More precisely, Model 1 has same or higher efficiency score than Model 2 and Model 3. For example: DMU01 has efficiency score equal to 1.000 (it is efficient DMU) by Model 1. By Model 2 and Model 3, the efficiency score is equal to 0.699 (it is inefficient DMU). This shows that the use of the risk component is important for the analysis. Another question is, how the risk component should be used. Generally, it is seen that the best use of the risk component is as the input variable. More precisely, there are 15 DMUs which use the risk component as input (DMU01, DMU02 etc.), 7 DMUs use the risk component as output (DMU07, DMU09 etc.) and 3 DMUs are in equilibrium (DMU16, DMU19 and DMU23). If the risk component behaves as output in Model 1, the efficiency score in Model 2 is same (this supports the credibility of the results). At same time, the efficiency score in Model 3 is lower (in case of DMU07 and DMU09, efficiency scores are same and they are equal to value one for all types of models). So it is seen that the use of the risk component is helpful for the model. If the risk component is in equilibrium then efficiency score for all models are equal to value one. DMUs are efficient and the use of the component is not important.

As it was mentioned, Model 2 has generally higher efficiency score than Model 3. So really, the risk component is helpful for the analysis. But the question is, if this specific risk component is the best demographic risk component for the analysis. For example, see the case of DMU05, DMU06 and DMU08. Model 3 (model without the risk component) has higher values of efficiency score than in case of Model 2. Still, these values are lower than efficiency score of Model 1.

Table 2: Results of all models for V5 in 2015.

	Origin	**Size**	**Model 1**	**β**	**γ**	**Dual-role**	**Model 2**	**Model 3**
DMU01	PL	medium	1.000	0.00008	0.00000	input	0.699	0.699
DMU02	PL	medium	1.000	0.00179	0.00000	input	0.360	0.360

[4] The list of banks on request form the author.

DMU03	CZ	large	1.000	0.00036	0.00000	input	0.436	0.436
DMU04	CZ	micro	1.000	0.00218	0.00000	input	1.000	1.000
DMU05	CZ	large	1.000	0.00009	0.00000	input	0.699	0.724
DMU06	HU	medium	1.000	0.00032	0.00000	input	0.898	1.000
DMU07	SK	micro	1.000	0.00000	0.00142	output	1.000	1.000
DMU08	HU	small	0.857	0.00027	0.00000	input	0.699	0.703
DMU09	CZ	small	1.000	0.00000	0.00005	output	1.000	1.000
DMU10	PL	large	1.000	0.00179	0.00000	input	0.352	0.352
DMU11	CZ	micro	1.000	0.00029	0.00000	input	1.000	1.000
DMU12	HU	medium	1.000	0.00025	0.00000	input	1.000	1.000
DMU13	PL	micro	0.547	0.00000	0.00073	output	0.547	0.375
DMU14	PL	large	1.000	0.00171	0.00000	input	0.413	0.413
DMU15	HU	large	0.914	0.00018	0.00000	input	0.600	0.600
DMU16	CZ	small	1.000	0.00000	0.00000	equilibrium	1.000	1.000
DMU17	HU	large	1.000	0.00009	0.00000	input	0.503	0.503
DMU18	SK	small	0.377	0.00000	0.00021	output	0.377	0.299
DMU19	CZ	micro	1.000	0.00000	0.00000	equilibrium	1.000	1.000
DMU20	SK	small	0.353	0.00000	0.00013	output	0.353	0.343
DMU21	HU	medium	1.000	0.00270	0.00000	input	1.000	0.642
DMU22	CZ	micro	0.752	0.00000	0.00043	output	1.000	0.729
DMU23	PL	micro	1.000	0.00000	0.00000	equilibrium	1.000	1.000
DMU24	SK	small	0.390	0.00000	0.00021	output	0.390	0.283
DMU25	SK	medium	0.428	0.00008	0.00000	input	0.315	0.315
Average			0.865				0.706	0.671
Min			0.353				0.315	0.283
Max			1.000				1.000	1.000

Table 3 shows results for financial institutions according to their origin. The most efficient financial institutions are in the Czech Republic. They are followed by financial institutions from Hungary, Poland and Slovakia. Results are same for all types of models. The only one difference is in the value of the efficiency score. The highest efficiency score is in Model 1 and lowest is in Model 3. Again, this again supports the theory that the use of the risk component is appropriate.

Table 3: Average results for banks by origin

Origin	Model 1	Model 2	Model 3
CZ	0.969	0.892	0.861
HU	0.962	0.783	0.741
PL	0.925	0.562	0.533
SK	0.509	0.487	0.448

Table 4 shows results for the financial institutions according to their size. It is seen that in this case, the results are different by each model. The results are not so clear. By Model 1, the most efficient financial institutions are the large financial institutions. They are followed by medium and micro financial institutions (efficiency score of financial institutions are close to each other). The least efficient financial institutions are small financial institutions. Model 2 and Model 3 give same results. Even the efficiency score is close to each other. The most

efficient financial institutions are micro, followed by medium, small and large financial institutions.

Table 4: Average results for banks by size

Size	Model 1	Model 2	Model 3
Micro	0.900	0.935	0.872
Small	0.663	0.636	0.605
Medium	0.905	0.712	0.669
Large	0.986	0.501	0.505

4. Conclusion

The nowadays situation in financial industry is discussed all around the world. There are many studies, which evaluate efficiency of financial industry in the Visegrad Group. Generally, these studies evaluate the efficiency under certainty and with the strictly set use of variables. This paper includes demography risk component (average salary, which had been chosen in previous research) and treats this variable as dual-role variable. Based on this, there had been defined three models – one with risk component as dual-role variable, one with risk component as output and one without the risk component. All these models had been calculated and analysed. It had been found that the use of the risk component is helpful for the analysis and it improves the results. Also, that the role of this special variable should be different than in previous research. Based on this research, the dual-role component is mainly seen in role of input variable. There are 15 DMUs, which use the risk component as input, 7 DMUs use the risk component as output and 3 DMUs are in equilibrium.

For future work, it would be good to introduce new variable which will more detect the risk. Also, it would be better to have larger database or try different types of DEA models. Nerveless, based on this study, there should be done relocation and another closer analysis of the reason why in some cases it is better to use the risk variable as input or output.

Acknowledgment

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Delta and Gamma for Gap Options

Iveta Ilavska[1], Marek Durica [2]

Abstract

The paper deals with calculation and analysis of Delta and Gamma parameter for the gap options price. A gap option is an option whose payoff is determined by the pre-specified exercise price, but another constant determines whether or not the payoff is made. This type of exotic option is suitable instrument for hedging of the option portfolio value in many cases. For effective using of the gap option as a hedging instrument there is necessary to check values of the Delta and Gamma for this options. Typical patterns for the variation of these parameters are shown in this paper, too.

Key words

Gap option, Underlying asset, Delta parameter, Gamma parameter, Delta-hedging.

JEL Classification: C00, G13, G23

1. Introduction

Today's uncertain world of finance forces the investors to use various strategies to ensure their portfolio keeps its value. One of the most frequently used methods is the use of various types of financial derivatives, especially options. This gives reason to the existence of a huge number of so-called exotic options and their function is determined by the need of hedging against a certain specific risk. These specific options or their functioning are different to the functioning of the classic (european) options in one or more aspects.

Standard (european) option is the kind of option that gives its owner the right, but not the obligation, to buy (call option) or sell (put option) the given underlying asset in a strictly set time period T (option expiration date) for a previously negotiated strike price X. Hence payment for the option is not obligatory the option can expire without being exercised. This type of option can be used to hedge against the expected drop or rise of the underlying asset price.

Exotic options have much more complex conditions compared to the standard call and put options. These options allow for a much more complex view of the future market trends than the classic options. They are usually traded on an over-the-counter (OTC) market. They basically have unlimited variability and can be adjusted to the needs of specific risk hedging. They therefore play a significant role as a hedging tool. They satisfy not only the need of investors to protect their portfolio value but also have relatively low cost. Exotic options are usually cheaper and more effective than standard options and can be used as an attractive investment and trade tools. (Whaley, 2006)

We can differentiate the options depending on whether immediate option realization would lead to profit or loss as follows. If the immediate realization leads to profit, we say that the option is so-called in-the-money (ITM). If by contrast the immediate realization leads to

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a loss, we say that the option is so-called out-of-the-money (OTM). The option is at-the-money (ATM) if it is located right between the ITM and OTM. (Hull, 2012)

2. Option pricing

2.1 Black-Scholes-Merton model (BSM model)

The analytic solution of this model still remains the most used approach for the classic european call and put option pricing. BSM model considers five key factors that influence the final option price: the underlying asset price, its volatility, risk-free interest rate, strike price and the option expiration date. These conditions are further added: transaction costs and taxes are not considered, arbitration chances are excluded, bond trade is continuous, the bonds are arbitrarily divisible, short selling is possible, risk-free interest rate is constant and remains the same for all expiration dates, option price is modelled by Geometric Brownian motion with constant drift and mean value.

The solutions of the BSM model in classic european call and put option pricing are as follows (Black and Scholes, 1973; Merton, 1973)

$$c_{BSM}(S,X,T) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2); \tag{1}$$

$$p_{BSM}(S,X,T) = Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1); \tag{2}$$

$$d_1 = \frac{ln(S/X)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}; \tag{3}$$

where: c_{BSM} – european call option price;
p_{BSM} – european put option price;
S – current underlying asset price;
X – strike price;
T – expiration date;
r – risk-free interest rate;
q – dividend yield on underlying asset;
σ – asset price volatility;
N – standard normal distribution function.

2.2 Alternative numerical pricing methods

Monte Carlo simulation method is a widely used approach to numerical determination of the option price. It is necessary to find the expected underlying asset price on the expiration date however this price is random variable. One way to estimate the predetermined underlying asset price is to simulate its process. This method can be used to set the price of almost all types of options. (Boyle, 1977; Boyle, Broadie and Glasserman, 1997; Adamko, 2015)

Another numeric option pricing technique is the binomial tree model. The point of it is simplifying the BSM approach in a way that the underlying asset price is considered only in discrete times. This model is usually used in european and american options pricing but it is also applicable in more complex exotic option pricing. (Cox, Ross and Rubinstein, 1979)

The Finite difference method is also one of the numerical methods used in option pricing. This method is based on a fact that the whole solution set is limited to finite number of points and also in substituting the partial derivations of the BSM model with the given differences. Thanks to this we get an equation or a system of equations where the solution is a numerical estimate of the option price. This method can also be used for more complex types of exotic options even in cases when it is impossible to price these options analytically. (Hull and White, 1990; Svabova, 2014; 2015a; 2015b)

3. Delta-hedging

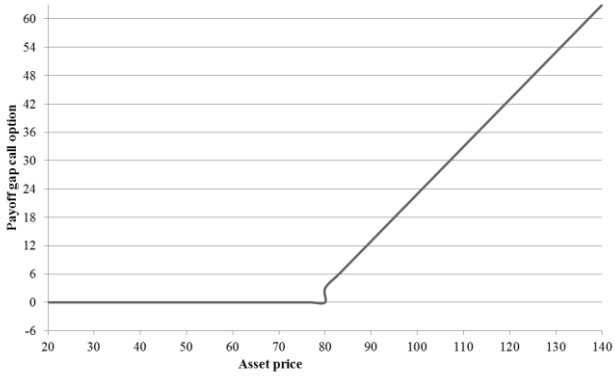
Investor who uses the derivatives in hedging tries to compose his portfolio so that it is resistant to small changes in underlying asset prices in the next short time period. This type of hedging is called delta-hedging. Delta parameter Δ is defined as the change of financial derivative price with respect to underlying asset price change. This means that the investor tries to build their portfolio to be delta-neutral. Such portfolio consists of assets and options in such proportion that the profit (or loss) from the option position tends to be offset by a loss (or profit) in asset position. Delta-neutral portfolio is created by selling one call option and then buying Δ underlying assets. The yield of this portfolio in a short term time frame will then be equal to risk-free interest rate.

Possible problem in this portfolio is the fact that the Delta parameter is time-dependent and if its value changes the investor has to re-evaluate his portfolio to stay delta-neutral, i.e. buy or sell part of the assets. In practice two types of delta-hedging are used: Dynamic delta-hedging, where the portfolio is being regularly re-evaluated to stay delta-neutral but that means it cannot be done continuously and demands high transaction costs. The opposite is static delta-hedging, where the delta-neutral portfolio is created at the start and is not further re-evaluated assuming that the value of Delta stays constant. This assumption is naturally oversimplified.

4. Gap option and its pricing

Gap option is an option where the payoff, i.e. profit of option holder coming from the option ownership, is determined by the strike price X_1, but another price X_2, also known as „trigger price“, determines whether or not the payoff is made. For example the gap call option has the payoff $S_T - X_1$, if the underlying asset price on the expiration date of option S_T is bigger than X_2, otherwise the payoff is zero. The difference between gap call option and a classic european option with strike price X_2 is that the payoff in case of the gap option is higher and equals the $X_2 - X_1$ (with the condition that $S_T > X_2$). The following figure illustrates the payoff of gap call option. (Whaley, 2006)

Figure 1: Payoff of gap call option; $X_1 = 77$ *a* $X_2 = 80$



Gap call option can be valued using a small modification in the BSM model. The gap call option price is then

$$c_{gap}(X_1, X_2) = Se^{-qT}N(d_1) - X_1e^{-rT}N(d_2); \quad (4)$$

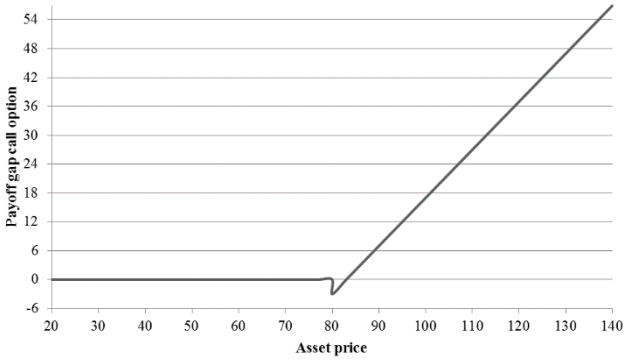
$$d_1 = \frac{ln(S/X_2)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}. \quad (5)$$

Provided that $X_2 > X_1$, the gap call option price will be higher than the corresponding european call option with strike price X_2 by a margin of $(X_2 - X_1)e^{-rT}N(d_2)$. This is

because the probability that the option will be exercised is $N(d_2)$ and in the case that option is exercised payoff is greater than payoff of the classical option by $X_2 - X_1 > 0$. (Hull, 2012)

It is important to note that there is no restriction to whether the trigger price X_2 is higher than the strike price X_1 or not. If $X_2 = X_1$ then the gap option is actually the classic option with this strike price. However in case $X_2 < X_1$ we get a very unusual payoff structure (Fig. 2). (Whaley, 2006)

Figure 2: Payoff of gap call option; $X_1 = 83$ *a* $X_2 = 80$



It is evident that there exists an interval of underlying asset price when the option holder may exercise his option but this would yield a loss. This of course makes no sense and therefore the holder will for sure not exercise the option hence we can consider the payoff in this interval a zero. However what we get after this is the standard call option payoff. For this reason we don't need to be concerned with this case any further.

The gap put option payoff is $X_1 - S_T$, if $S_T < X_2$, otherwise it is zero. We can derive a formula for the gap put option price by a small modification in the BSM model

$$p_{gap}(X_1, X_2) = X_1 e^{-rT} N(-d_2) - S e^{-qT} N(-d_1), \tag{6}$$

where d_1 and d_2 were defined in (5). (Hull, 2012)

5. Parameter Delta of Gap Option

This parameter describes the influence of change in price of underlying asset on the gap option price change. The asset price S on the spot market is one of the main factors influencing the gap call option price. If we assume that other input parameters remain constant then underlying asset price increase on the spot market while the strike price remains the same causes that the gap call option price must increase as well. Hence for the same strike price we are able to buy an asset with higher price and we have to pay more. The investor needs to know by how much the gap call option price changes with a certain asset price change on the spot market. This change, and therefore the Delta parameter can be expressed using a partial derivation of the gap call option with respect to parameter *S*.

After the gap call option function (4) derivation and some minor adjustments we get

$${}_c\Delta_{gap} = \frac{\partial c_{gap}}{\partial S} = e^{-qT} N(d_1) + \frac{e^{-qT} n(d_1)}{\sigma\sqrt{T}} \left(1 - \frac{X_1}{X_2}\right), \tag{7}$$

where all symbols have the same meaning as before and $n(.)$ represents the density of standard normal distribution. The gap call option price changes by this value when the underlying asset price changes by one unit. This relation makes it evident that the Delta parameter value is positive and in most cases less than 1. However it can gain much higher values in case the current underlying asset price is close to the trigger price X_2 but mainly in case of a short option expiration *T*. The following figure illustrates typical Delta parameter dependency on the asset price for this option type and various expiration dates.

Figure 3: Delta of gap call option ($X_1 = 80, X_2 = 85$)

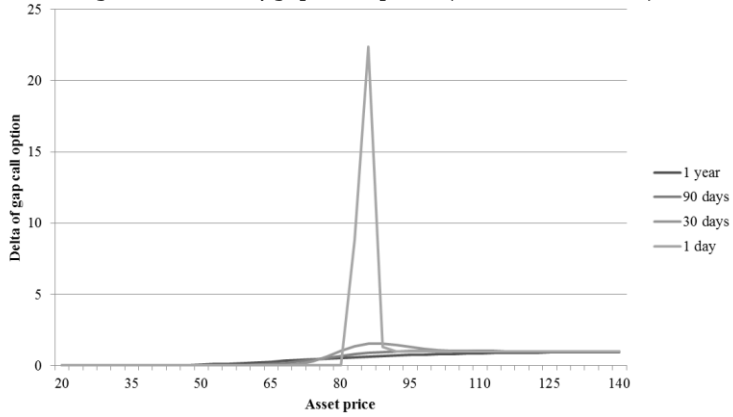


Figure 3 and formula (7) clearly show that the gap call option price is most sensitive to changes in the underlying asset price when this price is close to trigger price X_2. It also shows that Delta parameter sensitivity increases as the option expiration time decreases. The option price is less sensitive to changes in the underlying asset spot market price for the OTM and ITM options.

It can be proved that for the OTM option, i.e. in case $S \ll X_2$ (meaning the option is worthless), the value of ${}_c\Delta_{gap}$ is close to zero. That means when the underlying asset price changes by a unit, the corresponding gap call option price does not change or changes very little.

By contrast, when the gap call option is ITM, i.e. $S \gg X_2$ (meaning the option will very probably be exercised) the value of ${}_c\Delta_{gap}$ is close to one. That means when the underlying asset price increases by a unit, the corresponding gap call option price also increases by approximately one unit. The same applies in case the asset price decreases by a unit.

6. Parameter Gamma for gap options

Parameter Gamma Γ of the derivative portfolio is a value of the Δ parameter change depending upon the underlying asset price change. If the Gamma is small the Delta parameter change is only minimal therefore the investors don't need to often re-evaluate their portfolio if they want to keep it delta-neutral. By contrast, if the Gamma parameter value is higher, then this portfolio's Delta is sensitive to underlying asset price change. It is therefore risky to not re-evaluate this delta-neutral portfolio for a longer period of time.

If we want to calculate the Gamma parameters for the gap call option it is necessary to calculate the partial derivation of the Delta parameter for this option according to the underlying asset price S. It is clear from the way the Gamma parameter is defined. Therefore the Gamma parameter Γ for the gap call option equals

$$ {}_c\Gamma_{gap} = \frac{\partial\ {}_c\Delta_{gap}}{\partial S} = \frac{e^{-qT} n(d_1)}{S\sigma\sqrt{T}} - \frac{e^{-qT} n(d_1) d_1}{S\sigma^2 T}\left(1 - \frac{X_1}{X_2}\right), \qquad (8) $$

where the meaning of all the symbols remains same as before. This parameter dictates by how much the Delta parameter value changes when the underlying asset price changes by a unit.

The following figure illustrates typical Gamma parameter dependency on the underlying asset price and also how the option expiration date influences the Gamma parameter of this option. This figure clearly shows that the Delta parameter is most sensitive to changes in the underlying asset price when this price is close to X_2values, i.e. for the ATM option. This is valid independently on the expiration date of this option, although this sensitivity increases as the expiration time grows shorter.

The Delta parameter value is less sensitive to changes in the underlying asset price for OTM and ITM options and this sensitivity decreases as the expiration time decreases.

Figure 4: Gamma of gap call option ($X_1 = 80, X_2 = 85$)

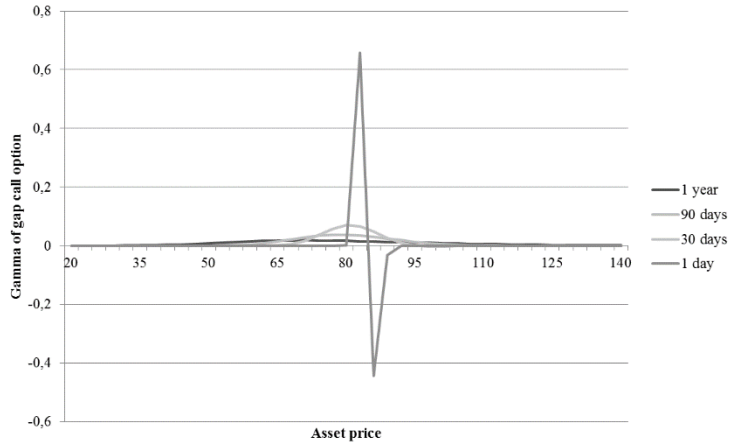


Figure 4 and formula (8) both show that the call option Delta parameter value is most sensitive to changes in the underlying asset price when this price is close to trigger price X_2 (option is ATM). It also shows that Delta parameter sensitivity increases as the option expiration time decreases. This implies that the Delta parameter value of the corresponding gap call option changes by a relatively large amount if the underlying asset price changes by a unit. Even in case when $T \to 0$, the value ${}_c\Gamma_{gap} \to \infty$.

For the OTM option, i.e. in case $S \ll X_2$, or for the ITM option, i.e. $S \gg X_2$, the Delta parameter value is less sensitive to the underlying asset price change. Furthermore, if the asset price is far from trigger price, then ${}_c\Gamma_{gap} \approx 0$. This means that the Delta parameter value of the corresponding gap call option changes only very little if the underlying asset price changes by a unit. Portfolio built from such options has a tendency to stay delta-neutral not only with minor changes in underlying asset price but with larger ones as well.

7. Conclusion

The aforementioned analyses make it evident that the gap option is sensitive to changes in the underlying asset price especially for the ATM option. It is also clear that this sensitivity increases as the expiration date of an option decreases. The reliability of the Delta parameter for larger changes in underlying asset price on the spot market is limited and a more precise option price change evaluation tool is needed. In such case we utilize the Gamma parameter using the properties of Taylor series. The expected change in option price is calculated as an aggregate (Durica, 2015; Durica and Svabova, 2013; Svabova, 2015b)

$$\Delta + \frac{1}{2}\Gamma. \tag{9}$$

This method is significantly more precise which is true especially of a high value portfolio and larger changes in underlying asset price.

If the gap option is ITM or OTM, i.e. underlying asset price being distant enough from the trigger price X_2, then static delta-hedging is enough for the investors to secure their portfolio. It is enough to create a delta-neutral portfolio and re-evaluate it only in case of larger changes in the underlying asset price. In the case that the underlying asset price gets closer to X_2, i.e. option is ATM, it is better to use the proposed Delta parameter modification (9) or to use dynamic delta-hedging with more frequent portfolio re-evaluation which on the other hand can bring higher transaction costs. As the considered gap option expiration date draws closer it naturally becomes necessary to watch the changes in underlying asset price more closely because the gap option price is very sensitive to these changes in such case.

Of course, it is necessary to observe the dependency of the gap option on its other parameters as well, for example the risk-free interest rate change, etc. Time influence is also important, i.e. observing the Theta parameter of a given option.

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Development and application of the Industry 4.0 principles in the selected firms and areas in the Czech Republic

Hana Janáčková, Kateřina Kořená [1]

Abstract

The article highlights the extensive digitization and use of robots in production. Individual countries have begun to take the initiative in this direction and have begun to prepare for a new technological future, including the Czech Republic, which has approved the National Industrial Initiative Industry 4.0 in 2016. The goal of the paper is to analyse the nature of this phenomenon and to show concrete examples in the Czech Republic. This article defines and describes the basic principles of Industry 4.0, too. Furthermore, their implementation is analysed in the selected major firms in the Czech Republic. In the next part, the article is focused on the reaction of universities based on the requirements of corporate practice with respect to the education of new specialists. The last part deals with issues related to ethics and safety of products produced within Industry 4.0 when society have to face necessary consequences of Fourth Industrial Revolution.

Key words

Industry 4.0, Cyber-Physical Systems, Digitization, Virtualization, Internet of things, Development and Utilization of Industry 4.0.

JEL Classification: L69

1. Introduction

Industry 4.0 is the name of the initially German initiative Industry 4.0, which goaled to point to the fact that large-scale digitization and the use of robots in production were currently in place, so it was possible to talk about the beginning of the so-called Fourth Industrial Revolution according to Schwab in World Economic Forum (2016). This vision was first presented at the Hanover trade fair in 2011 where, according to Harris in The Engineer (2013), the term Industry 4.0 was first used by a group of scientists and industrial workers. The concept of Industry 4.0 has then begun worldwide, other countries have begun to take the initiative in this direction and have begun to prepare for a new technological future, including the Czech Republic.2 In the Czech Republic, in 2015, the National Industry Initiative 4.0 was developed. It was approved by the Czech government in 2016 and the main coordinator was the Ministry of Industry and Trade.

In the researched sources, Industry 4.0 is defined in various ways. E.g. Sackey and Bester (2016, 104) report that Industry 4.0 is a common term for technology and value chain of the

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[2] E.g., in 2014, the Industrial Internet Consortium and Advanced Manufacturing Partnership 2.0 programmes emerged: Made-in-China 2025 in China, Manufacturing Industry Innovation 3.0 in South Korea, and Industrie du Futur in France in 2015.

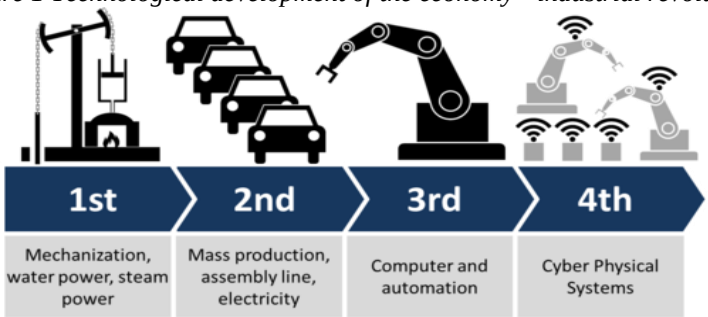
organization that combine cyber-physical systems, the Internet of things and the Internet of services. And according to Kamensky (2017) it is also a social system because it is necessary to construct their value matrix, too.

Because a lot of Czech authors don´t pay so much attention to this phenomenon the goal of this paper is to analyse the nature of this reality and to show concrete examples in the Czech Republic. In the first part the article explains the basic principles of its functioning in the form of the use of autonomous robots and multi-agent systems, digitization, virtualization and the creation of so-called Internet of things. Then it deals with the implementation of these principles in the selected major firms in the Czech Republic. Subsequently, it describes the response of higher education institutions to the demands of practice through the introduction of new accredited degree programmes. In the last part of the paper, it raises possible questions that will need to be addressed in the near future in connection with the introduction of the principles of Industry 4.0. The method of analysis, comparative and descriptive methods and method of induction at processing primary resources are used in this paper.

2. Outline of the development of industrial revolutions

In the course of the economic development, the technological progress gradually brought about four industrial revolutions. The First Industrial Revolution began in the 18th century in England, and it was based on a change in the organization of production and a change in the drive power of machines. At that time, the first manufactories were formed and there was a massive migration of workers from villages to towns and cities. The Second Industrial Revolution took place in the second half of the 19th century and meant the use of railways, telegraph and standardization of industrial products and individual components. It brought the beginning of assembly line production, the gradual disappearance of manufactories, and the emergence of large industrial firms and joint stock companies. New combustion engines and electricity became new sources of energy. The Third Industrial Revolution is based on microprocessors, the automation of individual production lines, and the growing use of computers. The beginning is connected with the 1970s. At present, a number of firms have begun to use the principles of the Fourth Industrial Revolution in production, so called Industry 4.0.

Figure 1 Technological development of the economy - industrial revolution



Source: Christoph Roser at AllAboutLean.com, https://commons.wikimedia.org/w/index.php?curid=47640595

3. The Fourth Industrial Revolution and the basic features of Industry 4.0

The fourth industrial revolution means the emergence of cybernetic-physical systems. CPS – Cyber-Physical Systems take on repetitive simple activities that people have done so far. These systems are an essential element of so-called intelligent factories, they are able to

exchange information autonomously, initiate the necessary actions in response to the current conditions, and check each other independently. Such interconnected cybernetic-physical systems interact with each other by means of standard communication protocols on the Internet basis, and analyse the data in order to predict possible errors or failures – i.e. they can configure themselves and adapt to altered conditions in real time (Mařík et al.).

CPSs are based on the following basic principles:

- use of autonomous robots and multi-agent systems,
- expanding digitization in production and use of virtualization,
- the emergence of the so-called Internet of things.

Use of autonomous robots and multi-agent systems

An autonomous robot (agent) is a robotic device that works independently – that is, it is not driven by people, but real-time programmes capable of separate analysis of the environment. The robots are able to learn on the basis of feedback. Multi-agent systems are made up of a multi-agent environment and the population with multiple agents, where the agents interact with each other and with the environment in which they are located.

Digitization and virtualization

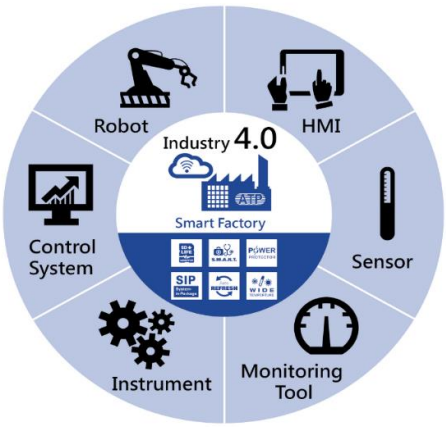
Digitization of industry is the process of introducing digital technologies into production. Based on the individual demand of specific customer requirements, a so-called digital twin is created, which is a virtual image of the future product. During production, this is subsequently connected with a real physical object. Production machines have access to these requirements, and the digital twin of the product tells the machines what they are supposed to do. Thus, the production logic rotates.

To create digital twins, it is necessary to make greater use of your computers' capacity, allowing virtualisation that is possible at different levels – it can be a whole machine (so-called virtual machine), individual hardware components (e.g. virtual processors, virtual memory), or possibly just software environment (operating system virtualization). Users can thus simulate and optimize the sequence of all individual processes before real production.

The Internet of things

The Internet of things means the existence of modern devices that are easy to control remotely via the Internet. In the case of industrial production, an "intelligent" factory is created through integrated applications that customers can use to optimize manufacturing systems and processes throughout the plant and product life cycle.

Figure 2 Elements of Industry 4.0



Source: Embedded-Computing Design http://embedded-computing.com/white-papers/white-0-challenges-solutions-storage-devices/.

4. Use of Industry 4.0

Production based on the principles of Industry 4.0 can now be used in a variety of areas ranging from the environment through home automation to industrial deployment where tailor-made products can be demanded and production can still respond to this demand without having to reduce its volume.

4.1 Changes in the consumers' demand

The fundamental change is that consumers propose products tailored to their specific requirements. This leads to the individualization of the demand for the product, where the manufacturer creates a digital twin for a particular consumer through visualization. Production can only take place when needed, which can both simplify the consumer's planning process and reduce storage requirements. At the same time, the buyer can obtain the product at a significantly shorter time than in the case of the conventional method of production.

4.2 Current possibilities of utilizing Industry 4.0

The possibilities of Industry 4.0 are really broad. In fact, the application of Industry 4.0 is possible in the collection of municipal waste where the containers themselves monitor the volume of waste, its structure and the need to transport or replace the entire container. An example in the building industry is the so called "intelligent house", which provides an optimal indoor environment for the comfort of people living in this house. Similarly, it is possible to use the elements of Industry 4.0 in the chemical industry for the manufacture of "self-mixing" mixtures, or for the production of sports shoes in the shoe industry. In prosthetics, prosthetic aids can be made specifically for a particular patient. Industry 4.0 finds a considerably wide application in the automotive industry. All of these products can be manufactured on the basis of individual customer demand, while increasing production.

4.3 Specific examples of Industry 4.0 use in the Czech Republic

In the Czech Republic, Industry 4.0 started to be used mainly in firms that are multinational and in which this element is therefore transferred to production in the Czech Republic. Significant representatives in terms of sales and number of employees are Siemens, ABB, and ŠKODA AUTO.

4.3.1 Siemens and Industry 4.0

In the near future, an intelligent factory, where the individual components on the production line communicate themselves and the machines and the semi-products deliver information to each other, will be introduced in all 8 branches in the Czech Republic by the Siemens parent company. In total, these factories have 10,000 employees and their sales were CZK 31 billion in 2015. Thanks to the introduction of smart elements into production, in particular, the production time should be reduced. Siemens Mohelnice plant in the Czech Republic has also begun to reduce emissions thanks to smart building management. Only one station has started to monitor and control the energy systems of the thirty-hectare plant with a number of production halls, since each led lamp in the illumination system has its own IP address through which it reports its operating status. In addition, the system controls compressors, solar panels, or water heaters, and it also monitors the consumption of water, natural gas, and compressed air.

In the area of digitization of production, Siemens also launched, in cooperation with IBM, its own MindSphere cloud solution to store data on the operation of its machines and equipment. They will then be able to analyse and optimize them to help optimize production. The analysis will also result in recommendations for timely maintenance.

4.3.2 ABB and Industry 4.0

ABB is the world's leading supplier of power and automation technology and employs more than 3,400 people in 8 plants in the Czech Republic; in 2015, the ABB's sales were CZK 14 billion. This firm has developed a YuMi industrial robot3 with two arms for mounting e.g. electrical outlets. The robot will be working together with people on a single production line designed to assemble electrical outlets at ABB Ltd., Elektro-Praga. The device is made up of flexible arms, parts feeding system, camera system for component placement, and robot control at the top level. Thanks to a consistently safe construction, this robot, working with people, ensures complete safety for the environment and increases the standard of factory production by accurately repeating operations4.

4.3.3 ŠKODA AUTO and Industry 4.0

The joint stock company ŠKODA AUTO is a major Central European passenger car manufacturer based in Mladá Boleslav, another 2 production plants are in Kvasiny and Vrchlabí. It has been part of the German Volkswagen Group since 1990. ŠKODA AUTO has been one of the largest employers with more than 28,000 employees in the long term; in 2015, its revenues exceeded CZK 314 billion, representing 1,055,000 cars. As part of the implementation of the Industry 4.0 elements, ŠKODA AUTO explores possibilities in logistics, component manufacturing, and the quality of delivered parts, final vehicle assembly and maintenance. An example is the ProGlove designed to optimize logistics work. This is an electronic glove with an implemented scanner that is currently used by the company ŠKODA AUTO. The smart glove is designed to make it easier for the wearer to make all the working steps easier, and it can show, for example, whether the correct part is used or whether the individual work steps have been performed correctly. ŠKODA brand's logistics currently employs more than 3,000 people who plan and provide automotive logistics around the world.

Overall, these individual projects are goaled at the concept of the so-called intelligent factory. An important part of this concept is the Transparent Factory project, which is an automatic data collection system from all workplaces to the central database. The main benefit is access to data from anywhere via a computer, tablet or smartphone, which will speed up all processes in the plants.

4.4 Education for Industry 4.0 in the Czech Republic

Education in the Czech Republic is beginning to reflect the current state resulting from the digital revolution – in the autumn of 2016, teaching in newly accredited Bachelor's and Master's degree programmes began. The Faculty of Electrical Engineering and Computer Science, VŠB-TU Ostrava, offers the study programme Computer Systems for the 21st Century, in which informatics is an effective tool for problem solving, and electrical engineering and electronics are the basis for developed and deployed computer systems in a number of application areas emerging in the digitally oriented world of the future. The accredited Master's Programme Industry 4.0 is open at the Faculty of Mechanical Engineering of the Czech Technical University in Prague.

5. Questions related to the principles of Industry 4.0

With the introduction of humanoid systems and robots, problems arise with regard to specific situations – such as who shuts down the robot, who will be responsible for how the

[3] YuMi means "You and Me".

[4] ABB is currently cooperating with the Faculty of Electrical Engineering and Computer Science at VŠB –Technical University of Ostrava.

robot anticipating future steps is programmed, what intelligence is inserted into it, where and who will have access the collected data, etc. It is also important who will establish the safety rules of operation and how to set the rules of ethics and the use of these systems. Besides these humanoid systems have artificial intelligence but it is intelligence without conscience. These are questions that go beyond this paper, but they have to be kept in mind and addressed in the near future.

Last but not least, reducing employment rates can lead to a decrease in the collection of direct taxes, health and social insurance contributions for public budgets, and thus, on the other hand, a reduction in spending these public budgets, or a reduction in the number of people who will have access to these funds from public budgets.

6. Conclusion

The paper focuses on the essence of Industry 4.0 and the basic principles of its operation in the form of the use of autonomous robots and multi-agent systems, digitization, virtualization, and the emergence of the so-called Internet of things. It also deals with the introduction of these principles in the selected major firms in the Czech Republic. It is clear that Industry 4.0 is a new challenge for the firms and thus they have to accommodate to it their whole internal firm system. Then the paper describes the response of universities to the requirements of practice through the introduction of new accredited degree programmes - this is the possibility how to change the education of new specialists. Last but not least it is evident that in future the society will face necessary consequences of implementation and rules of using of humanoid systems and robots which can change employment and fiscal situation.

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Impact of Banking Union on the Banking industry in Slovak Republic

Eva Jančíková[1], Janka Pásztorová[2], Leonid Raneta[3]

Abstract

The financial and debt crisis highlighted the importance of a healthy banking sector for the functioning of the economy. The globalisation in this sector means that problems of a bank can affect not only the economy of the parent country but also the economies of the daughter companies countries. In the Slovak Republic this problem is even more serious because of the level of internationalization of the banking sector which is the highest in EU. At present, the share of foreign capital in banks in Slovakia is about 95%. The aim of the article is to analyze the current state of the Slovak banking sector and to assess the impact of the Bank Union on its functioning.

Key words

Debt Crisis, Banking Union, Banking Supervision

JEL Classification: G01, G18

1. Úvod

Finančná kríza, ktoré vo veľkej miere otriasla ekonomikami krajín EÚ znovu upozornila na to, aký význam má zdravý bankový sektor na fungovanie ekonomiky. V posledných rokoch dochádza však v tomto sektore k takej globalizácie, že problémy niektorých bánk môžu ovplyvniť nielen ekonomiku materskej krajiny ale aj ekonomiky krajín, v ktorých majú svoje zastúpenie. V Slovenskej republike je tento problém o to vážnejší, že internacionalizácia bankového sektoru je najvyššia v celej EÚ. V súčasnosti dosahuje podiel zahraničného základného imania v bankách na Slovensku okolo 95%. Cieľom článku je analyzovať súčasný stav bankového sektora SR a zhodnotiť vplyv bankovej únie na jeho fungovanie.

2. Opatrenia na stabilizáciu ekonomík členských krajín EÚ

Priebeh finančnej krízy viedol k zvyšujúcej sa zadlženosti členských štátov EÚ, čo malo za následok rastúce náklady na verejné financie, neregulovaný pohyb kapitálu, špekulatívne investície do rizikových dlhopisov v neobmedzenom objeme, chýbajúca likvidita bánk, zvyšujúce zadlžovanie obyvateľstva, rýchlo klesajúci dopyt po výrobkoch a službách, spomalenie hospodárskeho rastu a stúpajúcu nezamestnanosť. Všetky tieto skutočnosti znamenali pre väčšinu krajín eurozóny problémy, na ktorých riešenie neboli pripravené a vo

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Rozdielnosť vo vyspelosti a výkonnosti národných ekonomík (HDP a jeho rast, produktivita pracovnej sily, mzdy) spolu s ich rozdielnou ekonomickou kondíciou (dlh verejných financií a štátny rozpočtový deficit) sa stali pre eurozónu veľmi zraniteľným prvkom. Tieto rozdiely boli dlho podceňované a akceptované. Okrem toho neplnenie dohodnutých a záväzných ukazovateľov a absencia sankcií a dôsledkov v minulosti sa stali pre eurozónu osudnými, keďže sa to negatívne odrazilo v ekonomicky slabších krajinách, ktoré boli ale dostatočne veľké na to, aby stiahli so sebou aj ostatné krajiny eurozóny. Táto nerovnováha medzi krajinami na jednej strane a tolerantný prístup ECB a ostatných európskych finančných inštitúcii nakoniec spôsobili značné problémy. Ekonomické problémy niektorých členských krajín (napr. Grécka) sa pritom negatívne vyvíjali počas dlhšieho obdobia a neboli dostatočne riešené zo strany ECB a Európskej komisie. Európska centrálna banka spolu s národnými centrálnymi bankami členských štátov eurozóny musela v relatívne krátkom čase predložiť návrh na riešenie vzniknutej situácie a opatrenia a nástroje v oblasti menovej a finančnej politiky na úrovni EMÚ na zmiernenie dopadu finančnej krízy.

V prípade menovej resp. finančnej politiky to boli opatrenia ECB hlavne v oblasti operácii na finančnom trhu ako zníženie úrokových sadzieb, ovplyvňovanie likvidity, vytvorenie záchranných balíkov pre pomoc ohrozených ekonomík a nastavenie pravidiel finančnej disciplíny na stabilizáciu verejného dlhu a obnovenie dôvery na finančnom trhu.

Okrem opatrení na riešenie vzniknutej situácie, európske orgány riešili aj to, ako eliminovať vznik podobnej situácie v budúcnosti. Išlo hlavne o tzv. *Európsky semester*, ktorý má za cieľ koordinovať hospodárske politiky EÚ a národných vlád platný pre všetky členské štáty a o *Pakt stability a rastu* s cieľom koordinovať hospodársku a rozpočtovú politiku, dohliadať na rozpočtovú disciplínu a zamedziť nadmernému deficitu a dlhom. Pre štáty EMÚ išlo aj o kontrolu národných rozpočtov a postihy za nedodržanie stanovených parametrov. Ďalším prvkom bola fiškálna dohoda, ktorá vznikla na základe Zmluvy o stabilite, koordinácii a správe v hospodárskej a menovej únie s cieľom udržiavať národné rozpočty v rovnováhe.

Ako sme už v úvode spomenuli, finančná a dlhová kríza v plnej miere preukázala nevyhnutnosť zabezpečenia zdravého bankového sektora. V tejto oblasti sa podarilo pripraviť projekt bankovej únie, ktorý mal zabezpečiť, aby v budúcnosti banky neboli destabilizačným faktorom a neohrozovali tak ekonomiky členských štátov EMÚ ale aj EÚ ako celku. Súbor pravidiel v rámci bankovej únie má zabezpečiť, aby v budúcnosti boli banky lepšie kapitalizované a mali lepšie kontrolované riziká.

Základnými piliermi bankovej únie sú Jednotný mechanizmus dohľadu, Jednotný systém riešenia krízových situácií a Jednotný fond ochrany vkladov. Jednotný dohľad umožní priamy dohľad nad dôležitými bankami, národné orgány dohľadu úzko spolupracujú v rámci integrovaného systému. Jeden z najviac chýbajúcich prvkov počas finančnej krízy bol krízový manažment. Ten by sa mal aplikovať prostredníctvom Jednotného systému riešenia krízových situácií, ktorý je nastavený tak, že v prípade zlyhania opatrení a pravidiel, jednotná rada pre riešenie krízových situácií môže rozhodnúť riešiť problémy krachujúcej banky s podporou fondu, do ktorého banky prispievajú. (Suchá, 2013)

Spoločný dohľad by mal zabezpečiť splnenie prísnych podmienok pre banky v oblasti držania dostatočnej kapitálovej rezervy a rezervy likvidity. To urobí z európskych bánk, banky silnejšie a odolnejšie voči prípadným stratám v budúcnosti. Prípadnou záchranou bánk sa tak neohrozia verejné financie a štátne rozpočty. Keďže banky budú pod rovnakým dohľadom aj v prípade pôžičiek, budú regulované a riadené rovnakými pravidlami, dosiahne sa tak jednotný stupeň kredibility a dôvery pre finančných investorov na strane jednej a domácnosti na strane druhej. To by malo umožniť bankám lepší prístup k pôžičkám a tým zabezpečiť lepšie podmienky pre svojich koncových klientov v rámci celej EÚ.

ECB prebrala 4. novembra 2014 povinnosť vykonávania jednotného bankového dohľadu v eurozóne. Cieľom jednotného mechanizmu dohľadu (SSM) so sídlom vo Frankfurte nad Mohanom separátne od sídla ECB, je lepšie predvídať a rýchlo riešiť krízové situácie v bankovej sfére. SSM bude sledovať 120 najväčších bankových inštitúcií, ktoré spolu predstavujú viac ako 80 % bankových aktív v tomto regióne. Počet sledovaných bánk sa zvýši, ak sa aj ďalšie členské štáty EÚ, ktoré nepoužívajú euro, rozhodnú pre vstup do eurozóny. Ďalších 3500 bánk menšieho strategického významu zostanú aj naďalej pod dohľadom zo strany národných regulátorov. ECB si však v prípade problémov vyhradzuje právo prevziať nad nimi kontrolu.(Euraktiv, 2014).

V roku 2013 sa vykonali v členských štátoch EÚ záťažové testy a komplexné hodnotenia vybraných komerčných bánk. Testovaných bolo 123 najdôležitejších bankových skupín v Európe a výsledky testov mali identifikovať zostávajúce nedostatky európskeho bankového sektora. Výsledky testu, ktoré sa uskutočnili pod dohľadom Európskej bankovej autority (EBA), ukázali, že až 25 bankových skupín by mohlo mať v budúcnosti problémy. Dvanásť z nich prijalo nápravne opatrenia. EBA vyčíslila chýbajúci kapitál 31. decembru 2013 na 24,6 mld. eur. (Euraktiv, 2014). Pozitívne je, že banky na Slovensku patria k najstabilnejším v EÚ. V ďalšej časti budeme analyzovať bankový sektor SR.

3. Analýza bankového sektora SR

Bankový sektor na Slovensku sa po nepriaznivom vývoji v 90. rokoch minulého storočia dokázal pretransformovať na jeden z najzdravších, najstabilnejších a najziskovejších spomedzi bankových sektorov v rámci Európskej únie. Slovenským bankám sa podarilo prekonať finančnú krízu bez akýchkoľvek záchranných opatrení zo strany štátu, vďaka tomu, že bankový sektor na Slovensku je orientovaný na relatívne konzervatívne bankové funkcie – prijímanie vkladov a poskytovanie úverov v domácej mene. (EBF, 2015).

Aj na základe výsledkov testovania je zrejmé, že slovenské banky nemajú výrazné problémy a hodnoty ich kapitálovej primeranosti sú nad stanovenou minimálnou hranicou. Pri normálnych podmienkach nemajú slovenské banky problémy v žiadnom aspekte. V prípade potenciálnej krízovej situácie, by sa pri prvej úrovni záťaže mohli vyskytnúť problémy v jednej banke a pri ďalšej úrovni najviac v dvoch bankách. Podmienky záťažových testov boli však stanovené veľmi prísne, takže pravdepodobnosť tejto situácie je minimálne.

V rámci bankového sektoru v Slovenskej republike pôsobí 28 finančných inštitúcií, z toho 10 bánk so sídlom v SR, 14 pobočiek zahraničných bánk, 3 stavebné sporiteľne a jedna sporiteľňa s organizačnou zložkou zahraničnej osoby. (NBS, 2016).

Podľa posledných dostupných údajov NBS (2016) počas tretieho štvrťroku 2016, sa podiel základného imania v rukách tuzemských subjektov pohyboval na úrovni 5,95%. Na slovenskom bankovom trhu pôsobili v tomto období len dve inštitúcie so 100% podielom tuzemského základného imania, a to Slovenská záručná a rozvojová banka, a.s. a ČSOB stavebná sporiteľňa, a.s.. Čo sa týka štruktúry zahraničného upísaného základného imania, najväčší podiel ku koncu septembra 2016 mali Česká republika 51,69%, Luxembursko 13,47%, Rakúsko 12,21% a Belgicko 7,99%.

Európska centrálna banka stanovila niekoľko kritérií, podľa ktorých rozdelila zúčastnené banky na systémovo významné a menej významné inštitúcie. Nakoľko slovenský bankový sektor je dlhodobo stabilný a zdravo fungujúci, zaobišiel sa bez priamej finančnej pomoci z Európskeho mechanizmu pre stabilitu. Preto sa v ďalšej časti nebudeme zaoberať kritériom, ktoré hovorí o zaradení banky medzi významné inštitúcie na základe tohto, že jej bola niekedy poskytnutá priama finančná pomoc. Na slovenské banky nie je aplikovateľné ani kritérium veľkosti aktív, pretože žiadna z bánk so sídlom v SR nepresahuje hodnotu 30 mld. EUR.

Podľa kritéria troch najväčších bankových inštitúcií v krajine boli pod priame riadenie ECB zaradené Slovenská sporiteľňa, a.s. (SLSP), Všeobecná úverová banka, a.s. (VUB) a Tatra banka, a.s. (TB). Tieto tri banky by boli zaradené pod dohľad ECB, aj bez ohľadu na túto skutočnosť na základe tohto, že ich zahraničné materské spoločnosti predstavujú systémovo významné inštitúcie s výškou aktív výrazne nad hodnotou 30 mld. EUR. Z titulu významnosti bankovej skupiny sú pod priamy dohľad ECB zaradené aj tieto banky: ČSOB, a.s., ČSOB stavebnú sporiteľňu, a.s. a Sberbank Slovensko, a.s.

Na Slovensku pôsobia relatívne malé banky a bankový sektor patrí k najstabilnejším v EÚ, čo by mohlo vyvolávať dojem, že banková únia nie je pre nás až taká relevantná. Význam bankovej únie pre Slovensko však spočíva práve v zahraničnej angažovanosti, ktorá sa dlhodobo pohybuje na úrovni okolo 95%. Teda môžeme konštatovať, že pre Slovensko je jednotný bankový dohľad dôležitý, nakoľko banky sú riadené veľkými bankovými skupinami zo zahraničia. Pred zavedením jednotných pravidiel pre dohľad sa každá krajina riadila svojimi národnými normami. Rozdielna regulácia bankového sektora umožňovala veľkým materským bankám využívať svoje dcérske spoločnosti na realizáciu takých rizikových transakcií, ktoré im regulácia v materskej krajine nedovoľovala. Potreba univerzálnych pravidiel pre dohľad nad bankovými inštitúciami je práve v takýchto prípadoch potrebná, aby sa obmedzili aktivity s vyšším stupňom rizikovosti. Pre Slovensko má pomerne veľký význam aj Jednotný mechanizmus riešenia krízových situácií, ktorý by mal zabezpečiť, že v prípade zlyhania niektorej z materských bánk nedôjde k odlevu kapitálu z dcérskej banky.

V tabuľke 1 uvádzame vybrané ukazovatele troch bánk v SR – VUB, TB a SLSP, ktoré spadajú pod priamy dohľad ECB. Požili sme údaje za r. 2013, ktoré sa použili aj pri hodnoteniach významnosti banky.

Tabuľka 1 Vybrané ukazovatele VUB, TB a SLSP v roku 2013

Ukazovateľ	VUB	TB	SLSP
Koeficient Tier 1	15,93%	15,44%	20,80%
Kapitálová primeranosť banky	16,67%	16,60%	25,20%
Základný kapitál banky v mil. eur	1086,13	915,77	967,18%
Vlastné zdroje banky v mil. eur	1141,62	894,64	1102,93
Rizikovo vážené aktíva (RVA) v mil. eur	6817,87	5388,49	4692,13

Zdroj: Vlastné spracovanie na základe výročných správ bánk

Komplexné hodnotenie bolo vo všetkých troch prípadoch pozitívne. V prípade VUB podľa výkazov skupina spĺňa minimálne kapitálové požiadavky v oboch hypotetických scenároch. K 31. 12. 2013 ukazovateľ kapitálovej primeranosti pre túto banku dosahoval 15,93%. Po vykonaní hodnotenia kvality aktív a následnom posúdení výkazov v rámci záťažových testov, mala VÚB v základnom scenári oproti stanoveným 8% koeficient CET1 na úrovni 15,92%. Rovnako aj v nepriaznivom scenári, v porovnaní s minimálnou hodnotou 5,5%, dosiahol koeficient kapitálovej primeranosti hodnotu 13,82%. V oboch prípadoch teda banka dosiahla lepšie výsledky, ako viaceré banky eurozóny.(VUB, 2014).

Aj ďalšia systémovo významná banka na Slovensku, Tatra banka, dosiahla v komplexnom hodnotení ECB pozitívne výsledky. Obe časti hodnotenia, hodnotenie kvality aktív spolu so stresovými testami ukázali, že Tatra banka má prudentný prístup v oblasti riadenia rizík a rovnako potvrdili jej odolnosť, ak by v budúcnosti došlo k nepriaznivému ekonomickému vývoju. Konkrétne banka dosahovala hodnotu CET1 v základnom scenári 14,79% a 11,68% v negatívnom scenári, čím splnila všetky stanovené kritéria, bez nutnosti doplnenia vlastných zdrojov.(Tatra banka, 2014).

Vo všeobecnosti môžeme pozorovať, že rovnako ako v prípade VÚB, aj Tatra banka vykazuje hodnoty kapitálovej primeranosti nad definovanou minimálnou hranicou počas uvádzaného obdobia.

Poslednou slovenskou bankou, ktorá bola priamo zapojená do komplexného hodnotenia v roku 2014, bola SLSP. Podľa výsledkov stresových testov komplexného hodnotenia dosiahol ukazovateľ CET1 hodnotu 19,9% v rámci základného scenára a rovnako priaznivý výsledok dosiahla SLSP aj na základe negatívneho scenára, keď hodnota CET1 dosiahla 19,5%. Tento výsledok zaradil banku medzi inštitúcie s najlepším výsledkom. Zároveň týmto hodnotením potvrdila pozíciu stabilnej banky, ktorá disponuje dostatočným množstvom kapitálu aj v prípade potenciálnych šokov na finančnom trhu.(Slovenská sporiteľňa, 2014).

Aj v prípade SLSP sa hodnoty kapitálovej primeranosti banky dlhodobo udržiavajú výrazne nad hodnotou 8%, stanovenou ešte v rámci bazilejských dohôd. Základnou myšlienkou kapitálovej primeranosti je, že ak by došlo v budúcnosti k zlyhaniu alebo stratám v ľubovoľnej banke, mali by byť kryté vlastnými zdrojmi danej banky. Inými slovami, straty by boli kompenzované kapitálom akcionárov finančnej inštitúcie, nie finančnými prostriedkami vkladateľov. Pri pohľade na výsledky, ktoré dosahuje SLSP môžeme konštatovať, že patrí nielen k najväčším bankám v SR, ale predstavuje aj jednu zo silných a zdravých inštitúcií, vďaka ktorým je slovenský bankový sektor uznávaný ako jeden z najstabilnejších v eurozóne.

V rámci komplexného hodnotenia boli však spomedzi všetkých slovenských bánk nepriamo zahrnuté aj ďalšie tri inštitúcie. Išlo o Československú obchodnú banku, a. s., ČSOB stavebnú sporiteľňu, a. s. a Sberbank Slovensko, a. s., ktoré boli zaradené medzi významné inštitúcie z titulu významnosti skupiny. Aj tieto prešli komplexným hodnotením a stresovými testami s pozitívnym výsledkom.

Neskoršie hodnotenie v roku 2015 však ukázalo, že v rámci nepriaznivého scenára vlastník Sberbank Slovensko, a. s., rakúsky Sberbank Europe AG, nadobúdal hodnotu kapitálovej primeranosti len 4,2%, čo pri stanovenej hodnote CET1 5,5% predstavovalo nedostatok vlastného kapitálu až vo výške 138 miliónov. Od začiatku roku 2015 však banka navyšovala svoj kapitál v celkovej výške 140 miliónov EUR, čiže v ich prípade nebolo potrebné vypracovať nápravné opatrenia pre doplnenie chýbajúceho kapitálu.(ECB, 2015).

3. Zhodnotenie a dopady činnosti bankovej únie

Vytvorenie bankovej únie bolo vyvolané snahou reagovať na situáciu na európskom finančnom trhu vyvolanú dlhovou krízou. Jej hlavným cieľom bolo zabezpečenie stability v bankovom sektore. V členských aj nečlenských krajinách eurozóny sa však objavujú aj kritické názory, ktoré vyčítajú tomuto riešeniu, že sa nezaoberá príčinami vzniknutej situácie ale rieši len negatívne dopady.

Objavujú sa aj obavy, že ECB urobí v budúcnosti všetko pre to, aby zachránila veľké a systémovo dôležité banky, ktorých pád by ohrozil ekonomiky krajín ale aj eurozónu ako celok. Tieto obavy vychádzajú z toho, že umelé zachraňovanie akejkoľvek banky z verejných zdrojov krajín, môže z dlhodobého hľadiska predstavovať oveľa škodlivejšie riešenie, ako jej pád a že tak dochádza k neefektívnej alokácii zdrojov a to zabraňuje prirodzeným trhovým efektom. Spoločnému fondu na ochranu vkladov vyčítajú, že prerozdelenie prípadnej straty jedného subjektu na všetky zúčastnené štáty v skutočnosti znamená, že na chyby jedného hráča, by doplatili aj krajiny, ktoré majú stabilný a zdravo fungujúci bankový sektor, teda napríklad aj Slovensko. (Pekárková, 2013).

Okrem členských krajín eurozóny, ktoré sa automaticky stali členmi bankovej únie, pozitívny postoj k pridruženiu zaujali niektoré nečlenské krajiny, a to Dánske kráľovstvo,

Bulharsko a Rumunsko. (Belke, 2016). Podľa vyjadrenia dánskej národnej banky, Danmarks Nationalbank, Dánsko očakáva, že ich účasť na bankovej únii bude pozitívne vplývať na ich finančnú stabilitu. (Danmarks nationalbank, 2015).

Na rozdiel od Dánska, ktoré má veľmi podobnú štruktúru bankového sektora ako Slovensko, Česká republika odmieta aj pričlenenie sa k bankovej únii a rovnako aj prijatie spoločnej meny euro. Podľa vyjadrenia Ministerstva financií ČR je zapojenie sa do bankovej únie prínosnejšie pre krajiny, ktoré nemajú stabilný bankový sektor. (Ministerstvo financií ČR, 2015). Aj Česká národné banka argumentuje proti pristúpeniu Česka k projektu bankovej únie. Napríklad jedným z argumentov je aj potenciálna preferencia záujmov veľkých zahraničných bankových skupín na rozdiel od uprednostňovania záujmov ich domácich bánk. (ČT, 2015).

Na základe stanoviska Ministerstva Financií SR (2012) však môžeme povedať, že Slovensko prijalo všetky konkrétne kroky, ktoré bolo potrebné prijať pre vytvorenie bankovej únie. V konečnom dôsledku sme od 1. januára 2009 jednou z krajín s jednotnou menou euro, pre ktoré bol tento projekt primárne vytvorený. Zároveň SR súhlasí s vyjadrením, že najväčšia hrozba pre celkovú stabilitu finančného trhu vyplýva z bankového sektora, ktorý je tvorený jednak zo systémových, ale aj z nesystémových bánk. Slovenskí odborníci si však uvedomujú aj množstvo rizík, ktoré so sebou prinášajú jednotlivé piliere bankovej únie, vzhľadom na to, aké miesto zohrávajú v našom bankovom sektore dcérske spoločnosti veľkých bankových domov so sídlom v eurozóne. Preto bola pre Slovensko eliminácia rizík spojených s týmto aspektom, v priebehu rokovaní prioritou. Čo sa týka Jednotného mechanizmu pre riešenie krízových situácií, Slovenská republika vyjadrila podporu aj pre druhý pilier bankovej únie. Slovensko súhlasí aj so spustením Systému ochrany vkladov, avšak len za spravodlivého určenia výšky príspevkov. Pripomienky SR spočívali predovšetkým v otázke časového harmonogramu uvádzania Jednotného mechanizmu dohľadu. Obavy sa týkali rýchlosti prijímania jednotlivých opatrení, ktorá by sa mohla neskôr prejaviť v zníženej kvalite dohľadu.

Z pohľadu Slovenskej republiky môžeme teda povedať, že od svojho vzniku prispieva k stabilizácii bankového sektora a obnoveniu dôvery obyvateľov voči bankám. Rovnako, môžeme medzi pozitívny prínos bankovej únie zaradiť aj zabezpečenie jednotného prístupu k dohľadu nad bankami v celej eurozóne, nakoľko ECB uplatňuje na základe jednotných pravidiel rovnaký postup a zaisťuje tak rovnocenné podmienky pre inštitúcie vo všetkých členských krajinách, keďže ako nezávislá inštitúcia je v rámci dohľadu objektívnejšia, na rozdiel od národných orgánov pre dohľad. Prínos vidíme aj v skutočnosti, že so zapojením sa do bankovej únie získavajú členské štáty aj možnosť podieľať sa na rozhodovacom procese a na vytváraní pravidiel. V prípade jednotného mechanizmu riešenia krízových situácií vidíme pozitívum, ktoré spočíva predovšetkým vo fonde na riešenie krízových situácií, pretože poskytuje štátom pocit väčšej stability a istoty, bez toho, aby prípadné straty znášali občania. Samozrejme, že v rámci fungovania bankovej únie sme mohli pozorovať aj niekoľko nedostatkov. Ide predovšetkým o zvýšenú administratívnu záťaž pri výkone priameho aj nepriameho dohľadu a s tým súvisiace zvýšenie nákladov.

Zároveň je však treba podotknúť, že projekt bankovej únie je stále nový a za dva roky od spustenia prvého piliera do praxe, ešte reálne nenastala situácia, v rámci ktorej by bolo možné otestovať účelnosť bankovej únie spolu s jej schopnosťou riešiť problémovú situáciu promptne, bez toho aby sa kríza preliala z jednej ekonomiky do ostatných a aby negatívne dôsledky znášali veritelia bánk.

4. Záver

Vytvorenie bankovej únie ako jedného z krokov k ochrane finančného sektora v EÚ je významným krokom a vďaka skúsenosti z nedávnej finančnej krízy, politickej vôli členských štátov eurozóny sme svedkami potrebnej hlbšej hospodárskej a menovej integrácie. Predstavitelia európskych inštitúcii a finanční odborníci sú presvedčení, že takýto nástroj Európskej únie bude prínosom pre väčšiu stabilitu, bezpečnosť a odolnosť nielen v eurozóne, ale i celej Európskej únii.

Bankový sektor predstavuje jednu z najdôležitejších súčastí fungovania trhovej ekonomiky. V súčasnosti dochádza k vytváraniu globálnych bankových inštitúcií, ktoré pôsobia vo viacerých štátoch a je pomerne náročné realizovať dohľad nad týmito inštitúciami národným regulátorom. Okrem toho problémy v komerčných bankách majú dopad na fungovanie celej ekonomiky, a to nielen v krajine, kde má banka sídlo. Finančná kríza v eurozóne bola dôkazom toho, že regulácia bánk nie je dostatočná. Aj keď v posledných rokoch došlo k určitému posunu pri dohľade na finančnými inštitúciami v rámci prijatia Bazilejských princípov. Tieto sa v minulosti dotýkali hlavne úverovania a investovanie do cenných papierov a obchodovanie s rôznymi derivátmi, ostalo mimo dosah regulátorov. Aj keď ešte stále zostáva rad problémov, ktoré sa musia riešiť v rámci bankovej únie a v mnohých členských štátoch sa objavujú kritické názory na niektoré prvky bankovej únie, už dnes môžeme hodnotiť tento proces ako pozitívny. V krátkej budúcnosti bude potrebné dobudovať všetky súčasti bankovej únie a vytvoriť tak predpoklady na to, aby sa eliminovali riziká vzniku ďalšej krízy.

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Inflation forecasting in company financial management (use and reliability)

František KALOUDA [1]

Abstract

The paper is devoted to the inflation prediction by CBS and the uselfuness of this prediction in the evaluation of the effectiveness of the investments. The aim of the paper is to examine the causality rate between the inflation prediction and actual future inflation. The article uses commonly available data published by the CNB. Methodology of the paper is principally based on the time series methods (first of all correlation and regression), trends analysis and graphs. The standard literary research, description, comparison and analytic-synthetic methods are used in this paper as well. The results of the article will more precisely clarify the ability of the CNB to predict inflation and, in a wider context, effectiveness of the inflation targeting.

Key words

Effectiveness of the investments, inflation, inflation prediction, inflation targeting

JEL Classification: C22, E22, E27, E58, G31

1. Úvod a cíl

Příspěvek v obecné rovině vychází z principiálního vnímání vlivu a významu inflace, což je ve významné míře ovlivněno subjektem, který vliv inflace hodnotí.

V konkrétní rovině se příspěvek orientuje především na podnikovou sféru (a její potřeby) v souvislosti s možnostmi predikce inflace v podmínkách ČR.

Základním cílem příspěvku je otestovat míru příčinných souvislostí mezi výstupy stávajícího systému predikce inflace prognózami České národní banky (dále jen ČNB) a realitou, tedy skutečnými hodnotami inflace v horizontech prognóz ČNB a prověřit využitelnost těchto prognóz pro procesy hodnocení a plánování efektivnosti podnikových investičních záměrů.

Volba témat předkládaného příspěvku byla do značné míry ovlivněna i jeho očekávanými výsledky.

K těm patří v první řadě zhodnocení schopnosti ČNB úspěšně prognózovat budoucí hodnoty inflace. Stejně tak však příspěvek usiluje (v širších souvislostech) i o zhodnocení efektivnosti procesů inflačního cílování jako takového.

2. Metodologie a data

2.1 Metodologie

Metodickým základem příspěvku je obvyklá množina: práce s prameny, popis, srovnání a analyticko-syntetické postupy. V těchto případech probíhaly aplikace zcela standardním

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způsobem, a proto se nejeví jako nezbytné podpořit využití těchto metodických přístupů uváděním pramenů.

Nicméně se jeví minimálně jako poctivé přiznat některým pramenům minimálně zřejmou názorovou originalitu, viz ku příkladu pramen (Flammant, 1995). Je i možné konstatovat v této souvislosti, že některým autorům nechybí dokonce ani odvaha jít (nejméně dílčím způsobem) proti hlavnímu proudu současné ekonomické teorie. Zde nechť slouží příkladem (snad i poněkud překvapivě) pramen Greenspan (2008).

S ohledem na povahu zkoumaných problematik je výše uvedené kvarteto ve významné míře doplněno aplikací teorie časových řad a vybranými prvky statistických metod.

Z teorie časových řad se pro potřeby příspěvku ukázalo jako potenciálně nejpřínosnější studium významu zpoždění pro hledání vztahů mezi časovými řadami (Arlt a Radkovský (2001) v podobě „opožděných korelací“, viz Cyhelský a Souček (2010, str. 57). Pro posílení vypovídací schopnosti získaných výsledků bylo rovněž zvažováno využití speciálního statistického instrumentária v podobě Durbin–Watsonova testu (Hindls, Hronová a Novák (1999)), které je schopno vyloučit „zdánlivou korelaci“ (Blatná, D. (2009, str. 59)). To obojí ale pro relativní nedostatek vstupních dat (viz dále) nebylo v plné podobě realizovatelné, získané výsledky mají proto charakter pouze výsledků pracovních a jako takové jsou i v tomto příspěvku presentovány (viz subkapitolu 4. …... Nicméně je zde zmiňujeme výslovně, neboť jakkoliv nejde o výsledky s definitivní platností, spatřujeme ve využití těchto metodik jednu z hlavních možností dalšího zkoumání problematik tohoto příspěvku.

Z prvků statistických metod nelze opomenout především korelaci (De VAUS, 2010, Davis, 1971) a charakteristiky souborů dat (Radová, Dvořák a Málek (2009) či Swoboda (1977)).

Pro zvýšení názornosti získaných výsledků využívá příspěvek i grafických vyjadřovacích prostředků, grafické analýzy (Arlt, Arltová a Rublíková (2002)). Jmenovitě jde o bodové a spojnicové grafy, které doplňují výsledky propočtů korelací – viz pramen Wisniewski (1996).

A konečně je třeba konstatovat, že pro presentaci získaných dat v grafické podobě byla využita SW podpora v podobě produktu MS EXCEL.

2.2 Data

Data čerpáme výhradně z otevřených zdrojů, především z informačních základem ČNB (http://www.cnb.cz/cs/menova_politika/prognoza/predchozi_prognozy/index.html – přístup 11.7.2017, 11:43) – to pokud jde o prognózy budoucích hodnot inflace ze strany ČNB.

Z citovaného pramene jsou bohužel použitelné pouze data prognóz pro roky 2010 až 2017. V tomto časovém intervalu jsou prognózy definovány následujícími parametry:

a) okamžik stanovení prognózy - vždy únor předcházejícího roku,
b) předstih prognózy - čtyři, respektive pět čtvrtletí (tedy pro první, resp. druhé čtvrtletí následujícího roku)
c) forma prognózy - bodové odhady pro všechny tři měsíce cílového čtvrtletí.

Pro cílové roky 2007 až 2009 jsou však již údaje prognóz ČNB nesrovnatelné s daty z období let 2010 až 2016 a to ve všech výše třech uvedených parametrech.

Inflační cíl je ve všech případech bezproblémový – vždy má podobu bodového odhadu.

Informace o skutečných hodnotách inflace jsou dostupné na stránkách Českého statistického úřadu (https://www.czso.cz/csu/czso/mira_inflace přístup - 17.8.2017, 12:23).

Protože v příspěvku je diskutována reálná úroková míra, byla jako ukazatel inflace zvolena „míra inflace vyjádřená přírůstkem indexu spotřebitelských cen **ke stejnému měsíci předchozího roku**.“ (https://www.czso.cz/csu/czso/mira_inflace – přístup 17.8.2017, 12:23).

Z publikovaných tří měsíčních údajů daného kvartálu byla podle jejich konkrétních hodnot jako representativní zvolena buď hodnota mediánu nebo modusu.

Teritoriálně je příspěvek jednoznačně zaměřen na ekonomiku ČR. Případné zobecnění závěrů příspěvku není v daný okamžik předmětem našeho zájmu.

3. Výsledky a diskuse

3.1 Zásadní vnímání významu inflace pro ekonomický život jedince i společnosti

Již v úvodu příspěvku zmíněné diskutabilní vnímání fenoménu inflace zcela zřejmě souvisí se subjektem, který se k inflaci vyjadřuje. V souladu se zaměřením příspěvku můžeme konstatovat, že především nás zajímá, jak inflaci vnímá podnikatelský subjekt (dále jen podnik).

Nicméně, s ohledem na reálné celospolečenské souvislosti, nebude možné opomenout stanovisko rozhodovacího subjektu, který má pro řízení inflace v současnosti klíčové kompetence. Obecně je to centrální banka, v podmínkách ČR ČNB, reprezzentovaná vybranou množinou špičkově kvalifikovaných ekonomických profesionálů.

A konečně, s cílem obsáhnout pokud možno celou šíři názorového spektra, nelze opomenout ani možné či pravděpodobné vnímání inflace jedincem bez speciální kvalifikace, občana či „muže z ulice“.

Tato část výsledků opouští dílčím způsobem (díky dostupným pramenům) deklarované zaměření pouze na kontext ČR. Protože jde o informace teoretického charakteru, chápeme tuto skutečnost jako vyjádření jisté míry univerzality presentované teorie.

3.1.1 Inflace a jedinec jako ekonomický subjekt

Zde vycházíme z předpokladu, že tento subjekt (jedinec, běžný občan, „muž z ulice“) nemá prakticky žádné možnosti jak inflaci řídit a stejně tak nemá příliš mnoho možností řešit důsledky inflace. Nicméně i v tomto případě jde o ekonomický subjekt, který je inflací ovlivňován.

V žádném případě zde neusilujeme o representativní sociologický průzkum. Presentované poznatky lze nejspíše považovat za literární reflexi v dostupných pramenech. V této souvislosti je zřejmé, že daný subjekt bude pro prameny méně zajímavý. To lze chápat jako důsledek jen omezené možnosti jedince fenomén inflace jak vnímat tak i ovládat.

V obecné rovině a v poněkud emotivním duchu presentuje pramen Flammant (1995, str. 118) vlivy inflace na občana takto: „Inflace byla – a stále zůstává – velkým problémem našeho století. Ať již svými pomalými tlaky, či svými víry, inflace rozdrobuje, ba i ničí peníze a ceny, mzdy a platy, jakož i jiné příjmy, stejně jako rodinné jmění a dědictví. Krátce: dotýká se všech hodnot, ke kterým člověk rád přilne.“ Není bez zajímavosti, že velmi podobné stanovisko k inflaci vyjádřil i člověk v informačně i mocensky extrémně silném postavení, když podle Greenspana (2008, str. 95) „President Carter na jaře roku 1980 podporoval Volckera, když prohlásil, že inflace je celostátním problémem číslo jedna.“

Průnik občanské a politické roviny jen zdůrazňuje zřetelně negativní vnímání fenoménu inflace i na takto definované „občanské“ úrovni. Ostatně právě citovaný P.A.Volcker, Jr., předseda FEDu v období presidentského mandátu presidentů Cartera a Reagana, „Od okamžiku, kdy složil přísahu, ….. věděl, že jeho úkolem je, jak řekl později, „zabít draka inflace“. (Greenspan, 2008, str. 94). Tato, opět do jisté míry, emotivní reakce rovněž ukazuje na do značné míry osobní vnímání svěřeného úkolu a tedy i daného problému.

V detailnější rovině lze hrozby inflace pro subjekty diskutovaného typu charakterizovat podle nám již známého pramene (Flammant, 1995, str. 42) například takto: „Inflace má četné a závažné nevýhody. …..

A) ..… Kupní síla mezd a platů má snižující se tendenci, … .
C) ….. Inflace, …..vede ekonomické subjekty k tomu, aby „prchaly“ před penězi; … Toto chování lze také označit jako „útěk k reálným hodnotám … .“

3.1.2 Inflace a její řiditelnost v konceptech centrální banky obecně a ČNB zvlášť

Zásadní úhel pohledu a vnímání této problematiky ČNB je zcela oficiálně vyjádřen naprosto jasně:

„ ..... při nulové inflaci velmi silně hrozí propad do deflace s velmi neblahými důsledky pro ekonomiku. …“ http://www.cnb.cz/cs/index.html (přístup 17.07.15, 14:58) .

Odtud je zřejmé, že podle ČNB mohou být k diskusi pouze otázky

a) principiální řiditelnosti inflace, zcela v duchu jednoduché otázky „Je vůbec možné, aby (libovolná) centrální banka inflaci řídila?“ respektive
b) jaká je optimální hodnota inflace, pokud ovšem vůbec existuje.

Jak uvidíme v dalším, obě tyto otázky jsou velmi těsně propojeny a v některých případech se diskutují praktiky současně a ve vzájemné podmíněnosti. Přesto se je budeme snažit strukturovat a diskutovat více méně odděleně.

Pokud jde o problematiku principiální řiditelnosti inflačních procesů centrální bankou, tak se ukazuje, že významná část autorit nahlíží na tuto možnost docela skepticky.

Kupříkladu i pramen Greenspan (2008, str. 507) můžeme řadit do této skupiny, když se nechal v roce 2007 slyšet, že „Ve skutečnosti četní ekonomové pokládají měnovou politiku centrální banky za klíčový faktor redukce inflace na celém světě. Rád bych tomu věřil.“ (Greenspan, 2008, 507). Podobně, jen snad ještě kritičtěji tentýž pramen na jiném místě uvádí: „Měnová politika centrálních bank nebyla primární příčinou trvalého poklesu inflace a dlouhodobých úrokových měn, ale my centrální bankéři jsme se rozhodli změnit svou politiku, abychom maximalizovali dlouhodobé zisky z těchto tektonických posunů v globálních financích. ….. Inflaci ve světě nekrytých peněz je obtížné potlačit.“ (Greenspan, 2007, str. 24).

Dokonce ještě striktněji odmítá možnost řídit inflaci pramen Flammant, M. (1995, str. 37), když tvrdí, že „Zkušenost ukázala, že snaha stabilizovat *míru* inflace je jen klamnou nadějí. Již ze své povahy je tato míra nestabilní. Kdo by ji chtěl zachovat na úrovni považované za přijatelnou, připravoval by tím vlastně příští explozi.“ Flammant, M. (1995, str. 37)

Čímž se vlastně dostáváme k druhé deklarované otázce, k možnosti určit přijatelnou (či dokonce optimální) míru inflace, speciálně pokud sledujeme její vliv na národní ekonomiku jako celek.

Ponecháme-li dále stranou posledně citovaný názor, který je fakticky neurčitý, lze v pramenech objevit kvantifikovaný názor o velmi přijatelné míře inflace: „Vždycky jsem se domníval, že uměřená inflace o nízké míře jednoho procenta se nedá udržet v ekonomice užívající papírové měny v konkurenční demokratické společnosti s nějakými zbytky populismu.“ (Greenspan, 2008, str. 405). Je to ovšem opět názor skeptický, pokud máme na mysli zásadní řiditelnost inflačních procesů.

Tentýž pramen nicméně presentuje nástroj, který podle jeho názoru inflační procesy ovládnout dokáže a to dokonce při řádově definované velikosti inflace: „V následujícím čtvrtstoletí (po druhé světové válce – pozn. FK) všeobecné přijetí kapitalismu volné soutěže pomohlo omezit globální inflaci a úrokovou míru na jednotky.“ (Greenspan, 2008, str. 22).

Nicméně potřeba přesněji kvantifikovat žádoucí či přijatelnou velikost inflace zůstává stále nezodpovězena. A v pramenech se objevuje ve velmi konkrétní podobě, viz ku příkladu

Flammant (1995, str. 36-37): „Stálá inflace je charakteristická pro rozvinuté kapitalistické země, které ji až do příchodu druhé světové války ignorovaly ….. vede k jedné otázce. Nebyla by možná nějaká „přijatelná“ inflace s určitým horním limitem, od kterého by již šlo o jev nepřijatelný? Tvrdívá se proto, že část inflace je třeba tolerovat a že zvýšení cen o několik bodů (od 3 do 4 % ročně) by bylo snesitelné, pokud by to bylo jen určité výkupné placené za to, abychom se vyhnuli příliš restriktivní politice, která by znamenala nebezpečí pro úroveň zaměstnanosti.“

Právě deklarovaná velikost inflace však byla v názorech části teorie (a nejen v těchto kruzích) považována za příliš nízkou: „Někteří zprava i zleva dokonce začali tvrdit, že inflace ve výši, řekněme, 6 procent ročně by se dala docela snést - … . Tohle sebeuspokojení se dokonce rozšířilo až na Wall Street.“ (Greenspan, 2008, str. 93)

Jak je tedy zřejmé, lze v diskusích řiditelnosti inflačních procesů objevit i velmi krajní názory. Podstatné pro nás je, že toto názorové spektrum teorie se rozhodně nenese v duchu bezbřehého optimismu.

3.1.3 Výkonnost podniku v inflačních podmínkách

Zde je situace docela jednoznačná. Je obecně známá představa, že majetek před inflací lze s jistotou větší než malou ochránit tím že bude využit v podnikatelských aktivitách.

Považujeme za vhodné (nikoliv za nezbytné) nezbytné, podepřít toto povědomí i o názor autority. Nám známý pramen například konstatuje, že: „Podniky jsou ohroženy inflací méně ve svých ziscích než v přesném odhadu těchto zisků; ….. .“ Flammant, M. (1995, 86). A nebo tvrdí na straně 42 dokonce ještě konkrétněji: „*Důchodové rozdělení* se inflací zcela změní. ..… . A tak inflace ….. dává zvýšené zisky podnikatelům.“.

Tyto informace považujeme na dané úrovni analýzy problému za postačující a dostatečně přesvědčivé. V širších souvislostech se tématu věnujeme v následující kapitole 3.2.

3.2 Vliv inflace na hodnocení efektivnosti investičních aktivit

Zásadní positivní vliv inflace na efektivnost podnikatelských aktivit byl (v obecné rovině) konstatován v paragrafu 3.1.4 Výkonnost podniku v inflačním prostředí. Nyní je vhodné věnovat se tomuto problému podrobněji a to jak z pohledu ekonomické teorie, tak i z náhledů pokud možno praktických.

3.2.1 Teoretická východiska hodnocení efektivnosti investičních aktivit – inflace a nominální a reálné úroková míra

Pro hodnocení efektivnosti podnikových investičních aktivit je rozhodující obecně známá metodika NPV (Kislingerová, E. at al. (2007). A pro tuto metodiku je opět kriticky významná triáda pojmů – úroková míra nominální a reálná a jejich vazba na inflaci.

Z důvodů obecné znalosti metodiky NPV připomínáme pouze tři detailní poznatky.
A sice především, že pro malé hodnoty inflace je reálná úroková míra „ ….. rovna nominální úrokové míře snížené o inflaci, očekávané v uvažovaném časovém období.” Flammant (1995, str. 33).

Ale nejen to. Neuvěřitelně aktuálním způsobem tentýž pramen tamtéž upozorňuje ne riziko faktických záporných úrokových sazeb: “Jestliže však skutečná inflace je vyšší než očekávaná, úroková míra skutečně vybíraná bude krácena a může klesnout až na nulu, nebo se dokonce stát negativní.“

A konečně je pro naše následující využití modelových přístupů poněkud paradoxně významný i fakt, že „Slabinou metody NPV je … vysoká citlivost na vývoj úrokových měr, která se promítá do výše diskontního faktoru a je v turbulentním prostředí obtížně predikovatelná.“ (Kislingerová a kol., 2007, str. 271-272).

3.2.2 Modelový příklad - vliv inflace na efektivnost investičního záměru

Výše uvedené úvahy převážně teoretického charakteru nyní uvedeme do prakticky co nejlépe uchopitelné polohy v podobě modelového příkladu.

Jeho vstupy odpovídají ekonomické realitě, alespoň pokud jde o nominální cenu kapitálu i míru inflace. Meziroční hodnoty CF_i jsou (bez uvedení jednotek, což je v tomto případě nepodstatné) ovšem dílem invence autora příspěvku a jsou voleny tak, aby byly výstupy modelového příkladu co nejpřesvědčivější – s uvažováním vlivem inflace jsou nastaveny tak, aby byl modelový investiční záměr přijatelný (byť i právě na hranici přijatelnosti a s přijatelnou nepřesností) - viz Table 1. Efektivnost téhož investičního záměru bez uvážení vlivu inflace je uvedena v Table 2.

Jako míru výkonnosti zde – v souladu s výše uvedenými teoretickými poznatky i zavedenou praxí - uvažujeme kritérium NPV. S ohledem na všeobecnou znalost této metodiky upouštíme od dalších komentářů, s jedinou výjimkou. Tou je předpoklad obecně racionálního chování podnikatelského subjektu, což se projevuje v následujícím:

► vstupní vektor veličin CF_i indexuje podle míry inflace a
► v diskontování vstupní vektor veličin CF_i uvažuje reálnou cenu kapitálu,

viz též podobná stanoviska pramene Radová, J., Dvořák, P. and Málek, J. (2009).

Zadání modelového příkladu:

Časový horizont investičního záměru – 7 let
Inflace – 2 %
Cena kapitálu nominální – 4 %
Cena kapitálu reálná – 2 %

Table 1: Modelový příklad – efektivnost investičního záměru na úrovni NPV s uvážením vlivu inflace

rok investičního záměru (i)	výkon investice (úroveň CF_i)	výkon investice (úroveň PV_i)
0	- 1 000	-1 000,00
1	50	50,98
2	100	99,96
3	200	196,00
4	300	288,24
5	252	237,37
6	138	127,44
	Suma CF_i = **40**	Suma PV_i = NPV = **0,01**

Pramen: Vlastní zpracování včetně vstupních dat

Table 2: Modelový příklad – efektivnost investičního záměru na úrovni NPV bez vlivu inflace

rok investičního záměru (i)	výkon investice (úroveň CF_i)	výkon investice (úroveň PV_i)
0	- 1 000	-1 000,00
1	50	48,08
2	100	92,46
3	200	177,80
4	300	256,44
5	252	207,13
6	138	109,06
	Suma CF_i = **40**	Suma PV_i = NPV = **- 109,03**

Pramen: Vlastní zpracování včetně vstupních dat.

Jednoduchost a průzračnost modelového příkladu nepřipouští pochybnosti – vliv inflace na efektivnost podnikatelského (investičního) záměru je podstatný. A tím se současně jasně ukazuje, jak významné pro podnik je (či může být) znalost budoucích úrovní inflace. Dostupnost těchto informací je v kontextu ČR definována jasně – jde o jeden z výstupů

rozhodovacích procesů ČNB. Příspěvek se těmto záležitostem věnuje v následující subkapitole.

Poznámka:

Případné zobecnění vlivu inflace na rentabilitu investičního záměru na (úrovni výpočetního algoritmu) jde již podle našeho vnímání nad rámec příspěvku a proto zde není provedeno.

3.3 Subjekt faktického řízení procesu inflace, jeho cíl a míra splnění tohoto cíle

3.3.1 Cíl, poslání a kompetence ČNB v procesech řízení inflace

Ve strategické rovině je v této rovině absolutním suverénem ČNB. Postavení ČNB v této oblasti řídí zákon, viz Zákon č. 6/1993 Sb.., O České národní bance, v aktuálním znění. Tento dokument definuje cílové zaměření ČNB takto:

„Hlavním cílem České národní banky je péče o cenovou stabilitu (§2 zákona o ČNB). ČNB, podobně jako většina centrálních bank, se soustřeďuje především na stabilitu spotřebitelských cen. ….. ČNB provádí svou měnovou politiku v režimu cílování inflace. ….. V praxi je inflace v oblasti spotřebitelských cen měřena jako přírůstek tzv. indexu spotřebitelských cen.“ http://www.cnb.cz/cs/statistika/inflace/index.html (přístup 11.7.2017, 11:33).

ČNB tedy zcela zjevně aspiruje na řízení inflačních procesů v rámci ČR a jako klíčové výstupy svých aktivit presentuje „ ….. využívání prognózy inflace a veřejné explicitní vyhlášení inflačního cíle ….. . “ (http://www.cnb.cz/cs/menova_politika/cilovani.html#c2 (přístup 17.7.2017, 11:36).

Toto obecné vymezení svého poslání ČNB konkretizuje a průběžně aktualizuje, přičemž aktuální podoba rozhodujících parametrů režimu cílování inflace má tuto podobu: „ ČNB umožní postupné snižování inflace ….. s dostatečným předstihem zhruba jednoho roku ….. **Po zvážení všech výše uvedených skutečností vyhlašuje ČNB nový inflační cíl platný od ledna 2010 jako meziroční přírůstek indexu spotřebitelských cen ve výši 2 %.** ČNB bude stejně jako doposud usilovat o to, aby se skutečná hodnota inflace nelišila od cíle o více než jeden procentní bod na obě strany.“
https://www.cnb.cz/miranda2/export/sites/www.cnb.cz/cs/menova_politika/strategicke_dokumenty/download/inflacni_cil_cnb_2010.pdf (přístup 18.8.2017, 9:50)

3.3.2 Empirický test plnění cílů ČNB v procesech řízení inflace

V souladu se základním cílem příspěvku na tomto místě s využitím reálných dat ověříme míru příčinných souvislostí mezi výstupy stávajícího systému predikce inflace (prognózami ČNB) a realitou (skutečnými hodnotami inflace v horizontech prognóz ČNB) a prověříme využitelnost těchto prognóz pro procesy hodnocení a plánování efektivnosti podnikových investičních záměrů.

Odpovídající vstupní data i výsledky porovnání prognózovaných a skutečných hodnot jsou pro horizont prognóz čtyři čtvrtletí v Table 3. Grafické znázornění klíčových hodnot)včetně korelačního koeficientu) se nachází ve Figure 1.

Pro druhý horizont predikce inflace, který ČNB používá (pět čtvrtletí) jsou vstupní data spolu s výsledky porovnání prognózy a skutečného stavu inflace dostupné v Table 4. Grafické znázornění klíčových hodnot (opět včetně korelačního koeficientu) je k disposici ve Figure 2.

Table 3: Prognózované (ČNB) a skutečné hodnoty inflace – vstupní data (horizont prognózy 4 čtvrtletí)

prognózovaný rok / inflační cíl	inflace – prognóza ČNB/skutečnost	odchylka inflační cíl - skutečnost
2010 / 3 %	1,4 % / 0,7 %	**+2,3 %**
2011 / 3 %	2 % / 1,7 %	**+ 1,3 %**
2012 / 2 %	2 % / 3,7 %	**- 1,3 %**

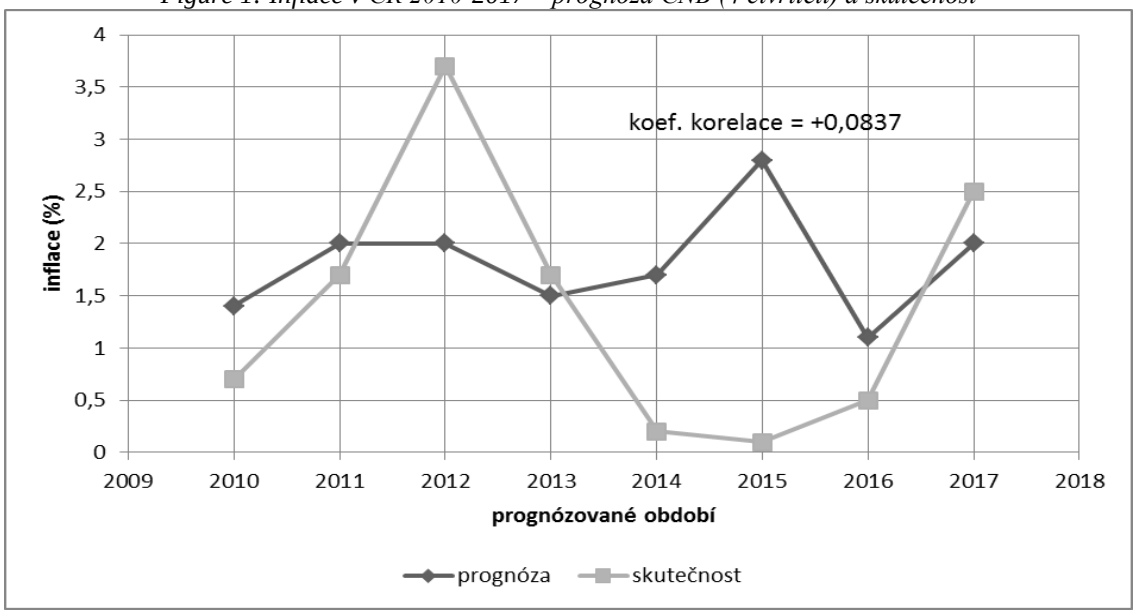
2013 / 2 %	1,5 % / 1,7 %	+ 0,3 %
2014 / 2 %	1,7 % / 0,2 %	**+1,8 %**
2015 / 2 %	2,8 % / 0,2 %	**+1,8 %**
2016 / 2 %	1,1 % / 0,5 %	**+1,5 %**
2017 / 2 %	2 % / 2,5 %	-0,5 %

Pramen: Vlastní zpracování s využitím vstupních dat
http://www.cnb.cz/cs/menova_politika/prognoza/predchozi_prognozy/index.html (přístup 11.7.2017, 11:43) a
https://www.czso.cz/csu/czso/mira_inflace (přístup 17.8.2017, 12:23)

Z Table 3 je zřejmé, že s plněním inflačního cíle to není nijak slavné – v šesti případech z osmi je z tohoto pohledu prognóza ČNB neúspěšná (vyznačeno tučně).

Korelace mezi prognózou formulovanou ČNB a skutečností, vyjádřená korelačním koeficientem má v tomto případě hodnotu + 0,0837. Grafické znázornění situace viz Figure 1. Takextrémně nízká honota korelačního koeficientu podle pramene De VAUS (2010, str. 272) vyjadřuje závislost sledovaných veličin jako: "nevýznamnou, velmi malou, nepodstatnou, drobnou, prakticky nulovou".

Figure 1: Inflace v ČR 2010-2017 – prognóza ČNB (4 čtvrtletí) a skutečnost



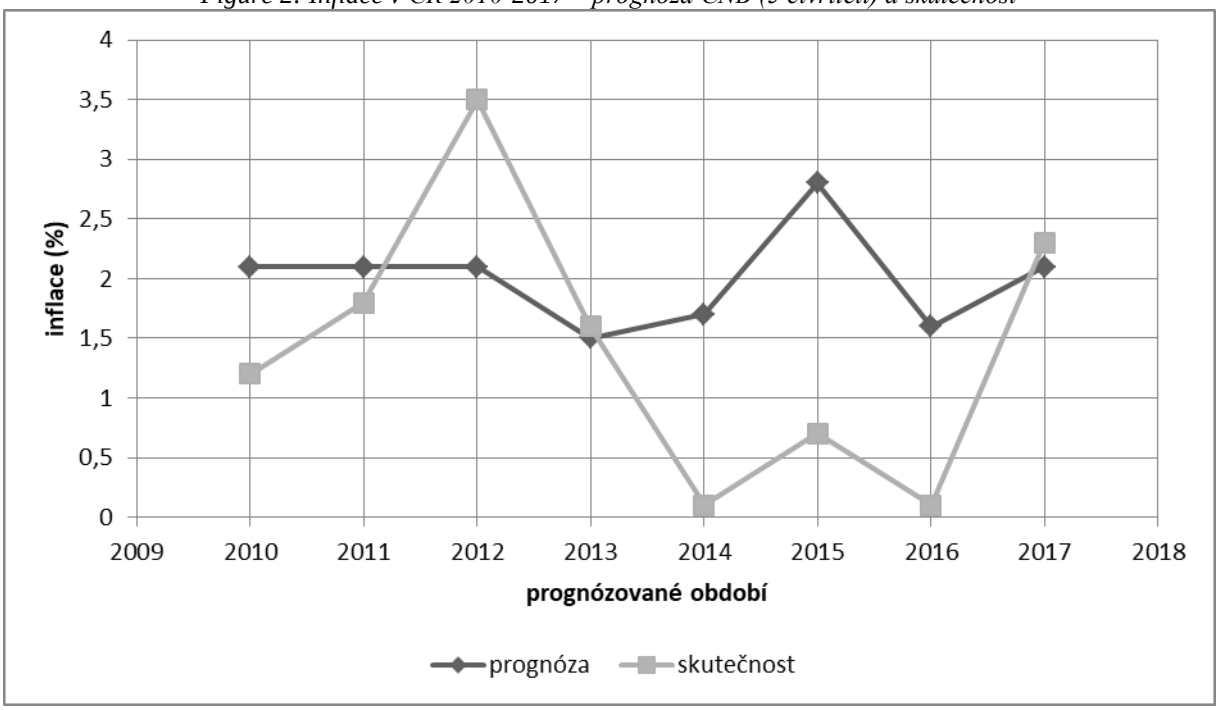
Pramen: Vlastní zpracování s využitím vstupních dat
http://www.cnb.cz/cs/menova_politika/prognoza/predchozi_prognozy/index.html (přístup 11.7.2017, 11:43) a
https://www.czso.cz/csu/czso/mira_inflace (přístup 17.8.2017, 12:23)

Table 4: Prognózované (ČNB) a skutečné hodnoty inflace – vstupní data (horizont prognózy 5 čtvrtletí)

prognózovaný rok / inflační cíl	inflace – prognóza ČNB/skutečnost	odchylka inflační cíl - skutečnost
2010 / 3 %	2,1 % / 1,2 %	**+1,8 %**
2011 / 3 %	2,1 % / 1,8 %	**+1,2 %**
2013 / 2 %	2,1 % / 3,5 %	**- 1,5 %**
2013 / 2 %	1,5 % / 1,6 %	+0,4 %
2014 / 2 %	1,7 % / 0,1 %	**+1,9 %**
2015 / 2 %	2,8 % / 0,7 %	**+1,3 %**
2016 / 2 %	1,6 % / 0,1 %	**+1,9 %**
2017 / 2 %	2,1 % / 2,3 %	-0,3 %

Pramen: Vlastní zpracování s využitím vstupních dat
http://www.cnb.cz/cs/menova_politika/prognoza/predchozi_prognozy/index.html (přístup 11.7.2017, 11:43) a
https://www.czso.cz/csu/czso/mira_inflace (přístup 17.8.2017, 12:23)

Figure 2: Inflace v ČR 2010-2017 – prognóza ČNB (5 čtvrtletí) a skutečnost



Pramen: Vlastní zpracování s využitím vstupních dat
http://www.cnb.cz/cs/menova_politika/prognoza/predchozi_prognozy/index.html (přístup 11.7.2017, 11:43) a
https://www.czso.cz/csu/czso/mira_inflace (přístup 17.8.2017, 12:23)

Z Table 4 je i v tomto případě zřejmé, že ČNB inflační cíl nijak zvlášť „netrefuje“ – opět v šesti případech z osmi je prognóza ČNB neúspěšná – inflační cíl je „podstřelen“ či „přestřelen“ (vyznačeno tučně).

Korelace mezi prognózou formulovanou ČNB a skutečností, vyjádřená korelačním koeficientem má v tomto případě hodnotu + 0,1717 (v grafu chybí díky záludnostem MS EXCEL). Grafické znázornění situace viz Figure 2.

Tato, rovněž nízká, hodnota korelačního koeficientu podle pramene De VAUS (2010, str. 272) znamená, že závislost sledovaných veličin lze považovat za: “nízkou až střední, resp. nepatrnou, slabou či menší”. Jde o svého druhu paradox – prognóza na delšší období (5 čtvrtletí se jeví jako sice nevýznamně, ale přece jen spolehlivější.

Tato skutečnost nám poslouží jako přechod k poslední části výsledků příspěvku v podobě subkapitoly 3.4

3.4 Hlavní metodický i datový problém – délka prognózovaného období.

Dnes v praxi ČNB standardní je časový horizont prognózy inflace na čtyři resp. pět čtvrtletí. To je dáno částečně „zvykovým právem“, částečně programovými deklaracemi ČNB (viz paragraf 3.3.1).

Současná teorie časových řad však dovoluje délku předstihu objektivizovat a fakticky i optimalizovat. Předpokladem úspěšné aplikace těchto přístupů je však odpovídající datová základna. Aktuálně dostupná data nezbytným však nezbytným požadavkům neodpovídají v potřebné míře (viz též subkapitolu 2.2)

Jsme nicméně schopni, nejméně s využitím elementárních metodik (srovnání) formulovat alespoň užitečné pracovní závěry.

Význam diskutovaného problému nelze podcenit, neboť „**Zjišťování délky zpoždění, s jakým se měnlivost v jedné ekonomické časové řadě odráží v měnlivosti řady druhé je velmi důležitou praktickou úlohou.** (Arlt a Radkovský, 2001,str. 14).

Pro úsporu rozsahu příspěvku presentujeme získané údaje v tabulkové formě (viz

Table 5: Empiricky stanovená délka prognózovaného období inflace (délka zpoždění)

horizont prognózy	teritorium	pramen
jeden rok	ČR	www.cnb.cz
dva až tři trimestry	USA	Flammant (1995)
dva roky	USA	Jílek (2004)
jeden až tři roky	USA	Jílek (2004), podle Fedu
tři roky	USA	Greenspan (2008)

4. Závěry

4.1 Význam predikcí inflace pro finanční řízení podniku

Příspěvek přesvědčivě prokazuje, že znalost budoucích hodnot inflace může mít pro podnik potenciálně naprosto mimořádný význam. Teorie i modelový příklad prokazují očekávatelný přínos v efektech investičních aktivit na úrovni kritéria NPV.

Podmínkou využití tohoto potenciálu je však dostupnost spolehlivých prognóz inflace v relevantních časových horizontech, aby je bylo možno zahrnout do výpočtů vstupní toků hotovostí i cen kapitálu (viz dále subkapitoly 4.2 a 4.3).

4.2 Schopnost ČNB predikovat resp. řídit inflaci

V příspěvku jsou uvažovány oba horizonty ČNB prognóz inflace, tedy čtyři a pět čtvrtletí. Shoda prognóz se skutečností je posuzována s využitím koeficientu korelace (r).

V prvním případě je shoda „nevýznamná či prakticky nulová“ (r= 0,0837), ve druhém případě pak „nepatrná či nízká“ (r = 0,1717). Střízlivě hodnoceno, jsou prognózy inflace ČNB pro podnikové potřeby nepoužitelné.

Pokud jde o procesy řízení inflace, ČNB zastává v zásadě optimistické stanovisko. V příspěvku presentovaná stanoviska teorie však takto optimistická ani zdaleka nejsou a v některých případech jsou vyloženě skeptická.

4.3 Efektivnost procesu cílování inflace ČNB ve vhodném předstihu

Praxe ČNB v podstatě neplní vlastní zadání ČNB, podle kterého by se prognózovaná hodnota inflace neměla od inflačního cíle odchylovat o víc než jeden procentní bod v obou směrech. Na naprosto minimálním souboru osmi roční dat je tato podmínky splněna pouze ve dvou případech, což je i při statisticky obtížné prokazatelnosti prostě alarmující. Jiná věc je ovšem praktický význam „trefování“ inflačního cíle.

Podle názoru ČNB je stávající praxe předstihu prognózy (přibližně jeden rok) v zásadě vyhovující. Teoretická stanoviska argumentují spíše pro prodloužení tohoto horizontu a to dokonce až na tři roky.

Exaktní řešení tohoto problému dosud není známo. V příspěvku nemohlo být provedeno pro nedostatek kvalitních dat. K tomuto, problému se v budoucnosti vrátíme.

Za stávajícího stavu věcí – omezený objem vstupních dat, pouze roční horizont předstihu prognózy a jen omezená spolehlivost prognózy – jsou i výstupy z procesu cílování inflace podle ČNB pro potřeby podnikové praxe nevyužitelné.

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Employees' Performance Management by Using MCDM Methods

Katerina Kashi [1]

Abstract

At this time, every organization or firm strive to make a profit, to keep their current employees, especially those that are the key employees, and to attract new ones. In order to know what competencies the key employees should have, the organization can apply competency models. The key question is what the key competencies in each position are, and what remuneration is preferred and, therefore, motivating for both the employees and the employer. Main objective of this paper is to present results from a case study aimed at utilization of the analytic hierarchy process method (AHP) in competency modeling and total remuneration. The main result of this case study presents the most important competencies for a selected position and, also the employees' preferences when it comes to the tangible and intangible remuneration.

Key words

Performance management, competency model, intangible remuneration, tangible remuneration

JEL Classification: C49, M12, M51

1 Introduction

In today's competitive world it is very important to build on the competitive activities of business. Companies today have to think about their business strategy, especially when it comes to the kind of competencies, which they need to have in order to compete in a specific environment. According to Berge and Verneil (2002) many organizations are now adopting competency-based models to meet the evolving goals and needs.

To be able to determine the key competencies the method of analytic hierarchy process (AHP), which belongs to multiple-criteria decision-making, can be used. The AHP method helps the managers to find the decision that best suits the goal and fulfill their understanding of the problem. The method provides rational framework to structure a decision problem, to represent and quantify its elements, which relate to the elements of the overall goal, and to evaluate alternative possible solutions. This method is also suitable for the competency modeling because of its possibility to decompose the goal/objective to several criteria and sub criteria Saaty (1994).

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2 Theoretical basis of competency models and total remuneration

A competency model is a framework, which provides a list of the competencies required for effective performance in a specific job or group of jobs. A competency is a human capability, which is required for effective performance. A competency can be comprised of personal characteristics, knowledge, skills and abilities.

2.1 Competency model

Completed competency model should include a list of competencies that are divided into groups by the type, (such as personal, technical, managerial, interpersonal etc.) with a definition and several examples of behavior for each competency, Marelli, Tondora, Hoge (2005). Competencies have to lead to effective performance, which means that the performance of a person with competency must be significantly better than that of a person without it. Competencies are components of a job, which are reflected in behavior that is observable in a workplace, Sanghi (2007). According to Vokoun (2015) the most hampering factors of innovation were among others lack of skilled personnel. Well-designed competency model can be used by the company's human resources in various ways. It can serve as a base for recruitment, performance management, succession planning, recognition and rewards, compensation, training and development, Marelli, Tondora, Hoge (2005). According to Novák, Vokoun, Stellner and Vochozka (2016) also the private personnel agencies stress the need for better qualifications and skills.

2.2 Developing a remuneration system

According to modern human resources management one part of total reward is apart from tangible reward the intangible reward i.e. reward which does not include any financial reward. Intangible remuneration is currently more important especially in the relationship to remuneration and motivation of e.g. key and talented employees (which is considered as a strategic priority of many organizations). Providing intangible remuneration can depend on the organization's top management, where the organization can contribute by enforcing the values which enable employees to express their opinions, implement and improve processes of work performance management, implement systems of formally recognizing good work and to take steps for improving balance between personal and work life Armstrong (2007).

3 Methodology

This chapter presents AHP and TOPSIS methods. AHP method will be used for decomposition of competencies and tangible and intangible rewards and their rankings. The TOPSIS method will be, consequently, used to link the most preferred rewards according to the employees' performance.

3.1 AHP Method

Analytic hierarchy process is a framework of logic and problem solving that spans from the spectrum from instant awareness to fully integrated consciousness by organizing perception, judgments and feelings into hierarchy of forces, which influence decision results. The hierarchy represents a complex problem in a multilevel structure which first level is the goal followed by levels of factors, criteria and subcriteria.

Users of AHP firstly decompose their decision problem into a hierarchy of more easily understood sub-problems, each of which can be analyzed independently. Once the hierarchy is

drawn, the decision makers systematically evaluate its elements by comparing them one to another two at a time, with the respect to their impact on the element above in the hierarchy. The AHP then converts these evaluations to numerical values, which can be processed and compared over the entire range of the initial problem. A numerical weight or priority, which is derived for each element of the hierarchy allows that often incommensurable elements can be compared to one another in a rational and consistent way.

Based on Zmeskal (2003) and (2012) the AHP methodology has three main phases: structuring the hierarchy, executing paired comparison between the elements and decision alternatives and synthesizing the results. Pair-wise comparison of all the attributes in a given level with the related attributes in the level just above to which it belongs. These priorities are unique within a positive multiplicative constant and therefore belong to an underlying ratio scale. They become unique through normalization. The resulting derived vector reflects the proportion of the decomposition of unity which each element receives.

The results of paired comparisons for n attributes is organized into reciprocal n x n matrix $\mathbf{S} = \left(s_{ij}\right)$:

$$\mathbf{S} = \begin{pmatrix} 1 & s_{12} & \dots & s_{1n} \\ 1/s_{12} & 1 & \dots & s_{2n} \\ \dots & \dots & \dots & \dots \\ 1/s_{1n} & 1/s_{2n} & \dots & 1 \end{pmatrix} \tag{1}$$

This matrix **S** contains the qualified information from expert user about the relations between all the pairs of attributes. The element s_{ij} can be described as the importance ratio of i-th and j-th attributes. So there are two basic properties for the elements of Saaty's matrix **S**:

$$\begin{aligned} s_{ii} &= 1 \\ s_{ij} &= 1/s_{ji} \end{aligned} \qquad i, j = 1, \dots, n \tag{2}$$

The importance of i-th attribute can be measured using the weight v_i. The sum of weights of all n attributes must be equal 1.

$$\sum_{i=1}^{n} v_i = 1 \quad i = 1, \dots, n \tag{3}$$

In ideal situation the Saaty's matrix **S** is consistent and each element s_{ij} can be defined as a ratio of the corresponding weights v_i and v_j:

$$s_{ij} = \frac{v_i}{v_j} \qquad i, j = 1, \dots, n \tag{4}$$

Using the laws of linear algebra it can be shown that the vector of weights $\mathbf{v} = \left(v_i\right)$ is the eigenvector of Saaty's matrix **S** belongs to its largest eigenvalue λ_{max}.

To quantify the relation between i-th and j-th attributes the Saaty's method uses the integer scale 1 to 9 for the elements of the matrix **S**, where: $s_{ij} = 1$ is used for indifference, 3 – weak preference, 5 – strong preference, 7 – very strong preference, and 9 – absolute preference. Even number of points (2, 4, 6, and 8) can be used for more precise differentiation. The impact of positive preference is expressed in the interval $s_{ij} \in (1; 9\rangle$, the scale of inverse preference belongs to an interval $s_{ij} \in \langle 1/9; 1)$, see Saaty (1994).

In real situations the Saaty's matrix **S** is not fully consistent. If inconsistency is not too large, the vector of weights **v** can be approximated by the vector **w** using nonlinear logarithmic – quadratic optimization to minimize the expression:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\log s_{ij} - \log\frac{w_i}{w_j}\right)^2 \quad i, j = 1,\ldots,n \tag{5}$$

It can be shown that vector **w** minimizing the expression (5) can be calculated using the weighted geometric mean of the elements of Saaty's matrix **S** rows:

$$w_i = \frac{\sqrt[n]{\prod_{j=1}^{n} s_{ij}}}{\sum_{i=1}^{n}\sqrt[n]{\prod_{j=1}^{n} s_{ij}}} \tag{6}$$

The inconsistency of Saaty's matrix **S** can be evaluated by the consistency index *CI* defined as:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{7}$$

The more accurate indicator of inconsistency is consistency ratio *CR* defined by Saaty (1994) as:

$$CR = \frac{CI}{RI} \tag{8}$$

where *RI* is the random index which values are derived from empiric research to make the values of consistency ratio *CR* independent of Saaty's matrix **S** rank, see table 1.

Table 1: Random index (*RI*)

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The inconsistency of Saaty's matrix measured by consistency ratio CR is considered as CR ≤ 0,1 to be able to interpret the elements of **w** vector as the good estimation (approximation) of attributes weights.

Using the geometric mean formula (6) the local weights **w** are calculated for each matrix **S** in AHP structure (each group of criteria / sub criteria). The global weights represent a relative distribution of weights across the whole group of sub criteria. To calculate global weights the local weights for *i*-th particular sub criteria group w_i has to be multiplied by relevant *j*-th criteria local weights w_j as follows:

$$w_{ij} = w_i \cdot w_j \tag{9}$$

where w_{ij} are the particular global weights. The sum of global weights in AHP structure has to be equal to 1:

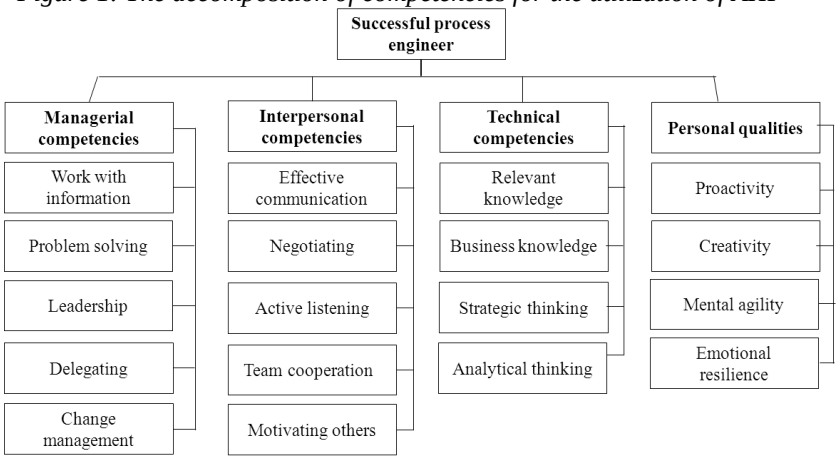
$$\sum_{i,j} w_{ij} = 1 \quad \text{for all possible combinations of } i, j \tag{10}$$

3.2 Applying AHP within the competency modeling

There are many models, constructions and frameworks for the competency models. Nevertheless, a question emerges whether they have any justification and whether they actually represent a contribution to corporate practice. Each of them can be said to have its role as to bring a new view of company performance, Mikusova (2011). For our model we have used a list of competencies, which derive from authors previous work, see Kashi, Friedrich (2013) from which we have selected the ones that were chosen most often by the employees as the most important competencies. Then based on the consultation with experts, i.e. company's HR manager, director and quality manager these competencies were divided

into the four following groups (criteria): managerial competencies, interpersonal competencies, technical competencies and personal qualities. Each group was then divided into several (4 or 5) sub criteria, which belong into the particular group. We have used this division because the individual competencies, in our opinion, belong into these groups. Proposal of the decomposition of competencies for the utilization of AHP is shown in figure 1.

Figure 1: The decomposition of competencies for the utilization of AHP



3.3 Applying AHP within the total reward

The two most used models for intangible remuneration are Aon Hewitt's Model and The Hay Group Performance Model. Nevertheless, the question is what intangible remuneration is most preferred by the employees. To show that AHP method can help to determine the employee's preference of intangible and tangible rewards in an organization, one position was chosen for the illustration. For our model we have used a modified list of rewards from both models, the proposed decomposition of intangible remuneration is shown in fig. 2 and tangible remuneration is shown in fig. 3.

Figure 3: The decomposition of intangible rewards for the utilization of AHP

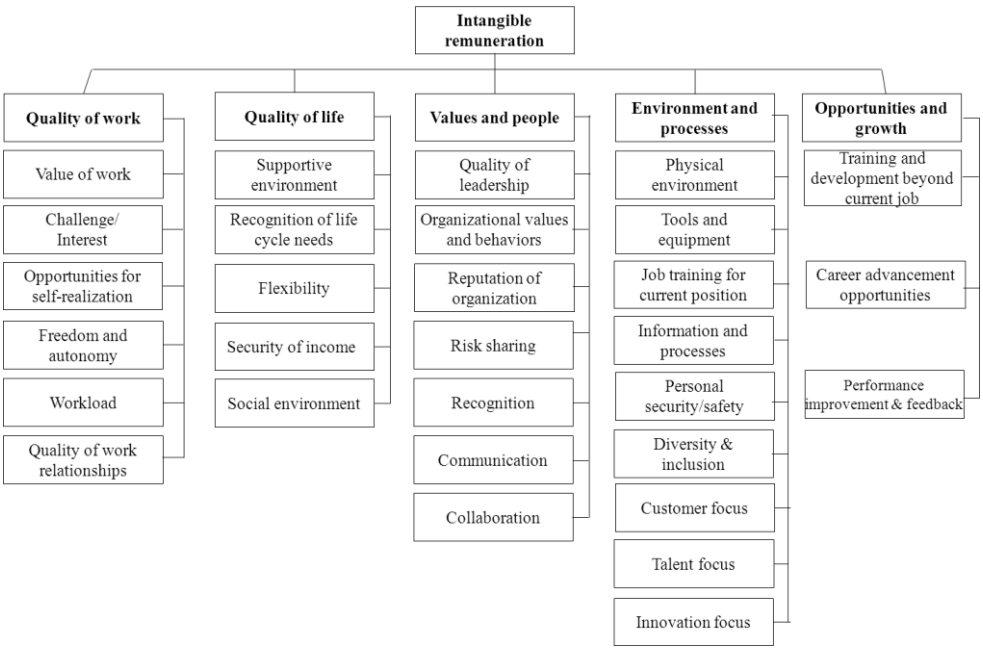
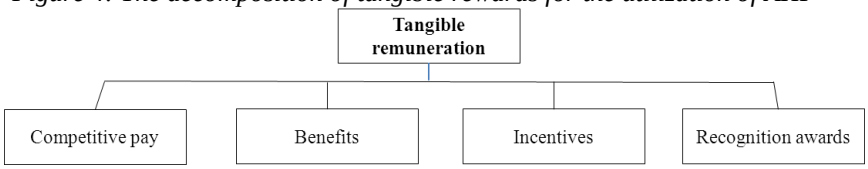


Figure 4: The decomposition of tangible rewards for the utilization of AHP



Next these competencies and rewards (tangible and intangible ones) were compared in pairs based on Saaty's scale (1-9) and a scale was assigned to each competency group (criteria) and each competency (sub criteria). This evaluation was done by a group of experts; the competency model evaluation has been executed by three experts, i.e. HR manager, director and quality manager; the rewards – both tangible and intangible ones were evaluated by four experts, i.e. the process engineers. Both evaluations were executed based on a majority vote. The assessors evaluated the criteria based on the following information. The assessors assigned the following values if the row was preferred before a column: A – the elements are indifferent, B – there is a slight preference, C – there is a strong preference, D – there is a very strong preference, E – there is an absolute preference. If the column was preferred before a row the reciprocal values were assigned, e.g. 1/A, 1/B, 1/C, 1/D, 1/E. The letter evaluation was intentionally proposed by the authors to eliminate the problem of assigning an average number to all comparisons. The letter scale A to E corresponds to the odd values of Saaty's scale 1 – 9.

4 Research's results

The evaluation of the criteria and sub-criteria has been done by three experts in a company, which produces automotive parts. Firstly, the HR managers/director/quality manager evaluated groups of competency based on pair comparison. Then the weights for individual group of competency were calculated based on the formulas, see (6). Then the consistency ratio was calculated, see (8) Consistency ratio index was fulfilled for all matrices. The results of individual competencies ranking are shown in table 2. Table 2 shows the results of competencies global weights and their rankings. The results for tangible and intangible remuneration are shown in table 3.

Table 2: Competencies global weights– analytical AHP

Ranking	**Specialists**	
1	Professional knowledge	**18.73%**
2	Analytic thinking	**17.59%**
3	Proactivity	**14.06%**
4	Mental agility	**9.22%**
5	Team cooperation	**9.08%**
6	Effective communication	**5.94%**
7	Stress resilience	**4.69%**
8	Business knowledge	**4.26%**
9	Active listening	**3.83%**
10	Motivating others	**2.77%**
11	Creativity	**2.38%**
12	Strategic thinking	**1.95%**
13	Problem solving	**1.66%**

14	Work with info	**1.50%**
15	Negotiating	**1.44%**
16	Change management	**0.50%**
17	Leadership	**0.20%**
18	Delegating	**0.20%**

The same, aforementioned procedure has been done for tangible and intangible remuneration; the results are shown in following table 3.

Table 3: Rewards global weights – analytical AHP

Ranking	Intangible rewards	Weight
1	Security of income	16,79%
2	Flexibility	12,17%
3	Performance improvement and feedback	11,81%
4	Challenge/interest	9,07%
5	Value of work	7,55%
6	Recognition of life cycle needs	6,74%
7	Workload	4,66%
8	Communication	3,57%
9	Supportive environment	3,54%
10	Training and development beyond current job	3,21%

Ranking	Tangible rewards	Weight
1	Competitive pay	45,99%
2	Benefits	27,34%
3	Incentives	18,77%
4	Recognition awards	7,89%

4 Conclusion

The companies' competitiveness is becoming very important today. One way how to remain competitive is to employ and keep quality people. One option how to establish this is to have well designed competency models, from which is clear what competencies are the most important in particular job position and which competencies are most preferred by the company and what rewards the employees can expect for a good performance. This paper briefly describes the competency models, its development and utilization; further, it describes the tangible and intangible rewards. It also deals with the description of AHP and its utilization in competency modeling, and total reward.

Competency models/total reward designed by the utilization of AHP method can help the human resource management in several areas, such as: planning training and development activities, employee appraisal, hiring new staff and compensation. The competencies are ordered based on importance, the HR department and the employees themselves will know on which competency they need to concentrate the most and what reward they will receive for their performance and also what knowledge/skills have to be improved by the employees the most. In our case the employees should mostly concentrate on improving professional knowledge, analytic thinking, proactivity, mental agility, team cooperation, effective communication and stress resilience, since these competencies are the most preferred by the company for the process engineer position.

If the employees reach a satisfactory appraisal then the company should provide the most preferred rewards, in our case the employees prefer security of income, flexibility and performance improvement and feedback, from tangible rewards the competitive pay is the most important for employees followed by benefits. If the company wants to keep quality people then the managers should give the people what they prefer in exchange for an excellent performance, this would keep the people satisfied and motivated. In addition, the company can use the employees who have received the highest score in the appraisal, especially for the key competencies for coaching other employees. If the employees will improve their core competencies, the whole organization knowledge will increase and the company, overall, should be able to deliver a better performance.

Acknowledgements

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Back-test of efficiency by combining technical indicators on the EUR/JPY

Andrea Kolková [1]

Abstract

The aim of the paper is to backtest the profitability of forex trading, using trades based on combinations of technical indicators. Empirical studies, confirmed by scientific work foe example (Mastný, 2003), state that most strategies based on separate technical indicators have been unsuccessful for a long time. The paper verifies the possibility of using a combination of individual indicators. Calculations will be made on currency pair EUR/JPY. Strategies based on combinations of SMA9, SMA20, EMA 9, EMA 20, Bollinger bands, Parabolic SAR, MACD, Stochastic Oscillator, CCI, ADX will be evaluated.

Key words

Forex, SMA, EMA, Bollinger bands, Parabolic SAR, MACD, Stochastic Oscilator, CCI, ADX

JEL Classification: G12, G14

Úvod

Mezi akademiky i obchodní veřejností stále panuje rozpor mezi efektivností technické analýzy a platností teorie efektivních trhů. Tomuto rozporu se věnuje například příspěvek Franek a Kresta (2016), který hodnotí klouzavé průměry. Nejširší komparaci jednotlivých výzkumů provedl Park a Irwin (2007), z tohoto příspěvku vyplynul fakt, že efektivitu technické analýzy nelze jednoznačně potvrdit. Rozdělil proto výzkumy zabývající se efektivitou technické analýzy do dvou skupin dle časového horizontu a to na studie z let 1960-1987 a studie novější tj. od roku 1987. Z celkového počtu 95 moderních studií, vyplynulo 56 jednoznačně potvrzujících efektivitu technické analýzy, 20 ji vyvrací a 19 poskytlo nejednoznačné výsledky. Lze tedy označit, že většina akademických výzkumů se kloní k možné efektivitě indikátorů technické analýzy. Je otázkou, zda použití indikátorů jednotlivě je dlouhodobě efektivní. Cílem tohoto příspěvku je potvrdit efektivnost využití kombinace technických indikátorů. Výzkum bude proveden na datech forexového páru EUR/JPY za období 10-ti let.

1. Technická analýza

Jedním z nástrojů odhadu potencionálního vývoje kurzu podkladového aktiva je technická analýza spolu s analýzou fundamentální a psychologickou.

Indikátory technické analýzy lze členit dle různých hledisek. Veselá a Oliva (2016) člení indikátory na trendové, cenové, objemové, sentiment indikátory a indikátory šíře a výkonnosti trhu. Elder (2006) člení indikátory na analýzu grafů a počítačovou technickou analýzu. Hartman (2014) pak indikátory rozděluje na trendové indikátory, oscilátory a predikující indikátory. Odlišná členění pak můžeme najít i u jednotlivých brokerů, potažmo u konkrétních obchodních platforem, program Metatrader specifikuje indikátory na ukazatele trendu,

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oscilátory a objemové indikátory. V tomto příspěvku bylo použito členění dle Metatraderu a testovány byli vybrané ukazatele trendu a oscilátory.

1.1 Ukazatele trendu

Ukazatele trendu je skupina indikátorů, které kopírují trend. Obvykle je z nich patrný nastolený bull či bear trend a některé jsou schopny odhadnout i jeho změnu. Ukazatele jsou často používány a patří k indikátorům první volby. Mezi nejznámější indikátory patří Simple Moving Average (dále jen SMA), Exponential Moving Average (dále jen EMA), Moving Average Convergence/Divergence (dále jen MACD), Bollinger bands, Parabolic Stop and Reversal (dále jen Parabolic SAR), Average Movenment Directional Index (dále jen ADX), které jsou vyžity i v tomto příspěvku, dále pak Envelopes, Average Directional Movement Index, Ichimoku Kingo Hyo, Trix, Vertical Horizontal Filter a řada jiných.

1.1.1 Klouzavé průměry

Efektivita klouzavých průměrů byla testována například v příspěvcích (Kresta, 2015), nebo (Kolková, 2017). Klouzavé průměry jsou velmi frekventovaným nástrojem a jsou také základem řady dalších indikátorů. Klouzavé průměry můžeme rozeznávat jako Jednoduché klouzavé průměry (SMA), Vážené klouzavé průměry (Wighted Moving Average), Exponenciální klouzavé průměry (EMA), Trojúhelníkové klouzavé průměry (Triangular Movina Average), Proměnlivé klouzavé průměry (Variable Movina Average), Klouzavé průměry přizpůsobené objemu (Volume- Adjusted Moving Average), Klouzavé průměry založené na předpovědi časových řad (Time Series Forecast Moving Average).

Klouzavé průměry pak můžeme ještě odlišovat dle délky časové periody, která se může pohybovat v rozmezí 5, ale také 200 obchodních dnů. Pro volbu časové periody je možno využít i další nástroje technické analýzy, jako například Fibonacciho posloupnost definovanou v příspěvku (Kolková, 2017).

V tomto příspěvku budou využity klouzavé průměry SMA 20, SMA9, EMA20 a EMA9. Indikátor SMA je definován vztahem,

$$SMA = \frac{P_M + P_{M-1} + ... + P_{M-(n-1)}}{n}, \text{ kde} \tag{1}$$

M je délka periody (v tomto případě 9 nebo 20), *P* jsou jednotlivé hodnoty kurzu.

Indikátor EMA je možno definovat upraveným vztahem,

$$EMA_{dnes} = (EMA_{vččer} \cdot (1 - ep)) + (P_{dnes} \cdot ep), \text{ kde} \tag{2}$$

ep je exponenciální procento neboli vyhlazovací faktor, který je možno vypočítat různými způsoby, projevuje se zde jistá nejednotnost v teoretických přístupech a komparací různých zdrojů, lze dojít k různým výpočtům. V tomto příspěvku je vyčíslena dle,

$$ep = \frac{2}{M+1}. \tag{3}$$

1.1.2 Parabolic SAR

Testování indikátoru Parabolic SAR na měnovém páru EUR/USD bylo realizováno také v příspěvku (Kolková, 2017), z jehož výsledku vyplynulo, že za použití samostatného indikátoru SAR bylo z 12 sledovaných let 7 ziskových. Celková výše ztráty po započtení transakčních nákladů ovšem převyšovala zisk. V kombinaci s indikátorem MACD ovšem zisk převýšil ztráty a obchodní systém byl označen za funkční.

Autorem indikátoru Parabolic SAR je J.W. Wilder (1978) v knize *New Concepts in Technical Treding Systems*. Parabolický systém není tvořen křivkou, ale sérií bodů odpovídající jednotlivým časovým intervalům. Indikátor je možno definovat vzorcem,

$$SAR_0 = SAR_{-1} + \alpha(P_{\exp} - SAR_{-1}), \text{ kde} \quad (4)$$

SAR_0 je hodnota bodu indikátoru pro daný časový rámec, SAR_{-1} je hodnota bodu indikátoru pro předešlý čas, α je akcelerační koeficient, P_{ex} je maximální zaznamenaný kurz během aktuálního trendu.

Specifikou tohoto indikátoru je akcelerační koeficient. Tento nástroj určuje, jak rychle se mění hodnoty ve směru trendu. Základní nastavení, které je využito i v tomto příspěvku, je vhodné pro minimální úroveň růstu 0,02 a maximální hodnota by se měla pohybovat do 0,2. Systém je možné nastavit i na citlivější reakci a to snížením akceleračního koeficientu. Naopak jeho zvýšením lze docílit pomalejší reakce systému na změny kurzu. V praxi se tak akcelerační koeficient pohybuje od 0,015 do 0,025 v jeho minimu a od 0,18 do 0,23 v maximální velikosti.

1.1.3 Bolinger Bands

Konstrukce Bolinger bands byla definována například v příspěvku (Kolková, 2016). Autorem indikátoru je John Bollinger, který tuto metodu publikoval v 80.-90. letech 20. století (Bollinger, 1992). Patří do skupiny Kanálových systémů, které popsal už Kaufman (1987). Technický indikátor pracuje s volatilitou a relativní úrovní cen v určitém období. Čím vyšší je volatilita, tím širší je Bollingerovo pásmo. Ceny finančního instrumentu se obvykle pohybují v rámci Bollingerových pásem a odrážejí se z jedné strany na druhou.

Bollingerova pásma tvoří tři křivky, jak uvádí například Elder (2006). Střední je vyjádřením SMA o délce *n* obchodních dnů (obvykle 20 denní klouzavý průměr). Horní pásmo je rovno SMA plus *r* krát standardní odchylka a dolní pásmo mínus stejný násobek standardní odchylky.

Volba násobku SMA pak určí pravděpodobnost, že se kurz bude pohybovat mezi těmito křivkami. Pokud dosadíme 1násobek standardní odchylky, pak platí, že 68% všech dat v daném časovém úseku se bude pohybovat v rozmezí horního a dolního Bollingerova pásma. V případě, že dosadíme 2násobek standardní odchylky, bude se mezi horním a dolním pásmem pohybovat 95% všech hodnot a pokud dosadíme 3násobek, získáme 99,7% hodnot v rozmezí obou pásem. V tomto příspěvku je Bollinger Bands konstruován na základě 20 denního klouzavého průměru s dvounásobnou standardní odchylkou.

1.1.4 MACD

Dalším z použitých ukazatelů trendu je MACD. Tento indikátor byl vyvinut Geraldem Applem (2005), a je dán rozdílem (výjimečně i podílem) mezi dvěma klouzavými průměry. Hodnota MACD pak osciluje kolem nulové linie. V tomto příspěvku je použita základní verze a to rozdíl mezi 26 denním a 12 denním indikátorem EMA. Tímto je vykreslena křivka MACD, s ní je ještě nutno zakreslit křivku 9 denní EMA, která je nazývána signalizační křivkou.

1.1.5 ADX

Tento indikátor sestrojil opět J. Wilder (1978) a jeho prvotní určení je pro obchody s komoditami. Umožňuje odhalit, zda se kurzy pohybují ve zřetelných trendech, či pouze v úzkém rozpětí bez jasného trendu. Je tedy schopen odhalit sílu trendu, nikoli však jeho směr. Výpočet indikátoru vychází v tzv. True Range, který je možno určit dle,

$$True\ Range = Max(max.kurz_t, uzavírací\ kurz_{t-1}) - Min(min.kurz_t, uzavírací\ kurz_{t-1}). \quad (5)$$

Z tohoto indikátoru je pak vyhlazením pomocí klouzavých průměrů stanovena hodnota Average True Range (dále ATR). Samotný ADX pak tvoří dvě křivky +DI a –DI.

$$+DI = \frac{EMA(+DI,t)}{ATR} \cdot 100 \text{ a} \tag{6}$$

$$-DI = \frac{EMA(-DI,t)}{ATR} \cdot 100, \text{ kde} \tag{7}$$

t je časová perioda klouzavého průměru EMA, v tomto příspěvku je použit 14 denní EMA.

1.2 Oscilátory

Oscilátory jsou indikátory, které pracují s kurzem podkladového aktiva, a lze na jejich základě posoudit sílu nastoupeného trendu. Díky tomuto pak lze odhadnout pravděpodobnost změny či setrvání trendu. Jejich specifickým rysem je kolísání kolem určené hodnoty, či kolísání v rozmezí určitého pásma.

1.2.1 Stochastic Oscilator

Stochastik oscilátor, jehož autorem je George Lane (1984), je určen k vyjádření možnosti otočení stávajícího trendu. Tento oscilátor porovnává uzavírací cenu měnového kurzu a jejich cenové rozpětí v daném časovém období. Citlivost oscilátoru k vývoji na trhu lze snížit úpravou časového období. Stochastický oscilátor je zobrazován jako dvě křivky a zároveň má vymezené pásmo oscilace. Hlavní křivka se nazývá *%K* a druhá je označována jako *%D*. Je možno je zapsat vzorci,

$$\%K = \frac{\textit{uzavírací kurz} - \textit{minimální kurz za \%K period}}{\textit{maximální kurz za \%K period} - \textit{minimální kurz za\%K period}}, \tag{8}$$

$$\%D = \frac{\textit{3 – denní suma(uzavírací kurz – minimální kuzr za\%K period)}}{\textit{3 – denní suma(maximální kurz za \%K period – minimální kurz za\%K period)}}. \tag{9}$$

1.2.2 Comodity Channel Index (dále jen CCI)

Autorem indikátoru CCI je Donald Lambert (1980), který tento indikátor, stejně jako ADX, vytvořil pouze na trh s komoditami, ačkoli prakticky ho nikdy neaplikoval. Jeho využívání pro jiné podkladové aktiva začalo až mnohem později. Používá nejčastěji k určování síly a rychlosti trendu. Je možno ho vypočítat dle,

$$CCI = \sum \frac{(TP_i - SMA_{n,TP})}{0{,}015 \cdot \sigma_{n,TP}{}^2}, \text{ kde} \tag{10}$$

$$TP = \frac{maximum_i + minimum_i + uzavíraci\ kurz_i}{3}, \tag{11}$$

σ^2 je směrodatná odchylka.

2. Metodika výzkumu

Zvolené technické indikátory jsou aplikovány na digitální opce diskutované již v předchozích výzkumech (Kolková, 2016). Tyto opce umožňují ukázat efektivitu zvolených indikátorů i bez vlivu money managementu. Umožňují pouze long nebo short obchod (up nebo down opce) uzavřené na předem definovanou dobu (v toto příspěvku 15 minut), kdy po uzavření obchodu již není možno pozici měnit a je možno pouze vyčkat jejího uzavření.

Digitální opce jsou podloženy měnovým párem EUR/JPY. Data jsou získána z programu Metatrader 4 prostřednictvím brokera XTB. Jsou analyzovány kurzy v pětiminutových časových intervalech v období od 20.11.2006 do 26.5.2017. Celkem je tedy podrobeno analýze 65 000 hodnot kurzu měnového páru EUR/JPY, ve tvaru OHLC (otevírací a uzavírací kurz, maximální a minimální kurz). K výpočtům jednotlivých technických indikátorů byl

využit program TA-Lib pro Excel, výpočty samotných ziskových pozic jsou realizovány v programu Excel.

Pozice jsou nejprve testovány s využitím indikátorů samostatně. Následně jsou v souladu s cílem příspěvku testovány jednotlivé kombinace indikátorů. Jednotlivé obchodní systémy jsou voleny dle logiky konkrétních indikátorů.

Pro klouzavé průměry a to jak SMA tak EMA platí, že nákupním signálem, respektive signálem ke vstoupení do long pozice, je situace protnutí linie klouzavého průměru a kurzu zespodu nahoru. Vstup do short pozice, nebo prodej digitální opce, je realizován v situaci protnutí klouzavého průměru a kurzu EUR/JPY shora.

Pro systémy založené na Bollinger bands platí, že nákupní signál (long pozice) je realizován v případě protnutí kurzu a horní linie Bollinger Bands. Naopak v případě protnutí dolní linie a kurzu, lze uvažovat o prodejním signálu, tedy vstoupení do short pozice.

Systém Parabolic SAR definuje vstup do long pozice, tedy nákup měnového páru, v situaci vykreslení bodů systému sdola, v opačné situaci je nutno pomýšlet na short pozici, detailní popis lze najít v předchozích výzkumech (Kolková, 2017).

Indikátor MACD je použit v obchodním systému, kdy signály generuje protnutí křivky MACD se signalizační křivkou. V případě protnutí zdola se jedná o signál k nákupu (long pozice), v opačném případě investor může vstoupit do short pozice, tedy prodat digitální opci.

Indikátory Stochastic Oscilátor, CCI a ADX oscilují v předem vymezeném pásmu. Překročení pásma shora je pak signál k short pozici, překročení zdola k long pozici. Konkrétně u indikátoru Stochastic Oscilátor je pásmo definováno na hranici 80 a 20 bodů, u CCI indikátoru je to +100 a -100 bodů. U indikátoru ADX je zvolen obchodní systém, který vyhodnocuje v souladu s logikou indikátoru pouze sílu trendu, nelze tedy určit jeho směr. Indikátor proto nelze použít samostatně k určení signálu, nicméně je používán v kombinaci s ostatními indikátory a to tak, že potvrzuje signál pouze v situaci silného trendu. Silný trend je u ADX definován v tomto příspěvku hranicí 40.

3. Výsledky výzkumu

Výsledky testování indikátorů samostatně jsou uvedeny v tabulce 1. Jednoznačně z nich vyplývá, že na testovaném měnovém páru EUR/JPY v daném testovaném období ani jeden z indikátorů není schopen nabídnout 60% ziskových obchodů. Indikátory Parabolic SAR, Bollinger bands, Stochastic oscilátor, CCI dokonce neposkytly ani 50% ziskových obchodů a lze je tedy považovat za ztrátové v delším období. Nejzajímavějšího výsledku dosáhl indikátor MACD, který na daném kurzu v desetiletém intervalu poskytuje 54,4% ziskových signálů.

Tabulka 1: Vyhodnocení ziskových signálů u indikátorů použitých samostatně

indikátor	celkový počet signálů	počet ziskových signálů	procenta	procento sell signálů	procento signálů buy	ziskovost sell signálů	ziskovost buy signálů
SMA 9	11702	6039	51,61%	50%	50%	51,29%	48,70%
SMA 20	6891	3568	51,78%	50%	50%	51,45%	48,54%
EMA 9	12164	6317	51,93%	50%	50%	51,60%	48,38%
EMA 20	7507	3893	51,86%	50%	50%	50,98%	49,01%
MACD	4257	2316	54,40%	50%	50%	48,66%	51,42%
SAR	7053	3474	49,26%	50%	50%	47,55%	52,44%
Bollinger Bands	29155	14499	49,73%	50,28%	49,72%	49,53%	50,47%
Stochastic Oscilátor	5832	2741	47,00%	50%	50%	52,02%	47,97%
CCI	6881	3377	49,08%	50%	50%	48,80%	51,19%
ADX	indikátor samostatně nelze použít						

Zdroj: vlastní výzkum

V případě, že se jednotlivé indikátory začnou vzájemně kombinovat, vznikne tak 45 různých obchodních systémů, v nichž vzájemně kombinujeme jak ukazatele trendu, tak oscilátory. Z tabulky 2 lze v horní části vyčíst počet ztrátových obchodů, dolní část tabulky definuje počty ziskových obchodů z kombinace konkrétních indikátorů.

Tabulka 2: Počet ztrátových a ziskových obchodů při systémech založených na kombinaci indikátorů

indikátor	Bollinger bands	SAR	SMA9	SMA20	EMA9	EMA20	Stochastic Oscilátor	MACD	CCI	ADX	počet ztrátových obchodů
Bollinger bands	X	1278	1334	1547	2688	1669	1120	713	1125	1408	
SAR	1325	X	1134	539	1088	587	325	211	262	580	
SMA9	1428	704	X	1581	4079	1799	415	1356	863	638	
SMA20	1674	376	1599	X	1951	2160	272	800	431	268	
EMA9	2840	734	4261	2021	X	2904	549	1428	2122	593	
EMA20	1813	377	1854	2288	3119	X	320	877	1807	203	
Stochastic Oscilátor	953	349	476	296	673	358	X	166	1083	393	
MACD	840	246	1400	828	1485	918	211	X	825	227	
CCI	1054	417	837	442	2182	1902	992	798	X	442	
ADX	1409	631	673	267	598	207	392	297	469	X	
počet ziskových obchodů											

Zdroj: vlastní výzkum

Tabulka 3 definuje stejné informace v jiném formátu, a to v horní části tabulky jsou celkové počty signálů, které daný obchodní systém poskytl. V dolní části pak lze vidět procento ziskových obchodů v rámci daného systému.

Tabulka 3: Počet signálů a procenta ziskových obchodů z nich

indikátor	Bollinger bands	SAR	SMA9	SMA20	EMA9	EMA20	Stochastic Oscilátor	MACD	CCI	ADX	celkový počet signálů
Bollinger bands		2603	2762	3221	5528	3482	2073	1553	2179	2817	
SAR	50,90%		1838	915	1822	964	674	457	679	1211	
SMA9	51,70%	38,30%		3180	8340	3653	891	2756	1700	1311	
SMA20	51,97%	41,09%	50,28%		3972	4448	568	1628	873	535	
EMA9	51,37%	40,29%	51,09%	50,88%		6023	1222	2913	4304	1191	
EMA20	52,07%	39,11%	50,75%	51,44%	51,78%		678	1795	3709	410	
Stochastic Oscilátor	45,97%	51,78%	53,42%	52,11%	55,07%	52,80%		377	2075	785	
MACD	54,09%	53,83%	50,80%	50,86%	50,98%	51,14%	55,97%		1623	524	
CCI	48,37%	61,41%	49,24%	50,63%	50,70%	51,28%	47,81%	49,17%		911	
ADX	50,02%	52,11%	51,33%	49,91%	50,21%	50,49%	49,94%	56,68%	51,48%		
procento ziskových obchodů											

Zdroj: vlastní výzkum

Je patrné, že výsledky jsou u některých systémů lepší, než při použití indikátorů samostatně, u některých ovšem nikoli. U indikátoru Bollinger bands dosáhne systém lepších výsledků v kombinaci s Parabolic SAR, všech klouzavých průměrů, indikátoru MACD a ADX. Nikde ovšem nepřekročí ani hranici 60%.

Indikátor Parabolic SAR dosahuje lepších výsledků u všech indikátorů mimo klouzavých průměrů, velmi zajímavých výsledků dosahuje v kombinaci s indikátorem CCI a to 61,41%, což je nejlepší výsledek ze všech zvolených obchodních systémů.

Indikátory na bázi klouzavých průměrů poskytují v kombinaci s jinými indikátory přibližně stejné výsledky, jako jejich samostatné použití. Vytvářet obchodní systém kombinací klouzavých průměrů tedy nemá opodstatnění. Což je dáno zejména tím, že ostatní ukazatele do svých výpočtů velice často zahrnují právě i průměry.

Při vzájemné kombinaci oscilátorů poskytuje zajímavý výsledek pouze kombinace MACD a ADX, respektive MACD a Stochastic Oscilátorem.

Procentuálně nejvíce ziskových obchodů poskytuje systém SAR v kombinaci s CCI, dále MACD v kombinaci s ADX a MACD v kombinaci s Stochastic Oscilátorem.

Dalším důležitým krokem je zhodnocení počtu generovaných signálů, protože i při nižším procentu ziskových obchodů (ale vyšším než 50%), může být schopen systém generovat vyšší zisky, v případě velkého počtu signálů. Počty signálů ve zvolených systémech se výrazně liší, nejvyššího počtu dosahují systémy v kombinaci s klouzavými průměry. Naprosto nejvíce signálů poskytuje kombinace EMA9 a SMA9. Naopak nejméně signálů poskytla kombinace MACD se Stochastic Oscilátorem, která byla vyhodnocena jako třetí nejčastěji generující ziskové signály. Celkově je možno říct, že strategie generující nejčastěji ziskové signály, patří mezi ty, které signálů generují méně.

Pro zhodnocení obou veličin, a to jak procenta ziskových obchodů, tak celkového počtu generovaných signálů, musí tedy být vyčíslen i bodový zisk z obchodních systémů. Výsledky naznačuje tabulka 3. Celkový zisk je vyčíslen jako hrubý zisk v bodech měnového kurzu EUR/JPY. Čistý zisk by pak byl závislý na celkové velikosti vloženého kapitálu na jeden obchod a transakčních nákladech.

Tabulka 4: Výše hrubého bodového zisku obchodních systémů

indikátor	Bollinger bands	SAR	SMA9	SMA20	EMA9	EMA20	Stochastic Oscilátor	MACD	CCI	ADX
Bollinger bands										
SAR	8,76									
SMA9	-28,41	-152,07								
SMA20	3,35	-52,19	-53,18							
EMA9	-8,43	-134,02	-18,95	-37,90						
EMA20	16,08	-88,87	-28,02	-30,12	-0,07					
Stochastic Oscilátor	-28,73	2,00	15,51	-6,54	22,40	0,95				
MACD	-92,44	5,04	4,04	-14,39	125,27	-3,32	1,92			
CCI	-0,13	69,26	29,87	30,88	48,14	54,02	-10,79	-1,59		
ADX	-11,97	53,53	-15,71	-13,85	95,51	-4,81	1,14	30,29	9,96	
výše hrubého zisku v bodech										

Zdroj: vlastní výzkum

Velikost zisku byla nejvyšší u strategie EMA 9 a MACD, která poskytovala jen 50,98% ziskových obchodů, nicméně signálů poskytla 2 913. Další, co do zisku zajímavou kombinací, je ADX v kombinací s EMA9 a také systém procentuálně nejčastěji generující zisk Parabolic SAR a CCI.

4. Závěr

Příspěvek potvrdil, že kombinace indikátorů mohou poskytnout investorům lepší výsledky než použití indikátorů samostatně. Hranici 60% ziskových obchodů ovšem přesáhla pouze kombinace Parabolic SAR a CCI. Indikátory na bázi klouzavých průměrů poskytují samostatně přibližně stejné výsledky, jako za použití kombinací s jinými indikátory.

Nejvyššího počtu signálů poskytují obchodní systémy založené na klouzavých průměrech a to nejvíce EMA9 v kombinaci s SMA9, obecně lze říci, že strategie poskytující nejvíce ziskových signálů, generují signály nejméně často. Nejvyššího hrubého bodového zisku ovšem dosáhl systém Parabolic SAR a CCI.

Pro další výzkum by bylo vhodné vypočítat celkový zisk se zahrnutím transakčních nákladů. U digitální opce na EUR/JPY tyto transakční náklady mohou dosáhnout až na 24,09% ze zisku. Transakční náklady tak mohou výrazně ovlivnit výsledný efekt z obchodního systému. Další směry výzkumu by mohly vést k porovnání výsledků s jinými měnami, případně s jinými podkladovými aktivy. Případně zahrnout kombinaci více než dvou indikátorů.

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SD portfolio enhancement with and without short sales

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Abstract

The paper deals with portfolio optimization problems which maximize portfolio mean return under constraints that the random return outperforms a given random benchmark. The outperformance can be understood in several different ways. In this paper, we focus on the first, second and third order stochastic dominance constraints. The first order stochastic dominance enhancement leads to mixed-integer problem, while the second order stochastic dominance enhancement can be formulated as a linear program. The third order stochastic dominance constraints are approximated by the so called super-convex third order stochastic dominance constraints which compare the semivariance functions in various grid points. In all problems, we compare two cases: models with and without short sales, using US stock market data with or without risk free asset. US market portfolio approximated by CRSP is used as a benchmark.

Key words

Portfolio selection, stochastic dominance enhancement, short sales

JEL Classification: D81, G11

1. Introduction

Every investor seeks for investments with high mean returns. Starting from a seminal paper of Markowitz (1952), a risk measuring and controlling has become another important goal of the investor. The risk of the investment can be modelled in various ways using risk measures, deviation functionals, etc. Alternatively, the risk attitude of the decision maker can be captured by a utility function. If so, the investor wants to maximize expected utility of the return or of the final wealth. However, if the utility function of the decision maker is not know, the maximizing expected utility problem cannot be solved. If only a partial knowledge about the utility function is available (for example concavity) then one can consider a set of admissible utility functions which leads to so called stochastic dominance approach. Stochastic dominance is, in general, a relation between two random variables. This relation allows for comparison of random returns or losses of the investments.

The basics of stochastic dominance go back to 1960th, see Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970). In these papers, the most popular stochastic dominance relations, so called the first-order (FSD), the second-order (SSD) and the third-order (TSD) stochastic dominance were introduced and analyzed. We say that a random variable X dominates a random variable Y with respect to the N-th order stochastic dominance, $N = 1, 2, 3$ if

$$\mathbb{E}u(X) \geq \mathbb{E}u(Y) \quad \text{for} \quad \text{all} \quad u \in U_N$$

where the set of admissible utility functions for the N-th order stochastic dominance is defined as follows:

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$$U_N = \{u(x) \in D^N : (-1)^k u^{(k)} \leq 0, \quad \forall x, \quad k = 1, \ldots, N\}.$$

See Levy (2016) and references therein for more details.

As Kopa (2016) summarizes, in last years, a substantial development of the stochastic dominance applications in finance was realized, mainly in the following directions:

• portfolio efficiency with respect to stochastic dominance criteria was analyzed, see e.g. Post and Kopa (2013), Grechuk (2014), Kopa and Post (2015) or Post et al. (2015).

• portfolio enhancement using stochastic dominance rules, mainly the second-order stochastic dominance criterion, see e.g. Roman et al. (2013), Hodder et al. (2015), Post and Kopa (2017).

• more robust version of stochastic dominance relations and stochastic dominance efficiency tests, see e.g. Dentcheva and Ruszczynski (2010), Kopa (2010), Dupačová and Kopa (2012, 2014).

• more general stochastic dominance (ordering) rules, see e.g. Ortobelli et al. (2013, 2016) and references therein, or Post (2016).

• Data Envelopment Analysis equivalent models in the sense that a portfolio is classified efficient with respect to a stochastic dominance relation if and only if it is efficient with respect to a particular Data Envelopment Analysis model, see Branda and Kopa (2012, 2014, 2016) for more details.

Moreover, the first and second order stochastic dominance constrains were also applied to an individual pension allocation multistage stochastic program in Kopa et al. (2017).

In this paper we follow Kopa (2016) in focusing on the portfolio selection problems which maximize mean return under stochastic dominance constraints of the first, second and third order. Contrary to Kopa (2016), for each of these orders we consider the following four models:

1. Without risk free asset and short sales are not allowed
2. Without risk free asset and limited short sales are allowed
3. With risk free asset and short sales are not allowed
4. With risk free asset and limited short sales are allowed.

In all these models, we analyze the outperformance of the mean return of the solution portfolio relative to benchmark mean return. We provide the empirical analysis using annual data (returns) of 10 industry US representative portfolios from the Kenneth French library in period 1966 – 2015. We consider the US market index proxied by CRSP as the benchmark. The risk free asset is proxied by the US one-year bond.

The remainder of this paper is structured as follows. Section 2 presents the portfolio selection models with FSD, SSD and TSD constraints with/without risk free asset and/or short sales. Section 3 shows the results of the numerical study. The paper is summarized and concluded in Section 4.

2. Stochastic dominance enhancement models

Let us consider a random vector $\mathbf{r} = (\mathrm{r}_1, \mathrm{r}_2, \ldots, \mathrm{r}_\mathrm{N})'$ of returns of N assets in T equiprobable scenarios. The returns of the assets for the various scenarios are collected in matrix:

$$X = \begin{pmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^T \end{pmatrix}$$

where $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_N^t)$ is the t-th row of matrix X. A vector of portfolio weights is denoted by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_N)'$, the weight of risk free asset is λ_0. In this paper, we focus on stochastic dominance constraints between random return $\mathbf{r}'\boldsymbol{\lambda}$ of a portfolio $\boldsymbol{\lambda}$ and random return $\mathbf{r}'\boldsymbol{\tau}$ of the benchmark portfolio $\boldsymbol{\tau}$.

The set of feasible portfolios Λ depends on the including/excluding risk free asset and short sales. To guarantee the existence of an optimal portfolio, we assume only limited short sales. Summarizing:

1. In model without risk free asset and short sales:
$$\Lambda = \{\boldsymbol{\lambda} \in R^N \big| \textstyle\sum_{n=1}^{N} \lambda_n = 1, \quad \lambda_n \geq 0, \ n = 1,2, \dots, N\}. \tag{1}$$
2. In model without risk free asset and with limited short sales allowed:
$$\Lambda = \{\boldsymbol{\lambda} \in R^N \big| \textstyle\sum_{n=1}^{N} \lambda_n = 1, \quad \lambda_n \geq -1, \ n = 1,2, \dots, N\}. \tag{2}$$
3. In model with risk free asset where short sales are not allowed:
$$\Lambda = \{\boldsymbol{\lambda} \in R^{N+1} \big| \textstyle\sum_{n=0}^{N} \lambda_n = 1, \quad \lambda_n \geq 0, \ n = 0,1,2, \dots, N\}. \tag{3}$$
4. In model with risk free asset and with limited short sales allowed:
$$\Lambda = \{\boldsymbol{\lambda} \in R^{N+1} \big| \textstyle\sum_{n=0}^{N} \lambda_n = 1, \quad \lambda_n \geq -1, \ n = 0,1,2, \dots, N\}. \tag{4}$$

The optimization models with stochastic dominance constraints were firstly proposed in Dentcheva and Ruszczynski (2003) for the second-order stochastic dominance relation. Later on, second order stochastic dominance was applied to portfolio enhancing in e.g. Roman, Mitra and Zverovich (2013). Similarly, Hodder, Jackwerth and Kolokolova (2015) applied Kopa and Post (2015) model for the portfolio enhancement based on the second-order stochastic dominance. Post and Kopa (2017) derived a portfolio enhancement model emloying third order stochastic dominance. In this paper, we follow these papers and consider the following three models:

$$\begin{gathered} \max \mathbb{E}(\mathbf{r}'\boldsymbol{\lambda}) \\ \text{s.t. } \mathbf{r}'\boldsymbol{\lambda} \succ_{FSD} \mathbf{r}'\boldsymbol{\tau} \\ \boldsymbol{\lambda} \in \Lambda \end{gathered} \tag{5}$$

$$\begin{gathered} \max \mathbb{E}(\mathbf{r}'\boldsymbol{\lambda}) \\ \text{s.t. } \mathbf{r}'\boldsymbol{\lambda} \succ_{SSD} \mathbf{r}'\boldsymbol{\tau} \\ \boldsymbol{\lambda} \in \Lambda \end{gathered} \tag{6}$$

$$\begin{gathered} \max \mathbb{E}(\mathbf{r}'\boldsymbol{\lambda}) \\ \text{s.t. } \mathbf{r}'\boldsymbol{\lambda} \succ_{TSD} \mathbf{r}'\boldsymbol{\tau} \\ \lambda \in \Lambda \end{gathered} \tag{7}$$

where τ is a given benchmark (reference) portfolio and FSD, SSD, TSD denote the first, second, third order of stochastic dominance, respectively. Finally, the set of feasible portfolios is given by one of the formulas (1)-(4). Since we consider a discrete distribution of returns with equiprobable scenarios we can follow Kuosmanen (2004) and reformulate (5) as a linear mixed-integer programming problem:

$$\begin{gathered} \max_{p_{i,j}, \boldsymbol{\lambda}} \frac{1}{T} \textstyle\sum_{t=1}^{T} \mathbf{x}^t \boldsymbol{\lambda} \\ \text{s.t. } X\boldsymbol{\lambda} \geq PX\boldsymbol{\tau} \end{gathered} \tag{8}$$

$$P = \{p_{i,j}\}_{i,j=1}^{T}, \sum_{i=1}^{T} p_{i,j} = \sum_{j=1}^{T} p_{i,j} = 1$$
$$p_{i,j} \in \{0,1\}, i,j = 1,\dots,T$$
$$\boldsymbol{\lambda} \in \Lambda.$$

Similarly, following Kopa and Post (2015) or Hodder, Jackwerth and Kolokolova (2015), problem (6) can be reformulated as follows:

$$\max_{d_s, v_{t,s}, \boldsymbol{\lambda}} \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}^t \boldsymbol{\lambda} \tag{9}$$
$$\text{s.t. } -T^{-1}\mathbf{x}^t\boldsymbol{\lambda} + \frac{1}{s}d_s - v_{t,s} + \frac{1}{s}\sum_{k=1}^{T} v_{k,s} \le -\frac{1}{Ts}\sum_{k=1}^{s} \mathbf{x}^k \boldsymbol{\tau}, \qquad t,s = 1,2,\dots,T$$
$$d_s, v_{t,s} \ge 0, \ t,s = 1,2,\dots,T$$
$$\boldsymbol{\lambda} \in \Lambda$$

Assuming a partition $(z_1,\dots,z_K)$ such that $z_1 \le \mathbf{x}^t\boldsymbol{\tau}$ and $z_K \ge \mathbf{x}^t\boldsymbol{\tau}$ for all $t = 1,2,\dots,T$, Post and Kopa (2017) proposed a SCTSD model with a tight sufficient condition for $\mathbf{r}'\boldsymbol{\lambda} \succ_{TSD} \mathbf{r}'\boldsymbol{\tau}$ in order to approximate (7):

$$\max_{\theta_{k,t}, \boldsymbol{\lambda}} \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}^t \boldsymbol{\lambda} \tag{10}$$

$$\begin{aligned}
&\text{s.t.} \quad \frac{1+e_k}{2T}\sum_{t=1}^{T} \theta_{k,t}^2 \le F_{\mathbf{r}'\boldsymbol{\tau}}^{(3)}(z_k), \quad k = 1,\dots,K\\
&\theta_{k,t} \ge z_k - \mathbf{x}^t\boldsymbol{\lambda} \quad k = 1,\dots,K, t = 1,\dots,T\\
&\frac{1}{T}\sum_{t=1}^{T} \mathbf{x}^t\boldsymbol{\lambda} \ge \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}^t\boldsymbol{\tau}\\
&\boldsymbol{\lambda} \in \Lambda\\
&\theta_{k,t} \ge 0, \quad k = 1,\dots,K, t = 1,\dots,T
\end{aligned}$$

where $e_1 = e_2 = 0$ and

$$e_k = \frac{F_{\mathbf{r}'\boldsymbol{\tau}}^{(3)}(z_k)}{F_{\mathbf{r}'\boldsymbol{\tau}}^{(3)}(z_{k-1}) + F_{\mathbf{r}'\boldsymbol{\tau}}^{(2)}(z_{k-1})(z_k - z_{k-1})} - 1, \quad k = 3,\dots,K.$$

In this paper, we consider *K=100* equidistant partition points. Alternatively, one could consider, for example: *K=T*.

3. Empirical study

In this section we apply problems (8)-(10) to annual returns of 10 Fama-French industry representative portfolios (base assets) from the Kenneth French library. We denote them IND1 to IND10. Moreover we include returns of one-year US t-bill as the risk free returns. The reference (benchmark) portfolio $\boldsymbol{\tau}$ is the US market portfolio, proxied by the CRSP index. We consider 50 annual returns for each asset and benchmark from 1966 to 2015. Hence, we consider N=11 and T=50. The following table summarizes the descriptive statistics of returns of the assets (including the risk free asset (RF) and the benchmark (CRSP)).

Table 1: Basic descriptive statistics of the returns

	mean	st. dev.	min	max	skewness	kurtosis
RF	5.01%	3.28%	0.02%	14.71%	0.43	0.46
CRSP	11.30%	17.88%	-36.74%	38.24%	-0.61	-0.11
IND1	13.95%	17.91%	-27.65%	53.36%	-0.29	0.24
IND2	11.16%	27.47%	-58.31%	77.89%	0.06	0.14
IND3	11.57%	17.55%	-40.34%	43.22%	-0.67	0.35
IND4	12.45%	20.47%	-33.98%	67.97%	0.05	0.60
IND5	12.52%	27.02%	-42.50%	82.81%	0.19	0.30
IND6	11.84%	21.05%	-39.29%	58.45%	-0.22	0.08
IND7	13.35%	22.30%	-36.33%	66.27%	0.11	0.25
IND8	13.51%	19.71%	-22.59%	61.38%	0.31	-0.26
IND9	10.67%	17.57%	-28.79%	51.76%	-0.22	-0.22
IND10	11.65%	20.83%	-47.21%	45.23%	-0.57	0.10

We start with the first-order stochastic dominance constraints. We find that the optimal solution portfolio of (8) coincides with the benchmark no matter which set of feasible portfolios is used. It means that there is no possible enhancement in the sense of the first order stochastic dominance. Therefore we proceed with the second order stochastic dominance constraints. Solving (9) we find some dominating portfolios (different from benchmark) in all four cases. The magnitude of the SSD enhancement in terms of the mean returns is depicted in Table 2.

Table 2: Mean return outperformance for the second order stochastic dominance constraints

SSD - benchmark	no short sales	allowed short sales
no risk free	2.65%	9.66%
with risk free	2.65%	15.12%

Finally, we apply (10) to our data and the TSD outperformance results are summarized in Table 3.

Table 3: Mean return outperformance for the third order stochastic dominance constraints

TSD - benchmark	no short sales	allowed short sales
no risk free	2.65%	9.72%
with risk free	2.65%	15.77%

4. Conclusion

In this paper we analyzed the portfolio selection models which maximize mean return under stochastic dominance constraints. In particular, first, second and third order stochastic dominance constraints were considered. The main goal was to compare the difference between mean return of the optimal portfolio and of the benchmark for the three orders of stochastic dominance and for four different sets of feasible portfolios (with/without risk free asset and short sales).

We found that the benchmark is efficient with respect to first order stochastic dominance, no matter which set of feasible portfolios is considered. It means that no enhancement with respect to the first order stochastic dominance was possible. Moving to the second order stochastic dominance constraints, some enhancement was observed in all four models. Of course the highest one in case of the model with risk free asset and short sales allowed.

Perhaps surprisingly, if short sales are not allowed, SSD enhancement with the risk less asset was as large as in the case without the risk less asset. The reason is that the optimal portfolios are the same in both cases. Finally, TSD enhancement was found very similar to SSD enhancement, the small differences were observed only in the case when the short sales are allowed.

For future research, this study can be improved in various ways. For example, longer historical data can be used or different frequency data can be considered (mainly with monthly or daily returns). In addition, one can consider the stochastic dominance relations in a more robust way as it was done in Kopa (2010) or Dentcheva and Ruszczynski (2010) or Dupačová and Kopa (2012, 2014) for the first and the second-order stochastic dominance, using contamination techniques and the worst-case approach. Alternatively, one can compare the results also with the case when the DARA (decreasing absolute risk aversion) stochastic dominance is used, see Post, Fang and Kopa (2015) for more details. Unfortunately, all these improvements would lead to much more computationally demanding optimization problems.

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Bayesian Estimation of Probability of Incidences of the Most Serious Oncological Diseases in the Czech Republic

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Abstract

The aim of this article is to estimate the probability of incidences of the most serious two oncological diseases in the three selected five-year age groups in the Czech Republic by using Bayesian theory of credibility, specifically binomial/beta model. These diseases are occurred in early childhood. During the lifetime, these incidences grow rapidly. The next aim of this article is to determine the parameters of a priori beta distribution, specifically in the case that the priory information on estimated parameter of binomial distribution is known. These parameters of beta distribution are determined by using basic characteristics of beta distribution. Possibilities of permanent update of estimates can be used in commercial and health insurance companies. Data on cancer incidences are obtained from database of ÚZIS.

Key words

oncological diseases, Bayesian theory of credibility, binomial/beta model, prior distribution

JEL Classification: C11, C13, I10

1 Úvod

Rakovina tlustého střeva a konečníku a rakovina průdušnice, průdušek a plic patří mezi nejčastěji se vyskytující závažná onkologická onemocnění v České republice. Výskyt těchto onkologických onemocnění je zaznamenán u dětí již v jejich raném věku a s přibývajícím věkem narůstá.

Komerční a zdravotní pojišťovny nabízejí množství produktů týkajících se pojištění závažných onemocnění, které chrání klienta před finančními důsledky způsobenými výskytem těchto onemocnění. Každá pojišťovna musí umět správně odhadnout rizika, aby byla v budoucnu schopna plnit své závazky, a to aplikováním vhodných metod odhadu pravděpodobností. Bayesovská teorie kredibility se jeví jako vhodná metoda stanovování a permanentní aktualizace těchto odhadů, protože se pojišťovny zaměřují na data z vlastních i cizích pojišťoven, a to za minulá období. (Jindrová a Kopecká, 2017; Pacáková, Šoltés a Šoltésová, 2009)

Cílem tohoto článku je odhad pravděpodobností výskytu dvou výše uvedených nejčastějších onkologických onemocnění, a to vždy ve třech vybraných pětiletých věkových skupinách 20-24, 40-44 a 60-64 let v rámci České republiky. Odhady pravděpodobností výskytu těchto onemocnění jsou konstruovány pomocí bayesovského modelu binomické/beta.

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Dále je věnovaná pozornost stanovení parametrů apriorního rozdělení beta za pomoci základních charakteristik (střední hodnoty a rozptylu) beta rozdělení, tj. v případě, že apriorní informace o odhadovaném parametru binomického rozdělení je známá, podle (Kotlebová, 2009).

2 Data a metodologie

Jak již bylo výše zmíněno, jedním z cílů tohoto článku je odhadnout pravděpodobnosti výskytu rakoviny tlustého střeva a konečníku a rakoviny průdušnice, průdušek a plic ve vybraných pětiletých věkových kategoriích v rámci České republiky. Data o incidencích těchto onkologických onemocnění pocházejí ze stránek ÚZIS ČR, konkrétně jsou čerpána ze stránek Epidemiologie zhoubných nádorů v České republice. Tato databáze poskytuje údaje o incidencích onkologických onemocnění od roku 1977 do roku 2014 za jednotlivé věkové kategorie. Dále data o počtu obyvatel byla čerpána vždy k 1. červenci daného roku ze stránek UNITED NATIONS. Tato data jsou k dispozici od roku 1950 do roku 2015 a také za jednotlivé věkové kategorie. OECD databáze poskytuje data o incidencích rakoviny tlustého střeva a plic pro jednotlivé členské státy, avšak bez ohledu na členění dle věkových kategorií. Jako apriorní informace je považována informace o pravděpodobnostech těchto incidencí v jednotlivých evropských státech, která jsou přepočtena na jednoho obyvatele.

Pro výpočet odhadů pravděpodobností výskytu dvou výše zmíněných onkologických onemocnění byla zvolena bayesovská teorie kredibility, konkrétně model binomické/beta.

2.1 Bayesovská teorie kredibility

Základem bayesovké teorie kredibility je bayesovská teorie odhadu. Bayesovská teorie odhadu nezahrnuje pouze údaje z vlastního výběru, jako je tomu u klasických metod odhadu, ale zahrnuje také jiné dostupné porovnatelné informace, které bývají často k dispozici, jak zmiňuje (Pacáková, 2004; Kotlebová, 2009).

Dalším rozdílem mezi klasickou a bayesovskou statistikou je neznámý parametr θ, který je v případě bayesovské statistiky považován za náhodnou proměnnou, a to na rozdíl od klasické statistiky, kde je tento parametr považován za neznámou konstantu. V případě bayesovské statistiky má tedy parametr θ (náhodná proměnná) rozdělení pravděpodobnosti $f(\theta)$. Toto rozdělení pravděpodobnosti se nazývá apriorní, protože poskytuje první informaci o odhadované parametru θ a tato informace zatím nepochází z vlastního výběrového souboru podle literatury (Pacáková, 2004).

I v případě, že apriorní informace o odhadovaném parametru θ není nejlepší, je možné ji zlepšit pomocí informace aposteriorní, tedy z vlastního výběrového zjišťování. Aposteriorní rozdělení odhadovaného parametru θ využívá informací jak jeho apriorního rozdělení, tak výsledek vlastního výběrového zjišťování a značí se $f(\theta / \boldsymbol{x})$. Jestliže $\boldsymbol{x}$ je náhodný výběr z rozdělení pravděpodobnosti $f(\boldsymbol{x} / \theta)$ a neznámý parametr θ má apriorní rozdělení $f(\theta)$, pak pro aposteriorní rozdělení podle spojité verze bayesovské věty platí vztah:

$$f(\theta / \boldsymbol{x}) = \frac{f(\boldsymbol{x}/\theta)\, f(\theta)}{\int_{\theta} f(\boldsymbol{x}/\theta)\, f(\theta)\, \mathrm{d}\theta} \tag{1}$$

Výraz ve jmenovateli ve vztahu (1) bude označen jako marginální hustota $f(\boldsymbol{x})$, která nezávisí na parametru θ a představuje konstantu, jenž bude pro zjednodušení vynechána. Aposteriorní rozdělení je možné nyní zapsat vztahem:

$$f(\theta / \boldsymbol{x}) \propto f(\boldsymbol{x} / \theta)\, f(\theta) \tag{2}$$

Dá se říci, že aposteriorní rozdělení odhadovaného parametru θ je proporcionální se součinem funkce věrohodnosti a apriorním rozdělením, jak uvádí (Pacáková, 2004).

V případě, že apriorní a aposteriorní rozdělení je stejného typu, ale různých parametrů, jsou tato rozdělení označena jako konjugovaná, a to k rozdělení, ze kterého pochází náhodný výběr. Model binomické/beta patří ke konjugovaným rozdělení využívaných zejména v pojišťovnictví, jak uvádí (Kotlebová, 2009; Pacáková a Kotlebová, 2014).

2.1.1 Model binomické/beta

V modelu binomické/beta je rozdělení beta konjugovaným apriorním rozdělením pro rozdělení binomické s neznámým parametrem θ. Náhodná veličina X má binomické rozdělení s parametrem θ. Podmíněná funkce pravděpodobnosti má tedy tvar:

$$f\left(x/\theta\right)=\binom{n}{x}\theta^{x}\left(1-\theta\right)^{n-x}, \qquad x=1,\ldots,n \tag{3}$$

Apriorní rozdělení parametru θ je výše zmíněné beta, a to s parametry α a β. Pro hustotu tohoto rozdělení na intervalu (0,1) platí pro $\theta \in (0,1)$ vztah:

$$f\left(\theta\right)=\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)+\Gamma\left(\beta\right)}\cdot\theta^{\alpha-1}\cdot\left(1-\theta\right)^{\beta-1}\propto\theta^{\alpha-1}\cdot\left(1-\theta\right)^{\beta-1} \tag{4}$$

Po dosazení do vztahu (2) dostaneme hustotu aposteriorního rozdělení vyjádřenou vztahem (5), která vyjadřuje hustotu rozdělení beta s parametry $\alpha^{*}=\alpha+x$ a $\beta^{*}=\beta+n-x$: (Kotlebová, 2009; Pacáková, 2004; Pacáková a Kotlebová, 2014)

$$f\left(\theta/x\right)\propto\theta^{x}\left(1-\theta\right)^{n-x}\cdot\theta^{\alpha-1}\left(1-\theta\right)^{\beta-1}=\theta^{\alpha+x-1}\left(1-\theta\right)^{\beta+n-x-1} \tag{5}$$

Bayesovským odhadem θ_B parametru θ bude v tomto případě vzhledem na kvadratickou ztrátu střední hodnota aposteriorního beta rozdělení, která je vyjádřená vztahem:

$$\theta_B=\frac{\alpha+x}{\alpha+\beta+n} \tag{6}$$

Vztah (6) je možné přepsat do tvaru:

$$\theta_B=\frac{n}{\alpha+\beta+n}\cdot\frac{x}{n}+\frac{\alpha+\beta}{\alpha+\beta+n}\cdot\frac{\alpha}{\alpha+\beta}=Z\cdot\frac{x}{n}+\left(1-Z\right)\cdot\mu, \tag{7}$$

kde Z vyjadřuje faktor kredibility, $\frac{x}{n}$ maximálně věrohodný odhad parametru θ binomického rozdělení a μ znázorňuje střední hodnotu beta rozdělení. (Gogola, 2013; Pacáková, 2004).

3 Modelování odhadů pravděpodobností výskytu onkologických onemocnění ve vybraných věkových kategoriích

V rámci této kapitoly je již pozornost věnovaná praktickému využití bayesovských odhadů, konkrétně modelu binomické/beta, který se jeví pro odhady pravděpodobností výskytu onkologických onemocnění a tím i pro potřeby pojišťoven jako vhodný. Data získaná ze stránek ÚZIS ČR a UNITED NATIONS jsou čerpaná za období 15 let, a to od roku 2000 do roku 2014.

Důležitou součástí bayesovkých odhadů je stanovení parametrů apriorního rozdělení v tomto případě beta, což vyžaduje znalost apriorní informaci o odhadovaném parametru θ binomického rozdělení. V případě, že tato informace není známá, což v praxi nebývá časté, využívá se rovnoměrného beta rozdělení, pro něhož jsou parametry α a β rovny 1 pro první

rok podle (Kotlebová, 2009). Jde o případ, kdy bayesovský odhad výskytu onkologického onemocnění pro první rok je roven 50 %. Představa, že pravděpodobnost výskytu onkologického onemocnění byla v roce 2000 takto vysoká, však není reálná.

Z pravděpodobností výskytu určitého onkologického onemocnění (rakoviny tlustého střeva nebo plic), které byly odhadnuty na základě dat OECD pro rok 2012 popsaných výše, je možné získat představu o apriorním rozdělení, a to výpočtem střední hodnoty μ^* a rozptylu $(\sigma^*)^2$ těchto pravděpodobností. Parametry apriorního rozdělení beta pro rok 2000 jsou stanoveny pomocí dvou základních charakteristik beta rozdělení, a to střední hodnoty a rozptylu, které jsou vyjádřeny vztahy podle (Kotlebová, 2009):

$$\mu^* = \frac{\alpha}{\alpha + \beta} \tag{8}$$

$$\left(\sigma^*\right)^2 = \frac{\alpha \cdot \beta}{\left(\alpha + \beta\right)^2 \cdot \left(\alpha + \beta + 1\right)} \tag{9}$$

V následujících dvou podkapitolách jsou již konstruovány odhady pravděpodobností výskytu dvou nejčastějších onkologických onemocnění v ČR ve třech vybraných pětiletých věkových kategoriích pomocí modelu binomické/beta, a to pro rok 2015.

3.1 Odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku

Nyní již jsou odhadovány pravděpodobnosti výskytu rakoviny tlustého střeva a konečníku pro rok 2015 pomocí modelu binomické/beta. Postup výpočtu bayesovských odhadů při využití modelu binomické/beta je popsán v podkapitole 2.1.1 a stanovování parametrů apriorního rozdělení beta pomocí základních charakteristik beta rozdělení je popsáno na začátku kapitoly 3. Za apriorní informaci je brána představa o výskytu rakoviny tlustého střeva v jednotlivých evropských státech v posledním známém roce 2012. V následující tabulce 1 je uveden postup konstrukce těchto odhadů, a to pro věkovou kategorii 20-24 let.

Tabulka 1: Odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku věkové kategorie 20-24 let

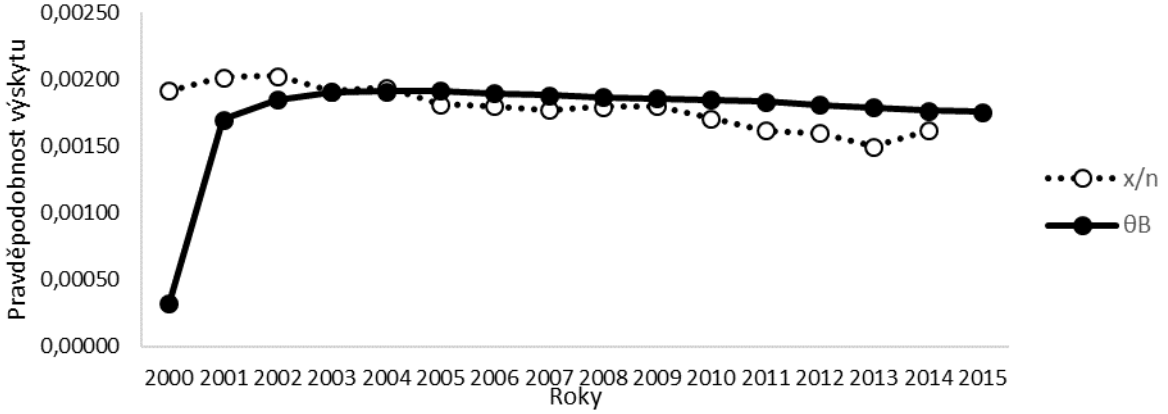
Roky	***n***	***x***	***x/n***	α	β	θ_B
2000	895 278	7	0,0000078	23	71 541	0,0003205
2001	869 335	1	0,0000012	30	966 812	0,0000310
2002	826 445	3	0,0000036	31	1 836 146	0,0000168
2003	775 603	3	0,0000039	34	2 662 588	0,0000127
2004	730 159	5	0,0000068	37	3 438 188	0,0000107
2005	698 010	7	0,0000100	42	4 168 342	0,0000101
2006	682 500	5	0,0000073	49	4 866 345	0,0000101
2007	680 367	1	0,0000015	54	5 548 840	0,0000097
2008	686 301	0	0,0000000	55	6 229 206	0,0000088
2009	691 891	3	0,0000043	55	6 915 507	0,0000079
2010	691 582	4	0,0000058	58	7 607 395	0,0000076
2011	683 265	13	0,0000190	62	8 298 973	0,0000075
2012	672 760	9	0,0000134	75	8 982 225	0,0000083
2013	658 672	4	0,0000061	84	9 654 976	0,0000087
2014	639 439	7	0,0000109	88	10 313 644	0,0000085
2015				95	10 953 076	**0,0000087**

V tabulce 1 označuje sloupec *n* počet obyvatel k 1. červenci daného roku ve věkové skupině 20-24 let a sloupec *x* znázorňuje počet výskytů rakoviny tlustého střeva a konečníku v této věkové skupině. Čtvrtý sloupec x/n vyjadřuje maximálně věrohodný odhad parametru θ binomického rozdělení, který je možné konstruovat pouze pro roky, pro která jsou známá data. Sloupce α a β znázorňují parametry beta rozdělení. Poslední sloupec θ_B vyjadřuje hledaný bayesovský odhad pravděpodobností výskytu rakoviny tlustého střeva a konečníku. Tento odhad má dvě hlavní výhody oproti maximálně věrohodnému odhadu. První z nich je výše zmiňované zachycení apriorní informace, což v případě maximálně věrohodného odhadu není možné. Druhou podstatnou výhodou je ovšem možnost konstrukce odhadu pravděpodobnosti pro rok následující po posledním známém roce, tzn. v tomto případě pro rok 2015. Parametry beta rozdělení jsou pro rok 2000 a věkovou kategorii 20-24 let stanoveny řešením soustavy rovnic (8) a (9). V tomto případě parametr $\alpha = 23$ a $\beta = 71541$. Při těchto parametrech je $\theta_B = 0{,}0003205$ pro rok 2000. Bayesovský odhad pravděpodobnosti výskytu rakoviny tlustého střeva a konečníku ve věkové kategorii 20-24 let klesá do roku 2011, avšak od tohoto roku začíná narůstat a pro rok 2015 tento odhad činí 0,0000087.

Výpočty bayesovských odhadů pomocí modelu binomické/beta pro ostatní uvedené věkové kategorie jsou prováděny analogicky. Na následujících dvou obrázcích 1 a 2 jsou graficky znázorněny odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku, a to nejprve ve věkové kategorii 40-44 let a následně ve věkové kategorii 60-64 let.

Obrázek 1: Odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku věkové kategorie 40-44 let

Obrázek 2: Odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku věkové kategorie 60-64 let



Na obrázku 1 a obrázku 2 je možné porovnat bayesovský odhad θ_B s maximálně věrohodným odhadem x/n. Bayesovský odhad má tendenci přibližovat se k maximálně věrohodnému odhadu v průběhu let, protože apriorní informace (z cizích porovnatelných

rizik) je postupem času vylepšována informacemi aposteriorními (z vlastního výběrového zjišťování). Bayesovský odhad má také hladší průběh na rozdíl od maximálně věrohodného odhadu, což může být považováno za jeho další výhodu. Na základě bayesovského odhadu má pravděpodobnost výskytu rakoviny tlustého střeva a konečníku ve věkových kategoriích 40-44 let a 60-64 let mírně klesající tendenci v průběhu posledních let.

Pro obě tyto věkové kategorie nabývají parametry beta rozdělení pro rok 2000 stejných hodnot jako v případě věkové kategorie 20-24 let. V případě věkové kategorie 40-44 je bayesovský odhad pravděpodobnosti pro rok 2015 roven 0,0001473. Dále pak pro věkovou kategorii 60-64 let bayesovský odhad pravděpodobnosti pro rok 2015 činí 0,0017555.

3.2 Odhady pravděpodobností výskytu rakoviny průdušnice, průdušek a plic

Dalším závažným onkologickým onemocněním, které se vyskytuje již v brzkém věku, je rakovina průdušnice, průdušek a plic. Veškeré výpočty týkající se bayesovských odhadů pravděpodobností výskytu pro jednotlivé věkové kategorie pomocí modelu binomické/beta jsou prováděny analogicky jako v předchozí podkapitole. V tabulce 2 jsou konstruovány odhady pravděpodobnosti výskytu rakoviny průdušnice, průdušek a plic ve věkové kategorii 20-24 let.

Tabulka 2: Odhady pravděpodobností výskytu rakoviny průdušnice, průdušek a plic věkové kategorie 20-24 let

Roky	***n***	***x***	***x/n***	α	β	θ_B
2000	895 278	1	0,0000011	20	65 215	0,0003038
2001	869 335	2	0,0000023	21	960 492	0,0000217
2002	826 445	1	0,0000012	23	1 829 825	0,0000125
2003	775 603	1	0,0000013	24	2 656 269	0,0000090
2004	730 159	1	0,0000014	25	3 431 871	0,0000072
2005	698 010	3	0,0000043	26	4 162 029	0,0000062
2006	682 500	2	0,0000029	29	4 860 036	0,0000059
2007	680 367	1	0,0000015	31	5 542 534	0,0000056
2008	686 301	0	0,0000000	32	6 222 900	0,0000051
2009	691 891	4	0,0000058	32	6 909 201	0,0000046
2010	691 582	1	0,0000014	36	7 601 088	0,0000047
2011	683 265	0	0,0000000	37	8 292 669	0,0000044
2012	672 760	4	0,0000059	37	8 975 934	0,0000041
2013	658 672	0	0,0000000	41	9 648 690	0,0000042
2014	639 439	3	0,0000047	41	10 307 362	0,0000040
2015				44	10 946 798	**0,0000040**

V tabulce 2 je možné vidět klesající trend bayesovských odhadů pravděpodobností výskytu rakoviny průdušnice, průdušek a plic pro věkovou kategorii 20-24 let. Pro tuto věkovou kategorii nabývají parametry beta rozdělení pro rok 2000 řešením soustavy rovnic (8) a (9) hodnot $\alpha = 20$ a $\beta = 65215$. Při těchto parametrech je pro tento rok $\theta_B = 0{,}0003038$, což platí i pro následující dvě věkové kategorie. Bayesovský odhad pro rok 2015 pro tento případ činí 0,0000040. Na obrázku 3 a 4 jsou graficky znázorněny opět maximálně věrohodné i bayesovské odhady pravděpodobností výskytu rakoviny průdušnice, průdušek a plic pro 2 věkové kategorie: 40-44 let a 60-64 let.

Obrázek 3: Odhady pravděpodobností výskytu rakoviny průdušnice, průdušek a plic věkové kategorie 40-44 let

Obrázek 4: Odhady pravděpodobností výskytu rakoviny průdušnice, průdušek a plic věkové kategorie 60-64 let

Na obrázku 3 lze pozorovat klesající trend bayesovských odhadů pravděpodobností ve věkové kategorii 40-44 let. Bayesovský odhad pro rok 2015 činí 0,0000736. V poslední věkové kategorii 60-64 let, pro které jsou odhady pravděpodobnosti znázorněny na obrázku 4, je možné vidět také klesající trend bayesovských odhadů, ale až od roku 2008. Bayesovský odhad pravděpodobnosti pro rok 2015 činí 0,0018003.

4 Závěr

Cílem tohoto článku bylo stanovit odhady pravděpodobností výskytu rakoviny tlustého střeva a konečníku a rakoviny průdušnice, průdušek a plic, a to vždy ve třech vybraných pětiletých věkových skupinách 20-24, 40-44 a 60-64 let v rámci ČR. Pro odhady pravděpodobnosti výskytu těchto onemocnění byl použit bayesovský model binomické/beta. Dále byla věnovaná pozornost stanovení parametrů apriorního rozdělení beta za pomoci střední hodnoty a rozptylu tohoto rozdělení, a to v případě, že apriorní informace o odhadovaném parametru binomického rozdělení je známá. Data byla čerpána ze stránek ÚZIS ČR, UNITED NATIONS a OECD.

Bayesovské odhady mají 3 hlavní výhody oproti maximálně věrohodným odhadům, jak již bylo uvedeno výše. Za první z nich je považována schopnost zahrnout kromě aposteriorní informace (z vlastního výběrového souboru) i informaci apriorní (z cizích porovnatelných rizik). Stanovení vhodné apriorní informace je tedy důležitým krokem pro přesnější konstrukci těchto odhadů. Mezi další výhody patří to, že bayesovské odhady jsou konstruovány pro rok následující po posledním známém roce v našem případě pro rok 2015. Poslední zde zmíněnou výhodou je hladký průběh těchto odhadů oproti maximálně

věrohodným odhadům, což je zjevné z obrázků 1-4. Ve světle těchto výhod se jeví bayesovské odhady vhodnými především pro využití v oblasti pojišťovnictví, konkrétně při pojištění závažných onemocnění.

V případě rakoviny tlustého střeva a konečníku byl zjištěn v posledních letech rostoucí trend bayesovských odhadů ve věkové kategorii 20-24 let. Naopak ve věkových kategoriích 40-44 let a 60-64 let byl tento trend v posledních letech mírně klesající. U rakoviny průdušnice, průdušek a plic jsou ve všech věkových kategoriích zaznamenány klesající trendy bayesovských odhadů v posledních letech. U obou vybraných onkologických onemocnění bylo zjištěno, že dochází k rapidnímu nárůstu pravděpodobnosti výskytu těchto onemocnění mezi jednotlivými věkovými kategoriemi, a to pro odhadovaný rok 2015.

Poděkování

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Use of Financial Instruments in the Czech Republic within the European Structural and Investment Funds in the Programming Period 2014-2020

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Abstract

During the programming period 2014-2020, the European Commission is seeking to extend its financial instruments in providing support from the European structural and investment funds. Also the Partnership Agreement between the Czech Republic and the European Commission declares the effort to use financial instruments in some operational programmes for financing viable projects. Since 2016, grant calls have been announced, yet the declared effort to distribute a significant proportion of aid with the use of financial instruments has not yet been fulfilled. The aim of the author is to evaluate the state of preparation for the use of financial instruments in those operational programmes where the use of the financial instruments has been planned since the beginning of this programming period.

Key words: financial instruments, European structural and investment funds, programming period, operational program

JEL Classification: H29, H81

1. Introduction

Among the key priorities of the European Commission in the programming period (PP) 2014-2020, there is a wider use of financial instruments (FIs) in the implementation of the European structural and investment funds (ESIF) in individual member states. An objective of the European Commission during the PP 2014-2020 is, for example, to double the volume of resources provided to small and medium-sized enterprises (SMEs) through FIs (European commission, 2014). The reason for using the FIs is to gain a number of benefits. Unlike the direct grant support, with the use of FIs, financial resources can be recycled to obtain sources of funding for a period when the support will no longer be available to a similar extent as in PPs 2007-2013 and 2014-2020 to some of the member states of the European Union (EU) currently receiving the support. Extending the support with FIs will also make the member states less dependent on external support from the ESIF and will combine financial resources with national, public and private sources. Besides the benefits, the use of FIs also has some limitations. Their application is only possible for viable projects, i.e. projects of an investment nature, which will generate sufficient income and are expected to have a good financial return, yet they cannot be financed or co-financed by common sources of financing due to the market situation.

Also, at the level of the Czech Republic (CZ), the Partnership Agreement for the PP 2014-2020 declares a plan to increase the use of FIs to ensure greater investment efficiency. It has been provisionally considered to use the FIs, for example, in the area of the SME support

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under the OP Enterprise and Innovation for Competitiveness, or in the area of research support under the OP Research, Development and Education (MRD CZ, 2014). It is therefore appropriate to assess to what extent the declared effort to use FIs in the current PP has reflected in practice on a more massive scale.

2. Literatury Review

According to the Ministry of Regional Development (MRD CZ), the FIs represent a form of the use of ESIF funds in a repayable manner (MRD CZ, 2015). These instruments may take the form of a credit or loan, equity or quasi-equity investments, guarantees or other risk-sharing instruments and, where appropriate, they can be combined with subsidies (European Court of Auditors, 2016).

Within the EU, FIs were used in the PP 2007-2013 to a wider extent than in the previous PPs. During the PP 2007-2013, EUR 21.5 billion was earmarked for FIs at the EU level, and 25 EU member states used them, most notably Italy and Denmark, and the Baltic States as for the new members. By the end of 2014, support for these instruments was paid in the amount of about EUR 16 billion. Compared to EUR 1.3 billion allocated to these instruments in the PP 2000-2006, this is a significant increase. (European Court of Auditors, 2016)

In the Czech Republic (CZ), FIs have not been much used in the previous PPs. In the last PP, the FIs were used only in the Integrated Operational Programme in the form of a credit under the JESSICA programme. The programme was launched in 2013 and focused on the modernization and reconstruction of residential buildings and social housing with a total allocation of CZK 612 million. Under this programme, 153 credit agreements were concluded in a total volume of CZK 601 million. (Mazal, 2016) The overall assessment of the use of FIs in the previous PP accentuates the late commencement of preparations, a problematic procedure with the choice of the FIs manager, so some of the resources originally intended for distribution using FIs were provided in the end of the PP in the form of non-repayable financial aid due to the fear of under-use. (MRD CZ, 2014)

In the Czech Republic, the use of FIs in the PP 2014-2020 is mainly based on Government Resolution No. 85/2015. This resolution addresses the implementation of FIs at the national level, primarily in terms of the technical solution of the implementation itself. (Government of CZ, 2015) A working group for innovative financial instruments of the ESF Council was set up to prepare the rules for the use of FIs. The ESF Council working groups address compliance with strategies and achievement of synergies in their respective areas of focus. (MRD CZ, 2012) The working group for innovative FIs was designed to co-ordinate the use of FIs and their implementation in individual OPs but it is not significantly involved in the preparation of rules in individual OPs, so the assessment of the areas of support and the possible selection of FIs takes place at the level of the individual OPs.

The basic methodological guide for the use of FIs in individual OPs is the current second version of the Methodological Recommendation for the Implementation of FIs in PP 2014-2020, which entered into force on March 1, 2017. Within the framework of individual OPs, this methodical document recommends first to make an analysis, assess the possibilities of using the FIs and their added value, calculate the possible effects of FIs and assess the absorption capacity. Subsequently, in the case of OPs where it will be possible to expect a meaningful use of the FIs, to prepare an investment strategy, select areas of support, select appropriate FIs, a manager of FIs or financial intermediary, discuss the possibility of co-financing, and translate the conditions into the Financing Agreement and subsequently make use of the FIs in practice in the form of specific support programmes. (MRD CZ, 2016)

3. Methodology

The assessment of the current state of preparations for the use of FIs in PP 2014-2020 was carried out on the basis of study of available materials, interviewing of responsible staff of ministries, FIs managers and other interested institutions, publicly available data on ESIF, FIs, OPs and plans for use of FIs in individual OPs. The assessment also took into account data on the macroeconomic situation in CZ, research on interest in the use of FIs as well as related materials presented at the European level. Results of research for individual OPs are presented in Results and Discussion.

4. Results and discussion

Within the OPs, which declared the effort to use FIs at the beginning of the PP, the analysis has been made or is being prepared or is in progress, in line with the methodological recommendation. On the basis of this analysis in OP Transport, where the use of FIs for combined transport terminals was planned, they have withdrawn from using FIs due to the low absorption capacity (Kokeš, 2017). An analysis with a positive conclusion about the use of FIs has already been completed in the OPs: Enterprise and Innovation for Competitiveness, Integrated Regional Operational Programme and OP Prague – Growth Pole of the Czech Republic. The other OPs (Employment, Research, Development and Education, Environment and Rural Development Programme) analyses of the use of FIs are being made or prepared.

4.1 Operational Programme Enterprise and Innovation for Competitiveness

Under the OP Enterprise and Innovation for Competitiveness (EIC), extensive use of different types of FIs is planned. In March 2015, the results of an analysis by the Ministry of Industry and Trade CZ (MIT CZ) were presented in the document "Preliminary Assessment of the Financial Instruments of the Operational Programme Enterprise and Innovation for Competitiveness 2014-2020" (MIT CZ, 2015). This document shows that within the EIC the objective is to use the FIs under Priority Axes 1 - 3 for the following specific objectives (MIT CZ, 2015):

- 1.1 Increase innovation performance of businesses
- 1.2 Increase the intensity and effectiveness of collaborative research, development and innovation
- 2.1 Enhance the competitiveness of start-up and developing small and medium-sized enterprises (SME)
- 2.3 Increase business infrastructure exploitability
- 3.1 Increase the share of renewable energy production in gross final consumption of CZ
- 3.2 Increase the energy efficiency of the business sector
- 3.5 Increase efficiency of heat supply systems.

The market situation analysis confirmed the possibility of using FIs in all the EIC's specific objectives analyzed, with the exception of the specific objective 4.1 aimed at increasing high-speed access coverage where the potential was found to be inadequate. Approximately 10 percent of EIC allocations to FIs (total EIC allocation: CZK 117 billion) are anticipated. In the framework of the analysis, the FIs were divided into two groups, the so-called First Generation FIs, which represent credits and guarantees usable in all selected specific objectives, and Second Generation FIs, i.e. the new instruments to support capital inputs in SC 1.2 and 2.1 and junior loans for regeneration of brownfields in SC 2.3. For implementation of the First Generation FIs, the Czech-Moravian Guarantee and Development Bank (CMZRB)

was selected as an intermediary, and for the implementation of second-generation FIs, the National Innovation Fund (NIF), investment company, a.s. (MIT CZ, 2015). In Figure 1, there is a proposal to use FIs in specific objectives.

Figure 1: Proposal for the use of FIs within specific EIC objectives

Specific Objective	**Support Programme**	**1st Generation FI**	**2nd Generation FI**	**Suggested FI allocation range (CZK million)**
1.1	Innovation	Guarantees		1 400-1 800
1.2	Proof of Concept		Capital Inputs	550-1 100
2.1	Expansion	Soft Loans and Guarantees		8 850-12 000
	Risk Capital		Capital Inputs	800-1 600
2.3	Immovable Assets	Soft Loans	Junior Loans	0-5505
3.1	Sources of Renewable Energy	Soft Loans		400-600
3.2	Energy Savings	Soft Loans		550-2 000
3.5	Energy Savings in the Heat Supply System	Soft Loans		300-700

Source: (MIT CZ, 2015)

From the aforementioned planned support programmes using FIs, on June 1, 2017, the programme Expansion for SMEs was launched in cooperation with CMZRB. Under this programme, interest-free credits and financial contributions are provided for investment projects implemented throughout the Czech Republic except for the capital city of Prague (CMZRB, 2017). In addition, in cooperation with CMZRB, the Energy Saving Programme is being prepared for SMEs as well as large enterprises, expected to be launched in autumn 2017 (Rohlíčková, 2017). Other programmes are in preparation.

4.2 Integrated Regional Operational Programme

In the framework of the Integrated Regional Operational Programme (IROP), based on an analysis made, support is planned mainly in Priority Axis 2 in the specific objective 2.5 Reduction of Energy Intensity in the Housing Sector. This support is similar to that in the previous PP (JESSICA programme, see Chapter 2). The support is expected to be paid starting from the end of 2017 or at the beginning of 2018. Unlike the JESSICA programme in the previous PP, the current programme will only be used to reduce the energy intensity of apartment buildings, using FI Soft Loans with a reduced interest rate. The estimated allocation is CZK 2.1 billion. The soft loan will be provided through the FI manager - commercial partner. At present, the tender dossier for the public procurement contract is being prepared. Due to the current market situation where interest rates are very low, it is being considered to supplement the FI with a grant component or to finance the project preparation in the form of a grant. (Deloitte CZ, 2015; Mazal, 2016)

4.3 Operational Programme Prague – Growth Pole of the Czech Republic

Under the OP Prague – Growth Pole of the Czech Republic (GPCZ), the results of an analysis made were presented in October 2016: Preliminary assessment of the use of financial instruments in the OP Prague – Growth Pole of the Czech Republic under the Cohesion Policy 2014-2020. This analysis was to identify areas that could be supported with FIs. they were identified as areas with a certain potential in Priority Axis 1: Strengthening Research, Technological Development and Innovation, specifically in Specific Objective 1.1.1

Promoting activities contributing to the commercialization of research results through verification of feasibility and commercial potential and their introduction into practice - ("proof-of-concept") and Specific Objective 1.2.3 Developing innovative companies at earlier stages of their life cycle. Within the two specific objectives, the use of FIs is considered in the form of a combination of subsidies and capital inputs, capital inputs as such or soft loans. Based on the results of the analysis in accordance with the type of support, the selection of the FIs manager was initiated. Negotiations with the European Investment Fund, the originally recommended manager, did not lead to any agreement. Negotiations are underway with the CMZRB and the Ministry of Industry and Trade CZ, which will be the founder of the NIF, and, due to the replacement of the originally planned FIs manager, it is foreseeable to initiate support with the help of FIs at the earliest in 2018. (Arthur D. Litle, GmbH, 2016).

4.4 Operational Programme Employment

Within the OP Employment, selected for support using the FIs was the area of social entrepreneurship in Investment Priority 2.1 intended for active inclusion, including inclusion with a view to promoting equal opportunities and active participation and improving employability to meet Specific Objective 1 Increasing the employability of people at risk of social exclusion or socially excluded in society and on the labour market (MLSA CR, 2014). Support under OP Employment is financed from the European Social Fund and is intended for non-investment expenditure. Social entrepreneurship is supported not only within the framework of OP Employment, but also within the framework of the IROP in Priority Axis 2 Improving public services and living conditions for the inhabitants of the region; investment expenditures are then supported from this OP. In the IROP, however, the use of FIs for social entrepreneurship is not planned. In June 2017, an agreement was concluded between the Ministry of Labour and Social Affairs of the Czech Republic (MLSA CR) and a supplier for making an analysis of the use of FIs within OP Employment. The expected completion date for the analysis is September 2017. Based on the results, the implementation of the FIs into practise will be initiated if the usability of FIs is confirmed. It is anticipated that the FIs will be used in the form of soft loans and guarantees, but the final form is yet to be revealed in the analysis. The expected date of commencement of support is planned for the beginning of 2019. (Maršíková, 2017)

4.5 Operational Programme Research, Development and Education

Within the OP Research, Development and Education (RDE), the identification of the support areas is still ongoing. PWC prepared a preliminary analysis for the Ministry of Education, Youth and Sports CR on the use of FIs for the area of research, development and education (PWC, 2017). An analysis in line with the Methodological Recommendation for the Implementation of FIs in the PP 2014-2020 is yet to be made. Based on a preliminary analysis of the environment of research, development and education in the CZ and communication with individual types of OP RDE beneficiaries, PWC has identified four types of projects suitable for pilot verification in terms of implementation of financial instruments (PWC, 2017):

1. Under Priority Axis 1: Strengthening Capacity for Quality Research, Investment Priority 1.1 Strengthening Research and Innovation Infrastructure and Capacities to Develop Excellence in Research and Innovation and Supporting Professional Centres, especially those of Europe-wide Interest:
    a. Purchase of new technologies and devices within the framework of the development of research organizations focused on cooperation with the application sphere, i.e. a project aimed at acquiring infrastructure, part of which would be pre-determined for use for contracted research.

   b. Implementation of research plans with potential for future use of the result in practice where pre-application projects that have the potential for commercialization are supported by follow-up support in the form of FN (i.e. in a returnable form).
2. Under Priority Axis 2: Developing higher education institutions and human resources for research and development, Investment Priority 1 Improving the quality and efficiency and access to tertiary and equivalent education, especially in disadvantaged groups, in order to increase participation and the level of education received:
   a. Supporting postgraduate researchers to enable the system to fund talented young scientists and motivate them to achieve above-average results in their work.
   b. Improving the quality of education at higher education institutions (support for foreign students) that would be supported in a returnable form of support, using the experience and mechanisms of the ERASMUS+ programme.

In the above areas of support, it is envisaged to use these types of FIs: credit, guarantee, quasi-capital and loans to individuals, and combinations with subsidies considered for all FIs. Therefore, the use of FIs within the OP RDE is still being prepared, the OP RDE managing body is preparing a selection procedure for the analysis of the use of FIs within the OP RDE in accordance with the Methodological Recommendation (MRD CZ, 2016), and it is thus not possible to set a timetable for the implementation of FIs and their availability in practice.

4.6 Operational Programme Environment

In the OP Environment, an assessment of the use of FIs was carried out, and then the preparation of support was started in the form of preferential loans, which the applicants could use in combination with subsidies. To provide support and manage the soft loans, the State Environmental Fund of the Czech Republic was chosen as a partner. The first planned step will be publishing a pilot call under Priority Axis 3 of OP Environment: Wastes and material flows, environmental burdens and risks and Specific Objective 3.5 To reduce environmental risks and develop systems for their management. The utilization of FIs is also envisaged in other OP priority axes where conditions for the support will be set according to the results of the pilot call. (Früblingová, 2017)

4.7 Operational Programme Rural Development Programme

Although the use of FIs under the OP Rural Development Programme (RDP) has been considered since the beginning of the PP, the analysis of the use of FIs within the OP has not yet been made. In general, in the agricultural sector, the possibility of using credits, securing credits or other forms of support through the Supporting and Guarantee Agricultural and Forestry Fund is available. Nevertheless, it is desirable to provide part of the support from the OP RDP in the form of FIs mainly in the form of credits. However, given the market situation, availability of funding sources and many other supports for farmers, their use in the coming years is not foreseen. (Tabery, 2017)

5. Counclusion

The FIs are still not used in the 2014-2020 PP to the extent planned so far and coordination has not been sufficiently applied in preparing the use of FIs across OPs (e.g. support for social entrepreneurship is addressed through IROP and OP Employment, the use of FIs in this area is only planned in OP Employment). It is possible to assume that due to the growth of gross domestic product in the CZ it will no longer be possible to obtain support in the other PPs to a similar extent as it has been. FIs are one of the means that can help to restore financial

resources for development in the future, but they are not sufficiently utilized in the current PP. This is due to the insufficient preparedness of the managing bodies of the respective OPs and the lengthy procedures for the analysis and implementation of the FIs in practice, see more in (Košťálová, Tetřevová, Paták, 2015). However, it is also necessary to mention the fact that the use of FIs is also significantly limited by macroeconomic conditions (low interest rates, high availability of standard sources of financing). It can therefore be assumed that half of the PP in 2018 will not achieve the planned objectives of the ESF-funded support with the use of FIs, and it will be problematic to achieve these objectives even at the end of the PP.

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Multivariate Dominance among financial sectors

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Abstract

In this paper, we propose a multivariate stochastic dominance comparison among different sectors from the point of view of non-satiable risk-averse investors. In particular, we consider different distributional hypotheses for the multivariate distribution of financial sectors and we examine if there exist some dominance among them. In this framework we also discuss the asymptotic dominance between financial sectors. Finally, we empirically examine the choices of some non-satiable investors taking into account the proposed studies.

Key words

Multivariate preferences, stochastic dominance, stable distribution, financial sectors comparison.

JEL Classification: G11, G12.

1. Introduction

In the financial literature, it is will-known that asset returns are not normally distributed and show heavy tails, for deeper discussion see, among others, Mandelbrot [4], Fama [1] and Rachev and Mittnik [9]. Empirical research has established some stylized facts about asset returns: (a) clustering of volatility, (b) skewness, and (d) fat tails (see for instance Kim et al. [3]). In this context, the assumption of finite variance is not always appropriate in real world problem (e.g. several random variables are generally not square-integrable). For this reason, the stable Paretian distribution has been proposed as an alternative model to the Gaussian distribution in many different frameworks, especially in finance.

Conceptually, the stable Paretian distributions are a natural generalization of the Gaussian distribution. In the non-Gaussian case, the stable distribution exhibits fatter tails than the normal and it is more peaked around its center (i.e., leptokurtic). For the purposes of portfolio analysis the most important feature of these distributions is that their variances are infinite (and all higher moments as well). We recall that stable distributions depend on location and scale parameters, moreover they are identified by a parameter which specifies the shape of the distribution in terms of skewness and, more importantly, a parameter that determines the asymptotic behavior of the tails.

The purpose of this paper is focused on the issue of ranking different financial sectors from the point of view of a risk averse investor. This problem of decision under uncertainty can be dealt with by relying on different approaches, such as utility theory (see Ingersoll [2]). Comparably, game theory may provide suitable tools for dealing with many allocation problems under uncertainty (see Moretti et al. [6]). Furthermore, from portfolio theory perspective, the individual preferences can be summarized by maximum gain and minimum

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risk and, in this setting, several reward-risk performance measures have been proposed in the literature, see Rachev et al. [10].

Unlike these approaches, and following Ortobelli et al. [8], we rely on the theory of stochastic orderings. In a univariate context, stochastic dominance rules, establish a partial order in the space of distribution functions, and quantify the concept of one random variable being 'preferable' to another. Since the decision problem is concerned with multivariate random elements (i.e., financial sectors), our main goal is to apply a dominance rule that can be simple and applicable to a multivariate framework. Due to the importance of stable Paretian distribution for approximating financial random variables, we use a rule that is aimed at ranking sub-Gaussian random vectors. In essence, we apply a criterion that selects the best financial sector to invest in.

The rest of the paper is organized as follows. In Section 2, we first define a simple and applicable multivariate dominance rule based on the mean–variance approach. Then, we propose a definition of a multivariate stochastic ordering aimed at ranking different financial sectors whose vectors of returns are in the domain of attraction of a stable sub-Gaussian law. In Section 3, we present an empirical comparison in order to evaluate possible dominance relations between several financial sectors of the S&P 500. Finally, our conclusions are summarized in Section 4.

2. Stochastic dominance between financial sectors

In this section, we present a possible financial application of multivariate stochastic orderings. Our main goal is to compare financial sectors from the point of view of investors. To this end, we introduce a multivariate ordering rule that is especially useful for ranking different financial sectors.

In the univariate case, we generally compare two random variables X and Y with respect to a given univariate order of preferences, represented by the symbol $\succ$. In particular, we say that X dominates Y with respect to $\succ$ and we write $X \succ Y$ when suitable conditions are satisfied. In general, these conditions involve the distribution functions of X and Y, say F_X and F_Y. One important ordering is the second order (or increasing concave) stochastic dominance (SSD): we say that X SSD Y if and only if $\int_{-\infty}^{t} (F_X(z) - F_Y(z))\, dz \leq 0$ for any t in $\mathbb{R}$ (see, among, others Levy (1992)).

In general, the extension of a given order of preferences $\succ$ to the multivariate case is not trivial, because in some practical cases it could be very difficult to satisfy the conditions of multivariate dominance. In this context, the natural generalizations of the first- and second-degree stochastic orderings can be found, for example, in Muller and Stoyan [7], but their corresponding dominance rules are far from being applicable, except to some specific cases. Indeed, multivariate stochastic orders consider the dependence structure of random vectors and cannot be simply based on component-wise comparisons of the marginal distributions. It is very rare to find a multivariate ordering rule that is based on comparisons between functionals. For this reason, following Ortobelli et al. [8], we use a simple and applicable dominance rule aimed at comparing multivariate random elements, such as different financial sectors.

Suppose that there are two sectors A and B, composed, respectively, of n and s assets. Denote by $x = [x_1, x_2, \dots, x_n]'$ and $y = [y_1, y_2, \dots, y_s]'$ the vectors of non-negative allocations among risky assets of sectors A and B, respectively. Moreover, assume that no short sales are allowed.

Definition 1. We say that a sector A with n assets strongly dominates sector B with s assets with respect to a multivariate preference ordering $\succ$ if, for any vector of returns Y_B

composed of $t \leq u = min(s, n)$ assets from sector B, there exists a vector X_A of sector A such that $X_A \succ Y_B$. Similarly, we say that a sector A with n assets weakly dominates another sector B with s assets with respect to the multivariate preference ordering $\succ$ if for any given portfolio of sector B with return $y'Y_B$, there exists a portfolio of the sector A with return $x'X_A$ such that $x'X_A \succ y'Y_B$.

Example 1. Suppose that the returns of sectors A and B are jointly elliptically distributed with finite variance. Suppose also that the two sectors have the same number of assets n, vector of averages μ_A and μ_B, and variance-covariance matrices Q_A and Q_B such that $\mu_A \geq \mu_B$ and $(Q_A - Q_B)$ is negative semi-definite. Then, sector A strongly dominates sector B with respect to the increasing concave multivariate order (see Muller and Stoyan [7]). Moreover, under these assumptions sector A weakly dominates sector B with respect to the concave order, because $x'\mu_A \geq x'\mu_B$ and $x'Q_Ax \leq x'Q_Bx$ for any vector $x \geq 0$. Note that this weak dominance between elliptically distributed vectors is also known in the literature as the increasing positive linear concave multivariate order (see Muller and Stoyan [7]).

The increasing positive linear concave multivariate ordering defined in Example 1 is strictly related to the mean–variance rule, which is widely used in finance to solve the portfolio optimization problem. According to this principle, investors generally aim at minimizing the variance (risk) for some fixed level of mean (reward).

Example 1 and Definition 1 are based on the mean–variance approach (see Markowitz [5]). However, we argue that a suitable dominance rule for ranking financial sectors should depend on the most reliable model. In doing so, we propose a second approach to the dominance problem that is based on stable Paretian (sub-Gaussian) distributional assumptions.

Let $r_{t+1} = [r_{1,t+1}, \dots, r_{n,t+1}]'$ denotes the vector of gross returns on date $+1$. In the empirical finance literature it is well-known that assets returns follows heavy tailed distributions. In this framework, we can assume that the returns are in the domain of attraction of a stable law, that is,

$$P(|r_i| > t) \approx t^{-\alpha_i} L_i(t) \text{ as } t \to \infty,$$

where $0 < \alpha_i < 2$ and $L_i(t)$ is a slowly varying function at infinity, i.e.

$$\lim_{t\to\infty} \frac{L_i(ct)}{L_i(t)} \to 1 \text{ for all } c > 0,$$

(see Rachev and Mittnik [9] and the references therein). The tail condition implies that any gross return r_i follows an α-stable distribution $S_\alpha(\sigma, \beta, \mu)$, where $\alpha \in (0,2)$ is the so-called stability index, which specifies the asymptotic behavior of the tails, $\sigma > 0$ is the dispersion parameter, $\beta \in [-1,1]$ is the skewness parameter and $\mu \in \mathbb{R}$ is the location parameter. Typically, we can assume that the vector of returns r is α-stable sub-Gaussian distributed. We remind that an α–stable sub-Gaussian distribution is an elliptical distribution symmetric around the mean (since $\beta = 0$) and when $\alpha = 2$ we obtain the Gaussian distribution.

Sub-Gaussian returns are elliptically distributed for a fixed value of α. Therefore, according to Ortobelli et al. [8], it is important to define a suitable order of preferences that deals with distributions with different values of α. In the sequel, we determine a ranking criteria aiming to compare sub-Gaussian distributions according to SSD. In particular, it has been found that SSD can be verified by comparing the values of the stability, dispersion and location parameters. In this regard, we recall the following Theorem for order of preferences, which is due to Ortobelli et al. [8]:

Theorem 1. Let $X_1 \sim S_{\alpha_1}(\sigma_1, 0, \mu_1)$ and $X_2 \sim S_{\alpha_2}(\sigma_2, 0, \mu_2)$. Suppose that $\alpha_1 > \alpha_2 > 1$, $\sigma_1 \leq \sigma_2$ and $\mu_1 \geq \mu_2$. Then $X_1 \; SSD \; X_2$.

The results of Theorem 1 can be extended to a multivariate setting, generalizing the multivariate mean–variance approach presented in Example 1, considering the asymptotic behaviour of the tail distributions. This yields the next definition of asymptotic multivariate dominance between financial sectors.

Definition 2. Assume that sectors A and B have an equal number of assets n. Assume that the returns of sectors A and B are stable sub-Gaussian distributed with stability indices α_A and α_B, vectors of averages μ_A and μ_B, and dispersion matrices Q_A and Q_B. We say that sector A dominates sector B with respect to the asymptotic increasing concave multivariate order if $\alpha_A > \alpha_B$, $\mu_A \geq \mu_B$ and $(Q_A - Q_B)$ is negative semi-definite. We say that sector A dominates sector B with respect to the weak asymptotic increasing concave multivariate order if $\alpha_A > \alpha_B$ and, for any vector $y \geq 0$, there exists a vector $x \geq 0$ such that $x'\mu_A \geq y'\mu_B$ and $x'Q_A x \leq y'Q_B y$.

3. Empirical analysis

In this section, we apply the asymptotic multivariate weak dominance rule in order to compare empirically the S&P 500 sectors. We compare the results of this method with those obtained by the weak concave multivariate order described in example 1. To this end, we consider the returns of the stock sectors of the S&P 500, through the period January 2005 till January 2017, namely, Information Technology (IT), Financials (FI), Health Care (HC), Consumer Discretionary (CD), Industrials (IN), Consumer Staples (CS), Energy (EN), Utilities (UT), Real Estate (RE), and Materials (MA).

Firstly, we verify the dominance rules proposed over the decade 2005–2017. In particular, we determine the so-called alpha–mean–dispersion efficient frontier, computing the portfolio with minimum dispersion $x'Qx$ for any fixed mean $x'\mu_A$, and then we compare the efficient frontiers determining whether the condition for the weak asymptotic increasing concave multivariate holds. Secondly, we present a similar analysis, assuming that the returns of each sector are normally distributed and we compare the mean–variance efficient frontiers, as suggested in Example 1. Finally, we examine the statistical characteristics of the returns of each sector.

In practice, we compare the efficient frontiers of the SPA 500 sectors starting from January 1, 2005, to January 1, 2017. In particular, every month (20 trading days) we estimate the mean–variance efficient frontiers and the stable–mean–dispersion efficient frontiers of the sectors, by using the assets which were active during the last 4 years (1000 daily historical observations). Therefore, at every recalibration time, we fit the efficient frontier solving the optimization problem for 40 levels of the mean. The results of this empirical analysis are reported in Tables 1, 2 and Figures. 1 and 2.

In Table 1, we report the average of the percentage invested in each sector over different periods (e.g. Subprime crisis, EU credit crisis and post crisis period) either using the Gaussian distributional hypothesis or the stable distributional assumption.

In Table 2, we present the results in terms of the average values for the McCulloch estimates of the stable Paretian parameters $(\alpha, \beta, \sigma, \mu)$ and summary statistics for the asset returns on average of each sector and for all sectors together: mean, standard deviation, skewness, kurtosis, number of times we get zero investment, number of dominated times by other sectors.

Table 1: reports the mean of the percentage invested in each sector over different periods, using a moving average window of 1000 observations

From	To	MA 1000	IT	FI	HC	CD	IN	CS	EN	UT	RE	MA
Gaussian distribution hypothesis												
December 2008	January 2017	**all period**	0,1227	0,0080	0,2794	0,1891	0,0430	0,2286	0,0031	0,0364	0,0905	0,0082
December 2008	September 2009	**Subprime crisis**	0,0675	0,0350	0,3917	0,1603	0,0182	0,3161	0,0000	0,0086	0,0000	0,0090
September 2009	January 2013	**EU credit crisis**	0,1207	0,0060	0,2455	0,2776	0,0000	0,1808	0,0000	0,0200	0,1491	0,0086
January 2013	January 2017	**post crisis**	0,1333	0,0051	0,2887	0,1215	0,0822	0,2534	0,0062	0,0544	0,0575	0,0076
Stabe distribution hypothesis												
December 2008	January 2017	**all period**	0,1100	0,0054	0,2561	0,1367	0,0298	0,2235	0,0080	0,0489	0,1737	0,0150
December 2008	September 2009	**Subprime crisis**	0,0238	0,0054	0,4765	0,0747	0,0346	0,2591	0,0078	0,0201	0,0398	0,0613
September 2009	January 2013	**EU credit crisis**	0,0659	0,0005	0,2700	0,1825	0,0001	0,1756	0,0013	0,0376	0,2547	0,0188
January 2013	January 2017	**post crisis**	0,1601	0,0094	0,2089	0,1094	0,0533	0,2567	0,0134	0,0630	0,1295	0,0042

From Table 1 we observe that:

- The percentage invested in each sector differs from Gaussian to Stable case.
- IT, HC, CD and CS sectors generally have higher weights than FI, IN, EN, UT, RE and MA counterparts, over all examined period, either for Stable or Gaussian case.
- IT sector get a higher weight post crisis than during the period of market distress (subprime and European credit crisis), which is not the case for CD sector that losses significant weights after European credit crisis.
- HC and CS sectors obtained higher weights during subprime crisis and less throughout European credit crisis (even though they have the highest percentage).
- After suffering from the subprime mortgage crisis, which was triggered by a large decline in home prices after the collapse of a housing bubble, RE get more weight throughout European credit crisis.
- FI, IN, EN, UT and MA sectors show lower percentages over the examined period (even playing important roles). These results could be explained by the effect of financial crises on these sectors.

Table 2: Statistics for the asset returns on average: mean, standard deviation, skewness, kurtosis, number of dominated times by other sectors, and McCulloch Estimates of the stable Paretian parameters $(\alpha, \beta, \sigma, \mu)$

	IT	**FI**	**HC**	**CD**	**IN**	**CS**	**EN**	**UT**	**RE**	**MA**	**all sec**
Gaussian distribution hypothesis											
Mean	0,038%	0,009%	0,052%	0,046%	0,038%	0,046%	0,022%	0,029%	0,035%	0,034%	0,036%
St. dev.	2,262%	2,830%	1,958%	2,323%	2,093%	1,501%	2,624%	1,455%	2,658%	2,208%	2,232%
Skew.	-0,1201	-0,2891	-0,2203	-0,0276	-0,2780	-0,0752	-0,2368	-0,0433	-0,1789	-0,2102	-0,1703
Kurt.	11,362	12,935	12,634	9,964	9,857	11,815	9,036	8,788	10,657	9,023	10,869
J–B rejected	1	1	1	1	1	1	1	1	1	1	1
Zero invest. time	1	29	0	0	42	0	69	0	22	1	n.c.
Stabe distribution hypothesis											
Alpha α	1,565	1,444	1,575	1,550	1,533	1,585	1,603	1,630	1,449	1,527	1,542
Beta β	0,0088	-0,0257	0,0296	0,0472	-0,0280	-0,0143	-0,0495	-0,0912	-0,0563	-0,0119	-0,0097
Sigma σ	1,156%	1,160%	0,986%	1,176%	1,066%	0,760%	1,380%	0,762%	1,132%	1,108%	1,085%
Mu μ	0,0519%	0,0280%	0,0633%	0,0744%	0,0354%	0,0477%	0,0304%	0,0079%	0,1063%	0,0503%	0,0507%
Zero invest. time	3	36	0	0	46	0	54	0	0	2	n.c.
# dominated	0	36	0	0	28	0	11	0	0	2	n.c.

From Table 2 we observe that:

- These parameter estimates suggest the presence of a slight skewness (since the asymmetry parameter β and the skewness are close to zero) and a quite remarkable tail-weightiness (the kurtosis exceeds three whilst the stability parameter α is less than two).
- We observe that FI returns present, on average, the lowest mean and α and the highest standard deviation.
- Computing the Jarque–Bera statistic with a 95 % confidence level, in order to test whether asset returns follow a normal distribution, we find that the Gaussian hypothesis is rejected.
- The number of times we have zero percentage invested is relevant for FI, IN, EN, and RE sectors, which is confirmed by the number of dominated times with respect to the other sectors.

Overall, from this ex ante analysis, the best performing sectors are IT, HC, CD and CS sector. This is also confirmed by Figures 1 and 2, which report two examples of the mean–variance and alpha–mean–dispersion dominance. In particular, Figure 2 shows the case when the FI, IN and MA sectors weakly dominated by the rest of sectors (IT, HC, CD, CS etc.) with respect to the asymptotic increasing concave multivariate order.

Figure 1: Example of Mean-Variance dominance.

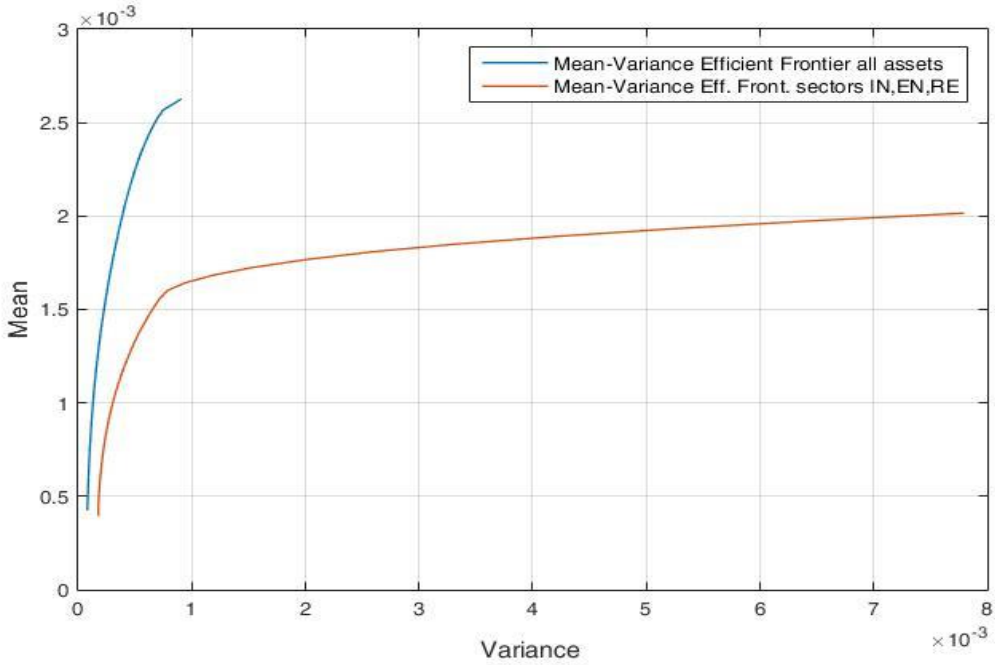
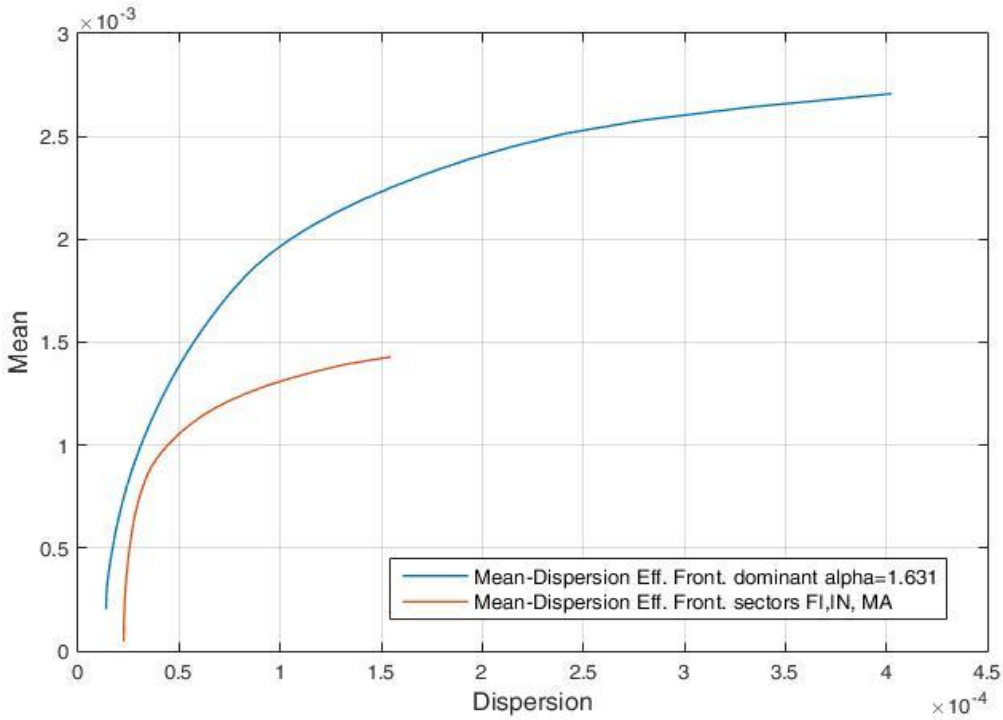


Figure 2: Example of alpha-Mean-Dispersion dominance.



4. Conclusion

In this paper, we introduced a methodology to compare different financial sectors based on asymptotic multivariate stochastic dominance and presented a possible application aimed at ranking S&P 500 sectors (namely, IT, FI, HC, CD, IN, CS, EN, UT, RE, and MA). In practice, the proposed dominance rules can be used by non-satiable risk averse investors in order to identify the best financial sectors to invest in. We showed that this method yields remarkably different results from a more traditional mean–variance approach. The primary contribution of the empirical comparison presented in this paper is the analysis of the impact of the distributional assumptions on asset allocation decisions. In particular, the great differences observed between the mean–variance approach and the stable Paretian approach suggest that portfolio selection models must consider the asymptotic (tail) behaviour of the returns.

Further research could involve theoretical and empirical studies. On the one hand, a natural extension of this research would be a multivariate stochastic dominance rule that also takes skewness into account. On the other hand, the use of nonparametric approaches could be considered. Future research will investigate these aspects. Another promising direction for future research is to study alternative ranking criteria (finer than SSD).

Acknowledgements

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Banking sector

Jan Krajíček[1]

Abstract

The contribution is dedicated to the banking sector. Conditions that affect it and the possibilities for further development. Emphasis is placed on the development of interest rates and their impact on the economy and management of banks. The decline in interest rates would itself be a problem for banks such as resources grow in the banking sector. Growth in deposits not only in banking, but in the financial sector, and has its consequent reflection in both the declining risk aversion and the need to create new products and services. The decline in risk aversion is also reflected in the design of new products, where risk is transferred to raise immediate profits.

Key words

Bank, Deposits, Liabilities, Loans, Revenue, Risk

JEL Classification: C15, E37, E42, G20, G21

1. Introduction

Management of banks is crucial for the management of assets and liabilities, which is an essential part of the Bank's financial management and its basis. The basis is a way of managing the structure of the bank balance sheet, which is an overview of the bank's assets and sources of funding (Krajíček 2016).

Asset and Liability Management applies to the Bank as a whole. The bank is guided by bank risk, diversification, risk limitation by lower and upper limits, while respecting the profitability criterion and banking profitability. In Asset and Liability Management, the Bank always operates in the area defined by the Central Bank and must respect its measures in terms of prudent banking business and monetary policy.

The interest of shareholders who have put capital into the bank is that the bank achieves maximum profit rates while increasing the market value of the bank. On the other hand, the client, represented by the central bank as a regulator, aims mainly at the stability, liquidity and solvency of the bank. The art of managing the bank means satisfying, on the one hand, the shareholder, demanding the greatest return on investment, while meeting the requirements of the central bank requiring the stability and security of the banking business.

The most common goal of the banking business is to achieve profit, which is also a prerequisite for ensuring the bank's stable market position.

The bank should not be about maximizing profits at a given time (in a short period of time). The level of profit achieved should respect the bank's long-term perspective (long-term profitability) and its continuing competitive ability.

The decisive goals of banking business can be considered:

- Profit growth.
- The maximum market value.

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- A strong position in the banking market.
- Stability of the bank.
- Security for the client and the owner of the bank.

The primary objective (making a profit) is necessary for banks to align with the secondary objectives (profitability, growth of the bank and its reputation). The Bank's certainty and liquidity are the necessary conditions for achieving these goals. The bank must seek a compromise between profit and certainty (stability).

The activity of each bank is associated with certain risks. Some of the risks arise from the economic environment in which banks operate, the second part is determined by the specific features of the bank's activity. The bank always operates in a particular economic environment that is subject to change, with little or no control over the bank's development.

The risks that the bank undergoes usually appear first to be a commercial risk, they are gradually transformed into a performance risk and eventually a financial risk.

Influence of credit risk on the bank's economy:

- Has a direct impact on the bank's profit, which is reduced as a result of provisioning, write-off of loans granted, creation of reserves and reduction of interest income of the bank.
- It is closely related to liquidity:
  - Instead of becoming a liquid asset from an illiquid asset, the balance remains an illiquid item.
  - Interest payments for missing funds drain bank liquidity.
  - The resources used in the credit relationship are not valued due to default interest.
  - The credit principal is not returned, causing the bank's cash flow.
- There is also a fundamental relationship between credit risk and the risk of interest rate movements. When changing interest rates to the detriment of the borrower, the credit risk increases.
- The ultimate consequence of credit risk growth is the downward pressure on employees. A lower number of employees reduces wage costs and consequently increases undistributed earnings during the year and profit at the end of the year. This has a positive impact on banks' regulatory indicators.

The risk of a loan portfolio in the area of credit operations of a bank is necessary to manage and monitor not only the risk of specific loans but also the risk of the entire loan portfolio. The more diversified the structure of the loan portfolio, the lower the bank's credit risk. High concentration per client, An economically linked group of clients, to one economic sector or region, the Bank's credit risk is always substantially increased.

Profit and profitability of a banking business

The main motive of a banking business is profit and profitability. Their measure expresses the efficiency of the bank's business activities, the level of banking management, and the success of business decisions. Although we are talking about profit, the bank can also make losses. Profit and loss are two possible forms of the Bank's financial performance.

Bank Profit

These are indicators that are the focus of the bank's success rate. From the point of view of shareholders, it means the rate of appreciation of their investments, from the point of view of the creditors, it is a proof of the credibility of the bank. For the bank itself, it represents a critical source of cover for the expected losses from the bank's active trades and the cheapest source of its economic growth.

Profit generation significantly influences the quality of the bank's financial decisions and the quality of the management of existing and potential risks, which is reflected in the difference between the bank's income and expenses. In their inadequate quality, the difference between revenue and costs may be negative, then the bank will lose.

Efficiency of bank transactions is expressed in particular in the profit margin. Other earnings are earned by the Bank from foreign exchange operations and from securities trades that it executes for clients or for its own account.

The final amount of the Bank's profit or loss significantly affects the creation and eventual use of provisions and provisions and its taxation in the form of income taxes.

Profitability of banking business

The absolute amount of profit or loss has a limited ability to provide information, in particular as regards comparisons with other banks, or analysis of factors that have affected the bank's profit or loss. Therefore, the Bank calculates the ratio of economic performance indicators, in particular the ROE and ROA ratios.

While it is necessary to understand the management of assets and liabilities as a whole, the management of liabilities has its own rules and peculiarities, as well as the management of assets, which is followed by risk management.

Management of bank liabilities

Its aim is to source resources while respecting the principle that the cost of obtaining resources should be as low as possible, and at the same time at least partly aligning the requirements of the asset side with the liabilities side as to their maturity, the commitment deadlines, interest rates and their sensitivity. Fundraising is an active process supported by marketing measures in the bank.

Each bank resource has its own specifics. The bank is the total source for their business in a certain structure relative to the asset side, the current situation on the resource market, the risk and cost assessment, or other aspects.

The management of the bank's liabilities is important not only because of the profit motive of obtaining resources at the lowest possible cost and with their longest possible use, but also in view of the liquidity problem of the bank.

Bank Asset Management

Its aim is to use the financial resources obtained so that the proceeds are as high as possible with appropriate risk reduction, taking into account the liabilities side up to maturity, the commitment period, etc.

When creating a bank asset structure, two basic objectives - the bank's return on the basis of profitable assets and the bank's stability based on risk management - must be respected.

Compared to other enterprises, the share of equity capital in the Bank's total liabilities is relatively low, while banks use large amounts of foreign capital in their business. The bank's balance sheet has significant information, based on its analysis we can estimate the Bank's orientation, strength and health.

Both the business plan and the bank's strategy are included in the balance sheet.

Recently, banking vulnerability has grown. Because of growing competition, banks are trying to more carefully distribute the resources they have gained - between different clients, products, markets. Increasing competition forces the emergence of new products and opportunities, while increasing the number of market participants. As a result, there is a better decision-making and management of banks, which contributes to the latest theoretical knowledge and experience.

Asset and Liability Management

Has evolved in banks, especially since the 1980s, and is the most significant in the last 20 years. By the 1950s, banks had a passive approach to obtaining funds, the deposits that were then borrowed in the form of a loan or invested on a conservative basis. Emphasis was placed on the assets side, on the management of the corporate banking portfolio.

At present, as a new significant element, assets and liabilities are actively accumulating a sharp increase in bank balances, influenced in particular by the growth of client deposits, when clients' deposits grow significantly. Client deposits grow very strongly.

Bank revenue management and bank orientation.

Interest rates are significantly reflected in the performance of banks. Due to the orientation of the bank, it is possible to assume different impacts on banks depending on their size. The paper is devoted to the analysis of the impact of the decline in interest rates in recent years on the results of banks. The fall in interest rates on the interbank market (PRIBOR 3M – as a significant benchmark) is reflected not only in the bank's economy but also in the risks involved. It is perhaps the objection that this rate is not the most important. Theoretically, it is also possible to consider the rates announced by the central bank (the discount rate repo rate), which have a decisive impact on bond prices. Rates announced by the central bank are reflected in the interbank market rates. Therefore, in the following calculations, the interest rate PRIBOR 3M is considered.

Its decisive influence is also the orientation of the bank, which is dependent on the size of the bank. In particular, small banks are oriented towards retail clients, which bring more profits at current interest rates. At a disadvantage, banks of medium size are engaged in the same way as large banks, both retail and corporate clients.

2. The Impacts of Interbank interest rates in Banks' risk a profit

The analysis is based solely on available data published by the CNB. It focuses exclusively on the Czech Republic, for which data are available in sufficiently long time series, which allow me to take relevant conclusions.

The decisive focus is the influence of the development of interbank interest rates on the development of risk loans in the banking sector. Attention is paid mainly to the effect of the size of the bank on its earnings results, the impact of the size of the bank on the administrative and personnel costs, and the risk of default credit in both the total and the client. In order to maintain the objectivity of banks' comparisons, the comparisons are always based on the benchmark of the average assets.

Chart 1 illustrates the development ROE (Return of Equity).

When assessing banks size, their orientation is decisive. While large and, in part, medium-sized banks are primarily oriented to corporate clients, small banks are primarily oriented to retail clients. In particular, credit card and consumer credit interest rates have also been subject to high interest margins in recent years, as documented in the following Chart 2, while Chart 3 clearly illustrates how interest rates on Commercial Loans have declined significantly. There is also a significant drop in the PRIBOR 3M interbank interest rate, which is linked to interest rates on Commercial Loans.

The evolution of interest rates is linked to the development of individual banks' profitability. Improves the profitability of small banks. Nevertheless, medium-sized banks achieve the best results. It can be assumed that this is a consequence of their orientation for both retail and commercial clientele.

Of course, overall developments are reflected in the risk management approach of banks. The highest proportion of receivables with default is reported by medium-sized banks. Small and large banks are starting to get roughly the same level. This is documented in Chart 4.

The situation in medium-sized banks stems from their orientation on both retail and commercial clientele, and they do not at the same time have such high-quality commercial clientele as large banks.

Small banks, together with large banks, achieve relatively good results due to their orientation - small banks are retail clients and large banks with their portfolio.

This analysis also signals the possibility of problems associated with illiquid loans, especially in medium-sized banks, while large and small banks are not at risk of defaulting loans.

The current developments, when interest rates on deposits have declined significantly, while the current liquidity surplus in Czech banks is significantly reflected in their orientation. In particular, small banks are trying to find a trick on the market. Critical for them is the provision of retail loans. Here, however, they are beginning to get more and more involved in battles with non-bank providers of consumer credit.

Large banks have the advantage both in their distribution of clients' portfolio and are thus not endangered.

The crucial problem remains with medium-sized banks, which are trying to compete with large banks by taking over corporate clients and pushing for consumer credit. As documented in Figure 6, the share of default credit is more than double that of the average and nearly three times larger than large banks.

It is crucial for further discussion to set the direction towards the orientation of individual parts of the banking sector in line with risk management.

The current favorable development in the portion of the loan in default in the loan portfolio is also influenced by the increase in the absolute value of the loan portfolio. The portion of defaulting credit is also reflected in a relatively short credit expansion period, particularly in the area of retail loans and housing loans. With a prolonged credit expansion period, risk aversion will be declining in banks as a result of high deposits growth. It can be assumed that this will result in a significant increase in default credit.

The ROE trend is noticeable as it grows in small banks, while in medium and large banks there is a systematic decline.

Chart 1: Development of ROE in %

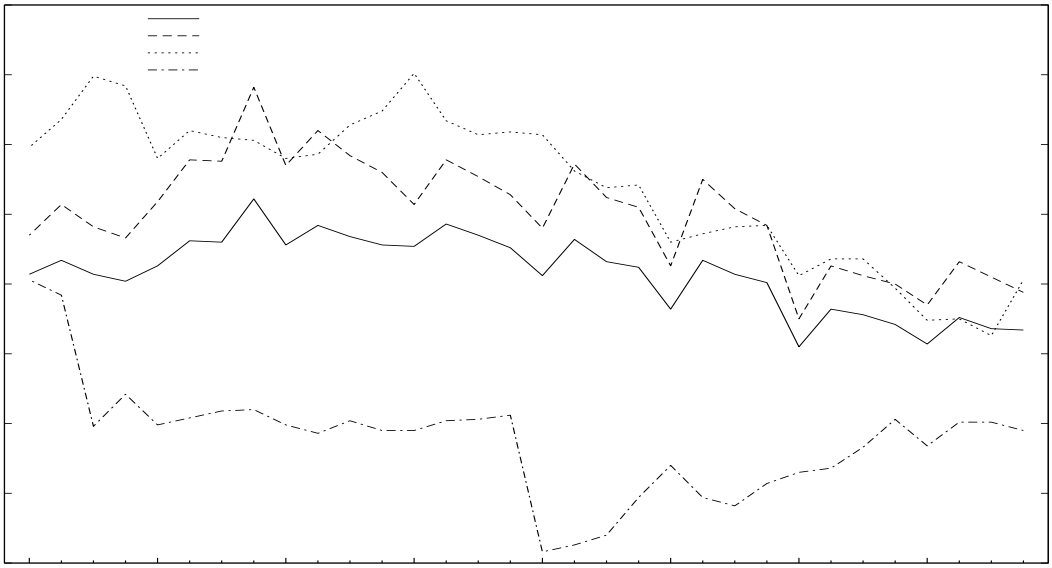
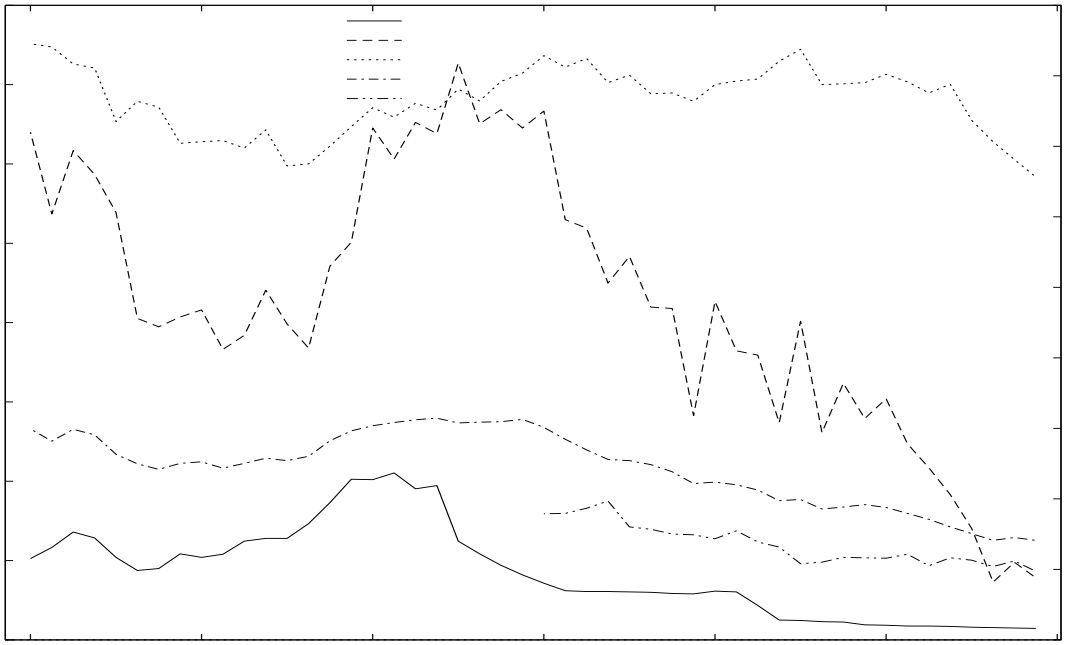


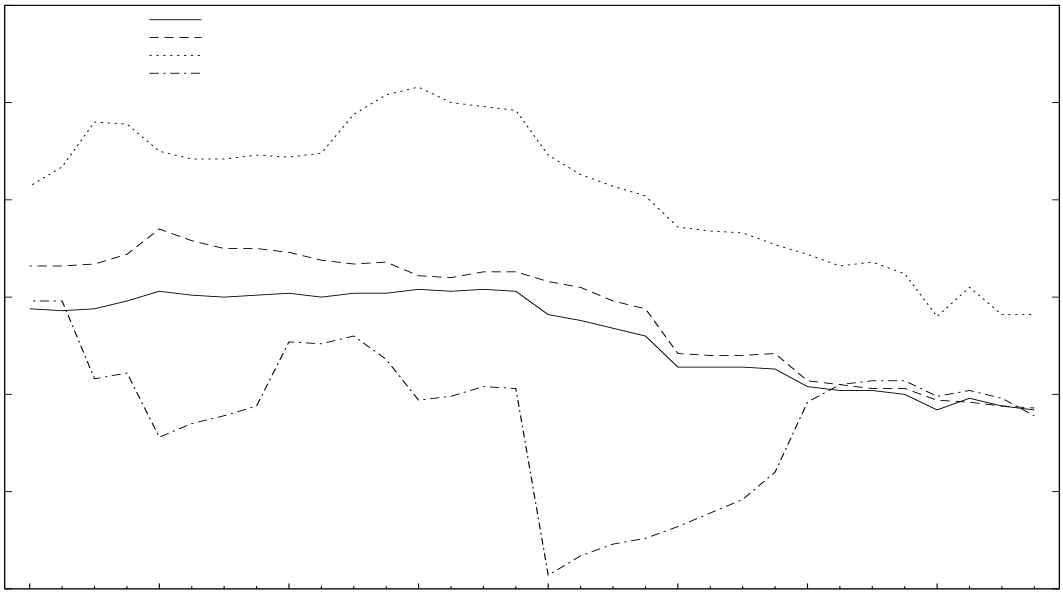
Chart 2: Development interest rates in %



The decisive factor affecting ROE is the Bank's orientation. The large banks are mainly focused on corporate clients, although the share of retail and mortgage loans is not insignificant, but small banks tend to focus on retail clients and show better results.

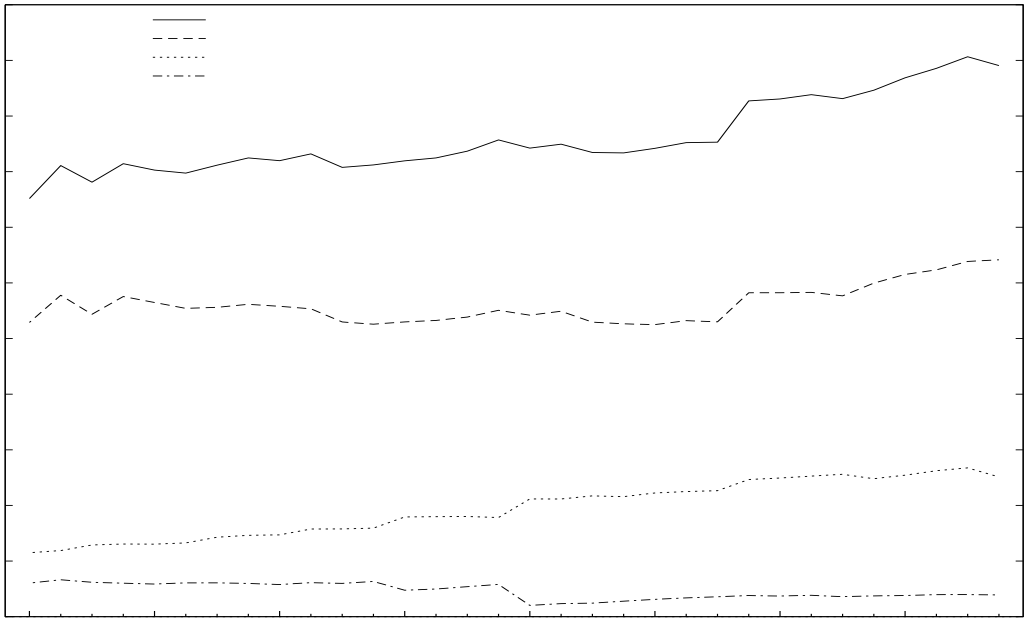
This development is documented in the following Chart 3, which is focused on the development of the average interest rates provided in individual loan categories. It is clear from the following chart that the highest average interest rates are achieved for consumer loans, which are mainly provided by smaller banks that do not reach corporate clients and are focused on providing consumer loans at average interest rates above 12%, while the average Interest rates on corporate corporations are around 2%. PRIBOR 3 M is at 0.29%.

Chart 3: Development of the share of interest income on assets in %



The fall in interest income on assets is affected by appreciable increases in risk exposures in the banking sector, which is particularly significant for large and medium-sized banks, while small banks are still keeping their risk exposures stable.

Table 4 Development of Risk Exposure in Mil. CZK



3. Results and discussion

The development of interest rates, depending on the size of the bank, is also linked to the development of the share of interest cost on assets. The share of interest costs on assets has declined in recent years, but the best results are achieved by medium-sized banks. In line with the small banks' orientation, they are performing better when they are in line with developments across the banking sector.

Bank results are influenced by a risk management approach that:

- Has a direct impact on the bank's profit, which is reduced as a result of provisioning, write-off of loans granted, creation of reserves and reduction of interest income of the bank.
- It is related to liquidity because.
- In the balance sheet still illiquid item.
- Interest payments for missing funds drain bank liquidity.
- The resources used in the credit relationship are not valued due to default interest.
- The credit principal is not returned, causing the bank's cash flow.
- There is also a fundamental relationship between credit risk and the risk of interest rate movements. When changing interest rates to the detriment of the borrower, the credit risk increases.
- The ultimate consequence of credit risk growth is the downward pressure on employees. A lower number of employees reduces wage costs and consequently increases undistributed earnings during the year and profit at the end of the year. This has a positive impact on banks' regulatory indicators.

4. Conclusions

For further development of the banking sector and individual banks, it is crucial:

- Client orientation of the bank, whether the bank is oriented towards corporate clientele, retail, mortgage lending.
- The size of the bank, which is closely linked to the ability of the bank to focus on client demand.
- The size of the bank at the same time also has a decisive influence on the Bank's potential for further development.
- Bank's approach to risk management:
  - Has a direct impact on the bank's profit, which is reduced as a result of provisioning, write-off of loans granted, creation of reserves and reduction of interest income of the bank.
  - Closely related to liquidity because.
  - Instead of becoming a liquid asset from an illiquid asset, the item still remains illiquid.
  - Interest payments for missing funds drain bank liquidity.
  - The resources used in the credit relationship are not revalued as a result of interest payments.
  - The credit principal is not returned, causing the bank's cash flow.
  - There is also a fundamental relationship between credit risk and the risk of interest rate movements. When changing interest rates to the detriment of the borrower, the credit risk increases.

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Break-even analysis under normally distributed input variables

Aleš Kresta[1], Karolina Lisztwanová[2]

Abstract

In traditional break-even analysis we assume that all the input variables (variable costs per unit, total fixed costs and price of the product) are known with certainty. However, these variables may be random and thus not known in advance. For instance, a firm can be price-taker – the price of the product is random variable determined by market; variable costs per unit depend on the price of raw materials, which again cannot be known in advance with certainty. In our paper, we discuss the break-even analysis introducing randomness. We focus on two input variables – the price of the product, which influences the revenues, and the variable costs per unit, which influence the costs. In the paper, we assume that these random inputs follow normal distribution.

Key words

break-even analysis; randomness;

JEL Classification: G31

1. Úvod

Tradičně pojatá analýza bodu zvratu je vhodným nástrojem krátkodobého rozhodování o objemu produkce, jež pokryje veškeré náklady související s výrobou a odbytem. K základním předpokladům patří prodej veškeré vyrobené produkce, tržby a celkové náklady měnící se jen s ohledem na změnu objemu produkce, náklady, jež lze klasifikovat na fixní a variabilní, lineární vývoj celkových a variabilních nákladů, neměnné fixní náklady v rámci dané výrobní kapacity, dopředu známá prodejní cena včetně neměnné technologie a organizace výroby za předpokladu nepřetržitého výrobního procesu. Díky těmto předpokladům lze popsat tržby a celkové náklady lineárními rovnicemi či v případě grafické prezentace přímkou, blíže viz Král a kol. (2010), Mruzková a Lisztwanová (2013) nebo Cafferky a Wentworth (2014).

Dle těchto vstupních předpokladů lze stanovit kritický objem výroby (bod zvratu), tj. objem produkce, při kterém jsou celkové tržby rovny celkovým nákladům, kdy vstupními proměnnými jsou cena produktu, variabilní náklady na jednotku produkce a celkové fixní náklady. Tradiční analýza bodu zvratu pracuje s těmito proměnnými jako předem známými, což jednoznačně platí pouze v případě fixních nákladů (obvykle opakující se náklady, jež jsou známy předem). Nicméně i v případě krátkodobého rozhodování nemusí být cena produktu a výše variabilních nákladů předem známa. Tento fakt je patrný například v případě firem obchodujícím se zahraničím. V tomto případě závisí výše jejich tržeb vyjádřená v domácí měně na směnném kurzu. Podobný problém je zřejmý i v případě firem, které nejsou tvůrci trhu v oblasti cen, ale jsou přinuceny akceptovat cenu danou trhem, jako je tomu například

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v případě firem produkujících a prodávajících elektřinu na spotovém trhu. V případě variabilních nákladů může mít podobný dopad i dopředu neznámá cena za spotřebované suroviny a materiál, jež se stávají součástí výrobku. Přestože nelze v těchto případech určit hodnotu těchto vstupních veličin s jistotou, můžeme alespoň odhadnout rozdělení pravděpodobnosti jejich hodnot.

Cílem příspěvku je modifikovat teorii analýzy bodu zvratu za předpokladu rizika spojeného se vstupními veličinami a porovnat výsledky obou přístupu – tradičního a modifikovaného na zvoleném příkladu. Pro modelování náhodných vstupních veličin v příspěvku uvažujeme normální (Gaussovo) rozdělení pravděpodobnosti.

Příspěvek je členěn následovně. V následující části je čtenář seznámen se základními aspekty tradiční analýzy bodu zvratu a jejím rozšířením pro modelování rizika. V další (třetí) části je představen reálný příklad, z něhož jsou patrné hlavní rozdíly obou přístupů, které jsou shrnuty v poslední části příspěvku.

2. Analýza bodu zvratu

Kritický objem produkce (bod zvratu) představuje takovou úroveň objemu výroby a prodeje, při kterém jsou celkové tržby (*TR*) rovny celkovým nákladům (*TC*), což znamená, že není dosaženo ani zisku, ale ani ztráty,

$$TR = TC\,. \tag{1}$$

Celkové tržby jsou ovlivněny prodejní cenou (*p*) a objemem prodaných výkonů (*Q*). Celkové náklady jsou tvořeny fixními náklady (*FC*) a variabilními náklady (*VC*). Variabilní náklady jsou zjištěny jako součin variabilních nákladů na jednotku produkce (*vc*) a předpokládaného objemu produkce (*Q*). S ohledem na výše uvedené lze rovnici (1) vyjádřit jako,

$$p \cdot Q = FC + vc \cdot Q\,, \tag{2}$$

kde neznámou je objem produkce *Q*,

$$Q = \frac{FC}{p - vc}\,. \tag{3}$$

Nahrazením vztahu *p* – *vc* jednotkovou marží (*m)* lze rovnici (3) upravit do následujícího tvaru,

$$Q = \frac{FC}{m}\,. \tag{4}$$

Předpokladem ve vztahu (3) je, že cena produktu *p* i variabilní náklady na jednotku produkce *vc* jsou dopředu známy. Pokud tyto vstupní proměnné nejsou dopředu známy, lze je na základě vývoje z minulého období popsat alespoň prostřednictvím rozdělení pravděpodobnosti. V tomto případě nelze určit kritický objem produkce s jistotou, lze jej určit pouze na základě určitého intervalu spolehlivosti (dále hladina spolehlivosti), tj. pravděpodobnosti s jakou nebude dosaženo ztráty. Například při 85% hladině spolehlivosti zjištěný bod zvratu znamená, že tento objem prodané produkce nevytvoří ztrátu s 85% pravděpodobností. Na druhou stranu existuje 15% šance, že při tomto objemu prodané produkce bude dosaženo ztráty.

V příspěvku je uvažováno Gaussovo (též normální) rozdělení pravděpodobnosti, které je velmi dobře známé rozdělení pravděpodobnosti snadno rozpoznatelné podle tvaru tzv. zvonové křivky. Toto rozdělení může být charakterizováno následující funkcí hustoty pravděpodobnosti,

$$f_N\left(x;\mu,\sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,, \tag{5}$$

a kumulativní distribuční funkcí,

$$\Phi(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{(t-\mu)^2}{2\sigma^2}}dt, \tag{6}$$

kde μ a σ jsou parametry, které určují tvar rozdělení. První parametr definuje polohu (průměr, medián a modus rozdělení se rovná tomuto parametru) a druhý parametr definuje variabilitu rozdělení (směrodatná odchylka se rovná tomuto parametru). Normální rozdělení je symetrické.

Pokud uvažujeme vstupní proměnné mající sdružené normální rozdělení, tj. $p \sim N(\mu_p,\sigma_p)$ a $vc \sim N(\mu_{vc},\sigma_{vc})$ s korelačním koeficientem $\rho_{p,vc}$, pak jednotkovou marži m lze popsat normálním rozdělením pravděpodobnosti,

$$m \sim N\left(\mu_p-\mu_{vc},\sqrt{\sigma_p^2+\sigma_{vc}^2-2\cdot\sigma_p\cdot\sigma_{vc}\cdot\rho_{p,vc}}\right). \tag{7}$$

Pro hladinu spolehlivosti α lze určit bod zvratu za rizika obdobně vztahu (4) následujícím způsobem,

$$Q=\frac{FC}{\Phi^{-1}\left(1-\alpha;\mu_p-\mu_{vc};\sqrt{\sigma_p^2+\sigma_{vc}^2-2\cdot\sigma_p\cdot\sigma_{vc}\cdot\rho_{p,vc}}\right)}. \tag{8}$$

Výhodou předpokladu normálního rozdělení vstupních proměnných je možnost analytického vyjádření kritického objemu produkce, viz rovnice (8). V případě předpokladu jiného rozdělení pravděpodobnosti (např. umožňujícího modelovat zešikmení, těžké konce) nebo nelineární závislosti vstupních proměnných by bylo potřeba kritický objem produkce získat simulačně. Postup by pak byl následující:

1. simulace náhodných jednotkových variabilních nákladů a ceny za jednotku produkce s ohledem na strukturu závislosti a marginální rozdělení pravděpodobností,
2. kalkulace kritického objemu produkce pro každou provedenou simulaci,
3. určení α kvantilu simulovaných kritických objemů produkce.

Pro simulaci náhodných proměnných lze použít např. kopula přístup, kdy jsou nejdříve simulovány náhodné prvky s požadovanou strukturou závislosti a tyto jsou následně pomocí inverzních distribučních funkcí převedeny na požadovaná rozdělení pravděpodobnosti. Blíže například Kresta (2010).

3. Reálný příklad

V této části je uveden příklad, ve kterém bude srovnán výpočet kritického objemu produkce za jistoty a za rizika. Uvažované hodnoty vstupních proměnných jsou uvedeny v tabulce 1. V případě jistoty tedy uvažujeme prodejní cenu 100 Kč a variabilní náklady na jednotku produkce ve výši 60 Kč. Ve druhém případě, tj. za rizika, vycházíme z toho, že prodejní cena má normální rozdělení pravděpodobnosti se střední hodnotou (μ_p) 100 Kč a směrodatnou odchylkou (σ_p) 10 Kč, což odpovídá situaci, kdy se cena s 95% pravděpodobností pohybuje mezi 80 Kč a 120 Kč. Obdobně uvažujeme u variabilních nákladů normální rozdělení pravděpodobnosti s parametry 60 Kč (μ_{vc}) a 5 Kč (σ_{vc}), tzn. variabilní náklady jsou s 95% pravděpodobností mezi 50 Kč a 70 Kč. Dále uvažujeme lineární nezávislost těchto dvou náhodných proměnných, tedy $\rho_{p,vc}=0$. Hodnota fixních nákladů je v obou případech rovna 1 000 000 Kč.

3.1 Bod zvratu za jistoty

Doplněním hodnot z tabulky 1 do rovnice (3) lze zjistit následující kritický objem produkce (bod zvratu) za jistoty:

$$Q = \frac{1\,000\,000 \text{ Kč}}{100 \text{ Kč} - 60 \text{ Kč}} = 25\,000. \tag{9}$$

Pokud se chceme vyhnout ztrátě, je potřeba vyrobit a prodat minimálně 25 000 ks produktu (za prodejní cenu 100 Kč a při variabilních nákladech 60 Kč).

Tabulka 1: Hodnoty vstupních proměnných

Vstupní proměnná	Za jistoty	Za rizika
Prodejní cena (*p*)	100 Kč	*N*(100Kč; 10 Kč)
Jednotkové variabilní náklady (*vc*)	60 Kč	*N*(60 Kč; 5 Kč)
Fixní náklady (*FC*)	1 000 000 Kč	

3.2 Bod zvratu za rizika

Doplněním hodnot z tabulky 1 do rovnice (8) lze na 85% hladině spolehlivosti zjistit následující kritický objem produkce (bod zvratu) za rizika:

$$Q = \frac{1\,000\,000}{\Phi^{-1}\left(1-0.85; 100-60; \sqrt{10^2+5^2-2\cdot 10\cdot 5\cdot 0}\right)} = \frac{1\,000\,000}{\Phi^{-1}\left(0{,}15; 40; 11{,}18\right)} = 35\,196. \tag{10}$$

Na základě tohoto výpočtu lze tvrdit, že s 85% pravděpodobností nebude dosaženo ztráty v případě, pokud bude vyrobeno a prodáno 35 196 kusů produktu.

Z rovnice (8) je zřejmé, že vyšší hladina spolehlivosti znamená vyšší hodnotu kritického objemu produkce. Obdobně lze analyzovat i vliv ostatních proměnných. Růst hodnoty kritického objemu produkce zapříčiní pokles μ_p, $\rho_{p,vc}$ a růst μ_{vc}, σ_p, σ_{vc}. Lze předpokládat, že nejméně přesný bude odhad lineární korelace, proto je v tabulce 2 uvedena citlivostní analýza kritického objemu produkce pro různé hladiny spolehlivosti a hodnoty lineární korelace.

Tabulka 2: Citlivostní analýza

$\alpha \setminus \rho_{p,vc}$	–1	–0,5	0	0,5	1
50 %	25 000	25 000	25 000	25 000	25 000
55 %	26 236	26 084	25 910	25 699	25 399
60 %	27 624	27 286	26 905	26 451	25 818
65 %	29 223	28 651	28 017	27 275	26 265
70 %	31 120	30 245	29 294	28 202	26 754
75 %	33 464	32 178	30 808	29 275	27 302
80 %	36 529	34 642	32 690	30 570	27 939
85 %	40 894	38 038	35 196	32 233	28 721
90 %	48 131	43 390	38 953	34 600	29 769
95 %	65 244	54 823	46 275	38 827	31 471
99 %	195 895	108 397	71 476	50 370	35 251
99,5 %	733 912	168 777	89 275	56 521	36 872

Z tabulky 2 je zřejmé, že čím vyšší kladná lineární korelace tím nižší kritický objem výroby a naopak čím vyšší záporná lineární korelace tím vyšší kritický objem výroby. Obdobně lze vidět, že kritické objemy výroby jsou vyšší pro vyšší hladiny spolehlivosti (tzn. vyšší jistotu, že nebude dosaženo ztráty). Závislost však není lineární a růst kritického objemu výroby se v koncích pravděpodobnostního rozdělení zrychluje. Konkrétní doporučení hodnoty hladiny spolehlivosti závisí na potřebách podniku, tzn. jak vysoká pravděpodobnost nedosažení ztráty je požadována. Lze však konstatovat, že při tomto typu rozhodování je vhodnější uplatnit nižší hladiny spolehlivosti, než jaké se uplatňují při regulaci finančních institucí. Jak lze vidět z tabulky 2, v případě uvažování 99% (respektive 99,5%) hladiny spolehlivosti by výsledkem byl kritický objem produkce ve výši trojnásobku kritického objemu produkce za jistoty a očekávaný zisk (průměr i medián) 1 859 tis. Kč (respektive 2 571 tis. Kč), tedy cca dvounásobek výše fixních nákladů.

4. Závěr

Analýza bodu zvratu je vhodným nástrojem krátkodobého rozhodování o objemu produkce, jež pokryje veškeré náklady související s výrobou a odbytem. V tradiční analýze předpokládáme, že všechny vstupní proměnné (variabilní náklady na jednotku, celkové fixní náklady a cena produktu) jsou známy s jistotou. Tyto proměnné však mohou být náhodné a předem neznámé. Příkladem může být podnik exportující výrobu do zahraničí, výrobce elektrické energie nebo podnik závislý na ceně vstupních surovin.

V příspěvku je prezentována analýza bodu zvratu za předpokladu rizika spojeného se vstupními veličinami. Pro modelování je v příspěvku uvažováno normální (Gaussovo) rozdělení pravděpodobnosti, což umožňuje vyjádřit kritický objem výroby analyticky. Konkrétně jsou uvažovány dvě náhodné vstupní proměnné – cena produktu, která ovlivňuje výnosy, a variabilní náklady na jednotku, které ovlivňují náklady. Fixní náklady jsou uvažovány jako nenáhodné.

Z analytického vyjádření kritického objemu produkce vyplývá, že tento bude vyšší, čím vyšší bude nejistota ohledně vstupních veličin (směrodatné odchylky) a čím nižší bude hodnota koeficientu korelace náhodných proměnných. V souladu s tradiční teorií bodu zvratu platí, že růst střední hodnoty jednotkových variabilních nákladů kritický objem produkce zvyšuje a růst střední hodnoty prodejní ceny naopak kritický objem produkce snižuje. Rovněž hladina spolehlivosti, tzn. pravděpodobnost nedosažení ztráty, ovlivňuje kritický objem produkce – čím vyšší pravděpodobnost nedosažení ztráty požadujeme, tím vyšší bude kritický objem produkce.

Poděkování

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A corporation structure draft for the selected Slovak companies

Peter Krištofík, Michal Ištok [1]

Abstract

This article is based on a simulation model comprising the data from the selected Slovak companies after which a beneficial outcome arises from engaging in onshore and offshore business. The basic criterion presenting favourability is the amount of potential tax savings calculated by contrasting the costs of creating a completely new corporation structure with the expenditures needed to set up or restructure an existing one. Provided the costs of creating a new corporation structure and the resulting tax liability are lower than the original tax liability, from the point of view of tax it is beneficial for the selected group of companies to engage in onshore and offshore business. Our simulation model is based on current legislation of selected laws in force in the Slovak Republic and abroad. Current trends in tax and bank information exchange agreements have also been considered. The prices of single elements are calculated in accordance with the reference market prices

Key words

Onshore and offshore business. Tax planning and tax optimization. Trade-off model. Ultimate beneficial owner (UBO).

JEL Classification: F23, H25, H26

Introduction

The use of tax havens has become increasingly popular among Slovak companies. In 2005, the number of Slovak companies using tax havens for a place of business was 1,510, whereas by the end of 2016 the number of these companies arose to 4,777 (Seliak, 2017). Minnick and Noga (2010, p. 708) define tax planning as the "ability to pay low amount of taxes for a long period". They consider tax planning not exactly being linked to unethical or illegal behaviour. Wilson (as cited in Armstrong et al., 2012) qualifies aggressive tax planning as the operation with taxable profit using tax planning, which may or may not be qualified as tax avoidance". The importance of incorporating the coordination of tax and non-tax factors (from the microeconomic and legal point of view), the effects of the tax rates on reducing the value of assets and international trade taxation are described by Shackelford and Shevlin (2001). They have presented a trade-off model, which contrasts the tax and non-tax benefits or costs, resulting from the specific company's choice of individual variants. They further discussed the questions concerning the design of individual researches, such as simulation model specification, data accessibility limiting and measurement errors.

Onshore and offshore business does not comprise only of a corporate structure that is an excellent fit, but also of individual flows and transactions and of creating a fit ownership structure or of a predominant or a significant impact of individual areas. The further

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influential areas in onshore and offshore business are economic substance creation, retaining the purpose of individual entities unchanged, behaving in line with the scenarios prepared in advance and lastly, a regular revision of corporate structures.

Tax liability planning for the selected Slovak companies

According to the database Cribis, (filtered on 03/15/2016) there are 971 Slovak companies which transact business in the NACE Information and Communication Services Sector and their EBIT is 40,000 EUR[2] and higher. Therefore, our expected hypothesis for the simulation modelling is following: We assume that a suitably drafted onshore and offshore business corporate structures enable the selected Slovak companies in the category of their annual gross earnings of minimum 40,000 EUR and engaging themselves in international business, to plan their tax burden in terms of corporate income tax. From all the NACE sectors, there are 18,537 companies residing in Slovakia, which fulfill the EBIT condition.

Modelling conditions. There is a Slovak company with the name "ABC, Ltd.", with a limited liability and one shareholder (owner), set up in 2012. The company operates in the Information and Communication Technologies NACE Sector[3] (programming, web sites creating and server management, etc.). The company has recorded a high on-year increase in turnover, especially despite its short operation. Moreover, the company reached the EBIT set to 40,000 EUR. This company offers its services mostly to clients from the EU member states (a foreign element) and employs less than 10 employees[4]. When taking into account the substantial legal and tax instability in Slovakia, the company's predetermined objectives are to eliminate legal insecurity, to improve the image (by cooperation with foreign countries or by for successful sales), to facilitate flexible ownership relationships (i.e. flexible transactions and flows management) and to provide the possibility of anonymity of the owner, to create a gateway for possible foreign investors and finally to reinvest the possible tax savings into business environment. Individual modelling levels or steps and also the restrictions stemming primarily from Slovak and international legislation, and therefore it is possible to generalise our conclusions to the selected group of companies and corporations.

Modelling. Current trends in tax and bank information exchange agreements have the effect of a continual avoidance of direct capital ties among Slovak enterprises and foreign companies via capital shares, mainly as a result of adhering to the arm's length principle. It is often necessary to maintain transfer prices documentation, which determines the limiting conditions. Considering the contemporary development and the inevitable need of transfer pricing documentation, in our simulation model we will work with a condition that the Slovak

[2] Testing limit of EUR 40,000 has been chosen as the average mean of the limits listed by the selected corporate service providers (in Slovakia and the Czech Republic) on their websites.

[3] This business sector is chosen for several reasons: According to corporate service providers, their customers from this industry comprised a significant part of their portfolio in 2014, it is a dynamically growing industry with high potential and growth in the future and by comparison of the databases Cribis and BISNODE, we have seen basic conditions being fulfilled by nearly a thousand Slovak companies (eight hundred in the case of purification of businesses that have an owner in one of the selected onshore and offshore jurisdictions).

[4] If we use the criterion of number of employees, the number of Slovak companies which fulfil the following criteria in response to the filtering criteria is 608. The number of companies is for the whole NACE sector *Information and Communication.* If we contemplated only the number in NACE sector 61 (computer programming, consultancy and related activities), NACE sector 62 (information services) and NACE sector 63 (telecommunications), this number would be 511.

company ABC Ltd. should not have any capital ties to another foreign company (no holding structure will be created).

Based on the established criteria, we pinpoint a possible solution by the creating new companies in the Czech Republic ("New CZ Ltd."), in the USA ("New US, LLC"), an offshore company ("OFFSHORE") and a new company in Slovakia ("New SK Ltd."). Since we use a trade-off model in the simulation, it should be clear that we compare the possible tax savings with the costs of creating a corporate structure[5]. It is necessary to calculate the costs in relation to the setup or start up process and constant improvement of its functioning. We do not aim for the lowest price offers in our model, because these offers often involve hidden costs, (e.g. consulting, transactions regulation or paperwork costs). For the simulation, we base on the assumption that we work either with a small or a medium-sized company, and therefore a corporate service provider is unavoidable; there is a need to consult a specialist, due to their specific knowledge of a sophisticated organisation of the flows to minimise the risk factors, e.g. formation of a permanent establishment in the Czech Republic or deductible expense invoices from the new Czech company New CZ Ltd.

The costs of creating a new company in the Czech Republic is illustrated at the level of 800 EUR[6] and at the same time we use the nominee services[7] (so the ultimate beneficial owner does not figure as a formal owner of the company). The minimal price for these services on the market represents 500 EUR. The founding costs for the American limited liability company owned by the OFFSHORE company are often offered as a package of both companies (expected to be bought at once). We estimate the costs to 3,000 EUR. In Slovakia, the cost of funding a company will be at the same level as in the Czech Republic. But in New SK Ltd., the owner figures as the co-owner and therefore we do not need the nominee service.

Besides the costs for individual companies (5,100 EUR), it is necessary to allocate approximately 1,500 EUR to the services regarding Service Agreement[8]. The direct costs of creating a corporate structure are approx. 6,600 EUR.

New CZ Ltd. is going to bill services for the mediation of orders[9], support in computer programming area, for server space, data processing, web hosting services, consultations, computer diagnostics and similar services. The billing between New CZ Ltd. and the existing Slovak company ABC Ltd., is going to be VAT excluded but with a reverse charge and the liability of VAT self-assessment (eligible for tax deduction). Cost invoices from the Czech Republic should not be evaluated as suspicious and should not be questioned by Slovak tax authorities (the transactions are neither from an offshore jurisdiction nor from related entities),

[5] Although there are no direct capital ties, given individual - entrepreneur takes control of individual transactions and companies.

[6] Prices are excluding VAT.

[7] E.g. a Cypriot, who does not figure in more than 50 companies. A person who has influence over a given structure may refer the nominee person (owner and managing director) at any time withdraw for example using the undated resignation of manager, or other instruments like transfer of shares. Nominee services are legal services, for example in the Czech Republic in accordance with the Act. 89/2012 Coll., the Civil Code and the Law No. 90/2012 Coll., on business corporations.

[8] This is a legal document by which New CZ, Ltd. can provide the services designated by the applicable service contract, namely for itself and on behalf of New US LLC, with whom a given service contract is being concluded. This agreement is an innominate, that unnamed, so the law does not determine its essential terms. In general, this type of contracts subject to the rules that are applicable to the most similar contracts, in our case it will be a commission contract.

[9] The demonstration is shown in figure.1 It is necessary to ensure economic substance (individual contractual relations and transactions are created solely for the purpose of reducing tax liability).

but the tax office often does not allow to be comprised in the tax-deductible expenses[10] and in some cases scope forward fines and penalties arise as well. The New CZ Ltd. should not experience any pressures arising from the Czech tax administrator, because the company is going to reflect relatively mild economic activity in its home country and at the same time pay taxes to the state budget.

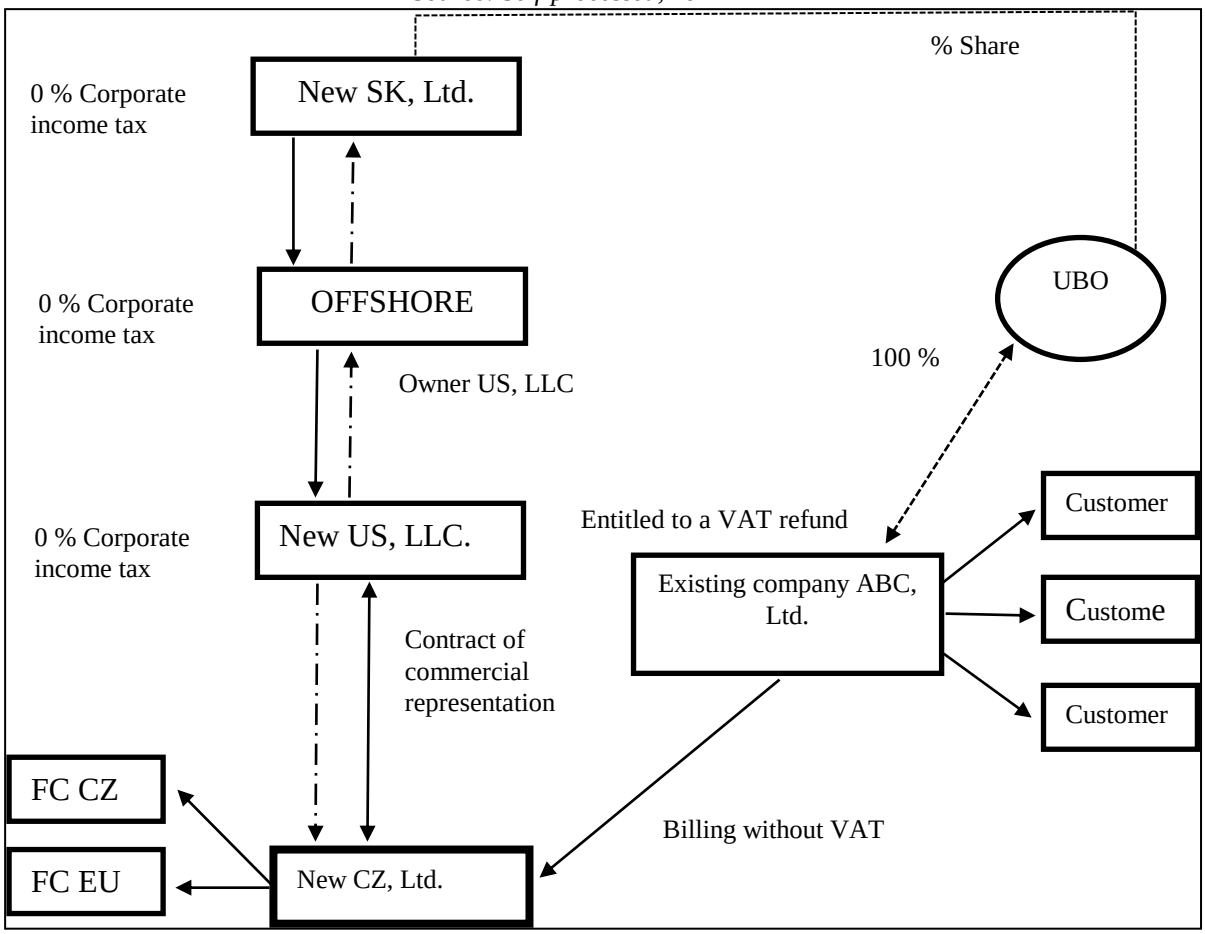
The Service Agreement serves as a framework agreement for the provision of services between the Czech company New CZ Ltd. and a foreign company New US LLC. The services will be stated in the Agreement provided by New CZ Ltd. in the account of a foreign company (Contract of commercial representation). It is a very sophisticated legal basis of the relationship between the Czech and American companies and at the same time also a proof of the existence of that relationship[11]. The opportunity to bill only one summary invoice for the selected period and the fact that there is no need to bill an invoice for every single service, which is an advantage (the American company will bill the sale of its IT solutions to the Czech company and simultaneously pay commissions for sales). A high level of anonymity is provided to the American company and its owners (OFFSHORE) and the detection of the real owner is highly unlikely[12].

[10] Slovakia has no signed double tax agreements with the most common offshore jurisdictions, such as the Seychelles, Panama, Gibraltar, British Virgin Islands and Belize on avoidance of double taxation. This means that in the case of invoices received from companies located in those jurisdictions is a Slovak company required to transfer withholding taxes (in some cases up to 35 %). From 1.7.2014 Slovak transactions of financial institutions conducted in the United States are not subject to withholding tax of 30 %.

[11] Ensure greater legal certainty when necessary advocacy transactions before the tax authorities.

[12] A publicly inaccessible register of shareholders and managers (one of several reasons for the high ranking of the US in Financial Secrecy Index ranking), (Tax Justice Network, 2015).

Figure 1: Corporate model of ABC. Ltd.
Source: Self-processed, 2017



Regarding the legal form abbreviated as LLC, or limited liability company, in case of the American company New US Ltd., the advantage of the limited liability to taxation at the level of business associates. If the companies within the legal form of limited liability are established by Non-United States Residents, who do not perform business operations in the US Territories and do not earn profits from American resources, they are not obliged to submit Tax Returns Statement and to pay the American income tax. The income at the level of the OFFSHORE company (only one annual government fee) will be also tax-free. If the company was paying dividends to a Slovak individual, a real threat of the liability to pay 14% for health insurance would appear. On behalf of this factor, we will use the new Slovak company[13]. This one will accept the profit sharing from the OFFSHORE company (New SK Ltd. is not subject to health insurance and taxes). These funds can be further re-invested into the business for different purposes at the discretion of an individual[14]. Funds re-investment appears to be a rational decision, particularly when considered that the bank secrecy merely exists.

After the corporate structure setting and the individual transactions, it is vital to set the amount of these transactions. In general, the Slovak company ABC, ltd. is limited by the

[13] A Slovak company New SK Ltd. establishes an offshore company, and will have its share of the business (e.g. investment in real estate). The connection between the US company New US, LLC. and offshore companies will be difficult to identify.

[14] Type of business or the number of owners of New SK, Ltd. is at the discretion of the owner of a Slovak ABC Company, Ltd. and in practice it depends on the individual situation.

Service Agreement in the transactions at the level between the Czech and American companies (some proportion is going to be taxed in the Czech Republic and the rest in the USA) and at the level between the existing Slovak company ABC Ltd. by the chosen level of aggressivity, i.e. what proportion of the gross profit will be shifted to other jurisdictions.

Contrast between the potential tax savings and the costs.[15] If the Slovak company is rather aggressive and only 5,000 EUR is subject to taxation (which is 12,5% of EBIT)[16], it will pay the tax in the amount of 1,100 EUR. The remaining 35,000 EUR will be shifted to the Czech Republic. The existing Slovak company will not be aggressive and will keep the ratio, fixed in the Service Agreement and the review (in our model case the ratio is 90:10[17]). In other words, 3,500 EUR will be taxed at 19 % rate, so the paid tax liability will represent 655 Eur. At the level of the New US LLC. and OFFSHORE company, a 0% tax rate will be applied. Even the capital transfer to New SK Ltd. will be tax-free.

The total costs[18] will be comprised of: the costs of creating a new corporate structure and the fees needed for the Service Agreement draw up (6,600 EUR[19]) and of the tax liability paid in Slovakia and the Czech Republic (1,765 EUR). The total cost represents the value of 8,365 EUR. If the existing Slovak company would tax 40,000 EUR only in Slovakia, the tax would represent 8,800 EUR[20] being transferred to the state budget. The costs are calculated as total sum in the first year of the structure's existence and for the individual transactions separately. In the following years, the costs will fall to the level of approximately 5,000 EUR[21]. The outcome of the tax savings is in comparison to the costs positive not only in the first but also in the upcoming years. It is important to consider also the fact that the existing Slovak company ABC taxes only 12.5% of the gross profit. If the transactions would be set less aggressively[22], we can state that the EBIT limit of 40,000 EUR is the imaginary break-even point[23], when it is relatively beneficial for the Slovak companies fulfilling criteria for engaging in onshore and offshore business from the point of view of the overall tax savings in comparison to the cost of a corporate structure. This imaginary break-even point is evaluated and calculated owing to the possible tax savings, therefore since

[15] The model takes only simplified terms into account, so only transactions are shown in figure no. 1. Calculation of potential taxes, respectively the potential tax savings is for 2015.

[16] American LLC company has allowed managers to be also non-residents (or a legal person) and has no obligation to audit financial statements (Ištok, 2016).

[17] The most common ratio in practice is 90:10, respectively 95:5.

[18] We compare the two alternatives, the total costs when a new corporate structure is or is not going to be created.

[19] These are only direct costs associated with the construction a new corporate structure, they do not include the costs of the additional business that shall be performed due to expectations. The prices include costs associated with opening of bank accounts for individual companies.

[20] We assume 22 % corporate income tax rate in Slovakia.

[21] The costs are still relatively higher, but there is some reserve for the case the corporate service providers would need to be consulted.

[22] The difference in 8,800 EUR (taxable only in Slovakia) minus 8,365 EUR (costs associated with the modelled simulation of a corporate structure) is of EUR 435. This means that to reach the potential break-even point in our case under ceteris paribus conditions in Slovakia, additional income can be taxed up to 6,977.27 EUR.

[23] In this article, we use in addition to an imaginary break-even point also the concept of "imaginary break-even point". The attribute imaginary used because that is the tipping point for each different subset of firm for each undertaking because of the individual characteristics of the company or the selected setting of a corporate structure. If a Slovak company uses the setting example only one offshore jurisdiction and the associated costs are significantly lower, but the tax savings potential remains questionable.

accomplishing this financial criterion (for the given NACE sector and the companies, which fulfill the conditions set in the simulation model) it is relatively advantageous to take interest in a corporate structure setting under the conditions of the recently accepted tax and bank information exchange agreements. This imaginary break-even point does not have to be valid for Slovak companies pursuing other indicators[24] than the possible tax savings when engaging in onshore and offshore business. Considering the amount of costs related to creating and adjusting of the given corporate structure[25], we further assume that the amount of 40,000 EUR illustrating the limit of the financial EBIT criterion is circa the break-even point for other Slovak companies pursuing business in other NACE Sectors.

It is again necessary to emphasize the importance of cost calculations related to setup of a corporate structure. If we had used lower market price quotations in the simulation model, we would have been able to reduce the costs under the level of approximately 5,000 EUR, which would have created a further 1,600 EUR in tax savings. However, these prices often do not include any specialist consultations and the risk of a completely non-functional corporate structure or a structure not achieving the planned and calculated tax savings therefore rises.

The results of the simulation enabled us to confirm the investigated hypothesis, where we had assumed that with a suitable organisation of the onshore and offshore businesses, the Slovak companies involved in international business and achieving at least 40,000 EUR of annual gross profit would able to plan their tax burden from the point of view of corporate income tax[26]. The model situation enables a rather flexible reaction to possible legislative changes in the corporate income tax area with the size of shifted costs or profits. The company can then re-invest the saved funds back into its own business. Corporate structures are not only created for reducing tax liability, which can later function for personal use of the UBO[27], but also because of a continual growth of business activities, in this case even in more jurisdictions.

Deciding to create a new corporate structure consists of decision-making at various levels.

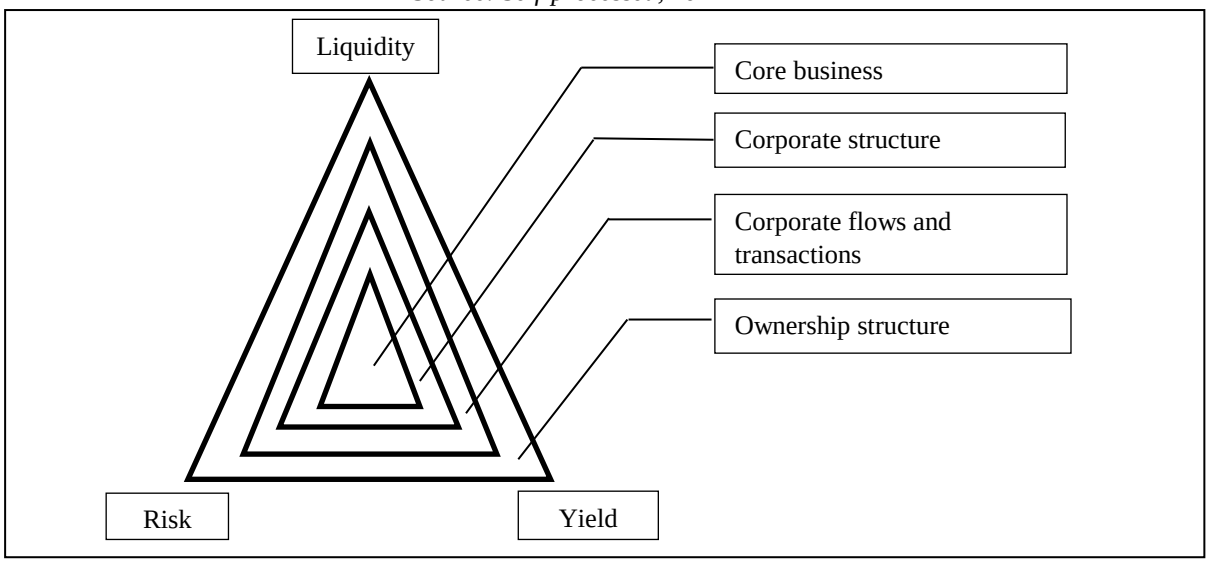
[24] Other incentives for involvement in onshore and offshore business are anonymity of ownership, flexible arrangement of ownership structures, asset management and other motives (e.g. establishment of new branches or business regulation). Other designs and their benefits are difficult to measure due to the fact that it is often a subjective assessment or very particular cases.

[25] Given the current trends in tax and bank information exchange agreements. As shown in our research, in the context of tightening the conditions in international tax planning, gradually starts to increase the costs of onshore and offshore business. Thus, the upward pressure on the cost (e.g. the necessary complementing supplementary elements or levels) will affect height of the break-point itself as well.

[26] The estimated corporate structure price is 6,600 EUR and the tax savings are at the level of 7,035 EUR (8,800 EUR – 1,765 EUR).

[27] According to the definition of the AML/CTF Act (Anti-Money Laundering and Counter-Terrorism Financing Act), the UBO is the person who is the final recipient of benefits (income and benefits), which proceed from holding property or control (FATF and APG, 2015).

Figure 2: Decision-making in the onshore and offshore business at different levels
Source: Self-processed, 2017



The decision-making in onshore and offshore businesses is at each level linked to thinking considering the interrelated factors, similarly to the magical investment triangle - risk, yield and liquidity. As shown in figure 2, onshore and offshore businesses do not consist only of the corporate structure setting.

Summary

To meet the conditions of modeling, we have calculated that the break-even point for Slovak companies operating in the NACE sector of information and communication services oscillates close to 40,000 EURO (financial indicator EBIT). The proposed corporate structure model for a selected group of Slovak companies includes shifting of taxable income to jurisdictions where there is a lower, respectively no taxable profit. The setting onshore and offshore businesses brings out the tax benefits and benefits in a flexible arrangement of ownership structures and achievement the anonymity of the ultimate beneficial owner. Although an important prerequisite for better functionality and real use of the model in practice is strict adherence to the different levels and flows of the proposed solutions, some intermediate elements (mainly in the distribution of the accumulated funds for the selected natural or legal person) and their relationships can vary according to individual requirements of the real owner. An important determinant in the use of such models (or modified) in practice is the type of business and its specific features. The market prices of each of the individual elements are known in advance, as well as various legal provisions in force in selected jurisdictions. The combination of these two basic areas gives suitable conditions for using onshore and offshore business in international tax planning and international tax optimization.

The decision-making process of the setup of a corporate structure and of the choice of individual elements and their combinations (deciding for one out of many alternatives) is most often affected by two basic areas - the price and the image of the structure (known also under the term performance). In most cases, more than one jurisdiction is suitable for a certain purpose and the choice then depends on the requested image, that the ultimate beneficial owner awaits. It is the stepping stone for the price. Corporate service providers consider the price and image ratio to be the essential criterion in the choice of a corporate structure

of onshore and offshore businesses. Only by various combinations of this ratio can the owners choose the best alternative for them. Another significant decision-making indicator is the choice of a corporate structure because of the price and the degree of anonymity, some owners even compare the ratio of the price versus legal stability offered in the selected jurisdiction.

Acknowledgment

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Application the Binomial Distribution, Hypergeometric Distribution and Poisson Probability Distribution in Accounting

Zuzana Kubaščíková [1]

Abstract

This paper presents the genetic formula for computing the mean and standard deviation of any type of probability distribution, as well as introduced three widely applicable discrete probability distributions: the binomial, the Poisson, and the hypergeometric distributions. The three types of probability distributions presented in this paper are of great relevance in auditing and forensic accounting. A forensic accountant has to sift through large amounts of financial data and is required to make probabilistic inferences. At times, it may not only be cumbersome but impossible to examine each data element. In such situations, a subset of the data is examined, such as in the case of multi-location inventory audits. When making inferences based on the properties of a specified probability distribution it is imperative to assess the underlying probability distribution correctly. Employing an incorrectly specified probability distribution would lead to fallacious inferences. For an accountant and auditor it is perhaps more important to know the applicability of various probability distributions that the computational mechanics associated with each other.

Key words

audit, sample size, binomial distribution, Poisson probability distribution, hypergeometric distribution

JEL Classification: C1

1. Introduction

The purpose of auditing – internal or external, is to identify failures in the system. It is impossible to audit every document, record or process in a company. (Juhászová, Užík, 2015) Usually we need to make a decision and select the sample that will be observed in a detail. Selection the sample is important to minimize the bias, the representative of the population must be sufficient to detect abnormalities if they exist. (Tumpach, 2009) There are several different strategies that can help to meet these objectives. The most common method is the Poisson distribution, but it is used also binomial and hypergeometric distribution. (Krišková, Užík, 2016)

2. Methodology and Data

Most auditing sampling plans are based on either the binomial or Poisson distribution, rarely it is used also hypergeometric distribution. (Pakšiová, 2014) This paper will compare show how the binomial, Poisson and hypergeometric distribution relate, a review and comparison of different sampling strategies will be presented.

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The binomial distribution is used when the outcome of an event has only two possible outcomes. For auditing this is either compliant or noncompliant. Typically one can use the binomial to calculate the probability of finding a set number of noncompliant findings during an audit. (Ondrušová, 2011) The formula for the binomial distribution is in Figure 1 where p is the percent noncompliant in the sample of size n. The main advantage of the binomial distribution is that it is sample size dependent.

Figure 1 Binomial distribution formula

$$\alpha = \binom{n}{x} \cdot p^{x} \cdot (1\text{-}p)^{(n-x)}$$

For computations related to binomial distribution in Excel, we can use the BINOMDIST worksheet function. The result is the probability value of fraud or mistake. The function is presented as BINOMIDIST (X, sample size, cumulative). X is the number of frauds or mistakes that is, the specific value of the random variable X, in these illustrations presented as P(X = x). „Sample size“ is self-evident. R s the probability of the event of fraud or mistake. „Cumulative“ is a „True“ or „False“ value.

The Poisson distribution describes the number of times an event occurs in a finite observation space. The Poisson distribution is defined by one parameter: lambda. This parameter equals the mean and variance. (Parajka, 2015) Using a Poisson distribution, a confidence interval can be calculated using the following one-sided upper interval:

Figure 2 Poisson distribution formula

$$\lambda \leq \frac{1}{2}\chi^{2}_{0.05;2(x+1)}$$

The main drawback of using the Poisson distribution is that it usually requires a large sample size to estimate the mean. Poisson distribution can be calculated in Excel with worksheet function to compute Poisson probabilities - function POISSON (X, lambda, cumulative), where X is the number of frauds or mistakes. Lambda, is the only parameter required to specify the Poisson distribution. Recall treat is both the mean and variance of the Poisson distribution over unit time. “Cumulative” is a “True” or “False! Value.

The hypergeometric distribution is used to calculate probabilities when sampling without replacement. Hypergeometric distribution differs from the binomial in that the population is final and the selection from the population is without repetition. Experiments are not independent. For binomial distribution, the likelihood of success is constant, whereas in the hypergeometric distribution, the probability of success changes after each experiment. (Baesens, Vlasselaer, Verbeke, 2015)

Figure 3 Hypergeometric distribution formula

$$p = \frac{({}_{k}C_{x})({}_{(N-k)}C_{(n-x)})}{{}_{N}C_{n}}$$

Hypergeometric distribution we can calculate in Excel with function as HYPGEOMDIST (X - sample size, A – population size). X is the number of frauds or mistakes, that is the specific value of the random variable X. Sample size is self-evident and in our formulation it was dented by “n”. A is the number of frauds or mistakes in the population.

Population size	Tolerable error rate	Expected population error rate	Confidence level
500	5%	1%	95%
500	10%	1%	90%
500	10%	5%	90%
5000	5%	1%	95%
5000	10%	1%	90%
10000	5%	1%	95%
15000	5%	1%	95%
35000	5%	1%	95%

For calculation sample sizes were used excel functions – BINOMDIST, POISSON, HYPGEOMDIST,

3. Results

Using Excel there were calculated these results:

Table 1: Calculated sample size

Population size	Tolerable error rate	Expected population error rate	Confidence level	Poisson distribution	Binomial distribution	Hyper-geometric distribution
500	5%	1%	95%	73	75	78
500	10%	1%	90%	163	152	195
500	10%	5%	90%	214	191	259
5000	5%	1%	95%	58	59	59
5000	10%	1%	90%	77	77	76
10000	5%	1%	95%	92	94	95
15000	5%	1%	95%	76	76	76
35000	5%	1%	95%	154	154	154

Using either the binomial distribution or Poisson distribution will give comparable confidence intervals for large sample sizes. For smaller sample sizes, the binomial gives tighter intervals. Poisson distribution is not population size (or sample size) dependent. Poisson distribution will allow a better acceptance range for an audit where the population size might not be known exactly.

The comparison between the binomial and hypergeometric distributions shows that there is a notable difference in the sample size when the population is small (e.g. 500), however, when the population size is large (i.e. > 5,000) there is minimal or no difference between the binomial and hypergeometric sample sizes.

The main advantage of binomial distribution is that the computation is mathematically simpler which enables comprehensive tables to be prepared. The binomial distribution is used for sampling large populations (i.e. > 5,000).

The hypergeometric distribution can be used to sample both small and large populations, however, the computation is complex as the population size gets larger. Using Excel or audit sampling software would be required to calculate the sample size.

4. Conclusion

This paper is focused on statistical distributions to show differences between three most common approaches. The Poisson distribution calculates the probability of having a finding in highly compliant systems where the actual number of errors on the system is a low number. More compliant systems require larger sample sizes since the likelihood of finding an error is smaller.

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Offshore Financial Centres, Tax Havens and Location of Banks' Claims

Jana Kubicová[1]

Abstract

This paper aims are twofold. First, the aim is to examine whether OFCs are identical countries as countries that are labelled as tax havens. Second, the aim is to learn, whether countries that are both, OFCs and tax havens attract more bank sector claims. Section two describes the process of securitisation of banks' assets and potential role of OFCs and tax havens in this process. Section three makes empirical research about the relationship between the location of banks' assets and OFCs and tax havens. It describes employed data and methodology and provides results. Finally, conclusion summarises research results.

Key words

tax haven, offshore financial centre, securitisation, bank sector claims

JEL Classification: F2, H2, Y1

1. Introduction

Commercial banks are profit oriented business corporations. One of the most important source of banks' profits is interest received from bank loans sold to debtors. To provide loans, commercial bank needs to raise funds. For decades the most usual way how to generate commercial banks' funds was collection of deposits from individuals, companies and other entities. However, regulation measures applied by central banks, among them a required proportion of reserves commercial banks must keep for financial safety reasons, have had profound impact on the portion of capital that commercial banks can use in their active operations.

To raise banks' financial capacity to sell more loans, commercial banks adopted a new innovative way: securitisation of banks' assets and financial innovations. The last financial crisis differs from previous crises as securitisation of banks' assets and channelling played innovative tactic to expand commercial banks' capital (Altunbas, Gambacorta, & Marqués, 2007), (Gambacorta & Marquers-Ibanez, 2011). The role of financial innovations, securitisation and derivatives in the outburst of the 2008 financial crisis was described in numerous works, among them by Somanathan and Nageswaran (Somanathan, a iní, 2015), Gambacorta, and Marquers-Ibanez (Gambacorta & Marquers-Ibanez, 2011), Drezner and McNamara (Drezner & McNamara, 2013).

From 1985 to 2008, banks reached a significant degree of interconnection, which is referred to as „too-interconnected-to-fail" or too-big-to-fail (Haldane, 2011). Haldan points out that the largest points of interconnection are located in the following states: USA, UK, Cayman Islands, Netherlands, Luxembourg, Ireland, and Switzerland. Shaxon adds that countries that act as nodes in a network of interconnected banks in the world are at the same

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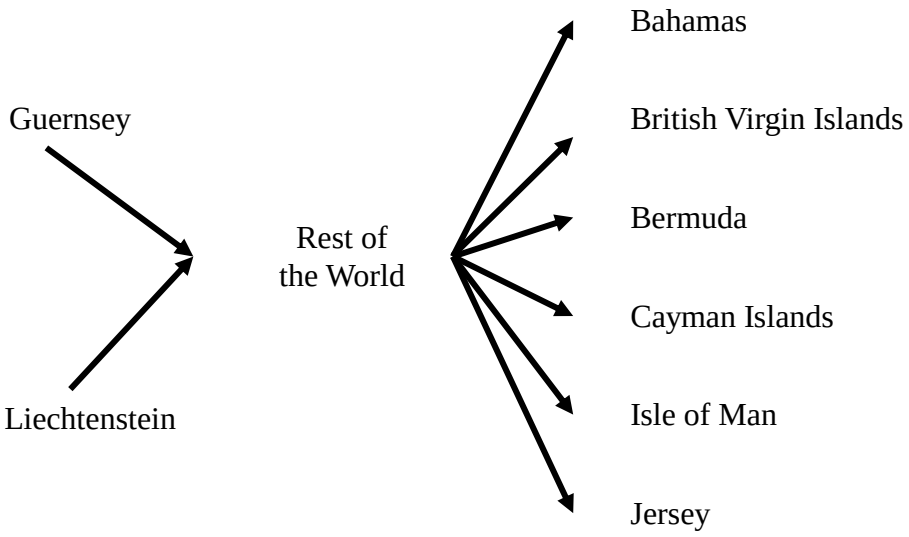
time countries considered to be the most important tax havens (Shaxson, 2017). Figure 1 illustrates relationship between rest of the world and tax havens.

Figure 1 Offshore Financial Centres: Pass-through of Funds



Source: *(Thomson Reuters Lipper, 2010)* and *(IMF, 2010, p.18*)

By observation, OFCs have some common characteristics: they are often tiny countries, with a land area just a few square kilometres, with population counting just several hundred or thousand people. OFCs are often islands located hundred kilometres from the mainland. OFCs are often former colonies, some of them are now sovereign states fully independent from their former mother states. On the other hand, many other OFCs are not independent states, to the contrast, they still remain under administration of the other sovereign state. Thus their legal status is kind of dependent territory – the exact legal form varies. Frequently countries labelled as OFCs and tax havens are not sovereign states but they have close administrative tights with some of the most developed countries or global players, e.g. with the United Kingdom, the USA, China, Netherland, New Zealand, and Australia.

This paper aims are twofold. First, the aim is to examine whether OFCs are identical countries as countries that are labelled as tax havens. Second, the aim is to learn, whether countries that are both, OFCs and tax havens attract more bank sector claims. Section two describes the process of securitisation of banks' assets and potential role of OFCs and tax havens in this process. Section three makes empirical research about the relationship between the location of banks' assets and OFCs and tax havens. It describes employed data and methodology and provides results. Finally, conclusion summarises research results.

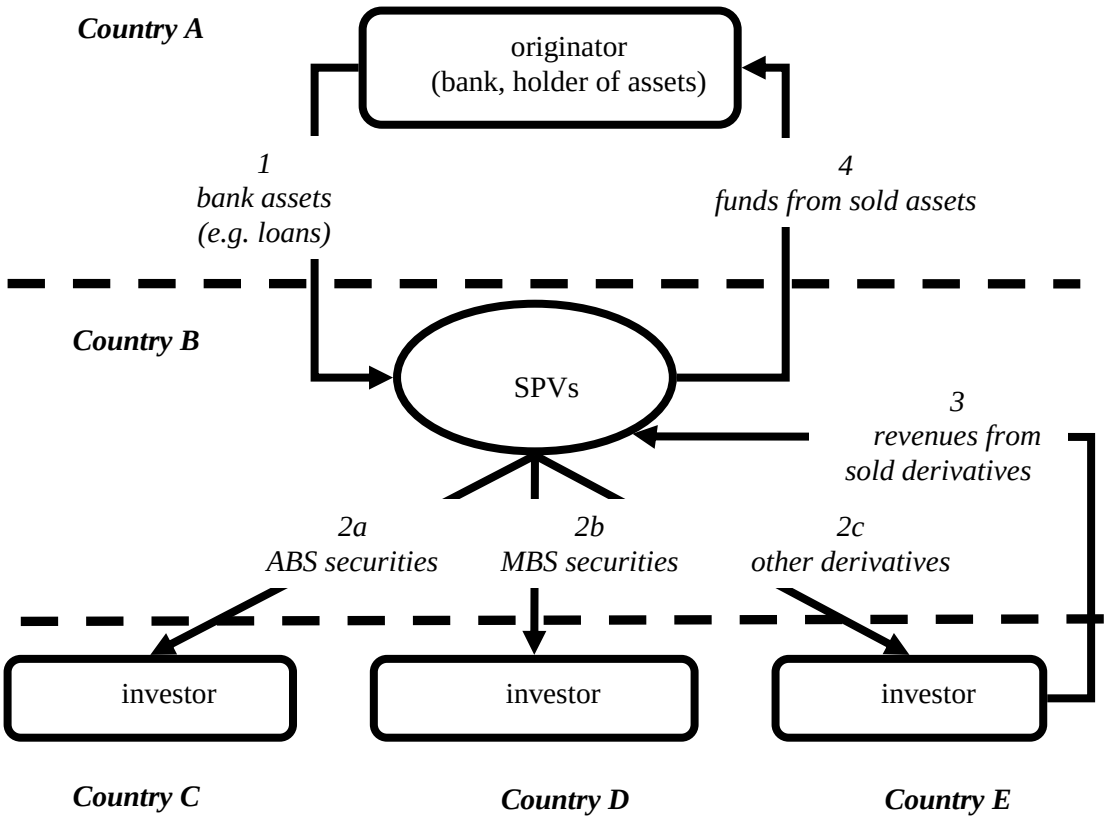
2. The Role of Tax Havens in the Securitisation of Banks' Assets

Securitisation of bank assets involves numerous parties and financial transactions/payments, and proceeds through several steps.

Figure 2 depicts parties and transactions in the process of securitisation of banks' assets, where 1 represents selling of bank assets to special purpose vehicles (or, SPVs); 2 shows selling of financial derivatives, e.g. asset backed securities (or, ABS) or mortgage backed securities (or, MBS) packed in tranches with different ratings to investors; 3 illustrates

payments investors pay to SPVs for received financial derivatives; 4 represents receivables commercial bank gets back from SPVs as compensation assets sold to them. The process of securitisation is explained in the financial literature, for reference see for example Fabozzi (Fabozzi & Kothari, 2007).

Figure 2 Securitization of bank's assets



Legend: SPVs states for special purpose vehicles; ABS states for asset based securities; MBS states for mortgage based securities.
Source: author's own work

To comprehend better the process of securitisation of banks' assets the understanding of the commercial bank's balance sheet is a key (see

Table 1). By selling bad loans (assets) to the third parties bank gets clear of bad loans. In turn for sold assets it receives compensations (liabilities) from thirde parties, often SPVs. Received funds (liabilities) then can be used in active operations, e.g. in selling new loans (assets). This cycle creates opportunity to escalate banks' business and produce more profits.

Table 1 Commercial Bank Balance Sheet

Assets	Liabilities
Securities in bank credit	Large time deposits
Loans	Other deposits
Commercial and industrial loans	Borrowings
Real estate loans	Borrowings from U.S. banks
Consumer loans	Borrowings from others
All other loans and leases	Trading liabilities
(Allowance for loan and lease	Net due to related foreign offices

losses)	
Fed funds	Other liabilities
Loans to commercial banks	
Cash assets	
Trading assets	
Other assets	Capital
Total assets	Total liabilities and capital

Source: (Board of Governors of FRS, 2017)

Several parties are involved in the process of securitisation of banks' assets, the most important are: originator, SPVs, and investors (see

Figure 2). All of them enter securitisation process supposing that overall benefits of the securitisation will considerably exceed its transaction costs. The assumption of low costs involves also assumption of the low *tax* costs. Parties to the securitisation assume that securitisation will be either low-tax cost or tax neutral process.

To make securitisation of banks' assets tax neutral, parties involved in securitisation are frequently domiciled in the offshore financial centres (or, OFCs). OFCs offer tax regimes that make tax costs of financial transactions acceptable for all involved parties. Characteristics of OFCs include: 1. low or no taxes on business or investment income; 2. no withholding taxes; 3. light and flexible incorporation and licensing regimes; 4. incorporation and licensing regimes; 4. light and flexible supervisory regimes; 5. flexible use of trusts and other special corporate vehicles; 6. no need for financial institutions and/or corporate structures to have a physical presence; 7. an inappropriately high level of client confidentiality based on impenetrable secrecy laws; and 8. unavailability of similar incentives to residents (Crockett, 2000, p.9). OFCs often provide favourable tax regimes which safeguard tax neutrality of securitisation. Favourable tax regimes offered by OFCs make them *both, offshore financial centres and tax havens at the same time*.

By being both, offshore financial centres and tax havens, these countries attract multinational companies (or, MNCs), including financial institutions who establish their headquarters, subsidiaries or branches in these countries. According Phillips, most of America's largest corporations maintain subsidiaries in offshore tax havens. He writes, that "Fortune 500 companies are holding nearly $2,5 trillion in accumulated profits offshore for tax purposes. Just 30 Fortune 500 companies account for 66 percent or $1.65 trillion of these offshore profits. "(Phillips, et al., 2016).

3. The Relationship between the Location of Banks' Assets and OFCs and Tax havens

3.1 Data and Methodology

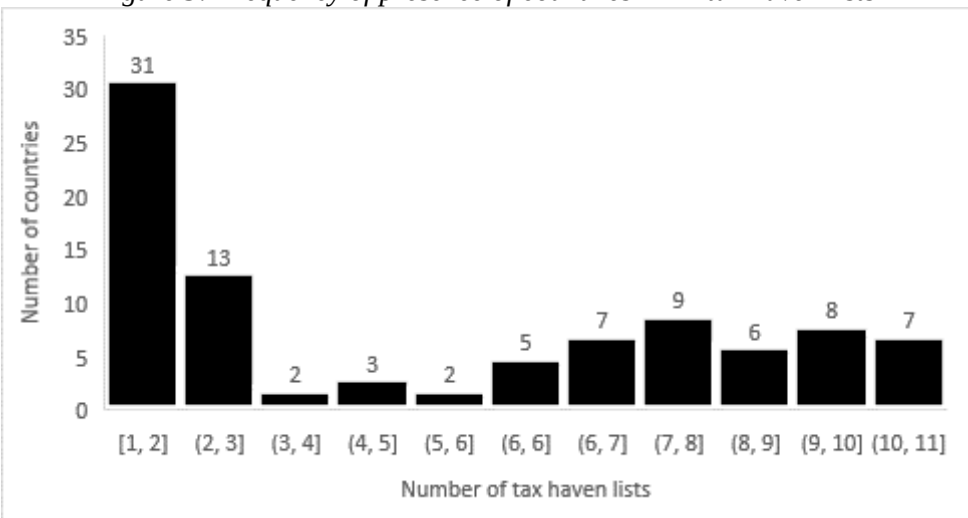
The aim of this research is, first, to examine, whether OFCs coincides with countries labelled as tax havens. Second, the aim is to learn, whether jurisdictions which possess both characteristics, i.e. they are both, OFCs and tax havens, attract more financial assets. To accomplish the research tasks, a *pivot table* is employed. The choice of the pivot table is supported by the fact, that it enables researcher to summarise in one table several data and by doing so to show several dimensions of measured items at the same time. To build a pivot table which will facilitate us in answering aforementioned research questions, a data described below are employed.

Cross-border positions, by residence and sector of counterparty, bank sector claims. To indicate location of banks' assets data on the cross-border positions, by residence and sector of counterparty were chosen. Data for 187 countries were retrieved from the Bank for International Settlements Database. However, in the Table 2 are shown just first 34 countries with the highest bank sector claims in the world.

Financial secrecy index. To indicate whether certain country is an offshore financial centre, it is used financial secrecy score (or, FSS) published by the Tax Justice Network (Tax Justice Network, 2015). The calculation of the overall FSS is based on fifteen key financial secrecy sub-indicators (or, KFSIs). For full list of them consult Tax Justice Network (2015, p.11). The method of calculation of the overall FSS is "by adding the values of each of the assessed KFSIs and divide the sum by the number of assessed KFSIs, expressing the resulting value (between 0 and 1) as a percentage score (0% to 100%). As a consequence, a jurisdiction can always achieve a maximum value of 0% secrecy (equivalent to 100% transparency)." (Tax Justice Network, 2015, p.9). The higher the FSS is, the more intensive financial secrecy certain country provides.

Lists of tax havens. To indicate, whether certain country is tax haven, it is used the list of tax havens prepared by Murphy (Murphy, 2009). He uses 11 lists of tax havens which were developed and published by various authors. Murphy prepared a table which shows 91 countries in rows. In its 12 columns then it is indicated whether country appears on the concrete tax haven list. Thus, certain country may appear at most on the 11 tax haven lists. Entire table is published by Murphy (Murphy, 2009, p.2).

Figure 3: Frequency of presence of countries in 11 tax haven lists



Source: Data are retrieved from (Murphy, 2009). Histogram is author's own.

3.2 Results

To combine all data together, a pivot table is employed. Countries are shown in rows, the name of each country is accompanied with the data on FSS. The title of each column of the table displays number that indicates how many times the country appears on the tax haven lists. Finally, for each country there is shown a bank sector' claims. Results are shown in the Table 2.

Table 2 Cross-border positions, by residence and sector of counterparty: Bank sector claims[1]

Country	**FSS**[2]	**Position of a Country on 11 Tax Haven Lists**[3]											
		11	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**
United Kingdom	40,8									3891			

United States	60,0										3182		
Germany	56,4											1383	
China, P.R.: Hong Kong	72,0			**1151**									
France	42,5											1081	
Japan	57,5												834
Luxembourg	55,1				**800**								
China, P.R.: Mainland	54,3												674
Switzerland	72,6			**624**									
Netherlands	48,0										588		
Singapore	69,0			**585**									
Ireland	40,4				**511**								
Belgium	40,9											420	
Canada	45,8												290
Italy	35,0											259	
Australia	43,5												252
Sweden	36,0												239
Saudi Arabia	61,1												196
Spain	33,0											167	
United Arab Emirates	77,4										165		
Denmark	30,9												160
Bahamas, The	79,0	**144**											
Mexico	45,0												132
Korea, Republic of	44,1												119
Norway	38,5												108
Russian Federation	54,0											101	
Austria	53,7												94
Brazil	51,8												88
China, P.R.: Macao	69,8								86				
Finland	31,4												74
Panama	72,4	**72**											
India	39,2												59
Barbados	78,3		**50**										
Cyprus	49,8		**47**										
Total*		**231**	**96**	**2362**	**1349**	**44**	**43**	**46**	**117**	**3892**	**4122**	**3600**	**4074**

Data sources: 1 Cross-border positions, by residence and sector of counterparty Amounts outstanding at End-December 2016, in billions of US dollars, (BIS, 2017); 2 Financial secrecy score (Tax Justice Network, 2015); 3 Position of a country on 11 tax haven lists, (Murphy, 2009).
Note: * The number states for summary of the 187 countries.
Source: Pivot table is author's own.

The pivot table above (see Table 2) shows that the highest portion of bank sector claims was located in the United Kingdom. This country possess moderate characteristics of both, FSS and presence on tax haven lists. The same keeps for the USA and Germany. This means, that the first three countries with the highest portion of the total world's bank sector's claims possess moderate or below avarage characteristics of OFCs and tax havens. The same keeps

for exapmle for France and Japan. However, the oposit is true when speaking about the forth highest portion of bank sector claims which is located in Hong Kong (China). It has much above average FSS (i.e. 72), and at the same time it appears on 9 out of 11 tax haven lists. Also Luxembourg , Switzerland, Singapore, Ireland, The Bahamas, Panama, Barbados and Cyprus possess both characteristics, namely at least average FSS and at the same time presence at least on 8 out of 11 tax haven lists. Some developed countries show moderate FSS but they do not appear on any of 11 tax haven lists, for example Japan, Mainland China, Canada, Australia, Sweden, Denmark, Mexico and others.

4. Conclusion

The research goal of this paper was twofold, first, to examine, whether countries which are offshore financial centres coincide with countries that are labelled as tax havens. The research goal was to examine, whether countries that possess above average features of OFC and tax havens attract above average portion of world banks' claims. The empirical analysis employed pivot table and used data on financial secrecy index, data on a presence of countries on 11 lists of tax havens, and data on banks' claims in 187 countries. The result shows first 34 countries with the highest portion of banks' claims located there. Out of 34 countries 9 countries show coincidence between their FSS and at least 8 times of their presence on tax haven lists, they are China, Hong Kong, Luxembourg, Switzerland, Singapore, Ireland, the Bahamas, Panama, Barbados, and Cyprus. On the other hand, the first three countries with the highest portion of banks' claims, i.e. United Kingdom, United States, and Germany are present on 3 or less of 11 tax haven lists, however show average FSS. Fourteen countries with high portion of banks' claims on their territories do not appear on any of the 11 tax haven lists, however maintain at least average FSS. They are: Japan, China, Mainland, Canada, Australia, Sweden, Saudi Arabia, Denmark, Mexico, Republic of Korea, Norway, Austria, Brazil, Finland, and India.

Acknowledgements

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Greening the Green - Environmental and Financial Aspects of the American Green Bond Market Development

Anetta Kuna-Marszalek,[1] Jakub Marszalek [2]

Abstract

The purpose of the article is to analyze the circumstances for green bonds issues in the context of changes in the environment and financial market conditions. The study was devoted to the American market. The environmental changes in the United States before and after the green bonds debut were analyzed. Financial characteristics of 140 issued green bonds and other the US corporate bonds was also included. Using the difference significance test and multiple regression, it has been shown that impact of green bonds on the environment is very controversial. These instruments, however, are very attractive source of risky investments financing. Therefore, the authors assume that the environmental premise of issuances is only to attract investors and realize the project that would be more difficult to implement without the green label.

Key words

green bonds, eco-investments, American market, climate change, ecology

JEL Classification: F21, G15, O13, O16, Q50

1. Introduction

The green bond market ends ten years. The first green bonds were issued by the European Investment Bank in 2007 and one year later by the World Bank. Other entities followed suit and the green bond market gradually started to grow, reached a tipping point in 2013. Since then, there has been an exponential increase in supply (An Introduction to green bonds ..., 2016), first of all because of that private sector issuers, including corporates and banks joined the market. In 2016 the green bond market ranged from USD 70 to 100 billion, with much of this growth being the impact of Chinese issuers (Cochu *et al.*, 2016). This growth also includes the first ever green government bond issue, a EUR 750 million deal made in December 2016 by Poland (www.mf.gov.pl, accessed 1 August 2017). At the beginning of 2017 the second issue was made by the state authorities - this time it was France, which issued green bonds with a record value EUR 7 billion (www.gouvernement.fr, accessed 1 August 2017).

Integrity of the green market remains robust so far (Ceres, 2015), and its geography is expanding and diversifying (OECD, 2015). Over the last years we can observe an increasing numbers of specialized green bond funds as well as institutional investors and financial institutions, which have intended to increase green bond holdings. Green bonds, nevertheless, account for a small fraction of the overall bond market (Cochu *et.al.*, 2016; Shishlov, Morel

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and Cochran, 2016), still represented about 1.4% of global debt capital markets issuances in 2016, and according to economists it was well below the scale needed to play a significant role in the transition to a new green economy (Record year for..., 2017).

Green bonds should be seen in the context of climate change and other sustainability challenges that have emerged in recent years. Transition towards a climate resilient, resource-efficient economy is now a motion and it seems unavoidable because of many international agreements in this regard (e.g. the Paris Agreement). In order to reach environmentally challenges green bonds can be used (Green Bonds Climate Capital..., 2015). It is a new financial instrument that combines both "bonds" and "green" features. Green bonds have basic function and characteristics of conventional bonds, e.g. for issuers, capital cost of issuing bond is low and interest is pre-tax charge; for investors, bonds tend to provide higher profitability, liquidity and stability (Wang and Zhi, 2016). “Green” means that their proceeds are invested in an environmentally friendly way (Ehlers and Packer, 2016) with projects with sustainable benefits. In other words, a “green” bond differentiates from a conventional bond by its label, which signifies a commitment to exclusively use the funds raised to finance or re-finance in part or in full new and/or existing eligible green projects or business activities (ICMA, 2015). It means that this financial instrument could be tied to mitigating the effects of climate change (e.g. climate bonds) or to a specific environmental issue or technology, such as wind and solar energy efficiency projects, energy retrofits and clean transportation (e.g. renewable energy bonds, energy efficiency bonds and green transportation bonds) (Green bonds: victory ...., 2013). Such initiatives also include renewable energy, construction of energy efficient buildings, reforestation, sustainable waste management, sustainable land use, biodiversity conservation, and other investments (Ahuja and Mackay, 2016; Chiang, 2017).

According to van Renssen (2014) and Clapp (2014) the main factor that drives the increasing demand for green bonds is that many organizations are also under intense pressure to be investing green and that future legislation is likely to benefit green investments. If the green bond offers the same yield as the conventional ones investors might choose the green alternative.

Green bond issuers usually are diversified, including government, financial institutions and private enterprises, etc. In the case of some green bonds, investors may apply for a refund for the project with the help of national or local government subsidies. Moreover, the government can also launch preferential green bond policies such as lower investment thresholds or tax reduction taxes for investors and issuers, etc. (Wang and Zhi 2016; Cochu *et al.*, 2016). To support the development of the green bond market, governments can levy taxes for brown or conventional bonds.

The green bond market can offer several important benefits for green investment. They include e.g.: providing an additional source of green financing; enabling more long-term green financing by addressing maturity mismatch; facilitating the “greening” of traditionally brown sectors; making new green financial products available to responsible and long-term investors (Ross, 2015; Green bonds: country experiences ..., 2016). It can be assumed that in the coming years the green bond market will be growing in size and expanding in geographies. First of all we can observe a tremendous change in the attitude of businesses, consumers and investors to environmental issues that increasingly determine daily and strategic decisions. Moreover technological progress makes low-carbon solutions (from alternative energy sources to electric vehicles) more and more accessible. Thanks to these "green" investments they are not only ethically but also - increasingly - economically justified (Dmuchowski, 2017).

One of the world's leading issuers of green bonds is the United States, with USD 345 billion in green bonds in 2016. Less than one-tenth of one percent of bonds outstanding in the United States are green (Chiang, 2017). It means a significantly lower percentage than in other

countries that also issue green bonds, e.g. China, India, the Nordic countries, Germany, the Netherlands, and France. It may result from the fact that in the United States there is no policy consensus on climate and official support for green bonds is fragmented. U.S. corporations, public agencies, and local and state governments can feel less pressure to practice environmentally friendly investment, implement sustainable projects than their counterparts in other countries (Chiang, 2017). What is more on 1 June 2017 the United States would withdraw from the Paris Agreement dealing with greenhouse gas emissions.

Today the U.S. green bond market is largely driven by U.S. states and municipalities. Their bonds rank among the most significant on the climate bond market. In the United States, green bonds have primarily been used to finance renewable energy and energy efficiency projects, and initiatives related to water quality improvement, water infrastructure, and wastewater and water efficiency (Cochu *et al.*, 2016).

Despite the lack of a favorable climate for emissions of green bonds in the United States the market of these bonds is expecting to grow in next years. There are several reasons: 1. State level climate plans are far-reaching. 2. California has declared a green bond action plan: is promoting green finance and will hold a Climate Summit in 2018. 3. Renewable energy growth and policy support remain very important. 4. Green municipal bond issuance has been growing up. 5. Companies and investors are very interested in green policy. Over 1,200 signatories, including representatives from major tech industries, investors, cities, states, and universities, have publicly pledged their commitment to the Paris Agreement (Green Bonds Policy..., 2017).

Taking the above into consideration the main goal of our article is to analyse the circumstances for green bonds issues in the context of changes in the environment and financial market conditions on the United States' green market. The topic of green bonds is new. So far, researchers have rarely dealt with this problem, because this market is relatively immature and still developing. In addition, the available data describing the green bond market are based on short time series, and analysis have lot of problems associated.

2. Methodology and Data

Thomson Reuters Eikon data base data was used for the study of green bonds. At the end of 2016, there were 239 greenbacks registered in the market in 2010-2016, amounted to USD 345 billion. In this group, 220 bonds were issued by the companies. The group of 93 corporate green bonds was listed on stock exchange – mainly on German stock exchanges, Luxembourg Stock Exchange and MarketTaxess. Only 34 was guaranteed. More than half of the instruments (128) had an investment rating and 15 were rated as high-yield bonds. Other bonds were not given a rating. The characteristics of US green bond issuance are shown in Table 1.

Table 1: Characteristics of the US green bonds issues in 2010-2016.

Specification	2010	2011	2012	2013	2014	2015	2016
Value of issuances (in mln USD)	309 740	12	1 458	1 084	6 540	15 282	11 207
Number of issuances	5	2	9	5	17	162	39

Source: Own elaboration based on Thompson Reuters Eikon Database

The market of these instruments is still very young. The corporate-level instruments has been developing since 2011, as 5 issues from 2010 were carried out by the International Bank

for Reconstruction and Development. None of the bonds have the conversion option or put/call early redemption option. All instruments are denominated in USD indeed (excluding 2 issues in CAD and 5 in EUR). Most of the bonds have plain vanilla fixed coupon. Only 16 has a floating rate and 4 are zero coupon. The vast majority of green bonds issues are aimed at financing the investments related to acquisition or conversion of energy. 74 issues are targeted to alternative energy and 55 eligible green bond projects. Others do not have a clearly defined purpose.

The analysis of the green bonds issues circumstances was divided into two stages. The first one was focused on analyzing changes in selected parameters of environmental pollution and energy consumption. The article compares the changes in emissions of several basic variables determining the degree of environmental pollution - carbon monoxide, nitrogen oxides, PM2.5, and sulfur dioxide. Multiple regression (multivariable) was used to describe the quantitative evolution of the explanatory variable (Y) taking into account two or more explanatory variables (Xj). Using the regression to describe changes in emission levels, two variables (Xj) were used:

• dummy, specifying the green bonds introduction to the market,

• trend, determining the trend in emissions' levels.

Similar calculations were made for changes in US electricity consumption. The least squares method (LSM) was used to estimate the values of the regression equation. The evaluation of the determination significance coefficient was performed by ANOVA. The significance of the individual regression coefficients were evaluated by the t test. The normality of the random component distribution was assessed by the Shapiro-Wilk test. The study was conducted for the period 1990-2016. Data were obtained from U.S. Environmental Protection Agency and the World Bank.

In the second stage, the investment attractiveness of green bond was assessed by comparing them to similar corporate debt instruments. The profitability (YTM) of the instruments offered so far by corporations have been analyzed as well. Statistical analysis was conducted using tests to assess the significance of the difference (t-tests). The selection of tests was not unequivocal, as the main assumption for the test was to analyse the results for pairs of bonds (green or non-green) with the same ratings. Hence, despite the fact that these are different subjects in physical terms, we used paired difference tests. Student's t-test is the basic test for comparing the two paired populations. In order to perform it, we needed differences between paired measurements, which represent normal distribution. The assumption was not always met in the case of examined companies, and in such cases we used the non-parametric equivalent of t-Student's test for paired samples, i.e. the Wilcoxon signed rank test. For both tests the null hypothesis assumes a lack of differences between both types of bonds (measured with the expected value of random variable for the t-Student test or the distribution function for the Wilcoxon test), and thus the alternative hypothesis is: there are differences. It was decided that the variable in comparable populations of bonds is statistically significant if the probability in the test, *p*, was below the assumed level of significance (α=0.05). Calculations were made in IBM SPSS Statistics 22.0.

3. Results

Table 2 presents the results of regression analysis of the relationship between change in emission of selected variables determining the degree of environmental pollution and electricity consumption changes in the context of the green bonds introduction to the bond market. The results obtained are not clear and make general conclusions impossible.

Table 2: Changes in environmental pollution and energy consumption in the face of the green bonds introduction in the US in 2000-2016.

	Regression coefficients	*p*
Model 1: Carbon monoxide (R-squared = 0.939)		
Constant	114882.814	0.000
Trend	-3947.263	0.000
Dummy	8453.702	0.115
Model 2: Nitrogen oxides (R-squared = 0.963)		
Constant	24839.58579	0.000
Trend	-847.568252	0.000
Dummy	184.7687406	0.031
Model 3: PM2.5 (R-squared = 0.296)		
Constant	6583.083	0.000
Trend	-80.181	0.000
Dummy	786.452	0.078
Model 4: Sulphur dioxide (R-squared = 0.979)		
Constant	17899.145	0.000
Trend	-813.450	0.000
Dummy	-1459.785	0.000
Model 5: Energy consumption per capita (R-squared = 0.905)		
Constant	8137.730	0.000
Trend	-77.318	0.000
Dummy	-149.182	0.000
Model 6: Renewable energy consumption (R-squared = 0.830)		
Constant	4.538	0.000
Trend	0.216	0.035
Dummy	0.830	0.169

Source: Own elaboration based on Thompson Reuters Eikon, U.S. Environmental Protection Agency and World Bank databases

The first model examines the impact of introducing green bonds on changes in carbon monoxide emission. Although the model has good representation the assessment of this impact for both variables is questionable in the context of content. The dummy variable is not statistically significant. Trend turned out to be significant and has a negative linear factor. This means that the introduction of green bonds is irrelevant to carbon monoxide emission, while emission is regularly lowering. In the second model, both variables are statistically significant, but the dummy linear coefficient is positive. With a negative trend factor, such a result cannot be taken as the emission of nitrogen oxides regularly decreases. Therefore the introduction of green bonds cannot raise the emission. The same value have trend coefficient in model 3. The results of the analysis cannot, however, be accepted also because the model representation is too low. The best in terms of substance is model 4. It is well matched and both variables are statistically significant. Linear coefficients are negative, which means that sulfur dioxide emission is lowered, and the introduction of green bonds is conducive to this tendency. Similar dependencies are observed for model 5. Per capita energy consumption is decreasing over time, and the appearance of green bonds can stimulate this direction of change. Positive changes are also observed in model 6. The model has a satisfactory level of representation, but only the trend variable is statistically significant and indicates the growing

trend of renewable energy consumption. Green bonds, despite the correct linear factor, do not play a significant role in changing this parameter.

Table 3: Yield to maturity (YTM) of corporate American green bonds in 2011-2016 (in %).

Specification	2011	2012	2013	2014	2015	2016
Average	3.692	2.990	6.567	4.406	7.409	4.845
Median	1.108	2.435	7.540	3.299	8.052	2.880
Standard deviation	5.410	1.807	2.675	3.482	1.921	3.608

Source: Own elaboration based on Thompson Reuters Eikon Database

Yields of green bonds in the analysed period should be considered as relatively low if we take into account that they finance rather risky ventures (Table 3). Apart from 2012, when the yields were close to the level of government debt, their average value is similar to that observed in the corporate bond market. They are not particularly varied. Median and arithmetic mean values are similar and standard deviations are not high. The median is more often lower than the average, indicating several high above-average observations. They deal with cases of unsecured green bonds. There is no trend of YTM in the studied period. It also does not depend from the number of bonds issued.

Table 4: Selected features of American green and corporate bonds of the same risk in 2011-2016.

Specification	Type of bond	N	Arithmetic average	Median	Standard deviation	p
YTM (%)	Green	128	5.824	6.402	11.118	0.017**
	Non-green	186	5.831	6.543	9.394	
Coupon (%)	Green	128	3.451	3.427	2.009	<0.001**
	Non-green	186	3.298	3.501	2.632	
Payback period (days)	Green	128	2 485	2 193	780	0.063*
	Non-green	186	3 068	2 556	1 485	

The significance of differences was assessed using Mann-Whitney's or t test.
* - statistically significant differences at α = 0,05;
** - statistically significant differences at α = 0,10
Source: Own elaboration based on Thompson Reuters Eikon Database.

A comparison of the basic characteristics of green and non-green corporate bonds with the same ratings indicates a slight difference between those two groups of securities. Green bonds have slightly lower YTM value. This applies both to the arithmetic average and to the median. However, in both groups, a high standard deviation is observed. Green bonds have an average higher coupon value. However, such a relationship does not occur in the median value. The higher median is observed in the group of non-green bonds. Relatively low is Standard deviation is relatively low, suggesting low variation of the coupon values in both instrument groups. Green bonds are issued for a significantly shorter time in comparison to non-green ones. It is observed for average and median values. Both groups of parameters are also the least varied, what is indicated by the lowest values of standard deviations. It is worth to remember, however, that in the case of payback periods the differences between green and non-green bonds are the least statistically significant.

4. Conclusions

One of the main premise for green bond issuance is the environmental protection. This is done through the implementation of investment projects aimed at implementing advanced technologies that allow most to reduce energy consumption or produce it in new way, often better than the previously used solutions. It is difficult, of course, to expect the immediate and direct impact of such investments on the environment. The value of issued bonds is too low for projects to be able to effectively influence the level of pollution. It must be borne in mind, however, that green bonds have a special status in the financial market, confirmed by a specific certification. For this reason, it seems reasonable to question about the actual ecological goal of the bonds offered. Presented research shows that level of environmental pollution decreases regularly as well as the energy consumption and the use of renewable energy increases in the United States. This is despite the opposite political declarations of the US authorities ignoring international climate agreements and when the US green bond market remains the largest in the world. Analysis of these instruments from the financial perspective can indicate a number of similarities to the typical corporate bonds. They have similar YTM and coupon. Therefore they are equally attractive to the investor. They are issued for a shorter time, which allows more flexible management of the investor's portfolio. From his point of view they may be more attractive instruments than traditional non-green corporate bonds. They carry an additional emotional charge - they support environmental protection. In fact, the state of the American environment is improving irrespective of the green bonds presence on the financial market.

Therefore there are reasons to believe that green bond status is primarily intended to support green investment, even when their green influence is difficult to quantify. The impression is that the American green bond issuers, like others in the rest of the world, try to convince investors of the magnitude of their projects, while they are no different from their non-green counterparts. Energy-saving or alternative-generation projects are usually based on technologically advanced ideas. This generates a significant investment risk. The green status decreases it somewhat subjectively. This is due to the aforementioned emotional load, hope of the supported technology development, and lack of financial consequences compared to traditional debt. Ecological projects, also for political reasons, protecting umbrella of over-confidence. They are often supported by government or international organizations. Therefore, it seems that the main advantage of issuing green bonds is to obtain this premium that would be unavailable under non-green bonds. The green label of the issued bonds becomes a decoy for investors ready to support fashionable investments, although the impact on the improvement of the environment remains debatable.

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Performance management of business entities in light of comprehensive income concept

Anna Kuzior, Małgorzata Rówińska[1]

Abstract

The paper presents the concept of comprehensive income adopted in international financial reporting and its consequences for users of financial reports. The profit or loss and other comprehensive income as well as management possibilities for these categories will be characterized. In particular, attention will be paid to the principles and manner of presenting information on the effects of completed business transactions and the effects of balance sheet valuations. For the empirical illustration of the research problems described above, the scope of the disclosed financial information of companies listed on the Warsaw Stock Exchange in the food sector will be analyzed. The research will be conducted in terms of elements affecting the comprehensive income resulting from completed and unrealized transactions.

Key words

Comprehensive income, profit/loss, other comprehensive income

JEL Classification: M41, M48

1. Introduction

The current IAS 1 "Presentation of Financial Statements" implies a broad approach to disclosing income and expense in the financial statements. In addition to income and expense shaping profit or loss, the annual report also includes income and expenses created and settled through equity. Recognition of income and expenses created profit and loss as well as other comprehensive income in a single statement – statement of profit or loss and other comprehensive income – allows for disclosing the entity's comprehensive income for a given period. Comprehensive income, also referred to as a comprehensive result, represents a change in equity during the period that resulted from business operations and other accounting events not forming part of transactions with owners acting as shareholders.

Reporting reflection of the operation results of the business entity, under the applicable IFRS regulations, is subordinated to the concept of comprehensive income. On the one hand, it is the result of broad application for balance sheet valuation of fair value in IFRS (even preference), and, on the other hand, is considered to be a response to investors' demand for reliable and useful financial information [1].

The purpose of this paper is to conduct a critical analysis of the existing solutions for presenting the components of comprehensive income in financial statements and to examine to what extent the comprehensive income reflects the actual performance of an entity and includes the unrealized results arising from the pursued accounting policy. According to the authors, the proper assessment of the entity's operations requires an in-depth knowledge of the entity's accounting policies, with additional information disclosed in numerous explanatory notes, and the fact that the comprehensive income also influences the balance sheet valuation (potential income and expense). An empirical study into the relationship

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between comprehensive income and profit or loss was conducted on the basis of annual reports of companies from food industry listed on the Warsaw Stock Exchange.

For the attainment of the purpose, the method of analyzing the accounting regulations (IFRS), analysis of the literature in the area being studied, and the method of synthesis and inference were applied.

2. Background

The issue of reporting comprehensive income is the subject of scientific discussion and research, assessing the scope and possibilities of use of this information by stakeholders.

Many scientific publications point to the need to ensure optimum quality of reporting information [2], [3], [4], [5]. In practice, due to the possibility to use individually selected accounting policy tools by entities, which provide the opportunity to manage the presented results, there is a lack of objective guarantee of high quality information. This has implications for the possibility to use information by stakeholders (to make decisions, conduct credible analyzes and assess the entity's situation), as highlighted in their research by Ponce, Hlaciuc, Mates, Maciuca [2]. On the other hand, the asymmetry of information is also emphasized. Those who prepare financial statements undoubtedly have more information than the stakeholders. Apart from the individual selection of balance sheet tools, the authors point out the very way in which information about comprehensive income is presented, which also influences the decisions taken by users of financial statements.

The necessity to report comprehensive income, the structure of the statement of profit or loss and other comprehensive income resulting from IAS 1 regulations is also subject to scientific discussion and research. Mechelli and Cimini [6] conducted a study among companies listed on regulated markets in Europe on the use of information on profit or loss and comprehensive income. It turned out that the net result (profit or loss) is perceived as a more important category than the value of comprehensive income. In turn, studies by Demir, Bahadir, Oncel [7], Du, Stevens, McEnroe [8], and Gazzola, Amelio [9] provide evidence of an information advantage of comprehensive income over profit or loss. Nevertheless, the publications highlight the impact of different reporting structures on users' ability to use information. Similar conclusions have been made by Pascan [10], who worked out on the basis of the research that comprehensive income represents a new and relevant category of performance measurement. The problem of presenting completed and unrealized income and expenses in one report is noted by, for example, Luty [11], Hodgson, Russell [12], Rówińska [13].

The conducted review of the literature shows that the studies into the usefulness of the comprehensive income report take little account of the fact that only some of the components of comprehensive income are amounts reflecting the entity's performance. Research focuses more on the comparison of categories: profit or loss, comprehensive income and indication of possibilities of their use.

According to the authors of this paper, from the point of view of the usefulness of information, it is very important to take into account the fact that the profit or loss and comprehensive income presented in the financial statements reflect the accounting policy of the entity which allows for the performance management of the business activities (earnings management).

3. Problem formulation

The comprehensive income presented in the financial statements reflects twofold elements: the effects of the completed transactions and the effects of the balance sheet valuation. These

latter are due in part to the need to adhere to the prudence principle (impairment of non-current assets, write-downs on inventories and receivables, provisioning) and to the implementation of the imposed measurement of most financial assets at their fair value. However, in many cases, the final amount of comprehensive income depends on the valuation model adopted by the entity, e.g. property, plant, equipment (PPE), intangible assets or investment property. Also, the application of obligatory accounting policies, in particular the prudence principle mentioned above, gives entities the ability to manage their profits by applying appropriate judgments and estimates. Therefore, comprehensive income is the category that ultimately represents the virtual outcome of an entity's activity, which includes not only the actual financial effects of the completed economic transactions. In part, it also reflects the potential result that the entity would have had at the balance sheet valuation if it had realized all economic benefits from the assets that were measured. Again, it should be emphasized that the potential outcome depends on the adopted accounting policy, in particular on the models of asset and liability valuation.

As a result, comprehensive income also represents, in addition to the actual result achieved, the expected future economic benefits of the entity.

4. Problem solution

The information presented in the consolidated financial statements prepared by food companies listed on the Warsaw Stock Exchange for the financial year 2016 was used for this analysis. The analysis includes companies (22) that prepare financial statements in accordance with IFRS, and therefore present and disclose information on items shaping comprehensive income. In the case of companies reporting in foreign currency, the average NBP exchange rate for a given currency was used to convert the amounts into Polish currency, valid at the balance sheet date. The table below (1) presents the components of comprehensive income reported in the financial statements (profit or loss and other comprehensive income) and their percentage structure.

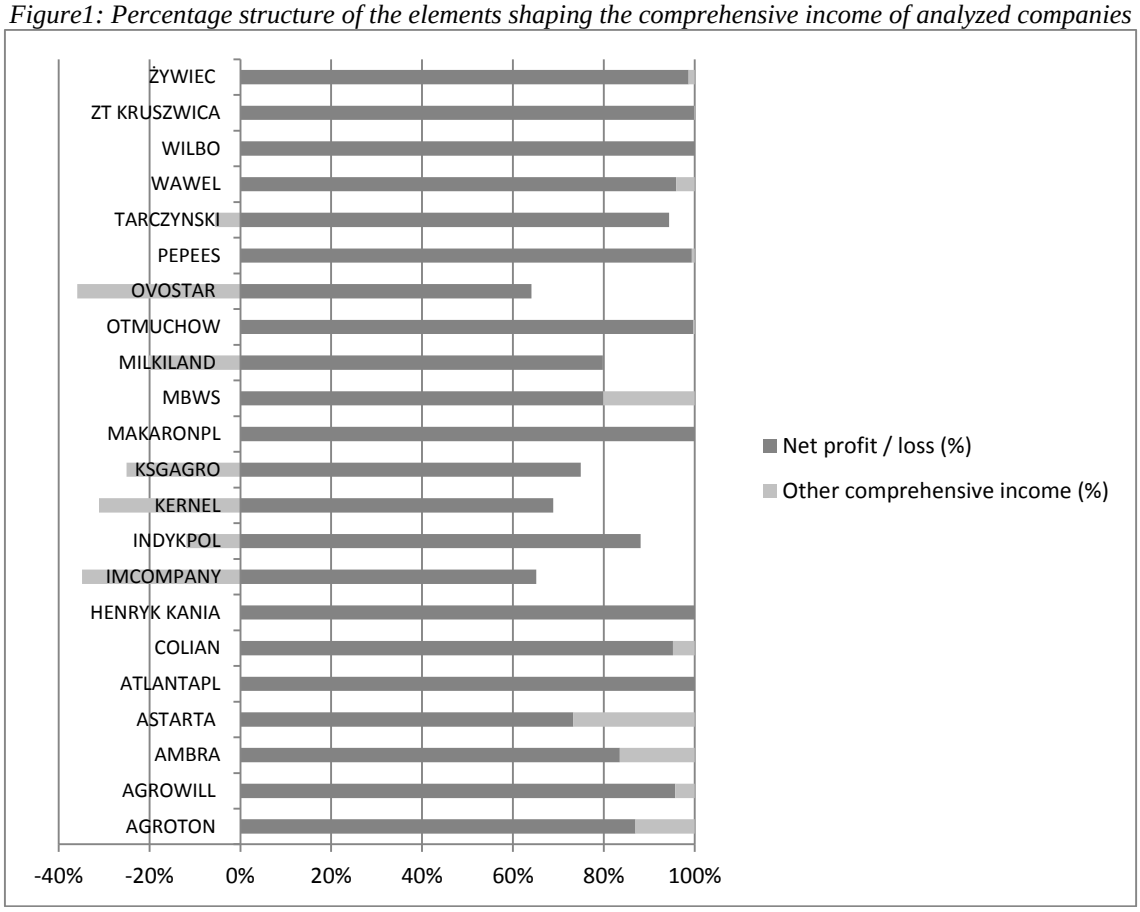
Table 1: Structure of comprehensive income in food companies listed on the Warsaw Stock Exchange for the financial year 2016

No.	Company	Comprehensive income (PLN)	Profit or loss (PLN)	Other comprehensive income (PLN)	Profit or loss (%)	Other comprehensive income (%)
1.	AGROTON	104 708 182.20	91 029 333.30	13 678 848.90	87%	13%
2.	AGROWILL	5 047 784.00	4 831 008.00	216 776.00	96%	4%
3.	AMBRA	30 285 000.00	25 288 000.00	4 997 000.00	84%	16%
4.	ASTARTA	498 748 488.00	365 612 632.00	133 135 856.00	73%	27%
5.	ATLANTAPL	4 533 000.00	4 533 000.00	0.00	100%	0%
6.	COLIAN	41 687 000.00	39 689 000.00	1 998 000.00	95%	5%
7.	HENRYK KANIA	46 176 000.00	46 176 000.00	0.00	100%	0%
8.	IMCOMPANY	44 890 328.00	96 611 312.00	-51 720 984.00	215%	-115%
9.	INDYKPOL	8 260 000.00	9 546 000.00	-1 286 000.00	116%	-16%
10.	KERNEL	494 922 442.90	902 907 173.20	-407 984 730.00	182%	-82%
11.	KSGAGRO	10 887 076.50	16 353 600.90	-5 466 524.40	150%	-50%
12.	MAKARONPL	5 092 000.00	5 092 000.00	0.00	100%	0%
13.	MBWS	37 404 920.00	29 866 424.00	7 538 496.00	80%	20%
14.	MILKILAND	-128 415 448.00	-172 120 144.00	43 704 696.00	134%	-34%
15.	OTMUCHOW	-46 272 000.00	-46 128 000.00	-144 000.00	100%	0%

16.	OVOSTAR	41 362 532.10	94 172 166.90	-52 809 634.80	228%	-128%
17.	PEPEES	18 011 000.00	17 898 000.00	113 000.00	99%	1%
18.	TARCZYNSKI	9 280 370.00	9 865 939.00	-585 569.00	106%	-6%
19.	WAWEL	88 725 000.00	85 086 000.00	3 639 000.00	96%	4%
20.	WILBO	-5 581 000.00	-5 581 000.00	0.00	100%	0%
21.	ZT KRUSZWICA	107 755 000.00	107 581 000.00	174 000.00	100%	0%
22.	ŻYWIEC	276 333 000.00	272 573 000.00	3 760 000.00	99%	1%

The percentage structure of other comprehensive income, showing the share of the profit or loss and other comprehensive income, is presented in Figure 1.

Figure1: Percentage structure of the elements shaping the comprehensive income of analyzed companies



From Table 1 and Figure 1, it is clear that the profit or loss definitely has the biggest share in the comprehensive income. This includes realized and unrealized results arising from the balance sheet valuation. Based on the analysis of explanatory notes to the statement of profit or loss and other comprehensive income, an attempt was made to determine what part of the result is the result of economic transactions carried out and what the result of the applied balance sheet valuation principles is. The results of the analysis are presented in the following table (2).

Table 2: Gross result of food companies listed on the Warsaw Stock Exchange for the financial year 2016 and its structure (data in PLN)

No.	Company	Profit/loss before tax	Unrealized revenue	Unrealized expenses	Realized result
1.	AGROTON	91 041 871.20	3 025 813.20	137 641 066.20	225 657 124.20
2.	AGROWILL	4 746 952.00	0.00	291 984.00	5 038 936.00
3.	AMBRA	31 353 000.00	2 390 000.00	5 703 000.00	34 666 000.00
4.	ASTARTA	376 340 832.00	210 595 672.00	11 785 536.00	177 530 696.00
5.	ATLANTAPL	5 635 000.00	1 543 000.00	2 335 000.00	6 427 000.00
6.	COLIAN	42 250 000.00	4 479 000.00	4 687 000.00	42 458 000.00
7.	HENRYK KANIA	57 956 000.00	0.00	0.00	57 956 000.00
8.	IMCOMPANY	95 067 336.00	8 357 763.29	2 194 304.00	88 903 876.71
9.	INDYKPOL	10 207 000.00	267 000.00	1 746 000.00	11 686 000.00
10.	KERNEL	986 274 556.70	170 902 141.10	46 374 475.30	861 746 890.90
11.	KSGAGRO	16 073 587.80	44 279 683.50	26 296 155.60	-1 909 940.10
12.	MAKARONPL	6 567 000.00	10 000.00	118 000.00	6 675 000.00
13.	MBWS	-19 195 736.00	63 718 872.00	90 846 840.00	7 932 232.00
14.	MILKILAND	162 466 976.00	119 448.00	16 457 280.00	178 804 808.00
15.	OTMUCHOW	-37 211 000.00	272 000.00	43 324 000.00	5 841 000.00
16.	OVOSTAR	94 903 544.40	2 704 007.10	309 268.20	92 508 805.50
17.	PEPEES	22 675 000.00	69 000.00	233 000.00	22 839 000.00
18.	TARCZYNSKI	12 587 157.00	319 506.00	444 328.00	12 711 979.00
19.	WAWEL	105 279 000.00	1 358 842.00	1 068 000.00	104 988 158.00
20.	WILBO	-5 581 000.00	115 000.00	3 869 721.00	-1 826 279.00
21.	ZT KRUSZWICA	124 273 000.00	247 000.00	4 069 000.00	128 095 000.00
22.	ŻYWIEC	344 897 000.00	43 254 000.00	57 309 000.00	358 952 000.00

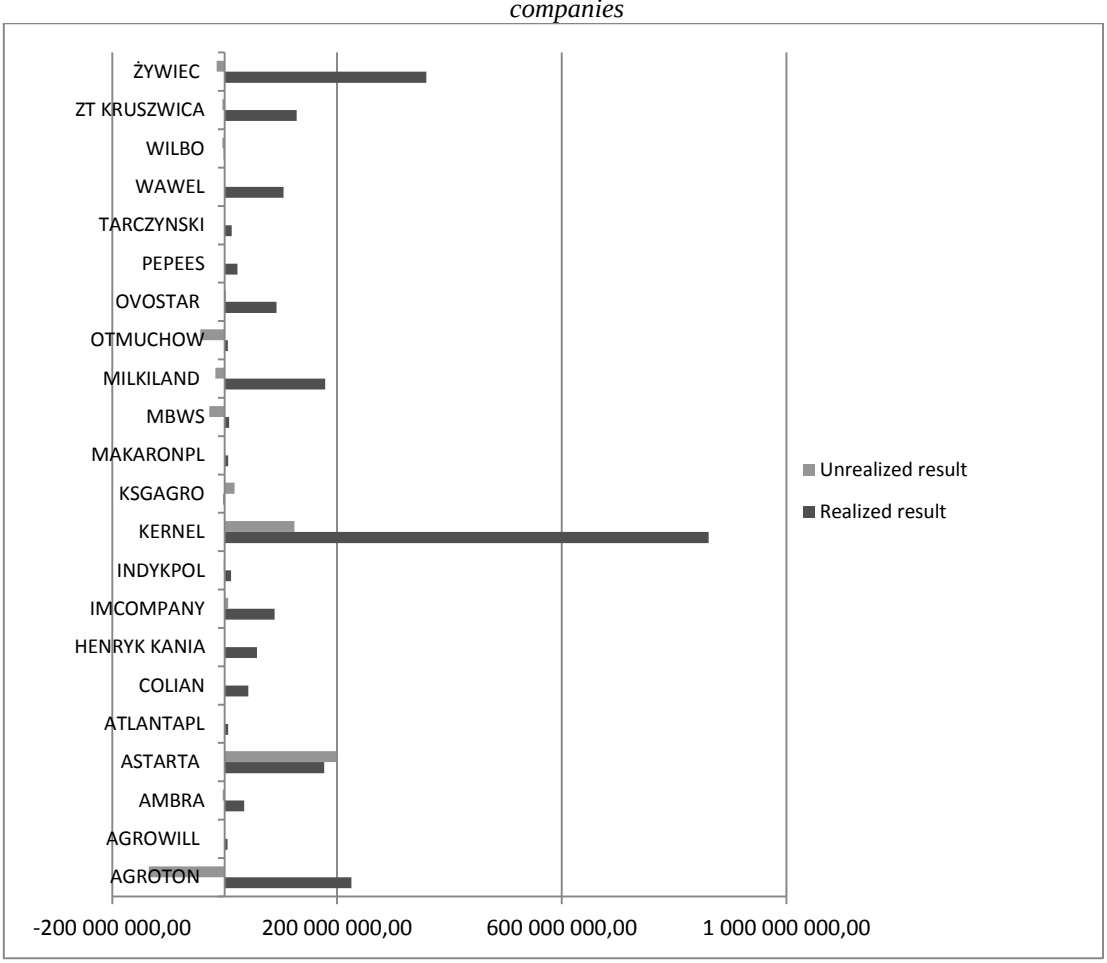
Elements shaping the profit or loss before tax of the analyzed companies, i.e. the result on completed and unrealized transactions is graphically presented in Figure 2.

As shown in Table 2, only in one case (Henryk Kania) unrealized income is zero. Knowing the principles of balance sheet valuation, it is difficult to agree with this situation. It seems reasonable to assume that this is due to the lack of precise information on this subject in the explanatory notes. The analysis of notes shows that the most frequently occurring unrealized expenses are those arising from write-downs of receivables and inventories, and the amount of provisions created. The effects of valuation of financial assets primarily occurred in the case of disclosed revenues. In no case was there any revenue associated with the valuation of property investments; in most cases the studied entities do not own such assets, those that own investment property value them at the cost model. As far as interest and exchange rate differences are concerned, in many cases there is no detailed and specific information on the realized and unrealized amounts. Very significant differences occurred in the case of 5 companies (the gross result is several times higher or closer to the result taking into account only expense and realized revenue). The differences between the realized and the unrealized result can be considered negligible in most entities.

In general, other comprehensive income represents a relatively small percentage of the comprehensive income of the surveyed entities (about 15%). In the case of four companies, there was no other comprehensive income. In addition, the titles of their origins are not much different. For the most part, these are the effects of the valuation of provisions for employee benefits that are mandatory and valued in accordance with the principles set out in IAS 19

Employee Benefits. In the case of companies with subsidiaries outside Poland (in particular foreign parent companies listed on the Warsaw Stock Exchange), the significant item of other comprehensive income is the exchange rate differences on consolidation. In several cases, the effects of the valuation of financial instruments arising from the applied hedge accounting occurred. In the case of Polish companies, there was no other comprehensive income arising from the measurement of operating non-current assets in the revaluation model, as these entities only use the historical cost model. Foreign companies value non-current assets at fair value, and therefore report other comprehensive income related to valuation in the revaluation amount model.

Figure 2: Elements of the profit or loss before tax achieved for the financial year 2016 in the analyzed companies



5. Discussion

Transparency of reporting information, which has a decisive influence over its usefulness, should require business entities to disclose any material adjustments to the values resulting from the balance sheet valuation. The effects of balance sheet valuation are often an essential element shaping entity's performance, although it is in fact an unrealistic element that represents the potential gains or losses resulting from the ownership of the asset at the balance sheet date.

The "lack of transparency" of the IAS 1 comprehensive income statement may result in misinterpretation of the information presented in the financial statements.

In authors' opinion, the clear visibility of the result on completed and unrealized transactions would better reflect the financial position of the entity. Separate presentation of income and expenses resulting from the balance sheet valuation would allow regarding earning management policies adopted by the entity.

6. Conclusion

The analysis of the financial statements of food companies listed on the Warsaw Stock Exchange showed the dominant share of the profit or loss in the comprehensive income presented in the financial statements. Other comprehensive income mainly results from business entities' obligations regarding the valuation, to a lesser extent, from voluntary valuation models. In particular, it is noticeable that Polish entities do not use the revaluation model. This may be justified by the lack of tradition in this regard; this model was introduced to the Polish legislation in 2005. The situation is different in the case of foreign companies. It should also be emphasized that the profit or loss is mainly driven by completed transactions. In fact, no excessive reserves or write-downs on inventories or receivables were observed. Those that occurred can undoubtedly be regarded as a reliable effect of the prudent valuation principle. The authors did not notice any obvious attempts in the analyzed reports of performance management. There is also a noticeable lack of precise information on transactions made and not realized in the explanatory notes to the statement of the statement of profit or loss and other comprehensive income (in particular to the profit or loss). The allegation mainly concerns interest and exchange rate differences that may be twofold and often represent significant amounts.

The requirement to separately present completed and unrealized transactions would greatly improve the usefulness of this report and allow the business entity to assess the extent of performance management by the business entity.

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Income inequality and intersecting Lorenz curves: an empirical study

Tommaso Lando[1], Lucio Bertoli-Barsotti[2]

Abstract

In this paper, we review some ranking criteria for comparison of income distributions in terms of inequality, based on the Lorenz curve. Then, we describe a general method for obtaining classes of inequality measures that are isotonic, or order-preserving, with the orderings studied. Finally, we perform an empirical analysis with which we analyze and compare the income distributions of 33 European counties, including the EU-28.

Key words

Lorenz ordering, income inequality, disparity, stochastic dominance.

JEL Classification: C44, E24, O52

1. Introduction

The Lorenz curve (LC) is a basic tool for the representation of the inequality of income distributions. The LC evaluated in $p \in [0,1]$ gives the proportion of the total wealth corresponding to the $p100\%$ poorer part of the population. If the LC of a distribution is uniformly higher than the LC of another distribution, we can rank unambiguously such distributions in terms of inequality, i.e., the first distribution definitely has . This criterion is referred to as the *Lorenz dominance* (LD). In an economic context, the LD is theoretically justified by the Pigou-Dalton condition, i.e., the so called "principle of transfers". Such principle states that the higher of two non-intersecting Lorenz curves can be obtained from the lower one by an iteration of income transfers from "richer" to "poorer" individuals (the so called *elementary transfers* or *T-transforms*, Marshall et al., 2009, p. 32, also called *progressive transfers*, Shorrocks and Foster, 1987). For this reason, the "coherence" with the LD represents a fundamental property for all inequality (or concentration) measures. We say that all the inequality indices that satisfy such basic property are *isotonic* with the LD.

The LD and the corresponding isotonic indices, such as the Gini index, are commonly used in the practice, although many empirical studies revealed that Lorenz curves often intersect, so that is not rare to find couples of distributions that cannot be ranked with the LD. In this paper we extend our approach in order to deal with intersecting LCs. In such cases, we can compare the intersecting LCs by relying on weaker orders of inequality, such as the *second-degree Lorenz dominance*, 2-LD (Aaberge, 2009). The objective of the 2-LD is to move from the (first-degree) LD to a dominance relation of higher degree by integrating LCs 1) *upward* or 2) *downward*, which gives rise to a Lorenz dominance of second degree (2-LD). In the first case, we obtain the upward 2-LD, which consists in cumulating the LC from the bottom (the left). This idea has been studied in several works, also related to the concept of third-degree inverse stochastic dominance (Muliere and Scarsini, 1989; Aaberge, 2009). All such kinds of orderings, basically emphasize the left tail of the distribution in spite of the right one. Indeed, many authors

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(see e.g. Shorrocks and Foster, 1987; 1988; Atkinson, 2008) agree that an elementary transfer should be more equalizing the "lower" it occurs in the distribution, that is, the principle of aversion to downside inequality (Davies and Hoy, 1995). On the other hand, in an economic context, where a lot of attention is recently given to those variations occurring at the top of the income distribution (Makdissi and Yazbeck, 2015). Thus, the second idea 2), introduced by Muliere and Scarsini (1989) and more recently developed by Aaberge (2009), consists in cumulating the LC from the top (the right) and can be termed as downward 2-LD (Aaberge, 2009). This method emphasizes the right tail of the distribution in spite of the right one. In order to combine the approaches of the upward and downward 2-LD into a single preorder, Lando and Bertoli-Barsotti (2016) introduced the *disparity dominance*, which is based on the cumulated difference between the upper and the lower parts of the LC and it is aimed at emphasizing inequality in both tails of the distribution.

Once we have determined an ordering that represents our attitude towards income inequality, our objective is generally to define indices that are isotonic with such ordering. This can be done, for instance, by relying on some well known results of majorization theory (Marshall et al. 2009).

In this paper, we review the dominance relation discussed above, we present a method for obtaining classes of isotonic indicators and finally we perform an empirical analysis. The data consist of the income distribution of 33 European countries in 2015, and have been downloaded from Eurostat's website. Our objective is twofold: i) we aim at identifying the ordering which has a higher discriminating power; ii) we analyze the impact of different types of indices on the ranking.

2. Theory and methods

We recall that a preorder is a binary relation $\prec$ over a set S that is reflexive and transitive. In particular, observe that a preorder $\prec$ does not generally satisfy the antisymmetry property (that is, $a \prec b$ and $b \prec a$ does not necessarily imply $a = b$) and it is generally not total (that is, each pair a, b in S is not necessarily related by $\prec$). We say that the preorder $\prec$ is stronger than the preorder $\prec'$ (or, equivalently, $\prec'$ is weaker, or finer, than $\prec$) if, for every pair a, b in S, $a \prec b$ implies $a \prec' b$.

Let F be a distribution with positive and finite expectation μ_F. The (generalized) inverse or *quantile function* of F is given by

$$F^{-1}(p) = \inf\{z: F(z) \geq p\}, p \in (0,1) \tag{1}$$

The Lorenz curve $L_F: [0,1] \to [0,1]$ is defined as follows [16]:

$$L_F(p) = \frac{1}{\mu_F}\int_0^p F^{-1}(t)dt\,, p \in (0,1). \tag{2}$$

We recall that the Gini index is given by twice the area between the Lorenz curve and the 45° line:

$$\Gamma(F) = 1 - 2\int_0^1 L_F(t)dt. \tag{3}$$

Let us also define the complementary curve [13], $\bar{L}_F: [0,1] \to [0,1]$, given by

$$\bar{L}_F(p) = \frac{1}{\mu_F}\int_0^p F^{-1}(1-t)dt = 1 - L_F(1-p), p \in (0,1) \tag{4}$$

Actually, for a given percentage p, $L_F(p)$ represents the percentage of total wealth possessed by the low $100p\%$ part of the distribution, while $\bar{L}_F(p)$ represents the percentage of total wealth corresponding to the top $100p\%$ part of the distribution. From a geometrical point of view, $\bar{L}_F(p)$, increasing and concave, is the 180° rotation of $L_F(p)$, increasing and convex, with respect to the point (0.5,0.5). Now we can introduce

$$\Delta_F(t) = \bar{L}_F(t) - L_F(t),\ t \in [0,1/2]. \tag{5}$$

The interpretation of Δ_F is quite simple. As $\bar{L}_F(t) \geq L_F(t)\ \forall t \in [0,1/2]$, the difference between the Lorenz curves expresses the disparity between the "higher" and the "lower" parts of the distribution. In terms of income distributions, Δ_F equals the difference between the proportion of the society's overall wealth that is held by the society's top (rich) $100t\%$, and the proportion of the society's overall wealth that is held by the society's low (poor) $100t\%$.

Definition 1. We write $F \leq_L G$ if and only if $L_F(p) \geq L_G(p), \forall\, p \in (0,1)$.

When the LD is not fulfilled, i.e. when LCs intersect, we need to introduce some weaker criteria in order to obtain unambiguous rankings. Muliere and Scarsini (1989) and Aaberge (2009) suggest cumulating LCs from the left or from right: that is, attaching more weighting to low or top incomes. Such two parallel approaches give rise to the upward and downward second-degree Lorenz dominance (Aaberge, 2009), defined as follows.

Definition 2. We say that G second-degree upward Lorenz dominates F, and write $F \leq_L^2 G$ iff:

$\int_0^t L_F(p)dp \geq \int_0^t L_G(p)dp, \forall t \in [0,1]$.

We say that G second-degree downward Lorenz dominates F, and write $F \leq_{\bar{L}}^2 G$ iff any of the following equivalent conditions is true:

$\int_0^t \bar{L}_F(p)dp \leq \int_0^t \bar{L}_G(p)dp, \forall t \in [0,1]$

$\int_t^1 1 - L_F(p)dp \leq \int_t^1 1 - L_G(p)dp, \forall t \in [0,1]$

Observe that $F \leq_L G$ implies $F \leq_L^2 G$ and $F \leq_{\bar{L}}^2 G$, but the converse is not necessarily true, i.e. $F \leq_L^2 G$ and $F \leq_{\bar{L}}^2 G$ do not imply the LD. Moreover, note that $F \leq_L^2 G$ implies that L_F starts above L_G and presents a larger (underlying) area: that is, a lower (or equal) value of the Gini index. Differently, $F \leq_{\bar{L}}^2 G$ implies that L_F starts below L_G but still has a lower (or equal) Gini. Hence it is apparent that in both cases the condition $\int_0^1 L_F(p)dp \geq \int_0^1 L_G(p)dp$ (i.e. $\Gamma(F) \leq \Gamma(G)$) is necessary to establish a dominance. In particular, if LCs cross only once, the one with higher Gini index dominates the other with respect to upward 2-LD (if the latter starts below, see Zoli, 1999) or downward 2-LD (if the latter starts above, Lando and Bertoli-Barsotti 2016). As an alternative approach, Lando and Bertoli-Barsotti (2016) combined the basic ideas (preferences) expressed by the $\leq_L^2$ and $\leq_{\bar{L}}^2$ orderings into a single preorder, which emphasizes inequality in both the tails of the distribution. This is done by symmetrically cumulating the Lorenz curve from both sides, through the disparity curve Δ_F, described above. In fact, it can be reasonable to wish that F is preferable to G if Δ_F is (uniformly) as small as possible, compared to Δ_F: this concept can be expressed as follows in terms of an integral inequality.

Definition 3. We say that F second-degree disparity dominates (2-DD) G and write $F \leq_D^2 G$ iff:

$$\int_0^t \Delta_F(p)dp \leq \int_0^t \Delta_G(p)dp\ \forall t \in [0,0.5].$$

Like $F \leq_L^2 G$, also $F \leq_D^2 G$ implies the condition $\int_0^{1/2} \Delta_F(p)dp \leq \int_0^{1/2} \Delta_G(p)dp$, which is equivalent to $\Gamma(F) \leq \Gamma(G)$ ($\int_0^{1/2} \Delta_F(p)dp = \int_0^1 (p - L_F(p))dp$).
Similarly to what has been discussed above, with regard to single-crossing LCs, if the curves are double-crossing, a theorem of Lando and Bertoli-Barsotti (2016) determines a sufficient condition for the 2-DD based on the positions of the crossing points and the values of the Gini index. Thus, 2-LD and 2-DD are especially suitable for ranking single-crossing and double-crossing LCs, respectively.

Given the preorder $\prec$ (defined over a set S), a mapping $M: S \to \mathbb{R}$ is said to be *isotonic*, consistent or order-preserving with respect to $\prec$ whenever, for every $a, b \in S$ such that $a \prec b$, we obtain that $M(a) \leq M(b)$. Clearly, if the preorder $\prec'$ is finer than the preorder $\prec$ we obtain that every mapping that is isotonic with $\prec'$ is isotonic with $\prec$ as well. Moreover, we say that M is strictly isotonic with $\prec$ if $a \prec b$ and $b \nprec a$, imply $M(a) < M(b)$.

Based on some well known results of Majorization theory (Marshall et al. 2009), it is possible to define classes of concentration indices that are consistent with the orderings defined above. It can be easily seen that the Gini index, Γ, in consistent with all the dominance relations. Moreover, with regard to the second-degree Lorenz dominance, the functional

$$\Phi(F) = \int_0^1 \phi(L_F(t))dt, \tag{6}$$

where we assume that $\phi(0) = 0$ and $\int_0^1 |\phi(L_F(t))|dt < \infty$, has the following properties (see Lando and Bertoli-Barsotti, 2016).

Properties.

1. If ϕ is decreasing and convex and $F \leq_L^2 G$, then $\Phi(F) \leq \Phi(G)$.
2. If ϕ is decreasing and concave and $F \leq_L^2 G$, then $\Phi(F) \leq \Phi(G)$.

Similarly, the functional

$$\Psi(F) = \int_0^{1/2} \psi(\Delta_F(t))dt, \tag{7}$$

where we assume that $\psi(0) = 0$ and $\int_0^{1/2} |\psi(\Delta_F(t))|dt < \infty$, has the following property.

3. If ψ is increasing and concave and $F \leq_D^2 G$, then $\Psi(F) \leq \Psi(G)$.

In our empirical analysis we shall propose the following indices:

a. $\Phi_1(F) = \int_0^1 (1 - L_F(t))^2 dt$ (Property 1)
b. $\Phi_2(F) = \int_0^1 (1 - L_F(t))^{0.5} dt$ (Property 2)
c. $\Psi_3(F) = \int_0^{0.5} (1 + \Delta_F(t))^{0.5} dt$ (Property 3)

Note that i) the Gini index can be obtained from $\phi(x) = x$ and $\psi(x) = x$ in (6) and (7) respectively and; ii) for the sake of simplicity, Φ_1, Φ_2, Ψ_1 are not normalized, unlike Γ.

3. Empirical analysis

One of the objectives of this study is to investigate the discriminating power of the dominance relations described in section 2. Logically, as a general rule, the stronger the

preorder, the less it is applicable, so that we may find, in practice, many couples of intersecting LCs that are not comparable with LD. In such cases, the main advantage of using a weaker criterion is that we can significantly increase the number of pairs of LCs that are comparable. In this paper, we refer to the proportion of couples that are ranked according to a given preorder as the *rate of completeness* of such preorder. By increasing the rate of completeness, we reduce the ambiguousness and we provide a normative justification for the use of an index of inequality that is coherent with the preorder considered, such as the Gini index. To perform our empirical analysis, we downloaded the income distributions of the EU-28 countries, plus other five European countries, in the year 2015. The dataset is described as follows.

Data have been retrieved online from Eurostat's website. In particular, Eurostat provides the "distributions of income by quantiles" with two options, in terms of income and living conditions indicator, namely: i) *top cut-off point*, which represent the income of the individual at the right end of the given quantile and; ii) *share of national equivalized income*, which is the share of the total income which belongs to a given interval. Eurostat provides i) and ii) for the three quartiles, the four quintiles, the nine deciles and the first (and last) five percentiles. Unfortunately, we note that some countries present negative incomes in the first 2-3 percentiles. The presence of negative incomes hampers the applicability of properties 1,2 and 3, discussed in the previous section. Moreover, smaller percentile values are generally less reliable and accurate. Then, we decided to consider the LCs starting from $p = 0.05$. Indeed, by properly cumulating the shares of national equivalized income we can obtain the values of the LC for

$$p = 0,0.05,0.1,0.2,0.25,0.3,0.4,0.5,0.6,0.7,0.75,0.8,0.9,0.94,0.95,0.96,0.97,0.98,0.99,1,$$

that is, a LC with 20 nodes.

For the year 2015, we computed the LCs of 33 countries, and compared each pair of LCs (i.e. 33×32/2=528 pairs) based on the four dominance relations studied in section 2, namely LD, upward 2-LD, downward 2-LD and 2-DD. The rates of completeness are shown in Table 1.

Table 1: Rates of completeness

LD	*Up. 2-LD*	*Down. 2-LD*	*2-DD*
0.56	0.77	0.73	0.75

Fist of all, we must take into account that the left tails of our LCs start from 0.05 rather than 0.01, whilst the right tails are evauated up to 0.99. Thus, the results will be logically slightly unbalanced, because the right tails contain more detailed information than the left tails. Table 1 reveals that the LD ranks approximately half of the pairs. Indeed, we observe that the 33 countries present similar concentration patterns as well as similar values of the Gini index (reporoted below), so that LCs cross most of the times. We also observe that, generally, cumulating from the left (upward 2-LD) is more critical (with regard to the discriminating power) than cumulating from the left (downward 2-LD). However, by taking into account the first five percentiles (i.e. negative incomes) we obtain just the opposite result (this result can be gound in Lando et al., 2017). For this reason, it seems to be wiser to adopt the 2-DD, in order to emphasize both tails at the same time, avoiding to rely just on one of them. In this regard, the 2-DD yields a rate of completenes which lies between that of the upward and downward 2-LD. Further analyses, reported in Table 2, show the effectiveness of the different ranking criteria according to the number of times a couple of LCs intersect. The results show that upward 2-LD has more discriminating power, especially when dealing with double-crossing LCs. We also note that the maximum number of times a LC crosses another is 3 (it happens

only in one pair over 528), where all of the weak dominance relations analyzed are able to rank such pair.

Table 2: Proportions of single and double-crossing LCs ranked by 2-LD and 2-DD

No. of intersections	*Up. 2-LD*	*Down. 2-LD*	*2-DD*
1	0.52	0.47	0.51
2	0.36	0.12	0.18

Table 3 shows the values of the different concentration indices, namely, the Gini index and Φ_1, Φ_2, Ψ_1 presented in section 2 and the different rankings yielded. As it can be seen, by changing the concentration index we do not modify the ranking substantially. More importantly, the results shown in Table 1 provide justification for the use of a concentration index. Indeed, when dealing with European countries, ranking based on the Gini index is supported by the LD only in half of the cases approximately, whilst e.g. the 2-DD supports the Gini index in almost 75% of the cases.

Table 3: Concentration indices and rankings for the EU-28 countries

Country	Γ	Φ_1	Φ_2	Ψ_1	*Country*	Γ	Φ_1	Φ_2	Ψ_1
Austria	0.541,{24}	0.957,{24}	1.543,{24}	1.126,{24}	*Latvia*	0.705,{5}	1.058,{5}	1.599,{5}	1.162,{5}
Belgium	0.519,{26}	0.947,{26}	1.533,{26}	1.121,{26}	*Lithuania*	0.754,{2}	1.082,{3}	1.619,{1}	1.172,{2}
Bulgaria	0.732,{4}	1.07,{4}	1.611,{4}	1.168,{4}	*Luxembourg*	0.564,{20}	0.97,{20}	1.551,{20}	1.131,{20}
Croatia	0.61,{14}	1.004,{14}	1.564,{15}	1.141,{14}	*Malta*	0.561,{21}	0.969,{22}	1.55,{21}	1.131,{21}
Cyprus	0.668,{11}	1.024,{11}	1.592,{7}	1.154,{11}	*Netherlands*	0.53,{25}	0.948,{25}	1.54,{25}	1.124,{25}
Czech Republic	0.502,{27}	0.93,{29}	1.53,{27}	1.118,{27}	*Norway*	0.479,{31}	0.923,{31}	1.52,{31}	1.113,{31}
Denmark	0.547,{23}	0.959,{23}	1.547,{23}	1.128,{23}	*Poland*	0.607,{15}	0.997,{15}	1.565,{14}	1.141,{15}
Estonia	0.689,{6}	1.049,{7}	1.593,{6}	1.158,{6}	*Portugal*	0.675,{9}	1.037,{10}	1.59,{10}	1.155,{9}
Finland	0.501,{28}	0.933,{28}	1.528,{28}	1.117,{28}	*Romania*	0.744,{3}	1.084,{2}	1.613,{3}	1.17,{3}
Macedonia	0.672,{10}	1.045,{8}	1.584,{11}	1.154,{10}	*Serbia*	0.759,{1}	1.096,{1}	1.617,{2}	1.173,{1}
France	0.581,{19}	0.974,{19}	1.56,{19}	1.135,{19}	*Slovakia*	0.47,{32}	0.923,{32}	1.514,{33}	1.111,{32}
Germany	0.597,{16}	0.991,{16}	1.563,{16}	1.139,{16}	*Slovenia*	0.488,{30}	0.929,{30}	1.522,{30}	1.115,{30}
Greece	0.681,{8}	1.045,{9}	1.591,{8}	1.157,{8}	*Spain*	0.687,{7}	1.052,{6}	1.591,{9}	1.158,{7}
Hungary	0.561,{22}	0.97,{21}	1.549,{22}	1.131,{22}	*Sweden*	0.499,{29}	0.938,{27}	1.525,{29}	1.117,{29}
Iceland	0.469,{33}	0.917,{33}	1.516,{32}	1.11,{33}	*Switzerland*	0.586,{18}	0.98,{18}	1.56,{17}	1.136,{18}
Ireland	0.592,{17}	0.987,{17}	1.56,{18}	1.137,{17}	*United Kingdom*	0.64,{13}	1.014,{13}	1.579,{12}	1.148,{13}
Italy	0.642,{12}	1.023,{12}	1.576,{13}	1.148,{12}					

We note that most countries mantain the same ranks even by changing the index. The main differences, in terms of ranking, can be found for a few countries such as: Spain, Macedonia, Lithuania, Czech Republic and Cyprus. Such countries generally exhibit slightly uncommon shapes of the LC, and present higher (or smaller) degrees of inequality in the left or in the right tail, compared to the others. This can be seen simply by observing the indices. For instance, Czech Republic present quite an even left tail (i.e., the poor part of the population), compared to the other countries with similar values of the Gini index, since its ranking changes substantially by moving from $\mathbf{\Phi_1}$ to $\mathbf{\Phi_2}$. Spain and Cyprus present similar values of the Gini coefficient (approximately 0.69) but Spain exhibits a more even left tail, whilst Cyprus has a more even right tail. In conclusion, we argue that indicators that are isotonic with dominance

relations that are finer than the LD contain more information about the shape of the LC, making it possible to determine which part of the population is more critical, in terms of inequality.

4. Conclusion

Our analysis has shown that European countries generally present very similar concentration patterns, and, consequently, their LCs often cross. On the one hand, this feature represents the outcome of European policy and other cosiderations of economic type. On the other hand, such result suggests that the LD and the Gini index are not always able to capture the differences between income distributions with sufficient accuracy. Thus, it seems to be more appropriate to employ weaker dominance relations and their corresponding isotonic indicators, which can give more detailed information about the shape of the LC, with particular emphasis towards the tails.

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Comparison of valuation approaches of finished goods and work in progress

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Abstract

The paper is concentrated on main aspects of valuation of stocks of finished goods and works in progress. The value of these items must be stated respecting the Accounting act. no. 563/1991 Coll. and implementing decree of this act which state main principle of valuation. Nevertheless act and decree do not explane lot of aspects that real economic life brings like the impact of fixed costs, clear definition of direct costs or costs that should not influence final value of goods. Therefore our paper compare attitude of Czech accounting legislation and attitude of International accounting standards, emphasize impact of fixed costs on final value of goods, and moreover determine details of valuation presented by National Accounting Council.

Key words

Valuation of stocks, Fixed costs, International Accounting Standards

JEL Classificaton: M21, G31

1. Základní přístup v oceňování vytvořených výkonů

V souvislosti se změnou účetních předpisů v roce 2016 došlo ke změně přístupů v oblasti zásob ať už hotové či nedokončené výroby, a to nejenom ve způsobu zachycení jejich přírůstků či úbytků ale rovněž i v případě jejich oceňování. Obě tyto změny byly projevem snahy o přiblížení české účetní legislativy legislativě vymezené mezinárodními účetními standardy. V případě zachycení přírůstků či úbytků zásob vlastní činnosti tak došlo k tomu, že se upustilo od jejich zachycení prostřednictvím výnosových účtů a změna stavu dokončené či nedokončené výroby se již uskutečňuje prostřednictvím nákladových účtů, konkrétně účtů účtové skupiny 58. Tato změna přispěla k lepší vypovídací schopnosti výkazu zisku a ztráty, kdy se tak prostřednictvím této účtové skupiny vyjadřuje hodnota nákladů spojená s realizovanými výstupy, přesněji výše nákladů spotřebovaných ve vyhotovených prodaných výkonech. Zbývající hodnota nákladů, která má podobu nákladů období, nicméně i nadále zůstává uznávána jako náklady aktuálního období a ovlivňuje tak výsledek hospodaření už v období svého vzniku, a ne v období realizace výkonů na trhu. Mimoto byl změněn rovněž přístup ke stanovení vlastních nákladů výkonů při oceňování zásob vytvořených vlastní činností podniku. Cílem příspěvku je objasnit základní aspekty novelizovaného postupu tohoto ocenění, vymezení případných problémových oblastí a jeho srovnání s přístupem vymezeným mezinárodními účetními standardy.

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2. Vlastní náklady u zásob vytvořených vlastní činností

Způsob ocenění zásob hotových výrobků a nedokončené výroby byl změněn v souvislosti s novelou zákona o účetnictví č. 563/1991 Sb. a s novelou prováděcí vyhlášky k tomuto zákonu č. 500/2002 Sb. Z následující tabulky 1 jsou patrné základní rozdíly mezi přístupem ve stanovení hodnoty dokončené a nedokončené výroby před výše uvedenými legislativními změnami a již zmíněný nový přístup.

Tabulka 1: Komparace stanovení vlastních nákladů u zásob vytvořených vlastní činností

Vlastní náklady u zásob vytvořených vlastní činností	
2015	2016
Vlastními náklady se rozumí buď skutečná výše nákladů nebo výše nákladů podle způsobu kalkulace výroby stanoveného účetní jednotkou. Výrobou se rozumí i jiná činnost, při které nevznikají hmotné produkty. Odchylně mohou účetní jednotky oceňovat zásoby vlastní výroby, jimiž jsou nedokončená výroba, polotovary a výrobky: a) ve výrobě s krátkodobým nepřetržitým cyklem nedokončenou výrobu pouze přímými materiálovými náklady a výrobky nebo polotovary přímými materiálovými a mzdovými náklady, b) v hromadné a velkosériové výrobě pouze přímými náklady, jimiž jsou náklady na přímý materiál, polotovary, přímé mzdy a ostatní přímé náklady, c) v malosériové a kusové nebo zakázkové výrobě a ve výrobě s dlouhodobým cyklem přímými náklady, výrobní režií a v případě, že výrobní cyklus přesahuje dvanáct měsíců výjimečně i správní režií.	**Vlastní náklady zásob** vytvořených vlastní činností se oceňují ve skutečné výši nebo na základě kalkulace výroby stanovené účetní jednotkou. • Vlastní náklady zahrnují přímé náklady a mohou zahrnovat také poměrnou část variabilních a fixních nepřímých nákladů, jež jsou příčinně přiřaditelné danému výkonu a jež se vztahují k období činnosti. • Náklady na prodej se do těchto nákladů nezahrnují. Volbu metody ocenění provede účetní jednotka s ohledem na povinnosti stanovené zákonem, zejména respektováním principu významnosti a věrného a poctivého zobrazení majetku.

Pojem zásoby je přesně vymezen v § 9 vyhlášky č. 500/2002 Sb. (dále jen vyhláška), kde je specifikován majetek, jež lze považovat za zásoby. Dle § 25 odst. 1 zákona o účetnictví (dále jen zákon) se zásoby vytvořené vlastní činností oceňují vlastními náklady. V § 49 odst. 5 vyhlášky je vymezen způsob oceňování zásob pořizovaných včetně zásob vytvořených vlastní činností. Zásoby vytvořené vlastní činností mají být oceňovány ve skutečné výši nebo na základě kalkulace výroby stanovené účetní jednotkou. Z tohoto znění není jasné, pokud má být jejich výše odvozená od stanovené kalkulace, o jakou konkrétní kalkulaci jde. Dá se předpokládat, že pojem stanovená znamená kalkulace propočtena předem, blíže se ale nedozvíme, zda je to kalkulace plánovaná nebo kalkulace operativní. Pokud si uvědomíme, že s ohledem na věrný a poctivý obraz, který má účetnictví poskytovat, by měly být přírůstky zásob nedokončené a dokončené výroby pravidelně evidovány, dospějeme k závěru, že díky změnám ve výrobním procesu či v poskytování činností a rovněž v souvislosti se změnami tržních podmínek, může dojít snadno k situaci, kdy věrný a poctivý obraz poskytne spíše předběžná operativní než plánovaná kalkulace.

2.1 Vymezení vlastních nákladů výkonů

V § 49 odst. 5 vyhlášky je dále uvedeno, že vlastní náklady zahrnují přímé náklady a mohou zahrnovat také poměrnou část variabilních a fixních nepřímých nákladů, příčinně přiřaditelných danému výkonu a vztahující se k období činnosti. V tomto znění je tak zmíněno

několik pojmů, které nejsou nikde jinde blíže legislativně vymezeny jako například vlastní náklady, přímé náklady, nepřímé náklady, fixní a variabilní náklady.

Co je ale jednoznačné dle zákona, zásoby vlastní výroby případně nedokončenou výrobu je potřeba ocenit. A to z toho důvodu, abychom prostřednictvím jejich ocenění upravili hodnotu nákladů souvisejících s realizovanými výkony. Rovněž také i proto, abychom byli schopni stanovit výši základu daně, který je ovlivněn výši nákladů. Pro ocenění je tedy nezbytné vyhotovit kalkulaci. V kalkulaci bychom se měli dopátrat hodnoty vlastních nákladů. Ta by měla být ovlivněna výší přímých případně nepřímých nákladů souvisejících s vytvořenými výkony a vztahujících se k období činnosti. Pokud jde o definici přímých nákladů tak dle (Fibírové, 2015), s ohledem na příčinné vazby nákladů k výkonu, který je jednoznačně vymezen druhem, jakostí a objemem a s ohledem na efektivní možnosti přiřazení nákladů výkonu, lze považovat za přímé náklady ty, které bezprostředně souvisejí s konkrétním druhem výkonu jako například základní materiál, práce výrobního dělníka na konkrétním výkonu včetně odvodů zdravotního a sociálního pojištění, odpis specializovaného zařízení, dodávky subdodavatelů související s konkrétním výkonem apod. Při bližší analýze podstaty přímých nákladů lze tak usuzovat, že přímými náklady mohou být nejenom jednicové náklady, které jsou zapříčiněny každou jednotkou výkonu (Mruzková, Lisztwanová, 2013), ale i náklady režijní, které jednoznačně a bezrozporně souvisejí s konkrétním druhem výkonu a jež tomuto výkonu mohou být bez výhrad přiřazeny. Součástí ocenění zásob tak mohou být i náklady režijní povahy. Pokud si uvědomíme, že režijní náklady vytvářejí podmínky pro zabezpečení a vykonávání činnosti podniku, můžeme minimálně identifikovat tyto náklady v oblasti zásobování, výroby, prodeje a správy.

V pokračujícím znění § 49 odst. 5 je uvedeno, že náklady na prodej, byť by měly přímou, tzn. jednoznačnou souvislost s prodanými výkony, se do oceňování zásob nezahrnují. Pokud připustíme, že s hotovými výrobky na skladě nebylo ještě manipulováno samotnými prodejci, je tento fakt pochopitelný. Nicméně bližší eliminace přiřaditelných nákladů chybí. Pokud bychom se striktně drželi znění vyhlášky, mohli bychom se domnívat, že i část správních nákladů, protože ty jsou nepřímými náklady ve vztahu k výkonu, a navíc jsou i náklady fixními a de facto mohou představovat náklady vztahující se k období činnosti, by mohla ovlivnit ocenění hotových výrobků případně nedokončené výroby s ohledem na respektování principu významnosti a věrného a poctivého zobrazení majetku v případě výrob s dlouhým výrobním cyklem.

2.2 Posouzení vlivu fixních nákladů na ocenění zásob

Další věc, na kterou narazíme v souvislosti s praktickou aplikací těchto ustanovení zákona a vyhlášky je to, že není blíže stanoven postup při přiřazení fixních nákladů ať už přímých či nepřímých. Dle základní definice fixních nákladů (Fibírová, 2005), za fixní náklady lze považovat takové náklady, jež zajišťují podmínky pro činnost v určitém rozsahu a jsou limitovány potřebnou kapacitou. Nejsou tedy závislé na objemu výkonů a lze je považovat pro dané rozmezí objemu výkonů za neměnné. Vlastní náklady výkonu, jež mají být použity pro ocenění dokončené či nedokončené výroby jsou však určeny s ohledem na výši přiřazených nákladů na jednotku výkonu. Vzhledem k tomu, že se jednotkové fixní náklady přiřazené výrobku mění s objemem produkce, stěžejní informací pro určení výše vlastních nákladů výkonů bude také údaj týkající se objemu těchto výkonů. Otázkou tak je, jaké množství výkonů je rozhodující pro stanovení vlastních nákladů výkonů. Pokud je totiž vyroben menší objem výrobků, než je stanovená normální (běžná) úroveň využití kapacity, podíl fixních nákladů aktivovaných do ceny zásob může být vyšší a hodnota majetku je díky tomu nadhodnocena, jak dokládá tabulka 2. Naopak v případě, že je vyrobeno větší než plánované

množství výrobků, je hodnota těchto zásob nižší. Legislativní znění zákona případně vyhlášky však tento problém vůbec neřeší.

Tabulka 2: Vliv fixních nákladů na ocenění zásob

	A (1 ks)	A (10 000 ks)	B (1 ks)	B (12 000 ks)	C (1 ks)	C (8 000 ks)
Zásoby počáteční stav		0 ks		0 ks		0 ks
Objem výroby		10 000 ks		12 000 ks		8 000 ks
Přímý materiál (Kč)	8	80 000	8	96 000	8	64 000
Přímé mzdy (Kč)	6	60 000	6	72 000	6	48 000
Nepřímé var. náklady (Kč)	1	10 000	1	12 000	1	8 000
Nepřímé fixní náklady (Kč)	5	50 000	4,17	50 000	6,25	50 000
Vlastní náklady výroby (Kč)	**20**	**200 000**	**19,17**	**230 000**	**21,25**	**170 000**
Objem prodeje		5000 ks		5000 ks		5000 ks
Tržby (Kč)	30	150 000	30	150 000	30	150 000
Výsledek hospodaření (Kč)		50 000		54 150		43 750
Zásoby konečný stav		5000 ks		7000 ks		3000 ks
Hodnota konečných zásob (Kč)		**100 000**		**134 150**		**63 750**

3. Komparace ocenění zásob dle české účetní legislativy a dle mezinárodních účetních standardů

Z výše uvedeného textu je zřejmé, že provedená změna zákona včetně navazující vyhlášky příliš nepřispěla k výkladové jednoznačnosti dané problematiky, naopak spíše ještě prohloubila nejasnosti, které je potřeba v řešení reálných problémů přesně vymezit. Jak bylo v úvodu zmíněno, novela zákona a vyhlášky měla mimo jiné přiblížit českou účetní legislativu mezinárodním účetním standardům. Dle mezinárodního účetního standardu IAS 2, který je věnován zásobám, lze nicméně najít mnohem precizněji formulovaná pravidla. Oceňování vytvořených zásob je vymezeno v části věnované nákladům na přeměnu, kde jsou za náklady na přeměnu považovány náklady na nedokončenou výrobu a dokončené výrobky, tj. například mzdy a další výrobní náklady. Režijní náklady související s výrobou jsou v tomto případě rozlišovány na variabilní a fixní. Přičemž fixní část režie se alokuje s ohledem na úroveň normální kapacity za běžných podmínek včetně času potřebného k údržbě. V případě, že je vyrobeno menší množství výrobků, podíl fixní režie aktivované do ceny zásob se nezvyšuje. Díky tomu vzniká nepřiřazená část fixní výrobní režie, která se pak stává součástí nákladů období a ovlivní tak výsledek hospodaření tohoto období. Pokud je vyrobeno vyšší množství výrobků než daná běžná kapacita, naopak se přiřazuje nižší hodnota fixní režie, což se následně projeví i nižší hodnotou nákladů přiřazených jednotlivému výkonu.

Rozdíl v přístupu dle české účetní legislativy a mezinárodních účetních standardů je zachycen v následující tabulce 3. Je zřejmé, že vymezení ocenění zásob hotové a nedokončené výroby dle IFRS ve srovnání s českou legislativou přesněji specifikuje, co je možno považovat za náklady vstupující do ceny hotových výrobků a nedokončené produkce, pracuje s jednotlivými identifikovanými druhy nákladů tj. náklady na pořízení vstupů a mzdovými a výrobními náklady související s přeměnou vstupů a navíc deklaruje v případě výpůjčních nákladů související s pořízením zásob, že je lze považovat za součást ocenění výrobních zásob a ne za náklady období, jak je tomu v případě české účetní legislativy.

Tabulka 3: Komparace ocenění zásob dle české účetní legislativy a dle MÚS

Vlastní náklady u zásob vytvořených vlastní činností	
České účetní legislativa	Mezinárodní účetní standardy
Vlastní náklady zásob vytvořených vlastní činností se oceňují ve skutečné výši nebo na základě kalkulace výroby, stanovené účetní jednotkou.	Náklady na pořízení zásob zahrnují náklady na nákup zásob a náklady na jejich další přeměnu v rámci daného (výrobního) procesu.
Součástí pořizovací ceny zásob jsou náklady související s pořízením, zejména přepravné, clo a pojistné. Součástí pořizovací ceny zásob nejsou úroky z úvěrů a zápůjček poskytnutých na jejich pořízení, kurzové rozdíly, smluvní pokuty, úroky z prodlení a další sankce ze smluvních vztahů.	Náklady na nákup zahrnují cenu pořízení, dovozní cla a nerefundovatelné daně, dopravné, náklady na manipulaci a ostatní náklady přímo přiřaditelné pořizovaným zásobám. Úroky z cizího kapitálu použitého k financování oběžného majetku jsou považovány za součást ocenění tohoto majetku. Naopak do ocenění vyrobené produkce nesmějí být zahrnuty náklady na neobvyklé množství odpadu, neúčelně vynaložené práce a náklady na skladování mimo skladování vstupů.
Vlastní náklady zahrnují přímé náklady a mohou zahrnovat také poměrnou část variabilních a fixních nepřímých nákladů, příčinně přiřaditelných danému výkonu a vztahujících se k období činnosti.	Náklady na přeměnu zahrnují náklady vynaložené na přeměnu materiálu na nedokončenou výrobu a poté na dokončené výrobky, tj. například mzdy a další výrobní náklady, které musí být rozděleny na variabilní a fixní část.
Náklady na prodej se do těchto nákladů nezahrnují.	Do ocenění vyrobené produkce nesmí být zahrnuta správní režie a náklady na prodej, tj. tedy náklady nesouvisející s procesem zhotovení výkonů a neovlivňující tvorbu hodnoty zásob.

4. Přístup k ocenění zásob dle národní účetní rady

Na poměrně nejednoznačně formulované požadavky na oceňování výrobních zásob dle českého zákona a vyhlášky tak zareagovala Národní účetní rada, která svým posouzením v interpretaci I-35 prezentovala praktický přístup ke stanovení hodnoty výrobních zásob. Jak je uvedeno v úvodu této interpretace, jejím cílem je stanovit jednotný postup oceňování zásob vytvořených vlastní činností v účetnictví vedeném dle zákona o účetnictví. Odborné posouzení nákladů souvisejících s výrobními zásobami lze shrnout do následující tabulky č. 4.

Tabulka 4: Vymezení nákladů ocenění zásob dle Národní účetní rady

Vlastní náklady výkonu	
1. Přímé náklady	Náklady účelně a účelově vynaložené na výrobu výkonu, jež mohou být přiřazeny přímo bez potřeby jejich rozvrhování (alokování, rozpočítávání) mezi více druhů výkonů, jako např. spotřeba materiálu a práce, jež jsou použity k výrobě výkonu, případně přijaté služby subdodavatelů, které mohou být považovány za náklady variabilní tedy měnící se s objemem produkce.
2. Nepřímé náklady	Náklady, které jsou účelně a účelově vynaloženy na výrobu výkonů a lze je přiřadit pouze pomocí rozvrhování (alokování, rozpočítávání) mezi více druhů výkonů za použití vhodné rozvrhové základny. K těmto nákladům patří například spotřeba vody, elektřiny a plynu ve výrobě, výkonové nebo časové odpisy výrobního zařízení, odpisy budov používaných k výrobě, nájemné výrobních prostor, náklady na technickou přípravu výroby, údržbu nebo kontrolu kvality. Tyto náklady mohou být povahy fixní či dokonce variabilní (např. výkonové odpisy).

Za náklady účelově vymezené související s výrobou se dle interpretace národní účetní rady považují náklady, které vedou k uvedení zásob na jejich současné místo a do požadovaného stavu, přičemž se připouští, že náklady související s výrobou mohou souviset i s délkou výrobního cyklu, který nutně nemusí být vždy kratší než jeden rok. Do ocenění zásob nicméně ale nelze zahrnout administrativní náklady, finanční náklady, prodejní náklady a náklady na přesun zásob v rámci skladů.

Účelně vynaloženými náklady jsou náklady, které splňují požadavky na zásady hospodárnosti a efektivity. To znamená, že neobvyklé množství odpadu, vyhnutelné škody, neúčelně vynaložené práce a náklady související s nevyužitou kapacitou tyto předpoklady nesplňují.

V případě využití kapacity je důležité umět posoudit tzv. běžný stupeň využití výrobní kapacity, která odpovídá průměru dosaženému během několika předcházejících období za běžných okolností s ohledem na objem plánované údržby. V případě nízkého objemu výroby je potřeba vzít v úvahu při oceňování zásob běžnou úroveň využití kapacity. Což znamená, že takto je eliminováno nadhodnocování zásob. Nepřiřazené nepřímé fixní náklady jsou pak považovány za náklady období. Pokud je vyrobeno vyšší než kapacitní množství výrobků, je nutno jednotlivým výkonům přiřadit takovou úroveň fixních nákladů, která odpovídá skutečnému vyrobenému objemu produkce.

Národní účetní rada věnuje pozornost i vyrovnání případných rozdílů mezi předpokládanou a skutečnou hodnotou nákladů přiřazených vyrobených výkonům. V případě významného rozdílu mezi nimi je potřeba upravit ocenění zásob k rozvahovému dni tak, aby tyto zásoby byly oceněny na úrovni skutečných vlastních nákladů.

Z posouzení vyhotoveného Národní účetní radou je zde zřejmá snaha o přiblížení se zásadám v oceňování zásob danými mezinárodními účetními standardy. Národní účetní rada se shoduje s mezinárodními účetními standardy v tom, že jednoznačně se přiřazují přímé náklady výkonu, navíc je nutno brát v potaz i účelnost vynaložení těchto nákladů, náklady administrativní povahy, správní povahy či náklady nesouvisející s výrobou včetně prodejních nákladů nemohou být součástí tohoto ocenění. Navíc stejně je posuzována Národní účetní radou i problematika vlivu fixních nákladů na ocenění zásob a to v tom smyslu, že v případě že je vyrobeno nižší než plánované množství dané běžnou kapacitou, nezvyšuje se hodnota zásob díky vyšší hodnotě jednotkových fixních nákladů a naopak v případě, že je vyrobeno vyšší množství výrobků, nižší fixní náklady na jednotku mají vliv na cenu výkonu. V čem se oba přístupy liší, je posuzování dopadu výpůjčních nákladů na ocenění zásob. Úroky z cizího kapitálu v případě české účetní legislativy nelze považovat za náklad ovlivňující ocenění, kdežto v případě mezinárodních účetních standardů tyto do ocenění vstupují.

5. Závěr

V rámci příspěvku byl podroben analýze přístup české účetní legislativy daný zákonen o účetnictví č. 563/1991 Sb. a novelou prováděcí vyhlášky k tomuto zákonu č. 500/2002 Sb. ke stanovení hodnoty zásob hotových a nedokončených výrobků. Byly vymezeny základní problémové oblasti praktické aplikace tohoto zákonného znění včetně uvedení přístupu k ocenění výkonů vymezených dle interpretace I-35 Národní účetní rady. Tento zákonem stanovený postup byl rovněž srovnán s přístupem dle mezinárodních účetních standardů, kdy byly vymezeny oblasti, v nichž se oba přístupy shodují a oblasti, kde je přístup v ocenění odlišný.

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Assessment of Impact of Items Reducing Tax Base and Tax on Total Amount of Corporate Income Tax in the Czech Republic in Selected Sectors

Karolina Lisztwanová, Iveta Ratmanová [1]

Abstract

Regarding corporate income tax, the aim of this paper is to state a value of tax savings with respect selected items reducing tax base and with respect tax credits as items decreasing enterprises` final tax liability. We concentrate our attention not only on total summarized data but moreover on selected sectors. To be able to express and to compare changing of above-mentioned items, we use base and chain indexes, method of pyramidal decomposition of the top indicator i.e. the final tax liability and functional method that provides information about intensity of observed changes. As source data are used data provided by Ministry of Finance of the Czech Republic of the selected period between years 2005 and 2015. Regarding results of our research, we can confirm that impact of above mentioned items changed in the selected periods and that differ among selected sectors.

Key words

corporate income tax, total tax liability, tax savings, pyramidal decomposition, functional method

JEL Classification: H20, G30, K34, H25, G39

1. Introduction

Tax policy can be described as an important part of fiscal policy of countries. Good tax system must be convenient with main tax principles, i.e. fairness, effectiveness, flexibility, political transparency and legal perfection or administrative simplicity. Moreover, tax system should eliminate distortion effects of taxation and support desirable behaviour of tax subjects respecting stimulating function of taxes. There are different possibilities how to adjust individual taxes to these requirements, there are different ways how tax system can take into account above-mentioned aspects.

The aim of corporates` taxation is to tax taxable profit but also to encourage desirable activities of the enterprises. These intentions are not only related with individual adjustments of accounting profit to taxable profit. The corporate income taxation may use other aspects that may influence the final tax liability of the tax subjects. The individual items reducing tax base or tax reliefs for instance are considered the items with impact of changes in corporates` behaviour. Regarding these facts, the aim of our paper is to concentrate on development of the main items influencing and reducing the tax base, on the tax reliefs and on evaluation of their impact on corporates` final tax liability respecting differences among individual selected sectors within certain specified period.

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2. Theoretical Aspects of Determination of Tax Liability and Main Points of Methodology

With regard development and relationship between decreasing of statutory tax rate of corporate income tax and changes of corporate tax revenues, it is clear that despite declining of statutory tax rate, the ratio of total corporate tax revenues on total tax revenues increased (Lisztwanová, Ratmanová, 2017). Thanks to this can be mentioned that there are other aspects that are able to influence the total tax liability.

Nevertheless, the Czech tax legislation connected with corporate income tax that is stated with The Act. 586/1992 Coll., on Income Taxes has been amended more than hundred times since its inception. The most of these changes are connected with the area of the items reducing tax base or in the tax reliefs as well. Reductions from the tax base and the tax reliefs are considered as the way how to use stimulation function of the taxation that concentrates on supporting of specific purposes and is able to decrease the adjusted tax base. It can be defined that the items reducing tax base (IRTB) cut down the taxable profit whereas the tax reliefs (allowances) influence the calculated tax.

Relationship among the tax base, the deductions of the tax base, the tax reliefs and the final value of the tax can be determined with following formula (Lisztwanová, Ratmanová, 2017):

$$ATB.STR - IRTB.STR - TR = TTL, \quad (1)$$

where ATB is the adjusted tax base, STR is the statutory tax rate, IRTB are the items reducing the tax base, TR are the tax reliefs and TTL is the total tax liability. Respecting above-mentioned equation, we can define general rule of application of the items reducing tax base and the tax reliefs respecting long-term decisions (Lisztwanová, Ratmanová, 2017):

$$IRTB.STR \geq TR. \quad (2)$$

The conditions of application of the items reducing corporates` tax base changed for several times over the last twenty-five years. Within observed period only the one kind of deductions is used in all the time - the tax loss. The carry forward tax loss refers of applying previous year`s loss to reduce the current year`s profits for tax purposes (The Act. 586/1992 Coll., on Income Taxes). The period that may apply this type of the deductions decreased from seven years to five years.

The research and development expenses of enterprises that were used for the first time in the year 2005 are first example of item reducing tax base. Specific attribute of this item is usage of possibility to decrease the taxable earnings as taxable expenses and secondly as the item reducing tax base. This reality means that by this way the expenses influence the final tax liability twice. The taxpayers may reduce their tax base by implementing projects, which are in the form of experimental or theoretical works of design or construction work, calculations, designs, technologies, production of functional samples or product prototypes (Hamáček, 2011). This deductible item can be applied for up to three tax years that follow immediately the taxable period with incurring of the deduction.

The aim of the third reduction of the tax base via the value of gratuitous transactions (donations) is to support financing of education, culture, social, medical, environmental, humanitarian or charitable purposes. The value of the donations is able to decrease the tax base in the taxable period that the gift is demonstrably provided in. Moreover, the tax law specifies the minimum value of the donations.

The tax reliefs, allowances affect the final tax differently then above-mentioned adjustments of the tax base. The tax reliefs respecting the Czech tax law concern investment incentives. The incentives are the one of the form of public business supporting and can be applied up to the amount of the tax liability. This kind of the tax reliefs may be used for five consecutive tax period. The first taxable period for which the reliefs can be applied is the tax

period in which the taxpayer fulfilled the general conditions under the legislation connected with the investment incentives and the special conditions set out in The Act. 586/1992 Coll., on Income Taxes.

Finally, thanks to this tax act we can identify the possibility to apply the tax reliefs connected with employing persons with disabilities. According to this, employers may reduce their tax liability for each employee with the disability.

Assessment of changes of individual earlier mentioned items is way how to understand to their impact on the total tax liability. We applied specific statistic approach in order to calculate it. Thanks to this, we used methodology of pyramidal decomposition. The pyramidal decomposition of individual indicator is derived from idea that a top indicator can be decomposed on partial indicators (Dluhošová, 2010). Moreover, it is possible mathematically to identify the certain operations among these individual indicators. Finally, can be stated that the change of the top indicator is explained through the changes of the individual indicators:

$$\Delta y_x = \sum_i \Delta x_{a_i}, \tag{3}$$

where x is the analysed indicator, Δy_x is increment in the influence of the analysed indicator, a_i is the indicator by which Δy_x can by partially explained, Δx_{a_i} is the influence of the indicator a_i on analysed indicator x.

In such systems, two types of the operations can be identified - additive or multiplicative. The additive operation is stated as:

$$\Delta x_{a_i} = \frac{\Delta a_i}{\sum_i \Delta a_i} \cdot \Delta y_x, \tag{4}$$

where $\Delta a_i = a_{i,1} - a_{i,0}$, $a_{i,0}$, and $a_{i,1}$ is the value of the indicator i respecting starting (0) and ending (1) state.

The multiplicative operations can be calculated with different methods. The one of them is the functional method that expresses the combined simultaneous impact of the all indicators explaining of their influence on the top indicator (Zmeškal, Dluhošová, Tichý, 2004). As input information, the relative changes of the items are used. Respecting the multiplicative operation between two indicators, the influences can be formulated as:

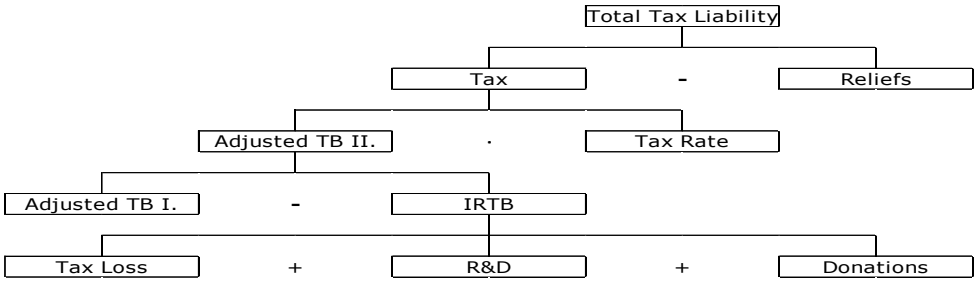
$$\Delta x_{a_1} = \frac{1}{R_x} \cdot R_{a_1} \cdot \left(1 + \frac{1}{2} \cdot R_{a_2}\right) \cdot \Delta y_x, \tag{5}$$

$$\Delta x_{a_2} = \frac{1}{R_x} \cdot R_{a_2} \cdot \left(1 + \frac{1}{2} \cdot R_{a_1}\right) \cdot \Delta y_x. \tag{6}$$

where R_{a_i} and R_x are relative changes of indicators.

We created the following pyramidal decomposition of the top indicator (figure 1) so that we could calculate the power of influences of the changes of the individual items on the changes of the total tax liability. The main aim of this decomposition was to state the influences of the changes of the items reducing tax base, the tax reliefs,the tax rate and the adjusted tax base I. on the total tax liability. The item of the adjusted tax base I. represents corrected accounting profit respecting adjustments mentioned in section mark 23, 24 and 25 ot The Act. 586/1992 Coll., on Income Taxes.

Figure 1: Pyramidal decomposition of total tax liability



Source: Authors` processing according data of Finanční správa

3. Assessment of Influence of Individual Indicators

Regarding information mentioned above the items reducing tax base and the tax reliefs can be considered as the indirect form of business` support. Intensity of these ones changed in the Czech Republic within the observed period between years 2005 and 2015. As input data, we used information provided by Finanční správa that provides annual value of the earnings before taxation (EBT), the tax base, and the items reducing tax base, the tax reliefs and the total tax liability. The following table 1 describes the changes of all items that were determined in our structure of the pyramidal decomposition of the total tax liability.

Table 1: Changes of selected items within period 2005 – 2015 in v %

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
TTL	100.0	114.1	132.7	117.5	95.7	93.4	92.4	93.6	97.9	106.8	115.9
STR	100.0	92.3	92.3	80.8	76.9	73.1	73.1	73.1	73.1	73.1	73.1
ATB I.	100.0	118.3	134.3	139.5	121.8	124.9	128.5	126.2	143.1	151.2	152.6
Tax Loss	100.0	81.5	69.1	107.7	105.7	97.9	125.0	84.5	105.6	84.3	85.9
R&D	100.0	131.7	160.1	147.9	157.2	213.4	305.2	325.3	385.2	375.9	421.5
Donations	100.0	116.2	116.2	111.9	110.4	114.0	122.5	121.8	171.5	170.4	172.0
Reliefs	100.0	118.6	139.5	88.7	85.4	108.4	127.1	167.0	413.5	431.1	171.0

Source: Authors` calculation according data of Finanční správa

As can be seen in the table 1 that used the base index and the basic values were data of the year 2005 of individual items, the final total tax liability changed during selected period.

Table 2: Year-on-year changes of selected items within period 2005 – 2015 in %

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
TTL	100.0	114.1	116.3	88.5	81.5	97.6	99.0	101.3	104.5	109.1	108.5
STR	100.0	92.3	100.0	87.5	95.2	95.0	100.0	100.0	100.0	100.0	100.0
ATB I.	100.0	118.3	113.5	103.8	87.3	102.5	102.9	98.2	113.4	105.6	101.0
Tax Loss	100.0	81.5	84.9	155.9	98.1	92.6	127.7	67.6	124.9	79.9	101.9
R&D	100.0	131.7	121.6	92.3	106.3	135.8	143.0	106.6	118.4	97.6	112.1
Donations	100.0	116.2	100.0	96.3	98.7	103.2	107.5	99.4	140.8	99.4	100.9
Reliefs	100.0	118.6	117.6	63.6	96.3	126.9	117.2	131.5	247.5	104.2	39.7

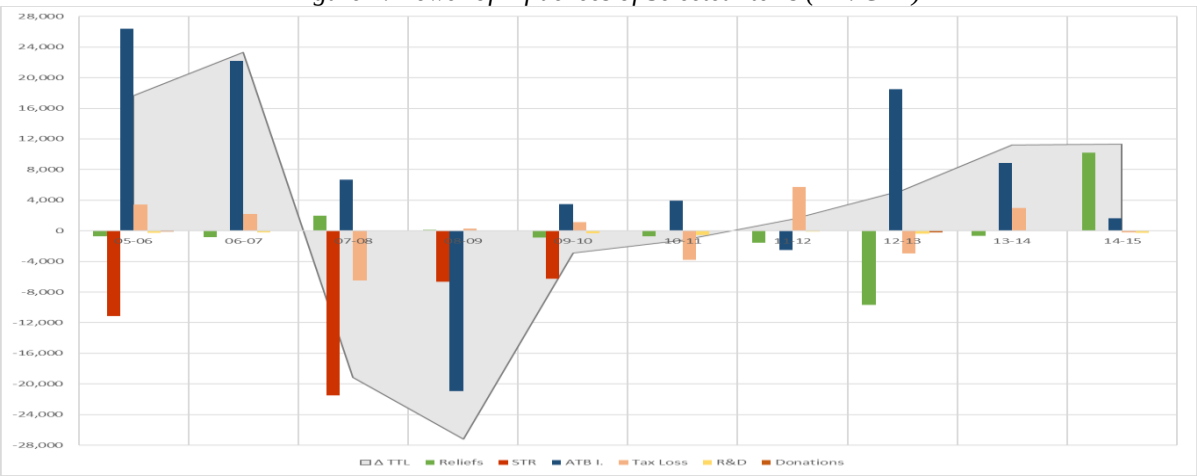
Source: Authors` calculation according data of Finanční správa

The data stated in the table 2 differs from the data of the table 1 thanks to calculation of year-on-year changes. We can state that statutory tax rate has not been changed since the year 2012. Moreover, the year 2012 was the first year after crisis period that generated higher value of the total tax liability. Impact of financial crisis is observed in case of loss carry-forward

allowance as well. On the other hand, possible trouble with usage and proving of research expenses can be seen recently. When it comes donation is clear that corporates use it in good times and use them less or not at all in bed times.

The figure 2 shows detailed results of our analysis. Their data express power of inluence of the individual indicators of selected periods in mil. CZK and moreover as percentage values. As input information we use the changes of the total tax liability between two following periods. Moreover, the impact of the individual items is expressed as decreasing or increasing of the total tax liability. Consequently, the total sum of changes of the individual items is equal to the change of the total tax liability. Respecting presented data can be confirmed that increasing of the total tax liability was not only influenced by increasing of all items. Explanation of this development we can find with usage of the functional method as tool of evaluating of the final impact of the partial indicators on the top indicator.

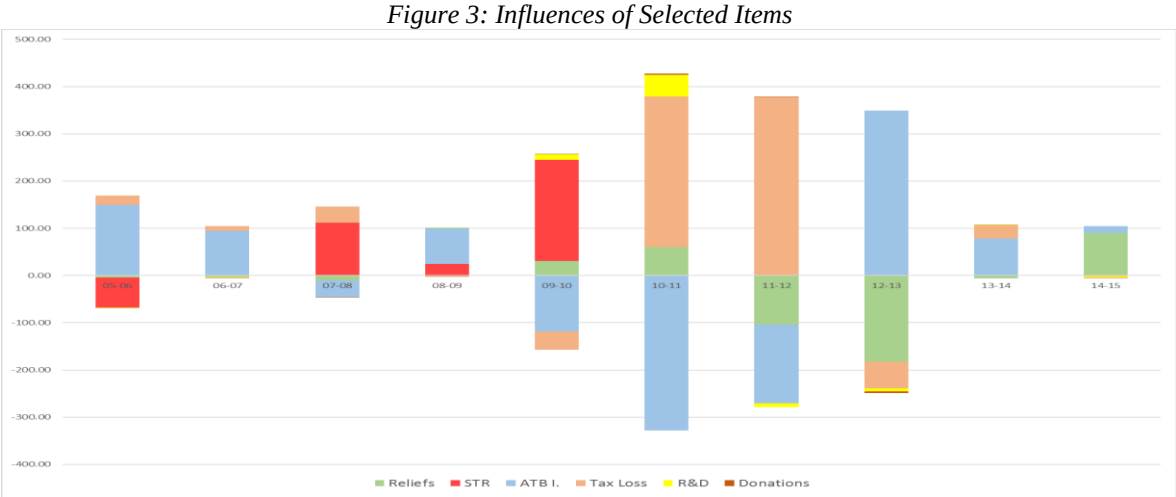
Figure 2: Power of Influences of Selected Items (mil. CZK)



Source: Authors` calculation according data of Finanční správa

The power of individual indicator can be described moreover by percentage value as can be seen in Figure 3.

Figure 3: Influences of Selected Items



Source: Authors` calculation according data of Finanční správa

Instead of expressing percentage and absolute value of impact of changes of the individual indicators we were able to state moreover order of their influences as can be seen in table 3.

Table 3: Order of influences of individual indicators

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Reliefs	4	3	4	4	4	4	3	2	3	1
STR	2	6	1	2	1	6	6	6	6	6
ATB I.	1	1	2	1	2	2	2	1	1	2
Tax Loss	3	2	3	3	3	1	1	3	2	4
R&D	5	4	5	5	5	3	4	4	4	3
Donations	6	5	6	6	6	5	5	5	5	5

Source: Authors` calculation according data of Finanční správa

With regard data of above mentioned tables 3, and in figures 2 and 3 is clear that not only changes of tax loss and the tax reliefs impacted the total tax liability. Nevertheless, the adjustment tax base I. was the most important indicator that brought the highest value of corporates` tax savings. The final impact of changes of the statutory tax rate was not important for all selected time period. We can confirm its impact only in time period when this indicatar had been changed and later its importancy decreased. Taking into account the total ratio of donations the value of the total tax savings does not change significantly and can be stated that its development is the approximately same. The item that required our attention was the tax saving arising thanks to the deductible item to support research and development expenses of enterprises. As can be seen, its influencing on the total value of the tax savings increased especially in the last five years. This increasing can be explained, for example, thanks to changes made by Ministry of Finance of the Czech Republic that reevaluated research not only as subject of interest of research-development organizations, but can also took place in ordinary businesses. Ministry of Finance of the Czech Republic in the year 2010 also identified other criteria for the assessment of the research and development activities, and thanks to these the taxpayers are not obliged to meet all these criteria at the same time. Moreover, can be claimed (Hamáček, 2011) that the administrative burden of this deduction is much lower compared to direct grant.

4. Assessment of Influence of Individual Indicators of Selected Sectors

Except evaluation of changes of all subjects, we moreover evaluated agriculture, processing industry and construction industry as well. Following tables describe order of the impact of the individual indicators. Regarding data mentioned in table 4 we can see that statutory tax rate the most influenced the total tax liability in case of the construction industry. Moreover, can be observed because we evaluated six indicators, that the influence of this one was the lowest in the last three periods.

Table 4: Order of influences of individual indicator: statutory tax rate

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	2	6	1	2	3	6	5	6	6	6
Processing Industry	2	6	2	2	3	6	6	6	6	6
Construction Industry	2	6	1	1	1	6	6	6	6	6

Source: Authors` calculation according data of Finanční správa

When it comes to the indicator adjusted tax base I. we can see in the table 5 that the total tax liability of the processing sector is the most influenced by this one. It means that this sector the most intense used the items that are able to influence the adjustments of the tax base. Moreover, comparing the power of impact of this indicator of the rest of sectors it is

clear that the indicator adjusted tax base I. plays the significant role in case of the rest of sectors.

Table 5: Order of influences of individual indicator: adjusted tax base I.

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	1	1	3	1	1	1	2	1	1	1
Processing Industry	1	1	1	1	1	1	1	2	1	1
Construction Industry	1	1	2	3	2	1	1	1	1	1

Source: Authors` calculation according data of Finanční správa

Following tables 6, 7 and 8 are connected with the items that are able to reduce the tax base. The items reducing the tax base are used to support the desirable activities of the enterprises and finally may influence the final tax liability. Their impact can be indirectly expressed throughout decreasing of value of the tax base. The table 6 provides information about the effect of the first of them: the loss carry-forward allowance on the total tax liability. Thanks to our results can be claimed that the agriculture sector was been the most influenced with the changes of this one.

Table 6: Order of influences of individual indicator: tax loss

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	3	2	2	3	2	2	1	2	2	2
Processing Industry	4	3	4	3	2	3	3	4	3	2
Construction Industry	3	2	3	5	4	3	4	2	2	2

Source: Authors` calculation according data of Finanční správa

In addition to the tax loss expenses, expenses on the research and the development as the following items reducing the tax base data presented in table 7 surprisingly show that the construction industry the most utilized this item in the observed period. On the other hand, we can state that the level of the research expenses are low in case of the agriculture sector. Moreover, assessing impact of this factor on the total tax liability, it is clear that this indicator is not often used. However, respecting its trend it is possible to confirm that the research expenses applied by companies grows in the observed period.

Table 7: Order of influences of individual indicator: research and development expenses

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	5	5	5	6	6	4	4	5	4	3
Processing Industry	5	5	5	5	5	4	4	3	5	4
Construction Industry	4	3	4	4	5	2	3	3	3	5

Source: Authors` calculation according data of Finanční správa

The gratuitous transactions are items that are able to decrease corporates` tax base but neverhtheless, they the least affected the total tax liability in our selected periods. This fact can be explained with the willingness of the corporates to finance or to support the specific subjects respecting certain purposes mentioned by the tax legislation. The final results and order of the selected sectors are presented in table 8.

Table 8: Order of influences of individual indicator: donations

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	6	4	6	5	5	5	6	4	5	5
Processing Industry	6	4	6	6	6	5	5	5	4	5
Construction Industry	6	5	6	6	6	4	5	5	5	3

Source: Authors` calculation according data of Finanční správa

Comparing the impact of the items reducing tax base and the tax reliefs they differ each other. It is understandable because the items reducing the tax base directly influence only the tax base as opposed to the reliefs that directly change the value of the final tax liability.

Table 9: Order of influences of individual indicator: reliefs

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15
Agriculture	4	3	4	4	4	3	3	3	3	4
Processing Industry	3	2	3	4	4	2	2	1	2	3
Construction Industry	5	4	5	2	3	5	2	4	4	4

Source: Authors` calculation according data of Finanční správa

Thanks to data mentioned in table 9 we can observe the highest influence in case of the processing industry. These reliefs were connected first of all with the investment incentives and with the employment of handicap people.

5. Conclusion

In our research we concentrated our attention on the way, how to express the impact of the individual indicators on the final tax liability in case of the corporate income tax of the selected period with respect not only total data provided by Finanční správa but moreover we tried to express the impact of these indicator of three selected sectors. We used statistical approach so that we could be able to state the power of the influence of the individual indicators. With regard data coming from our research we can generally confirm that the impact of these ones differ within the selected period. We identified the biggest impact of the adjustment of tax base I. following by application of the tax loss and by the tax reliefs. The impact of changes of the statutory tax rate was lower then the impact of previous items but was higher then usage of decreasing of the tax base with the expenses on research. The smallest impact was connected with the donations.

The same proceeding was used to evaluate the impact of individual items of the selected sectors. Thanks to our calculations, we could identify the sectors that were the most influenced by the changes of the selected items. In case of the agriculture sector we identified the greater impact of the adjusted tax base I. and the tax loss, in case of the processing industry the biggest impact of the adjusted tax base I. and the reliefs and in case of the construction industry the greater impact of the adjusted tax base I. and the tax loss. On the other hand, the donations and the research and development expenses were identified as items with the weakest impact on the final tax liability in the case of agriculture and processing sector. The donations and the reliefs were determined as items with the weakest influence when it comes the construction industry.

Acknowledgments

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Semiparametric Tests for Behavioral Finance Efficiency

Matteo Malavasi [1], Sergio Ortobelli[2]

Abstract

We propose semiparametric tests for portfolio efficiency, with respect to different Behavioral Finance orderings. In particular, we focus on Markovitz order and Prospect order. We assume that return distributions belong to a scale invariant family, weakly determined by a finite number of parameters: a reward measure, a risk measure and other distributional parameters. We recall stochastic dominance rules for such family of distributions and provide efficiency conditions when the reward measure is isotonic with Markovitz or Prospect type of investors' preference. Finally, we empirically test portfolio efficiency (in the sense of Markovitz and prospect orderings) when return distribution is uniquely determined by four parameters, using estimation function theory.

Key words

Semi-parametric, Behavioral Finance, Stochastic Dominance Efficiency

JEL Classification: C12, C13, C15, C44, D81, G11.

1. Introduction

In Economics and Finance, dealing with decisions under uncertainty is of most importance. Most of the models falls into the Von Neumann and Morgenstern Expected Utility framework (Von Neumann and Morgenstern (1944)). Nevertheless, many experimental studies point out contradiction in this paradigm. Prospect theory, proposed by Kahneman and Tversky in (1979) relies on four pillar: investors take decision according to change of wealth, not total or final wealth, are risk averse for gains and risk seeker for losses(have S-shaped utility), look at subjective probabilities, rather than objective ones and suffers from the "framing" effect (see Kahneman and Tversky (1979) and Tversky and Kahneman (1985)). Markovitz, in 1952 suggests that investors' utility function has a fourfold behavior: convex-concave-convex-concave. These investors are risk averse for losses and risk seeker for gains, while in case of extreme events are risk seeking for losses and risk averse for gains (inverse S-shaped utility). (Markovitz (1952)).

The most used tool, in assessing optimal choices under uncertainty is Stochastic dominance, which establish general rules valid for all the investors with utility function belonging to the same class. In particular, First Order of Stochastic Dominance (FSD), to which correspond optimal choices for non satiable investors, Increasing and Concave Stochastic Dominance (ICV), to which correspond optimal choices of non satiable and risk averse investors and Increasing and Convex Stochastic Dominance (ICX) to which correspond optimal choices for

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non satiable and risk seeking investors. In this study we focus on semiparametric test for portfolio efficiency. Similarly, there exist also coherent orderings with Kaneman and Tverskty and Markovitz utility functions, namely Prospect Stochastic Dominance(PSD) and Markovitz Stochastic Dominance (MSD).(Levy and Levy (2002)). All these ordering can be addressed into the class of FORS orderings (Ortobelli et al. (2009) (2013)).

In this paper, we focus on portfolio efficiency testing and propose and empirically compare MSD and PSD semiparametric tests. To do so, we first assume that all portfolio gross return distributions belong to a scale invariant family, weakly determined by a finite number of parameters and recall sufficient and necessary dominance conditions valid for distribution belonging to this family. This class is rich enough also to include higher moment of the distribution such as skewness and kurtosis. Then, using efficient function theory we proposed a methodology to semiparametric test for PSD and MSD and finally we empirically compare their results.

In section 2 we briefly describe Markovitz Stochastic Dominance and Prospect Stochastic Dominance, in section 3 we present the family of scale invariant distributions and discuss its properties linked to stochastic dominance. In section 4 we combine MSD and PSD, the properties of the scale invariant family of distributions and estimation function theory to present semi parametric MSD and PSD efficiency hypothesis tests.

2. Markovitz and Prospect Stochastic Dominance

In this section, we describe Markovitz Stochastic Dominance (MSD) and Prospect Stochastic Dominance (PSD). (Levy and Levy (2002) and Baucells and Heukamp (2006)). Let $X$and Y be two random variables defined on a common probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with distribution functions. F and G respectively, we say that F dominates G in the Markovitz Stochastic Dominance, i.e. $F\ MSD\ G$, if and only if $\int_{-\infty}^{y} G(t) - F(t)dt \geq 0, \forall y \leq 0$ and $\int_{x}^{\infty} G(t) - F(t) \geq 0, \forall x \geq 0$. A MSD non dominated choice correspond to an efficient choice for investors with so called inverse S-shaped utility function, which exhibit a fourfold behavior: convex-concave-convex-concave. These investors are risk averse for losses and risk seeker for gains, while\doubleE in case of extreme events are risk seeking for losses and risk averse for gains (Markovitz (1952) and Levy and Levy (2002)). Analogously, we say that F dominates G in the sense of Prospect Stochastic Dominance, i.e. $F\ PSD\ G$ if and only if $\int_{y}^{0} G(t) - F(t)dt \geq 0, \forall y \leq 0$ and $\int_{0}^{x} G(t) - F(t) \geq 0, \forall x \geq 0$. A PSD non dominated choices correspond to efficient choices for investors with the so called S-shaped utility function, concave for gain and convex for losses (Kahneman and Tversky (1979)). Aforementioned orderings can be addressed into the class of FORS orderings (Ortobelli et al. (2009) (2013)) In particular, MSD admits as a FORS risk measure the following functional, $\forall y \leq 0, x \geq 0\ \lambda \in [0,1]$:

$$m_{\lambda,y,x}(X) = \lambda \mathbb{E}\left[(y - X)\mathbb{I}_{X \leq y}\right] + (1 - \lambda)\mathbb{E}[(x - X)\mathbb{I}_{X \geq x}] \tag{1}$$

and PSD admits as FORS risk measure the following functional, $\forall y \leq 0, \lambda \in [0,1]$:

$$p_{\lambda,y,x}(X) = \lambda\left(y\mathbb{P}[X \geq 0] + \mathbb{E}\left[(y - X)\mathbb{I}_{X \in [y,0]}\right]\right) + (1 - \lambda)\left(x\mathbb{P}[X \leq 0] + \mathbb{E}\left[(x - X)\mathbb{I}_{X \in [0,x]}\right]\right) \tag{2}$$

The functionals in (1) and (2) identifies the distribution of X varying the parameters y, x, λ, and are consistent with monotony order (in the sense that are implied by the FSD). Moreover, by the consistency of FORS risk measure, whenever we have $F\ MSD(PSD)\ G$ then $m_{\lambda,y}(X) \leq$

$m_{\lambda,y}(Y)$ ($p_{\lambda,y}(X) \leq p_{\lambda,y}(Y)$) (Ortobelli et al. (2009)). Thus, they are good candidate as reward measure in the following context.

3. Classification of Distribution Functions

Consider a portfolio $X = Zx$, where $Z_{N\times T} = \left(Z_{i,j}\right)_{i=1,\dots,N,j=1,\dots,T}$ is the matrix of historical observations of the gross returns, where $Z_{i,j}$ is ther gross return of the i asset at time and $x \in \Delta^n$, with Δ^n being the n-dimensional standard simplex. Under institutional restrictions, such as limited liabilities and no short sales allowed, portfolios gross returns can be considered as realizations of positive random variables (see Ortobelli (2001)). In particular, we assume that portfolio gross return distributions belong to a scale invariant family, called $\sigma\tau_q^+(\bar{a})$, weakly determined by a finite number of parameters such that for every $F_X \in \sigma_q^+(\bar{a})$ there exists a random variable X with distribution function F_X identified by q parameters $(\mu(X), \rho(X), a_1(X), \dots, a_{q-2}(X)$ where $\mu(X)$, is a reward measure and $\rho(X)$ is a risk measure.

By definitions of $m_{\lambda,y}(X)$ and $p_{\lambda,y}(X)$, $\mu(X) = -m_{\lambda,y}(X)$ and $\mu(X) = -p_{\lambda,y}(X)$ are legitimate reward measures for the class $\sigma\tau_q^+(\bar{a})$. Moreover, in this way μ is a reward measure isotonic with MSD or PSD.

The $\sigma\tau_q^+(\bar{a})$ has some interesting properties associated with reward measures as reported by Ortobelli et al. (2017). Assuming translation invariant risk measure (i.e. $\rho(X + t) = \rho(X)$), if there exist a random variable with distribution belonging to the class with reward measure greater than the mean, then all random variables with distribution in $\sigma\tau_q^+(\bar{a})$ have reward measure grater than the mean. Second, let X and Y be two portfolios with distribution functions in $\sigma\tau_q^+(\bar{a})$ with translation invariant risk measure, then $\mu(X) \geq \mu(Y)$ and $\rho(X) \leq \rho(Y)$ implies that $X\ ICV\ Y$. Third, when the reward measure is isotonic with given investors' preference, we can even more characterize investors' behavior. In fact, when a reward measure is greater than the mean, it is isotonic with risk seeking prevalent behavior, while those lower than the mean are isotonic with risk averse prevalent behavior. In other words, maximizing any reward measure greater than the mean will give a ICX non dominated portfolio, while maximizing a reward measure lower than the mean will give a ICV non dominated portfolio.

3.1 Non Dominated choices in the family $\boldsymbol{\sigma\tau_q^+(\bar{a})}$

Combining the properties of the family $\sigma\tau_q^+(\bar{a})$ with the MSD or PSD orderings we can construct portfolio dominating a given benchmark. Assume all portfolio gross return distributions belong to $\sigma\tau_4^+(\bar{a})$ weakly determined by $(\mu(X), \rho(X), a_1(X), a_2(X))$, where ρ is translation invariant. Suppose to have a benchmark portfolio P with parameters $(\mu(P), \rho(P), s, k)$ where s is skewness and k is the kurtosis and $\rho(X)$ is translation invariant. If the portfolio P is not FSD dominated, then all non satiable risk seeker investors prefers the solution of the following optimization problem:

$$\begin{gathered} \max \boldsymbol{\mu}(\boldsymbol{X}) \\ \text{s.t. } \mathbb{E}(X) \geq \mathbb{E}(P) \\ \rho(X) \leq \rho(P) \\ a_1(X) = s \\ a_{2(X)} = k \\ x \in \Delta^n \end{gathered} \tag{3}$$

with $\mu(X) = -m_{\lambda,y,x}(X)$ or $\mu(X) = -p_{\lambda,y,x}(X)$. Similarly, all non satiable and risk averse investors, prefers the solution of the following optimization problem:

$$\max \boldsymbol{\mu}(\boldsymbol{X}) \tag{4}$$
$$\text{s.t. } \mathbb{E}(X) \leq \mathbb{E}(P)$$
$$\rho(X) \leq \rho(P)$$
$$a_1(X) = s$$
$$a_{2(X)} = k$$
$$x \in \Delta^n$$

with $\mu(X) = -m_{\lambda,y,x}(X)$ or $\mu(X) = -p_{\lambda,y,x}(X)$. Moreover, portfolio solutions of problems (3) and (4) are also preferred by investors with Markovitz or Prospect type of Utility functions.

4. Estimation Function Theory

In this section we describe how estimation function theory can be combined with previous section results, in order to obtain test statistic for the hypothesis tests. Following Godambe e Thompson (1989) we consider the following unbiased and mutually orthogonal estimation functions, based on the first four distribution moments.

Consider a portfolio $X = Rx$, with distribution $F_X \in \sigma_4^+(\bar{a})$ with parameters $(\mu(X), \rho(X), a_1(X), a_2(X))$ where $\mu(X)$ is a reward mesure, $\rho(X)^2 = \mathbb{E}\left[\left(f\left(R_t\right) - \mu(X)\right)^2\right]$, $a_1(X) = \frac{\mathbb{E}\left[\left(f\left(R\right)_t - \mu(X)\right)^3\right]}{\mathbb{E}\left[\left(f\left(R_t\right) - \mu(X)\right)^2\right]^{3/2}}$ and $a_2 = \frac{\mathbb{E}\left[\left(f\left(R_t\right) - \mu(X)\right)^4\right]}{\mathbb{E}\left[\left(f\left(R_t\right) - \mu(X)\right)^2\right]^2}$. Then estimators for $\mu(X)$ and $\rho(X)$ can be obtained as follows. Consider as estimation functions:

$$\boldsymbol{h_1(R_t, \mu) = f(R_t) - \mu(X)}, \tag{4}$$
$$h_2(R_t, \mu, \rho) = (f(R_t) - \mu(X))^2 - \rho^2(X) - a_1(X)\rho(X)(f(R_t) - \mu(X)) \tag{5}$$

where f is a real valued measurable function, such that $\mathbb{E}[f(R_t)] = \mu(X)$. The estimators $\hat{\mu}(X), \hat{\rho}(X)$, are the solutions of the system of equations (see Crowder (1986)):

$$l_\mu = \sum_{t=1}^{T} (d_{1,t} h_1(R_t, \mu) + b_{1,t} h_2(R_t, \mu, \rho)$$
$$l_\rho = \sum_{t=1}^{T} b_{2,t} h_2(R_t, \mu, \rho)$$

where $d_{1,t} = \frac{\mathbb{E}\left[\frac{\partial h_1(R_t,\mu)}{\partial \mu}\right]}{\mathbb{E}\left[h_1^2(R_t,\mu)\right]} = \frac{-1}{\rho^2(X)}$, $b_{1,t} = \frac{\mathbb{E}\left[\frac{\partial h_2(R_t,\mu)}{\partial \mu}\right]}{\mathbb{E}\left[h_2^2(R_t,\mu)\right]} = \frac{a_1(X)}{\rho^3(X)(a_2(X) - 1 - a_1(X)^2)}$ and $b_{2,t} = \frac{\mathbb{E}\left[\frac{\partial h_2(R_t,\mu)}{\partial \rho}\right]}{\mathbb{E}\left[h_2^2(R_t,\mu)\right]} = \frac{-2}{\rho^3(X)(a_2(X) - 1 - a_1(X)^2)}$. The joint estimators $\hat{\mu}(X)$ and $\hat{\rho}(X)$, are the following:

$$\boldsymbol{\hat{\mu}(X) = \frac{1}{T} \sum_{t=1}^{T} f(R_t)}, \tag{6}$$

$$\hat{\rho}(X) = \left(\frac{1}{T} \sum_{t=1}^{T} (f(R_t) - \hat{\mu}(X))^2 \right)^{1/2} \tag{7}$$

Then, the random vector $T^{\frac{1}{2}}\big(\hat{\mu}(X) - \mu(X), \hat{\rho}(X) - \rho(X)\big)$ converges in distribution to a normal distribution with mean vector (0,0) and covariance matrix V^{-1}, where $V = \big(v_{k,h}\big)$, $k, h = 1,2$, with $v_{1,1} = \mathbb{E}\left[\frac{\partial l_\mu}{\partial \mu}\right] = \frac{a_2 - 1}{\rho\text{^}2(a_2 - 1 - a_1^2)}$, $v_{1,2} = v_{2,1} = \mathbb{E}\left[\frac{\partial l_\mu}{\partial \rho}\right] = \frac{-2a_1}{\rho\text{^}2(a_2 - 1 - a_1^2)}$ and $v_{2,2} = \mathbb{E}\left[\frac{\partial l_\rho}{\partial \rho}\right] = \frac{4}{\rho\text{^}2(a_2 - 1 - a_1^2)}$. Thus, by choosing the suitable function f in the estimators function (4) and (5) we get estimators for the set of parameters, weakly determining portfolio distributions. When the reward measure is $\mu(X) = -m_{\lambda,y}(X)$, we use $f(R_t) = -\lambda(y - R_t x)\mathbb{I}_{y - R_t x \geq 0} - (1 - \lambda)(\bar{x} - R_t x)\mathbb{I}_{\bar{x} - R_t x \geq 0}$. When the reward measure is $\mu(X) = -p_{\lambda,y}(X)$, we use $f(R_t) = -\lambda\big(y\mathbb{I}_{R_t x > 0} + (y - R_t x)\mathbb{I}_{y \leq R_t x \leq 0}\big) - (1 - \lambda)\big(\bar{x}\mathbb{I}_{R_t x \leq 0} + (\bar{x} - R_t x)\mathbb{I}_{0 \leq R_t x \leq \bar{x}}\big)$.

4.1 PSD and MSD efficiency test

In this subsection, we describe a methodology to test for efficiency of a given portfolio, say P, in the sense of PSD or MSD, with respect to a benchmark.
Assume all portfolio distributions belong to the family $\sigma\tau_4^+(\bar{a})$. Let P be a portfolio with parameters $(\mu(X),\ \rho(X),\ s,\ k)$. First, we run the testing procedure in Scaillet and Topaloglou (2010) to test whether the portfolio is not FSD dominated. Then we want to test whether the portfolio P is stochastically dominated, in the sense of Markovitz dominance or Prospect dominance, by a portfolio with paramters $\mu(X)$, equal to $-m_{\lambda,y,x}(X)$ or $-p_{\lambda,y,x}(X)$, $\rho(X)^2 = \mathbb{E}\left[\big(f(R_t) - \mu(X)\big)^2\right]$, $a_1(X) = \frac{\mathbb{E}\left[(f(R)_t - \mu(X))^3\right]}{\mathbb{E}\left[(f(R_t) - \mu(X))^2\right]^{3/2}}$ and $a_2 = \frac{\mathbb{E}\left[(f(R_t) - \mu(X))^4\right]}{\mathbb{E}\left[(f(R_t) - \mu(X))^2\right]^2}$
To do so we solve optimization problems (3) and (4), with both reward measures, to construct dominating portfolio candidates X^{rs} and X^{ra}, respectively.
Consider X^{rs}, with reward measure $\mu(X^{rs})$ and risk measure $\rho(X^{rs})$. When the reward measure $\mu(X^{rs}) = -m_{\lambda,y}(X^{rs})$, to test for MSD we perform the efficiency test for ICX order, as proposed by Ortobelli (2001) and Ortobelli et al. (2017). The null and alternative hypothesis then can be formulated as:

$$\boldsymbol{H_0^1}\text{: Portfolio P is not ICX dominated} \qquad \boldsymbol{H_1^1}\text{: } \boldsymbol{\mu(P) - \mu(X) \geq 0}$$
$$\rho(P) - \rho(X) \geq 0$$

The test statistic is the vector $\hat{\theta} = (\hat{\mu}(X) - \hat{\mu}(P), \hat{\rho}(X) - \hat{\rho}(P))$ and the rejection region can be defined as $C^1 = \left\{T^{1/2}\hat{\theta} \in \left[c_{1,\alpha}^1, \infty\right) \times \left[c_{2,\alpha}^1, \infty\right)\right\}$ where, for a give test size α, the values $c_{1,\alpha}^1$, $c_{2,\alpha}^1$ are taken such that $\lim_{T \to \infty} \mathbb{P}\left[T^{1/2}\hat{\theta} \in C^1 \mid H_0\text{^}1\right] \leq \int_{c_{1,\alpha}^1}^{\infty} \int_{c_{2,\alpha}^1}^{\infty} \phi(x,y)dxdy = \alpha$, where by the properties of estimation functions, $\phi(x,y)$ is the density of a bivariate Gaussian distribution, with variance covariance matrix described previously. In the case of X^{ra}, with reward measure $\mu(X^{ra})$ and risk measure $\rho(X^{rs})$, following Ortobelli (2001) and Ortobelli et al. (2017), when the reward measure is $\mu(X^{rs}) = -m_{\lambda,y}(X^{rs})$, to test for MSD we perform the following ICV test:

$$\boldsymbol{H_0^1}\text{: Portfolio P is not ICV dominated} \qquad \boldsymbol{H_1^2}\text{: } \boldsymbol{\mu(X) - \mu(P) > 0}$$
$$\rho(X) - \rho(P) < 0$$

In this case, the rejection region is defined as $C^2 = \{T^{1/2}\hat{\theta} \in [c_{1,\alpha}^2, \infty) \times (-\infty, c_{2,\alpha}^2]\}$ and for a given test size α, $c_{1,\alpha}^2$ and $c_{2,\alpha}^2$ are chosen such that $\lim_{T\to\infty} \mathbb{P}[T^{1/2}\hat{\theta} \in C^2 | H_0\text{^}1] \leq \int_{c_{1,\alpha}^2}^{\infty} \int_{-\infty}^{c_{2,\alpha}^2} \phi(x,y)dxdy = \alpha$, with $c_{2,\alpha}$^2 being a non positive real number.
Analogously, the PSD test can be performed using $\mu(X) = -p_{\lambda,y}(X)$.

5. *Empirical Studies*

In this section we present an empirical comparison of the aforementioned tests. We investigate the possibility of constructing a portfolio able to dominates the index in the sense of MSD or PSD of stocks of S&P 500. Since the efficiency of market portfolio and investors preferences may change over time, we perform a rolling window type of analysis every 21 days starting from January 3, 2000 to June 21, 2017. We proceed as follows. Under the assumption that all portfolio gross return distribution belongs to the $\sigma\tau_4^+(\bar{a})$ weakly determined by $(\mu(X), \rho(X), a_1(X), a_2(X))$, where when we test for MSD $\mu(X) = -m_{\lambda,y}(X)$, when we test for PSD $\mu(x) = -p_{\lambda,y}(X)$ and the other parameter are defined as in section 4. We solve the optimization problems in (3) with both reward measures, and then perform the hypothesis test H_0^1 vs H_1^1. Then, we solve optimization problem (4) and perform hypothesis test H_0^1 vs H_1^1. We estimate the test statistics using estimation function theory described in Section 4, and evaluate the evaluate the p-values. We reject at 5%.

Type of investors	*Reward measure*	*Ordering*	*Confidence level (α)*	*% Dominance*
Risk seeker	m_{λ^*,y^*x^*}	MSD	95%	100
Risk averse	m_{λ_*,y_*,x_*}	MSD	95%	100
Risk seeker	p_{λ^*,y^*,x^*}	PSD	95%	100
Risk averse	p_{λ_*,y_*,x_*}	PSD	95%	93.0233

Table 1. Percentage of acceptance of the null in both cases MSD or PSD.

Table 1 shows results of the empirical part: in the last column are reported the acceptance reate for the null hypothesis in both case MSD and PSD. In the case of MSD we accept the null the 100% of times both for risk seeker and risk averse investors. While for PSD the acceptance rate is a bit lower, approximately at 93%, for risk averse investors.

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6. Conclusion

In this paper we proposed a semiparametric test for Stochastic dominance efficiency with respet to behavioral finance order. We focused in Prospect Stochastic Dominace and Markovitz Stochastic Dominance. PSD refers to investors with S-shaped utility function, while Markovitz refers to investors with inverse S-shaped utility function. We showed the FORS associated to both order and under the assumption that return distribtuion belogn to a scale invariant family, weakly determined by four parameters, we provided optimization problems to obtain optimal

choices for both the aformetnioned ordering. Then, exploiting estimation function theory we provide hypothesis tests and related test statitistcs for efficiency of a given portfolio with repsect to a benchmark. We empirically implemented these tests on portfolios constructed on S&P 500, finding evidence of efficiency with respect both orders in almost all cases.

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Investigating the Distributional Properties of Highly Volatile Bitcoin Exchange Rate

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Abstract

In this paper we use two well-known fat-tailed and semi-fat-tailed distributions, i.e. alpha stable distribution and normal inverse Gaussian distribution, to characterize the probabilistic distribution of daily yields of bitcoin exchange rate from 2011 to 2017. All parameters of these two distribution capturing the fat tails, skewness and other moments are estimated. As its evolution over time has been uneven, the analysis is performed not only for the whole period, but also for its individual phases.

Key words

Bitcoin, stable distribution, normal inverse Gaussian distribution

JEL Classification: G10, G120

1. Introduction

Bitcoin (BTC) has undergone major changes in recent years. The bitcoin exchange rate to USD started somewhere around 2 USD and it has currently reached the height about 3000 USD. However, this rise has been accompanied by high volatility making it a very risky investment. In this article we will concentrate on the probability characteristics of the bitcoin exchange rate development. By using the alpha stable and Normal inverse Gaussian (NIG) distribution we try to find the best approximation of the BTC / USD log-returns. A stable distribution is determined by four parameters and each has its own interpretation (tail power, skewness, etc.). Similarly, this is true of NIG, although the interpretation of the parameters is not so straightforward. Parts 2 and 3 give definitions of alpha stable and NIG distribution, in the following one the Maximal Likelihood Estimation method (MLE) that was used to estimate the parameters is briefly described. Finally, the results of the estimates and their interpretation are given.

2. Stable distributions

Definition of stable distribution

Let, $X, X_1, X_2, X_3, \dots, X_n$ i.i.d.
A random variable X is said to have the α-stable distributions if there is a positive number c_n and a real number d_n for any $n \geq 2$ such that
$X_1 + X_2 + X_3, + \cdots + X_n = c_n X + d_n$ in distribution.
Thus, any sum of independent equally distributed random variables have the same distribution except for the „mean“ and „variance“. Unfortunately there is no general form of the probability density function, we know only the general form of the characteristic function:

$$\Phi(t) = \exp\left\{-\sigma^\alpha |t| \left(1 - i\beta sign\,(t) \tan\frac{\pi\alpha}{2}\right) + i\mu t\right\} \quad \text{for } \alpha \neq 1 \tag{1}$$

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$$\Phi(t) = \exp\left\{-\sigma|t|\left(1 - i\beta\frac{2}{\pi}sign(t)\log|t|\right) + i\mu t\right\} \text{ for } \alpha = 1 \quad (2)$$

where parameters $\alpha \in (0,2], \beta \in [-1,1], \mu \in \mathbb{R}$ and $\sigma \in [0,\infty)$ and we will label this distribution as $S\ \alpha,\beta,\sigma,\mu$ [3]

So the α -stable distributions require four parameters. These parameters may be interpreted as

α… tail power (tail index), as α decreses tail thicknes increases

β… skewness parameter, determines asymmetry, a positive β indicates that right tail is fatter then left tail and vice versa, $\beta = 0$ corresponding to a symetric distributions

μ … location parameter (corresponding to a mean for $\alpha > 1$)

σ… scale parameter, generalized a standard deviation, for $\alpha = 2$ corresponding to a standard deviation of normal distribution

Power of the tails

The power of the tail is the index α which approximately means that[4] $P(X < x) \approx c_\alpha |x|^{-\alpha}$ as $x \to -\infty$

3. Generalized hyperbolic distribution and its special cases

This distribution was introduced by Barndorff-Nielsen (1977) and at first applied them to model grain size distributions of wind blown sands. Eberlein and Keller (1995) were the first to apply these distributions to finance. The probability density function is as follows:

$$f(x) = \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\left(\lambda-\frac{1}{2}\right)}\delta^\lambda K_\lambda\left(\delta\sqrt{(\alpha^2-\beta^2)}\right)}(\delta^2 + (x - \mu)^2)^{(\lambda-1/2)/2}K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)\exp\left(\beta(x-\mu)\right) \quad (3)$$

Where $K_\lambda(x)$is modified Bessel function of the third (sekond) kind with index $\lambda \in \mathbb{R}$. It can be defined as

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty s^{\lambda-1}\exp\frac{x(s+s^{-1})}{2}ds$$

The following distributions are the special cases of the generalized hyperbolic distribution (GH). For $\lambda = 1$ we get the hyperbolic distribution, for $\lambda = 1/2$ we get the normal inverse Gaussian distribution (NIG). So the probability density function of NIG is (using some properties of Bessel functions):

$$f(x) = \frac{\alpha\delta\, K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}}\exp\left(\delta + \beta(x-\mu)\right) \quad (4)$$

Fat tails of GH

The asymptotic probability of GH is the following

[3] There are other equivalent representations of the characteristic function, which may be advantageous in certain situations (see Nolan 2003).

[4] The exact formula for $c_\alpha$can be found in Nolan 2003.

$P(X \leq x) \approx |x|^{\lambda-1} \exp[(\alpha+\beta)x]$ as $x \to -\infty$

4. Estimation of Parameters using MLE

Because closed-form formula for the probability density function of NIG distribution exists using MLE is straightforward. In the case of stable distributions there is no explicit for of density function, so we have to find it by using inverse Fourier transformation.
According Borak, Hardle, Weron (2005), after substitution $\zeta = -\beta \tan\frac{\pi\alpha}{2}$ the density of standard $\alpha -$ stable random variable (μ=0, σ=1) for $\alpha \neq 1$ can be expressed as:
for $x > \zeta$:

$$f(x;\alpha,\beta) = \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi|\alpha-1|}\int_{-\xi}^{\frac{\pi}{2}} V(\theta;\alpha,\beta)\exp\left(-(x-\zeta)^{\alpha/\alpha-1}\, V(\theta;\alpha,\beta)\right)d\theta, \qquad (5)$$

for $x = \zeta$:

$$f(x;\alpha,\beta) = \frac{\Gamma\left(1+\frac{1}{\alpha}\right)\cos\xi}{\pi(1+\zeta^2)^{\frac{1}{2\alpha}}}$$

and for $x < \zeta$:

$$f(x;\alpha,\beta) = f(-x;\alpha,-\beta)$$

where

$$V(\theta;\alpha,\beta) = \quad (\cos\alpha\xi)^{\frac{1}{\alpha-1}}\left(\frac{\cos\theta}{\sin\alpha(\xi+\theta)}\right)^{\alpha/\alpha-1}\frac{\cos[\alpha\xi(\alpha-1)\theta]}{\cos\theta}$$

$$\xi = \frac{1}{\alpha}\arctan(-\zeta)$$

In MLE we have to find from observation data x_i a maximum of the likelihood function

$$\sum_{i=1}^{n} \log f(z_i;\alpha,\beta,\delta,\mu)$$

with respect to parameters α,β,δ,μ , where $z_i = \frac{x_i-\mu}{\delta}$.

5. Shortly about bitcoin

Bitcoin (BTC) was proposed by an unknown person or persons, under the name Satoshi Nakamoto in October 2008, as a combination of a digital asset and a peer-to-peer payment system in his study: Bitcoin: A Peer-to-Peer Electronic Cash System.9 The first bitcoin was minted on January 4, 2009, and the first payment was January 11, 2009 . The software was released as an open source on January 15, 2009, allowing anyone with sufficient technical skills and computer equipment to engage in development. For a long time, the bitcoin was a little interested. From the second quarter of 2012, transaction volumes began to grow dramatically. The current daily average volume of bitcoin transactions during their lifetime (from January 4, 2010 to January 1, 2017) is 19,301,677 USD.

Bitcoin "coins" are created by a network of computers with specialized software programmed to release new coins at a steady but still declining pace. The number of coins in circulation should reach 21 million in year 2140 when the coinage should be terminated.

There also exist the other cryptocurrencies e.g. Dash, Etherum (ETH), Monera (XMR)[5] to name at least the best known. At present, there are about 30 cryptocurrencies but bitcoin is the most liquid, so a big trade has only a little influence on its price and its volatility.

[5] https://en.wikipedia.org/wiki/List_of_cryptocurrencies

There is lot of Bitcoin exchanges where you can trade bitcoin (and other cryptocurrencies) with "classical" currencies and also with each other. On the website https://www.bestbitcoinexchange.io/ you can find 41 exchanges with their characteristics.

Transactions take place between users directly, without an intermediary. These transactions are verified by network nodes and recorded in a *blockchain*. Since the system works without a central repository or single administrator, bitcoin is called the first decentralized digital currency

Blockchain is a public list (or rather a database) where all transactions that have been made with bitcoin (or other cryptocurrency), are anonymously recorded. It is a chronological string of transaction blocks that is constantly growing and shared among all users - meaning that each computer connected to the network's bitcoin gets a copy of a blockchain that will automatically download at the next logon. At present, more than 90% of banks are considering using blockchain. Possibilities of applying this technology to payment systems, risk management, etc. are considered.

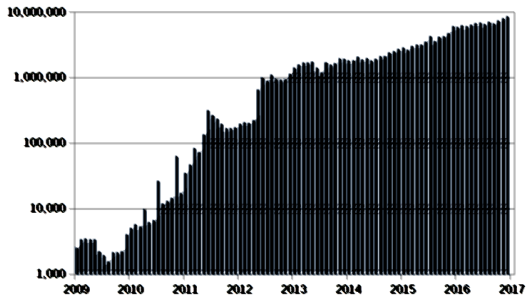
Bitcoin price (in USD) and its volatility

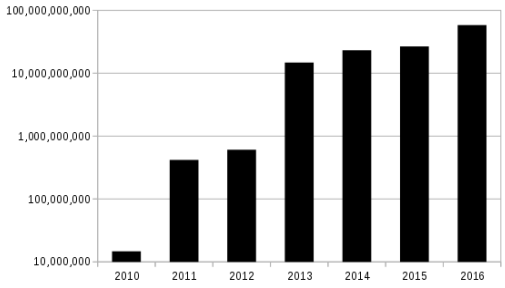
Bitcoin weekly volatility reached up to 60% per annum during the 2016 but other cryptocurrencies shows even bigger swings. Bitcoin is, however, still more volatile than any fiat currency pairs.

Bitcoin has volatility seven times greater than gold, eight times greater than the S&P 500, and 18 times greater than the U.S. dollar.

Figure 1: *History of bitcoin price in USD (Source Bitstamp)*

Figure 2 : *Number of bitcoin transactions per month (left) and liquidity (estimated, USD/year (logarithmic scale). Source Wiki*





Bitcoin volatility can be traded using options. Currently there are only a few places where you can trade with options on Bitcoin (e.g. Deribit). In general, the more liquid the derivatives markets are, the more deeply liquid the underlying product is. The appearance of options of Bitcoin shows that liquidity of Bitcoin increased.

Figure 3 : Bitcoin 30 day annualized volatility compared with USD /EUR and S&P 500 (Source https://bitcoinmagazine.com/articles/bitcoin-volatility-analysis-1377627247/)

Bitcoin price over the last 1 year was highly correlated with DASH and XMR but shows no correlation with Ethereum (see Table 1) It can be assumed that Bitcoin trading activities boost volumes of other coins as well, and it might well be that news shocks on Bitcoin may affect prices of other coins.

*Table 1: **Correlation** Matrix (Source https://www.cryptocompare.com/coins/guides/5-things-you-need-to-know-about-bitcoin-volatility/)*

	DASH/USD	**XMR/USD**	**ETH/USD**	**BTC/USD**
DASH/USD	1	0.78	0.19	0.75
XMR/USD	0.78	1	-0.11	0.80
ETH/USD	0.19	0.80	1	-0.04
BTC/USD	0.75	0.80	-0.04	1

6. Results

The used daily data includes the exchanged rates BTC/USD in the period 13.9. 2011 to 16.6. 2017. taken from Bitstamp, which is one of the most popular exchanges where bitcoin is traded. Exchange rates have been transformed into a series of log-returns and using the MLE method the parameters for stable and NIG distributions have been estimated. These estimates were made both for the whole period and also for the two periods 2011-2014 and 2013-2017. The

reason was that in the first period the bitcoin was more volatile than in the second one. In Table 2 the basic descriptive statistics are shown and in the following tables the final parameter estimates for both distributions are given. In these tables standard error (S.E.) of the parameter estimation is also shown.

Figure 4: Daily log-returns of BTC/USD

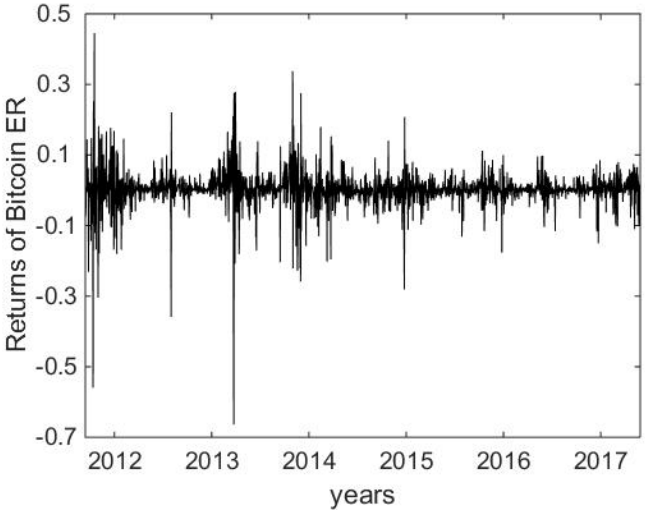


Table 2: Descriptive statistics of analyzed data

Characteristic	**Period 2011 - 2017**		**Period 2011 - 2014**		**Period 2013 - 2017**	
	Ex rate	Returns	Ex rate	Returns	Ex rate	Returns
Mean	384,1609	0,0029	208,4726	0,0040	580,3186	0,0009
Median	285,45	0,0023	69,9	0,0027	458,48	0,0015
Maximum	2954,22	0,4455	1132,01	0,4455	2954,22	0,2756
Minimum	2,24	-0,6639	2,24	-0,6639	171,41	-0,2809
Std. Dev.	412,9197	0,0524	275,0835	0,0651	402,8693	0,0380
Skewness	2,2695	-1,6114	1,1500	-1,5294	2,7386	-0,7701
Kurtosis	11,2880	31,0041	2,9574	24,4348	12,9987	13,5890
Observations	2083	2082	1083	1082	1283	1282

Table 3: Parameter estimation result for bitcoin returns series, period 2011 - 2017

	NIG distribution		Alpha Stable Distribution	
	value	S.E.	value	S.E.
alpha	5,8080	0,0308	1,1866	0,0193
beta	0,1556	0,0661	0,0977	0,0155
sigma	0,0166	0,0005	0,0152	0,0004
mu	0,0024	0,0003	0,0074	5,31E-06

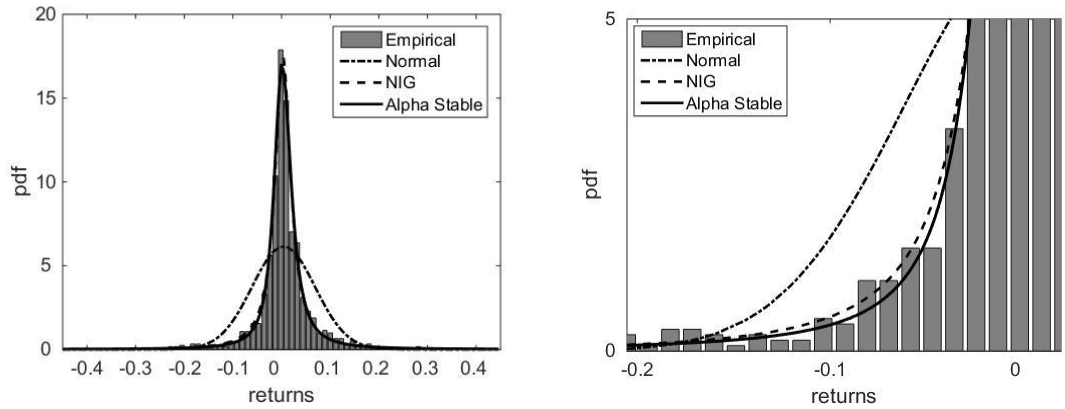
Table 4: Parameter estimation result for bitcoin returns series, period 2011 - 2014

	NIG distribution		Alpha Stable Distribution	
	value	S.E.	value	S.E.
alpha	4,2332	0,3601	1,1479	0,2209
beta	0,3244	0,0932	0,1327	0,0524
sigma	0,0196	0,0041	0,0180	0,0043
mu	0,0025	0,0019	0,0126	0,1121

Table 5: Parameter estimation result for bitcoin returns series, period 2013 - 2017

	NIG distribution		Alpha Stable Distribution	
	value	S.E.	value	S.E.
alpha	10,4982	0,8791	1,2837	0,0406
beta	-0,6481	1,3364	0,0421	0,0142
sigma	0,0160	0,0120	0,0140	0,0008
mu	0,0019	0,0005	0,0030	7,38E-04

Figure 5: Histograms of bitcoin/USD returns compared with stable, NID and normal distribution with estimated parameters (*on the right is the detail of the left tail*)



7. Conclusion

The results show that the stable distribution approximates the empirical BTC / USD exchange rate distribution more precisely than the NIG one. The estimates of all parameters have a smaller S.E. Alpha of a stable distribution is close to 1, which means that dis distribution is close to Cauchy one distribution. In the first (volatile) period, alpha is smaller than in the second one, the alpha for the entire period is between this two. It can be seen from the figures that normal distribution is not appropriate and has not been further investigated. Sample volatility is very high, and high is the sigma coefficient of stable distribution (which we interpret as generalized volatility). The problem occurs with beta (generalized skewness). In the case of a stable distribution, in all three cases, it is positive but the sample skewness in Table 2 is negative. However, all these values differ little from zero, so it can be said that the empirical distribution is very little different from the symmetrical one.

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Optimal Strategy for Homeowners about Mortgages in Eurozone in 2017

Dušan Marček [1], Lukáš Falát [2]

Abstract

In this paper, we review the current situation on mortgages market in Eurozone. After discussing the theoretical background, we discuss factors which are important in setting up mortgages rates by commercial banks. We then present the current situation about mortgages in Eurozone. Afterwards we suggest a strategy for homeowners about their current mortgages. Our strategy is based on qualitative analysis. As for quantitative predictions, we use two approaches time series models represented by exponential smoothing and results from published regression analysis. We suggest the 10-year fixation for conservative homeowners while the 5-year fixation for the risky ones.

Key words

Mortgages, Eurozone, time series, exponential smoothing,

JEL Classification: E4, E47, D1, C22

1. Introduction

In recent years, the situation in Eurozone has changed a lot. The world experiences the global financial crisis and Europe had not gone through that without impacts. In Eurozone, the group of countries using the euro currency, the most important player as for monetary policy is the European Central Bank. Among other functions, this bank control the interest rates bank can lend money from the ECB. The level of interest rates in mainly based on economic activity in the country or union. "Central banks such as ECB alter the cost of borrowing in a bid to influence economic activity. The aim is to control inflation - the rate of increase in the prices we pay for goods and services - at a typical target of 2% per year. When economic growth is robust, demand for goods typically becomes greater than supply and, as a result, inflation typically rises, reducing consumers' purchasing power. If such a scenario plays out for too long, interest rates can be hiked to help cool inflation. However, this is not an issue troubling developed world economies right now. Since the global financial crisis of 2008, developed nations have not only cut the cost of borrowing to record lows but they've also electronically pumped money into their respective systems via quantitative easing (QE) to help stimulate demand." (Barclays, 2016)

The interest rates of ECB are in strong correlation with mortgage rates, therefore, it is no surprise that mortgage rates across Eurozone countries have developed over the recent years, too. Also, as mortgage is one of most used and most affordable way how to live in own property, mortgage rates are the interest of many ordinary people. Mortgage interest rates have a very significant impact on the overall long-term cost of purchasing a home through financing. Mortgage borrowers seek the lowest possible rates, but on the other hand, mortgage lenders

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must manage their risk through the interest rates they charge. (Maverick, 2015)

The paper is divided into 5 chapters. The section two present internal and external factors which influence the mortgage rate. Section 3 describes the current situation in Eurozone. Section 4 quantitatively analyzes the current situation and. Section 6 concludes the paper and suggest optimal strategy for lenders.

2. Mortgage Rate Determinants

When buying a home, or even refinancing an existing mortgage, one need to be aware of the numerous factors that can influence their mortgage interest rate. There are quite a lot of factors which influence the rate of mortgages for homeowners. One can divide these factors into two groups from the homeowner point of view. The first group are internal factors and the second group are external factors.

The most influencing internal factors are (Williams, 2016): credit score, expected down payment, loan term, the length of fixation and home location.

Person's credit score, also known as FICO score, determines how probable you are to pay back your loan. "If you make your payments on-time, use less than 20% to 30% of your available credit, keep good-standing accounts open for long periods of time, avoid opening too many new accounts, and demonstrate that you can handle both installment loans (e.g., auto or student loans) and revolving credit (e.g., department store credit cards), chances are you'll have a respectively high credit score." (Williams, 2016) In general, people who have higher credit score may also receive lower mortgages rates compared to people who do not have such a high score. This is because there is a lower risk for financial institutions that the loan will not be paid back.

Another factor which influences the rate of a mortgage is the amount of money you put down on a home (expected down payment). The more money a person puts down, the lower the risk for a bank and hence, the lower the interest rate of a mortgage. In general, if more than twenty percent of the loan is put down, the buyer usually gets lower mortgage rate.

The number of year one plans to pay back their loan is another factor which is often considered. Some lenders prefer shorter loan term. In the past, the shorter the loan term, the lower the mortgage rate was. However, today, most banks do not make an exception and usually 30-year loan term is preferred.

The length of fixation – this means the sureness that one is going to pay the same amount of payment for a fixed term. In general, there are two main types of interest rates: fixed and adjustable. As for fixed rate, there are various lengths of fixation offered by banks. Usually, there are 3, 5, 10, 15 and 30-year fixation periods. For example, with 30-year fixation term, the payment of a mortgage loan is not going to move over the whole term of your payment. The disadvantage of such a guarantee is usually the higher payment. In Eurozone, the mortgage rates are adjusted based on EURIBOR.

One of other factors of the mortgage process is the location of a home. In some countries, the location of a home can influence a mortgage rate. However, the more often the location of the house determined the maximum amount of money the bank is willing to lend. For example, there is much more chance the banks will give a 100 percent mortgage for an apartment in large city than for a small house in a countryside.

"While the financial health of borrowers affects the specific interest rates they can obtain, the general level of mortgage interest rates is also influenced by many critical economic factors." (Maverick, 2015). These are factors a person is not able to influence or change as these are usually based on economic and political situation of the country or union. The most influencing external factors are the following ones: inflation, monetary policy, housing market conditions.

The rise of prices, i.e. inflation is a very important factor in the overall economy and a critical factor for mortgage lenders. Inflation lowers the purchasing power of money over time. The rate of inflation also influences the rate of mortgages over time.

Inflation correlates with economic growth, i.e. if there is a high economic growth of the country, the inflation is usually rising too. Therefore, gross domestic product (GDP) and the employment rate have also influence on the mortgage rates. In good times, the mortgages rates are higher. "Naturally, the opposite effect results from a weakening economy. Employment and wages decline, leading to decreased demand for home loans, which in turn puts downward pressure on the interest rates offered by mortgage lenders." (Maverick, 2015)

The monetary policy is one of the most important factors which influences the mortgage rates. In Eurozone, the monetary policy is performed by the European Central Bank headquartered in Frankfurt. The ECB however, does not set the specific interest rates in the mortgage market. What ECB does is that it adjusts the money supply by increasing and decreasing the interest rates for commercial banks. The mortgage rates and interest rates of ECB (EURIBOR) are usually highly correlated.

Finally, "trends and conditions in the housing market also affect mortgage rates. When fewer homes are being built, or offered for resale, the decline in homes being purchased leads to a decline in the demand for mortgages and pressures interest rates downward." (Maverick, 2015).

3. Analysis of Mortgage-Rates in Eurozone

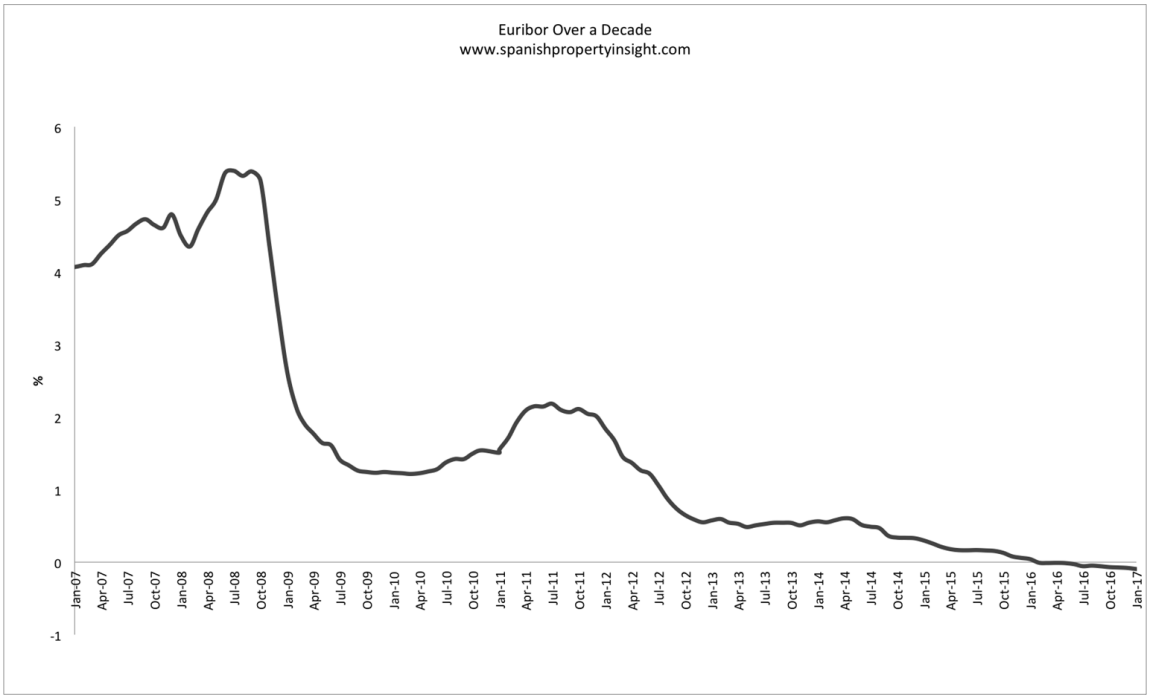
The situation about mortgage rates in Eurozone is in great from the homeowners point of view. The reason are low interest rates. Moreover, to support the embattled Eurozone economy, in June 2014, the European Central Bank lowered interest rates for banks from 0% to -0.1%. Since then the rate has lowered even more, to -0.4%, while its main interest rate is at 0% (Barclays, 2016). In 2015, ECB started off its own quantitative easing.

In 2017 Eurozone interest rates dropped to a record low in January, which was the 12th consecutive month of negative interest rates. 12-month Euribor – the rate used to calculate most mortgage rates in Eurozone countries lowered to -0.095 in January. (Stucklin, 2017) And as interest rates directly influence the rate for which banks are willing to lend money, the low interest rates also mean low mortgage rates.

There are already signs that rates for fixed-interest mortgages are starting to rise. In January, several banks across Europe announced they plan to raise the fixed rate they offer (Stucklin, 2017). "However, some economists expect that its QE strategy could be extended further and ECB president Mario Draghi is unlikely to be in a hurry to tighten monetary policy, given that said he believes inflation will take longer than he initially expected to pick up." (Barclays, 2016).

Resulting from this, "there is unlikely to be a rise in European Central Bank interest rates for the coming years, a development that will be a huge boost to thousands of mortgage holders. Andrew Milligan said international markets had not priced in any rise before 2019, and he did not see Eurozone rates rising." (Weston, 2016). The chief economist at Investec Ireland, says that the ECB's main interest rates will only start to rise in the spring of 2019 because inflation in the Eurozone remains muted. "My personal view is that the ECB won't countenance a rate rise for at least two years," said David McNamara, economist at Davy Stockbrokers (Quinn and Percival, 2016).

Figure 1: The development of EURIBOR rates in last 10 years (Stucklin, 2017)



As for future situation, the mortgage rates will certainly go up. The question is when and how fast. Some analysts predict that with Eurozone inflation staying stubbornly low the ECB may only start hiking rates as late as 2019. Resulting from this, costs for homeowners paying off their mortgages will stay low for a couple of years (Quinn and Percival, 2016). So, it could be two to four years before interest rates rise. The question is: Is it good to take a mortgage? If so, is it good to take a mortgage with a fixation rate? If so, what is the optimal fixation rate for homeowners today, in 2017? What seems to be the best strategy?

4. Quantitative Analysis

To give the answer for the question stated above, one must know the future state in the Eurozone and in the ECB interest rates market. Of course, the exact value is impossible to determine, however, there exist prediction models which are very helpful in situation like this. As said, there are various approaches how to estimate the future values of a time series. The two most popular are autoregressive models (Box and Jenkins, 1976) and exponential smoothing (Brown 1956 and 1963), Holt (1957).

At first, forecasted interest rates of ECB themselves as there is a strong correlation between ECB interest rates and mortgages rates in Eurozone countries. This correlation was also confirmed by regression models performed by Allianz (Euler Hermes, 2017). In their study, they used regression analyses to estimate how an increase in the ECB key interest rate will impact average interest rates on bank loans for households and the corporate sector.

We performed two approach to be able to select the optimal strategy on mortgage rates market. The first approach was modelling of the interest rates of ECB (marginal lending facility) with time series models. Due to the fact that the system of determining in apriori unknown as well as due to the fact that the current interest rate is independent of the previous one we decided to use exponential smoothing models rather than Box-Jenkins models. The goal of our models was to perform 10-periods ahead forecasts, i.e. we tried to model what is going to be the actual interest rate in Eurozone in five years (i.e. in 2022) according to our exponential smoothing models. These models are black box models and so they let the analysts to model a

time series without apriori knowledge about the process itself. The is a big advantage compared to autoregressive Box-Jenkins models. Table 1 states our data - the interest rates of the ECB. For our purpose of modelling, we chose marginal lending facility rates as these are the most important in influencing mortgage rates in Eurozone countries.

Table 1: Changes in interest rates of the European Central Bank (National Bank of Slovakia, 2017)

Valid since	Marginal lending facility	Main refinancing operations	Deposit facility
1.1.2009	3,00%	2,50%	2,00%
21.1.2009	3,00%	2,00%	1,00%
11.3.2009	2,50%	1,50%	0,50%
8.4.2009	2,25%	1,25%	0,25%
13.5.2009	1,75%	1,00%	0,25%
13.4.2011	2,00%	1,25%	0,50%
13.7.2011	2,25%	1,50%	0,75%
9.11.2011	2,00%	1,25%	0,50%
14.12.2011	1,75%	1,00%	0,25%
11.7.2012	1,50%	0,75%	0,00%
8.5.2013	1,00%	0,50%	0,00%
13.11.2013	0,75%	0,25%	0,00%
11.6.2014	0,40%	0,15%	-0,10%
10.9.2014	0,30%	0,05%	-0,20%
9.12.2015	0,30%	0,05%	-0,30%
16.3.2016	0,25%	0,00%	-0,40%

Before modelling, we first edited the data so that the periods would be equal; we noted interest rates two times per year – in January, 1st and in July, 1st. The dataset contained 36 periods, from Jan, 1st, 1999 to July, 1st, 2017. The numerical characteristics we used for evaluating our models was MAPE (mean absolute percentage error). As for exponential smoothing, we implemented the simple exponential smoothing model as well as the Holt model. For both of the models we tried to optimize values of parameters (alpha in simple exponential smoothing and alpha and beta for Holt model). Unfortunately, in both models the best MAPE characteristic was achieved with alpha equal to one and beta equal to zero. The reason for this was time series models of exponential smoothing was unable to catch the pattern. The most probable reason for this is that the pattern is non-existent and therefore cannot be modelled. This can be due to the fact that interest rates are determined by political decisions as well as by the fact that the interest rates are close to constant in recent years.

To be able to suggest the optimal strategy for homeowners with mortgages, we had to use the second approach. Usually the regression approach is the alternative to standard time series modelling. As this regression analysis was already performed, we used the results from regression analysis performed by Allianz (Euler Hermes, 2017). In Allianz, they simulated three scenarios of how interest rates would develop. These three scenarios describe the development of the ECB key interest rate up to 2022. Their base scenario was one of “soft normalization”, with the ECB only starting to hike the key rate from 2019 onwards; the other two scenarios were called “moderate” and “hard normalization”. The key rate would reach 2 percent at 2022 year-end in the base scenario, 3 percent in the second and 4.25 percent in the third scenario.

Except for this, we also used the information that in November, 2016, in euro zone average interest rate on new mortgages was 1.72%, i. e. approximately 1,5 percent above the ECB

interest rates (marginal lending facility) (RTE, 2017). So, on base of Allianz analysis, the three scenarios of mortgages rates in 2022 would be

- Base scenario: 2 per cent + 1,5 per cent = 3,5 per cent
- Moderate normalization: 3 per cent + 1,5 per cent = 4,5 per cent
- Hard normalization: 4,25 per cent + 1,5 per cent = 5,75 per cent

As a standard, we used 5-year fixation period. The Table 2 and 3 illustrates four different scenarios, the first to third one are the same in the first 5 years, the owner takes the mortgage for a current average value 1,72 per cent. After five years three possibilities are evaluated – basic normalization, moderate and hard. Option 4 is an alternative to the first three options and it is the case when the owner take the 10-year fixation in 2017. The disadvantage is that 10-year fixation are more expensive than the 5-year, approximately by 1 per cent. The results are in Table 2 and 3.

Table 2: The average payment during first ten years of the 30-year mortgage

Amount of €	soft	moderate	hard	10-year fixation
50000	197,745	209,74	225,665	203,33
100000	395,485	419,485	451,335	406,65
150000	593,225	629,22	676,995	609,98
200000	790,97	838,965	902,66	813,31
250000	988,715	1048,705	1128,325	1016,63
300000	1186,455	1258,445	1353,995	1219,96
500000	1977,425	2097,41	2256,655	2033,27

Table 3: The leftover after first ten years of 30-year mortgage

Amount of €	soft	moderate	hard	10-year fixation
50000	37 627,21	38 285,66	39 033,71	37 723,20
100000	75 254,42	76 571,31	78 067,40	75 446,39
150000	112 881,62	114 856,97	117 101,09	113 169,60
200000	150 508,84	153 142,64	156 134,79	150 892,79
250000	188 136,04	191 428,28	195 168,49	188 615,99
300000	225 763,25	229 713,95	234 202,19	226 339,19
500000	376 272,08	382 856,57	390 336,97	377 231,98

We can see in Table 2 and 3 that the average payment is the lowest one in soft normalization. However, the average payment in 10-year fixation period is better than the payment that a payer would pay if the ECB interest rates would develop in hard but also in moderate normalization period.

5. Conclusion

In this paper, we analyzed the current situation on the Eurozone mortgage-rates market. Today, in 2017, the ECB interest rates which influence the mortgage rates in Eurozone, are very low, in some cases they are even negative.

The reason why the ECB decided to put negative interest rates on some loans is that negative base rates are a golden opportunity for investors as borrowing money is probably as cheap as it is ever going to get (Stucklin, 2017). These rates should encourage both consumers and businesses to borrow, spend and invest more. This in turn can potentially lead to stronger economic growth, higher share prices and rising inflation, as more money is pumped into the economy.

A backdrop of such loose monetary policy means consumers with mortgages pegged to the base rate will enjoy smaller monthly mortgage payments too, and therefore a greater level of disposable income (Barclays, 2016). There are various scenarios what is going to happen in 5 years on the mortgage rates market. However, regardless of what is going to happen, "the days of extremely low interest rates are numbered," said Michael Heise, chief economist at Allianz." Even in the least favorable scenario of "hard normalization", the total interest payments will 'only' increase by roughly EUR 280bn 14 years later - in 2022: their absolute value will still fall short of the level reached at the 2008 peak of the credit boom. In the more likely event of ''soft normalization", interest payments are only expected to increase by around EUR 160bn, almost one quarter lower than in 2008 in absolute terms. (Euler Hermes, 2016).

Except for the macroeconomics point of view we also looked at these scenarios from the microeconomics point of view. We used two approaches to suggest the optimal strategy for a homeowner on mortgage market in 2017. However, using the exponential smoothing models we found out interest rates of ECB cannot be modelled by time series models.

On base of regression analysis performed by Allianz we found out that for a homeowner the best situation would be if the soft normalization would happen, In that case to take 5-year fixation on their mortgage now, in 2017 would be the most beneficial. However, it is not 100 per cent sure that interest rates would developed by soft normalization. Therefore we recommend this strategy only for risk-takers. For homeowners who do not like risk we recommend to take 10-year fixation now, in 2017 as in total, it is less expensive than to take 5-year fixation now subject to moderate or hard normalization in 5 years.

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Optimization and Testing of RSI

Patrice Marek[1], Blanka Šedivá[2]

Abstract

This paper deals with the Relative Strength Index (RSI), a popular oscillator. It answers the question whether – almost 40 years after its publication – the RSI can be still useful in trading. To answer this question, we compare four strategies: RSI with recommended parameters, RSI that uses parameters optimized each trading day, simple buy-and-hold strategy, and strategy based on Kelly gambling. Companies that were included with the largest weights in the S&P 500 Index in years 2006–2009 are used for simulations. Totally, 10,000 simulations of investment are performed for each strategy. Start of each simulation is randomly chosen between 15 February 2007 and 12 February 2010. The end is randomly chosen between 18 February 2014 and 14 February 2017. Companies and dates are used so they include as little information that we possess nowadays as possible. Later, simulations with shorter time interval are performed to verify findings.

Key words

Relative Strength Index, RSI, Kelly gambling, buy-and-hold, investment, comparison, optimization, stocks

JEL Classification: G110

1. Introduction

Relative Strength Index (RSI) – published in Wilder (1978) – is popular oscillator that is used in many trading systems. The equation for the RSI using period p is

$$RSI_t = 100\left(1 - \frac{D_t}{D_t + U_t}\right), \tag{1}$$

where D_t, or *average downward change*, and U_t, or *average upward change*, are computed using exponential moving averages of closing prices C_t as shown in equations (2) and (3).

$$D_t = \frac{1}{p} \cdot \max(0; C_{t-1} - C_t) + \left(1 - \frac{1}{p}\right) \cdot D_{t-1} \tag{2}$$

$$U_t = \frac{1}{p} \cdot \max(0; C_t - C_{t-1}) + \left(1 - \frac{1}{p}\right) \cdot U_{t-1} \tag{3}$$

The first values of D_t and U_t, i.e. D_p and U_p, are computed using simple average of closing prices $C_1, C_2, \dots, C_p$. Originally, Wilder (1978) used period $p = 14$ (days).

The RSI can be used to obtain buy and sell signals. Anson, et al. (2012) list three possibilities:

- Buy signal occurs when the RSI reaches 30 from above, and sell signal occurs when the RSI reaches 70 from below.

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- Buy signal occurs when the RSI returns to 30 from below, and sell signal occurs when the RSI returns to 70 from above.
- More sophisticated analysis of the RSI is made to obtain buy and sell signals.

Originally, Wilder (1978) used the third option as he searched for a sell or buy signal with thresholds at 30 and 70. For example, a sell signal is obtained when the RSI moves as follows:

- forms a top above 70,
- decreases to form a bottom called *fail point* (this can be even under 70),
- starts to increase its value to form another top that is lower than the first one,
- crosses the value of *fail point* from above so the sell point – called *failure swing point* – is obtained.

Moreover, Wilder (1978) obtained buy and sell signals from divergence between the price and RSI and from some chart formations (for example pennant).

This paper explores whether there are benefits of continual day-to-day optimization of the RSI where the second rule of the previous list is used to obtain buy and sell signals. This rule is significantly easier for implementation than the third rule of the list. It is also more similar to the Wilder's strategy than the first rule of the list because it waits for a buy or sell signal after the RSI enters the interval above 70 or below 30.

The main goal of this paper is to investigate how useful can RSI be. We do not use only original parameters, but we continually optimize parameters of RSI each trading day. Results of this day-to-day optimization are compared with three strategies:

- RSI that uses original parameters (14 days long period and signal lines at 30 and 70),
- simple buy-and-hold strategy,
- strategy based on the Kelly gambling with continually optimized parameters.

Kelly gambling – that is usually used with continually optimized parameters – is chosen so the strategy with day-to-day optimization of the RSI can be compared to strategy with similar attitude.

2. Data

First, companies that will be used in simulations have to be chosen. This step has to be done with adding as little information that is known nowadays as possible, e.g., it is necessary to avoid conscious inclusion or exclusion of companies that performed well or bad after the start of simulations, and the selection has to be (partly) left on chance. We decided to use large companies of that time as investors can consider these companies more stable than small ones.

S&P 500 Index is used to identify large companies. Its historical composition is not available for free, and it is possible to obtain only top 10 companies with the largest weights in the index for each year at ETF Database (2017). Our simulations start randomly between years 2007 and 2010 (explained afterwards); therefore, we decided to include top 10 companies of the index in 2006, i.e. before the start of all simulations. These companies are Exxon Mobil (ticker symbol: XOM), General Electric (GE), Microsoft (MSFT), Citigroup (C), Bank of America (BAC), Procter & Gamble (PG), Wal-Mart Stores (WMT), Johnson & Johnson (JNJ), Pfizer (PFE), and American International Group (AIG). Next, top 10 companies of the index in years 2007–2009 were also included. Usually, there are no big changes of S&P 500 in two consecutive years; therefore, in 2007, the top 10 companies of the index contained only two new records – AT&T (T) and Google (GOOG). Next four obtained companies – Chevron Corp (CVX) and JPMorgan Chase (JPM) for 2008, and Apple (AAPL) and IBM (IBM) for 2009 – were not included in the list of the top 10 companies before the start of all simulations, but they were large enough even in the year 2007 to include them. Mentioned 16 companies are used in all simulations.

Data of all companies were available up to 14 February 2017 and were obtained from website Yahoo! Finance (2017). Ticker symbol of each company listed above can be used to find historical data on this website. These data include dates, close prices, dividends, and splits. Usually, it is also possible to obtain adjusted close prices for dividends and splits, but at the time of writing this paper, the adjusted close prices were not available; therefore, they were reconstructed using historical dividends and splits. We used the same procedure that is used by Yahoo! Finance (2017), i.e. if a dividend D_t is distributed on ex-date t, and the previous closing price of stock in time $t-1$ was C_{t-1}, then all prices before t (i.e. starting with C_{t-1}) are multiplied by multiplicator M_t obtained as

$$M_t = 1 - \frac{D_t}{C_{t-1}}. \tag{4}$$

This procedure is repeated for all recorded dividends. It ensures that returns of stocks include dividends, i.e. distributed dividends are immediately reinvested in the stock. When a split occurs, then all prices and dividends before the split have to be multiplied by the ratio given by the appropriate split.

3. Methods

Data were available up to 14 February 2017. The earliest date for starting simulations is set on 15 February 2007. We want to avoid influence of selecting one specific date for the start and the end of simulations. Therefore, simulations use three years long interval between 15 February 2007 and 12 February 2010 as a start of investment. The start of each single simulation is randomly chosen from this interval, i.e. the interval that includes some time before and after the peak of financial crisis in 2008.

The end of investment in each single simulation is also randomly chosen from three years long interval, in this case between 18 February 2014 and 14 February 2017. Therefore, the investment lasts at least four years, and the start and the end of investment can occur randomly in time span of three years before and after this four years long interval.

To see how the chosen time interval influences simulations we performed next simulations, this time with data between 15 February 2012 and 14 February 2017, i.e. half of the original time interval. We used the same ratio of subintervals for the start and the end of each simulation. Therefore, the start of each investment in a simulation is randomly chosen between 15 February 2012 and 14 August 2013, and the end of each investment is randomly chosen between 17 August 2015 and 14 February 2017.

The main goal of this paper is to study how useful can the RSI be when parameters of this indicator are optimized each trading day. Results of day-to-day optimization are compared with three strategies. The first strategy uses the RSI with original parameters (14 days long period and signal lines at values 30 and 70). The second strategy is simple buy-and-hold strategy, i.e. strategy that buys at the beginning of each simulation and sells only at the end of each simulation. The third strategy is based on Kelly gambling, i.e. it uses log-optimal portfolio, see Kelly (1956) and Cover and Thomas (2006). Instead of the original criteria, we used its approximation – proposed by Marek, Ťoupal, and Vávra (2016) – that is easier for implementation and produces almost the same results as the original criteria. This strategy also uses day-to-day optimization; therefore, it offers good comparison to method based on day-to-day optimization of RSI.

In this paper, day-to-day optimization of RSI means that parameters of RSI that would produce the highest one-year capital appreciation are found each trading day. To avoid extreme changes in parameters we compare the actual value of capital to the average value of capital in one-week time span exactly before one year, i.e. with usual 252 trading days in a year, the

average is computed from values of capital before 250–254 days. The one-year capital appreciation is chosen so it includes only relevant history of companies that can significantly change their strategies in longer periods.

In the optimization of RSI, the period p can take values in interval $[7,30]$, i.e. half of the original value of 14 and little bit more than double of the original value. The upper signal line is set to $70 + \Delta$, and the lower signal line is set to $30 - \Delta$, where Δ can take values in set $\{-10, -9, \dots, 10\}$, i.e. lines maintain their symmetric distance from value of 50. Parameters out of the presented set can be considered as extreme – they would produce extreme number of trades or, on the contrary, no trades.

Later, when optimized parameters are used for trading, it is necessary to pay special attention to days when optimal parameters are changed. The problem is illustrated by the following example. Let us denote the RSI's value at time $t - 1$ based on parameter $p = 20$, upper signal line at 75, and lower signal line at 25 as $RSI_{t-1}(20; 75{,}22)$. Now, let us consider that we are in a long position at time t, and $RSI_{t-1}(20; 75{,}22)$ was 78. Next, new optimal parameters are obtained for time t. With these new optimal parameters, we obtain $RSI_t(15; 72{,}28) = 65$, and $RSI_{t-1}(15; 72{,}28) = 71$. As can be seen, $RSI_t(15; 72{,}28)$ and $RSI_{t-1}(15; 72{,}28)$ do not produce sell signal as the decline from 71 to 65 does not cross the upper signal line at 72. Obviously, this leads to a situation where the sell signal is missed as $RSI_{t-1}(20; 75{,}22)$ has already entered the interval where it waits for the confirmation of sell signal, i.e. crossing upper signal line from above. Therefore, it is necessary to consider also value of $RSI_{t-1}(20; 75{,}22)$. Then, the sell signal is obtained as $RSI_{t-1}(20; 75{,}22)$ with value 78 declines to $RSI_t(15; 72{,}28)$ with value 65. This means that current value of RSI has to be compared with two values from yesterday. One obtained by the RSI with current optimal parameters, and one obtained by RSI with yesterday's optimal parameters. The same holds for buy signal. These problems are rare, but they can occur in some simulations.

All trades, simulations, and optimizations are made as real as possible; therefore, we use commission 0.2% of trade volume per each trade (this corresponds to trade size around 2,500 USD with a common fee around 5 USD per trade; see NASDAQ (2017)). Next, adjusted close prices are used for trading, i.e. they are used for optimization and, after the optimization is performed, the system uses the same prices also for trading on the same day, which is – obviously – not possible. However, this simplification is not crucial, as prices usually do not change a lot within minutes that are necessary for performing the optimization of all methods, i.e. it is possible to run optimization 10 minutes before the end of trading, and it will be still possible to buy or sell a stock before the end of trading day. The second option would be to use adjusted open prices that are available for the next day, but adjusted open price can significantly differ from the previous day adjusted close price.

4. Results

We performed 10,000 simulations for each of 16 stocks. First, in each simulation, the start date and the end date was generated. Then, trading between these two dates was evaluated for all four methods. Detailed results are presented only for the main time interval, i.e. trading between 15 February 2007 and 14 February 2017. Figure 1 shows – along with adjusted close price of Exxon Mobile – how the time interval is divided. Two horizontal lines divide time interval into three parts. The start of each simulation is randomly chosen from the first interval, then trading continues during the second part, and it ends at randomly chosen time from the third interval.

Results of simulations performed within the second time interval – starting on 18 February 2014 and ending on 14 February 2017 – are presented only in limited form as they serve mainly to determine influence of the chosen time interval on results.

Figure 1: Time intervals and adjusted close price for Exxon Mobile

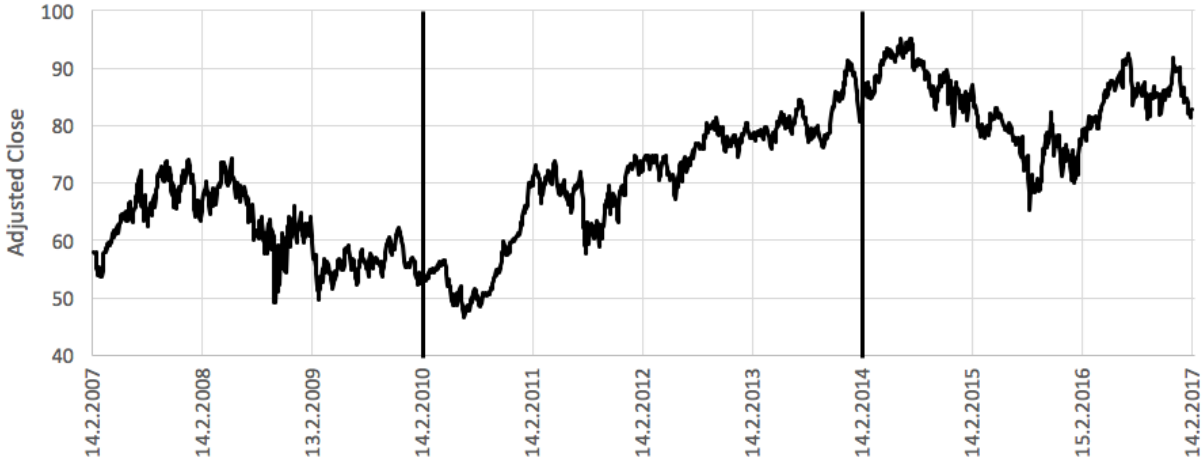


Figure 2 shows full results of all simulations in form of boxplots (observation is shown as an outlier if its distance from the box is more than three times the distance of interquartile range). Returns shown in figure 2 are not annualised; therefore, presented results give information about the final value of one invested unit at the beginning, and time of investment of each simulation can vary from 4 years to 10 years. This can be done because in each simulation only one time interval is randomly chosen, and all four strategies are evaluated within this interval. Moreover, the time interval is not standardized because each of 10,000 simulations can be performed in different time; therefore, the usage of annualised returns can cause some interpretation problems.

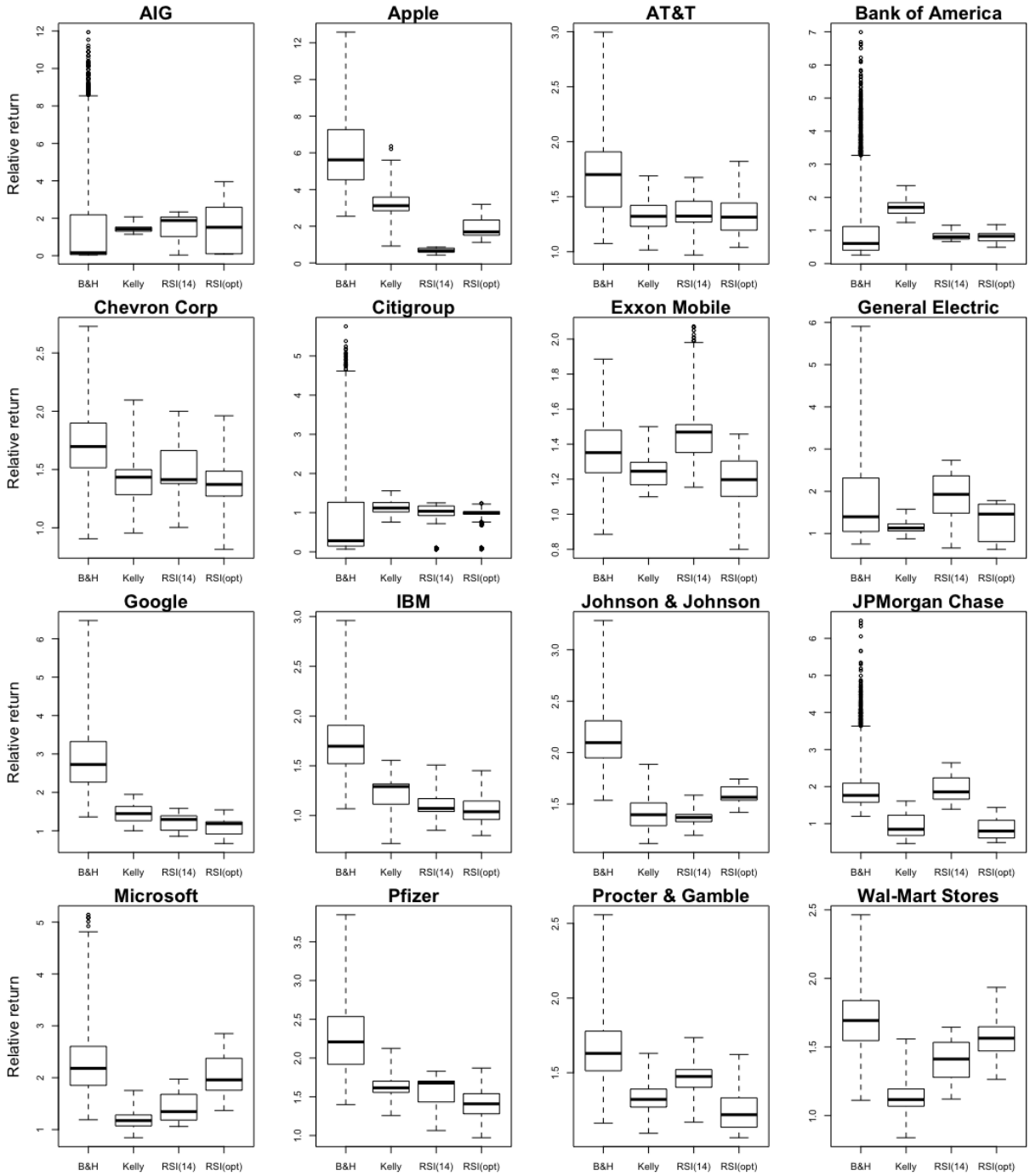
The strategy with the highest final value of capital was found in each simulation, and final results are presented in Table 1, e.g., the highest final value of capital for AIG was obtained by buy-and-hold strategy (B&H) in 1,538 simulations, by the Kelly gambling in 2,279 simulations, etc.

Table 1: Number of simulations with the best results for buy-and-hold strategy (B&H), Kelly gambling (Kelly), RSI with original parameters (RSI(14)), and RSI with optimized parameters (RSI(opt)); the earliest start on 15 February 2007 and the latest end on 14 February 2017

Company	B&H	Kelly	RSI(14)	RSI(opt)
AIG	1,538	2,279	**3,802**	2,381
Apple	**10,000**	0	0	0
AT&T	**8,639**	904	344	113
Bank of America	987	**9,013**	0	0
Chevron Corp	**7,450**	728	1,796	26
Citigroup	2,793	**4,400**	2,805	2
Exxon Mobile	3,381	139	**6,408**	72
General Electric	**4,870**	1,452	3,678	0
Google	**10,000**	0	0	0
IBM	**9,940**	60	0	0
Johnson & Johnson	**10,000**	0	0	0
JPMorgan Chase	3,333	0	**6,667**	0
Microsoft	**5,337**	0	0	4,663
Pfizer	**9,708**	284	8	0
Procter & Gamble	**8,644**	146	1,210	0
Wal-Mart Stores	**7,466**	0	103	2,431

Table 2 presents results of simulations performed within the second time interval, i.e. the earliest start on 15 February 2012 and the latest end on 14 February 2017.

Figure 2: Boxplots of relative returns for buy-and-hold strategy (B&H), Kelly gambling (Kelly), RSI with original parameters (RSI(14)), and RSI with optimized parameters (RSI(opt)); the earliest start on 15 February 2007 and the latest end on 14 February 2017



5. Discussion

Results presented in Table 1 suggest that the best strategy – out of four presented strategies – was the simplest one, i.e. buy-and-hold. This strategy produced the most cases with the highest value of the final capital for 11 companies. It also recorded the highest median of the final capital value for 10 out of 16 companies as can be seen in Figure 2. The buy-and-hold strategy also recorded the highest sample standard deviation of the final capital value for all companies; however, this was usually due to high value of the final capital. Only for three companies – AIG, Bank of America, and Citigroup – buy-and-hold strategy recorded the lowest

median of the final capital value that was, moreover, bellow the initial invested capital. The main reason is that these three companies were significantly influenced by the financial crisis in 2008, and values of their stocks are still not recovered.

Surprisingly, the RSI with original parameters (RSI(14)) produced better results than the RSI with optimized parameters (RSI(opt)) for Chevron, Exxon Mobile, General Electric, Google, IBM, JPMorgan Chase, Pfizer, and Procter & Gamble, i.e. for 8 companies. The opposite can be found only for Apple, Johnson & Johnson, Microsoft, and Wal-Mart Stores, i.e. for 4 companies. This is not possible to explain by some special behaviour of stocks; therefore, we are probably dealing with overfitting, and day-to-day optimization of RSI can cause more problems than benefits due to too frequent changes of parameters.

The RSI with optimized parameters (RSI(opt)) produced better results than Kelly gambling (Kelly) for General Electric, Johnson & Johnson, Microsoft, and Wal-Mart Stores, i.e. 4 companies. The opposite can be found for Apple, Bank of America, Citigroup, Exxon Mobile, Google, IBM, Pfizer, Procter & Gamble, i.e. 8 companies.

Simulations for shorter time interval – with the earliest start on 15 February 2012 and the latest end on 14 February 2017 – were performed to evaluate influence of chosen time interval on simulations. Results are presented in Table 2 that can be easily compared with Table 1. There are some changes but the overall result is still the same, i.e. buy-and-hold strategy offers usually the best results, now for 10 out of 16 companies. Both, the RSI with original parameters and RSI with optimized parameters, were evaluated as the best strategy for three companies. Unlike the results in the longer time interval, results of all simulations stayed in narrower intervals with less extreme values; the lowest value of the final capital value (61%) was recorded by the buy-and-hold strategy for IBM and the highest value (345%) was recorded by the buy-and-hold strategy for Bank of America. In the longer time interval – full results are presented in Figure 2 – both most extreme values were recorded for AIG by the buy-and-hold strategy; the lowest value was 3% and the highest value was 1194%. Overall, there are no big changes in conclusions when the shorter interval is used. However, the main change is lower standard deviation of results and not so extreme values. There are two main, and connected, reasons. The first reason is that the shorter interval even with data of a similar nature will usually produce less extreme values. The second reason is that we are not dealing with data of similar nature because of the financial crisis that is involved only in the first interval.

Table 2: Number of simulations with the best results for buy-and-hold strategy (B&H), Kelly gambling (Kelly), RSI with original parameters (RSI(14)), and RSI with optimized parameters (RSI(opt)); the earliest start on 15 February 2012 and the latest end on 14 February 2017

Company	B&H	Kelly	RSI(14)	RSI(opt)
AIG	**9,167**	0	328	505
Apple	**7,390**	2,220	0	390
AT&T	3,569	0	**6,431**	0
Bank of America	**6,693**	703	2,604	0
Chevron Corp	1,024	1,751	491	**6,734**
Citigroup	**5,463**	239	4,298	0
Exxon Mobile	57	1,405	1,006	**7,532**
General Electric	1,823	2	**5,677**	2,498
Google	**10,000**	0	0	0
IBM	422	13	**9,565**	0
Johnson & Johnson	**9,772**	13	0	215
JPMorgan Chase	**5,768**	278	3,954	0
Microsoft	**9,100**	13	887	0
Pfizer	**7,526**	0	1,095	1,379
Procter & Gamble	**8,398**	188	1,414	0
Wal-Mart Stores	457	0	9	**9,534**

6. Conclusion

This paper presented the Relative Strength Index (RSI) and its day-to-day optimization that was compared with three strategies: RSI with recommended parameters, simple buy-and-hold strategy, and Kelly gambling. Simulations were performed with randomized time interval with the earliest start on 15 February 2007 and the latest end on 14 February 2017. Sixteen companies of S&P 500 that were among the top 10 largest companies for years 2006–2009 were chosen for simulations.

Results show that the best strategy was usually the simplest one, i.e. buy-and-hold strategy. However, this strategy recorded the highest sample standard deviation, and it produced the lowest sample median out of all strategies for companies that were significantly influenced by the financial crisis.

The RSI usually produced value of the final capital above the initial invested capital; therefore, it can be evaluated as useful indicator that can be used in building of trading systems. However, day-to-day optimization of its parameters is – according to obtained results – not recommended as it usually produced worse results that the RSI with original parameters.

Simulations on the shorter time interval confirmed findings, and no essential differences to results on the longer time interval were obtained.

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The impact of macroeconomic indicators on sovereign rating

David Mareš, Jana Kotěšovcová[1]

Abstract

Countries' credit rating is affected by political and economic factors. In this paper, the authors look solely at economic factors. They chose gross domestic product and inflation to make an analysis of the mutual relations between sovereign rating and macroeconomic factors. They used data from the Czech Republic over the 2005 – 2015 period for verification. The objective of this paper is to verify the dependence of the selected macroeconomic factors on the Czech Republic's rating and to measure their impact. The results the authors receive can then be used to estimate future ratings of the country, and thus also ensure a better predication of the rate of return on government bonds.

Key words

Macroeconomic factors, sovereign rating, gross domestic product, inflation.

JEL Classification: L21, O12, O31, O33

1. Introduction

This paper investigates the influence of selected macroeconomic factors on a country's credit rating. The paper's authors selected the two fundamental macroeconomic factors of gross domestic product and inflation. The paper's objective is to discover the principal factors which are most involved in the Czech Republic's credit rating and measure their influence on this rating. The results the authors receive can then be used to estimate future ratings of the country, and thus also ensure a better predication of the rate of return on government bonds.
The authors used rating agency Standard and Poor's scale which has a 23-point scale for long-term rating from AAA as the best rating to D as the worst. Each grade was assigned a weight, with the best AAA rating given a weight of 23, and the weight progressing arithmetically downwards to the worst rating of D, which was assigned a weight of 1. Thus weight was assigned to each point on the scale in accordance with its rating. These weights were then assigned to a particular year and the relationship between the weight and macroeconomic factor was investigated using a correlation analysis. Řezanková and Löster (2009, p. 47-58) say that, "the objective of a correlation analysis is to investigate a linear correlation between variables where it makes sense to look at such an association. The intensity of the correlation is assessed on the basis of the correlation coefficient which has a range of <-1; 1>. A value of 0 indicates linear independence, a value of 1 indicates a direct relationship (one variable is a linear combination of the other) and the value -1 indicates an indirect correlation. Positive values indicate a positive correlation (when the value or order of one variable increases, so the value or order of the second variable rises), while negative values indicate a negative correlation (when the value of order of one variable increase, the value or order of the second

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variable falls). The correlation coefficient expresses the level of mutual correlation. The Pearson correlation coefficient is used for quantitative variables, as is demonstrated."

Relationships between macroeconomic values and sovereign rating have been studied, e.g., by O Canuto, P F Pereira dos Santos, P C de sá Porto (2012). An analysis of sovereign rating factors has also been described by E Bissoondoyal-Bheenick (2005) and A Afonso (2003). Some authors have looked at sovereign rating's influence on credit default swaps (CDS), e.g. C Mellios, E Paget Blanc (2006), R Blanco, S Brennan, I W Marsh (2005) and I Ismailescu, H Kazemi (2010). A Czech author who has investigated relations between CDS and sovereign rating is Petr Budinský (2015). S A. Alfonso (2003) have investigated the reliability of credit rating agencies.

1.1 GDP and sovereign rating

In this part of the paper, we investigate the relationship between rating weight and GDP. One would expect that the higher the GDP (its growth), the higher the rating weight will be – better rating. The table below shows gross domestic product in bn CZK and individual rating weights for particular years. It shows the period between 2005 and 2015.

Table:1 GDP/sovereign rating

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
GDP in mld. CZK weight	3 258,0	3 507,1	3 831,8	4 015,3	3 921,8	3 953,7	4 033,8	4 059,9	4 098,1	4 313,8	4 554,6
Rating	17	17	18	18	18	18	20	20	20	20	20

Source: own processing based on the Czech Statistical Office.

For the above data, we would expect a linear correlation. We can undertake a correlation analysis to calculate the linear correlation.

Formula 1: Pearson's Correlation Coefficient

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{1}$$

Source: (Škaloudová, 2015, p. 28)

In the above table, we can expect a linear dependence of rating weight to GDP, which we verify using the Pearson Correlation Coefficient – rating weight – GDP.

Formula 2: Pearson's Correlation Coefficient, rating – GDP

$$r = \frac{\sum_{i=1}^{n}(RW_i - \overline{GDP})(GDP_i - \overline{GDP})}{\sqrt{\sum_{i=1}^{n}(RW_i - \overline{\mathrm{RW}})^2 \sum_{i=1}^{n}(GDP_i - \overline{\mathrm{GDP}})^2}} \tag{2}$$

Source: adapted from (Škaloudová, 2015, p. 28)

Wherein:
r – Pearson's Correlation Coefficient
RW – Rating weight
GDP – Gross domestic product

The Pearson Correlation Coefficient rating weight - GDP came to a value of 0.82507, which represents a direct relationship between GDP and the rating weight (rating). Thus, we can state that the rating weight (rating) is influenced to a degree of 82.507 % by GDP. The relationship between gross domestic product and credit risk within the countries of Central and Eastern Europe between 1996 and 2010 has been investigated, e.g., by M Dudian and P A Popa (2012 p.22).

1.2 Inflation rate and sovereign rating

In the book Analytical Corporate Finance (2016), Angelo Corelli looks at the relationships between inflation, interest rate and government bond yields. "Inflation risk is the risk that the value of the currency a bond pays out will lose purchasing power over time due to an increase in inflation. Investors expect some amount of inflation, so the risk is that the inflation rate will be higher than expected. Some governments issue inflation-linked types of bonds, protecting investors against upper movements of the inflation rate. These bonds link both the interest payment and the face value to some inflation index. There is an inverse relationship between interest rates and bond prices, with bond prices rising when the market interest rates go down. It happens because when interest rates are on the decline, investors try to capture or lock in the highest rates they can for as long as they can. This can be achieved by scooping up the existing bonds that carry an interest rate higher than the market. The surplus of demand implies an increase in bond price. If the prevailing interest rate were on the rise, the reaction of investors would force bond prices down."

In this section, we will look at the effect of inflation rate in % (increase in average annual consumer prices index on rating weight (rating).

Table:1 Inflation rate % / sovereign rating

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Inflation rate in %	1,9	2,5	2,8	6,3	1,0	1,5	1,9	3,3	1,4	0,4	0,3
Rating weight	17	17	18	18	18	18	20	20	20	20	20

Source: Own processing based on the Czech Statistical Office.

We can expect an indirect relationship in the table above, meaning that with growth in inflation rate in % we would expect a lower (worse) rating. Thus we are investigating the impact of inflation rate in % on rating (rating weight).

Formula. 3 Pearson's Correlation Coefficient – rating – inflation rate %

$$r = \frac{\sum_{i=1}^{n}(RW_i - \overline{IR})(IR_i - \overline{IR})}{\sqrt{\sum_{i=1}^{n}(RW_i - \overline{\mathrm{RW}})^2 \sum_{i=1}^{n}(IR_i - \overline{\mathrm{IR}})^2}} \tag{3}$$

Source: adapted from Škaloudová (2015, p.28)

Wherein:
r – Pearson's Correlation Coefficient
RW – Rating weight
IR – Inflation rate

The Pearson Correlation Coefficient rating weight - IR came to a value of -0.311692, meaning that an indirect relationship between inflation rate and rating was confirmed. Growth in inflation rate led to a fall in rating. The rating is influenced to a degree of 31.16 % by the inflation rate.

2. Conclusions

The impacts of the selected macroeconomic factors on rating were analysed. The macroeconomic factors of gross domestic product and inflation rate were selected and it was demonstrated that gross domestic product had the greatest impact on rating, at 82.507 %. It was further demonstrated that there is a linear direct relationship between gross domestic product and rating such that increasing gross domestic product leads to a better rating. The greater the growth in GDP, the greater the future state budget revenue and thus also the lower the risk of insolvency or unwillingness to pay sovereign debt. In regard to inflation rate, an indirect relationship between inflation rate and rating was seen such that we can state that an increased inflation rate leads to a poorer rating. The rating is influenced to a degree of 31.16 % by the inflation level. Lower inflation represents stable prices and a sustainable monetary policy. If a country is unable to pay off its debts through government revenue, it may resort to "printing money".

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Multidimensional phenomena analysis with the use of Hölder function properties

Adrianna Mastalerz-Kodzis [1]

Abstract

The paper proposes the use of multidimensional, economic random fields and Brownian motion processes in financial analyses. The stock exchange data feature a multidimensional character and it is dependent on time, therefore, an approach with the use of multidimensional stochastic processes is fully justified. A comparison of the variability of different characteristics concerning the same stock market value has been carried out on the base of estimated pointwise Hölder exponents. The paper is composed of two main parts, theoretical basics and empirical analysis together with the conclusions.

Key words

random fields, multidimensional stochastic process, time series modelling, Hölder function, estimation of the pointwise Hölder exponent, risk analysis.

JEL Classification: C21, C58, D14, E44

1. Introduction

Common access to economic data, including data coming from stock exchanges allows to conduct comprehensive analyses on the base of historical data. Stock exchange quotes of each of the stock market values can be treated as multidimensional stochastic processes, as each company possesses a couple of its own descriptive characteristics.

One of the possible mathematical tools, which can be used in quantitative analyses, seem to be multidimensional random fields. On their base we can consider multidimensional Brownian motion processes as realizations of stock market values. The proposed approach delivers new cognitive possibilities, it uses the so called memory in time series and allows for the generation of short-term forecasts of series' variability.

The aim of the article is to present a stochastic approach, based on multidimensional Brownian motions, to model the variability of stock market values with the use of the memory effect, as well as the estimation of pointwise Hölder exponents and the measurement of the variability of each of the multidimensional random fields' components in context of stock exchange analyses.

2. Multidimensional random fields

In most of the cases, economic phenomena, including the phenomena taking place on the stock exchange markets, show changes of random character, therefore, they should be analyzed using random fields. Stock market values are described with the use of many different characteristics dependent on time, we can, therefore, apply a multidimensional approach.

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Phenomena which occur in the surrounding us reality are mostly of random character[2]. Processes which describe these phenomena are *random processes*. The realization of a random process is determined with the use of a function *Y(x)*, where the function can feature real or vector values. A function which is the result of a random experience is called *the realization of a random process.* If an argument x_0 is determined, then the value of the function $Y(x_0)$ is a random variable. Then *the random process* is understood as a set of random variables $\{Y(x_0)\}$ for x_0 from a certain set *A*.

When *A* is any set, then the random process relates to the random function. Whereas the term *stochastic process* is used in the case when the variable *x* is interpreted as time parameter. When a time variable is the argument of the function, then the function is called *a random field.*

Let $Y_i, i = 1, \ldots, k$ be a variable, which undergoes random fluctuations. Then $Y = [Y_1 \ \ldots \ Y_k]^T$ is *k - dimensional random field*, e.g. function of form $Y : D \times \Omega \to R^k, D \subset R^n$, such, that for each $x_0 \in D, n \in N$ the value $Y(x_0, \omega)$ is a k - dimensional random vector determined in a set probabilistic space..

Each of the components of vector *Y* is a random field, e.g. function $Y_i : D \times \Omega \to R^1, i = 1, \ldots, k$ such, that for each $x_0 \in D$ value $Y_i(x_0, \omega)$ is a random variable. If we determine a time variable, then the random field $Y(x, \omega)$ is a static field, if, on the other hand, the variable is subject to fluctuations in time, then the random field also describes the dynamics of the studied phenomenon.

The components of the above mentioned vector are *one-dimensional random fields.* Their basic characteristics are: the expected value and the function of covariance. We can differentiate the following random field classes: homogeneous (invariable in regard to translation) and isotropic (the medium value function and the covariance function are invariable in regard to turnover)[3].

3. Application of pointwise Hölder exponents in multidimensional financial analyses

A time series of, for example, stock closing prices can be treated as a function of one variable - parameter *t*, where *t* is a time variable. Adding arguments, for example other characteristics of stock market values i.e. opening price, minimum and maximum price, volume of turnover, we obtain a function of many variables dependent on time. We can treat such a type of function as a multidimensional random field, then, in consideration of a random character of variables, we obtain a multidimensional stochastic process.

The analyzed in the further part of the paper methods used in the study of time series possess the feature of using in the process of modelling the so called memory effect. It is assumed that historical data influences the present and future values. The length of the memory in time series and the variability of values is studied. It seems reasonable to question whether this methodology can be transferred to a multidimensional case.

Numerous theoretical and empirical studies show that in economic sciences, and not only there, historical data possesses an influence on the present and the future. Transferring the methodology with preservation of the so called memory effect from time series to multidimensional data is therefore justified.

[2] Sobczyk (1996), Przybycin (2004), Anselin (1988), Mastalerz-Kodzis (2003A).
[3] Przybycin (2004).

In literature concerning financial markets there exist two below mentioned hypotheses [4]:

- **The effective market** hypothesis. An effective market is a market where the price discounts all information, all important information which can influence the price are already included in it. There exists a lack of memory
- **The fractal market** hypothesis. A fractal market takes into account influence of information and the length of time horizon on the behaviour of market participants. Information is valuable according to the length of the investment horizon, information has a different influence on different time horizons. The market preserves memory (short- or long-term).

We can question the validity of these hypotheses in context of the processes taking place in economy in a multidimensional term and to what extent do the time series describing the same stock market value equally preserve memory.

Stochastic processes are defined on probabilistic space using the term: random variable[5]. Random variables are functions dependent on parameter. In economic phenomena and processes time *t* is most frequently the parameter.

We assume that *T* is a subset of a set of real numbers and that $t \in T$. *Stochastic process* $X(t)$ is a family of random variables $\{X_t(\omega), t \in T, \omega \in \Omega\}$ defined on probabilistic space (Ω, Σ, P) and dependent on parameter *t*.

Stochastic models which apply Brownian motion processes have already been present in literature concerning econometrics and finances for a long time. Time series, inter alia, as realizations of fractional and multi-fractional Brownian motion processes are modelled. The data on a plain can be modelled with the use of Hurst exponent and Hölder function[6]. In this case, series' graph on a plain possesses a topological dimension equal to 1, while the capacitive (fractional) dimension belongs to the range (1,2). The variability of the graph are measured by pointwise Hölder exponents dependent only on time.

In the process of modelling time series with the use of fractional and multi-fractional Brownian motion processes the only argument of the process is time. Fractional processes are a special case of the multi-fractional ones, that is, constant in time Hölder function is the value of Hurst exponent. Processes with stationary increments, like fractional Brownian motion processes, are dependent on a constant parameter - Hurst exponent. This exponent belongs to the range $(0,1)$ and divides the series into:

- persistent $H \in (\frac{1}{2}, 1)$– with positive correlation between the following realizations of the process;
- random series $H = \frac{1}{2}$ for which the probability that after an increase in the value there will be another increase equals 1/2 and
- anti - persistent $H \in (0, \frac{1}{2})$, in which the correlation is negative.

Let x_0 be any point from the function domain f $(x_0 \in D \subset \Re)$. Function $f : D \to \Re$ is at point x_0 Hölder function of class $C_{x_0}^{\alpha}$, if there exist constants $\varepsilon, c > 0$ such, that for each $x \in (x_0 - \varepsilon, x_0 + \varepsilon)$ the following equity is fulfilled:

[4] Fama (1965), Mandelbrot (1982), Mastalerz-Kodzis (2002A, 2003B, 2013, 2016B), Mastalerz-Kodzis & Zawadzki (2001).

[5] Sobczyk (1996), Mastalerz-Kodzis (2016A).

[6] Ayache and Lévy Véhel (1999), Daoudi, Lévy Véhel and Meyer (1998), Mandelbrot (1982), Mastalerz-Kodzis (2003A), Peltier and Lévy Véhel (1995), Miśkiewicz-Nawrocka (2016), Miskiewicz-Nawrocka and Zeug-Zebro (2016), Zeug-Zebro (2015).

$$| f(x) - f(x_0) | \le c | x - x_0 |^{\alpha} . \tag{1}$$

By pointwise Hölder exponent of function f at point x_0 we call the number $\alpha_f(x_0)$ defined by the formula $\alpha_f(x_0) = \sup\left\{\alpha : f \in C_{x_0}^{\alpha}\right\}$. By Hölder function of function f we call the function, which to each of the points $x \in D$ assigns the number $\alpha_f(x)$.

Hölder function is of class C^{α} in the surrounding of any point from the domain. Since parameter $\alpha \in (0,1)$, we can consider fractional differentiation. When the value of the derivative is interpreted then the speed of the value change at argument change is determined. Therefore, when time is the argument, then the value of the derivative is the speed of the change in time.

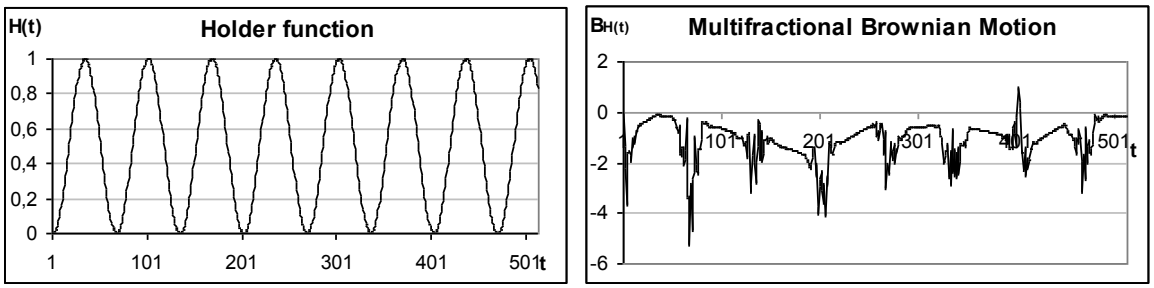
By the end of the 20th century a process was proposed in literature called the multifractional Brownian motion process[7]. Let $H_t : [0,\infty) \to (0,1)$ be Hölder function of exponent $\alpha > 0$. By multifractional Brownian motion process with function parameter H_t we call a stochastic process $B_{H_t}(t)$ defined for $t \ge 0$ with the formula

$$B_{H_t}(t) = \frac{1}{\Gamma\left(H_t + \frac{1}{2}\right)} \left\{ \int_{-\infty}^{0} \left[(t-s)^{H_t - ½} - (-s)^{H_t - ½} \right] dB(s) + \int_{0}^{t} (t-s)^{H_t - ½} dB(s) \right\} \tag{2}$$

where B is a standard Brownian motion process.

Pointwise Hölder exponents inform about the process properties. We can, among others, note that the process does not feature stationary increments when Hölder function is not a constant function and the closer the values of Hölder function to zero, the higher the variability of the graph, for function values close to one the process is smoother. Local capacity dimension and Hausdorff dimension of process trajectory $B_{H_t}(t)$ for each $t_0 \ge 0$ is $2 - H(t_0)$ and with probability equal to one, Hölder point exponent of process trajectory $B_{H_t}(t)$ for each $t_0 \ge 0$ equals $H(t_0)$. Process regularity measured with pointwise Hölder exponents alters in the range (0,1) (figure 1).

Figure 1: Hölder function and computer generated multifractional Brownian motion process.



Stochastic processes are frequently used in empirical analyses in technical, medical, as well as socio-economic sciences, and in stock market analyses. Time series can be treated as a realization of a stochastic process. Estimation of pointwise Hölder exponents allows for an analysis of process variability in the surrounding of any point from the domain.

[7] Peltier & Lévy Véhel (1995); Banassi, Jeffard & Roux (1997), Ayache & Lévy Véhel (1999); Daoudi, Lévy Véhel & Meyer (1998), Mastalerz-Kodzis (2001A, 2001B, 2002B, 2014, 2016A).

There is a method of estimation of pointwise Hölder exponents present in literature[8]. Let us define with the symbol $\left\{B_{i,n} = B_H\left(\frac{i}{n}\right), \quad 0 \le i \le n\right\}$ - Brownian motion process with a constant in this range exponent H. Let $1 < k < n$ be the length of the neighbourhood (range) used to estimate function H. We will estimate a function for t from the range $[k / n, 1 - (k / n)]$.

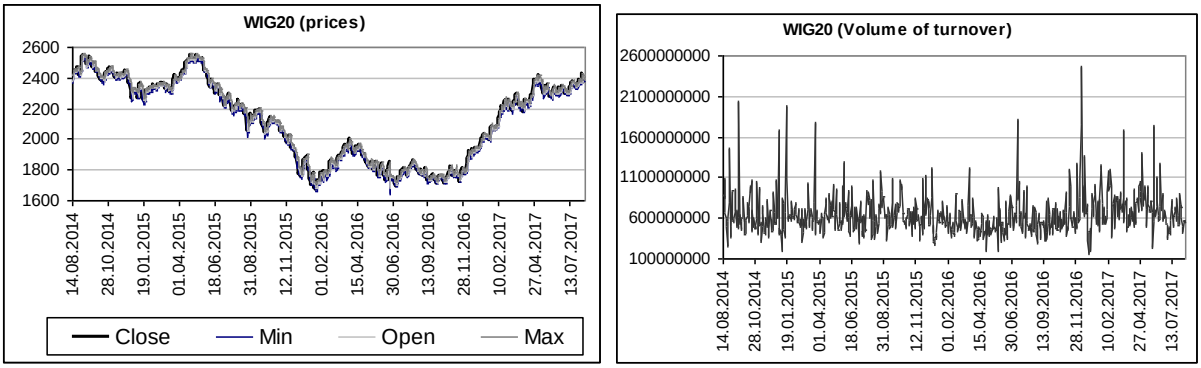
The estimator for $S_{k,n}(i) = \frac{m}{n-1} \sum_{j=i-k/2}^{i+k/2} \left| B_{j+1,n} - B_{j,n} \right|$ is the following $\hat{H}_{i/(n-1)} = -\frac{\log\left(\sqrt{\pi/2}\, S_{k,n}(i)\right)}{\log(n-1)}$.

4. Empirical analysis

Stock market data time series were treated as realizations of stochastic processes (dependent on time).

Empirical analysis was conducted on the base of WIG20 index data recorded on The Warsaw Stock Exchange. The data used came from the period covering the last three years, taken into consideration were the recordings from the period: 14.08.2014-11.08.2017 (751 recordings). Five time series were analyzed; four of them were prices: opening, closing, minimum, maximum and the fifth one - the volume of turnover. (figure 2).

Figure 2: WIG20 index, prices and turnover size graph . Data covering the period 14.08.2014-11.08.2017.



Basic characteristics i.e. average historical rate of return and its standard deviation were defined for the data. It can be stated that the series feature similar values and statistic measures, they also possess very low values of the coefficients of variation.

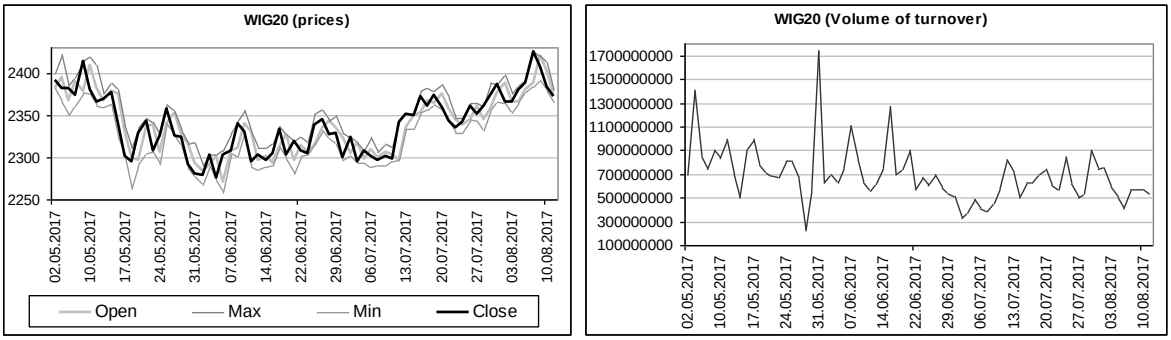
Table 1: Average historical rate of return and standard deviation of the rate of return for the WIG20 index value in the period 14.08.2014-11.08.2017.

Price	open	close	min	max
mean	7,03E-05	4,15E-05	6,27E-05	3,99E-05
standard deviation	0,0107	0,0093	0,0108	0,0108
coefficient of variation (%)	1,5220	2,2461	1,7185	2,7058

In the WIG20 graph we can notice, that the differences in values between the minimum, maximum, opening and closing price are very low, hardly noticeable on the graph. Therefore, the graphs below show a selected subset of the data from the period 2.05.2017-11.08.2017 . The differences of the daily recordings of the index prices and the direction of changes of the turnover volume can be clearly noticed (figure 3).

[8] Ayache and Lévy Véhel (1999), Peltier and Lévy Véhel (1995).

Figure 3: WIG20 index, prices and turnover size graph. Data from the period 2.05.2017-11.08.2017.



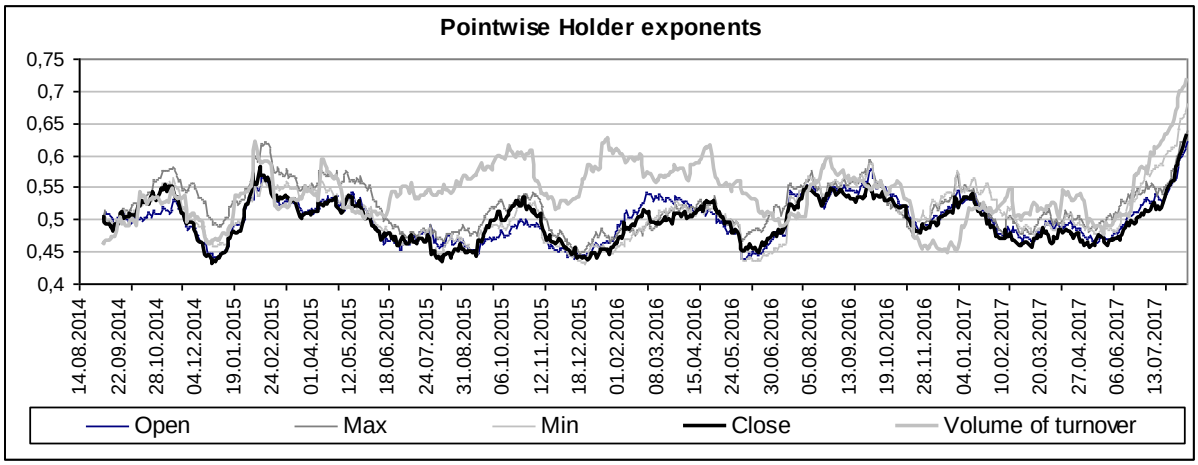
A question concerning the existence of relationship between the realizations of the analyzed processes was set. The values of Pearson correlation coefficients are shown in table 2. It can be stated that between the prices (closing, opening, min and max) there exist very strong, positive relationships (which could have been expected), whereas between the prices and the volume of turnover there is a minimal, positive relationship.

Table 2: Pearson coefficient values for prices' series of WIG20 index in the period 14.08.2014-11.08.2017.

Value	open	close	min	max	volume of turnover
open	1	0,9970	0,9987	0,9988	0,1292
close		1	0,9989	0,9988	0,1340
min			1	0,9988	0,1234
max				1	0,1243
volume of turnover					1

Questioning the variability of the series (global and local) and the stationarity of increments, pointwise Hölder exponents were defined for the five analyzed series. The result was illustrated in figure 4.

Figure 4: Estimated pointwise Hölder exponents for WIG20. Data for the period 14.08.2014-11.08.2017.



The following questions can be set:

- Whether the estimated series possess stationary increments,
- Whether the pointwise Hölder exponents of the series vary significantly,
- Whether the series feature memory,
- What is the relationship between the pointwise Hölder exponents for the series,
- Whether there exists a relationship between the price and the volume of turnover for the values and for pointwise Hölder exponents?

Tests concerning the stationarity of series were conducted for the data from WIG20 in the Statistica programme. On the base of the analyses, it can be stated that all the analyzed series which characterize this index do not feature stationary increments. Studying the values of pointwise Hölder exponents, it can be stated that, for the analyzed prices, they differ insignificantly (maximally by absolute value 0,07), whereas between the prices and the size of turnover the differences are significant (absolute value reaches 0,18). In most cases, pointwise Hölder exponents for the turnover size are of higher values than for the prices. This proves that the memory in the series is stronger in the volume of turnover series than in the prices series (index values). The values of Hölder function for the prices belong to the range (0.439; 0.683), while for the volume of turnover to the range (0.449; 0,719). Therefore, there exists memory in the series. With probability *H(t)* after an increase in the value of the series, another increase takes place, with probability *1-H(t)* there is a decrease in the value.

The direction of changes of pointwise Hölder exponents in time is similar for the analyzed series. For the series of turnover size realization in the period 06.2015-06.2016 pointwise Hölder exponents present significantly higher values than for the rest of the series (slight variation in figure 2, 4). Analyzing the values of the estimated pointwise Hölder exponents, it can be stated that, in the following recordings, an upward direction of the changes in prices will be preserved.

5. Summary

The application of multidimensional random fields together with Brownian motion stochastic processes seems to be the right complement to classical methods of stock market analyses. Observing the changes taking place on stock markets not only with the use of time series methods is of great value, as it allows for a more precise description of the studied phenomena and enables us to notice the relationships which cannot be recorded during the analyses of one-dimensional stochastic processes.

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The Quantification of Effectiveness as Precondition for Facility Management

Jaroslav Mazanec, Viera Bartošová

Abstract

Nowadays, facility management represents the up-to-date trend in optimizing operating costs in private, public and non-profit organizations. Facility management is management of support activities to increase the effectiveness in organization. However, the definition of facility management varies depending on national associations. This article focuses on the quantification of effectiveness rate in organization as a precondition for the potential application of facility management because identifies the weaknesses of organization. The best-know tools of facility management are insourcing and outsourcing. Many organizations apply facility management to increase competitiveness in the market.

Key words

Data Envelopment Analysis, Effectiveness, Facility Management, Hospital

JEL Classification: C00, I1

1. Úvod

V súčasnom období mnohé subjekty uplatňujú facility management za účelom zvýšenia dlhodobej efektívnosti a prosperity. Dôvodom je, že facility management prispieva k ekonomickému rastu a celkovému úspechu organizácie z hľadiska konkurencieschopnosti.

2. Teoretické východiská

Facility management (FM) vznikol v 70. rokov minulého storočia v USA, a následne v 80. rokoch vznikla medzinárodná asociácia International Facility Management Association (IFMA) v Houstone. Na druhej strane, v krajinách západnej Európy sa facility management rozvíja až od začiatku 90. rokov minulého storočia. Ďalej, na území Slovenskej republiky vznikla asociácia zaoberajúca sa facility managementom až v roku 2009, a to pod názvom Slovenská asociácia facility managementu (SAFM). K primárnym úlohám slovenskej asociácie patrí presadzovanie facility managementu do procesov rozhodovania a riadenia organizácie na strategickej, taktickej a prevádzkovej úrovni. Navyše, umožňuje výmenu informácií medzi odborníkmi z danej oblasti (Somorová, 2014).

Definície facility managementu sa líšia v závislosti od národných asociácií. IFMA uvádza, že FM predstavuje metódu harmonizácie pracovníkov, pracovných činností a pracovného prostredia. K základným aspektom facility manažmentu patria, napr. riadenie podporných procesov, zlúčenie procesov, apod. British Institute of Facilities Management (BIFM) uvádza, že facility management je integrácia multidisciplinárnych aktivít v stavebnom prostredí, ktoré vplývajú na pracovníkov a pracovisko. Vo všeobecnosti, BIFM zdôrazňuje pracovisko ako súčasť stavebného objektu. Ďalej, German Facility Management (GEFMA) charakterizuje facility management z hľadiska nákladového (ekonomického) princípu procesov s cieľom podporiť základnú činnosť organizácie. Kicova (2016) poukazuje na to, že Európska únia

(EÚ) prispela k zosúladeniu národných noriem v oblasti facility managementu z dôvodu rastúceho významu. Dodáva, že facility management má ekonomické prínosy, pretože umožňuje dosiahnúť podstatné zníženie nákladov na podporné činností. Európska komisia (EK) uvádza v dokumente „*Konkurencia medzi poskytovateľmi obchodných služieb a prínos k výkonu európskych organizácií*“, že facility manažment zvyšuje výkonnosť organizácie.

Podľa Somorovej (2014) FM predstavuje integráciu činností, ktoré zabezpečujú a rozvíjajú vybrané služby za účelom zvýšiť mieru efektívnosti základných činností. Vyskocil & Kuda (2011) uvádzajú, že manažment podporných procesov predstavuje manažérsku aktivitu, ktorá ovplyvňuje výsledky a efektivitu hodnotového reťazca ako celku v hospodárskej oblasti, vo verejnej správe a v nepodnikateľskej sfére. Ďalej, Kuda & Beránková (2012) uvádzajú, že prínosom facility managementu je zvýšenie efektívnosti pomocou koncentrácie na hlavnú činnosť, zodpovednosť za riadenie podporných činností, vyššia miera riadenia podporných činností, zlepšenie procesu riadenia a kontroly, zvýšenie informovanosti, optimálne previazanie pracovníkov, prostredia a procesov, redukcia konfliktov medzi internými a externými dodávateľmi služieb, integrácia a koordinácia všetkých požadovaných podporných služieb, apod.

V každej organizácií prebiehajú primárne (hlavné) činnosti a sekundárne (podporné) činnosti. Hlavná činnosť súvisí s primárnym predmetom podnikania, pričom ostatné činnosti predstavujú tzv. podporné činnosti. K podporným činnostiam (angl. non-core processes) patrí správa majetku, prevádzka objektu, údržba objektu, energetický manažment, priestorový manažment, upratovanie, ochrana životného prostredia, apod. K základným formám facility managementu patrí insourcing a outsourcing, ktorého cieľom je znížiť náklady a zvýšiť kvalitu poskytovaných služieb. Vyskocil & Kuda (2011) tvrdia, že outsourcing predstavuje vyčlenenie vedľajších činností na externého dodávateľa, ktorý zodpovedá za realizáciu. Napr. Hrusecka, et al. (2015) skúmali uplatnenie outsourcingu v logistike v rámci českých výrobných podnikov. A zároveň, FM môžeme považovať za inovačný nástroj v riadení organizácie. Janoskova & Kral (2015) uvádzajú, že v súčasnom období globálnych zmien zohráva významnú úlohu práve riadenie inovácií.

Na základe tvrdení, pod facility manažmentom rozumieme kvalitné integrované riadenie podporných činností v organizácií, čím prispieva k plynulému priebehu hlavnej činnosti organizácie a pomáha vytvárať kvalitné pracovné prostredie pre zamestnancov. Úspešný FM má pozitívny dopad na ekonomické a neekonomické indikátory organizácie prostredníctvom znižovania energetickej náročnosti a prevádzkových nákladov s cieľom sústrediť sa na hlavné aktivity subjektu a zvýšiť spokojnosť zákazníkov. Tzn., že FM vedie k zvýšeniu efektívnosti organizácie. Navyše, FM môže prispieť k zvýšeniu kvality poskytovaných služieb (Gutkevych & Punchak, 2016). Vykopal (2012) považuje facility management za efektívnu stratégiu manažérskeho riadenia, pričom Naranjo-Gil (2015) sa zaoberá úlohou riadiacich systémov pri tvorbe týchto stratégií.

FM sa uplatňuje najmä v komerčnom, ale aj vo verejnom a súkromnom sektore. Tento článok sa zameriava na posúdenie miery efektívností za účelom potenciálneho uplatnenia facility managementu vo vybraných slovenských nemocniciach. Janisch & Lennerts (2011) uvádzajú, že nemocnice čelia zvyšujúcim sa nákladom na poskytovanie zdravotnej starostlivosti, čo vyžaduje opatrenia na zníženie výdavkov. Dodávajú, že približne 30 % nákladov nemocníc pochádza zo sekundárnych činností. Ďalej, považujú benchmarking za východiskový bod pri identifikovaní príležitosti v oblasti úspor nákladov. Lennerts, Abel, Pfrunder & Sharma (2005) uvádzajú, že zdravotná starostlivosť je veľmi nákladná, pričom významnú časť nákladov tvoria náklady na lôžkovú liečbu. Dodávame, že primárnym cieľom zdravotníctva je zvyšovanie efektívnosti pri zabezpečení kvalitnej zdravotnej starostlivosti.

Na posúdenie miery efektívnosti existujú rôzne nástroje, napr. Ruckova & Roubickova (2012) uvádzajú, že finančná analýza je vhodný nástroj na posúdenie miery efektívnosti subjektov, resp. nástroj na identifikovanie slabých miest prostredníctvom pomerových ukazovateľov, ktoré vychádzajú zo štandardných účtovných výkazov. Je potrebné si uvedomiť, že finančná analýza je dôležitá pri posúdení finančného zdravia subjektu. Na druhej strane, nevýhodou pomerových ukazovateľov je, že sa zaoberá menším počtom faktorov, ktoré vplývajú na celkovú efektivitu posudzovanej jednotky. Ďalej, Jablonsky & Dlouhy (2015) tvrdia, že predpokladom pre zlepšenie konkurenčnej schopnosti je kvantifikácia výkonnosti a efektívnosti ekonomických jednotiek a identifikácia zdrojov neefektívnosti pomocou Data Envelopment Analysis (DEA). Pod ekonomickou jednotkou rozumieme jednotku, ktorá vytvára výstupy pri určitom množstve vstupov, napr. nemocnice, vysoké školy, apod. Tzn., že je možné hodnotiť homogénne jednotky, ktoré vykonávajú rovnakú činnosť. Dodávajú, že na efektívne fungovanie vplývajú rôzne vstupy a výstupy, pričom niektoré vstupné a výstupné veličiny sú považované za dôležitejšie a iné za menej dôležité.

DEA je viackriteriálna metóda, ktorá slúži na meranie a porovnanie efektívnosti v rámci príslušnej skupiny homogénnych jednotiek. Ide o metódu lineárneho programovania, ktorá sa spočiatku využívala na meranie efektívnosti neziskových organizácií. Pôvod DEA metódy súvisí s Debreuom (1951), Koopmansom (1951), Shepardom (1951) a Farrellom (1957). Ďalej, Charnes, Cooper a Rhodes (1978) vytvorili CCR DEA model, ktorý je pomenovaný na základe iniciálok mien autorov (Kliestik & Zvarikova, 2013, Farrell, 1957). Jablonsky (2011) uvádza, že tento model maximalizuje mieru efektívnosti, ktorá je vyjadrená podielom vážených výstupov a vážených vstupov za predpokladu, že miera efektívnosti všetkých ostatných posudzovaných jednotiek je menšia alebo rovná 1. V našom prípade sme aplikovali vstupovo-orientovaný CCR DEA model, ktorý stanoví príslušný objem vstupov z hľadiska zabezpečenia 100 %-nej úrovne efektívnosti. Tento model je možné formulovať ako úlohu lineárneho lomeného programovania. V tomto prípade má nasledujúcu podobu

$$e\,(DMU_j) = \sum_{k=1}^{r} u_k\, y_{kq} \rightarrow \max, \tag{1}$$

$$\sum_{i=1}^{m} v_i\, x_{iq} = 1, \tag{2}$$

$$-\sum_{i=1}^{m} v_i\, x_{ij} + \sum_{k=1}^{r} u_k\, y_{kj} \leq 0,\ j = 1, 2, \ldots, n, \tag{3}$$

$$u_k \geq \varepsilon,\ i = 1, 2, \ldots, r, \tag{4}$$

$$v_i \geq \varepsilon,\ j = 1, 2, \ldots, m, \tag{5}$$

kde: „e“ je miera efektivnosti hodnotenej DMU, „ε“ je konštanta, ktorá zabezpečuje nezápornú váhu, „u_k“ je príslušná váha výstupu, „v_i“ je príslušná váha vstupu, „x_{ij}“ je množstvo vstupov, „y_{kq}“ je množstvo výstupov, „i“ je počet vstupov a „k“ je počet výstupov.

Vo všeobecnosti platí, že výstupné veličiny majú maximalizačný charakter. Tzn., že väčšie množstvo výstupov pri nezmenenom množstve vstupov spôsobuje vyššiu mieru efektívnosti. Avšak, vstupné veličiny sú charakteristické minimalizačným charakterom. Tzn., že menšie množstvo vstupov pri nezmenenom množstve výstupov smeruje k vyššej miere efektívnosti. Samotný výsledok efektívnosti sa pohybuje v rozmedzí od 0 do 1, resp. od 0 do 100 %. DEA model je založený na množine prípustných možností, ktorý sa skladá zo všetkých efektívnych rozhodovacích jednotiek (DMUs). Výhody DEA spočívajú v tom, že porovnávajú efektívnosť v rámci posudzovanej skupiny, vstupné a výstupné veličiny môžu byť v rôznych merných jednotkách, apod. Na druhej strane, nevýhodou je citlivosť na počet posudzovaných jednotiek

v súvislosti s počtom veličín v danom modeli. Z toho dôvodu, vychádzame z návrhu Bankera, et al. (1989), ktorý presadzuje 3-násobne vyšší počet DMUs než je počet vstupných veličín.

3. Metodológia

Cieľom článku je posúdiť mieru efektívnosti náhodnej vzorky nemocničných zariadení, ktoré sa skladajú z príspevkových a neziskových organizácií prostredníctvom neparametrickej metódy DEA. Pre výpočet miery efektívnosti sme vybrali sedem vstupných premenných, a to aktíva v € (*1*), náklady na spotrebu energií v € (*2*), náklady na prevádzkovú činnosť v € (*3*), osobné náklady v € (*4*), tržby z predaja služieb v € (*5*), počet ukončených hospitalizácií vrátane osobitne hradených výkonov (OHV) (*6*) a počet vykonaných operačných zákrokov vrátane OHV (*7*). Skúmaná vzorka sa skladá z 22 nemocničných zariadení na území Slovenskej republiky. Pre meranie efektívnosti sme použili finančné informácie zo zverejnených účtovných závierok na oficiálnej stránke v správe Ministerstva financií Slovenskej republiky (MF SR), interné údaje nemocníc a ďalšie informácie o realizovaných výkonoch sú získané z databázy Inštitútu pre ekonomické a sociálne reformy (INEKO) za obdobie 2015. Výpočet vstupo-orientovaného CCR DEA modelu je uskutočnený pomocou softvéru Data Envelopment Analysis.

4. Výsledky

Na základe výpočtu efektívnosti náhodnej vzorky nemocníc na území Slovenskej republiky pomocou viackriteriálnej metódy sme zistili, že len sedem z 22 posudzovaných nemocníc dosahuje 100 % mieru efektívnosti. Zaujímavosťou je, že štyri zo siedmich nemocníc má právnu formu neziskovej organizácie, pričom ostatné nemocnice majú právnu formu v podobe príspevkovej organizácie v správe štátu alebo VÚC. Na základe týchto výsledkov môžeme predpokladať, že nemocnice – neziskové organizácie sú efektívnejšie riadené v porovnaní so „štátnymi“ nemocnicami. K efektívnym nemocničným zariadeniam patria Dolnooravská NsP Dolný Kubín (DMU 1), FN Nitra (DMU 2), FNsP Nové Zámky (DMU 5), Forlife, n. o. (DMU 17), Všeobecná NsP Lučenec, n. o. (DMU 20), NsP sv. Jakuba, n. o. (DMU 21) a NsP, n. o. Levice a Topoľčany (DMU 22). Na druhej strane, k najmenej efektívnym nemocniciam UN Bratislava (DMU 13, 73,32 %), UN L. Pasteura Košice (DMU 14, 74,42 %), NsP Prievidza so sídlom v Bojniciach (DMU 12, 71,18 %), Liptovská NsP Liptovský Mikuláš (DMU 10, 77,85 %) a NsP Brezno, n. o. (DMU 19, 78,46 %). Na základe týchto výsledkov môžeme tvrdiť, že najväčšie slovenské nemocnice sú menej efektívne než menšie nemocnice. V prípade najmenej efektívnej nemocnice je potrebné znížiť objem aktív z pôvodných viac než 142 mi. € na približne 105 mil. €, a zároveň znížiť náklady na energie z viac než 6,5 mil. € na menej než 5 mil. €, znížiť náklady na prevádzkovú činnosť z z viac než 130 mil. € na približne 95 mil. € a znížiť osobné náklady z viac než 125 mil. € na menej než 92 mil. €. Navyše, je potrebné zvýšiť počet ukončených hospitalizácií na takmer 120-tisíc. Tabuľka 1 zobrazuje vstupné údaje pre výpočet CCR DEA modelu, ktoré sú kvôli prehľadnému znázorneniu zobrazené v mil. eurách, a to ukazovatele (1), (3), (4) a (5) a ukazovateľ (2) je zaokrúhlený na celé číslo.

Tabuľka 1: Vstupné údaje pre výpočet miery efektívnosti na základe vstupovo-orientovaného CCR DEA modelu

DMU	(1)	(2)	(3)	(4)	(5)	(6)	(7)
DMU 1	1,02	690 204	8,59	8,53	12,91	10 606	2 210
DMU 2	54,54	922 300	20,98	20,95	40,65	25 822	7 870
DMU 3	55,22	2 785 380	37,52	36,52	66,05	29 306	9 024
DMU 4	49,73	1 263	38,63	38,49	57,70	39 727	8 347
DMU 5	22,52	711 605	20,52	20,43	33,24	24 698	7 682
DMU 6	20,08	1 760 458	29,02	28,73	41,88	27 320	7 582
DMU 7	22,57	952 970	27,67	27,34	40,83	24 972	8 079
DMU 8	32,57	1 156 528	21,56	21,05	32,55	23 014	6 045
DMU 9	16,06	658 366	11,51	11,20	18,96	12 108	2 206
DMU 10	15,72	662 986	8,85	8,76	13,56	9 094	2 057
DMU 11	13,09	656 531	12,94	12,71	14,09	14 669	3 053
DMU 12	14,58	752 265	15,62	15,40	20,26	13 887	2 887
DMU 13	142,35	6 428 216	130,26	125,15	164,08	85 889	31 686
DMU 14	77,88	4 762 502	60,38	57,79	86,58	45 774	14 175
DMU 15	57,51	1 694 106	40,11	39,96	65,32	31 016	10 768
DMU 16	79,09	1 486 183	26,27	25,88	42,51	16 640	5 181
DMU, n. o. 17	3,74	810 775	10,16	8,84	14,24	12 625	3 429
DMU, n. o. 18	6,10	266 482	7,08	6,78	9,94	8 710	1 168
DMU, n. o. 19	7,87	352 269	8,15	7,87	10,61	6 941	1 251
DMU, n. o. 20	8,52	543 054	10,97	10,57	16,71	14 810	5 780
DMU, n. o. 21	16,35	426 339	8,18	7,14	14,84	11 352	1 880
DMU, n. o. 22	8,17	1 504 999	13,44	12,14	26,92	20 480	5 443

Zdroj: autor podľa INEKO (2015), MF SR (2017) a interných údajov nemocničných zariadení

Tabuľka 2 ukazuje poradie nemocníc v závislosti od miery efektívnosti a odporúčané hodnoty vstupov a výstupov, ktoré by mali nemocnice dosiahnúť kvôli 100 % miere efektívnosti.

Tabuľka 2: Usporiadanie posudzovaných nemocníc podľa miery efektívnosti na základe CCR DEA modelu

DMU	Efektívnosť	(1)	(2)	(3)	(4)	(5)	(6)	(7)
DMU 1	1,00	1,02	690 204	8,59	8,53	12,91	10 606	2 210
DMU 2	1,00	54,54	922 300	20,98	20,95	40,65	25 822	7 870
DMU 5	1,00	22,52	711 605	20,52	20,43	33,24	24 698	7 682
DMU, n. o. 17	1,00	3,74	810 775	10,16	8,84	14,24	12 625	3 429
DMU, n. o. 20	1,00	8,52	543 055	10,97	10,57	16,71	14 810	5 780
DMU, n. o. 21	1,00	16,35	426 339	8,18	7,14	14,84	11 352	1 880
DMU, n. o. 22	1,00	8,17	1 504 999	13,44	12,14	26,92	20 480	5 443
DMU 7	0,99	22,45	947 953	27,16	26,84	40,83	31 541	8 079
DMU, n. o. 18	0,99	6,04	263 830	6,80	6,60	10,32	8 710	3 126
DMU 15	0,93	53,58	1 578 405	37,37	36,29	65,32	45 490	11 546
DMU 6	0,88	17,59	1 542 592	25,43	24,57	41,88	31 871	7 582
DMU 11	0,83	10,91	547 421	10,79	10,20	17,10	14 669	5 050
DMU 16	0,83	48,64	1 233 444	21,80	21,39	42,51	28 023	8 301

DMU	Efektívnosť	(1)	(2)	(3)	(4)	(5)	(6)	(7)
…								
DMU, n. o. 9	0,78	6,18	276 389	6,39	6,17	10,61	7 713	1 795
DMU 10	0,78	11,68	516 113	6,89	6,58	13,56	9 402	2 679
DMU 12	0,77	11,25	580 560	12,06	11,60	20,26	14 803	3 481
DMU 14	0,74	57,96	3 544 219	44,94	42,35	86,58	62 005	16 851
DMU 13	0,73	104,37	4 713 096	95,03	91,76	164,08	119 206	31 686

Zdroj: autor podľa INEKO (2015), MF SR (2017) a interných údajov nemocničných zariadení

5. Diskusia

V poslednom období zaznamenávame rozšírené uplatňovanie viackriteriálnej metódy DEA na meranie efektívnosti v zdravotníckych zariadeniach po celom svete. Younis, a kol. (2017) použili neparametrickú metódu DEA spolu s Malmquistov indexom na výpočet produktivity tureckých nemocníc pre posúdenie zdravotnej reformy. Flokou, Aletras & Niakas (2017) použili tri vstupy (počet nemocničných lôžok, počet lekárov a iného zdravotného personálu) a tri výstupy (počet hospitalizovaných prípadov, počet chirurgických zákrokov a počtu ambulantných návštev na výpočet efektívnosti gréckych nemocníc v rokoch 2009-2013. Samut & Cafri (2016) poukazujú v článku „*Analysis of the Efficiency Determinants of Health Systems in OECD Countries by DEA and Panel Tobit*“ na to, že takmer väčšina krajín čelia znižovaniu rozpočtov na zdravotníctvo, ktoré motivujú verejné a súkromné nemocnice k účinnému využívaniu zdrojov. Ďalší výskum v súvislosti s facility managementom sa môže zamerať na posúdenie potenciálnych ponúk od podnikateľov za účelom znížiť náklady na prevádzku a súčasne energetickú náročnosť. Vzhľadom na to, že väčšina skúmaných nemocníc sú príspevkovými organizáciami v správe štátu je možné uplatniť jednotný FM vo všetkých nemocniciach za účelom zníženia nákladov v súvislosti s rozsahom požadovanej služby pre všetky nemocnice. Pri posudzovaní miery efektívnosti pomocou DEA považujeme za obmedzenie, že nebolo možné v tomto prípade použiť ľubovoľný ukazovateľ rentability (napr. ROA z EBITDA), pretože všetky ukazovatele dosahujú záporné hodnoty, čo je v rozpore s DEA modelom, ďalším limitom bolo nemožnosť použiť kategoriálnu premennú v súvislosti s hodnotením spokojnosti pacientov s nemocnicou na základe údajov INEKO. Táto spokojnosť bola hodnotená od 1 – 5 (najhoršie), pričom konečný výsledok modelu by mohol viesť k zvýšeniu spokojnosti na viac než 12, čo nemá vypovedaciu schopnosť vzhľadom na stanovenú hodnotiacu stupnicu. Navyše, k ďalším nedostatkom patria obmedzené údaje o počte doktorov, zdravotných sestrách, lôžok za jednotlivé nemocnice, ktoré zahraniční autori bežne používajú v štúdiách. Ďalšou možnosťou je výskum s cieľom porovnať väčší počet zdravotníckych zariadení na území Slovenskej republiky z časového hľadiska prostredníctvom Malmquistovho indexu alebo realizovať medzinárodné porovnanie zdravotníckych zariadení v rámci Vyšehradskej skupiny.

6. Záver

Kvantifikáciu miery efektívnosti môžeme považovať za východiskový bod pre možné uplatnenie FM. V prípade, že posudzované subjekty dosahujú nepriaznivé výsledky z hľadiska miery efektívnosti, môžu sa rozhodnúť pre jeden z mnohých úsporných nástrojov, napr. pre facility management. FM sa zameriava na úsporu prevádzkových nákladov s cieľom zlepšiť

ekonomické a neekonomické ukazovatele prostredníctvom spravovania podporných činností. Tzn., že príslušná organizácia sa môže zamerať na základný predmet podnikania. Je zrejmé, že nemocničné zariadenia majú potenciál na znižovanie prevádzkových nákladov v súvislosti s energetickou náročnosťou budov, nákladov na údržbu a opravy, náklady na prevádzkovú činnosť, osobné náklady, a pod. A práve FM môže zabezpečiť prevádzku bez potenciálnych porúch a údržbu technických a technologických zariadení. Ďalší, výskum môže byť zameraný na posúdenie konkrétnych ponúk podnikov, ktorých hlavný predmet podnikania spočíva v podporných aktivitách nemocnice.

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Benefits and risks of using outsourcing of economic activities

Jitka Meluchová, Martina Mateášová [1]

Abstract

The paper focuses on assessment the effectiveness of outsourcing in economic activities such as IT, accounting and financial services. One analyzes the efficiency of outsourcing from the point of view of individual processes and activities in the enterprise. The paper examines the selected activities and processes most frequently provided by external contractors and examines current trends in outsourcing in Slovakia. One evaluates the advantages and disadvantages of outsourcing and assesses the possible risks of outsourcing and offshoring under the current globalization trend. In the conclusions, the paper focuses on interpreting of the findings and proposing possible procedures for achieving optimal solutions.

Key words

Efficiency, Risk Management, Shared Services Centers, Financial Accounting, Outsourcing, Offshoring

JEL Classification: M48, G15, G17

1. Možnosti strategického využitia externých služieb

Cieľom príspevku je analyzovať efektivitu využívania outsourcingu vybraných činností, najmä z oblasti IT, účtovníctva a finančných služieb, ktoré sú vytesnené z podnikových aktivít a vykonávané externým dodávateľom. Skúmať vybrané činností a procesy, ktoré sú najčastejšie zabezpečované externým dodávateľom, ako aj súčasné trendy v oblasti poskytovania outsourcingu na Slovensku. Sumarizovať tak možnosti a prínosy ako aj možné riziká vyplývajúce z outsourcingu a offshoringu v pokračujúcom trende globalizácie. Outsourcing, označovaný ako *Business Process Outsourcing* (ďalej BPO), je pojem americkej obchodnej angličtiny a označuje strategické využitie externých zdrojov. Je to novotvar odvodený z anglických slov outside, resource, using a jeho preklad do slovenčiny je iba opisný - ako vyčlenenie, vytesnenie, presunutie alebo využívanie externých služieb. Outsourcingom sa obecne rozumie prax, kedy podnik určité svoje ekonomické činnosti nevykonáva sám ale si ich zaisťuje prostredníctvom externého poskytovateľa, ktorý sa špecializuje na realizáciu týchto činností. Ide o prevzatie zodpovednosti za ucelenú časť aktivít smerom od objednávateľa k poskytovateľovi na základe zmluvy. Zabezpečenie externou formou však musí vhodným a efektívnym spôsobom dopĺňať činnosti podniku, ktorý si ich tak nebude obstarávať resp. zaisťovať sám vo vlastnej réžii, ale spoľahne sa na externý subjekt. Spravidla ide o odovzdanie takých činností a aktivít, ktoré nesúvisia s hlavnou

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činnosťou podniku (core business), tzn. že podnik vyčlení rôzne podporné a vedľajšie činnosti a zverí ich inému subkontraktórovi, špecializovanému na príslušnú činnosť. Tento druh deľby práce tak umožní efektívnejšie využívanie zdrojov, dosiahnutie úspory nákladov, vyššiu špecializovanosť a upriamenie pozornosti na primárne činnosti podniku v záujme zvýšenia konkurencieschopnosti. Externý poskytovateľ (servisná firma) je obvykle nezávislá na podniku, pre ktorý danú činnosť zabezpečuje. Jednou z častých činností, ktoré sú na Slovensku outsourcované je vedenie účtovníctva. Podnik vytesní kompletné účtovníctvo zo svojej činnosti a deleguje ho na špecializovanú firmu, ktorá spravidla sídli v rovnakom meste čo má výhody ako rýchlu dostupnosť, osobné kontakty, operatívne odovzdávanie podkladov a blízkosť inštitúcií kam podnik patrí (daňový úrad, poisťovne). Outsourcing teda predstavuje presun určitých činností a aktivít na externého poskytovateľa v domácej krajine. V prípade, ak je externý dodávateľ umiestnený v zahraničí dochádza k offshoringu a už sa však nerozlišuje či ide o prepojený alebo nezávislý podnik.

Offshoring predstavuje vyčlenenie určitých aktivít v rámci jednej spoločnosti (spravidla nadnárodnej) do inej krajiny, opäť s hlavným cieľom dosiahnuť úspory. Presunom do zahraničia chce podnik získať konkurenčnú výhodu v podobe lacnejšej pracovnej sily, nižších cien vstupov alebo ľahšej dostupnosti k prírodným zdrojom. Ďalším dôvodom, ktorý ovplyvňuje strategické rozhodovanie, sú napríklad nižšie ceny nehnuteľností, výhodnejší prenájom, lacnejšie energie, daňové úľavy alebo nižšie dane. Ako uvádza Dvořáček a Tyl (2010) offshore centrá sú priamo miesta, ktoré majú okrem špecifického daňového režimu pre investorov taktiež priaznivé podmienky pre rozvoj podnikania na svojom území. Offshoring je považovaný za rizikovejší ako outsourcing, nakoľko sa s ním spájanú niektoré špecifické riziká, ako je geopolitické (legislatíva, politický režimu, vládne regulácie), právne (vymáhateľnosť obchodných zmlúv, ochrana duševného vlastníctva a priemyselných práv), IT (bezpečnosť a ochrana dát, zraniteľnosť informačných systémov), podnikateľské a finančné (podnikateľská etika, transakcie v cudzej mene, pohyb kurzov) a ďalšie. Finálnemu rozhodnutiu o prenose podnikateľských činností za hranice materského štátu musí predchádzať hĺbková analýza a zváženie všetkých výhod, ako aj rizík s tým spojených.

Podnik funguje za účelom dosiahnutia určitého výsledku. Tento výsledok je vždy merateľný, vždy s ním súvisia určité náklady a určité riziko. Podnik k dosiahnutiu svojich cieľov využíva svoje nehnuteľnosti, stroje, zariadenia, technológie a zamestnancov, čo prináša vznik nákladov a existenciu podnikateľských rizík. Práve outsourcing a offshoring dáva možnosť zefektívniť určité podporné procesy a činnosti a dosiahnuť okrem úspory nákladov aj prenos podnikateľských rizík práve tým, že všetko čo nie je hlavnou činnosťou sa prenesie na externého poskytovateľa. Tento proces bol typický najmä pre veľké nadnárodné podniky, ale v súčasnosti je využívaný rôznorodým spektrom podnikov čo do veľkosti aj predmetu činnosti, ako vyplýva z mnohých prieskumov.

2. Outsourcing ako strategický nástroj riadenia

Outsourcing je chápaný ako súčasť strategického riadenia podniku. Hlavné problémy, ktoré musí manažment podniku riešiť sú otázky: aké činnosti vyčleniť, finančné efekty vyčlenenia, konkurenčná výhoda vyčlenenia, dlhodobé dôsledky vyčlenenia a to je potrebné flexibilne prispôsobovať aktuálnej stratégii podniku, ktorá je odrazom vnútorných ako aj vonkajších vplyvov. Manažment pri rozhodovaní zvažuje všetky atribúty a integruje ich do životaschopnej stratégie a akčných plánov, s cieľom dosiahnuť kľúčových ukazovateľov ako sú obrat, zisk, profitabilita, návratnosť investícií atď. To ovplyvní aj stupeň outsourcingu, ktorý podnik nakoniec zvolí.

Table 1: Stupeň využitia outsourcingu

Stupeň spolupráce	Charakteristika	Výhody pre podnik
1. Jednorazový outsourcing	Využitie outsourcingu na individuálny resp. jednorazový projekt.	• riešenie jednej parciálnej situácie resp. činnosti • získanie skúseností s outsourcingom • potvrdenie či zmena dodávateľa príp. potvrdenie rozhodnutia zriadiť vlastné centrum zdieľaných služieb
2. Opakovaný outsourcing či preferovaný dodávateľ	Opakované využívanie outsourcingu pre menšie či väčšie vybrané činnosti či procesy.	• overenie riešení pre rôzne činnosti či projekty • analýza hlavných výhod a nevýhod resp. rizík outsourcingu • výber vhodného dodávateľa príp. zriadenie vlastného centra zdieľaných služieb
3. Strategické partnerstvo	Spoločné dlhodobé strategické plánovanie vyčlenených vedľajších alebo podporných činností či procesov. Kombinácia rôznych druhov outsourcingových služieb. Previazanosť vlastných a outsourcovaných činností a aktivít. Štruktúrované zdieľanie rizík a zodpovednosti	• outsourcing činností ako dlhodobý strategický nástroj a taktický nástroj flexibility • detailná znalosť výhod a nevýhod resp. rizík outsourcingu • vývoj služieb na mieru a vytváranie pridanej hodnoty • využitie synergie obidvoch subjektov (objednávateľa a dodávateľa) na dlhodobej báze a prehĺbenie spolupráce

Zdroj: Vlastné spracovanie

Ďalej je možné skúmať variantnosť oblastí zabezpečovaných prostredníctvom outsourcingu. Využíva sa od jednoduchých podporných činností až k odovzdaniu komplexných činností, pri ktorých dochádza k vzájomnému prelínaniu dvoch podnikov.

Table 2: Druhy outsourcingu

Označenie	Popis
Selektívny outsourcing	• označovaný ako outtasking, odovzdané sú len jednotlivé úlohy, nie celé procesy
Plný outsourcing	• vytesnenie celých funkcií (napr. účtovníctvo, logistika)
Obchodno-procesný outsourcing	• ak sa jednotlivé funkcie nedajú vyčleniť izolovane, odovzdá sa často celý procesný reťazec
Backsourcing/insourcing (integrácia)	• skôr vytesnená služba alebo služba, ktorá doposiaľ nebola uskutočňovaná v podniku, je importovaná do podniku
Single-sourcing	• vytesnenú úlohu preberá jeden samotný poskytovateľ
Multi-sourcing	• vytesnenú úlohu preberajú viacerí poskytovatelia
Nearshoring-outsourcing	• presun vybranej činnosti do inej lokality v blízkosti podniku
Offshoring-outsourcing	• presunutie vybranej činnosti do inej krajiny (nízko nákladové zahraničné oblasti)
Interný outsourcing	• poskytovanie služby nákladovým strediskom podniku, ktoré je ale ekonomicky, právne alebo teritoriálne samostatné. Táto forma vytesnenia býva najčastejšie organizovaná vo forme Shared Service Center
Externý outsourcing	• presunutie vybranej činnosti na poskytovateľa, ktorý nie je nijako zviazaný s objednávateľom

Zdroj: Spracované podľa Thiel a Cawelius, 2007

Problematika outsourcingu zahŕňa spravidla tie funkčné oblasti, ktoré bezprostredne nesúvisia s hlavným predmetom činnosti podniku, napr. stravovanie zamestnancov alebo oblasti, ktoré doteraz boli výhradne vykonávané priamo samotným podnikom, napr. výroba. Najčastejšie však ide o oblasti, ktoré majú obidve uvedené charakteristiky - teda bezprostredne nesúvisia s hlavným predmetom činnosti podniku a doposiaľ bolo obvyklé vykonávať ich samotným podnikom (napr. účtovníctvo, telemarketing, prevádzka informačného systému) ako uvádza Bruckner a Voříšek (1998). Najčastejšie činnosti, ktoré sú predmetom outsourcingu alebo offshoringu sú oblasť informačných technológií napr. správa technického vybavenia, správa počítačových sietí, zálohovanie dát, správa webových portálov, servis, upratovanie, ochrana majetku a osôb, HR manažment, právne služby, stravovanie, marketing, obchod, logistika, development, mzdové a finančné účtovníctvo, správa investícií, reporting a pod. Ak by sme mali identifikovať primárne dôvody využitia outsourcingu je možné ich zhrnúť do štyroch oblastí:

- *konkurenčné dôvody* sú výsostne strategické, získanie výhody nad konkurenciou s cieľom zabezpečiť dlhodobé prínosy,
- *vecné dôvody* sa týkajú zdokonalenia v hlavnej oblasti činnosti, väčšinou ide o rozvoj, v niektorých prípadoch však môže byť vecným dôvodom udržanie tradície alebo dokonca prežitie,
- *finančné dôvody* sú zníženie nákladov a/alebo zvýšenie výnosov, finančné hľadisko je dôležitým hodnotiacim faktorom úspešnosti outsourcingu aplikovaného z iných dôvodov ako finančných,
- *organizačné dôvody* vedú k zjednodušeniu manažérskej práce a redukcii organizačnej štruktúry podniku, to súvisí s rastúcou špecializáciou podniku, pracovníkov, ale i stredného manažmentu.

Prostredníctvom outsourcingových kontraktov sa diverzifikujú riziká a podnik, ktorý si kupuje externé služby, tak zdieľa riziká s ostatnými zákazníkmi svojho poskytovateľa. Súčasne vznikajú aj nové riziká, plynúce zo závislosti na poskytovateľovi, na jeho serióznosti a stabilite. Riziko je súčasťou každej činnosti, teda aj outsourcingu, nemožno ho odstrániť a preto je potrebné s ním pracovať. Riziko zvyšuje najmä dĺžka obdobia partnerstva, zmluvné podmienky, nedostatočná kontrola procesov a činností prenášaných na poskytovateľa, nedostatočný rast úrovne služieb a kvalifikácie personálu poskytovateľa. Avšak podnik je vystavený aj možnej strate kľúčových znalostí a kompetencií, riziku úniku a zneužitia informácií a v neposlednom rade aj strate efektivity v prípade nesprávnej analýzy nákladov a prínosov z outsourcingu a pod.

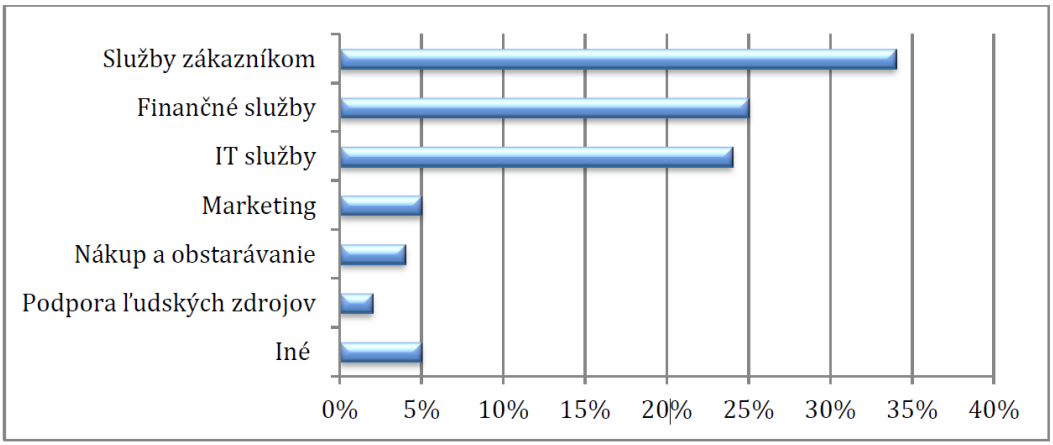
3. Centrá zdieľaných služieb v oblasti účtovníctva

Z dôvodu potreby zabezpečiť presunutie resp. vytesnenie uvedených činností mimo podnik, začali vznikať servisné centrá ako poskytovatelia týchto služieb, ktoré v druhej polovici osemdesiatych rokov minulého storočia zakladali najmä nadnárodné spoločnosti po celom svete. *Shared Service Center* (ďalej SSC) je špecializovaná entita podniku, ktorá preberá a vykonáva vedľajšie prevádzkové a podporné činnosti s cieľom ich štandardizácie a zjednodušenia procesov, s cieľom dosiahnutia vyššej efektivity a nižších nákladov. SSC je spravidla umiestnená v krajine s nízkou cenou prác, ale vysokokvalifikovanou pracovnou silou, s možnosťou optimalizácie daňového zaťaženia či výhodnej strategickej polohy. Na rozdiel od outsourcingu sú tieto centrá zakladané vlastným podnikom, a to buď vo forme spoločného podnikania alebo sú založené ako samostatná dcérska spoločnosť, do ktorej je daná činnosť delegovaná. Nejde už teda o presun na nezávislú tretiu firmu, ale o presunutie

vnútropodnikových činností a procesov na špecializované servisné centrum spravidla v rámci nadnárodnej skupiny.

Z hľadiska vzťahu ku klientom existujú na Slovensku dva typy centier podnikových služieb označované ako *Business Service Center* (ďalej BSC). Sú to centrá zdieľaných služieb (SSC), ktoré realizujú vybrané vnútropodnikové procesy, čím podporujú kľúčové činnosti v rámci skupiny podnikov a centrá externého poskytovania služieb (BPO), ktoré poskytujú služby v oblasti podnikových procesov pre tretie strany. Oba typy BSC vykonávajú na Slovensku široké spektrum činností. Podiel jednotlivých činností na celkových aktivitách BSC na Slovensku je znázornený v nasledujúcom grafe.

Figure 1: Služby poskytované BSC na Slovensku



Zdroj: BSCF, 2016

V súčasnosti SSC vykonávajú činnosti najmä z oblasti účtovníctva, správy aktív, IT, ľudských zdrojov, zákazníckeho servisu, centrálneho nakupovania vstupov, čím podporujú kľúčové činnosti materskej a dcérskych a sesterských spoločností. SSC prešli svojim prirodzeným vývojom. Vo svojich začiatkoch sa venovali predovšetkým financiám a účtovníctvu. Vykonávali sa v nich iba jednoduché transakčné a operatívne aktivity bez veľkej pridanej hodnoty. Postupne sa ale prepracovali k intelektuálne náročnejším funkciám s vyššími nárokmi na odborné zručnosti a prax, centralizovali sa v nich ucelené aktivity a procesy, a v neposlednom rade aj tzv. vlastníctvo štandardizovaných procesov (Mateášová, Meluchová, 2017). V roku 1998 otvorila spoločnosť IBM svoje prvé SSC mimo svoj región v Indii. V tom istom roku vzniklo aj účtovné centrum *European Accounting Center* (EAC) v Zürichu pre spoločnosť Sony a nasledovalo ďalšie centrum účtovných služieb (EAS) v Amsterdame rovnako pre Sony. Medzi ďalších priekopníkov patrila aj spoločnosť Nike, ktorá otvorila finančno-logistické centrum NSC v Hilversume. Výrazný boom SSC predovšetkým v krajinách strednej a východnej Európy nastal po roku 2004, po prijatí týchto krajín do Európskej únie, kedy sa SSC stávajú bežnou realitou a budúcnosťou najmä veľkých nadnárodných koncernov. V súčasnosti je na Slovensku etablovaných viac ako 200 SSC a tento počet stále rastie prieskumu agentúry Sario (2017).

3.1 Nastavenie procesu dodávateľských faktúr v SSC

Štandardizácia procesov a uniformita postupov sú kľúčové pri predchádzaní problémom. Štandardizácia umožňuje identické nastavenie činností, rotáciu zamestnancov medzi SSC, čo je východiskové už pri zakladaní centier. Z vykonanej analýzy vyplýva, že SSC spravidla vznikajú podľa identického modelu, čím je možné dosiahnuť nastavenie požadovanej uniformity podniku, zamestnancov a činností, avšak napriek tejto uniformite existujú rozdiely v ich interných procesoch. Príkladom môže byť proces spracovania dodávateľských faktúr, ktorý by mohol byť relatívne identický v rámci všetkých SSC. Spracovanie dodávateľských

faktúr je možné zjednodušene charakterizovať ako postupnosť krokov, ktoré sa začínajú prijatím faktúry, pokračujú jej skenovaním, nahratím údajov z faktúry do systému, kontrolou vecnej a formálnej správnosti údajov na faktúre, jej schválením kompetentnými osobami, jej zaúčtovaním, následne zaplatením a končia archiváciou faktúry. Podniky dostávajú „rovnaké" faktúry od rovnakých dodávateľov bez ohľadu na ich predmet podnikania. Výsledkom rozdielnych postupov sú odlišná efektivita, kvalita a doba spracovania. Tieto faktory majú za následok rozdielne náklady na spracovanie faktúr. Cieľom SSC je tento postup štandardizovať tak, aby proces spracovávania rovnakých faktúr bol identický, teda optimálny, pretože iba efektívny model je jednoduchý.

V SSC je proces spracovania dodávateľskej faktúry atomizovaný na jednotlivé kroky, kde každý krok procesu (prijatie, skenovanie, nahratie atď.) vykonáva určitý pracovník, často netušiaci, čo jeho operácii s faktúrou predchádzalo a čo sa s danou faktúrou udeje následne, ako on vykoná svoju činnosť. Bežná je aj situácia, že „rovnaké" faktúry účtujú rôzni pracovníci podľa vlastného „účtovníckeho názoru", často konajú mechanicky, bez potrebného odborného nadhľadu a uvedomenia si špecifík ako napríklad formálnych a obsahových náležitostí danej faktúry. To zvyšuje chybovosť v SSC. Príkladom môžu byť zle uvedené náležitosti faktúry alebo zle uvedený dátum pre vznik daňovej povinnosti, zlý dátum dodania tovaru/služieb alebo nesprávne nastavenie kódu položky z vnútropodnikového číselníka, čím sa automaticky zaúčtuje daná položka na nesprávny účet. Jednoduchosť procesu sa využíva pri rovnakých resp. podobných faktúrach od rovnakých dodávateľov, napríklad od mobilného operátora alebo dodávateľa energií. SSC sa snažia čo najviac zjednodušiť podstatu procesu a tak mnohé doplnili proces spracovania dodávateľských faktúr o objednávky. Celý proces tak začína od vopred vytvorenej a schválenej objednávky, kde je zo systému vybraný dodávateľ so všetkými správnymi náležitosťami, objednávka je doplnená o kód položky z číselníka účtov, ktoré následne došlú faktúru od dodávateľa spárujú s objednávkou a automaticky zaúčtujú na správny účet nákladov a do príslušného strediska podniku. Rovnako sa zmenil proces schvaľovania faktúr. Manažéri žiadajúci objednanie tovarov/služby a tak nechajú nahrať do systému objednávku, ktorú následne kompetentní schvália. Následne, ak dodávateľská faktúra súhlasí so schválenou objednávkou, prípadne je rozdiel v cene v rámci dohodnutej tolerancie, už sa duplicitné schvaľovanie nerobí. Došlé faktúry, ktoré sú odlišné od vytvorenej objednávky a vyžadujú osobitné schválenie, sa eliminujú a v súčasnosti podľa našich zistení predstavujú 5 až 10 %. Takýmto štandardným nastavením procesov sa šetrí čas zainteresovaných pracovníkov, znižuje sa percento chybovosti a celý proces sa tak výrazne zefektívňuje. SSC sa snažia dosiahnuť vyššiu efektivitu aj rôznymi metodikami ako je napríklad Operational Excellence, Lean, Six Sigma, Kaizen. Výber a zavedenie vhodnej metodiky ako aj zakúpenie licencií, zaplatenie konzultantov a zaškolenie zamestnancov je potrebné správne zanalyzovať, aby celkové vynaložené náklady priniesli reálny výsledok. Vyššiu efektivitu je možné dosiahnuť aj zamestnávaním vysoko kvalifikovaných zamestnancov a preto vysokoškolské vzdelanie, znalosť cudzích jazykov a vzdelanie v oblasti účtovníctva ako aj reálne skúsenosti s účtovníctvom sú v súčasnosti hlavné požiadavky na prijatie do SSC.

Sledovaným cieľom je štandardizácia operácií a zjednodušenie procesov, ktoré prinesú zvýšenie efektivity v podobe úspory času a nižších nákladov na výkon uvedených činností v SSC. Tento proces bol implementovaný na jednu z ekonomických činností a to dodávateľské faktúry. Uvedený postup pri spracovávaní dodávateľských faktúr je obecne uplatniteľný aj na iné procesy, ktoré SSC vykonávajú.

3.2 Výhody a riziká zdieľaných služieb v oblasti účtovníctva

Ak sa podnik rozhodne vytesniť určité činnosti a procesy na externého poskytovateľa, musí najskôr dobre zanalyzovať všetky výhody a nevýhody, ktoré tento významný krok podniku prinesie. Využiť môže outsourcing, offshoring alebo služby SSC a BSC. Iné pozitíva to bude mať pre veľkú nadnárodnú spoločnosť a iné pre stredný a malý podnik. Nadnárodná skupina podnikov využíva najmä vlastné SSC a BSC, ktoré sústredí do určitej krajiny, ktorá potom vykonáva tieto služby pre všetky ostatné entity. Z pohľadu malej a strednej firmy sa ako najvýhodnejší javí outsourcing pred offshoringom či centrom zdieľaných služieb. Dôvodom je najmä to, že poskytovateľ sa nachádza v blízkosti podniku a pozná podnikateľské prostredie, v ktorom operuje. Komunikácia je priama, bezprostredná a v prípade nesúladu informácií či podkladov reaguje operatívne priamo na mieste. Rutinných operácií býva menej a často riešenie neobvyklých či špecifických účtovných prípadov je vhodnejšie riešiť priamo za účasti outsourcingového partnera než priamo v samotnom podniku. Optimálne nastavenie procesov, odovzdávanie informácií a vykonávanie kontrol môže zabezpečiť flexibilné a operatívne riadenie pre podnik. Počiatočnou nevýhodou môže byť zvýšenie nákladov na tieto služby a nastavenie celého systému odovzdávania pokladov, komunikácie, kompetencií a zodpovednosti. U offshoringu a SSC býva primárna orientácia na účtovníctvo vedené podľa medzinárodných štandardov IFRS a US GAAP danej skupiny (matky), čo môže viesť k strate dôležitých detailov a špecifík národnej účtovnej legislatívy. Spravidla je potom lokálne účtovníctvo zabezpečované pomocou prevodových mostíkov alebo dvojitým vedením účtovníctva. To môže mať za následok, že nie všetky rozdiely musia byť identifikované a odhalené, ktoré sú však dôležité pre správne vyčíslenie výsledku hospodárenia, stanovenie základu dane z príjmov či pre účely výplaty podielov na zisku.

3.3 Koncepcia rozvoja servisných centier na Slovensku

Vláda SR prijala v júli 2016 *Koncepciu podpory centier podnikových služieb na Slovensku* (ďalej len Koncepcia), ktorá stanovuje opatrenia, s cieľom uľahčiť aktivity centier a rozvíjať priaznivé podmienky, motivujúce zahraničných i slovenských investorov zakladať a rozširovať BSC na Slovensku. Koncepcia predpokladá, že ak sa na Slovensku vytvoria priaznivé podmienky pre BSC, môže do roku 2020 vzniknúť až 20 000 nových pracovných miest v tejto oblasti. Koncepcia formulovala tri piliere, v rámci ktorých chce realizovať prijaté opatrenia. *Pilier 1 – Zabezpečenie dostatočného množstva pracovnej sily pre BSC, Pilier 2 – Vytvorenie prostredia podporujúceho rozvoj a rast BSC a Pilier 3 – Aktívna podpora nových investícií v oblasti BSC*. V zmysle Koncepcie vznikol pri Ministerstve hospodárstva SR (ďalej MH SR) zoznam centier podnikových služieb, na základe ktorého budú môcť BSC čerpať stanovené výhody.

Table 3: Zoznam centier strategických služieb k 30.6.2017 registrovaných pri MH SR

Spoločnosť		Sídlo	Dátum zaradenia
1.	IBM International Services Centre s.r.o.	Mlynské Nivy 49, 821 09 Bratislava	10.1.2017
2.	Johnson Controls International spol. s.r.o.	Štúrova 4, 811 02 Bratislava	6.2.2017
3.	Holcim Business Services s.r.o.	Protif. bojovníkov 11, 040 01 Košice	6.2.2017
4.	Henkel Slovensko spol. s.r.o.	Záhradnícka 91, 820 09 Bratislava	17.2.2017
5.	Orange Business Services Slovakia s.r.o.	Metodova 8, 821 08 Bratislava	7.3.2017
6.	T-Systems Slovakia, s.r.o.	Žriedlová 13, 040 01 Košice	13.3.2017
7.	AT&T Global Network Services SK, s.r.o.	Einsteinova 24, 851 01 Bratislava	31.3.2017
8.	Embraco Slovakia, s.r.o.	Odorínska cesta 2, 052 02 SNV	31.3.2017
9.	Fpt Slovakia, s.r.o.	Južná trieda 6, 040 01 Košice	7.4.2017
10.	Danfoss Power Solutions a.s.	Kukučínova 2148-84, 017 01 BB	7.4.2017
11.	Adient Slovakia s.r.o.	Štúrova 4, 811 02 Bratislava	11.4.2017

12.	KONE SSC, s.r.o.	Tomášikova 64, 831 04 Bratislava	15.5.2017
13.	Lenovo (Slovakia) s.r.o.	Einsteinova 21, 851 01 Bratislava	30.5.2017
14.	Lenovo Global Technology Slovakia s.r.o.	Einsteinova 21, 851 01 Bratislava	30.5.2017
15.	Deutsche Telekom Services Europe SK s.r.o.	Karadžičova 8, 821 08 Bratislava	31.5.2017
16.	Dell s.r.o.	Fazuľová 7, 811 07 Bratislava	30.6.2017
17.	IBM Slovensko spol. s.r.o.	Mlynské Nivy 49, 821 09 Bratislava	30.6.2017
18.	Zurich Insurance Company Ltd, org. zložka	Nám. 1. mája 18, 811 06 Bratislava	30.6.2017

Zdroj: Spracované podľa BSCF, 2016

Investičná pomoc môže byť poskytnutá vo forme dotácie na obstaranie dlhodobého majetku, úľavy na dani z príjmov, príspevku na vytvorenie nového pracovného miesta a vo forme prevodu alebo zámeny nehnuteľného majetku za cenu nižšiu, ako je všeobecná hodnota majetku.

V oblasti financií a účtovníctva Koncepcia predpokladá vytvorenie pracovných pozícií:

Financial Analyst – finančný analytik
Financial Advisor – finančný poradca
Accounting Specialist – špecialista na účtovníctvo
Invoicing Specialist – špecialista na fakturáciu
Payroll Specialist – špecialista na mzdy
Tax Assistant/Specialist – daňový asistent / špecialista
AR/AP Clerk – AR/AP pracovník

Zdroj: Spracované podľa BSCF, 2016

Podľa prieskumu agentúry Sario (2017) patrí medzi top dôvody, prečo sú na Slovensku zriaďované SSC a BPO najmä kvalifikovaná pracovná sila, nízka miera fluktuácie, dobré jazykové zručnosti a priaznivá geografická poloha. Ďalšími dôvodmi sú bezpečné investičné prostredie, politická a ekonomická stabilita, ekonomický rast, mena euro, deviate miesto na svete v oblasti adaptovania nových technológií a vysokého inovačného potenciálu, kontinuálny rast siete infraštruktúry, investičné stimuly a vysoká adaptabilita pracovníkov na rôzne podnikové kultúry a typy vedenia.

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The relationship between profitability and efficiency

Jiří Mihola, Petr Wawrosz[1]

Abstract

The article analyses the relationship between profitability and efficiency from the point of view how both indicators are able to express whether a firm develop extensively or intensively. We conclude that the indicator of efficiency does the task better and we explain why.

Keywords

dynamic parameter, effectiveness, intensity, extensity, profitability

JEL Classification L21, O12, O31, O33

1. Introduction

When measuring the intensity of development, we encountered a problem of meaningful and consistent definition of inputs and outputs of the analysed economic system by one aggregate input quantity and one aggregate quantity for output. When applying this methodology at the company level, we used microeconomic terminology. If we define the company outputs for the selected period by one number as the total ***TR*** revenue and the company inputs are expressed in one figure for the same period as the total ***TC*** costs, then the difference between these two ***TR-TC*** quantities will define economic profit ***EP***.

The development of economic profit is rightfully considered as an indicator of the quality of the business, however, as profit can grow both extensively and intensively, a reliable indicator of intensity has to be introduced, being the efficiency ***Ef***,, defined as the share of total revenues and total costs, i.e. ***Ef*** = ***TR/TC***, which expresses the amount of CZK outputs from one CZK costs incurred. A strictly extensive development changes the scope of production simply by changing the scope of economic activity, while intensive development is based on effective innovations of all kinds.

When presenting our results of intensity and extensibility measurements at the business level, objections have often been made in the sense that corporate finance discipline usually works with the profitability indicator of proceeds, i.e. ***R***=***EP/TR***, which represents the share of profit in total revenues. The profitability of sales ***R*** can easily be introduced into our relationships instead of effectiveness ***Ef***, but it is not appropriate with regard to the interpretation of results , and as shown in the default dynamic production function, there must be an effectiveness index. Although, there is a clear functional relation between the profitability and the effectiveness, i.e. ***EF***=f(***R***) and also the inverse relation ***R***=f(***Ef***), which does not contain any parameters, however this relation is not linear, but monotone increasing. Therefore, this study deals with this relationship, its derivation, representation and characteristics.

The main aim of this paper is to show why it is preferable to use effectiveness, not profitability, as an indicator of an intensive factor in company development.

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The result was accomplished as part of the resolution of a student project entitled The Improvement of pension system as an intensification factor of the economy with the use of the special-purpose support for a specific university research of the University of Finance and Administration (VŠFS)

2. Identification of the role of measuring the intensity of development

In order to analyse the intensity of the company's development, the time series of two of the three basic indicators, being ***TR*** total revenues (sales), ***TC*** total costs and ***EP*** profit, which is given by their difference, will suffice. If the efficiency of ***Ef*** is their share, then this special two-factor production function can be defined

$$\boldsymbol{TR} = \boldsymbol{Ef}\,.\boldsymbol{TC} \qquad (1)$$

If we change effectiveness for profitability, we would get a much less clear relationship that does not allow for a clear typology of all possible company developments including, but not limited to declines and compensations of both factors that allow for the assignment of appropriate well-interpretable values to all pure and mixed developments

$$\boldsymbol{TR} = \frac{\boldsymbol{TC}}{(1-\boldsymbol{R})} \qquad (2)$$

From the expression (1) we can derive a dynamic production function indicating that the total income index is given by the product of the effectiveness index and the total cost index.

$$\boldsymbol{I(TR) = I(Ef)\,.\,I(TC)} \qquad (3)$$

However, if we dynamize the expression (2), we will obtain a virtually inapplicable expression for subsequent derivation.

$$I(\boldsymbol{TR}) = \frac{\boldsymbol{R_0} - \frac{\boldsymbol{1}}{\boldsymbol{I(R)}}}{(\boldsymbol{R_0}-1)}.\boldsymbol{I(TC)} \qquad (4)$$

The fundamental problem of this expression is that it does not allow to separate identifiers of intense and extensive factors. We would need to obtain the relation between the dynamic characteristics (in this case, between the indexes) of the total revenues and the identifiers of the extensive and intensive factors, i.e. total costs and the effectiveness. The profitability index is not a suitable indicator of intensity as the size of profitability in the initial period $\boldsymbol{R_0}$. is still present in the expression (4).

What is important is that in a purely intensive development, effectiveness changes while costs remain unchanged, if***I(TC)***=1, then ***I(TR)*** = ***I(Ef)***. If, in a purely intense development, the production range changes to the next period ***k***-times, then this simple well-interpretable relation applies to the effectiveness index

$$\boldsymbol{I(Ef)} = \frac{\boldsymbol{Ef_1}}{\boldsymbol{Ef_0}} = \frac{\frac{TR_1}{TC_1}}{\frac{TR_0}{TC_0}} = \frac{\frac{k.TR_0}{TC_0}}{\frac{TR_0}{TC_0}} = \boldsymbol{k} \qquad (5)$$

The same k-times production extension will be reflected in in the profitability index in a purely intensive development in a much more complex manner.

$$\boldsymbol{I(R)} = \frac{\boldsymbol{R_1}}{\boldsymbol{R_0}} = \frac{1-\frac{TC_1}{TR_1}}{1-\frac{TC_0}{TR_0}} = \frac{1-\frac{TC_0}{k.TR_0}}{1-\frac{TC_0}{TR_0}} = \frac{1}{\boldsymbol{R_0}} - \frac{1}{\boldsymbol{k}}.(\frac{1}{\boldsymbol{R_0}} - 1) \qquad (6)$$

Such a complicated expression, in which the initial profitability is also present, is not a good indicator of intensity, and therefore not a good starting point for a full typology of developments.

To calculate the share of the influence of the development of the intensive factors indicated by the effectiveness index and the corresponding share of the extensive factors in the overall income development, it is necessary to start with an additive expression, which we obtain by calculating the logarithm of the expression (3).

$$\ln \boldsymbol{I(TR)} = \ln \boldsymbol{I(Ef)} + \ln \boldsymbol{I(TC)} \qquad (7)$$

The expression (7) allows the derivation of dynamic parameters of intensity and extensity, which express not only the share of the influence of intensive and extensive factors, but also

whether the factor influences the growth of the product (if positive) or the decrease of the product (if negative)[2].

The dynamic intensity parameter [3]expresses how the change in effectiveness is contributing to the change in revenue and is given by:

$$\boldsymbol{i} = \frac{ln\boldsymbol{I}(\boldsymbol{Ef})}{|\ln\boldsymbol{I}(\boldsymbol{Ef})|+|\ln\boldsymbol{I}(\boldsymbol{TC})|} \tag{8}$$

The dynamic extensity parameter extrapolates the extent to which the extensive factor has an impact on the resulting revenue development:

$$\boldsymbol{e} = \frac{ln\boldsymbol{I}(\boldsymbol{TC})}{|\ln\boldsymbol{I}(\boldsymbol{Ef})|+|\ln\boldsymbol{I}(\boldsymbol{TC})|} \tag{9}$$

For purely intensive development the expression (8) and (9) above generate values of dynamic parameters $\boldsymbol{i}$ = 1 a $\boldsymbol{e}$ = 0. For purely extensive development, they generate $\boldsymbol{i}$ = 0 a $\boldsymbol{e}$ = 1 in all other cases, the said pair of dynamic parameters gives clear information [4]about the type of development in a given partial or aggregate period. The advantage of dynamic intensity and extensity parameters is that they have no area constraints and make it possible to compare different companies at different times. This would not be the case if we used the profitability of the sales as an indicator of intensity.

3. Exact expression of the profitability and effectiveness relation

The exact expression of the relation between efficiency and profitability of sales in a static role is based on their defining expressions

$$\boldsymbol{Ef} = \frac{\boldsymbol{TR}}{\boldsymbol{TC}} = \frac{\boldsymbol{TR}}{\boldsymbol{TR}-\boldsymbol{EP}} = \frac{1}{1-\boldsymbol{R}} \tag{10}$$

The reciprocal relation expresses profitability of sales using the effectiveness

$$\boldsymbol{R} = \boldsymbol{1} - \frac{\boldsymbol{1}}{\boldsymbol{Ef}} \tag{11}$$

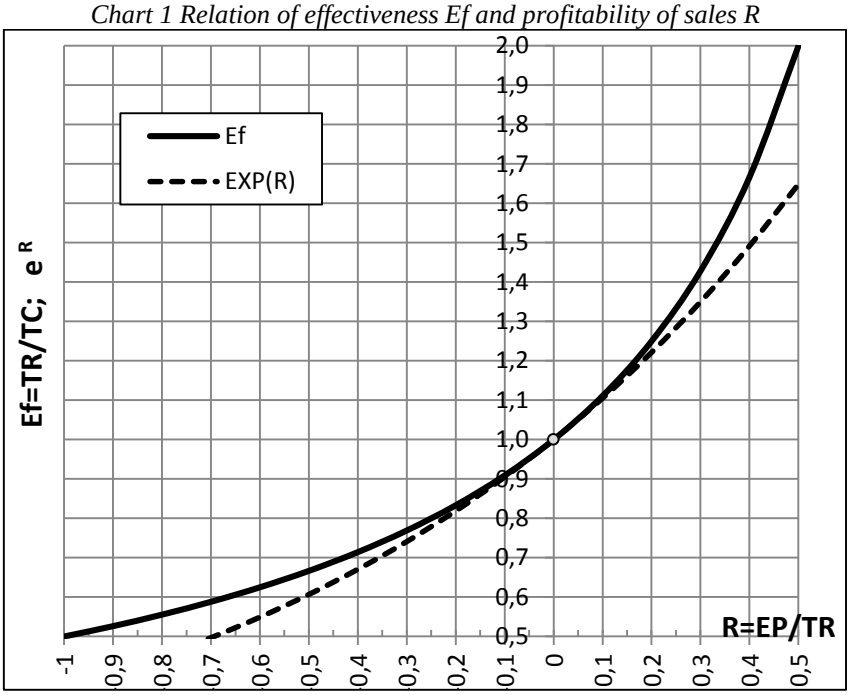
The relationship (10) is shown in Chart 1

[2] The derivation of dynamic parameters of intensity and extensity is analysed in the greatest detail in: (Mihola, 2007)

[3] For a detailed derivation of these relations, including the corresponding development typology at the national economy level please see (Mihola, Wawrosz, 2014) and at the business level please see (Kotěšovcová, Mihola, Wawrosc, 2015, p.41 to 54)

[4] The application of the said dynamic parameters when analysing the quality of the development of ŠKODA auto (Kotěšovcová, Mihola, 2013) may serve as an example.

Chart 1 Relation of effectiveness Ef and profitability of sales R



Source: Own calculations

It is a monotone-increasing, concave, continuous function that intersects y-axis at point [0; 1]. In the vicinity of this point, the function (10) is very well approximated by the exponential[5] $y=e^R$, which at this point has not only the same functional value, but also the derivation values of all degrees.

The relationship (11) representing an inverse function to the relationship (10) is shown in Chart 2.

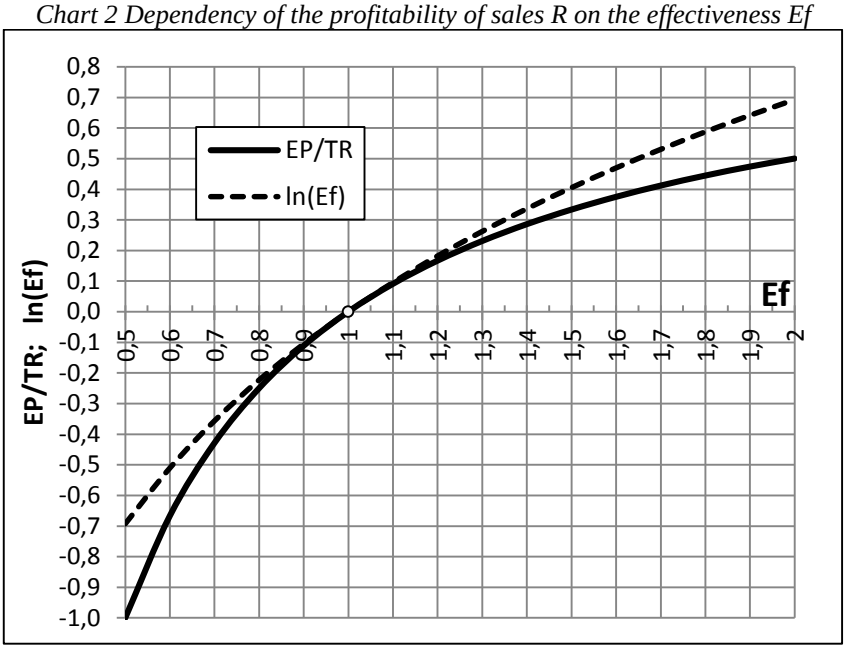
It is a monotone-increasing, convex, continuous function that intersects x-axis at point [1; 0]. In the vicinity of this point, the axis is very well approximated by the natural logarithm y=ln(Ef), which at this point has not only the same functional value, but also the derivation values of all derivations.

For us, the relationship of profitability and effectiveness in the dynamic role is fundamental, i.e. we are looking for some expression of the relationship between dynamic characteristics of effectiveness and profitability. By dynamising the expressions (10) and (11) we will obtain

$$\boldsymbol{I(Ef)} = \frac{\mathbf{1-R_0}}{\mathbf{1-R_1}} = \frac{1-R_0}{1-R_0.\boldsymbol{I(R)}} \qquad (12)$$

5 (Ball, 2011, p. 177 to 181) deals with this issue in great detail.

Chart 2 Dependency of the profitability of sales R on the effectiveness Ef

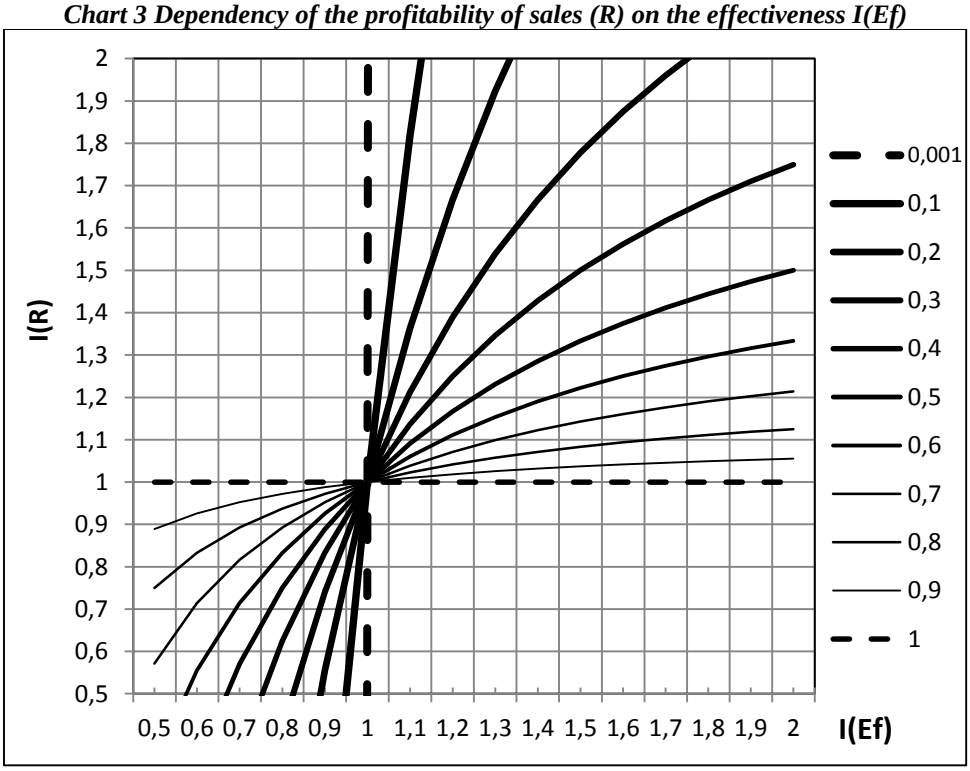


Source: Own calculations

The reciprocal relation expresses profitability of sales using the effectiveness

$$\boldsymbol{I(R) = \frac{1}{R_0} - \frac{1}{I(Ef)} \cdot \left(\frac{1}{R_0} - 1\right)} \tag{13}$$

Chart 3 Dependency of the profitability of sales (R) on the effectiveness I(Ef)



Source: Own calculations

In both cases, it is not possible to establish the exclusive relationship between dynamic characteristics (indexes or growth rates), since in both expressions also the profitability R_0 in the initial period appears. Chart 3 showing the expression (13) values R_0 = 0 to 1 to 0.1. All curves are growing, convex and continuous. The lower the initial profitability value, then the corresponding curve becomes steeper and becomes a limiting parallel to y-axis for I(Ef)=1. If, on the other hand, R_0 approaches 1, then this curve approach a limiting parallel to x-axis for I(R)=1.
The existence of these curves prevents the profitability index (or growth rate) from being use to derive dynamic parameters of intensity and extensity, and that this otherwise significant feature would be used as an indicator of intensity factors.

4. Conclusion

This paper shows the specific reasons why the profitability of sales ***R*** is not a more suitable indicator of intensity than the effectiveness ***Ef***, even though in the static role, these two variables uniquely determined each other by non-linear monotone-increasing inverse functions (10) or (11). The main reason is the dynamic version of the two-factor production function (3), where only the purely dynamic characteristics of the output ***I(TR)*** and the intensity and extensity factors, ***I(Ef)*** and ***I(TC)*** are present. The attempt to substitute the index of effectiveness ***I(Ef)*** by index ***I(R)*** or its growth rate fails due to the fact that the profitability in the initial year R_0 is always present in the relationship between the dynamic characteristics ***I(TC), I(R) and I(TC***. In analytical calculations and when compiling specific e.g. simulation models, it is possible to rely on the profitability of sales and to use it as an appropriate analytical tool.

Dedication

The result was accomplished as part of the resolution of a student project entitled The Improvement of pension system as an intensification factor of the economy with the use of the special-purpose support for a specific university research of the University of Finance and Administration.

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Determinants of value of tax shield in the Slovak Republic

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Abstract

The use of corporate debt financing is increasing nowadays. In addition to the interest rate, one of the reasons is the existence of tax deductibility of expenses, such as interest. The tax benefit is an important factor in the investment decision-making and is essential to know the variables that affect its value. The aim of the paper is to identify and analyze the determinants of value of tax shield in the conditions of the Slovak Republic. These determinants are identified by a linear regression model and the model includes asset turnover, liquidity, financial leverage, and profitability ratios as independent variables. Enterprise size and industry are used as dummy variables.

Key words: Tax shield, Debt, Interest, Corporate Finance

JEL Classification: C01, C51, G31, G32

1. Úvod

Problematika daňových štítov je čoraz intenzívnejšie objektom záujmu manažérov podnikov, ale aj odbornej verejnosti. Daňové výdavky generujú daňové úspory (daňové štíty) a tie významne ovplyvňujú rozhodovanie podnikov v tých oblastiach podnikateľskej činnosti, v ktorých tieto výdavky vznikajú. Za najdôležitejší zdroj daňových úspor sa považujú úroky (nákladové úroky) a odpisy, z tohto dôvodu sú daňové štíty rozdelené na dve základné kategórie a to úrokové a neúrokové daňové štíty. Zahraničná literatúra prináša značne rozsiahly pohľad na túto problematiku z teoretického aj praktického hľadiska. Domáci autori sa daňovým štítom venujú len okrajovo, pričom nezohľadňujú špecifiká podmienok slovenskej ekonomiky. Väčšina autorov sa zaoberá kvantifikáciou hodnoty resp. súčasnej hodnoty daňových štítov; absentuje dostatočne rozsiahla analýza vplyvu faktorov, ktoré determinujú hodnotu daňovej výhody. Naopak analýza ďalších veličín spojených daňovou problematikou je relatívne široká; analýza dane z pridanej hodnoty (Majerova, 2016) alebo daňového klinu (Paliderova, Bielikova, 2014).

Cieľom tohto príspevku je zhodnotiť vplyv vybraných premenných na daňovú úsporu v prostredí Slovenskej republiky pomocou modelu lineárnej regresie.

Príspevok je rozdelený na tri časti; prvá časť sa zaoberá súčasným stavom metód kvantifikácie hodnoty daňového štítu a názormi predovšetkým zahraničných autorov na faktory, ktoré ho ovplyvňujú. V druhej časti analyzujeme na výberovej vzorke vplyv vysvetľujúcich premenných na hodnotu daňového štítu. Posledná časť je zameraná na

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zhodnotenie získaných výsledkov a možných ďalších smerov výskumu problematiky daňových štítov.

2. Prístupy ku kvantifikácií daňového štítu

Ak berieme do úvahy daňový systém, potom podľa modelu Modiglianiho a Millera (1963) úrok ako daňovo odpočítateľná položka ovplyvňuje zisk pred zdanením a tým aj výšku daňovej povinnosti podniku. Z uvedeného vyplýva, že aj hodnota podniku sa zmení z dôvodu existencie úrokového daňového štítu (interest tax shield). Hodnota daňového štítu je jednoducho daná ako súčin:

$$TS = T \times k_D \times D \qquad (1)$$

Kde TSje hodnota daňového štítu, Tje sadzbou dane z príjmu právnických osôb, k_D predstavuje náklady cudzích zdrojov a Dje hodnota cudzích zdrojov.

V rámci príspevku sa zameriame primárne na úrokové daňové štíty vzhľadom na ich široké teoretické aj praktické využitie v oblasti finančného riadenia.

Brealey, Myers a Allen (2010) definujú úrokový daňový štít ako: úsporu dane spôsobenú daňovou odpočitateľnosťou úrokových platieb.

Damodaran (2010) sa vyjadruje v podobnom duchu: „Úroky sú daňovo uznateľné a výsledné daňové úspory znižujú náklady úverov firiem.“.

Tham a Veléz – Pareja (2002) definuje dva rôzne spôsoby výpočtu súčasnej hodnoty daňových štítov: Existujú dva spôsoby definovania súčasnej hodnoty daňového štítu (PVTS). Po prvé (súčasná) hodnota daňového štítu VTS je jednoducho daňový štít diskontovaný podľa ρ, čo je primeraná diskontná miera pre daňové štíty. Po druhé (súčasná) hodnota daňového štítu VTS je rozdiel medzi súčasnou hodnotou daňových platieb nezadlženého podniku a zadlženého podniku, pričom rizikové charakteristiky dvoch daňových platieb sú rovnaké. Je ukázané, že obe tieto definície sú ekvivalentné.

V prvom prípade, ak je hodnota cudzích zdrojov perpetuitná, daňový štít podniku závisí len od výšky daňovej sadzby a hodnoty cudzích zdrojov. Potom je súčasná hodnota daňového štítu rovná diskontovanej hodnote z rovnice (1). Ako diskontný faktor sa používa hodnota nákladov na cudzí kapitál k_D, vzhľadom ku skutočnosti, že riziko daňového štítu (ďalej TS) by malo byť zhodné s rizikom úroku, ktorý je základom pre výpočet daňového štítu.

$$PV(TS) = \frac{T \times k_D \times D}{k_D} = T \times D \qquad (2)$$

Predpokladá sa, že podnik dosahuje daňové úspory v danom období len vtedy, ak je schopný dosiahnuť taký zisk pred úrokmi a zdanením, aby boli pokryté úrokové platby (nákladové úroky) a daňový štít môže byť v danom období uplatnený len v prípade, že podnik má kladný výsledok hospodárenia pred zdanením, t.j. platí daň z príjmu právnických osôb.

Z predchádzajúceho vzťahov (1) a (2) vyplýva, že daňový štít je ovplyvnený troma premennými: daňovou sadzbou, nákladmi na cudzí kapitál, hodnotou spoplatnených cudzích zdrojov. Liu (2009) na rozdiel predchádzajúcich modelov považuje daňový štít za veličinu ovplyvnenú štyrmi premennými: Daňový štít je funkciou štyroch premenných ´čistého príjmu, úrokovej miery, dlhu a daňovej sadzby.´ Hodnota daňového štítu podľa MM obsahuje len dve premenné ´dlh a daňovú sadzbu´, je nezávislá od úrokovej sadzby a to nemôže byť pravda.

Z predchádzajúceho vyplýva, že rozpor medzi autormi vzniká nielen z hľadiska samotnej kvantifikácie hodnoty daňového štítu, ale aj faktorov, ktoré ho ovplyvňujú. Preto je potrebné tento vzťah analyzovať nielen v prostredí Slovenskej republiky, ale aj vo väčšom meradle.

3. Analýza vplyvu determinujúcich činiteľov na hodnotu daňového štítu

3.1 Metódy analýzy

Na analýzu vplyvu determinujúcich činiteľov na hodnotu daňového štítu sme použili regresnú analýzu. Táto analýza patrí medzi najdôležitejšie ekonometrické nástroje, tj. na popis vzťahov medzi ekonomickými a finančnými veličinami. V modeli vždy vystupuje vysvetľovaná premenná *y* a vysvetľujúca premenná *x* (resp. vysvetľujúce premenné x_{i1}, x_{i2},..., x_{ik}). Všeobecný tvar lineárneho regresného modelu je daný podľa rovnice (3).

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \varepsilon_i \tag{3}$$

Kde $y_i$je vysvetľovaná premenná, $\beta_0, \beta_1, \beta_2, \dots, \beta_k$ sú neznáme parametre modelu (regresné koeficienty), $x_{i1}, x_{i2}, \dots, x_{ik}$ sú vysvetľujúce premenné a ε_i jereziduálna (náhodná) zložka modelu.

Regresné koeficienty modelu sú odhadované pomocou metódy najmenších štvorcov.(Cipra, 2013) Na testovanie štatistickej významnosti modelu sa používa *F-test štatistickej významnosti lineárneho regresného modelu*. Nulovou hypotéza má tvar:

H_0: $\beta_i = 0$, oproti alternatívnej hypotéze: H_1: $\beta_i \neq 0$

Testovacím kritériom je *F-štatistika*

$$F = \frac{MSM}{MSR} = \frac{\frac{SSM}{k}}{\frac{SSR}{n-k-1}} \tag{4}$$

Nulová hypotéza sa zamieta, ak $F > F_\alpha(k; n-k-1)$, kde $F_\alpha(k; n-k-1)$ je α-kritická hodnota *F*-rozdelenia s *k* a *n-k-1* stupňami voľnosti.

Druhou testom lineárneho regresného modelu je *test významnosti regresného koeficientu.* Pomocou tohto testu sa zisťuje štatistická významnosť regresných koeficientov, pričom nulová hypotéza je daná ako:

H_0: $\beta_i = 0$, oproti alternatívnej hypotéze: H_1: $\beta_i \neq 0$

Testovacou štatistikou je *t-štatistika*

$$t = \frac{b_i}{s_{rez}\sqrt{|X^T X|^{-1}_{i+1;i+1}}} \tag{5}$$

Nulová hypotéza sa zamieta, ak $|t| > t_\alpha(n-k-1)$.

Najčastejším problémom analýzy lineárneho regresného modelu je výber optimálnej množiny vysvetľujúcich premenných. Výber vysvetľujúcich premenných v ekonomickej sfére vychádza z empírie a niektorých teoretických poznatkov. Optimalizácia počtu regresných koeficientov vychádza z *testovania nulového efektu dodatočnej premennej*. (Hendl, 2009) Nulová hypotéza hovorí o tom, že príspevok dodatočnej vysvetľujúcej premennej je štatisticky nevýznamný oproti alternatívnej hypotéze, že tento príspevok je štatisticky významný. Testovacou štatistikou je *F-štatistika*

$$F = \frac{\frac{\Delta var}{1}}{\frac{SSM_1}{n-k-1}} = \frac{\frac{SSM_1 - SSM_2}{1}}{\frac{SSM_1}{n-k-1}} \tag{6}$$

Jednotkou je označený model s dodatočnou premennou a číslom dva model bez dodatočnej premennej. Ak $F > F(1, n-k-1)$, tak sa zamieta nulová hypotéza a dodatočná premenná je pridaná do modelu.

Výber optimálnej množiny vysvetľujúcich premenných vychádza z niekoľkých algoritmov. Metóda *krokovej selekcie* (vzostupnej selekcie, forward selection) predpokladá tvorbu regresného modelu s lokujúcou konštantnou β_0 v prvom kroku a následnom pridávaní

premenných s testovaním štatistickej významnosti nulového efektu dodatočnej premennej kým vplyv žiadnej z premenných nie je štatisticky významný. *Stepwise selection* (hybridná kroková selekcia) je obdobná ako predchádzajúci postup, rozdiel je v opätovnom testovaní významnosti nulového efektu dodatočnej premennej zaradenom v predchádzajúcom kroku selekcie. Posledná metóda (Backward elimination) vychádza z opačného postupu ako prvá metóda; z úplného modelu sú postupne vyraďované tie premenné, ktoré štatisticky nevýznamný prínos, kým nie je vybraná optimálna množina premenných, ktorých efekt je štatisticky významný.

3.2 Výberový súbor a premenné modelu

V príspevku je na analýzu použitý výberový súbor skladajúci sa finančných ukazovateľov 419 podnikov za rok 2016 pôsobiacich na území Slovenskej republiky, ktorých vlastníctvo je tuzemské súkromné. Ďalšou podmienkou výberu bolo kritérium kladného výsledku hospodárenia pred zdanením. Toto kritérium je nutnou podmienkou existencie daňového štítu, v prípade záporného výsledku hospodárenia podnik neplatí daň z príjmu právnických osôb, teda daňový štít nie je v danom období uplatnený. Uplatnenie kritéria kladného výsledku hospodárenia pred zdanením zároveň eliminuje vplyv daňovej licencie na výsledky tejto analýzy. (Kramarova, Gregova, Cisko, 2014)

Zdrojom dát boli účtovné závierky predmetných podnikov za rok 2016, ktoré boli získané z databázy Finstat. Na základe súvahy a výkazov zisku a strát boli vypočítané pomerové ukazovatele finančnej analýzy zo štyroch kategórií: aktivita, likvidita, zadlženosť a rentabilita. Tieto ukazovatele boli použité v modeli lineárnej regresie ako vysvetľujúce premenné.

Tabuľka 1 Finančné ukazovatele použité vysvetľujúce premenné (zdroj: autor)

Ukazovateľ	**Algoritmus výpočtu**
Viazanosť aktív	$Aktíva/Tržby$
Viazanosť neobežného majetku	$Neobežné\ aktíva/Tržby$
Viazanosť obežného majetku	$Obežné\ aktíva/Tržby$
Obrat aktív	$Tržby/Aktíva$
Obrat neobežného majetku	$Tržby/Neobežné\ aktíva$
Obrat obežného majetku	$Tržby/Obežné\ aktíva$
Doba splatnosti krátkodobých záväzkov	$(Krátkodobé\ záväzky/Tržby)\times 360$
Doba inkasa pohľadávok	$(Pohľadávky/Tržby)\times 360$
Likvidita 1. stupňa	$Krátkodobý\ finančný\ majetok/Krátkodobé\ záväzky$
Likvidita 2. stupňa	$Obežné\ aktíva - zásoby - Dlhodobé\ pohľadávky/Krátkodobé\ záväzky$
Likvidita 3. stupňa	$Obežné\ aktíva - Dlhodobé\ pohľadávky/Krátkodobé\ záväzky$
Krytie krátkodobých záväzkov z cash flow	$Cash\ flow_{zo\ samofinancovania}/Krátkodobé\ záväzky$
Celková zadlženosť	$Záväzky/Aktíva$
Koeficient samofinancovania	$Vlastné\ imanie/Aktíva$
Bežná zadlženosť	$Krátkodobé\ záväzky/Aktíva$
Dlhodobá zadlženosť	$Dlhodobé\ záväzky/Aktíva$
Úverová zadlženosť	$Bankové\ úvery + Finančné\ výpomoci/Aktíva$
Koeficient zadlženosti	$Záväzky/Vlastné\ imanie$
Finančná samostatnosť	$Vlastné\ imanie/Cudzie\ zdroje$
Úrokové krytie	$EBIT/Úroky$
Tokové zadlženie	$Záväzky/Cash\ flow_{zo\ samofinancovania}$
Finančná páka	$Aktíva/Vlastné\ imanie$
Rentabilita aktív(ROA)	$EBIT + Úroky(1-T)/Aktíva$
Rentabilita vlastného imania (ROE)	$EAT/Vlastné\ imanie$

Ukazovateľ	Algoritmus výpočtu
Rentabilita tržieb (ROS)	$EAT/Tržby$
Rentabilita dlhodobých zdrojov (ROCE)	$EBIT/Vlastné\ imanie + Záväzky$
Rentabilita nákladov	$EAT/Náklady$

Okrem kvantitatívnych premenných uvedených v Tabuľke 1, boli v modeli použité aj kvalitatívne premenné. Jedná sa o premennú veľkosť podniku a odvetvie. Prvá premenná člení podniky podľa typu účtovnej jednotky na mikro účtovné jednotky, malé a veľké účtovné jednotky. Vzhľadom k tomu, že účtovná jednotka, ktorá spĺňa podmienky pre zatriedenie do skupiny mikro účtovných jednotiek, môže účtovať jako malá účtovná jednotka, tak v predmetnom modeli sme podniky členili len na dve skupiny: malá a veľká účtovná jednotka.

Odvetvie je reprezentované členením podnikov podľa SK Nace. Vo výberom súbore sa nachádzalo celkom 19 kategórií podnikov SK Nace z 21 možných. Prehľad týchto kategórií je uvedený v Tabuľke 2.

Tabuľka 2 Obmeny premennej odvetvie podľa SK Nace (SK NACE)

Kód	Kategorizácia	Kód	Kategorizácia
A	Poľnohospodárstvo, lesníctvo a rybolov	K	Finančné a poisťovacie služby
B	Ťažba a dobývanie	L	Činnosti v oblasti nehnuteľnosti
C	Priemyselná výroba	M	Odborné, vedecké a technické služby
D	Dodávka elektriny, plynu, pary a studeného vzduchu	N	Administratívne a podporné služby
E	Dodávka vody, čistenie a odvod odpadových vôd	O	Verejná správa a obrana, povinné sociálne zabezpečenie
F	Stavebníctvo	P	Vzdelávanie
G	Veľkoobchod a maloobchod, oprava	Q	Zdravotníctvo a sociálna pomoc
H	Doprava a skladovanie	R	Umenie, zábava a rekreácia
I	Ubytovacie a stravovacie služby	S	Ostatné činnosti
J	Informácie a komunikácia		

Uvedené kvalitatívne premenné sú v modeli lineárnej regresie uvedené pomocou tzv. umelých (binárnych) premenných. Počet týchto premenných *n*-, kde *n* je počet obmien znaku kvalitatívnej premennej. V prípade premennej veľkosť podniku vzniká jedna umelá premenná *malá účtovná jednotka*, v prípade premennej odvetvie vzniká 18 umelých premenných podľa Tabuľky 2, ich názov je odvodený z kódu SK Nace od písmena *A* po písmeno *R*. Celkovo bolo analyzovaných 46 vysvetľujúcich premenných, 27 kvantitatívnych premenných a 19 binárnych premenných.

Vysvetľovaná premenná je reprezentovaná daňovým štítom, ktorý je kvantifikovaný podľa definície Mareka (2009), tj. ako súčin hodnoty položky znižujúcej základ dane z príjmu PZZD a sadzby dane z príjmu SDzP. V rámci modelu uvažujeme o úrokovom daňovom štíte, pričom nutnou podmienkou jeho existencie je kladný výsledok hospodárenia pred zdanením. V opačnom prípade je jeho hodnota nulová.

$$TS = \begin{Bmatrix} T \times Nákladové\ úroky\ \ ak\ \ EBT > 0 \\ 0\ ak\ EBT \leq 0 \end{Bmatrix} \tag{6}$$

3.3 Výsledky a zhodnotenie analýzy

Vstupné premenné sme analyzovali pomocou štatistického programu SAS Enterprise Guide, ktorý poskytuje CENTRUM VEDECKO-TECHNICKÝCH INFORMÁCIÍ SR pomocou Integrovaného Systému Služieb. Vytvorili sme lineárny regresný model, pričom optimálnu množinu vysvetľujúcich premenných sme vyberali pomocou krokovej selekcie, hybridnej krokovej selekcie a metódy postupnej eliminácie. Všetky hypotézy boli testované

na hladine významnosti α=0,05. Výsledky analýzy rozptylu modelu na základe postupnej eliminácie sú prezentované v Tabuľke 3.

Tabuľka 3 Analýza rozptylu lineárneho regresného modelu (zdroj: autor)

Analýza rozptylu					
Zdroj variability	**DF**	**Suma štvorcov**	**Priemerný štvorec**	**F- testovacie kritérium**	**p-hodnota**
Modelová	5	14776137565	2955227513	23,94	<.0001
Reziduálna	413	50984838827	123449973		
Celková variabilita	418	65760976393			

Na základe p-hodnoty môžeme skonštatovať, že model je štatisticky významný na hladine 0,05. V ďalšej tabuľke sú uvedené optimálne vysvetľujúce premenné spolu s odhadom regresných koeficientov.

Tabuľka 4 Odhad regresných koeficientov (zdroj: autor)

Odhad parametrov regresného modelu						
Premenná	**DF**	**Odhad parametra**	**Štandardná chyba**	**t-testovacie kritérium**	**p-hodnota**	**Štandardizovaný odhad**
Lokujúca konštanta	1	649,80043	1476,29509	0,44	0,6601	0
Celková zadlženosť	1	9710,19319	3710,27221	2,62	0,00962	0,21118
Bežná zadlženosť	1	-9013,40326	3758,8936	-2,40	0,0169	-0,18862
Úverová zadlženosť	1	25702	4680,93899	5,49	<.0001	0,26436
D	1	13659	3073,43776	4,44	<.0001	0,21512
Malá účtovná jednotka	1	-2620,50010	1155,20108	-2,27	0,0283	-0,09967

Z Tabuľky 4 vyplýva, že štatisticky významné sú len päť premenných: celkové zadlženie, bežné zadlženie, úverové zadlženie a kvalitatívne premenné Sk Nace *D* a premenná malá účtovná jednotka. Lokujúca konštanta je štatisticky nevýznamná a v nasledujúcom kroku bola vyradená. Model bez lokujúcej konštanty je takisto štatisticky významný na hladine 0,05, v nasledujúcej Tabuľke 5 sú uvedené odhady regresných koeficientov tohto modelu.

Tabuľka 5 Odhady regresných koeficientov bez lokujúcej konštanty (zdroj: autor)

Odhad parametrov regresného modelu						
Premenná	**DF**	**Odhad parametra**	**Štandardná chyba**	**t-testovacie kritérium**	**p-hodnota**	**Štandardizovaný odhad**
Koeficient samofinancovania	1	-9021,68016	3806,79229	-2,37	0,0183	-0,37834
Bežná zadlženosť	1	-9150,93043	3768,72164	-2,43	0,0156	-0,35321
Úverová zadlženosť	1	25943	4689,10409	5,53	<.0001	0,30621
Koeficient zadlženosti	1	-9841,56196	3555,56692	-2,77	0,0059	-10,35822
Finančná páka	1	9880,89245	3548,12205	2,78	0,0056	10,66941
D	1	13548	3078,25544	4,40	<.0001	0,21205
Malá účtovná jednotka	1	-2580,18916	1154,78461	-2,23	0,0260	-0,16183

Z Tabuľky 5 vyplýva, že po odstránení lokujúcej konštanty z modelu, sa zmenila množina vysvetľujúcich premenných. Nový model obsahuje sedem premenných: koeficient zadlženia, bežnú zadlženosť, úverovú zadlženosť, koeficient zadlženosti, finančnú páku a kvalitatívne premenné SK Nace *D* a *Malá účtovná jednotka*. Kvantitatívne premenné patria do kategórie analýzy zadlženosti, resp. determinujú finančnú (kapitálovú) štruktúru podniku. To znamená,

že hodnota daňového štítu je determinovaná kapitálovou štruktúrou a ziskovosť nemá štatisticky významný vplyv na vysvetľovanú veličinu, za podmienky kladného výsledku hospodárenia pred zdanením.

Z hľadiska kvalitatívnych premenných nemajú skúmané odvetvia signifikantný podiel na hodnote úrokového daňového štítu okrem odvetvia *D*. Znamená to, že pokiaľ je podnik patrí do kategórie SK Nace *D*, tak podľa daného modelu by mala byť hodnota úrokového daňového štítu o 13 548 Eur vyššia ako hodnota tejto veličiny pri porovnateľnom podniku inej kategórie SK Nace. N adruhej strane veľkosť podniku má signifikantný vplyv na hodnotu daňového štítu. Ak je podnik malou účtovnou jednotkou, tak by úrokový daňový štít mal byť o 2 580,19 Eura nižší za inak nezmenených podmienok.

Okrem testov hypotéz prezentovaných vyššie, kvalitu regresného modelu hodnotí koeficient determinácie R^2. V prípade modelu bez lokujúcej konštanty má hodnotu 0,2665, ktorá vyjadruje, že 26,65 % celkovej variability vysvetľovanej premennej je vysvetlená modelom. Relatívne nízky koeficient determinácie indikuje, že napriek tomu, že model je signifikantný, tak model nie je navrhnutý správne. Jednak množina vysvetľujúcich premenných nebola kompletná; existujú premenné, ktoré neboli zahrnuté v modeli, jednak lineárny model nie je vhodným. Okrem toho je pri veľkom množstve ukazovateľov vhodné použiť niektorú z metód viacrozmernej štatistickej analýzy: faktorovú analýzu, analýzu hlavných komponentov alebo zhlukovú analýzu. (Simková, 2015)

4. Záver

V príspevku sme analyzovali determinanty hodnoty daňového štítu v Slovenskej republike. Bol skonštruovaný lineárny regresný model na základe finančných ukazovateľov 419 tuzemských súkromných podnikov za rok 2016. Na vytvorenie modelu bolo použitých 27 kvantitatívnych ukazovateľov a dva kvalitatívne ukazovatele, z ktorých bolo vytvorených 19 binárnych premenných. Z týchto vysvetľujúcich premenných bolo pomocou postupnej eliminácie vytvorená optimálna množina premenných. Jedná o kvantitatívne premenné determinujúce zadlženosť podniku. Z kvalitatívnych premenných je štatisticky významná veľkosť podniku (typ účtovnej jednotky), naopak odvetvie nemá signifikantný podiel na hodnote daňového štítu, okrem podnikov patriacich do kategórie *Dodávka elektriny, plynu, pary a studeného vzduchu.*

Na základe koeficientu determinácie môžeme povedať, že vytvorený model je štatisticky významný, ale nie je vhodne zvolený. Riešením je rozšírenie množiny vysvetľujúcich premenných alebo zmena typu regresného modelu. Inou modifikáciou modelu je použitie vysvetľovanej premennej podľa modelu Velez-Pareju (2013) alebo Fernandeza (2005). Podľa modelu rast ukazovateľov zadlženia spôsobuje rast daňovej výhody; preto by mal podnik maximalizovať svoje zadlženie. Na druhej strane v modeli nie je zahrnutý rast kreditného rizika (Valaskova, Zvarikova, 2014) a možnosť platobnej neschopnosti podniku. (Kliestikova, Misankova, 2016) Okrem toho existujú aj makroekonomické faktory ako fiškálna politika (Primorac, 2014) alebo legislatívne obmedzenia (Kramarova, Valaskova, 2015).

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Analysis of Blue-Collars and White-Collars Approach to Company's Attendance

Ondřej Mikulec [1]

Abstract

Designation of employees as blue-collars and white-collars and their different work redistribution has a significant influence also on their approach to company's attendance. There are large differences in the approach of the two employees' categories even though net nominal workday fund is the same for every employee. This paper aims to analyze behavior of the two employees' categories considering the development of not-worked hours divided into holidays, sick leave and injuries and other absenteeism and their relation with creation of overtimes. Multiple regression models explaining which part of overtimes is influenced by each form of absenteeism are presented in the second part of the paper separately for both blue-collars and white-collars. We conclude that blue-collar and white-collar approach to company's attendance shows significant differences as well as the creation of overtimes is influenced differently for blue-collar and white-collar employees.

Key words

Blue-collars, company‘s attendance, multiple regression, people analytics, white-collars.

JEL Classification: M1, M54

1 Introduction

Designation of employees as blue-collars (BC) and white-collars (WC) is non-official classification based on the colors of their collars worn at work which can commonly reflect one's occupation as described in Gallie D. (1996). WC employees are named for the white-collared shirts that were fashionable among office workers in the early and mid-20th century and WC nowadays typically refers to office employees. BC employees are referred to as such because in the early 20th century, they usually wore sturdy, inexpensive clothing that did not show dirt easily and BC nowadays typically refers to manually working employees.

This work is dedicated to analyze behavior of the two employees' categories considering the development of not-worked hours (i.e. absenteeism, closer in subchapter 2.1) divided into holidays, sick leave and injuries and other absenteeism and their relation with creation of overtimes even though nowadays also other colored-collars are described, Lips-Wiersma, M., Wright, S., Dik, B. (2016). Different approach of BC and WC to various issues was already identified and described by Groshen, E. L., Williams D. R. (1992) or Toppinen-Tanner, S., Kalimo, R., Mutanen, P. (2002).

Proper work redistribution and individual approach to each employee based not only on its BC or WC designation plays important role for the companies to be able to effectively manage their resources both tangible and intangible including human capital. Data from six production companies with in total 7688 employees (5367 BC and 2321 WC) were collected including information about their employees' approach to attendance from a period between January 2014 and December 2016 to analyze the development of company's attendance statistics,

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partly as in Allen, S. G. (1981), and to identify similarities and differences in BC and WC approach. Furtherly, multiple regression models explaining which part of overtimes is influenced by each form of absenteeism are presented in the second part of the paper separately for both BC and WC as absenteeism tends to influence overtimes as ws already described in Ehrenberg, R. G. (1970). Company financial planning and budgeting can be significantly improved by reflecting differences in BC and WC.

2 Methodology

Methodology used in this article includes the description of company's attendance components and their share on the net nominal workdays fund and multiple regression models, which are used to describe the relationship between overtimes and absenteeism attendance components both of BC and WC.

2.1 Company's attendance

Attendance of any employee can be generally distinguished on worked time and not-worked time caused by weekends (or in continuous operation free shift), bank holidays and absenteeism. Workplace, work activities, starting workday and information are part of employment contract according to §34 of Czech Labor Code while it usually contains also information about a worktime according to §78 of Czech Labor Code, otherwise agreed separately. Worked hours are easily explained as time which is spend in a workplace during time agreed with an employer. Overtimes are described in Czech Labor Code §78 i) as the work done by the employee on the order of the employer or with his consent over the specified weekly working hours (or outside of the working shift schedule). Overtimes are expensive for companies, their costs are 1,25 times regular wage per hour at minimum and thus companies are trying to minimize. This paper's focus is also on the causal relationship of overtimes and forms of absenteeism described below.

Nevertheless, there are many ways to look at absenteeism. Absenteeism generally represents all situations in which an employee is not present in the workplace during agreed worktime, Armstrong, M., Taylor, S. (2014). We can specify three types of absenteeism (excluding illegal or unexplained absenteeism which is not allowed and not included as a part of absenteeism time in this work):

- holiday,
- sick leave and injuries,
- other absenteeism.

Holiday represents an official, planned and paid form of absenteeism. Holidays are unavoidable and also employees should be given a possibility to properly take a vacation during the year in full length. Minimum length of holidays per year in Czech Republic per law is four weeks although many employers offer extra week(s) as a form of employee' benefits and to attract new employees.

Sick leave and injuries is the main involuntary absenteeism area. Occasional absences are inevitable because people are getting sick or injured. Some sick leave events can be even long term, leaving an employee out of a workplace for months or even longer than a year. Habitual sick leave or high frequency of absenteeism events are partly preventable and very well highlighted by the Bradford factor score analysis as described in Mikulec, Špačková (2017).

Other absenteeism includes events which are usually short term oriented, sometimes even just a part of the day. Some of them are planned, such as scheduled doctor visit, blood donations, compensatory time off for the holiday and compensations defined in Czech Labor

Code (wedding, funeral etc.), or unplanned, such as emergency doctor visit, unexpected obstacles etc.

A sum of shares of each form of absenteeism on the net nominal workday shows which amount of worktime which is actually spend at work and which is spend legally outside a workplace. We can express a share of each form of absenteeism on the net nominal work fund for the whole company, selected parts or even of an individual in a given time period as following:

$$\%\ absenteeism = \frac{total\ amount\ of\ "given\ absenteeism"\ workdays}{net\ nominal\ workday\ fund}. \tag{1}$$

where net nominal workday fund is equal to approximately 21,5 days per each month and it reflects average amount of workdays in a month (calendar days in a month excluding bank holidays and weekends).

Share of Overtimes on the work time is not compared to net nominal workday fund, but to the total amount of workdays (workhours) as following:

$$\%\ overtimes = \frac{total\ amount\ of\ overtime\ workhours}{total\ amount\ of\ worked\ hours}. \tag{2}$$

where total amount of worked hours equals to sum of regular worked hours and overtimes.

2.2 Multiple regression models

Multiple regression models are based on similar principles as simple linear regression, which is to find a causal relationship between dependent variable and explanatory independent variable. In the analysis based on multiple regression we look for values of the dependent variable from a linear combination of values of several (two and more) independent variables. The formula for the calculation is similar to the simple regression:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_i + \varepsilon_i, \tag{3}$$

where Y is a dependent variable which values we try to estimate, β_0 is constant, β_i for i = 1, 2, …, k represents regression coefficients (also called partial regression coefficients) and x_i for i = 1, 2, …, k are independent values and ε_i is error term.

The goal of multiple regression is to explain the variance in the dependent variable Y. For this reason we use R-squared statistic. R-squared is the proportion of variability dependent on Y, which can be explained by the common influence of explanatory dependent variables. We calculate R/squared statistic as following:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}, \tag{4}$$

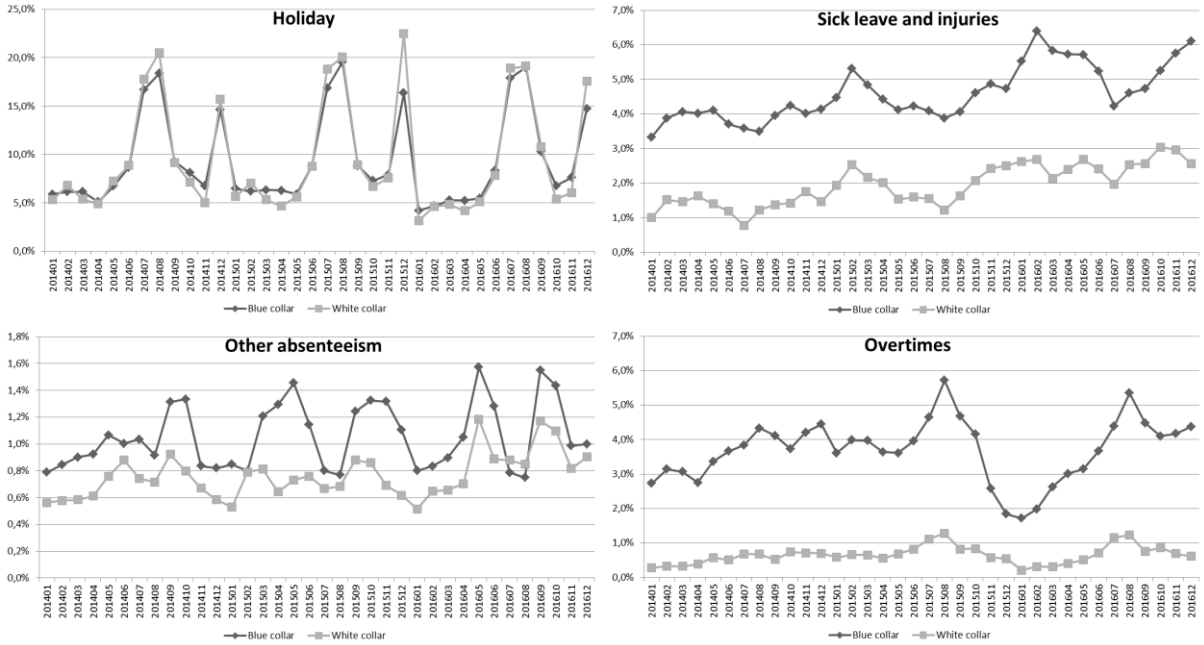
where SS_{res} is the sum of squares of residuals and SS_{tot} is the total sum of squares (proportional to the variance of the data).

Another result of the regression is to estimate (calculate) the influence of each of the independent variables x_i on the dependent variable Y. The influence of each independent variable is estimated by controlling the effect of the other independent variables entering the model. Multiple regression through standardized regression coefficients β_i also helps to determine the relative influence force of individual variables on the variable dependent - we then determine which variables have the greatest influence on the variance of the dependent variable and which, on the contrary, have the smallest influence. Estimated regression model can be used to predict the value of the dependent variable for each given scenario. Regression coefficients are estimated as described in Kutner, M. H., Nachtsheim, C., Neter, J. (2004) and regression model assembled as in Meloun, M., Militký, J., Hill, M. (2012). Results of multiple regression models are interpreted as in Cohen et. al. (2002).

3 BC and WC Approach to Company's Attendance

BC and WC approach to company's attendance is compared based on empirical observations of average of 7688 employees (5367 BC and 2321 WC) between January 2014 and December 2016 reaching maximum of 7928 employees (5656 BC and 2443 WC) and minimum of 7216 (5078 BC and 2132 WC). Because of the volatility in the amount of employees observed we transform each observed attendance absenteeism statistics as a relative share to net nominal work fund as described in (1) and overtimes express as in (2) to have a comparative evolution of each attendance statistic as shown in Figure 1.

Figure 1: BC and WC attendance statistic 2014 – 2016



Development of holiday share on net nominal work fund is very similar for both BC and WC because both categories have the same limit per each year. From Figure 1 we can observe that development of holiday share is very variable and periodically shows large dependency on season as summer months July and August shows large increase in holiday share as well as December. WC tend to reach higher extremes during both December and summer months.

Sick leave and injuries observation brings extremely important information as the difference between BC and WC move similarly for most of the observed period with differences beginning after February 2016. After this period the development of Sick leave and injuries ratio on the net nominal work fund differs for BC and WC. WC tends to have a greatly lower share of sick leave and injuries compared to BC, specifically around 2%, which can be explained by greater physical work participation. It was also observed that the share of sick leave and injuries is steadily increasing in time and requires active measurement, as described in Mikulec, O., Špačková A. (2017).

Other absenteeism of WC is steadily lower than of BC but the development is quite different than of sick leave and injuries. Average other absenteeism share is 2,3% lower than sick leave and injuries and also WC are reaching similar values as BC, nevertheless lower. Periods with significant increase in other absenteeism were identified to be periods in which companies activated convalescence stays.

Overtimes development is partly dependent on the development of absenteeism i.e. not-worked parts of the net work day fund. Exact relationship is described in following chapter while large part still remains unexplained by absenteeism components. This is caused dude to

production needs and work management. It is observable from Figure 1 that BC overtimes development is significantly higher than WC and has a large decrease between in winter 2015 and spring 2016 where management took action steps to decrease the overtimes, but statistics show that the solution was not long-term and since March 2016 overtimes continued to grow again and correspond more with the season and production needs.

3.1 Overtimes Multiple Regression Models

This subchapter is dedicated to define a causal relationship between overtimes and not-worked hours to define which of the parts of absenteeism plays the most (and the least) important role in causing the overtimes to grow. Overtimes are set as dependent variable with holiday, sick leave and injuries and other absenteeism as independent variables to define the common relationships.

Table 1: Multiple Regression Model for Blue-Collars' Overtimes

Source	SS	df	MS
Model	4246114	3	1415371
Residual	12471142	32	389723
Total	16717256	35	477636

Number of obs	=	36
F (3, 32)	=	3,6317
Prob > F	=	0,0231
R-squared	=	0,2540
Adj R-Squared	=	0,1841
Root MSE	=	624,28

	Coef.	Std. Err.	t	P > | t |	[95% Conf. Interval]	
Constant	2050,1	780,83	2,6255	0,0132	459,60	3640,6
Holiday	0,0752	0,0258	2,9216	0,0063	0,0228	0,1277
Sick Leave	-0,1242	0,1501	-0,8276	0,4140	-0,4299	0,1815
Other absenteeism	0,7871	0,4991	1,5771	0,1246	-0,2295	1,8037

BC overtime as described in Figure 1 and following comment in the subchapter is steadily higher than WC. Explained variability in Overtimes is explained only by 18,4% by not-worked time as shown in Table 1 even though the multiple regression model is statistically significant. Most of the unexplained variance in the variable should be explained by work-redistribution and production needs. Multiple regression model shows that other absenteeism is significant for overtimes creation and each hour/day spend on other absenteeism has to be replaced by approximately 0,8 hours/days of overtimes even though that it is not valid under 5% significance level as well as sick leave. Holidays have much weaker relationship with 0,075 hours of overtimes needed to cover one hour of holidays, but fulfil the 5% significance importance. Sick leave and injuries shows even reverse relationship which can be partly explained by the fact that sick leave is expected to be covered by more flexible workers and in the case of long term illnesses by agency workers.

Table 2: Multiple Regression Model for White-Collars' Overtimes

Source	SS	df	MS
Model	66887	3	22296
Residual	83164	32	2599
Total	150051	35	4287

Number of obs	=	36
F (3, 32)	=	8,5790
Prob > F	=	0,0003
R-squared	=	0,4458
Adj R-Squared	=	0,3938
Root MSE	=	50,98

	Coef.	Std. Err.	t	P > | t |	[95% Conf. Interval]	
Constant	13,023	38,611	0,3373	0,7381	-65,625	91,671
Holiday	0,0167	0,0049	3,3827	0,0019	0,0066	0,0267
Sick Leave	-0,0216	0,0523	-0,4139	0,6817	-0,1282	0,0849
Other absenteeism	0,5483	0,1886	2,9068	0,0066	0,1641	0,9325

WC overtimes creation is surprisingly better described by causal relationship as 39,4% of WC overtimes variance is explained by absenteeism components, which is two times more than in multiple regression model for BC. Table 2 shows that overtimes of WC are influenced differently than overtimes of BC. The main difference is in significantly lower constant value which is not significant for the model in 5% significance level unlike in BC model. WC overtime regression model is better described by other absenteeism and holiday while sick leave and injuries and constant are insignificant for the model.

4 Conclusion

Analysis of behavior of BC and WC and their approach to company's attendance considering the development of absenteeism components such holidays, sick leave and injuries and other absenteeism and their relation with creation of overtimes was performed. It was proved by empirical observations that BC and WC approach to company's attendance is significantly different. The most similar absenteeism component development of BC and WC are holiday because its amount is given and the same for both categories as well as seasonality preferences. Nevertheless, WC employees tend to reach higher extremes during winter and summer. Sick leave and injuries component shows that BC spend on average 2% of net nominal work fund more than WC and for both categories this statistic is steadily growing. Other absenteeism share on net nominal work fund is 1% on average which makes it the smallest absenteeism component from all observed but its influence on creation of overtimes proved to be very high for both BC (0,7871 hours of overtimes per each 1 hour of other absenteeism) and WC (0,5483 hours of overtimes per each 1 hour of other absenteeism), even though insignificant as a factor on 5% significance level. Variance in development of overtimes was explained by the absenteeism components for BC only by 18,4% and for WC by 39,4%. We conclude that absenteeism components have statistically significant influence on overtimes as both multiple regression model are important on 5% significance level, but large part of variance remains unexplained and caused by management decisions, production needs and other non-direct factors.

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The evaluation of the effectiveness of a long-term stocks investment strategy based on the largest Lyapunov exponent

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Abstract

The article *Portfolio Selection* by H. Markowitz (1952) started intensive development of scientific field which is a portfolio analysis. In recent years, in addition to classical methods of portfolio analysis have been developed tools that are both modifications of these concept as well as new, alternative diversification techniques of investment portfolio which take into account for example the indicators of fundamental analysis, taxonomic measure of investment attractiveness, value investing, growth investing, dividend investing, innovation investing. A new approach proposed in the paper is the use of the measures for identifying chaos, i.e. the largest Lyapunov exponent.
The paper aims to construct and evaluate the efficiency optimal portfolios determined based on the largest Lyapunov exponent. The test will be conducted on the basis of the financial time series in 1994-2015.

Key words

Portfolio analysis, largest Lyapunov exponent, Markowitz model

JEL Classification: C3, C8, G11, E4

1. Introduction

The construction of an optimal portfolio proposed by H. Markowitz [4,5] started intensive development of scientific field which is a portfolio analysis [6, 7, 13, 15, 12]. Portfolio analysis is one of the most important techniques for investing in the capital market, which main goal is to diversify the investment risk. Research conducted for many years in various scientific centers have provided new tools and approaches for estimating the shares in the optimal portfolio, e.g. taxonomic measure of investment attractiveness (TMAI) [13,14], value investing, growth investing, dividend investing [1, 15], innovation investing [12]. A new approach proposed by the authors is the use to construct the optimal portfolio of two measures for identifying chaos, i.e. the largest Lyapunov exponent [8, 9, 10, 11, 17]. Since determinism of chaotic time series indicates on potential possibility of their prediction, it is also expected that has a significant impact on the construction of optimal portfolio.

The aim of the paper will be an attempt to diversify the risk of the investment portfolio, based on the constructed optimal portfolios determined on the value of the largest Lyapunov exponent. In the study we used the financial time series of the WIG20 index, which at the time of portfolios building were listed on the Warsaw Stock Exchange for at least 10 years. The WIG20 is the stocks index of 20 largest companies listed on the Warsaw Stock Exchange, updated at the end of each quarter. The data cover a period from 1.01.1994 to 31.12.2016, and stocks portfolios were built at the end of each year between 2004 and 2015.

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2. The largest Lyapunov exponent

For the dynamic system with discrete time (X, f) described by first order recurrence equation:

$$x_{t+1} = f(x_t), \qquad t = 0,1,2,\ldots, \tag{1}$$

where x_t, x_{t+1} - state on the system respectively at the moments t and $t+1$Lyapunov exponents are defined as limits [16]:

$$\lambda_i(x_0) = \lim_{n\to\infty} \frac{1}{n} \ln\left|\mu_i(n, x_0)\right|, \qquad i = 1. \ldots. m, \quad \text{for } m \geq 1, \tag{2}$$

where $\mu_i(n, x_0)$ are the eigenvalues of the Jacobi matrix of mapping f^n. f^n is an *n*-fold submission of function *f*, and *f* is the function that generates a dynamic system.

The Lyapunov exponents measure the rate of divergence or convergence of neighboring trajectories, i.e. the level of chaos in a dynamic system. The largest (maximal) Lyapunov exponent allows to specify the extent of a change (an increase or a decrease) in the distance between the current state x_N of the system and its nearest neighbor x_i in the evolution of the system, and also estimate the distance between the vectors x_{N+1} and x_{i+1}. Based on this distance the value of the forecasts $\hat{x}_{N+1}$ is determined [2,8].

For real-time series, if you do not know a generator function *f*, the largest (maximal) Lyapunov exponent is estimated based on the relation [3]:

$$\Delta_n = \Delta_0 \cdot e^{n\lambda_{\max}}, \tag{3}$$

as the direction component of the regression equation [3, 10]:

$$\ln \Delta_n = \ln \Delta_0 + \lambda_{\max} n, \tag{4}$$

where Δ_0 is the initial distance between two initially close (in the Euclidean distance sense) points of the reconstructed state space, Δ_n is the distance between these points after *n* iterations and $\lambda_{\max}$is the largest (maximal) Lyapunov exponent.

Consider a one-dimensional time series, composed of *N* observation$(x_1, x_2, \ldots, x_N)$. Of all the vectors x_t^d of reconstructed state space we choose the vector closest to the vector x_N^d (in terms of Euclidean distance) and it is denoted by $x_{\min}^d$. Let $\Delta_{\min}$ denote the distance between x_N^d and $x_{\min}^d$, and Δ_1- the distance between x_{N+1}^d and $x_{\min+1}^d$. Assuming that $\Delta_1/\Delta_{\min}$ is a small change in the evolution of the system, the distance between vectors x_{N+1}^d and $x_{\min+1}^d$ is given by [2]:

$$\Delta_1 \approx \Delta_{\min} \cdot e^{\lambda_{\max}}, \tag{5}$$

where $\lambda_{\max}$is the largest (maximal) Lyapunov exponent.

3. The optimal stock portfolio

Optimal allocation of shares in portfolio is issued by solving the following optimization problems:

Table 1: Optimization problems

Model I	Model II	Model III
Markowitz portfolio: $\min S_p^2$ $R_p \geq R_0$	$\min S_p^2$ $R_p \geq R_0$ $\sum_{i=1}^{m} \lambda_{\max i} x_i \leq \lambda_0$	$\min S_p^2$ $R_p \geq R_0$ $\lambda_{\max i} \geq 0 \wedge \lambda_{\max i}$ - statistically

$\sum_{i=1}^{m} x_i = 1$ $x_i \geq 0, \quad i = 1,...,m$	$\sum_{i=1}^{m} x_i = 1$ $x_i \geq 0, \quad i = 1,...,m$	significant $\sum_{i=1}^{m} x_i = 1$ $x_i \geq 0, \quad i = 1,...,m$
Model IV	**Model V**	**Model VI**
$\min S_p^2$ $R_p \geq R_0$ $\lambda_{\max i} < 0 \vee \lambda_{\max i}$ - statistically insignificant $\sum_{i=1}^{m} x_i = 1$ $x_i \geq 0, \quad i = 1,...,m$	$\max\left(\sum_{i=1}^{m} \lambda_{\max i} x_i\right)$ $R_p \geq R_0$ $\sum_{i=1}^{m} S_i x_i \leq S_0$ $\sum_{i=1}^{m} x_i = 1$ $x_i \geq 0, \quad i = 1,...,m$	Portfolio of equal shares of companies

where: S_p - the risk of the portfolio m-shares:

$$S_p^2 = \sum_{i=1}^{m} x_i^2 S_i^2 + 2\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} x_i x_j S_i S_j \rho_{ij} \ , \tag{6}$$

S_i - the standard deviation for *i*-company,

ρ_{ij} - the correlation coefficient of *i*-share with *j*-share,

x_i - contribution of *i*-share in the portfolio,

R_p - the expected rate of return on the portfolio: $R_p = \sum_{i=1}^{m} x_i R_i$,

R_i - the expected rate of return for *i*-company,

R_0 - average rate of return for companies,

S_0 - mean standard deviation,

λ_0 - mean largest (maximal) Lyapunov exponents,

$\lambda_{\max i}$ - the largest (maximal) Lyapunov exponent for *i*-company.

4. The purpose and conduct of the study

In the study we used financial time series of the WIG20 index, which at the time of portfolios building were listed on the Warsaw Stock Exchange for at least 10 years – from 2004 to 2015. The optimal portfolios were built at the end of each year between 2004-2015.

The largest Lyapunov exponent for the analyzed companies was estimated on the basis of the algorithms described in point 2. For this were used financial time series which were set up with logarithms of daily returns of closing price indexes of selected companies in period from 1.01.1994 to 31.12.2015[3]. The value of the largest Lyapunov exponent allowed to choose the companies, whose time series are characterized by chaotic dynamics i.e. the largest Lyapunov exponent is positive and statistically significant (R^2>0,3).

In the next stage of the studies it was constructed 72 investment portfolios based on solving optimization problems: model I - model VI. Table 1-6 shows the value of the expected rate of return and risk of constructed portfolios. The "-" sign in tables 2-4 means that an optimal portfolio wasn't built, because the companies didn't meet the model assumptions.

[3] The data comes from stooq.com. Date of access: 12.29.2009, 2.12.2010, 10.30.2012, 4.24.2017.

Table 1: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model I

Model I	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	BPH BZW DBC MBK	0.1943 0.2229 0.3140 0.2688	BPH BZW MBK	0.3311 0.3655 0.3033	BPH BZW KTY MBK	0.2687 0.1864 0.2634 0.2815	BPH BZW KGHM MBK	0.2426 0.4000 - 0.3574	ACP BZW CERS KGHM MBK PEO PND TPSA	0.1785 0.1920 0.1218 0.0922 0.0011 0.2501 0.1632 0.0010	ACP BZW CERS CEZ KGHM MBK PEO PKN TPSA	0.1923 0.0628 0.0171 0.2170 0.0344 0.0000 0.2598 - 0.2166
Rate of return	-0.000015		-0.001342		-0.001615		0.006364		0.031266		0.021969	
Annual rate of return	0.389148		0.380898		0.093286		-0.598796		0.496466		0.054987	
Risk	0.008985		0.010576		0.010869		0.105603		0.014349		0.009518	
Model I	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	ACP BZW CEZ KGHM MBK PEO PKN PXM TPSA	0.1034 0.1340 0.1476 0.0358 0.0515 0.1218 0.0828 0.1658 0.1573	ACP BHW KGHM MBK PEO PKN TPSA	0.0736 0.2032 0.1179 0.0554 0.1140 0.2046 0.2314	ACP BHW BRS KGHM MBK PEO PKN TPSA	0.1683 0.0873 0.1050 0.0744 0.1580 0.2604 0.1465 -	ACP BHW BZW KGHM MBK PEO PKN	0.2262 0.0297 0.1172 0.0875 0.1302 0.2606 0.1484	ACP BZW KGHM LPP MBK PEO PKN PKO	0.2366 0.0806 0.1448 0.0505 0.0273 - 0.1030 0.3571	ACP BZW CCC EURO KGHM LPP LTS MBK PEO PGN PKN PKO	0.2282 0.0074 0.0556 0.0393 - 0.0686 0.0325 0.0633 0.3359 0.0218 0.1203 0.0270
Rate of return	0.157187		0.000910		-0.000455		-0.000181		0.000101		0.504960	
Annual rate of return	-0.041319		0.433026		0.251311		0.037384		-0.223241		0.033139	
Risk	0.006593		0.011638		0.008171		0.010311		0.008145		0.007513	

Table 2: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model II

Model II	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	BPH BZW	0.4485 0.5515	-	-	BPH	1.0000	BZW	1.0000	ACP BZW CERS KGHM MBK	0.1891 0.2998 0.3188 0.1569 0.0355	BZW CERS MBK PEO PKN	0.1563 0.2427 0.0694 0.3950 0.1366
Rate of return	-0.000651		-		-0.000536		-0.000122		0.003756		-0.000484	
Annual rate of return	0.472105		-		-0.001619		-0.550317		0.699715		0.154599	
Risk	0.013257		-		0.020619		0.025414		0.018301		0.016468	
Model II	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	BZW MBK PXM	0.3538 0.2666 0.3796	BHW KGHM PEO	0.4102 0.2678 0.3219	ACP BHW BRS KGHM MBK PEO PKN	0.1821 0.0861 0.1098 0.0706 0.1512 0.2567 0.1436	ACP BHW KGHM MBK PEO	0.2732 0.0520 0.1462 0.1677 0.3608	BZW KGHM LPP MBK PEO PKN	0.1550 0.2035 0.1513 0.1667 0.1719 0.1516	ACP BZW MBK PEO PKN	0.2972 0.0479 0.1019 0.4026 0.1503
Rate of return	-0.000222		0.000990		-0.000430		-0.000011		0.000136		0.552799	
Annual rate of return	-0.063208		0.543830		0.248567		0.041118		-0.236154		0.028747	
Risk	0.009950		0.014990		0.008175		0.010935		0.009778		0.008066	

Table 3: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model III

Model III	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	BPH BZW	0.4485 0.5515	-	-	BPH	1.0000	BZW	1.0000	ACP BZW CERS KGHM MBK	0.1891 0.2998 0.3188 0.1569 0.0355	BZW CERS MBK PEO PKN	0.0975 0.1635 0.1030 0.6360 0.0000
Rate of return	-0.000651		-		-0.000536		-0.000122		0.003756		-0.000484	
Annual rate of return	0.472105		-		-0.001619		-0.550317		0.699715		0.157290	

Risk	0.013257		-		0.020619		0.025414		0.018301		0.015469	
Model III	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	BZW MBK PXM	0.3538 0.2666 0.3796	BHW KGHM PEO	0.4102 0.2678 0.3219	ACP BHW BRS KGHM MBK PEO PKN	0.1682 0.0874 0.1050 0.0744 0.1579 0.2604 0.1466	ACP BHW KGHM MBK PEO	0.2732 0.0520 0.1462 0.1677 0.3608	BZW KGHM LPP MBK PEO PKN	0.1949 0.2166 0.1486 0.0865 0.1803 0.1731	ACP BZW MBK PEO PKN	0.2963 0.0533 0.0985 0.4018 0.1499
Rate of return	-0.000222		0.000990		-0.000456		-0.000011		0.000136		0.551666	
Annual rate of return	-0.063208		0.543830		0.251263		0.041118		-0.231576		0.030079	
Risk	0.009950		0.014990		0.008171		0.010935		0.009697		0.008065	

Table 4: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model IV

Model IV	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	DBC MBK	0.5712 0.4288	BPH BZW MBK	0.3311 0.3655 0.3033	BZW KTY MBK	0.2666 0.3920 0.3414	BPH KGHM MBK	0.3306 0.0000 0.6694	PEO PND TPSA	0.3166 0.3340 0.3494	ACP CEZ KGHM TPSA	0.2986 0.3274 0.0503 0.3237
Rate of return	0.000508		-0.001342		-0.001975		0.008526		0.002788		-0.000240	
Annual rate of return	0.142400		0.380898		0.226835		-0.618775		0.226935		-0.042228	
Risk	0.011356		0.010576		0.012585		0.143594		0.016661		0.011386	
Model IV	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	ACP CEZ KGHM PEO PKN TPSA	0.1594 0.2980 0.0862 0.1673 0.0741 0.2150	ACP MBK PKN TPSA	0.2275 0.1888 0.3070 0.2767	-	-	BZW PKN	0.5000 0.5000	ACP PKO	0.4842 0.5158	CCC EURO LPP LTS PGN PKO	0.1333 0.1429 0.1335 0.1360 0.1106 0.3434
Rate of return	0.214958		0.000850		-		-0.000606		0.000010		0.454501	
Annual rate of return	-0.045085		0.269870		-		0.017491		-0.013687		0.040159	
Risk	0.007738		0.012708		-		0.014214		0.009044		0.010604	

Table 5: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model V

Model V	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	BPH BZW DBC MBK	0.2000 0.4000 0.4000 -	BPH BZW MBK	0.3537 0.4000 0.2463	BPH BZW KTY MBK	0.2668 0.0358 0.4000 0.2974	BPH BZW KGHM MBK	0.24711 0.3630 0.3899 -	ACP BZW CERS KGHM MBK PEO PND TPSA	0.4000 0.0592 - - 0.1002 - 0.4 0.0406	ACP BZW CERS CEZ KGHM MBK PEO PKN TPSA	0.4000 - 0.1730 0.4000 - 0.0270 - - -
Rate of return	0,000178		-0.00135		-0.00161		0.006364		0.004118		-0.000420	
Annual rate of return	0,360064		0.362457		0.081926		-0.60455		0.341110		-0.081340	
Risk	0,010429		0.010753		0.011803		0.107246		0.019415		0.014344	
Model V	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	ACP BZW CEZ KGHM MBK PEO PKN PXM TPSA	0.0361 0.3810 0.0720 0.4000 - - - 0.1109	ACP BHW KGHM MBK PEO PKN TPSA	- 0.3320 0.2885 - 0.3655 - 0.0140	ACP BHW BRS KGHM MBK PEO PKN TPSA	0.1113 0.3012 0.1875 - 0.4000 - - -	ACP BHW BZW KGHM MBK PEO PKN	0.1402 - 0.4000 0.3853 0.0744 - -	ACP BZW KGHM LPP MBK PEO PKN PKO	0.0605 - 0.4000 0.0253 0.4000 - 0.0518 0.0624	ACP BZW CCC EURO KGHM LPP LTS MBK PEO PGN PKN PKO	- - 0.1685 - - 0.0991 - 0.4000 0.0459 - - 0.2866

Rate of return	0,110813	0.000910	-0.00082	-0.00018	0.000101	0.333798
Annual rate of return	-0,117820	0.537018	0.492476	-0.00139	-0.30431	0.050065
Risk	0,010827	0.014167	0.011897	0.014162	0.011493	0.012162

Table 1: The expected rates of return, annual rate of return, risk and compositions of constructed portfolios-model VI

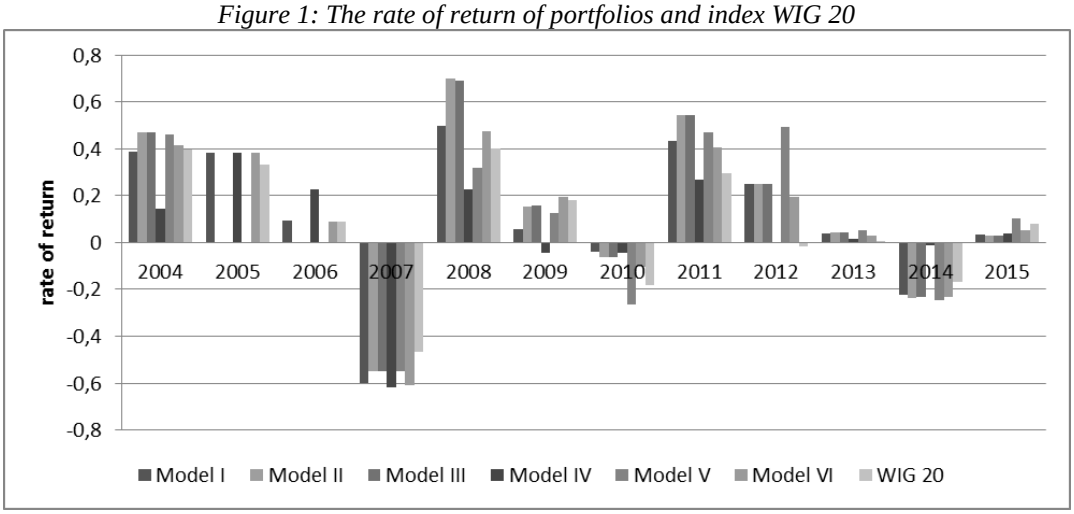
Model VI	**2004**		**2005**		**2006**		**2007**		**2008**		**2009**	
	BPH BZW DBC MBK	0.2500 0.2500 0.2500 0.2500	BPH BZW MBK	0.3333 0.3333 0.3333	BPH BZW KTY MBK	0.2500 0.2500 0.2500 0.2500	BPH BZW KGHM MBK	0.2500 0.2500 0.2500 0.2500	ACP BZW CERS KGHM MBK PEO PND TPSA	0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250	ACP BZW CERS CEZ KGHM MBK PEO PKN TPSA	0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111
Rate of return	-0.000152		-0.001342		-0.001615		0.006364		0.032003		0.028854	
Annual rate of return	0.415469		0.385048		0.090843		-0.610851		0.475094		0.193602	
Risk	0.009098		0.010601		0.011008		0.108453		0.014513		0.012712	
Model VI	**2010**		**2011**		**2012**		**2013**		**2014**		**2015**	
	ACP BZW CEZ KGHM MBK PEO PKN PXM TPSA	0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111	ACP BHW KGHM MBK PEO PKN TPSA	0.1429 0.1429 0.1429 0.1429 0.1429 0.1429 0.1429	ACP BHW BRS KGHM MBK PEO PKN	0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250	ACP BHW BZW KGHM MBK PEO PKN	0.1429 0.1429 0.1429 0.1429 0.1429 0.1429 0.1429	ACP BZW KGHM LPP MBK PEO PKN PKO	0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250	ACP BZW CCC EURO KGHM LPP LTS MBK PEO PGN PKN PKO	0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833 0.0833
Rate of return	0.110813		0.000910		-0.000816		-0.000181		0.000101		0.333798	
Annual rate of return	-0.103782		0.404857		0.194389		0.030919		-0.232657		0.051004	
Risk	0.007336		0.012190		0.008668		0.010958		0.008829		0.009536	

On the basis of the results presented in Tables 1-6 we can conclude that for the optimization model I and the optimization model VI most of portfolios are characterized by higher expected rate of return than for other optimization models. For each optimization model portfolio the highest expected returns were obtained in 2015. The highest level of expected return was characterized for portfolios built on the optimization models II and III in 2015 at one of the lowest risk level. Portfolios built on the optimization models II, III and VI are characterized by higher risk level. The exception are 2005 and 2007 in which the highest level of risk was obtained for optimization models I, IV and VI.

Analyzing the obtained annual rates of return for the designated portfolios (Table 1-6) it can be seen that the largest profit would give the investment in portfolios built on the optimization models II and III in 2008, and then in the same portfolios in 2011. The lowest annual rate of return obtained for the portfolio 2007 in all optimization models.

Figure 1 compares the rates of return of obtained portfolios with the return of the WIG 20 index for the years 2004-2015.

Figure 1: The rate of return of portfolios and index WIG 20



On the basis of the data in Fig. 1 we can conclude that most portfolios in the optimization models I, II, III, V and VI are characterized by higher rate of return than index WIG 20. The exception are years 2007, 2009, 2014 and 2015. The calculated 10-year return rates for the analyzed strategies showed that the most profitable strategy was to invest in the optimization model III (0.9301), then model II (0.9161), model I (0.9010), model VI (0.8320) and WIG20 index (0.6813). The worst long-term strategy was model IV (0.0975).

Figure 2: The cumulative rate of return of portfolios and index WIG 20

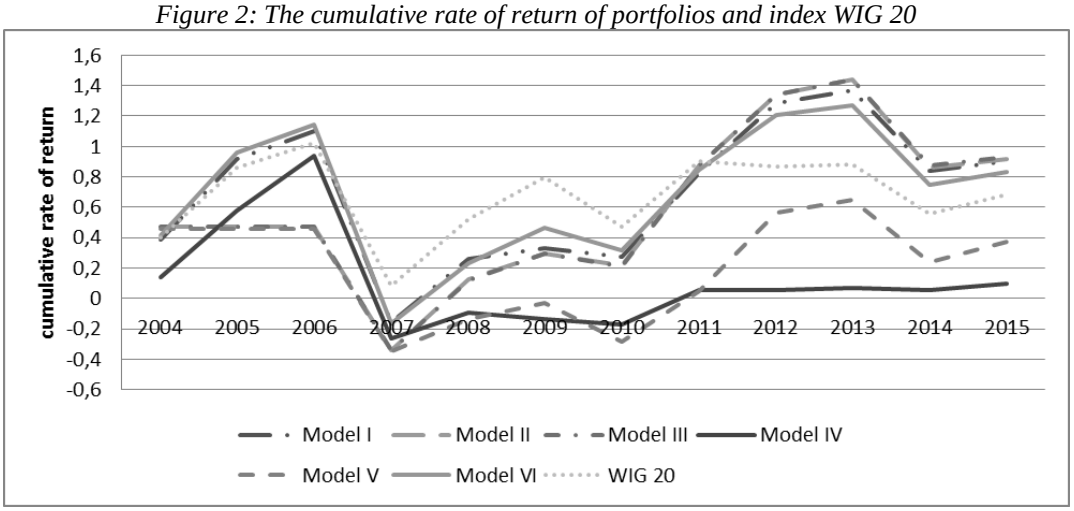


Figure 2 presents the cumulative rate of return of portfolios and index WIG20. Obtained results (fig. 2) show also that proposed in the paper approach gives generally better results during bull market (2005-2007 and 2009-2014) then bear (2008, 2015).

5. Conclusions

In the paper was presented an attempt to construct the optimal portfolios of shares based on the value of the largest Lyapunov exponent. The study has revealed that the portfolios built on the basis of the largest Lyapunov exponent (model III and model II) give better results due to the annual rate of return then the index WIG20 in 2005-2015. Obtained results show also that proposed in the paper approach gives generally better results during bull market then bear.

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Impact of changes in accounting regulation on sanctions for its violation in selected countries

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Abstract

Legislative changes are common in every area of law including accounting. At the national level the changes of accounting legislation could be connected with a membership of countries in different international organisations or unions which could rise their obligation to adopt new accounting rules or harmonize the currently valid regulation. This impact reflects not only on accounting rules but as well as on the range of fines and penalties in case of the breach of accounting legislation. The range and amount of fines and penalties could be impacted also by the national setters. The main purpose of this research paper is to investigate the influence of national and international requirements on the changes in sanctions for violation of accounting regulations and in the concepts of administrative offences in accordance with accounting legislation valid in Croatia, the Czech Republic, the Republic of Lithuania and Slovakia.

Key words

Accounting regulation, administrative offence, fine, penalty, sanction, violation, Croatia, Czech Republic, Republic of Lithuania, Slovak Republic

JEL Classification: M40, M41, K29

1. Introduction

The state assigns the presentation of correct and truthful accounting information based on the true and fair view and fulfilment of accounting legislation by a business under the possibility of a control and consequent sanctions in case of the breach of accounting rules (Hornická, R., 2014). The national concept of accounting delicts and the prescription and imposition of consequent fines and penalties could be conditioned as well as by requirements for international regulation. All from selected countries are members of the European Union (hereinafter: EU). The Czech Republic, Lithuania and the Slovak Republic have been members of the EU since 1 May 2004. Croatia joined the EU on 1 July 2013. One of the requirements of the EU for its member states is to fully harmonize legislation with the EU regulation. For accounting realm is in force Directive 2013/34/EU of the European Parliament and of the Council on the annual financial statements, consolidated financial statements and related reports of certain types of undertakings as amended (hereinafter: Accounting

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Directive) since 20 July 2013. All EU member states had to incorporate accounting principles and rules prescribed by the Accounting Directive into national accounting legislation until 20 July 2015. The EU does not fine directly the infringements of the national provisions adopted in accordance with the Accounting Directive, but in accordance with the Article 51 of this Directive all member states have the obligation to provide for penalties applicable to infringements and shall take all the measures necessary to ensure that those penalties are enforced. The OECD is another international organisation with the impact on the national prescription and imposition of fines and penalties for the breach of accounting legislation. The Czech Republic joined the OECD in 1995 and the Slovak Republic in 2000. Both countries accepted the 1997 OECD Convention on Combating Bribery of Foreign Public Officials in International Business Transactions (hereinafter: OECD Anti-Bribery Convention) with the force from 1999. Lithuania is in process for joining the OECD, it became the 42nd Party to the OECD Anti-Bribery Convention on 15 July 2017. The entrance into the European Monetary Union (hereinafter: EMU) could have a kind of influence on the penalties connected with their converting from the national currencies into the euro. The Slovak Republic gained the approval from the Council of the EU to join the EMU on 1 January 2009. Lithuania became a member of the EMU and adopted the euro on 1 January 2015.

2. The concept of sanctions for violation of accounting regulation in Croatia

The Ministry of Finance of Croatia determines penalties and sanctions for infringements of Croatian accounting regulations in the Act on Accounting (Official Gazette, No. 78/2015.). Since Croatian independence (1991) there was in force 4 different Acts on Accounting (1992, 2005, 2007 and 2015) and each of them have had few amendments. Some of amendments were related with the list of accounting offences and the amounts of penalties. Changes in the list of accounting offences were connected with the status of Croatia in process of the accession to the European Union before 2013 (especially from 2005 when started accession negotiations) and after 2013 with the implementation of the Accounting Directive.

Table 1 List of the accounting offences in Croatia

Legislation	**Accounting offences**
Act on Accounting (Official Gazette, No. 146/2005.)	Article 21.: false presentation of the positions of the financial statements Article 22.: legal entity (entrepreneur) does not prepare accounting documents, does not control accounting documents before entering in the business books, does not keep accounting documents, does not provide data entry to appropriate bearers, does not open, conduct, close and keep business books, does not perform the accounts in the general ledger according to the Chart of accounts, does not perform the list of assets and liabilities, does not prepare and keep the financial statements, does not use the International or Croatian financial reporting standards
Act on Accounting (Official Gazette, No. 109/2007.)	Article 31. false presentation of assets, financial position and financial performance of the entrepreneur in the annual financial statements Article 32. includes accounting offences listed in Article 22. of the Act on Accounting (2005) plus: legal entity does not prepare, present and keep the consolidated financial statements, does not revise the annual financial statements, does not keep the audit report in settled deadlines, does not prepare the annual report, does not provide annual financial statements, annual report, audit report and other documents to the Financial Agency for the purpose of public disclosure
Act on Accounting (Official Gazette, No. 78/2015.)	Article 42. includes accounting offences listed in Article 32. of the Act on Accounting (2007) plus: annual financial statements are not signed from the Chairman of the Board of directors and all members of the Board, as well as from all Chief Executive Officers, the limits for income distribution are not applied, nonfinancial report and consolidated nonfinancial report are not prepared, the statement on application of corporate governance code is not included in the annual report, the report on payments in the public sector is not prepared

Source: Act on Accounting (Official Gazette, No. 146/2005.); Act on Accounting (Official Gazette, No. 109/2007.); Act on Accounting (Official Gazette, No. 78/2015.)

List of accounting offences is significant, and with each new amendment the list is increasing. Furthermore, list of administrative offences in Croatia is also regulated by the General Tax Act (Official Gazette, No. 115/2016.) and the Company Act (Official Gazette, No. 111/1993.).

Table 2 Overview of fines for accounting offences listed in Table 1, 2005 – 2017

Year	Accounting offences	Fines for accounting offences
2005 2006	Listed in Article 21. (Act on Accounting, 2005)	• from 1 356 € to 6 780 € (from 10 000,00 to 50 000,00 HRK, as of 31.12.2005, 1 € = 7,375626 HRK) – for responsible person in the entrepreneur
	Listed in Article 22. (Act on Accounting, 2005)	• from 1 356 € to 13 560 € (from 10 000,00 to 100 000,00 HRK, as of 31.12.2005, 1 € = 7,375626 HRK) – for legal entity • from 1 356 € to 6 780 € (from 10 000,00 to 50 000,00 HRK, as of 31.12.2005, 1 € = 7,375626 HRK) – for responsible person in the entrepreneur
2007 2008 2009 2010 2011 2012 2013 2014	Listed in Article 31. (Act on Accounting, 2007)	• from 2 730 € to 13 650 € (from 20 000,00 to 100 000,00 HRK, as of 31.12.2007, 1 € = 7,325131 HRK) – for legal representative • from 6 825 € to 68 250 € (from 50 000,00 to 500 000,00 HRK, as of 31.12.2007, 1 € = 7,325131 HRK) – for legal entity
	Listed in Article 32. (Act on Accounting, 2007)	• from 1 365 € to 13 650 € (from 10 000,00 to 100 000,00 HRK, as of 31.12.2007, 1 € = 7,325131 HRK) – for legal entity • from 680 € to 2 730 € (from 5 000,00 to 20 000,00 HRK, as of 31.12.2007, 1 € = 7,325131 HRK) – for responsible person in the entrepreneur
2015 2016 2017	Listed in Article 41. (Act on Accounting, 2015)	• from 1 310 € to 13 100 € (from 10 000,00 to 100 000,00 HRK, as of 31.12.2015, 1 € = 7,635047 HRK) – for legal entity • from 655 € to 2 620 € (from 5 000,00 to 20 000,00 HRK, as of 31.12.2015, 1 € = 7,635047 HRK) – for responsible person in the entrepreneur

Source: Act on Accounting (Official Gazette, No. 146/2005.); Act on Accounting (Official Gazette, No. 109/2007.); Act on Accounting (Official Gazette, No. 78/2015.)

Although the list of accounting offences is significant and it is constantly increasing, the amounts of fines are very low for them. In the Act on Audit (Official Gazette, No. 146/2005.) are specified offences in the area of auditing and the amounts of fines related to them.

3. The concept of sanctions for violation of accounting regulation in the Czech Republic

Legal regulation of sanctions concerning accounting is included in Act No. 563/1991 Coll., on Accounting, as amended (hereinafter: the Czech Act on Accounting). In this respect, there was a change in the concept of administrative punishment in the Czech Republic when new Act No. 250/2016 Coll. on Liability for Offences and Relevant Proceedings entered into force as of 1 July 2017. The primary reason for implementing new legislation was the fact that the old legislation was outdated, not complex enough and sanction proceedings were not covered sufficiently.

The concept of current legal regulation concerning offences in the sphere of accounting is as follows: the Czech Act on Accounting defines the cases when duties required by the Czech Act on Accounting are breached and then sets the maximum amount of the relevant sanction; the sanctions imposed are in the form of a fine. Typically, the amount of the fine is derived from the value of total assets, while sanction limits are (depending on the seriousness of the breach) between 3% and 6% of total value of assets. In some cases, the maximum value of the fine is set with a flat rate.

When comparing the new with the old legislation on sanctions, changes were mainly driven by supranational organizations. Until the end of 2011, sanctions were more modest, namely penalty up to CZK 500,000 or CZK 1,000,000. After the paradigm shift on 1 January 2002, the maximum penalty increased to 3% or 6% of total assets which is still reflected in the

current legislation and which was driven by the desire to unify the legislation with the EU law and to react on recommendation of OECD Anti-Bribery Convention (Huleš, J., Jeřábek, A., 2001). Further clarification of the legislation was enacted as of 1 January 2011 also because of recommendations included in the assessment on fulfilment of the OECD Anti-Bribery Convention. The main aim was to apply sanctions to accounting entities that underestimated their assets used as a basis for calculation of sanctions so as to prevent this behaviour. More specifically, the specific measure allows state institutions (namely the tax authorities) to levy sanctions not based on the value of assets in the balance sheet but based on the real value of assets ascertained by the tax authorities.

To conclude, efforts of the Czech Republic in the area of implementation of recommendations of the OECD Anti-Bribery Convention are perceived rather positively as the Czech Republic is 3 out of 44 OECD member countries (after Finland and Great Britain) that underwent already all 4 assessment phases of Convention implementation. (Společná tisková zpráva Nejvyššího státního zastupitelství a Ministerstva spravedlnosti, 2017).

The Czech Act on Accounting is not the only legal regulation that defines sanctions concerning accounting. In a case when the breach of duty is very serious and other aspects are present, it may even be classified as a criminal act of "Misrepresentation of data relating to economic results and equity", where the defining aspect is that duties related to keeping accounts have been intentionally breached (for instance, stating untrue information in financial statements), which results in threatening a legal right or threatening the due and timely tax assessment. The relevant punishment depends on the seriousness of conduct and ranges from the ban on activity to imprisonment for 2 to 8 years.

4. The concept of sanctions for violation of accounting regulation in the Republic of Lithuania

Accounting in the Republic of Lithuania is regulated at three levels. First level includes the Law on Accounting, which is valid for all subjects in private and public sectors and regulates the arrangement of accounting. Second level is based on laws which regulate requirements for the information provided in the financial statements and preparation of financial statements (the Law on Financial Statements and the Law on Consolidated Financial Statements for private sector; the Law on Public Sector Financial Statements for public sector). Provisions of mentioned above laws were complied with the EU Directives that started integration of Lithuanian accounting system into accounting system of EU. The third accounting regulation level includes Business Accounting Standards or IFRS for private sector and Public Sector Accounting and Financial Reporting Standards, accordingly. (Lakis, V., Subačienė, R., 2014; Rudžionienė, K., Christauskas, Č., 2010).

Sanctions for violation of provisions on accounting regulation are generally defined in analysed laws, more detail in the Code of Administrative Offenses and the Criminal Code, and are presented in table 3. Lithuanian Criminal Code system of sanctions is based on the general rule that for serious and very serious crimes, imprisonment is the only available penalty (OECD, 2017).

Table 3 Responsibility and sanctions for violation of accounting regulation in Lithuania

Characteristics of Law	Number and title of the article	Definition of responsibility	Sanctions determined in the Codes
The Law on Accounting, No IX-574 (2001)	21. Responsibility for the arrangement of accounting, the preservation of accounting records	The entity's manager is responsible for the arrangement of accounting in accordance with the requirements of this Law, preservation of accounting documents and accounting registers and	Article No 205 of the Code of Administrative Offenses determines the size of fines from 30 € to 6000 € for various types of offenses in accounting arrangement and etc.

Characteristics of Law	Number and title of the article	Definition of responsibility	Sanctions determined in the Codes
	and accounting registers and the rendering of their data	the submission of their data according to legal acts regulating civil, administrative and criminal responsibility.	Articles No 222 and 223 of the Criminal Code determine punishment by fine, arrest, public works or imprisonment from up to 2 to 4 years for fraudulent and reckless accounting.
The Law on Financial Statements, No IX-575 (2001)	28. Responsibility for the preparation of financial statements, annual report and submission to the Register of Legal Entities	The entity's manager, members of the management and supervisory bodies, according to their level of assigned competences, are responsible for the preparation of financial statements, annual report, auditor's report (if it has to be prepared), statement for corporate social responsibility and submission to the Register of Legal Entities. They must compensate all the damage caused to the legal entity and (or) other persons.	Article No 223 of the Code of Administrative Offenses determines, that for submission of incorrect financial statements, annual report, auditor's report (if it has to be prepared), or consolidated financial statements, consolidated annual report, auditor's report, or other statements and not submission of above mentioned statements, impose a fine from 30 € to 3000 €. Article No 221, 222 and 223 of the Criminal Code determines punishment by fine or imprisonment up to 3 years for not submission of declaration, statement or other document. *The Criminal Code establishes fines from 38 € to 1 883 000 €.*
The Law on Consolidated Financial Statements for private sector, No IX-576 (2001)	14. Responsibility for the preparation of consolidated financial statements, consolidated annual report and submission to the Register of Legal Entities	The parent company's manager, members of the management and supervisory bodies, according to their level of assigned competences, are responsible for the preparation of consolidated financial statements, consolidated annual report, auditor's report, consolidated statement for corporate social responsibility and submission to the Register of Legal Entities. They must compensate all the damage caused to the parent company and (or) other persons.	
The Law on Public Sector Financial Statements for public sector, No X-1212 (2007)	35. Responsibility for the preparation of financial statements, submission and publication on time	The public sector entity's manager or his authorized head of administration is responsible for the preparation of financial statements or consolidated financial statements, their submission and publication on time according to the procedure established by legislation.	

Source: own processing according to the sources provided in the table

Tax authorities under the Law on Tax Administration (2017) may apply special tools for tax exaction, fines or penalties. Generally, the size of fines defined from 10 to 50 % of unpaid tax amount. Also, tax payers and other stakeholders who do not comply with the provisions of the law or comply improper way are responsible under the Code of Administrative Offenses and the Criminal Code.

5. The concept of sanctions for violation of accounting regulation in the Slovak Republic

The Ministry of Finance of the Slovak Republic determines penalties for infringements of Slovak accounting regulation in the Act No. 431/2002 Coll. on Accounting as amended (hereinafter: Slovak Act on Accounting). Since 2002 the Slovak Act on Accounting has been amended 21 times. Some of amendments were connected with the list of administrative offences or with the limits for penalties and their imposition. Breaking points could be identified for both of these issues. Since the entrance of the Slovak Republic to the OECD, its Working Group on Bribery has been monitoring the implementation of the OECD Anti-Bribery Convention. Amendments of the Slovak Act on Accounting on fines and penalties after June 2012 were connected with the results published in the "Phase 3 Report on

Implementing the OECD Anti-Bribery convention in the Slovak Republic". Although before the publishing of the OECD report, a significantly changed approach to the fines and penalties was implemented in the Slovak Act on Accounting in 2012, as it is presented in table 5. Amendments of Slovak Act on Accounting in 2013 and 2014 adopted by the Parliament led only to "light" changes of the list of administrative offences. In November 2014 the OECD published a report "Slovak Republic: Follow-up to the Phase 3 Report and Recommendations", which provided information on the progress made by Slovakia in implementing the recommendations of its Phase 3 report. The OECD Working Group summed up that one of the important recommendations that has not been implemented addressed the level of sanctions for accounting offences, which have been lowered since Phase 3 (OECD, 2014). With the respect to the OECD recommendations and the need of the EU´s Accounting Directive implementation the Parliament adopted Act No. 130/2015 Coll. of 6 May 2015 and later on Act No. 423/2015 Coll. on Statutory Audit (Domaracká, D. - Hunyady, K., 2016). Both acts amended Slovak Act on Accounting and affected the administrative offences and the level of sanctions. As in table 4 it is shown, the restriction for accounting entry was changed.

Table 4 Comparison of the restriction for accounting entry in accordance with the Slovak accounting legislation and OECD Anti-Bribery Convention

Legislation/Convention	**Restriction for accounting entry**
Act on Accounting before Act No. 130/2015 in force	§ 11 (3): Accounting entities may not make accounting entries outside their accounting books.
Act on Accounting after Act No. 130/2015 in force	§ 11 (3): An accounting unit cannot perform any accounting records outside the accounting books, perform any record on an accounting case which has not occurred, and conceal or not recognize any fact that is a subject to the accounting.
OECD Anti-Bribery Convention	Article 8 (1): In order to combat bribery of foreign public officials effectively, each Party shall take such measures as may be necessary, within the framework of its laws and regulations regarding the maintenance of books and records, financial statement disclosures, and accounting and auditing standards, to prohibit the establishment of off-the-books accounts, the making of off-the-books or inadequately identified transactions, the recording of non-existent expenditures, the entry of liabilities with incorrect identification of their object, as well as the use of false documents, by companies subject to those laws and regulations, for the purpose of bribing foreign public officials or of hiding such bribery.

Source: Act No. 431/2002 Coll. on Accounting and OECD (1997)

Amended wording of accounting entry sets new facts that could lead to the violation of accounting legislation. Except for the restriction for accounting entry which amended the list of administrative offences, there was added a new type of administrative offences referred to failure to keep accounts. Failure to keep accounts, violation of restriction for accounting entry along with failure to compile financial statements belong to the group of the most serious breaches of accounting legislation with the highest possible fine amounting up to 3,000,000 EUR. The amount of highest fine increased from 1,000,000 EUR. The system of fines is graduated and except for the maximum limit specified by a fixed number of euros there is maximum percentage of euro calculated from the total net assets. In table 5 the evolution of fines and their current status is demonstrated.

Table 5 Overview of fines for administrative offences, 2009 - 2017

Year	Maximum limit	Fines for administrative offences of lower seriousness
2009 2010 2011	Up to **3 319 391,88 €** (maximum limit recalculation 100,000,000 SKK as of 31.12.2008, 1 € = 30,126 SKK)	• up to 1% of the total amount of assets determined from the balance sheet (total net assets) or from statement of assets and liabilities, or • up to 3% of the total amount of assets determined from the balance sheet (total net assets) or from statement of assets and liabilities or consolidated balance sheet (total net assets)
2012		• up to 2% of the total amount of assets determined from the balance

2013 2014 2015	Up to **1 000 000 €**	sheet (total net assets) or from statement of assets and liabilities ≤ 1,000,000 € / 100,000 € / 1,000 €
2016 2017	Up to **3 000 000 €**	• up to 2% of the total amount of assets determined from the balance sheet (total net assets) or from statement of assets and liabilities ≤ 1,000,000 € / 100,000 € / 1,000 €

Source: Act No. 431/2002 Coll. on Accounting

The Ministry of Finance of the Slovak Republic has prepared a proposal for the amendments of the Slovak Act on Accounting with expected effectiveness as of 1 January 2018. For the more effective control of the observance of accounting legislation, the preservation of the accounting documentation will be prolonged from 5 years to 10 years. Besides it should be implemented the status of administrative offence having been committed repeatedly including the sanction for it. These proposals should assign some objectives of the Governmental Action Plan against Tax Fraud 2012-2016.

Fines for commitments of the administrative offences are imposed by the tax authority. Tax authority imposes fines for administrative offences in accordance with the Act No. 563/2009 Coll. on Tax Administration as amended after the control of the observance of the Slovak Act on Accounting and revelation of an administrative offence. If the tax authority finds out during the control that the administrative offence was an intentional act committed by the entity, then it will be considered as a criminal offence and the Act No. 300/2005 Coll. Criminal Code as amended applies.

The entrance of the SR into to the EMU did not have any impact on the level of the maximum limit of fine. The exchange rate for the Slovak Koruna into euro as of 31 December 2008 was used for the recalculation of the maximum fine valid at that time.

6. The comparison of the approach to sanctions in selected countries

Table 6 presents the comparison of selected features considering national legislation and impact of international organisations on accounting sanctions and the focus on the maximum limit of fines and the authority that is responsible for their imposition.

Table 6 Accounting regulation and sanctions for its violation - comparison

Country	Primary legislation covering accounting sanctions	Secondary legislation of accounting sanctions	Impact of international organisations and reasons for the amendments of accounting sanctions incl. their acceptation (✓) /not yet or refusal (×)	Maximum limit of fines (2017)	Authority responsible for imposition of fines
Croatia	• Act on Accounting (2015)	• General Tax Act (2016), • Company Act (1993), • Act on Audit (2005)	• EU via Accounting Directive (not directly): - effective, proportionate and dissuasive penalties (✓)	• up to **13 100 €**	• tax authority
Czech Republic	• Act No. 563/1991 Coll. on Accounting	• Act No. 250/2016 Coll. on Liability for Offences and Relevant Proceedings • Act No. 40/2009 Coll. Criminal Code	• OECD: - effective, proportionate and dissuasive sanctions (✓) - enforceability and proportionality of sanctions incl. confiscation, (✓) - low level of sanctions (✓) • EU via Accounting Directive (not directly): - effective, proportionate and dissuasive penalties (✓)	• 6% of the total amount of assets	• tax authority
Republic	• Law on	• Code of	• OECD (×)	• up to **1 883 000 €**	

Country	Primary legislation covering accounting sanctions	Secondary legislation of accounting sanctions	Impact of international organisations and reasons for the amendments of accounting sanctions incl. their acceptation (✓) /not yet or refusal (×)	Maximum limit of fines (2017) / Authority responsible for imposition of fines
of Lithuania	Accounting • Law on Financial Statements • Law on Consolidated Financial Statements • Law on Public Sector Reporting	Administrative Offenses No XII-1869 • Criminal Code No VIII-1968 • Law on Tax Administration No IX-2112	• EU via Accounting Directive (not directly): - effective, proportionate and dissuasive penalties (✓) • National requirements (✓)	• Tax authorities • the Financial Crime Investigation Service
Slovak Republic	• Act No. 431/2002 on Accounting	• Act No. 563/2009 Coll. on Tax Administration, • Act No. 300/2005 Coll. Criminal Code • Act No. 423/2015 Coll. on Statutory Audit	• OECD: - effective, proportionate and dissuasive sanctions (✓) - enforceability and proportionality of sanctions incl. confiscation, (✓) - low level of sanctions (✓) • EU via Accounting Directive (not directly): - effective, proportionate and dissuasive penalties (✓) • national requirements: - Governmental Action Plan (×)	• up to **3 000 000 €** • tax authority

Source: Own processing

On behalf of compared attributes there is possible to identify some similarities and differences. The primary legislation covering accounting sanctions except for the Republic of Lithuania is only act on accounting. All selected countries except for Croatia apply also Criminal Act in case of detection that the administrative offence was an intentional act committed by the entity. The influence of international organisations on the system of sanctions is demonstrable. In the Slovak Republic exists Governmental Action Plan, but this one is not fully adopted for the area of sanctions. The system of sanctions and their maximum limit vary comparing the selected countries. Tax authority is responsible for imposition of fines and in addition the Financial Crime Investigation Service in the Republic of Lithuania.

7. Conclusion

The national concept of accounting delicts and the prescription and imposition of consequent fines and penalties in Croatia, the Czech Republic, the Republic of Lithuania and the Slovak Republic is conditioned as well as by requirements for international regulation. All from analysed countries apply the requirement of the EU Accounting Directive for effective, proportionate and dissuasive penalties. The OECD has also a significant impact on effective, proportionate and dissuasive sanctions; enforceability and proportionality of sanctions incl. their confiscation and the level of sanctions in case it was considered as low. The Czech Republic and the Slovak Republic implemented the requirements of the OECD in this respect. The conditions of the Republic of Lithuania (as a new member of the OECD) were already verified in the Phase I. There may be necessary to apply changes requested by the OECD into the Lithuanian accounting legislation, as well as to properly complete the reforms of states forests and road maintenance state enterprises. The proposal for amendments of Slovak accounting legislation valid from 2018 considering this issue contains some objectives set in the Governmental Action Plan against Tax Fraud 2012-2016. So we can identify also the

influence of national requirements. The membership in the EMU did have any influence on the high of sanctions neither in the Republic of Lithuania nor in the Slovak Republic. The sanctions were just converted from national currencies into euro. The high of sanctions varied in the past. It is possible to identify the sine wave of the high of sanctions in case of Croatia and the Slovak Republic although the OECD requested to increase the limits in the Slovak Republic. The comparison indicates that Croatia has the lowest fines for accounting offences.

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Brand valuation and recognition of Sedita with using a licence analogue method and the possibility of its use in creating the value of the enterprise

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Abstract

The purpose of the contribution is to define the issue of branding by means of a licensing analogy method and at the same time to point out the possibilities of using quantified brand value as a very important factor that plays a significant role in the creation of the property business. The methodical part of the contribution describes the brand value and methods of rating the brands used in the Slovak Republic. The analytical part of the contribution is a licensing analogue method that includes calculating the production scale, license fee, revaluation coefficient, capitalization coefficient and intangible asset share for production, and calculating the Sedita brand for 351,204 € demonstrating the objective of the contribution. The data were supplemented by documentary analysis, including the documents and methods of marking the mark and by Decree No. 492/2004 Coll., which stipulates the marking of the mark.

Key words

brand value, brand valuation and recognition, brand valuation methods, licence analogue method

JEL Classification: M30, M31, M39, C10

1. Úvod

V dnešnom svete plnom globalizačných zmien a procesov zohráva hospodárska súťaž rozhodujúcu otázku pre tvorcov podnikateľských politík na všetkých úrovniach podniku. Etablovanie podniku na národných a medzinárodných trhoch si vyžaduje kodifikáciu obchodných stratégií na zlepšenie schopností a konkurencieschopnosti. Stratégie a úlohy vlastnej značky dokážu priniesť podniku nielen konkurenčnú výhodu, ale aj váhu ovplyvnenia zákazníkov, meranie vlastného kapitálu značky.

V skutočnosti, podstatnou časťou podnikových aktív nie sú tak hmotné aktíva, ako aktíva nehmotného charakteru zahrňujúce oblasti manažmentu, marketingu, financií a vlastností a povesti značky. Spoľahlivé meranie hodnoty značky a faktory v reputácii poskytuje správne opatrenie na posúdenie dlhodobých účinkov marketingových rozhodnutí (Jacková, 2012).

Jednou z povinností vrcholového manažmentu je vytvoriť silnú značku a zároveň zlepšiť schopnosť podniku splniť svoje záväzky. Posilnenie konceptuálneho rozdielu medzi produktmi prostredníctvom značky, získanie reputácie a viac verných zákazníkov vytvárajú to, čo je viac ako finančný zisk, a nazývame ho špecifickou hodnotou alebo mentálnym obrazom značky na verejnosti.

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2. Prehľad literatúry

Hoci má pojem značka niekoľko desaťročnú históriu, za posledné 2 desaťročia sa uvažuje o hodnote značky ako kľúčovej a základnej hodnote pre podnik. Vznik koncepcie obchodnej značky sa pridal k strategickej hodnote marketingu a vytvoril zaostrovací bod pre výskumníkov a manažérov (Križanová, Masárová, Štefániková, 2015).

2.1 Značka a hodnota značky

Slovo značka pochádza zo staro nórskeho slova brandr čo znamená vypáliť. Podľa definície Americkej marketingovej asociácie AMA predstavuje značka (brand) meno, pojem, symbol, dizajn alebo kombináciu uvedených prvkov, ktoré identifikujú produkty a služby určitého dodávateľa a odlišujú ich od konkurentov.

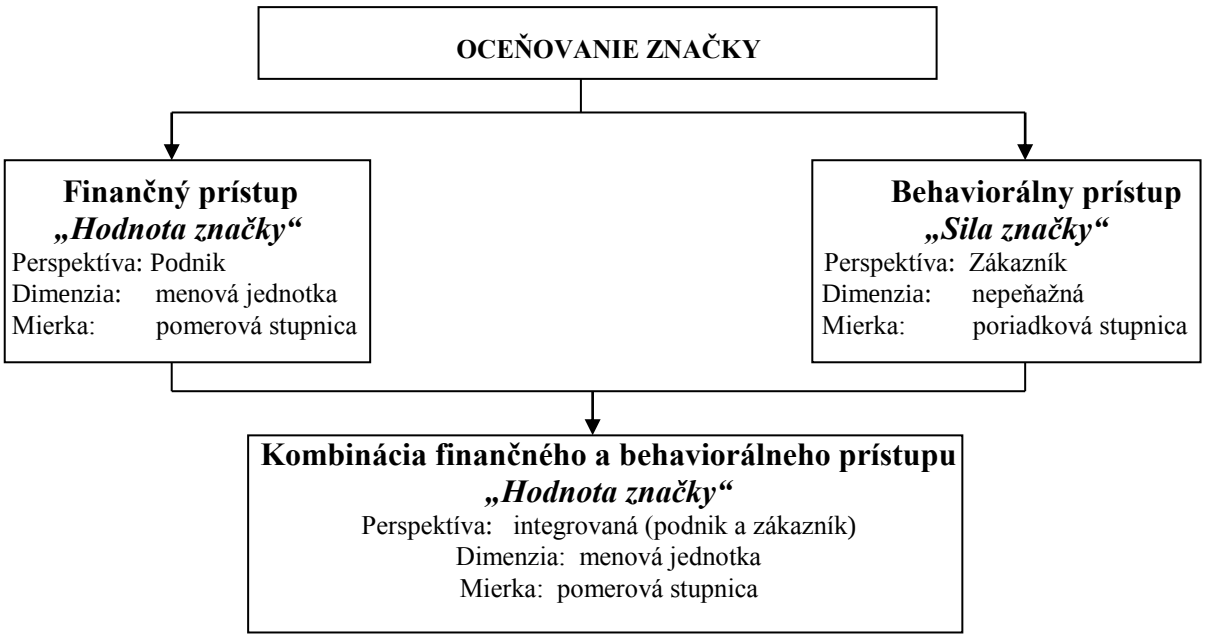
Predmetom tohto príspevku je pojem obchodná značka, čo je termín z oblastí marketingu a obchodu. Ekonomický slovník definuje Obchodnú značku ako návrh alebo slovo právne registrované, ako poznávacia značka výrobcovho alebo obchodníkovho výrobku.

Značka je nositeľom hodnoty a preto dobre zvolená značka môže byť ekonomicky ocenená viac než ostatne aktíva organizácie. Ak je značka dobre riadená, jej hodnota v priebehu času neklesá, ale naopak zvyšuje sa (Aaeker, 2003). Značka môže byť jediným predmetom záujmu o kúpu podniku a môže niekoľkonásobne zvýšiť jeho cenu. Hodnota značky má v rámci odborných zdrojov rôzne interpretácie alebo vymedzenia (Haigh, 2002). Z pohľadu marketingu hodnota značky predstavuje prísľub zážitku, ktorý ponúka značka svojou výnimočnosťou zákazníkovi. Z pohľadu podniku alebo podnikateľa znamená tento pojem záruku budúcich výnosov. Z pohľadu práva je hodnota značky v podobe hodnoty ochrannej známky súčasťou nehmotného majetku podniku (Kylianová, 2010).

Hlavnými kategóriami hodnoty značky sú znalosť mena značky, vernosť značke, vnímaná kvalita a asociácie spojené so značkou (Brand Finance, 2013).

Hodnotenie značky je proces hodnotenia značky, ktorý je reprezentovaný rozdielnymi metódami hodnotenia značky (Janošková, Križanová, 2017).

Obrázok 1: Pochopenie hodnoty značky



Obrázok 1. zobrazuje rozdielne perspektívy definovania hodnoty značky, ktoré sú kategorizované vo finančnom, behaviorálnom prístupe alebo ich kombináciou.

Behaviorálny prístup skúma reakciu na značku z hľadiska postojov zákazníkov, alebo z hľadiska nákupu. Zákazníci zvyčajne reagujú pozitívnejšie na silné značky, než na neznačkové produkty alebo slabšie značky.

Finančný prístup k hodnote značky definuje finančnú hodnotu značka pre podniky a investorov. Hodnota značky je v tomto prípade vymedzená ako dodatočný cash flow, ktorý prinášajú značkové produkty v porovnaní cash flow z predaja neznačkových produktov (Soto, 2008).

2.2 Metódy oceňovania značky

Metodika oceňovania značiek prešla zložitým vývojom, t. j. keď sa dokázal ekonomický význam značiek a následne ich zaradenie do nehmotného majetku, začali sa hľadať postupy čo najpresnejšieho vyjadrenie finančnej hodnoty značiek (Jacková, 2012). V podmienkach Slovenskej republiky, problematiku oceňovania značiek upravuje vyhláška č. 492/2004 Z. z, ktorá stanovuje oceňovanie dvoma spôsobmi:

- **metóda licenčnej analógie** - používa sa v prípade licencií, patentov, ochranných známok a iných obchodovateľných súčasti nehmotného majetku. Jej podstata spočíva v prevedení odhadu pravdepodobných budúcich tržieb a na základe stanovenia sadzby licenčného poplatku sa dá zistiť výška príjmov, ktoré by mohli byť vďaka týmto poplatkom získané (Mažík, 2005). Ročná všeobecná hodnota – HV sa vypočíta na základe vzťahu:

$$HV = RV.\ LP.\ KZ\ .\ KK\ .\ PM\ (€) \qquad (1)$$

RV – ročný rozsah výroby
LP – licenčný poplatok
KZ – koeficient zastarania/zhodnotenia
KK – koeficient miery kapitalizácie/diskontnej sadzby
PM – podiel nehmotného majetku na výrobe

V nasledujúcom texte príspevku budeme oceňovať konkrétnu značku metódou licenčnej analógie vrátane opisu jej základných ukazovateľov.

- **metóda kapitalizácie odčerpateľných zdrojov** – základom tejto metódy je stanovenie dĺžky obdobia v rokoch, počas ktorých sa nehmotný majetok bude používať (Kardoš, 2010).

Medzinárodné oceňovacie štandardy zaoberajúce sa problematikou oceňovania, uznávajú tri metódy a ich kategórie, ktorými je možné stanoviť hodnotu značky (Tabuľka 1).

Tabuľka 1: Metódy oceňovania značky

Metóda oceňovania značky	Kategórie metódy oceňovania značky
Nákladová metóda	- metóda nákladového prístupu - metóda ohodnotenia podľa údajov účtovnej evidencie
Porovnávacia (trhová) metóda	
Metóda príjmu	- metóda prírastku a výnosu - metóda predpokladanej straty alebo zisku - reziduálna výnosová metóda - metóda kapitalizácie odčerpateľných zdrojov - metóda licenčnej analógie

3. Metódy

Zhodnotenie značky začalo v 80. rokoch minulého storočia ako finančný nástroj na oddelenie hodnoty značiek od goodwillu na účtovné účely (Seddon, 2010, s. 9-57, Seddon, 2015, s. 146-161).

V tomto príspevku sú parametre vybrané na základe prehľadu literatúry (hodnota značky a metódy hodnotenia značky) od autorov Aaeker (2003), Haigh (2002), Janošková a Križanová (2017).. Informácie sa uskutočnenie ocenenia značky Sedita týkajúce sa kvantifikácie rozsahu výroby, licenčného poplatku, koeficientu zastaranie/zhodnotenia, koeficientu miery kapitalizácie a podielu nehmotného majetku na výrobe vychádzajú z interných podkladov spoločnosti Sedita a informácií publikovaných na oficiálnej webovej stránke http://www.finstat.sk/ poskytujúcej základné ekonomické hodnoty umožňujúce kvantifikáciu spomínaných faktorov.

4. Výsledky

Slovenská značka Sedita našla svoje uplatnenie nielen na slovenskom trhu, ale aj na trhoch krajín V4, slovinskom, nemeckom, rumunskom, francúzskom, švédskom apod., dokonca i na kanadskom trhu či trhoch Spojených Arabských Emirátov. Značka Sedita je pre zákazníka zárukou stabilne vysokej kvality výrobkov, ktorá je vyústením viac ako 50-ročnej tradície výroby.

Predmetom predkladaného príspevku je ocenenie značky Sedita metódou licenčnej analógie, ktorá je najviac používaná na Slovensku. Táto metóda je založená na princípe, že hodnota nehmotného majetku sa rovná cene, ktorá by na trhu bola s najväčšou pravdepodobnosťou zaplatená za súhlas s využívaním rovnakého alebo podobného riešenia, prípadne za jeho prevod. Za právo na jeho využívanie sa platí finančná náhrada, zvyčajne formou licenčných poplatkov. Vo väčšine prípadov sa výška licenčných poplatkov stanovuje v percentách z čistej predajnej ceny.

Celkové ohodnotenie daného nehmotného majetku získame sčítaním jednotlivých ročných ohodnotení za obdobie, ktoré odpovedá predpokladanej dobe užívania nehmotného majetku.

Z uvedeného vyplýva, že budeme vychádzať zo vzťahu (1) ročnej všeobecnej hodnoty. Pre jeho lepšie pochopenie postupne kvantifikujeme aj faktory, ktoré obsahuje.

Ročný rozsah výroby – RV – sú to tržby spoločnosti, ktoré dosahuje, resp. dosahovať bude v budúcnosti. Neuvažuje sa s čistým ziskom ani cash flow, pretože licenčné poplatky za používanie ochrannej známky závisia práve od tržieb spoločnosti (Haigh, 2002). Prognóza tržieb je dôležitým faktorom, ktorý má veľký vplyv na cenu oceňovaného nehmotného majetku. Je tiež potrebné uviesť uvádza vzťah, podľa ktorého môžeme vypočítať tempo prírastku –TP.

$$TP\ (\%) = (y_1 - y_0)/y_0 \,.\, 100 \qquad (2)$$

y_1 – hodnota ukazovateľa v súčasnosti
y_2 – hodnota ukazovateľa v minulosti

Tržby spoločnosti I. D. C. Holding, a. s., pod ktorú patrí aj značka Sedita. za obdobie 2010 – 2015 ako aj vypočítaný medziročný nárast tržieb sú uvedené v Tabuľke 2.

Tabuľka 2: Medziročný rast značky Sedita

Rok	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Príjmy (€)	5 028 811	4 779 309	4 809 934	5 183 416	5 730 748	6 020 341
Rast (%)	-	- 4,96	+ 0,64	+ 7,76	+ 10,56	+ 5,05

Na základe dostupných informácií je možné odhadnúť budúce tempo rastu. Z údajov v Tabuľke 2 vyplýva, že do roku 2015 bolo tempo rastu tržieb nesúmerné, ale malo tendenciu rásť. Predpokladáme, že od roku 2015 bude tempo rastu tržieb na úrovni 2 % a v ďalších rokoch sa bude zvyšovať o 0,5 %. Doba oceňovania 10 rokov.

Tabuľka 3: Odhadovaný rozsah výroby 2015 – 2025

Rok	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**
Príjmy (€)	6 020 341	6 140 748	6 294 267	6 483 095	6 710 003	6 987 403	7 292 431	7 657 053	8 078 191	8 526 882	9 119 469

Licenčný poplatok – LP – je to poplatok, ktorý musí zaplatiť používateľ licencie za jej užívanie. Výška licenčného poplatku pri ochranných známkach je stanovená len rámcovo. Platí jednoduché pravidlo: čím je známka známejšia, tým vyšší je licenčný poplatok (Kylianová, 2010). Výrobky pod značkou Sedita majú na slovenskom trhu 50-ročnú tradíciu. Stabilné postavenie na trhu je odrazom používania vlastných receptúr, dodržiavania technologických postupov výroby ako aj výstupnej kontroly. Ochrannú známku spoločnosti preto možno považovať za priemernú. Na výšku licenčného poplatku má veľký vplyv odvetvie, z ktorého budú plynúť tržby z licencie. Nosnou podnikateľskou činnosťou spoločnosti je výroba trvanlivého pečiva. Pre výrobu potravín je priemerná hodnota licenčného poplatku 2,8 %. Ak výrobu trvanlivého pečiva zaradíme do spotrebného priemyslu, potom sa odporúča používať licenčný poplatok v intervale 1-5 %. Licenčný poplatok stanovujeme na **1,2 %.**

Koeficient zastarania, prípadne zhodnotenia – KZ – medzi hodnotou a dĺžkou používania nehmotného majetku existuje vzťah, ktorý závisí od typu nehmotného majetku. Hodnota patentov, dizajnu a know-how používaním klesá (Soto, 2008). Dôvodom je najmä technický pokrok. V tomto prípade sa pri oceňovaní používa **koeficient zastarania**, ktorý postupne znižuje stanovenú hodnotu v rozmedzí niekoľkých percent až desiatok percent ročne. Hodnota ochranných známok sa časom zvyšuje. V tomto prípade sa pri oceňovaní používa **koeficient zhodnotenia**, ktorý postupne zvyšuje hodnotu označenia ročným prídavkom v rozmedzí 1-5 %.

Spoločnosť I. D. C. Holding, a. s. každoročne investuje do výskumu a vývoja nových výrobkov, ktorými pravidelne obohacuje sortiment ponúkaných výrobkov. Aj napriek existencii negatívny externých faktorov sa spoločnosť dlhodobo udržala v konkurenčnom boji s oveľa väčšími spoločnosťami. Preto volím koeficient zhodnotenia na úrovni **1 %**.

Koeficient miery kapitalizácie (prípadne koeficient diskontnej sadzby) - KK – slúži na prepočítanie vypočítanej výšky licenčného poplatku na čistú súčasnú hodnotu. V teórii financií sa tento koeficient používa pod názvom odúročiteľ. Mal by obsahovať základnú bezrizikovú sadzbu danej krajiny a riziko vyplývajúce zo straty likvidity či nedostatočného odbytu výrobkov (Mažík, 2005). Jeho hodnota sa v súčasnosti pohybuje v hraniciach 8-40 %.

Koeficient miery kapitalizácie (diskontnej miery) sa vypočíta zo vzťahu:

$$KK = 1/(1+d)^t \qquad (3)$$

d – sadzba miery kapitalizácie (%)
t – jednotlivé po sebe idúce obdobia, v ktorých sa zisťuje aktualizovaná hodnota

Výsledná diskontná miera, označovaná ako Yn, je kvantifikovaná ako súčet bezrizikovej diskontnej miery a rizikovej prirážky podľa nasledovného vzťahu:

$$Yn = fr + RP\ (\%) \qquad (4)$$

rf – bezriziková úroková miera
RP – riziková prirážka

$$RP = rf.(n\text{-}1)\ .\ (100\text{-}ßß)/100 \qquad (5)$$

(n-1) – koeficient rizikovej prirážky
ßß – počet bodov získaných pri ßrandßeta™

V prípade značky Sedita, je potrebné najskôr stanoviť ßß ako súčet bodov jednotlivých ukazovateľov nadobúdajúcich hodnotu od 1 do 10, t. j, maximum 100 bodov. V prípade, že podnik získa 0 bodov, pri ocenení bude použitá maximálna diskontná sadza, ak získa 50 bodov, priemerná diskontná sadzba, a Ak 100 bodov, bude použitá bezriziková diskontná sadzba.

Jednotlivé ukazovatele sú obodované nasledovne: market time (7), distribúcia (7), podiel na trhu (4), pozícia na trhu (6), miera rastu predaja (5), cenová prirážka (4), elasticita ceny (5), výdaje na marketing (6), povedomie o značke (8); spolu tieto ukazovatele tvoria hodnotu 52.

Na určenie bezrizikovej diskontnej miery - rf – použijem bude použitý desaťročný štátny dlhopis číslo ŠD 223 J, SK4120008871, vydaný 21. 10. 2015, splatný 15. 11 2024, ktorého úrokový výnos je 3,375 %.

Po dosadení príslušných veličín do vzťahu (5) dostávame rizikovú prirážku vo výške 14,6 %. Výsledná diskontná sadzba, t. j. miera kapitalizácie sa kvantifikuje zo vzťahu (4) a dostaneme hodnotu 17, 955 %.

Koeficienty miery kapitalizácie (diskontnej sadzby) pre roky 2015-2024 stanovíme postupným dosadzovaním do vzťahu, pričom za mieru kapitalizácie (diskontnú sadzbu) – d - dosadzujeme vypočítanú hodnotu 17,995 % a za ročné obdobia – t – postupne čísla od 1 po 10, pretože doba oceňovania je 10 rokov.

Koeficienty miery kapitalizácie (diskontnej sadzby) pre roky 2015 – 2024 sú uvedené v Tabuľke 4.

Tabuľka 4: Koeficienty sadzieb kapitalizácie za roky 2015 – 2024

Rok	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**
KK	0,847	0,718	0,609	0,516	0,437	0,371	0,314	0,266	0,226	0,191

Podiel nehmotného majetku na výrobe – PM – tento koeficient vyjadruje ako sa oceňovaný nehmotný majetok podieľa na hospodárskom výsledku, ktorý je u tejto metódy vyjadrený objemom výroby.

Stanovenie podielu nehmotného majetku na výrobe – PM pre značku Sedita

V prípade trvanlivého pečiva je ochranná známka kľúčovou, preto volíme podiel nehmotného majetku na výrobe **90 %**.

Výpočet hodnoty značky Sedita

Na základe aplikácie všetkých základných ukazovateľov hodnôt, ktoré vplývajú na hodnotu značky, bola vyčíslená hodnota značky Sedita vo výške ***351 204 €.***

Tabuľka 5: Výpočet hodnoty ochrannej známky pomocou metódou licenčnej analógie

Rok	RV (€)	LP 1,2 %	KZ 1 %	KK 17, 995 %	PM 90 %	HV (€)
2015	6 140 748	0,012	1,01	0,847	0,90	56 735
2016	6 294 267	0,012	1,02	0,718	0,90	49 784
2017	6 483 095	0,012	1,03	0,609	0,90	43 920
2018	6 710 003	0,012	1,04	0,516	0,90	38 889
2019	6 978 403	0,012	1,05	0,437	0,90	34 582
2020	7 292 431	0,012	1,06	0,371	0,90	30 972
2021	7 657 053	0,012	1,07	0,314	0,90	27 784
2022	8 078 191	0,012	1,08	0,266	0,90	25 064
2023	8 562 882	0,012	1,09	0,226	0,90	22 781
2024	9 119 469	0,012	1,10	0,191	0,90	20 693
SPOLU						**351 204**

Hodnota známky po 10 rokoch vypočítanej **metódou licenčnej analógie** sa rovná súčtu vypočítaných hodnôt za jednotlivé roky, t. j. **351 204 €.**

5. Diskusia

Z Tabuľky 5. výpočtu ocenenia značky Sedita je možné konštatovať, ž hodnota značky bola ocenená vo výške 351 204 €. Toto hodnotenie môžeme hodnotiť veľmi pozitívne a to nie iba z hľadiska dosiahnutého výsledku, ale aj z hľadiska potvrdenia dôležitosti vnútropodnikového hodnotenia značky. Spoločnosti ako Sedita častokrát nevnímajú skutočnosť, že nie iba ekonomické ukazovatele, ktoré sú často premenlivého charakteru sa podieľajú na hodnote značky, ale ako vidíme na príklade ocenenia značky Sedita, aj samotné historické pozadie spoločnosti s 50-ročnou históriou pridáva na jej hodnote. Preto môžeme zhodnotiť toto ocenenie ako hodnotné z hľadiska využitia i opodstatnenia oceňovania značky.

Význam, ktorý značka Sedita prináša spoločnosti, poskytuje množstvo cenných funkcií. Slúži nielen k identifikácii produktu pre spotrebiteľov, ale aj pre spoločnosť tvorí nehmotný majetok a dobrú povesť pridávajúcu hodnotu pre podnik, pretože hmotné aktíva môžu tvoriť okolo 10 % celkovej hodnoty, však až 70 % ich nehmotného majetku môžu tvoriť značky.

Metóda licenčnej analógie má svoje prednosti, ale aj nedostatky. Na jednej strane táto metóda umožňuje zohľadniť niektoré riziká budúceho vývoja priamo do miery kapitalizácie. Riziko vyjadrené touto hodnotou odpovedá rizikovosti potravinárskeho odvetvia ako aj pozícii značky podniku na trhu, ktorá je síce stabilná, ale konkurencia najmä zahraničných výrobkov je veľmi veľká. Túto skutočnosť potvrdzujú aj výsledky štúdie, ktoré zverejnila Potravinárska komora Slovenska. Štúdia mapovala tovar v slovenských obchodoch za posledné štyri roky a výsledky sú alarmujúce. Podiel slovenských potravín na pultoch tvorí len 40 % všetkých tovarov. Vzhľadom na všetky uvedené skutočnosti, sme toho názoru, že pri oceňovaní značky podniku by bolo vhodné použiť vždy aspoň dve, prípadne i viac metód oceňovania, s tým, že výsledná hodnota by nebola automaticky aritmetickým priemerom vypočítaných hodnôt. V závislosti od kvality vstupných informácií, situácie daného podniku a charakteru jednotlivých metód, by bolo vhodné určiť jednotlivým metódam váhu v rámci výsledného ocenenia. Tiež si myslíme, že by bolo veľkým prínosom pre túto oblasť, keby sa

zjednotil spôsob označovania jednotlivých veličín súvisiacich s výpočtom. Značne by to zrýchlilo aj sprehľadnilo celú oblasť oceňovania.

6. Záver

Účelom tohto príspevku bolo určiť hodnotu značky Sedita pri zohľadnení finančných a marketingových premenných. Hlavným prístupom príspevku bolo spojiť rôzne hľadiská finančných a marketingových odborníkov pri posudzovaní špecifickej hodnoty značky. Metóda licenčnej analógie sa môže použiť na meranie rôznych faktorov simulujúcich dopyt na daných trhoch. Metóda sa dá využiť pri odhadovaní dopytu a zároveň při plánovaní rozmiestnenia zdrojov na podporu faktorov k sledovaniu účinku rozmiestnenia zdrojov, ktoré ovplyvňujú dopyt po značke. Hodnotu značky Sedita je možné využiť aj na budovanie hodnoty samotnej spoločnosti v oblastiach vykazovania v podnikovej rozvahe, internej komunikácii, prideľovania rozpočtových zdrojov v oblasti marketingu, vnútorného podnikového riadenia, plánovania fúzií a akvizícií, uzatvárania licenčných a franšízingových dohôd, podkladov pre súdne spory alebo daňového plánovania.

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Comparative analysis of traditional and alternative financing of SMEs in Slovakia

Hussam Musa, Zuzana Stroková, Zdenka Musová [1]

Abstract

Access to finance is a key precondition for the development of small and medium sized enterprises (SMEs) in Slovakia as well as in the other EU countries. Bank lending is the most common source of external finance for many SMEs, which are often heavily dependent on traditional financing to fulfil their start-up, cash flow and investment needs. On the other side, alternative financing is available for newer, innovative and fast growing companies whose owners can´t get traditional financial resources as bank loans or overdrafts. Access to alternative financing together with traditional financial resources represents one of the most significant challenges for new SMEs not only in the Slovak Republic but all over the world. The aim of this paper is to characterize traditional and alternative financial resources of SMEs and to present the selected results of own questionnaire survey focused on financing of SMEs in Slovakia.

Key words

Traditional financing, Alternative financing, Bank loan, Small and medium sized enterprises.

JEL Classification: G20, G24, G30, M13

1. Teoretické východiská skúmanej problematiky

Finančné riadenie je hlavnou zložkou komplexného podnikového riadenia a v existencii a fungovaní podnikov zohráva jednu z najdôležitejších úloh (Cisko and Klieštik (2009), Hajdúchová, et al. (2011); Sedliačiková, Šatanová and Foltínová (2012); Kalouda, (2015); Banerjee, (2015)). Financovanie podnikov definujú Vlachynský, et al. (2009) a Polednáková, et al. (2011) ako proces získavania kapitálu, s ktorým podnik hospodári a vykonáva svoju podnikateľskú činnosť, pričom veľkosť a štruktúra tohto kapitálu je daná finančnými zdrojmi, z ktorých sa kapitál sústreďuje. Má svoje špecifiká aj z hľadiska veľkostných kategórií podnikov.

Malé a stredné podniky (ďalej len "MSP") vzhľadom na svoje charakteristické črty majú rozdielne potreby v oblasti financií v porovnaní s veľkými podnikmi. Nedostatok finančných zdrojov a obmedzený prístup k cudzím finančným zdrojom (najmä bankovým úverom) sú zároveň považované za najvýznamnejšie prekážky v rozvoji podnikania MSP. Podľa viacerých autorov (Musa, et al. (2015); Fetisovová, Nagy and Vlachynský (2014); Belanová

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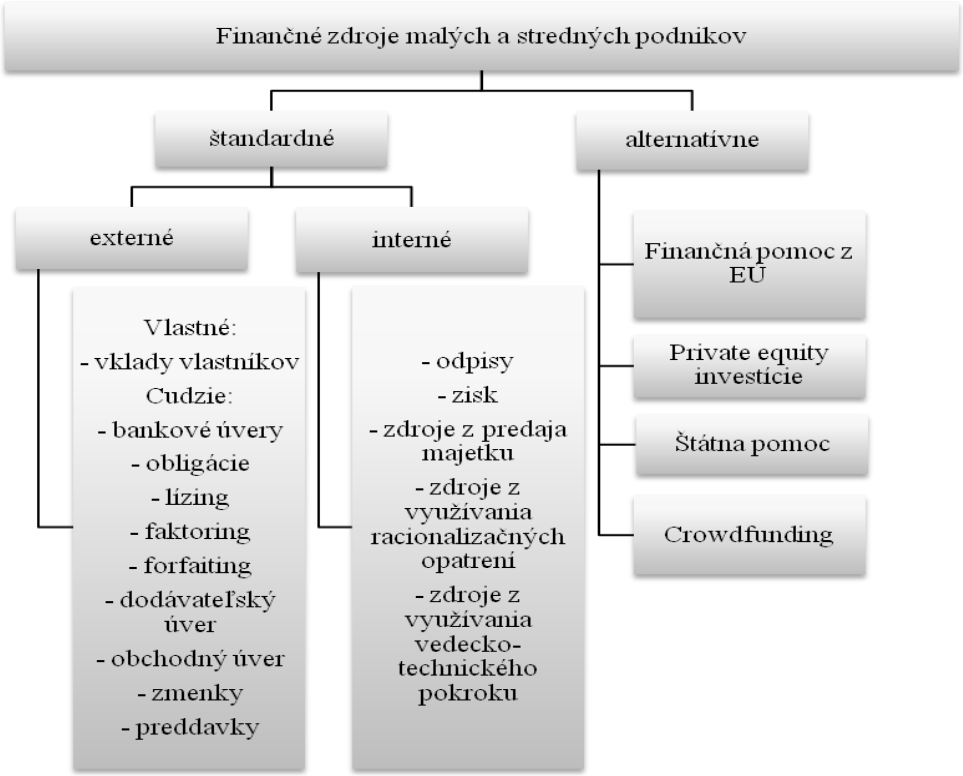
(2013); Sobeková Majková (2011)) patrí obmedzený prístup k finančným, resp. úverovým zdrojom k štandardným problémom MSP. Je vo veľkej miere zapríčinený najmä vysokou rizikovosťou MSP.

Finančné zdroje podnikov sa členia z hľadiska doterajšej tradície ich využívania na štandardné a alternatívne. Zo samotného názvu vyplýva, že štandardné finančné zdroje sú také, ktoré sa už určitú dobu využívajú a podniky ich poznajú (bankové úvery, lízing, faktoring, a forfaiting). Na druhej strane alternatívne finančné zdroje predstavujú nové – netradičné možnosti financovania ako napr. financie zo štrukturálnych fondov, pomoc zo strany štátu alebo rizikový kapitál (Sobeková, Majková, 2011). Ich využívanie vo väčšine prípadov predpokladá určité poznanie týchto zdrojov podnikateľmi. Rovnako možno konštatovať, že alternatívne finančné zdroje využívajú takí podnikatelia, ktorým na rozvoj a inovácie svojho podnikania štandardné zdroje nepostačujú, resp. pre vysoké riziko by ich nezískali. Nasledujúci obrázok 1 prezentuje klasifikáciu finančných zdrojov MSP.

Obrázok 1: Klasifikácia finančných zdrojov malých a stredných podnikov



Podľa Európskej komisie (2013) dopyt po rôznych druhoch financovania môže byť odlišný v závislosti od odvetvia, rastového profilu firiem a krajiny. MSP v Európskej únii nevnímajú financie ako hlavný limitujúci faktor ich rastu, hoci väčšina z týchto podnikov uvádza, že vyžadujú financovanie z externých zdrojov pre svoje prežitie a rast. Nedostatočný prístup k primeraným finančným zdrojom sa stáva závažnou prekážkou rozvoja malých a stredných podnikov, rovnako ako pre menšie podniky vyššie riziko úpadku. Investori sú opatrní v poskytovaní kapitálu malým firmám. Ak sa investori rozhodnú poskytnúť kapitál, podmienky sú väčšinou pre malé podniky nepriaznivé.

Iný pohľad na členenie finančných zdrojov MSP prináša Tekelová (2012), ktorá rozlišuje tieto zdroje podľa vývojovej fázy podnikania (Obrázok 2).

Obrázok 2: Formy financovania MSP podľa vývojovej fázy podnikania

2. Materiál a metodika skúmania

V snahe naplniť cieľ príspevku sme pri našom skúmaní využívali primárne a sekundárne zdroje informácií. Sekundárne zdroje informácií tvorila odborná a vedecká literatúra, zborníky vedeckých prác a periodiká z oblasti finančného manažmentu, financovania podnikov a finančnej štruktúry podnikov. Primárne dáta sme získali z vlastného dotazníkového prieskumu, ktorého čiastkovým cieľom bolo aj zhodnotenie využívania tradičných a alternatívnych foriem financovania MSP na Slovensku v období rokov 2008 až 2015. Pri spracovaní sme využili logicko-vedecké metódy, ako analýzu, syntézu, indukciu a dedukciu.

Subjektom skúmania boli malé a stredné podniky pôsobiace na Slovensku (základný súbor). Výberový súbor sme určili na základe náhodného výberu, pričom pri tvorbe databázy kontaktov sme využili členenie podnikov podľa troch pomocných znakov (veľkosť podniku – Smernica Európskej komisie č. 2003/361/EC, odvetvovej klasifikácie ekonomickej činnosti – SK NACE a regionálneho umiestnenia podnikateľskej jednotky) za účelom následného zisťovania reprezentatívnosti výberového súboru.

Dotazník tvorilo viacero druhov otázok - väčšinu otázok v dotazníku tvorili uzavreté otázky, polootvorené otázky s možnosťou výberu jednej alebo viacerých odpovedí a škálové otázky. V úvode dotazníka boli zaradené povinné identifikačné otázky, nasledovalo niekoľko tematicky zameraných sekcií. S ohľadom na zameranie nášho príspevku budeme prezentovať len vybrané výsledky.

Zo šesťtisíc oslovených podnikov sa vrátilo 214 správne a kompletne vyplnených dotazníkov. Štruktúru podnikov, ktoré sa zapojili do prieskumu, z hľadiska sídla ich podnikania (geografická štruktúra podľa krajov) zobrazuje nasledujúca tabuľka 1.

Tabuľka 1: Zastúpenie podnikov vo výskume podľa umiestnenia

Kraj	Výberový súbor		Základný súbor	
	Počet podnikov	Percentuálne zastúpenie	Počet podnikov	Percentuálne zastúpenie
Banskobystrický kraj	26	12,15 %	17 056	8,69 %
Bratislavský kraj	56	26,17 %	67 872	34,56 %
Košický kraj	20	9,35 %	20 032	10,20 %
Nitriansky kraj	21	9,81 %	21 486	10,94 %
Prešovský kraj	19	8,88 %	17 873	9,10 %
Trenčiansky kraj	19	8,88 %	15 258	7,77 %
Trnavský kraj	29	13,55 %	17 776	9,05 %
Žilinský kraj	24	11,21 %	19 011	9,68 %
Spolu	**214**	**[redacted]**	**196 364**	**100,00 %**

Pre overenie reprezentatívnosti výberového súboru podľa geografického hľadiska sme použili chí-kvadrát test dobrej zhody. Nulová hypotéza (H_0) predpokladala, že výberový súbor je z hľadiska geografického umiestnenia podnikov reprezentatívny. Na základe výsledkov testu dobrej zhody (hladina významnosti $\alpha = 0,05$; p-hodnota vo výške 0,064) nezamietame hypotézu H_0. Mohli sme prijať predpoklad, že pozorované a teoretické početnosti sa zhodujú a výberový súbor je reprezentatívny vzhľadom na umiestnenie podnikateľskej jednotky. Rozdelenie podnikov vo výberovom súbore sa zhoduje so zastúpením podnikov v základnom súbore.

Najviac dotazníkov sa nám podarilo získať z kategórie malých (94, t. j. 43,92 %) a mikro podnikov (82, t. j. 38,32 %). Najmenej dotazníkov sme získali z kategórie stredných podnikov (38, t. j. 17,76 %).

Z hľadiska odvetvovej štruktúry mali vo výberovom súbore najpočetnejšie zastúpenie priemyselné a stavebné podniky, ktoré boli zastúpené rovnakým podielom (25,70 %). Druhú najpočetnejšiu skupinu tvorili podniky v oblasti veľkoobchodu, maloobchodu, opráv motorových vozidiel a motocyklov (17,29 %). Treťou skupinou s najväčším zastúpením v súbore boli podniky v oblasti poľnohospodárstva, lesníctva a rybolovu (12,62 %). Pre overenie reprezentatívnosti výberového súboru podľa týchto kritérií sme taktiež použili chí-kvadrát test dobrej zhody, avšak na základe výsledkov sme reprezentatívnosť výberového súboru nepotvrdili.

3. Výsledky prieskumu

V dotazníkovom prieskume sme sa zamerali na využívanie rôznych foriem financovania MSP na Slovensku v období rokov 2008 až 2015. Z výsledkov dotazníkového prieskumu vyplynulo, že viac ako polovica (57,0 %) MSP zapojených do prieskumu využívala okrem vlastných finančných zdrojov aj cudzie finančné zdroje. Ak sa osobitne pozrieme na mikro podniky a osobitne na malé a stredné podniky, zistíme, že 63,71 % mikro podnikov zapojených do prieskumu využívalo cudzie zdroje financovania, zatiaľ čo v kategórii malých a stredných podnikov (s počtom zamestnancov od 10 – 249) využívalo cudzie zdroje financovania až 73,45 % podnikov.

Pri podrobnejšom skúmaní využívania tradičných cudzích foriem financovania (bankový úver, lízing, faktoring, forfaiting, atď.) sme zistili, že najintenzívnejšie boli našimi respondentmi (214 mikro, malých a stredných podnikov) využívané bankové úvery, kontokorentné úvery a lízing. Podrobnejšie výsledky uvádzame v tabuľke 2 (respondenti mali pomocou 5-stupňovej hodnotiacej škály určiť intenzitu využívania jednotlivých cudzích finančných zdrojov, pričom jednotlivým stupňom boli pridelené váhy: 1 = žiadna intenzita, 2 = slabá intenzita, 3 = stredne silná intenzita, 4 = vysoká intenzita, 5 = veľmi vysoká intenzita).

Tabuľka 2: Využívanie cudzích finančných zdrojov MSP

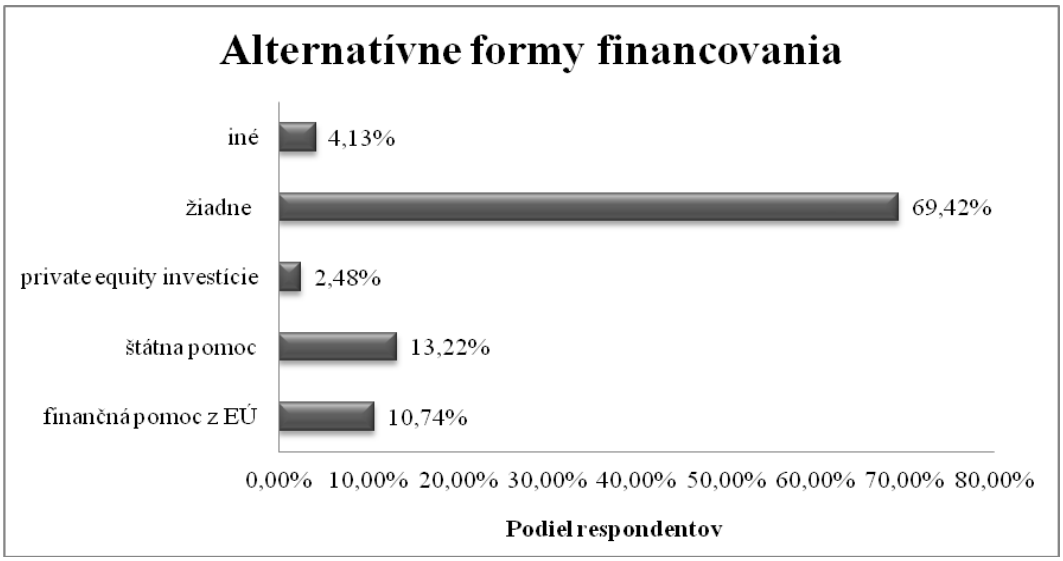
Finančný zdroj / Hodnotiaca škála	**1**	**2**	**3**	**4**	**5**	**Vážený priemer***
Povolené prečerpanie bankového účtu	43,69%	15,53%	18,45%	12,62%	9,71%	2,21
Bankový úver	25,00%	17,31%	36,54%	9,62%	11,54%	**2,65**
Ostatné pôžičky (od rodiny, známych)	63,37%	15,84%	10,89%	8,91%	0,99%	1,68
Lízing	36,89%	21,36%	31,07%	5,83%	4,85%	2,20
Faktoring	91,92%	4,04%	3,03%	1,01%	0,00%	1,13
Forfaiting	98,00%	2,00%	0,00%	0,00%	0,00%	1,02
Dodávateľský úver	75,76%	5,05%	13,13%	2,02%	4,04%	1,54
Obchodný úver	89,00%	3,00%	3,00%	4,00%	1,00%	1,25
Zmenka	95,00%	2,00%	2,00%	1,00%	0,00%	1,09
Preddavky od odberateľov	71,00%	18,00%	10,00%	1,00%	0,00%	1,41
Iné	98,00%	0,00%	2,00%	0,00%	0,00%	1,04

**Pri výpočte váženého aritmetického priemeru sme použili váhy a hodnoty sme následne vynásobili percentuálnym podielom odpovedí pri konkrétnom finančnom zdroji pri konkrétnej hladine významnosti*

Skúmali sme tiež závislosť medzi intenzitou využívania cudzích finančných zdrojov a veľkosťou podnikov. Náš predpoklad sme overovali pomocou testu nezávislosti dvoch kvalitatívnych znakov. Na základe výsledkov χ^2 testu nezávislosti (p hodnota 0,016 je menšia ako nami zvolená hladina významnosti $\alpha = 0,05$) sme mohli prijať predpoklad, že existuje významný rozdiel medzi intenzitou využívania bankového úveru a veľkosťou podnikov, t. j. mikro podnikmi, malými a strednými podnikmi. Vzhľadom na zamietnutie nulovej hypotézy H_0 sme použili niektorú z mier, charakterizujúcich intenzitu závislosti medzi dvojicou skúmaných kvalitatívnych znakov. Z výstupu programu SPSS vyplynulo, že hodnota Pearsonovho kontingenčného koeficientu je 0,426 a hodnota Cramerovho kontingenčného koeficientu je 0,301. Môžeme konštatovať, že medzi intenzitou využívania bankového úveru a veľkosťou podnikov je stredne silná závislosť. V prípade intenzity využívania ďalších cudzích finančných zdrojov a veľkosťou podnikov sa nepotvrdil predpoklad o závislosti medzi sledovanými kvalitatívnymi znakmi.

V ďalšej časti sme zisťovali, ktoré alternatívne finančné zdroje využívali MSP zapojené do prieskumu. Prehľad využívania alternatívnych finančných zdrojov MSP prezentuje nasledujúci obrázok 3.

Obrázok 3: Využívanie alternatívnych finančných zdrojov MSP



Najviac MSP (69,42 %) sa vyjadrilo, že v sledovanom období nevyužívali žiadne alternatívne finančné zdroje. Z tých podnikov, ktoré alternatívne formy financovania využívali

(65), uvádzali najmä štátnu pomoc (pomoc z ministerstiev, SZRB, Eximbanka...) a finančnú pomoc z fondov Európskej únie (štrukturálne fondy, kohézny fond...). Iba tri podniky využívali private equity investície (venture kapitál...). Päť podnikov využívalo a považovalo za alternatívne zdroje financovania aj lízing, okrem toho koncernové zdroje, pôžičky vlastníkov a pôžičky od spriaznených podnikov (konkretizovali to v možnosti „iné“). Žiadny z oslovených podnikov nevyužil crowdfunding, čo pripisujeme najmä nedostatku povedomia a informovanosti o tejto inovatívnej forme financovania.

Zaujímavým bolo tiež identifikovanie faktorov, ktoré ovplyvňujú malé a stredné podniky pri výbere cudzích finančných zdrojov. Najvýznamnejším faktorom podľa respondentov v našej vzorke boli náklady potrebné na získanie cudzích finančných zdrojov – za významný ho považovala viac ako polovica opytovaných podnikov (52,6 %). Druhým najvýznamnejším faktorom boli skúsenosti s predošlým využívaním daného finančného zdroja (45,6 % podnikov), nasledoval čas potrebný na získanie finančných zdrojov (43 % respondentov). Medzi významné faktory ovplyvňujúce rozhodovanie MSP pri výbere cudzích finančných zdrojov patrili ďalej administratívna náročnosť, prísnosť podmienok potrebných pre získanie daného finančného zdroja, či nedostatok informácií o nových finančných zdrojoch.

Čo sa týka skúmania využívania alternatívnych foriem financovania MSP, zaujímavé boli výsledky, ktoré sa týkali finančnej pomoci z fondov EÚ. Približne 23,07 % (15) MSP sa v našom prieskume vyjadrilo, že v sledovanom období využilo možnosť získať finančné zdroje z fondov Európskej únie. Naopak, 76,93 % MSP túto možnosť nevyužilo. Financovanie z fondov EÚ považujeme za príležitostnú formu financovania, ktorá je poskytnutá za predpokladu splnenia konkrétnych podmienok. Väčšina týchto MSP (až 80 %) získala finančnú pomoc zo štrukturálnych fondov EÚ. Najviac MSP (viac ako 33 %) získalo finančnú pomoc z fondov EÚ v rokoch 2014 – 2015, necelých 30 % v období rokov 2008 – 2009 a približne 25 % MSP v období rokov 2012 – 2013.

Pri finančnej pomoci z EÚ sme sa taktiež zaoberali ďalšími parametrami ako časová náročnosť jej získania a výška získanej pomoci. V tabuľke 3 prezentujeme súhrn našich zistení. Na základe uvedeného možno konštatovať, že čím vyššiu sumu finančnej pomoci z fondov EÚ podnik požaduje, tým dlhší je schvaľovací proces.

Tabuľka 3: Časová náročnosť získavania finančných zdrojov zo štrukturálnych fondov EÚ a ich výška

Výška pomoci z EÚ / Doba získania	**1 – 3 mesiace**	**4 – 6 mesiacov**	**7 – 9 mesiacov**	**10 – 12 mesiacov**
do 10 000 €	2	0	0	0
10 000 - 49 999 €	5	1	0	2
50 000 - 99 999 €	0	1	0	0
100 000 - 499 999 €	1	3	0	5
500 000 - 999 999 €	0	1	1	1
viac ako 1 000 000 €	0	1	1	0
Spolu	**8**	**7**	**2**	**8**

Zaujímal nás aj účel použitia finančných prostriedkov získaných z fondov EÚ. Najviac podnikov (41,3 %) využilo finančnú pomoc z fondov EÚ na obstaranie strojov a zariadení. Percentuálnu zhodu (27,6 %) sme zaznamenali pri MSP, ktoré využili finančné prostriedky z fondov EÚ rovnako prioritne na udržiavanie a podporovanie zamestnanosti a na „iné“ účely (napr. investície na ochranu životného prostredia, oprava lesných ciest, modernizácia a rekonštrukcia výrobných budov a pod.).

Pri zisťovaní dôvodov nevyužívania finančnej pomoci z fondov EÚ podniky najčastejšie uvádzali administratívnu náročnosť (34,4 % opýtaných). Pre 30,4 % respondentov boli

dôvodom nevyužívania tejto formy prísne podmienky pre jej získanie. Ako ďalšie dôvody podniky uvádzali nezáujem (16 %) a nedostatok informácií (11,2 %). V kategórii „iné dôvody“ (8,0 %) sa objavila korupcia, klientelizmus, súvisiace vysoké finančné náklady.

Pri otázke ohľadom potenciálnej možnosti využitie služieb poradenskej spoločnosti pri získavaní finančných prostriedkov z EÚ sa až 78 % respondentov vyjadrilo v jej prospech, zvyšných 22 % by takéto služby nevyužili. V minulosti bola poradenská spoločnosť nápomocná pri získavaní finančných prostriedkov z EÚ len v jednom podniku (z našich respondentov) – išlo o malý podnik v oblasti poľnohospodárstva, lesníctva a rybolovu a využil takéto služby opakovane. Z hľadiska kritérií pri výbere poradenskej spoločnosti by podniky v našej výskumnej vzorke brali do úvahy referencie a skúsenosti (približne 40 % opýtaných), cenu za poskytované služby (30 % MSP) a odporúčania od známych, príp. obchodných partnerov (26,4 %). Menší význam pripisovali podniky informáciám na internetovej stránke poradenskej spoločnosti či jej veľkosti.

Keďže (až na jeden malý podnik) naši respondenti doteraz služby poradenských spoločností pri získavaní finančných prostriedkov z EÚ nevyužívali, zaujímali sme sa aj o dôvody takéhoto správania. V odpovediach najčastejšie rezonovala obava z úniku citlivých firemných informácií (36,8 %) a špecifiká odvetvia, ktoré neumožňujú využívať postupy navrhované poradcami (32,4 %). Približne 21 % MSP uviedlo, že takýto druh poradenstva nepotrebujú. Niektoré podniky ako dôvod uviedli aj korupčné správanie a korupčné vybavovanie služieb, ako aj vysoké ceny.

Náš výskum potvrdil, že v malých a stredných podnikoch dominujú z cudzích finančných zdrojov bankové úvery. Oveľa menší záujem prejavujú tieto podniky o alternatívne formy financovania. Časť podnikov využila ponúkanú finančnú pomoc predovšetkým zo štrukturálnych fondov EÚ, avšak považujú ju len za špecifický a príležitostný zdroj financovania. Odstránenie bariér, ktoré podniky v tomto prípade financovania najčastejšie vnímajú, by mohli priniesť aj nové komunitárne programy Horizont 2020 a COSME. Veľmi zaujímavou a perspektívnou formou aj v našich podmienkach by mohol byť crowdfunding, ktorý je vo vyspelejších krajinách využívaný na financovanie rôznych projektov, výskumu a vývoja, charitatívnych a filantropických iniciatív.

V oblasti zlepšenia prístupu MSP k adekvátnym finančným zdrojom je nevyhnutné predovšetkým zvýšiť povedomie a informovanosť MSP o možnostiach získavania finančných zdrojov (napríklad prostredníctvom regionálnych a informačných centier, seminárov SBA a pod.), ako aj o možnostiach poskytovania záruk (napr. prostredníctvom Slovenskej záručnej a rozvojovej banky, či Eximbanky).

4. Záver

Malé a stredné podniky, ktoré tvoria viac ako 99 % všetkých podnikateľských subjektov na Slovensku, sú nezastupiteľné nielen z hľadiska tvorby pracovných miest, ale aj z hľadiska ich podielu na tvorbe HDP. Najmä vo svojich začiatkoch MSP bojujú s nedostatkom kapitálu potrebným na rozbehnutie ich podnikania. Vyriešenie otázky financovania je nevyhnutnou podmienkou pre ich prežitie a ďalší rozvoj.

Výsledky nášho skúmania potvrdili, že dominantným cudzím zdrojom financovania MSP je bankový úver, nasledujú kontokorentný úver a lízing. Bankové financovanie medzi MSP na Slovensku momentálne prevláda a zmeny sa neočakávajú ani v budúcnosti. Toto tvrdenie podporujú aj výsledky prieskumu Slovak Business Agency (2017), ktorý bol zameraný na využívanie externých foriem financovania MSP a realizovaný na vzorke 1000 podnikateľov. Z prieskumu vyplýva, že 21 % oslovených podnikateľov bude v najbližších dvoch rokoch určite potrebovať externý zdroj financovania. Takmer dve tretiny podnikateľov (64 %)

predpokladajú, že využijú najmä bankový úver. Ďalšia tretina podnikateľov (32 %) bude chcieť využiť lízing a štvrtina podnikateľov (24 %) verejnú podporu.

Záujem MSP o alternatívne formy financovania je podľa výsledkov nášho prieskumu výrazne nižší. Podniky v našom prieskume mali skúsenosť najmä s pomocou štátu a s finančnou pomocou z fondov EÚ. Pre menšie a začínajúce podniky sa môžu javiť ako perspektívnejšie nové alternatívne formy financovania, napríklad crowdfunding alebo peer-to-peer lending, ktoré sa čoraz viac dostávajú do povedomia slovenských podnikov. Spoločným znakom obidvoch foriem financovania je obchádzanie tradičného sprostredkovateľa a priame financovanie drobnými investormi cez online internetové platformy.

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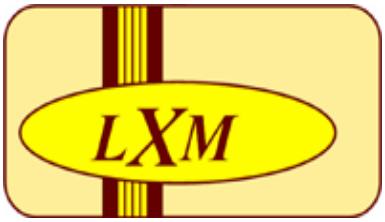
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